7. PROBABILITY

EXERCISE 7.1

1) State the sample space and $n(S)$ for the following random experiments.

(a) A coin is tossed twice. If a second throw results in a tail, a die is thrown.

Sol: A coin is tossed twice. If the second throw gives tail, a die is thrown.

∴ S = {HH, TH, HT1, HT2, HT3, HT4, HT5, HTG, TT1, TT2, TT3, TT4, TT5, TT6}

∴ n(S) = 14

(b) A coin is tossed twice. If a second throw results in head, a die thrown, otherwise a coin is tossed.

Sol: A coin is tossed twice. If a second throw gives head,

a die is thrown , otherwise a coin is tossed again.

∴ S = {HH1, HH2, HH3, HH4, HH5, HH6, TH1, TH2, TH3, TH4, THE, TH6, HTH, HTT, TTH, TTT}

∴ n(S) = 16

2) In a bag there are 3 balls; one black, one red and one green. Two balls are drawn one after another with replacement. State sample space and n(S)

Sol: To draw any two coloured balls out of given 3 balls.

(B, R, G) one after the other with replacement.

∴ S = {BB, BR, BG, RR, RB, RG, GG, GB, GR}

∴ n(S) = 9

3) A coin and a die are tossed. State sample space of following events.

(a) A: getting a head and an even number.

(b) B: getting a prime number.

(c) C: getting a tail and perfect square.

Sol: A coin and a die are tossed together.

∴ S = {H1, H2, H3, H4, H5, H6, T1, T2. T3, T4; T5, T6}
n(S) = 12

(a) Let A: Coin gives a head and die shows an even number.

A = {H2, H4, H6}

∴ n(A) = 3

(b) Let B: The die gives a prime number.

B = {H2, H3, H5, T2, T3, T5}

∴ n(B) = 6

(c) Let C: Coin gives a tail and die shows a perfect square.

C = {T1, T4}

∴ n(C) = 2

4) Find total number of distinct possible outcomes n(S) for each of the following random experiments.

(a) From a box containing 25 lottery tickets any 3 tickets are drawn at random.

Sol: 3 tickets are selected from a box containing 25 tickets at random.
3 tickets can be drawn from 25 tickets in 25C3 ways.
∴ $n(S) = 25C3 = \frac{25 \cdot 24 \cdot 23}{3 \cdot 2 \cdot 1} = 2300$

(b) From a group of 4 boys and 3 girls, any two students are selected at random.

Sol: Two students are selected at random from a group of 4 boys and 3 girls (total 7)
2 students can be selected from a group of 7 students in 7C2 ways.

∴ $n(S) = 7C2 = \frac{7 \cdot 6}{2 \cdot 1} = 21$

(c) 5 balls are randomly placed into 5 cells, such that each cell will be occupied.

Sol: 5 balls are arranged in 5 linear cells one in each.
Since the objects are linearly arranged in numbered places, each outcome is a permutation.

$\therefore n(S) = 5P5 = 5! = 120$

(d) 6 students are arranged in a row for a photograph.

Sol: 5 students are arranged in 6 linear places for a photograph.
Since a photograph is a linear arrangement each outcome is a permutation.

$\therefore n(S) = 6P6 = 6! = 720$

5) Two dice are thrown. Write favourable Outcomes for the following events.

(a) P: Sum of the numbers on two dice is divisible by 3 or 4.

(b) Q: Sum of the numbers on two dice is 7.

(c) R: Sum of the numbers on two dice is a prime number.
Also check whether
(I) Events P and Q are mutually exclusive and exhaustive.

(II) Events Q and R are mutually exclusive and exhaustive.

Sol: Let S: Two dice are rolled

∴ Sample space is,

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1),

(5, 2), (5, 3), (5, 4). (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore n(S) = 36$

(a) Let P: Sum of the numbers on two dice is divisible by 3 or 4

∴ P = {(1, 2), (1, 3), (1, 5), (2, 1), (2, 2), (2,4), (2, 6), (3, 1), (3, 3), (3, 5), (3, 6), (4, 2)
(4, 4), (4, 5), (5, 1), (5, 3), (5,4), (6, 2), (6, 3), (6, 6)}

∴ n(P) = 20

(b) Let Q: Sum of the numbers on two dice is 7.

∴ Q = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6,1)}

∴ n(Q) = 6

(c) Let R: Sum of the numbers on two is a prime number.

∴ R = {(1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4) (4, 3), (5, 2), (6, 1), (5, 6), (6, 5)}
∴ n(R) = 15

(I) $P \cup Q \neq S$ but $P \cap Q \neq \emptyset$

∴ P and Q are not exhaustive but they are mutually exclusive.

(II) $Q \cup R \neq S$ and $Q \cap R \neq \emptyset$

∴ Q and R are neither exhaustive nor mutually exclusive.

6) A card is drawn at random ' from an ordinary pack of 52 playing cards. State the number of elements in the sample space if consideration of suits

(a) Is not taken into account.

(b) Is taken into account.

Sol:

(a) If consideration of suits is not taken into consideration, then a card is selected at random from 52 cards.
i.e. To select one card out of 52.

∴ n(S) = 52C1 = 52

(b) If consideration of suits is taken into account then the desired suit is separated and a card is selected from that suit.
i.e. A card is selected at random from the desired suit containing 13 cards. (Either spades or clubs or hearts or diamonds).
∴ n(S) = 13C1 = 13

7) Box I contains 8 red (R111, R12, R13) and 2 blue (B11, B12) marbles while

Box II contains 2 red(R21, R22) and 4 blue (B21, B22, B23, B24) marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box I; if it turns up tails, a marble is chosen from Box II. Describe the sample space.

Sol: A coin is tossed, if the result is head, a marble is chosen from Box I, for tail, a marble is chosen from Box II.
S = {(H, R11), (H, R12), (H, R13),
(H, B11), (H, B12), (T, R21),
(T, R22), (T, B21), (T, B222),
(T, B23), (T, B24)}.

$\therefore n$(S) = 11

8) Consider an experiment of drawing two cards at random from a bag containing 4 cards marked 5, 6, 7 and 8. Find the sample Space if cards are drawn

(a) with replacement.

(b) without replacement.

Sol: Two cards are selected at random one after the other from a bag containing 4 cards (5, 6, 7, 8) with replacement. (i.e. repetition is allowed).
S = {(5,5),(5,6),(5,7),(5,8),(6,5), (6,6), (6, 7), (6, 8), (7, 5), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 7), (8, 8)}

$\therefore n$(S) = 16

(b) Two cards are selected at random one after the other from bag without replacement.
(i.e. repetition is not allowed).
S = {(5, 6), (5, 7), (5, 8), (6, 5), (6, 1), (6, 8), (7, 5), (7, 6), (7, 8), (8, 5), (8, 6), (8, 7)}
$\therefore n$(S) = 12

<u>EXERCISE 7.2</u>

1) A fair die 18 thrown two times. Find the chance that

(a) product of the numbers on the upper face is 12.

(b) sum of the numbers on the upper face is 10.

(c) Sum of the numbers on the upper face is at least 10.

(d) sum of the numbers on the upper face is at least 4.

(e) the first throw gives an odd number and second throw gives multiple of 3.

(f) both the times die shows same number (doublet).

Sol: A fair die 1s thrown two times.

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4. 3), (4, 4), (4, 5), (4, 6), (5, 1),

(5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore n(S) = 36$.

(a) Let event A: Product of the numbers on the uppermost face is 12.

A = {(2, 6), (3, 4), (4, 3), (6, 2)}

$\therefore n(A) = 4$.
$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{6}$

(b) Let event B: Sum of the numbers on the uppermost face is 10

B = {(4, 6), (5, 5), (6, 4)}

$\therefore n(B) = 3$.

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

(c) Let event C: Sum of the numbers on the upper most face is at least 10 (i.e. 10, 11
or 12)

C = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

$\therefore n(C) = 6$.

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(d) Let event D: Sum of the numbers on the uppermost face is at least 4.

∴ D': Sum of the numbers on the uppermost face is < 4 (i.e. 2 or 3).

D' = {(1, 1), (1, 2), (2, 1)}

∴ n(D') = 3

∴ $P(D') = \frac{n(D')}{n(S)} = \frac{3}{36} = \frac{1}{12}$

Required Probability,

∴ $P(D) = 1 - P(D') = 1 - \frac{1}{12} = \frac{11}{12}$

(e) Let event E: First throw gives an odd number (1, 3 or 5) and second throw gives multiple of 3 (3 or 6).

E = {(1, 3), (1, 6), (3, 3), (37 6), (5, 3), (5, 6)}

∴ n(E) = 6

∴ $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(f) Let event F: Both times die shows the same number (doublet).

F = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)} n(F) = 6

∴ n(F) = 6

∴ $P(F) = \frac{n(F)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

2) Two cards are drawn from a pack of 52 cards. Find the probability that

(a) Both are black.

(b) Both are diamonds.

(c) Both are ace cards.

(d) Both are face cards.

(e) One is spade and other is non-spade.

(f) Both are from same suit.

(g) Both are from same denomination.

Sol: Two cards are drawn at random from a pack of 52 cards.

$\therefore n(S) = 52C2$

(a) Let event A: Both are black. There are 26 black cards

$\therefore n(A) = 26C2$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{^{26}C_2}{^{52}C_2} = \frac{26.25}{52.51} = \frac{25}{102}$

(b) Let event B: Both are diamonds.
There are 13 diamond cards.

$\therefore n(B) = 13C2$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{^{13}C_2}{^{52}C_2} = \frac{13.12}{52.51} = \frac{1}{17}$

(c) Let event C: Both are ace cards.

There are 4 ace cards.

$\therefore n(C) = 4C2$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{^{4}C_2}{^{52}C_2} = \frac{4.2}{52.51} = \frac{1}{221}$

(d) Let event D: Both are face cards.
There are 12 face cards.

$\therefore n(D) = 12C2$

$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{^{12}C_2}{^{52}C_2} = \frac{12.11}{52.51} = \frac{11}{221}$

(e) Let event E: One is spade and other is non-spade.
There are 13 spades and 39 non-spades.

$\therefore n(E) = 13C1 \times 39C1$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{^{13}C_1 \times \ ^{39}C_1}{^{52}C_2} = \frac{13 \times 39}{\frac{52 \times 51}{1 \times 2}} = \frac{13}{34}$

(f) Let event F: Both are from the same suit.
A suit (out of 4) is selected then two cards are selected from that suit (13 cards).

$\therefore n(F) = 4C1 \times 13C1$

$\therefore P(F) = \frac{n(F)}{n(S)} = \frac{^{4}C_1 \times \, ^{13}C_1}{^{52}C_2} = \frac{4 \times 13 \times 12}{52 \times 51} = \frac{4}{17}$

(g) Let event G: Both are from same denomination.

A denomination is selected (from 13) and two cards are selected from that-denomination.

$\therefore n(G) = 13C1 \times 14C1$

$\therefore P(G) = \frac{n(G)}{n(S)} = \frac{^{13}C_1 \times \, ^{14}C_1}{^{52}C_2} = \frac{13 \times 4 \times 3}{52 \times 51} = \frac{1}{17}$

3) Four cards are drawn from a pack of 52 cards. What is the probability that

(a) 3 are kings and 1 is jack.

(b) All the cards are from different suit.

(c) At least one heart.

(d) All cards are club and one of them is jack.

Sol: 4 cards are drawn at random from a pack of 52 cards.

$\therefore n(S) = 52C4$

(a) Let event A: 3 cards are kings and 1 is jack. There are 4 kings and 4 jacks.
$\therefore n(A) = 4C3 \times 4C1$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{^{4}C_3 \times \, ^{4}C_1}{^{52}C_4}$

(b) Let event B: A11 4 cards are from different suits.
To draw 1 card each from every suit of 13 cards

$\therefore n(B) = 13C1 \times 13C1 \times 13C1 \times 13C1$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{^{13}C_1 \times \; ^{13}C_1 \times \; ^{13}C_1 \times \; ^{13}C_1}{^{52}C_4}$

(c) Let event C: At least, one heart.

$\therefore n(C')$: All 4 cards are non-hearts
There are 39 non heart cards.

$\therefore n(C') = 39C4$

$\therefore P(C) = \frac{n(C')}{n(S)} = \frac{^{39}C_4}{^{52}C_4}$
Required Probability,

$\therefore P(C) = 1 - P(C') = 1 - \frac{^{39}C_4}{^{52}C_4}$

(d) Let event D: All 4 cards are clubs and one of them is a jack.
There is one jack of club, out of total 13 club cards.

$\therefore n(D) = 1 \times 12C3$
$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{^{12}C_3}{^{52}C_4}$

4) A bag contains 15 balls of three different colours, Green, Black and Yellow. A ball is drawn at random from the bag. The probability of green ball is 1/3. The probability of yellow ball is 1/5.

(a) What is the probability of black ball?

(b) How many balls are green, black, and yellow?

Sol: Let event G = A Green ball is sleeted from the bag.
Given: $P(G) = \frac{1}{3}$
Event B = A Black ball is selected from the bag.
To find: P(B).

Event Y = A Yellow ball is selected from the bag.
Given: $P(Y) = \frac{1}{5}$

(a) Since the events are mutually exclusive and, exhaustive,
$P(G) + P(B) + P(Y) = 1$

$\frac{1}{3} + P(B) + \frac{1}{5} = 1$

$\therefore$ P(B) = 1 $-\frac{1}{3}-\frac{1}{5}$

$\therefore$ Probability of a black ball selected is $\frac{7}{15}$

(b) Let S: To draw 1 ball from the bag containing 15 balls.

$\therefore n(S)$ = 15C1 = 15

Let the number of. Green, Black and Yellow balls be x, y, z respectively.

$\therefore n(G) = x, \quad n(B) = y, \quad n(Y) = z$

Since,

$P(G) = \frac{1}{3} = \frac{n(G)}{n(S)} = \frac{x}{15} \Rightarrow x = 5$

$P(B) = \frac{1}{3} = \frac{n(B)}{n(S)} = \frac{y}{15} \Rightarrow y = 7$

$P(Y) = \frac{1}{3} = \frac{n(Y)}{n(S)} = \frac{z}{15} \Rightarrow z = 3$

i.e. there are 5 Green, 7 Black and 3 Yellow balls in the bag.

5) A box contains 75 tickets numbered 1 to 75. A ticket is drawn at random from the box. What is the probability that

(a) Number on ticket is divisible by 6.

(b) Number on ticket is a perfect square.

(c) Number on ticket is prime.

(d) Number on ticket is divisible by 3 and 5.

Sol: To select one ticket at random out of 75 tickets numbered from 1 to 75.

$\therefore n(S)$ = 75C1 = 75.

(a) Let event A: Number on the ticket is divisible by 6. A = {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72}

$\therefore n(A)$ = 12C1 = 12.

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{12}{75} = \frac{4}{25}$

(b) Let event B: Number on the ticket is a perfect square.

B = {1, 4, 9, 16, 25, 36, 49, 64}

$\therefore n(B) = 8C1 = 8.$

$\therefore P(B) = \frac{n(A)}{n(S)} = \frac{8}{75}$

(c) Let event C: Number on the ticket is prime.

C = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 51, 57, 71, 73}

$\therefore n(C) = 21C1 = 21.$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{21}{75} = \frac{7}{25}$

(d) Let event D: Number on the ticket is divisible by 3 and 5 i.e. by 15
D = {15, 30, 45, 60, 75}

$\therefore n(D) = 5C1 = 5.$

$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{5}{75} = \frac{1}{15}$

6) From a group of 8 boys and 5 girls, 8 committee of 5 is to be formed. Find the Probability that the committee contains

(a) 3 boys and 2 girls.

(b) at least 8 boys.

Sol: Douro select 5 from a group of 8 boys, 5 girls (total 13).

$\therefore n(S) = 13C5$

(a) Let event A: To select 3 boys from 8 and 2 girls from 5.

$\therefore n(A) = 8C3 \times 5C2$
$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{{}^{8}C_{3} \times {}^{5}C_{2}}{{}^{13}C_{5}}$

(b) Let event B: To select 5 consisting of at least 3 boys.
Committee can have 3 boys and 2 girls or 4 boys and 1 girl or all 5 boys.

$\therefore n(B) = 8C3 \times 5C2 + 8C4 \times 5C1 + 8C5$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{{}^8C_1\times\ {}^5C_2+\ {}^8C_4\times\ {}^5C_1+\ {}^8C_5}{{}^{13}C_5}$

7) A room has 3 sockets for lamps. From a collection of 10 light bulbs of which 6 are defective, a person selects 3 bulbs at random and puts them in socket. What is the probability that the room is lit?
Sol: To select 3 bulbs at random from the collection of 10 bulbs.

$\therefore n(S) = 10C3 = \frac{10\cdot5\cdot4}{3\cdot2\cdot1} = 120$
Let event A: Selected 3 bulbs are fixed in the sockets so that the room gets lit.

∴ A': The room is not lit.

(i.e.) A11 3 bulbs selected are defective (from 6 defective bulbs)

$\therefore n(A') = 6C3 = \frac{6\cdot5\cdot4}{3\cdot2\cdot1} = 20$

$\therefore n(A') = \frac{n(A')}{n(S')} = \frac{{}^6C_3}{{}^{10}C_3} = \frac{20}{120} = \frac{1}{6}$
Required Probability that the room is lit is

$\therefore P(A) = 1 - P(A') = 1 - \frac{1}{6} = \frac{5}{6}$
8) The letters of the word LOGARITHM are arranged at random. Find the probability that

(a) Vowels are always together.

(b) Vowels are never together.

(c) Exactly 4 letters between G and H.

(d) Begin with O and end with T.

(e) Start with vowel and end with consonant.

Sol: The word LOGARITHM contains 9 distinct letters consisting of 3 vowels and 6 consonants.
Since the letters are to be rearranged in linear places, each arrangement is a permutation.

To arrange all 9 letters in 9 linear places.

$\therefore n(S) = {}^{9}P_{9} = 9!$

(a) Let event A: In the arrangement, all 3 vowels are together.
To arrange 1 block of vowels + 6 consonants in 7 places.
Also, 3 vowels are rearranged amongst themselves.

$\therefore n(A) = 7! \times 3!$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{7!3!}{9!}$

$= \frac{7! \times 3 \times 2 \times 1}{9 \times 8 \times 7!} = \frac{1}{12}$

(b) Let event B: In the arrangement, no two vowels are together.
6 consonants to be arranged in 6 linear places and
3 consonants are inserted in any 3 of 7 alternate place as shown:

× [C] × [C] × [C] × [C] × [C] × [C] ×

$\therefore n(B) = 6! \times {}^{7}P_{3}$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6! \times {}^{7}P_{3}}{9!}$

$= \frac{6! \times 7 \times 6 \times 5}{9 \times 8 \times 7 \times 6!} = \frac{5}{12}$

(c) Let event C: In the arrangement, there are exactly 4 letters between G and H.
Such an arrangement is possible only if G and H occupy
1st and 6th
OR 2nd and 7th
OR 3rd and 8th
OR 4th and 9th places and remaining,
7 letters can be arranged 1n remaining 7 places.

$\therefore n(C) = 4 \times 7!$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{4 \times 7!}{9!}$

$= \frac{4 \times 7!}{9 \times 8 \times 7!} = \frac{1}{18}$

(d) Let event D: The arrangement begins with O and ends with T.
1st place is occupied by O and 9th place is occupied by T, remaining 7 letters are arranged in 7 remaining places.

$\therefore n(D) = 1 \times 7! \times 1 = 7!$

$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{7!}{9!}$

$= \frac{7!}{9 \times 8 \times 7!} = \frac{1}{72}$

(e) Let event E: The arrangement starts with a vowel and ends with a consonant.
1st place is occupied by any one of 3 vowels and 9th place is occupied by any one of 6 consonants, remaining 7 places are occupied by remaining 7 letters.

$\therefore n(E) = 3 \times 7! \times 6$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3 \times 7! \times 6}{9!}$

$= \frac{3 \times 7! \times 6}{9 \times 8 \times 7!} = \frac{1}{4}$

9) The letters of the word SAVITA are arranged at random. Find the probability that vowels are always together.

Sol: The word SAVITA contains 6 letters

∴ Let S: b All 6 letters of the word are rearranged in 6 linear places,
Since the letter 'A' is repeated twice.

$\therefore n(S) = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$

Let event A: In the arrangement, the vowels are together
To arrange 1 block of 3 vowels (2 identical) + 3 consonants in 4 linear places.
Also, the 3 Vowels can be arranged among themselves.
Considering the repetition of letter 'A',

$\therefore n(A) = 4! \times \frac{3!}{2!} = 4 \times 3 \times 2 \times 1 \times 3 \times \frac{2!}{2!}$

$= 72$

$\therefore n(A) = \frac{n(A)}{n(S)} = \frac{72}{360} = \frac{1}{5}$

EXERCISE 7.3

(1) Two dice are thrown together. What is the probability that the sum of the numbers on two dice is 5 or number on the second die is greater than or equal to the number-on the first die?

Sol: Let S: Two dice are thrown together.

$\therefore S = \{(1, 1), (1. 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), .(6, 5), (6, 6)\}$

$\therefore n(S) = 36.$

Let event A: Sum of numbers on two dice is 5.

$\therefore A = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

$\therefore n(A) = 4$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{36}$

Let event B: Number on the second die ≥ number on the first die.

$B = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 5), (5, 6), (6, 6)\}$

$\therefore n(B) = 21$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{21}{36}$

Now, $(A \cap B) = \{(1, 4), (2, 3)\}$

$\therefore n(A \cap B) = 2$

$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$

Required Probability is $P(A \cup B)$

By Addition Theorem:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{36} + \frac{21}{36} - \frac{2}{36}$

$P(A \cup B) = \frac{23}{36}$

2) A card is drawn from a pack of 52 cards. What is the probability that

(a) card is either red or black ?

(b) card is either red or face card?

Sol: Let S: A card is selected at random from a pack of 52 cards.

$\therefore n(S) = {}^{52}C_1 = 52.$

Let event A: A red card is selected.
There are 26 face cards.

$\therefore n(A) = {}^{26}C_1 = 26.$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

Let event B: A black card is selected.
There are 26 black Cards.

$\therefore n(B) = {}^{26}C_1 = 26.$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

Let event C: A face card is selected.
There are 12 face cards.

$\therefore n(C) = {}^{12}C_1 = 12.$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{12}{52} = \frac{3}{13}$

(a) A and B are mutually exclusive.

$\therefore$ Probability that the card is either red or black is
$P(A \cup B) = P(A) + P(B)$

$\therefore P(A \cup B) = \frac{1}{2} + \frac{1}{2} = 1$

(b) $A \cap C$: A red face card is selected.
There are 6 red face cards

$\therefore n(A \cap C) = {}^{6}C_1 = 6$

$\therefore P(A \cap C) = \frac{n(A \cap C)}{n(S)}$

$= \frac{6}{52} = \frac{3}{26}$

Probability that the card is a red card or face card is,

$P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$= \frac{1}{2} + \frac{3}{13} - \frac{3}{26}$

$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$

$= \frac{32}{52}$

$\therefore P(A \cup C) = \frac{8}{13}$

3) Two cards are drawn from a pack of 52 cards. What is the probability that

(a) both the cards are of the same colour?

(b) both the cards are either black or queens?

Sol: Let S: Two cards are selected at random from the pack of 52 cards.

$\therefore n(S) = {}^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 1326$

Let event A: Both cards are red. There are 26 red cards.

$\therefore n(A) = {}^{26}C_2 = \frac{26 \times 25}{2 \times 1} = 325$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{325}{1326} = \frac{25}{102}$

Let event B: Both cards are black. There are 26 black cards.

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{25}{102}$

Since A and B are mutually exclusive,

probability that both cards are of the Same colour is

$P(A \cup B) = P(A) + P(C)$

$= \frac{25}{102} + \frac{25}{102} = \frac{50}{102}$

$P(A \cup B) = \frac{25}{51}$

Let event C: Both cumin are queens there are 4 queens.

$\therefore n(B) = {}^{4}C_2 = \frac{4\times3}{2\times1} = 6$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{1326} = \frac{1}{221}$

Event $B \cap C$: Both cards are black queens.
There are 2 black queens.

$\therefore n(B \cap C) = {}^{2}C_2 = 1$

$\therefore P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{1}{1326}$

$\therefore$ Probability that both cards are bla6k or queens
$P(B \cup C) = P(A) + P(C) - P(A \cap C)$

$= \frac{25}{102} + \frac{1}{221} - \frac{1}{1326}$

$= \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}$

$= \frac{330}{1326}$

$\therefore P(B \cup C) = \frac{55}{221}$

4) A bag contains 50 tickets, numbered from 1 to 50. One ticket is drawn at random. What is the probability that

(a) number on the ticket is a perfect square or divisible by 4?

(b) number on the ticket is prime number or greater than 30?
Sol: Let S: One ticket is selected at random from a bag containing 50 tickets numbered from 1 to 50.

$\therefore n(S) = {}^{50}C_1 = 50$

(a) Let event A: Number on the selected ticket is a perfect square
$A = \{1, 4, 9, 16, 25, 36, 49\}$

$\therefore n(A) = {}^{7}C_1 = 7$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{7}{50}$

Let event B: Number on the selected ticket is divisible by 4

B $= \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$

$\therefore n(\mathrm{B}) = {}^{12}C_1 = 12$

$\therefore \mathrm{P(B)} = \frac{n(\mathrm{B})}{n(\mathrm{S})} = \frac{12}{50} = \frac{6}{25}$

$\mathrm{A} \cap \mathrm{B}$: Number on the ticket is a perfect square divisible by 4

$\mathrm{A} \cap \mathrm{B} = \{4, 16, 36\}$

$\therefore n(\mathrm{A} \cap \mathrm{B}) = {}^{3}C_1 = 3$

$\therefore \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{n(\mathrm{A} \cap \mathrm{B})}{n(\mathrm{S})} = \frac{3}{50}$

Probability that the number on the selected ticket is either a perfect square or divisible by 4 is

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P(A)} + \mathrm{P(C)} - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$

$= \frac{7}{50} + \frac{12}{50} - \frac{3}{50}$

$= \frac{16}{50}$

$\therefore \mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \frac{8}{25}$

(b) Let event C: Number on the selected ticket is a prime number

C $= \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$

$\therefore n(\mathrm{C}) = {}^{15}C_1 = 15$

$\therefore \mathrm{P(C)} = \frac{n(\mathrm{C})}{n(\mathrm{S})} = \frac{15}{50} = \frac{3}{10}$

Let event D: Number on the ticket is greater than 30

D $= \{31, 32, \ldots., 50\}$

$\therefore n(\mathrm{D}) = {}^{20}C_1 = 20$

$\therefore \mathrm{P(D)} = \frac{n(\mathrm{D})}{n(\mathrm{S})} = \frac{20}{50} = \frac{2}{5}$

$C \cap D$: Number on the selected ticket is a prime number > 30.

$C \cap D = \{31, 37, 41, 43, 47\}$

$\therefore n(C \cap D) = {}^{5}C_1 = 5$

$\therefore P(C \cap D) = \frac{n(C \cap D)}{n(S)} = \frac{5}{50} = \frac{1}{10}$

∴ Probability that the number on the selected ticket is either a prime number or greater than 30 is
$P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$= \frac{3}{10} + \frac{2}{5} - \frac{1}{10}$

$= \frac{15}{50} + \frac{20}{50} - \frac{5}{10}$

$= \frac{30}{50}$
$\therefore$ **$P(C \cup D) = \frac{3}{5}$**

5) Hundred students appeared for two examinations.
60 passed the first, 50 passed the second and 30 passed in both.
Find the probability that student selected at random has

(a) passed in at least one examination.

(b) passed in exactly one examination.

(c) failed in both the examinations.

Sol: Let S: A student is selected at random from 100 students appeared in two examinations.

$\therefore n(S) = {}^{100}C_1 = 100$
Let event A: The student selected has passed in first examination.
60 students passed in first examination.

$\therefore n(A) = {}^{60}C_1 = 60$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{60}{100} = \frac{6}{10}$
Let event B ': The student selected has passed in second examination.
50 students passed in second examination.

$\therefore\ n(\mathrm{B}) = {}^{50}C_1 = 50$

$\therefore\ \mathrm{P(B)} = \frac{n(\mathrm{B})}{n(\mathrm{S})} = \frac{50}{100} = \frac{5}{10}$

Event A ∩ B: The student has passed in both examinations
30 students passed 'both examinations.
$n(\mathrm{A} \cap \mathrm{B}) = {}^{30}C_1 = 30$

$\therefore\ \mathrm{P(A \cap B)} = \frac{n(\mathrm{A \cap B})}{n(\mathrm{S})} = \frac{30}{100} = \frac{3}{10}$

(a) Probability that a student has passed in at least one of the examinations is
$\mathrm{P(A \cup B) = P(C) + P(D) - P(A \cap B)}$

$= \frac{6}{10} + \frac{5}{10} - \frac{3}{10}$

$= \frac{8}{10}$

$\therefore\ \mathbf{P(A \cup B)} = \frac{4}{5}$

(b) Probability that a student has passed in exactly one of the examination is
P (A but not B or B but not A)

$= \mathrm{P(A \cap B') + P(A' \cap B)}$

$= \mathrm{P(A) - P(A \cap B) + P(B) - P(A \cap B)}$

$= \frac{6}{10} - \frac{3}{10} + \frac{5}{10} - \frac{3}{10}$

$= \frac{5}{10}$

$= \frac{1}{2}$

(c) Probability that the student has failed in bath the examinations.

i. e. P (Neither A nor B)

$= \mathrm{P(A' \cap B')}$

$= \mathrm{P(A \cup B)'}$ ... D' Morgan's Rule

$= 1 - \mathrm{P(A \cup B)}$ ... $\mathrm{P(A) = 1 - P(A')}$

$= 1 - \frac{4}{5}$

$= \frac{1}{5}$

6) If $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$

Find the values of the following:

(a) $P(A \cap B)$ (b) $P(A \cap B')$

(c) $P(A' \cap B)$ (d) $P(A' \cup B')$

(e) $P(A' \cap B')$

Sol: Given probabilities are:

$P(A) = \frac{1}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$

(a) By Addition Theorem

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{1}{4} + \frac{2}{5} - \frac{1}{2}$

$= \frac{5+8-10}{20}$

$\therefore P(A \cap B) = \frac{3}{20}$

(b) $P(A \cap B') = P(A) - P(A \cap B)$

$= \frac{1}{4} - \frac{3}{20}$

$= \frac{5-3}{20}$

$= \frac{2}{20}$

$\therefore P(A \cap B') = \frac{1}{10}$

(c) $P(A' \cap B) = P(B) - P(A \cap B)$

$= \frac{2}{5} - \frac{3}{20}$

$= \frac{8-3}{20}$

$= \frac{5}{20}$

$\therefore P(A' \cap B) = \frac{1}{4}$

(d) $P(A' \cup B') = P(A \cap B)'$

$\ldots D'Morgan's\ Rule$

$= 1 - P(A \cap B)$

$\ldots P(A) = 1 - P(A')$

$= 1 - \frac{3}{20}$

$\therefore P(A' \cup B') = \frac{17}{20}$

(e) $P(A' \cap B') = P(A \cup B)'$

$\ldots D'Morgan's\ Rule$

$= 1 - P(A \cup B)$

$\ldots P(A) = 1 - P(A')$

$= 1 - \frac{1}{2}$

$\mathbf{P(A' \cap B') = \frac{1}{2}}$

7) A computer software company is bidding for computer programs A and B. The probability that. the company will get software A is 3/5, the probability that the company will get software B is 1/3 and the company will get both A and B is 1/8. What is the probability that the company will get at least one software?

Sol: Let event

A: The company gets software A

B: The company gets software B

Then $A \cap B$: The company gets both software.

Now, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{8}$

Probability that the company gets at least one software is

$P(A \cup B) = P(A) + P(\ \ ) - P(A \cap B)$

$= \frac{3}{5} + \frac{1}{3} - \frac{1}{8}$

$= \frac{72+40-15}{120}$

$\therefore$ $\mathbf{P(A \cup B) = \frac{97}{120}}$

8) A card is drawn from a well shuffled pack of 52 cards. Find the probability of it being a heart or a queen.

Sol: Let S: A card is selected at random from a well shuffled pack of 52 cards. .

$\therefore n(S) = \ {}^{52}C_1 = 52$

Let event A: The card drawn is a heart.

There are 13 hearts.

$\therefore n(A) = \ {}^{13}C_1 = 13$

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

Event B: The card drawn is a queen.

There are 4 queens.

$\therefore n(B) = \ {}^{4}C_1 = 4$

$\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

Event $A \cap B$: The card drawn is a queen of hearts

$\therefore n(A \cap B) = 1$

$\therefore$ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52}$

Probability that the card selected is either heart or a queen is given as,

$P(A \cup B) = P(A) + P(B) - P(A \cap B$

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52}$

$= \frac{13+4-1}{52}$

$= \frac{16}{52}$

$\therefore \mathbf{P(A \cup B)} = \frac{4}{13}$

9) In a group of students, there are 3 boys and 4 girls. Four students are to be selected at random from the group. Find the probability that either 3 boys and 1 girl or 8 girls and 1 boy are selected.

Sol: Let S: 4 students are selected at random from a group of 7 students (3 boys, 4 girls)

$\therefore n(S) = {}^7C_4 = \frac{7\times6\times5\times4}{4\times3\times2\times1} = 35$

Let A: Event that the selected group has boys and 1 girl.

3 boys can be selected out of 3 in ${}^3C_3$ ways and 1 girl can be selected out of 4 in ${}^4C_1$ways.

$\therefore n(A) = {}^3C_3 \times {}^4C_1 = 1 \times 4 = 4$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{4}{35}$

Event B: The selected group has 1 boy and 3 girls.

1 boy can be selected out of 3 in ${}^3C_1$ ways and 3 girls can be selected out of 4 in ${}^4C_3$ ways.

$\therefore n(B) = {}^3C_1 \times {}^4C_3 = {}^3C_1 \times {}^4C_1$

$= 3 \times 4 = 12$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{12}{35}$

Since events A and B are mutually exclusive

Required probability,

$P(A \cup B) = P(A) + P(B)$

$= \frac{4}{35} + \frac{12}{35}$

$\therefore \mathbf{P(A \cup B)} = \frac{16}{35}$

EXERCISE 7.4

1) Two dice are thrown simultaneously. It at least one of the dice shows a number 5, what is the probability that, sum of the numbers on two dice is 9?

Sol: Two dice are thrown together.

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 3), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1),

(5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore n(S) = 36$

Let A: Event that one of the dice shows the number 5

$\therefore$ A = {(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 5)}

$\therefore n(A) = 11$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$

Let B: Event that the sum of the numbers of the dice is 9

B = {(3, 6), (4, 5), (5, 4), (6, 3)}

$\therefore n(B) = 4$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{4}{36}$

$\therefore A \cap B$ is the event that one dice shows a 5 and the sum of the numbers is 9.

$\therefore A \cap B$ = {(4, 5), (5, 4)}

$\therefore n(A \cap B) = 2$

$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$

Now, probability that sum of the numbers on the dice is
9 given that one dice shows 5 is given by,

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{2}{36}}{\frac{11}{36}}$

$P(B/A) = \frac{2}{11}$

2) A pair of dice is thrown. If sum of the numbers is an even number, what is the probability that it is a perfect square?

Sol: A pair of dice is thrown,

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 3), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1),

(5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore n(S) = 36$

Let A: Event that sum of the numbers is even

$\therefore A$ = {(1, 1), (1, 3), (1, 5) (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6),

(5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)}

$\therefore n(A) = 18$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

Let B: Event that sum of the numbers is a perfect square

B = {(1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (5, 4), (6, 3)}

$\therefore n(B) = 7$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{7}{36}$

$\therefore A \cap B$ is the event that the sum of the numbers is an even perfect square.

$\therefore A \cap B$ = {(1, 3), (2, 2), (3, 1)}

$\therefore n(A \cap B) = 3$

$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36} = \frac{1}{2}$

Now, Probability that the sum of the numbers is a Perfect square given that it is even is given by,

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{1}{12}}{\frac{1}{2}}$

$\mathbf{P(B/A) = \frac{1}{6}}$

(3) A box contains 11 tickets numbered from 1 to 11. Two tickets are drawn at random with replacement. If the sum is even, find the probability that both the numbers are odd.

Sol: Two tickets are drawn from 11 tickets numbered 1 to 11 with replacement

$\therefore$ $n(S)$ = 11C1 x 11C1 = 121.
Let A: Event that sum of the numbers on the tickets is even.
For this both tickets need to be even numbered or odd numbered.

Out of 5 even numbered tickets, 2 can be drawn in $5 \cdot 5 = 25$ ways.
Similarly, out of 6 odd numbered tickets, 2 can be drawn in $6 \cdot 6$

$= 36$ ways. ... (I)

$\therefore$ $n(A) = 25 + 36 = 61$

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{61}{121}$

Let B: Event that both tickets drawn are odd numbered.

$\therefore$ $n(B) = 36$ ... (from (I))

$\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{36}{121}$

$\therefore A \cap B$ is the event that the tickets drawn are odd numbered and their sum is even. .

$\therefore n(A \cap B) = 36$

$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{36}{121}$

Now, probability that the tickets drawn are odd numbered given that their sum is even is,

$P(B/A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{36/121}{61/121}$

$\mathbf{P(B/A) = \frac{36}{61}}$

4) A card is drawn from a well shuffled pack of 52 cards. Consider two events A and B.

A: a club 6 card is drawn.

B: an ace card 18 drawn.

Determine whether the events A and B are independent or not.

Sol: Let S: A card is drawn at random from a deck of 52 cards.

$\therefore n(S) = 52$

Event A: A club card is drawn.

There are 13 club cards.

$\therefore n(A) = 13$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$

Event B: An ace card is drawn.

There are 4 aces.

$\therefore n(B) = 4$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{4}{121} = \frac{1}{13}$

Event $A \cap B$: An ace of clubs is drawn.

$\therefore n(A \cap B) = 1$

$\therefore n(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52}$

Now, $P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{13}$

$= \frac{1}{52}$

i.e. $P(A \cap B) = P(A) \cdot P(B)$

Hence, events A and B are independent.

5) A problem in statistics; is given to three students A, B and C. Their chances of solving the problem are 1/3, 1/4 and 1/5 respectively. If all of them try independently, what is the probability that

(a) Problem is not solved?

(b) Problem is solved?

(c) Exactly two students solve the problem?

Sol: The probabilities that the students A, B and C solve the problem are 1/3, 1/4 and 1/5 respectively.

∴ Let $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(C) = \frac{1}{5}$

Hence, the probabilities the it the Students do not solve .the problem can be given as

$P(A) = \frac{2}{3}$, $P(B') = \frac{3}{4}$, $P(C') = \frac{4}{5}$

Since all the three students independently, events A, B, C, A', B', C are mutually independent.

(a) Probability that the problem is solved

$= P$ (At least one of the three Students solve the problem)

$= 1 - P$ (None of the students solve)

$= 1 - P(A' \cap B' \cap C')$

$= 1 - P(A') \cdot P(B') \cdot P(C')$

$= 1 - \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

(b) Probability that the problem is not solved

$= P$ (None of A, B and C solve the problem)

$= P(A' \cap B' \cap C')$

$= P(A') \cdot P(B') \cdot P(C')$

$= \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5}$

$= \frac{2}{5}$

(c) Probability that exactly two of them solve the problem.

$= P$(Two students solve and third does not)

$= P$ (A, B but not C or A, C but not B or B, C but not A)
$= P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(C') + P(A) \cdot P(B') \cdot P(C') + P(A') \cdot P(B) \cdot P(C)$

$= \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{4}{5} + \frac{1}{3} \cdot \frac{3}{4} \cdot \frac{1}{5} + \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}$

$= \frac{4+3+2}{60} = \frac{9}{60}$

$= \frac{3}{20}$

6) The probability that a 50-year old man will be alive till age 60 is 0.83 and the probability that a 45-year old woman will be alive till age 55 is 0.97. What is the probability that a man whose age is 50 and his wife whose age is 45 will both be alive after 10 years.

Sol: Let Event A: A 50 year old man is alive at 60.
Event B: A 45 year old woman is alive at 55.
Given: $P(A) = 0.83$ and $P(B) = 0.97$
Required probability

$= P$ (a man at 50 and a woman at 45 are alive after 10 years)

$= P$ (A and B)

$= P(A \cap B)$

$= P(A) \cdot P(B)$

... (events A and B are independent)

$= 0.83 \times 0.97$

$=$ **0.8051**

7) In an examination 30% of students have failed in subject I, 20% of the students have failed in subject II and 10% have failed in both subject I and subject II. A student is selected at random, what is the probability that the' student

(a) has failed in subject I, if it is known that he has failed in subject II ?

(b) has failed in at least one subject?

(c) has failed in exactly one subject?

Sol: Let event

A: A student selected has failed in subject I.

B: A student selected has failed in subject II.

$\therefore$ $A \cap B$: A student selected has failed in both subjects I and II.

$P(A) = 30\% = \frac{30}{100} = \frac{3}{10}$

$P(B) = 20\% = \frac{20}{100} = \frac{2}{10}$

And $P(A \cap B) = 10\% = \frac{10}{100} = \frac{1}{10}$

(a) Probability that a selected student has failed in subject I, knowing that he has failed in subject II

P (event A given that B has occurred)

i.e. $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/10}{2/10} = \frac{1}{2}$

$\therefore P(A/B) = \frac{1}{2}$ or 0.5

(b) Probability that a selected student has failed in at least one subject

i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{10} + \frac{2}{10} - \frac{1}{2}$

$= \frac{4}{10} = \frac{2}{5}$ or 0.4

$\therefore P(A \cup B) = \frac{2}{5}$ or 0.4

(c) Probability that a selected student has failed in exactly one subject.

i.e. P(A but not B. or B but not A)

$= P(A \cap B') + P(A' \cap B)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= \frac{3}{10} - \frac{1}{10} + \frac{2}{10} - \frac{1}{10}$

$= \frac{3}{10}$ or 0.3

8) One shot is fired from each of the three guns. Let A, B and C denote the events that the target is hit by the first, second and third gun respectively. Assuming that A, B and C are independent events and that $P(A) = 0.5$, $P(B) = 0.6$ and $P(C) = 0.8$, then find the probability that at least one hit is registered.

Sol: Let event

A: The target is hit by first gun.

B: The target is hit by Second gun.

C: The target is hit by third gun.

$P(A) = 0.5$, $P(B) = 0.6$, $P(C) = 0.8$

Hence, probabilities that the target is not hit are given as

$P(A') = 0.5$, $P(B') = 0.4$, $P(C') = 0.2$

Since A, B, C are independent

∴ A', B', C' are also independent.

Required Probability that at least one hit is registered

$= P$(At least one gun hits the target)

$1 - P$(None of the guns hits the target)

$1 - P(A' \cap B' \cap C')$

$1 - P(A') \cdot P(B') \cdot P(C')$

$1 - 0.5 \times 0.4 \times 0.2$

$1 - 0.04$

$= 0.96$

9) A bag contains 10 white balls and {5 black balls. Two balls are drawn in succession without replacement. What is the probability that

(a) first is white and second is black.

(b) one is white and other is black.

Sol: The bag contains 25 balls. Since two balls are drawn in succession without replacement, for the conditional second event a ball IS reduced in the bag.
Let event A: A white ball is selected.

event B: A black ball is selected.

$P(A) = \frac{10}{25}; P(B) = \frac{15}{12}$

$\therefore A/B$: Second ball is black, first bell being white.

B/A: Second ball is white, first ball being black.

$P(B/A) = \frac{15}{24}; P(A/B) = \frac{10}{24}$

(a) Probability4 that first ball 18 white and second ' is black

$= P$(event A and event (B/A))

$= P(A) \cdot P(B/A)$

$= \frac{10}{25} \times \frac{15}{24}$

$= \frac{1}{4}$

(b) Probability that one ball is white and the other is black.

$= P$ (First ball is white, second is black or First ball is black, second is white)

$= P(A$ and B/A or B and A/B)

$= P(A) \cdot P(B/A) + P(B) \cdot P(A/B)$

$= \frac{10}{25} \times \frac{15}{24} + \frac{15}{25} \times \frac{10}{24}$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

10) An urn contains 4 black, 5 white and 6 red balls. Two balls are drawn one after the other without replacement. What is the probability that at least one ball is black?

Sol: To select two balls from the bag containing 15 balls.

$\therefore n(S) = {}^{15}C_2 = \frac{15\times4}{2\times1} = 105$

Let event A: Selecting at least one black ball.
Balls drawn can be both black or one black and other non-black.

$\therefore n(A) = {}^{4}C_1 \times {}^{11}C_1 + {}^{4}C_2 = 4 \times 11 + \frac{4\times3}{2\times1}$

$= 44 + 6 = 50$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{50}{105} = \frac{10}{21}$

11) Two balls are drawn from an urn containing 5 green, 3 blue, 7 yellow balls one by one without replacement. What is the probability that at least one ball is blue?

Sol: Let S be the sample space-for two balls drawn from an urn one by one without replacement
There are total 15 balls,

$\therefore n(S) = {}^{15}C_2$

Let A: Event that at least one ball is blue.

∴ A': Event that none of the balls is blue.
Out of 15 balls, 3 balls are blue

∴ 2 balls can be drawn from 12 non - blue ball in $^{12}C_2$ ways

∴ $n(A') = {}^{12}C_2$

∴ $P(A') = \frac{n(A)}{n(S)} = \frac{^{12}C_2}{^{15}C_2} = \frac{12\times11}{15\times14}$

$= \frac{22}{35}$

∴ $P(A) = 1 - P(A')$

$= 1 - \frac{22}{35}$

∴ $\mathbf{P(A) = \frac{13}{35}}$

12) A bag contains 4 blue and 5 green balls. Another bag contains 3 blue and 7 green balls. If one ball is drawn from each bag, what is the probability that two balls are of the same colour?

Sol: First bag contains 4 blue + 5 green = 9 balls.
Let event A: A blue ball is drawn from bag I.
event A': A green ball is drawn from bag I.

∴ $P(A) = \frac{4}{9}$ and $P(A') = \frac{5}{9}$
Second bag contains 3 blue + 7 green 10 balls.
Let event B: A blue ball is drawnfrom bag II.

event B': A green ball is drawn from bag II.

∴ $P(B) = \frac{3}{10}$ and $P(B') = \frac{7}{10}$
Required probability

= P(both balls are of same colour)

= P(both are blue or both are green)

= P(A and B or A and B')

$= P(A \cap B) +$ or $P(A' \cap B')$

Now, A and B are independent .

$\therefore A'$ and B' are independent.

$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{4}{9} \cdot \frac{3}{10} = \frac{12}{90}$

and $P(A' \cap B') = P(A') \cdot P(B') = \frac{5}{9} \cdot \frac{7}{10} = \frac{35}{90}$

Required probability

$= P(A \cap B) + P(A' \cap B')$

$= \frac{12}{90} + \frac{35}{90}$

$= \frac{47}{90}$

13) Two cards are drawn one after the other from a pack of 52 cards with replacement. What is the probability that both the cards drawn' ere face cards?

Sol: For 2 cards to be drawn from a pack of 52 cards one after the other with replacement,

$n(S) = {}^{52}C_1 \cdot {}^{52}C_1$

Let A: Event that both the draws result in a face card

There are 12 face Cards.

$\therefore n(A) = {}^{13}C_1 \cdot {}^{13}C_1$

$\therefore P(A) = \frac{n(A)}{n(S)}$

$= \frac{{}^{13}C_1 \cdot {}^{13}C_1}{{}^{15}C_1 \cdot {}^{52}C_1}$

$= \frac{13}{52} \cdot \frac{13}{52}$

$= \frac{1}{4} \cdot \frac{1}{4}$

$\therefore P(A) = \frac{1}{16}$