Solved Paper 2008

JCECE
Engineering Entrance Exam

Physics

1. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(a) x^2 (b) e^x
(c) x (d) $\log_e x$

2. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection ?

(a) Yes, 60° (b) Yes, 30°
(c) No (d) Yes, 45°

3. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is

(a) directly proportional to R but inversely proportional to v
(b) directly proportional to both radius R and velocity v
(c) inversely proportional to both radius R and velocity v
(d) inversely proportional to R but directly proportional to velocity v

4. Nickel shows ferromagnetic property at room temperature. If the temperature is increased beyond Curie temperature, then it will show

(a) paramagnetism
(b) anti-ferromagnetism
(c) no magnetic property
(d) diamagnetism

5. In radioactive decay process, the negatively charged emitted β-particles are

(a) the electrons present inside the nucleus
(b) the electrons produced as a result of the decay of neutrons inside the nucleus
(c) the electrons produced as a result of collisions between atoms
(d) the electrons orbiting around the nucleus

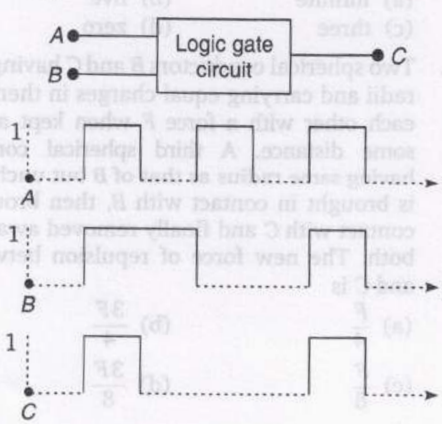
6. What is the value of inductance L for which the current is a maximum in a series LCR circuit with $C = 10\ \mu F$ and $\omega = 1000\ s^{-1}$?

(a) 100 mH
(b) 1 mH
(c) Cannot be calculated unless R is known
(d) 10 mH

7. Three point charges $+q$, $-2q$ and $+q$ are placed at points $(x = 0,\ y = a,\ z = 0)$, $(x = 0,\ y = 0,\ z = 0)$ and $(x = a,\ y = 0,\ z = 0)$, respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are

(a) $\sqrt{2}\,qa$ along $+y$ direction
(b) $\sqrt{2}\,qa$ along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(c) qa along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(d) $\sqrt{2}qa$ along $+x$ direction

8. The following figure shows a logic gate circuit with two inputs A and B and the output C. The voltage waveforms of A, B and C are as shown below



The logic circuit gate is

(a) AND gate (b) NAND gate
(c) NOR gate (d) OR gate

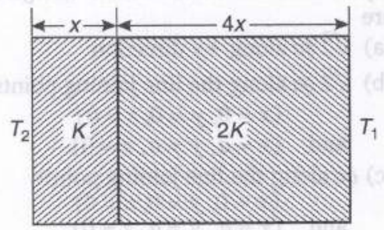
9. Assuming the sun to have a spherical outer surface of radius r, radiating like a black body at temperature t°C, the power received by a unit surface, (normal to the incident rays) at a distance R from the centre of the sun is

(a) $\frac{4\pi r^2 \sigma t^4}{R^2}$ (b) $\frac{r^2 \sigma (t+273)^4}{4\pi R^2}$

(c) $\frac{16\pi^2 r^2 \sigma t^4}{R^2}$ (d) $\frac{r^2 \sigma (t+273)^4}{R^2}$

where σ is the Stefan's constant.

10. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and T_1 $(T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equals to



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3

11. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment, is

(a) infinite (b) five
(c) three (d) zero

12. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged, is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is

(a) $\frac{F}{4}$ (b) $\frac{3F}{4}$

(c) $\frac{F}{8}$ (d) $\frac{3F}{8}$

13. In gamma ray emission from a nucleus

(a) both the neutron number and the proton number change
(b) there is no change in the proton number and the neutron number
(c) only the neutron number changes
(d) only the proton number changes

14. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the x-axis an angle of

(a) 30° (b) 45°
(c) 60° (d) 0°

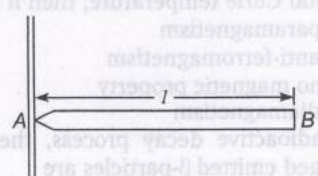
15. The resistance of an ammeter is 13 Ω and its scale is graduated for a current upto 100 A. After an additional shunt has been connected to this ammeter it becomes possible to measure currents upto 750 A by this meter. The value of shunt resistance is

(a) 20 Ω (b) 2 Ω
(c) 0.2 Ω (d) 2 kΩ

16. The primary and secondary coils of a transformer have 50 and 1500 turns respectively. If the magnetic flux ϕ linked with the primary coil is given by $\phi = \phi_0 + 4t$, where ϕ is in weber, t is time in second and ϕ_0 is a constant, the output voltage across the secondary coil is

(a) 90 V (b) 120 V
(c) 220 V (d) 30 V

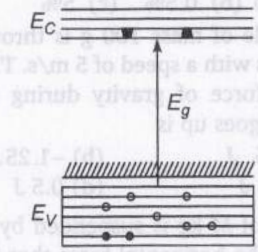
17. A uniform rod AB of length l and mass m is free to rotate about point A. The rod is released from rest in the horizontal position. Given that the moment of inertia of the rod about A is $\frac{ml^2}{3}$, the initial angular acceleration of the rod will be



(a) $\frac{2g}{3l}$ (b) $mg\frac{l}{2}$

(c) $\frac{3}{2}gl$ (d) $\frac{3g}{2l}$

18. In the energy band diagram of a material shown below, the open circles and filled circles denote holes and electrons respectively. The material is a/an

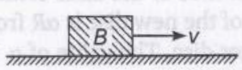


(a) p-type semiconductor
(b) insulator
(c) metal
(d) n-type semiconductor

19. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

(a) $\frac{T}{4}$ (b) $\frac{T}{8}$ (c) $\frac{T}{12}$ (d) $\frac{T}{2}$

20. A block B is pushed momentarily along a horizontal surface with an initial velocity v. If μ is the coefficient of sliding friction between B and the surface, block B will come to rest after a time



(a) $\frac{v}{g\mu}$ (b) $\frac{g\mu}{v}$ (c) $\frac{g}{v}$ (d) $\frac{v}{g}$

21. A transformer is used to light a 100 W and 110 V lamp from a 220 V mains. If the main current is 0.5 A, the efficiency of the transformer is approximately

(a) 30% (b) 50%
(c) 90% (d) 10%

22. A steady current of 1.5 A flows through a copper voltameter for 10 min. If the electrochemical equivalent of copper is 30×10^{-5} g C^{-1}, the mass of copper deposited on the electrode will be

(a) 0.40 g (b) 0.50 g
(c) 0.67 g (d) 0.27 g

23. Three resistances P, Q, R each of $2\,\Omega$ and an unknown resistance S form the four arms of a Wheatstone's bridge circuit. When a resistance of $6\,\Omega$ is connected in parallel to S the bridge gets balanced. What is the value of S?

(a) $2\,\Omega$ (b) $3\,\Omega$
(c) $6\,\Omega$ (d) $1\,\Omega$

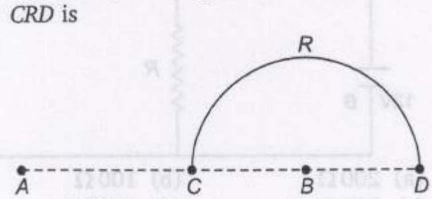
24. A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N/m. What should be the minimum amplitude of the motion, so that the mass gets detached from the pan ? (Take $g = 10\ m/s^2$)

(a) 8.0 cm
(b) 10.0 cm
(c) Any value less than 12.0 cm
(d) 4.0 cm

25. Two satellites of earth, S_1 and S_2, are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true ?

(a) The time period of S_1 is four times that of S_2.
(b) The potential energies of earth and satellite in the two cases are equal.
(c) S_1 and S_2 are moving with the same speed.
(d) The kinetic energies of the two satellites are equal.

26. Charges $+q$ and $-q$ are placed at points A and B respectively which are a distance $2L$ apart, C is the midpoint between A and B. The work done in moving a charge $+Q$ along the semicircle CRD is



(a) $\frac{qQ}{4\pi\varepsilon_0 L}$ (b) $\frac{qQ}{2\pi\varepsilon_0 L}$
(c) $\frac{qQ}{6\pi\varepsilon_0 L}$ (d) $-\frac{qQ}{6\pi\varepsilon_0 L}$

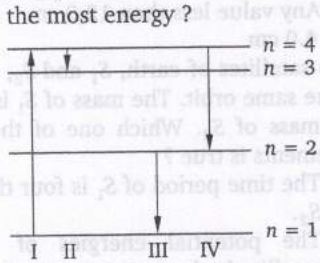
27. A beam of electrons passes undeflected through mutually perpendicular electric and magnetic fields. If the electric field is switched off, and the same magnetic field is maintained, the electrons move

(a) in an elliptical orbit
(b) in a circular orbit
(c) along a parabolic path
(d) along a straight line

28. Monochromatic light of frequency 6.0×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W. The number of photons emitted, on the average, by the source per second is

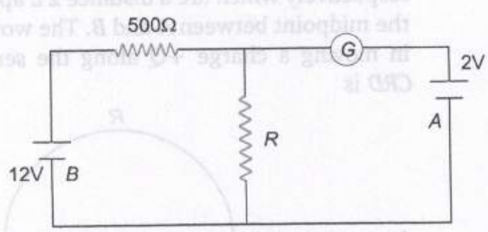
(a) 5×10^{15} (b) 5×10^{16}
(c) 5×10^{17} (d) 5×10^{14}

29. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy ?



(a) III (b) IV (c) I (d) II

30. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



(a) 200 Ω (b) 100 Ω
(c) 500 Ω (d) 1000 Ω

31. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is

(a) $\frac{1}{2} I_0$ (b) $\frac{1}{4} I_0$ (c) zero (d) I_0

32. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency ?

(a) Zero (b) 0.5% (c) 5% (d) 20%

33. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is

(a) – 0.5 J (b) –1.25 J
(c) 1.25 J (d) 0.5 J

34. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is

(a) $Mg(\sqrt{2} + 1)$ (b) $Mg\sqrt{2}$
(c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2} - 1)$

35. An electric charge 10^{-3} μC is placed at the origin (0, 0) of X-Y coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be

(a) 9 V (b) zero
(c) 2 V (d) 4.5 V

36. A circular disc of radius R is removed from a bigger circular disc of radius $2R$, such that the circumference of the discs coincide. The centre of mass of the new disc is αR from the centre of the bigger disc. The value of α is

(a) 1/3 (b) 1/2
(c) 1/6 (d) 1/4

37. A common-emitter amplifier has a voltage gain of 50, an input impedance of 100 Ω and an output impedance of 200 Ω. The power gain of the amplifier is

(a) 500 (b) 1000
(c) 1250 (d) 100

38. A vertical spring with force constant k is fixed on a table. A ball of mass m at a height h above the free upper end of the spring falls vertically on the spring, so that the spring is compressed by a distance d. The net work done in the process is

(a) $mg(h + d) + \frac{1}{2} kd^2$ (b) $mg(h + d) - \frac{1}{2} kd^2$
(c) $mg(h - d) - \frac{1}{2} kd^2$ (d) $mg(h - d) + \frac{1}{2} kd^2$

39. Dimensions of resistance in an electrical circuit, in terms of dimension of mass M, of length L, of time T and of current I, would be
(a) $[ML^2T^{-3}I^{-1}]$ (b) $[ML^2T^{-2}]$
(c) $[ML^2T^{-1}I^{-1}]$ (d) $[ML^2T^{-3}I^{-2}]$

40. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is
($R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$)
(a) diatomic
(b) triatomic
(c) a mixture of monoatomic and diatomic
(d) monoatomic

41. Diwali rocket is ejecting 50 g of gases/sec at a velocity of 400 m/s. The accelerating force on the rocket will be
(a) 22 dyne (b) 20 N
(c) 20 dyne (d) 100 N

42. A frame made of metallic wire enclosing a surface area A is covered with a soap film. If the area of the frame of metallic wire is reduced by 50%, the energy of the soap film will be changed by
(a) 100% (b) 75%
(c) 50% (d) 25%

43. A symmetric double convex lens is cut in two equal parts by a plane perpendicular to the principal axis. If the power of the original lens is 4D, the power of a cut lens will be
(a) 2D (b) 3D
(c) 4D (d) 5D

44. Two non-ideal batteries are connected in parallel. Consider the following statements.
(i) The equivalent emf is smaller than either of the two emfs.
(ii) The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Both (i) and (ii) are correct
(b) (i) is correct but (ii) is wrong
(c) (ii) is correct but (i) is wrong
(d) Both (i) and (ii) are wrong

45. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$, where l is about 100 cm and is known to have 1 mm accuracy. The period is about 2s. The time of 100 oscillations is measured by a stop watch of least count 0.1 s. The percentage error in g is
(a) 0.1% (b) 1% (c) 0.2% (d) 0.8%

46. The mass of the earth is 6.00×10^{24} kg and that of the moon is 7.40×10^{22} kg. The constant of gravitation $G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$. The potential energy of the system is -7.79×10^{28} J. The mean distance between the earth and moon is
(a) 3.80×10^8 m (b) 3.37×10^6 m
(c) 7.60×10^4 m (d) 1.90×10^2 m

47. If a rubber ball is taken at the depth of 200m in a pool, its volume decreases by 0.1%. If the density of water is $1 \times 10^3 \text{ kg/m}^3$ and $g = 10 \text{ m/s}^2$, then the volume of elasticity in N/m^2 will be
(a) 10^8 (b) 2×10^8
(c) 10^9 (d) 2×10^9

48. At 100K and 0.1 atmospheric pressure, the volume of helium gas is 10 L. If volume and pressure are doubled its temperature will change to
(a) 400 K (b) 127 K
(c) 200 K (d) 25 K

49. An organ pipe P_1 closed at one end vibrating in its first overtone and another pipe P_2 open at both ends vibrating in its third overtone are in resonance with a given tuning fork. The ratio of lengths of P_1 and P_2 is
(a) 1 : 2 (b) 1 : 3
(c) 3 : 8 (d) 3 : 4

50. Fraunhofer spectrum is a
(a) line absorption spectrum
(b) band absorption spectrum
(c) line emission spectrum
(d) band emission spectrum

Chemistry

1. How many unit cells are present in a cube shaped ideal crystal of NaCl of mass 1.00 g ? [Atomic masses : Na = 23, Cl = 35.5]
(a) 2.57×10^{21} (b) 5.14×10^{21}
(c) 1.28×10^{21} (d) 1.71×10^{21}

2. In acidic medium dichromate ion oxidises ferrous ion to ferric ion. If gram molecular weight of potassium dichromate is 294 g, its gram equivalent weight is
(a) 294 g (b) 127 g
(c) 49 g (d) 24.5 g

3. When $CH_2{=}CH{-}COOH$ is reduced with $LiAlH_4$, the compound obtained will be
(a) $CH_3{-}CH_2{-}COOH$
(b) $CH_2{=}CH{-}CH_2OH$
(c) $CH_3{-}CH_2{-}CH_2OH$
(d) $CH_3{-}CH_2{-}CHO$

4. Which one of the following compounds has the smallest bond angle in its molecule ?
(a) SO_2 (b) OH_2
(c) SH_2 (d) NH_3

5. For the redox reaction
$$Zn(s) + Cu^{2+}(0.1\ M) \longrightarrow Zn^{2+}(1\ M) + Cu(s)$$
taking place in a cell, E°_{cell} is 1.10 V. E_{cell} for the cell will be $\left(2.303\frac{RT}{F} = 0.0591\right)$
(a) 2.14 V (b) 1.80 V
(c) 1.07 V (d) 0.82 V

6. The rate law for a reaction between the substances A and B is given by rate $= k[A]^n[B]^m$. On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as
(a) $\frac{1}{2^{m+n}}$ (b) $(m+n)$
(c) $(n-m)$ (d) $2^{(n-m)}$

7. If at 298 K the bond energies of C—H, C—C, C=C and H—H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction
$$H_2C{=}CH_2(g) + H_2(g) \longrightarrow H_3C{-}CH_3(g)$$
at 298 K will be
(a) +250 kJ (b) −250 kJ
(c) +125 kJ (d) −125 kJ

8. Which one of the following characteristics is not correct for physical adsorption ?
(a) Adsorption on solids is reversible
(b) Adsorption increases with increase in temperature
(c) Adsorption is spontaneous
(d) Both enthalpy and entropy of adsorption are negative

9. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2NH$ is
(a) $CH_3NH_2 < NH_3 < (CH_3)_2NH$
(b) $(CH_3)_2NH < NH_3 < CH_3NH_2$
(c) $NH_3 < CH_3NH_2 < (CH_3)_2NH$
(d) $CH_3NH_2 < (CH_3)_2NH < NH_3$

10. Nylon threads are made up of
(a) polyvinyl polymer
(b) polyester polymer
(c) polyamide polymer
(d) polyethylene polymer

11. Due to presence of an unpaired electron, free radicals are
(a) cations
(b) anions
(c) chemically inactive
(d) chemically reactive

12. The highest electrical conductivity among the following aqueous solutions, is of
(a) 0.1 M difluoroacetic acid
(b) 0.1 M fluoroacetic acid
(c) 0.1 M chloroacetic acid
(d) 0.1 M acetic acid

13. Aluminium oxide may be electrolysed at 1000°C to furnish aluminium metal (atomic mass = 27 u; 1 F = 96,500 C). The cathode reaction is
$$Al^{3+} + 3e^- \longrightarrow Al^0$$
To prepare 5.12 kg of aluminium metal by this method would require
(a) 5.49×10^1 C of electricity
(b) 5.49×10^4 C of electricity
(c) 1.83×10^7 C of electricity
(d) 5.49×10^7 C of electricity

14. The molecular shapes of SF_4, SiF_4 and ICl_4^- are
(a) different with 1, 0 and 2 lone pairs of electrons on the central atoms, respectively
(b) different with 0, 1 and 2 lone pairs of electrons on the central atoms, respectively
(c) the same with 1, 1 and 1 lone pair of electrons on the central atoms, respectively
(d) the same with 2, 0 and 1 lone pairs of electrons on the central atoms, respectively

15. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives
(a) HgO
(b) Hg_2O
(c) $NH_2—Hg—Hg—Cl$
(d) $HgNH_2Cl$

16. Alkyl halides react with dialkyl copper reagents to give
(a) alkenyl halides
(b) alkanes
(c) alkyl copper halides
(d) alkenes

17. Acid catalysed hydration of alkenes except ethene leads to the formation of
(a) mixture of secondary and tertiary alcohols
(b) mixture of primary and secondary alcohols
(c) secondary or tertiary alcohol
(d) primary alcohol

18. An aqueous solution of 6.3 g of oxalic acid dihydrate is made upto 250 mL. The volume of 0.1 N NaOH required to completely neutralise 10 mL of this solution is
(a) 40 mL (b) 20 mL
(c) 10 mL (d) 4 mL

19. The wavelength of the radiation emitted, when in a hydrogen atom electron falls from infinity to stationary state one, would be (Rydberg constant = $1.097 \times 10^7\ m^{-1}$)
(a) 91 nm
(b) 192 nm
(c) 406 nm
(d) 9.1×10^{-8} nm

20. Which one of the following aqueous solutions will exhibit highest boiling point ?
(a) 0.01 M Na_2SO_4
(b) 0.01 M KNO_3
(c) 0.015 M urea
(d) 0.015 M glucose

21. Which among the following factors is the most important in making fluorine the strongest oxidising agent ?
(a) Electron affinity
(b) Ionisation enthalpy
(c) Hydration enthalpy
(d) Bond dissociation energy

22. What is the equilibrium expression for the reaction,

$$P_4(s) + 5O_2(g) \rightleftharpoons P_4O_{10}(s)\ ?$$

(a) $K_c = \dfrac{[P_4O_{10}]}{[P_4][O_2]^5}$ (b) $K_c = \dfrac{[P_4O_{10}]}{5[P_4][O_2]}$

(c) $K_c = [O_2]^5$ (d) $K_c = \dfrac{1}{[O_2]^5}$

23. The enthalpies of combustion of carbon and carbon monoxide are −393.5 and −283 kJ mol^{-1} respectively. The enthalpy of formation of carbon monoxide per mol is
(a) 110.5 kJ (b) 676.5 kJ
(c) −676.5 kJ (d) −110.5 kJ

24. Which one of the following ores is best concentrated by froth-floatation method ?
(a) Magnetite (b) Cassiterite
(c) Galena (d) Malachite

25. Excess of KI reacts with $CuSO_4$ solution and then $Na_2S_2O_3$ solution is added to it. Which of the statements is incorrect for this reaction ?
(a) Cu_2I_2 is formed
(b) CuI_2 is formed
(c) $Na_2S_2O_3$ is oxidised
(d) Evolved I_2 is reduced

26. Which one of the following does not have sp^2 hybridised carbon ?
(a) Acetone (b) Acetic acid
(c) Acetonitrile (d) Acetamide

27. Which of the following will have a *meso*-isomer also ?
(a) 2-chlorobutane
(b) 2,3-dichlorobutane
(c) 2,3-dichloropentane
(d) 2-hydroxypropanoic acid

28. Consider the acidity of the carboxylic acids
(i) PhCOOH
(ii) $o\text{-}NO_2C_6H_4COOH$
(iii) $p\text{-}NO_2C_6H_4COOH$
(iv) $m\text{-}NO_2C_6H_4COOH$

Which of the following orders is correct ?

(a) (i) > (ii) > (iii) > (iv)
(b) (ii) > (iv) > (iii) > (i)
(c) (ii) > (iv) > (i) > (iii)
(d) (ii) > (iii) > (iv) > (i)

29. The compound formed on heating chlorobenzene with chloral in the presence of concentrated sulphuric acid is

(a) gammexane (b) DDT
(c) freon (d) hexachloroethane

30. The smog is essentially caused by the presence of

(a) O_2 and O_3
(b) O_2 and N_2
(c) oxides of sulphur and nitrogen
(d) O_3 and N_2

31. The radioisotope, tritium (^3_1H) has a half-life of 12.3 yr. If the initial amount of tritium is 32 mg, how many milligrams of it would remain after 49.2 yr ?

(a) 4 mg (b) 8 mg
(c) 1 mg (d) 2 mg

32. In this reaction

$$CH_3CHO + HCN \longrightarrow CH_3CH(OH)CN \xrightarrow{H \cdot OH} CH_3CH(OH)COOH$$

an asymmetric centre is generated. The acid obtained would be

(a) 50% D + 50% L-isomer
(b) 20% D +80% L-isomer
(c) D-isomer
(d) L-isomer

33. The method of zone refining of metals is based on the principle of

(a) greater noble character of the solid metal than that of the impurity
(b) greater solubility of the impurity in the molten state than in the solid
(c) greater mobility of the pure metal than that of impurity
(d) higher melting point of the impurity than that of the pure metal

34. The temperature dependence of rate constant (k) of a chemical reaction is written in terms of Arrhenius equation, $k = Ae^{-E^*/RT}$. Activation energy (E^*) of the reaction can be calculated by ploting

(a) $\log k$ vs $\frac{1}{T}$ (b) $\log k$ vs $\frac{1}{\log T}$
(c) k vs T (d) k vs $\frac{1}{\log T}$

35. In a set of the given reactions, acetic acid yielded a product C.

$$CH_3COOH + PCl_5 \longrightarrow A$$

$$A \xrightarrow[\text{anhy } AlCl_3]{C_6H_6} B \xrightarrow[\text{ether}]{C_2H_5MgBr} C$$

Product C would be

(a) $CH_3CH(OH)C_6H_5$
(b) $CH_3-C(OH)(C_2H_5)C_6H_5$
(c) $CH_3CH(OH)C_2H_5$
(d) $CH_3COC_6H_5$

36. Which one of the following compounds, is not a protonic acid ?

(a) $SO(OH)_2$ (b) $SO_2(OH)_2$
(c) $B(OH)_3$ (d) $PO(OH)_3$

37. NaCl, NaBr and NaI mixture on heating with conc. H_2SO_4 gives gases, respectively

(a) HCl, HBr, HI (b) HCl, Br_2, I_2
(c) Cl_2, Br_2, I_2 (d) Cl_2, HBr, HI

38. The reaction between aniline and nitrous acid at low temperature yields

(a) N-nitrosamine (b) diazonium salt
(c) nitrile (d) amine-nitrite salt

39. The maximum number of molecules is present in

(a) 15 L of H_2 gas at STP
(b) 5 L of N_2 gas at STP
(c) 0.5 g of H_2 gas
(d) 10 g of O_2 gas

40. Ionic radii are

(a) inversely proportional to effective nuclear charge
(b) inversely proportional to square of effective nuclear charge
(c) directly proportional to effective nuclear charge
(d) directly proportional to square of effective nuclear charge

41. The radioactive isotope $^{60}_{27}Co$ which is used in the treatment of cancer can be made by (n, p) reaction. For this reaction the target nucleus is

(a) $^{59}_{28}Ni$ (b) $^{59}_{27}Co$
(c) $^{60}_{28}Ni$ (d) $^{60}_{27}Co$

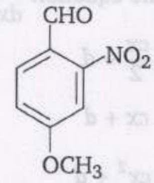
42. The vapour pressure of two liquids P and Q are 80 and 60 Torr, respectively. The total vapour pressure of solution obtained by mixing 3 moles of P and 2 moles of Q would be
(a) 140 Torr
(b) 20 Torr
(c) 68 Torr
(d) 72 Torr

43. The energy of second Bohr orbit of the hydrogen atom is -328 kJ mol^{-1}; hence the energy of fourth Bohr orbit would be
(a) -41 kJ mol^{-1}
(b) -1312 kJ mol^{-1}
(c) -164 kJ mol^{-1}
(d) -82 kJ mol^{-1}

44. In van der Waals' equation of state of the gas law, the constant 'b' is a measure of
(a) intermolecular repulsions
(b) intermolecular attractions
(c) volume occupied by the molecules
(d) intermolecular collisions per unit volume

45. Which of the following is responsible for depletion of the ozone layer in the upper strata of the atmosphere ?
(a) Polyhalogens
(b) Ferrocenes
(c) Fullerenes
(d) Freons

46. What is the correct IUPAC name of



?

(a) 4-methoxy-2-nitrobenzaldehyde
(b) 4-formyl-3-nitro anisole
(c) 4-methoxy-6-nitrobenzaldehyde
(d) 2-formyl-5-methoxy nitrobenzene

47. Which of the following is anhydride of perchloric acid ?
(a) Cl_2O_7 (b) Cl_2O_5
(c) Cl_2O_3 (d) HClO

48. The concentration of H_2O_2 solution of '10 volume' is
(a) 30% (b) 3%
(c) 1% (d) 10%

49. The dipole moment of HBr is 1.6×10^{-30} C-m and inter-atomic spacing is 1 Å. The % ionic character of HBr is
(a) 7 (b) 10
(c) 15 (d) 27

50. $CaC_2 + N_2 \longrightarrow A$, product A is
(a) $CaCN_2$ (b) $CaCN_2$ and C
(c) $CaCN_2 + N_2$ (d) None of these

Mathematics

1. If $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$, $|\vec{\mathbf{a}}| = 2$, $|\vec{\mathbf{b}}| = 3$, $|\vec{\mathbf{c}}| = 4$ and the angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is $\frac{2\pi}{3}$, then $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ is equal to
(a) $4\sqrt{3}$ (b) $6\sqrt{3}$
(c) $12\sqrt{3}$ (d) $18\sqrt{3}$

2. The general solution of $y^2\,dx + (x^2 - xy + y^2)\,dy = 0$ is
(a) $\tan^{-1}\left(\frac{x}{y}\right) + \log y + c = 0$
(b) $2\tan^{-1}\left(\frac{x}{y}\right) + \log x + c = 0$
(c) $\log(y + \sqrt{x^2 + y^2}) + \log y + c = 0$
(d) $\sin^{-1}\left(\frac{x}{y}\right) + \log y + c = 0$

3. The vector $\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{\mathbf{i}} + (4x - 2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The value of x is
(a) $\left\{-\frac{2}{3}, 2\right\}$
(b) $\left\{\frac{1}{3}, 2\right\}$
(c) $\left\{\frac{2}{3}, 0\right\}$
(d) $\{2, 7\}$

4. Three forces of magnitude 30, 60 and P acting at a point are in equilibrium. If the angle between the first two is 60°, then value of P is
(a) $30\sqrt{7}$ (b) $30\sqrt{3}$
(c) $20\sqrt{6}$ (d) $25\sqrt{2}$

5. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is

(a) $y = \frac{1}{4}e^{-2x} + \frac{cx}{2} + d$

(b) $y = \frac{1}{4}e^{-2x} + cx + d$

(c) $y = \frac{1}{4}e^{-2x} + cx^2 + d$

(d) $y = \frac{1}{4}e^{-2x} + cx^3 + d$

6. $\int \frac{dx}{x^2 + 4x + 13}$ is equal to

(a) $\log(x^2 + 4x + 13) + c$

(b) $\frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + c$

(c) $\log(2x + 4) + c$

(d) $\frac{2x+4}{(x^2 + 4x + 13)^2} + c$

7. The value of $\int_2^3 \frac{x+1}{x^2(x-1)} dx$ is

(a) $\log\frac{16}{9} + \frac{1}{6}$ (b) $\log\frac{16}{9} - \frac{1}{6}$

(c) $2\log 2 - \frac{1}{6}$ (d) $\log\frac{4}{3} - \frac{1}{6}$

8. $\int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x)\, dx$ is equal to

(a) $\sqrt{2} - 2$ (b) $2\sqrt{2} - 2$

(c) $3\sqrt{2} - 2$ (d) $4\sqrt{2} - 2$

9. The length of the chord of the parabola $x^2 = 4y$ passing through the vertex and having slope $\cot\alpha$ is

(a) $4\cos\alpha\,\mathrm{cosec}^2\alpha$ (b) $4\tan\alpha\sec\alpha$

(c) $4\sin\alpha\sec^2\alpha$ (d) None of these

10. The equation of tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, which are perpendicular to the line $3x + 4y = 7$, are

(a) $4x - 3y = \pm\sqrt{20}$

(b) $4x - 3y = \pm\sqrt{12}$

(c) $4x - 3y = \pm\sqrt{2}$

(d) $4x - 3y = \pm 1$

11. From the point $P(16, 7)$ tangents PQ and PR are drawn to the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. If c be the centre of the circle, then area of quadrilateral $PQCR$ is

(a) 450 sq unit (b) 15 sq unit

(c) 50 sq unit (d) 75 sq unit

12. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is

(a) $\frac{3}{2}$ (b) $\frac{3}{10}$

(c) 6 (d) None of these

13. In a ΔABC, right angled at C, the value of $\cot A + \cot B$ is

(a) $\frac{c^2}{ab}$ (b) $a + b$ (c) $\frac{a^2}{bc}$ (d) $\frac{b^2}{ac}$

14. The records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die, is

(a) 8748×10^{-5} (b) 1458×10^{-5}

(c) 1458×10^{-6} (d) 41×10^{-6}

15. Out of 40 consecutive natural numbers, two are chosen at random. Probability that the sum of the number is odd, is

(a) $\frac{14}{29}$ (b) $\frac{20}{39}$

(c) $\frac{1}{2}$ (d) None of these

16. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) None of these

17. If α, β are the roots of the equation $lx^2 + mx + n = 0$, then the equation whose roots are $\alpha^3\beta$ and $\alpha\beta^3$, is

(a) $l^4x^2 - nl(m^2 - 2nl)x + n^4 = 0$

(b) $l^4x^2 + nl(m^2 - 2nl)x + n^4 = 0$

(c) $l^4x^2 + nl(m^2 - 2nl)x - n^4 = 0$

(d) $l^4x^2 - nl(m^2 + 2nl)x + n^4 = 0$

18. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \dots$

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) 2 (d) 1

19. Out of 6 boys and 4 girls, a group of 7 is to be formed. In how many ways can this be done, if the group is to have a majority of boys ?
(a) 120 (b) 80 (c) 90 (d) 100

20. The domain of the function $\sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$ is
(a) $[-1, 2]-\{0\}$ (b) $[-2, 2]-(-1, 1)$
(c) $[-2, 2]-\{0\}$ (d) $[1, 2]$

21. Function $f(x)=\begin{cases} x-1, & x<2 \\ 2x-3, & x \geq 2 \end{cases}$ is a continuous function
(a) for $x = 2$ only
(b) for all real values of x such that $x \neq 2$
(c) for all real values of x
(d) for all integral values of x only

22. Differential coefficient of $\sqrt{\sec \sqrt{x}}$ is
(a) $\frac{1}{4\sqrt{x}} \sec \sqrt{x} \sin \sqrt{x}$
(b) $\frac{1}{4\sqrt{x}} (\sec \sqrt{x})^{3/2} \cdot \sin \sqrt{x}$
(c) $\frac{1}{2} \sqrt{x} \sec \sqrt{x} \sin \sqrt{x}$
(d) $\frac{1}{2} \sqrt{x} (\sec \sqrt{x})^{3/2} \cdot \sin \sqrt{x}$

23. The function $x^5 - 5x^4 + 5x^3 - 1$ is
(a) neither maximum nor minimum at $x = 0$
(b) maximum at $x = 0$
(c) maximum at $x = 1$ and minimum at $x = 3$
(d) minimum at $x = 0$

24. If the planes $x + 2y + kz = 0$ and $2x + y - 2z = 0$, are at right angles, then the value of k is
(a) 2 (b) -2
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

25. The ratio in which the line joining (2, 4, 5), (3, 5, – 4) is divided by the yz-plane is
(a) 2 : 3 (b) 3 : 2
(c) – 2 : 3 (d) 4 : – 3

26. The radical centre of the circles
$$x^2 + y^2 - 16x + 60 = 0,$$
$$x^2 + y^2 - 12x + 27 = 0$$
and $x^2 + y^2 - 12y + 8 = 0$ is
(a) $\left(13, \frac{33}{4}\right)$ (b) $\left(\frac{33}{4}, -13\right)$
(c) $\left(\frac{33}{4}, 13\right)$ (d) None of these

27. If the lines $3x + 4y + 1 = 0$, $5x + \lambda y + 3 = 0$ and $2x + y - 1 = 0$ are concurrent, then λ is equal to
(a) – 8 (b) 8
(c) 4 (d) –4

28. A ball falls from rest from top of a tower. If the ball reaches the foot of the tower in 3 s, then height of tower is (take $g = 10\ m/s^2$)
(a) 45 m (b) 50 m
(c) 40 m (d) None of these

29. The value of
$$1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 2)^3}{3!} + \ldots \text{ is}$$
(a) log 3 (b) log 2
(c) $\frac{1}{2}$ (d) None of these

30. The maximum value of $f(x) = \frac{x}{4 + x + x^2}$ on $[-1, 1]$ is
(a) $-\frac{1}{3}$ (b) $-\frac{1}{4}$
(c) $\frac{1}{5}$ (d) $\frac{1}{6}$

31. $\int \frac{e^x}{(2 + e^x)(e^x + 1)} dx$ is equal to
(a) $\log\left(\frac{e^x + 1}{e^x + 2}\right) + c$ (b) $\log\left(\frac{e^x + 2}{e^x + 1}\right) + c$
(c) $\left(\frac{e^x + 1}{e^x + 2}\right) + c$ (d) $\left(\frac{e^x + 2}{e^x + 1}\right) + c$

32. If the radius of a circle be increasing at a uniform rate of 2 cm/s. The area of increasing of area of circle, at the instant when the radius is 20 cm, is
(a) $70\ \pi\ cm^2/s$ (b) $70\ cm^2/s$
(c) $80\ \pi\ cm^2/s$ (d) $80\ cm^2/s$

33. The focus of the parabola $y^2 - x - 2y + 2 = 0$ is
(a) $\left(\frac{1}{4}, 0\right)$ (b) (1, 2)
(c) $\left(\frac{5}{4}, 1\right)$ (d) $\left(\frac{3}{4}, \frac{5}{2}\right)$

34. The equation of normal at the point (0, 3) of the ellipse $9x^2 + 5y^2 = 45$ is
(a) x-axis (b) y-axis
(c) $y + 3 = 0$ (d) $y - 3 = 0$

35. The equation of the tangent parallel to $y - x + 5 = 0$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

(a) $x - y + 1 = 0$
(b) $x - y + 2 = 0$
(c) $x + y - 1 = 0$
(d) $x + y + 2 = 0$

36. Let the functions f, g, h are defined from the set of real numbers R to R such that $f(x) = x^2 - 1$, $g(x) = \sqrt{(x^2+1)}$ and $h(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$, then $ho(fog)(x)$ is defined by

(a) x (b) x^2
(c) 0 (d) None of these

37. The argument of the complex number $\frac{13-5i}{4-9i}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{5}$ (d) $\frac{\pi}{6}$

38. If $\sin \alpha$ and $\cos \alpha$ are the roots of the equation $px^2 + qx + r = 0$, then

(a) $p^2 + q^2 - 2pr = 0$
(b) $p^2 - q^2 + 2pr = 0$
(c) $p^2 - q^2 - 2pr = 0$
(d) $p^2 + q^2 + 2qr = 0$

39. In the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$, the term independent of x is

(a) 8th (b) 7th (c) 9th (d) 10th

40. The general value of θ in the equation $\cos\theta = \frac{1}{\sqrt{2}}$, $\tan\theta = -1$ is

(a) $2n\pi \pm \frac{\pi}{6}, n \in I$
(b) $2n\pi + \frac{7\pi}{4}, n \in I$
(c) $n\pi + (-1)^n \frac{\pi}{3}, n \in I$
(d) $n\pi + (-1)^n \frac{\pi}{4}, n \in I$

41. In a ΔABC, if $r_1 = 2r_2 = 3r_3$, then

(a) $\frac{a}{b} = \frac{4}{5}$ (b) $\frac{a}{b} = \frac{5}{4}$
(c) $a + b - 2c = 0$ (d) $2a = b + c$

42. The value of $\lim_{x \to \infty} \left(\frac{x^2 + bx + 4}{x^2 + ax + 5}\right)$ is

(a) $\frac{b}{a}$ (b) 0 (c) 1 (d) $\frac{4}{5}$

43. Let $f(x) = \begin{cases} \frac{\sin \pi x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$, if $f(x)$ is continuous at $x = 0$, then k is equal to

(a) $\frac{\pi}{5}$ (b) $\frac{5}{\pi}$
(c) 1 (d) 0

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors with magnitudes 3, 4 and 5 respectively and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the values of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is

(a) 47 (b) 25
(c) 50 (d) −25

45. From a point a metre above a lake the angle of elevation of a cloud α and the angle of depression of its reflection is β. The height of the cloud is

(a) $\frac{a \sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ m (b) $\frac{a\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ m
(c) $\frac{a\sin(\beta-\alpha)}{\sin(\alpha+\beta)}$ m (d) None of these

46. If $A = \text{diag}(2, -1, 3)$, $B = \text{diag}(-1, 3, 2)$ then A^2B is equal to

(a) diag $(-4, 3, 18)$ (b) diag $(5, 4, 11)$
(b) diag $(3, 1, 8)$ (d) None of these

47. The negation of the proposition $(p \wedge \sim q) \Rightarrow r$ is

(a) $\sim p \vee q \Rightarrow \sim r$ (b) $\sim r \Rightarrow \sim p \vee q$
(c) $r \Rightarrow p \wedge \sim q$ (d) None of these

48. Negation of "$2 + 3 = 5$ and $8 < 10$" is

(a) $2 + 3 \neq 5$ and < 10
(b) $2 + 3 = 5$ and $8 < 10$
(c) $2 + 3 \neq 5$ or $8 \nless 10$
(d) None of the above

49. On the parabola $y = x^2$, the point least distance from the straight line $y = 2x - 4$ is

(a) $(1, 1)$ (b) $(1, 0)$
(c) $(1, -1)$ (d) $(0, 0)$

50. The number of terms which are free from radical sign in the expansion of $(y^{1/5} + x^{1/10})^{55}$ is

(a) 7 (b) 6
(c) 5 (d) None of these

Answers

PHYSICS

1.	(a)	2.	(a)	3.	(b)	4.	(a)	5.	(b)	6.	(a)	7.	(b)	8.	(a)
9.	(d)	10.	(d)	11.	(b)	12.	(d)	13.	(b)	14.	(c)	15.	(b)	16.	(b)
17.	(d)	18.	(a)	19.	(c)	20.	(a)	21.	(c)	22.	(d)	23.	(b)	24.	(b)
25.	(c)	26.	(d)	27.	(b)	28.	(a)	29.	(a)	30.	(b)	31.	(a)	32.	(d)
33.	(b)	34.	(d)	35.	(b)	36.	(a)	37.	(c)	38.	(b)	39.	(d)	40.	(a)
41.	(b)	42.	(c)	43.	(a)	44.	(c)	45.	(c)	46.	(a)	47.	(d)	48.	(a)
49.	(c)	50.	(a)												

CHEMISTRY

1.	(a)	2.	(c)	3.	(b)	4.	(c)	5.	(c)	6.	(d)	7.	(d)	8.	(b)
9.	(c)	10.	(c)	11.	(d)	12.	(a)	13.	(d)	14.	(a)	15.	(d)	16.	(b)
17.	(c)	18.	(a)	19.	(a)	20.	(a)	21.	(c)	22.	(d)	23.	(d)	24.	(c)
25.	(b)	26.	(c)	27.	(b)	28.	(d)	29.	(b)	30.	(c)	31.	(d)	32.	(a)
33.	(b)	34.	(a)	35.	(b)	36.	(c)	37.	(b)	38.	(b)	39.	(a)	40.	(a)
41.	(c)	42.	(d)	43.	(d)	44.	(c)	45.	(d)	46.	(a)	47.	(a)	48.	(b)
49.	(b)	50.	(b)												

MATHEMATICS

1.	(c)	2.	(a)	3.	(a)	4.	(a)	5.	(b)	6.	(b)	7.	(b)	8.	(d)
9.	(a)	10.	(a)	11.	(d)	12.	(b)	13.	(a)	14.	(b)	15.	(b)	16.	(b)
17.	(a)	18.	(c)	19.	(d)	20.	(b)	21.	(c)	22.	(b)	23.	(c)	24.	(a)
25.	(c)	26.	(d)	27.	(b)	28.	(a)	29.	(c)	30.	(d)	31.	(a)	32.	(c)
33.	(c)	34.	(b)	35.	(a)	36.	(b)	37.	(b)	38.	(b)	39.	(c)	40.	(b)
41.	(b)	42.	(c)	43.	(a)	44.	(d)	45.	(a)	46.	(a)	47.	(a)	48.	(c)
49.	(a)	50.	(b)												

Hints & Solutions

Physics

1. From given information $a = -kx$, where a is acceleration, x is displacement and k is a proportionality constant.

$$\frac{v\,dv}{dx} = -k\,x$$

$\Rightarrow$ $$v\,dv = -k\,x\,dx$$

Let for any displacement from 0 to x, the velocity changes from v_0 to v.

$\Rightarrow$ $$\int_{v_0}^{v} v dv = -\int_0^x k\,x\,dx$$

$\Rightarrow$ $$\frac{v^2 - v_0^2}{2} = -\frac{k\,x^2}{2}$$

$\Rightarrow$ $$m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mk\,x^2}{2}$$

$\Rightarrow$ $$\Delta K \propto x^2 \quad [\Delta K \text{ is loss in KE}]$$

2. Man will catch the ball if the horizontal component of velocity becomes equal to the constant speed of man *ie*,

$$v_0 \cos\theta = \frac{v_0}{2}$$

$\Rightarrow$ $$\cos\theta = \frac{1}{2}$$

$\Rightarrow$ $$\cos\theta = \cos 60°$$

$\therefore$ $$\theta = 60°$$

3. Retarding force acting on a ball falling into a viscous fluid

$$F = 6\pi\eta R v$$

where R = radius of ball,
v = velocity of ball,
and η = coefficient of viscosity.

$\therefore$ $F \propto R$ and $F \propto v$

Or in words, retarding force is directly proportional to both R and v.

4. Nickel exhibits ferromagnetism because of a quantum physical effect called exchange coupling in which the electron spins of one atom interact with those of neighbouring atoms. The result is alignment of the magnetic dipole moments of the atoms, inspite of the randomizing tendency of atomic collisions. This persistent alignment is what gives ferromagnetic materials their permanent magnetism.

If the temperature of a ferromagnetic material is raised above a certain critical value, called the Curie temperature, the exchange coupling ceases to be effective. Most such materials then become simply paramagnetic; that is, the dipoles still tend to align with an external field but much more weakly, and thermal agitation can now more easily disrupt the alignment.

5. β-decay can involve the emission of either electrons or positrons. The electrons or positrons emitted in a β-decay do not exist inside the nucleus. They are only created at the time of emission, just as photons are created when an atom makes a transition from higher to a lower energy state.

In negative β-decay a neutron in the nucleus is transformed into a proton, an electron and an antineutrino. Hence, in radioactive decay process, the negatively charged emitted β-particles are the electrons produced as a result of the decay of neutrons present inside the nucleus.

6. **Key Idea** *In resonance condition, maximum current flows in the circuit.*

Current in LCR series circuit,

$$i = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

where V is rms value of current, R is resistance, X_L is inductive reactance and X_C is capacitive reactance.

For current to be maximum, denominator should be minimum which can be done, if

$$X_L = X_C$$

This happens in resonance state of the circuit *ie*,

$$\omega L = \frac{1}{\omega C}$$

or $$L = \frac{1}{\omega^2 C} \quad \ldots\text{(i)}$$

Given, $\omega = 1000\ \text{s}^{-1}$, $C = 10\ \mu\text{F} = 10 \times 10^{-6}$ F

Hence, $$L = \frac{1}{(1000)^2 \times 10 \times 10^{-6}}$$

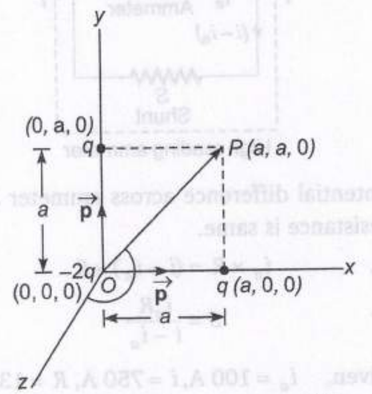
$$= 0.1\ \text{H} = 100\ \text{mH}$$

7. Key Idea *Electric dipole moment is a vector quantity directed from negative charge to the similar positive charge.*

Choose the three coordinate axes as x, y and z and plot the charges with the given coordinates as shown.

O is the origin at which $-2q$ charge is placed.

The system is equivalent to two dipoles along x and y-directions respectively. The dipole moments of two dipoles are shown in figure.



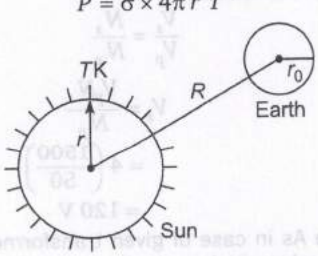
The resultant dipole moment will be directed along OP where $P \equiv (a, a, 0)$. The magnitude of resultant dipole moment is

$$p' = \sqrt{p^2 + p^2} = \sqrt{(qa)^2 + (qa)^2}$$
$$= \sqrt{2}\, qa$$

8. The Boolean expression which satisfies the output of this logic gate is $C = A \cdot B$, which is for AND gate.

9. From Stefan's law, the rate at which energy is radiated by sun at its surface is

$$P = \sigma \times 4\pi r^2 T^4$$



[Sun is a perfectly black body as it emits radiations of all wavelengths and so for it $e = 1$]

The intensity of this power at earth's surface (under the assumption $R >> r_0$) is

$$I = \frac{P}{4\pi R^2} = \frac{\sigma \times 4\pi r^2 T^4}{4\pi R^2}$$
$$= \frac{\sigma r^2 T^4}{R^2}$$
$$= \frac{\sigma r^2 (t + 273)^4}{R^2}$$

10. Let the temperature of common interface be T °C.

Rate of heat flow

$$H = \frac{Q}{t} = \frac{K A \Delta T}{l}$$

$$\therefore \quad H_1 = \left(\frac{Q}{t}\right)_1 = \frac{2K A (T - T_1)}{4x}$$

and $$H_2 = \left(\frac{Q}{t}\right)_2 = \frac{KA (T_2 - T)}{x}$$

In steady state, the rate of heat flow should be same in whole system *ie*,

$$H_1 = H_2$$
$$\Rightarrow \quad \frac{2KA (T - T_1)}{4x} = \frac{KA (T_2 - T)}{x}$$
$$\Rightarrow \quad \frac{T - T_1}{2} = T_2 - T$$
$$\Rightarrow \quad T - T_1 = 2T_2 - 2T$$
$$\Rightarrow \quad T = \frac{2T_2 + T_1}{3} \quad \text{...(i)}$$

Hence, heat flow from composite slab is

$$H = \frac{KA (T_2 - T)}{x}$$
$$= \frac{KA}{x}\left(T_2 - \frac{2T_2 + T_1}{3}\right) = \frac{KA}{3x}(T_2 - T_1) \quad \text{...(ii)}$$

[from Eq. (i)]

Accordingly, $$H = \left[\frac{A (T_2 - T_1) K}{x}\right] f \quad \text{...(iii)}$$

By comparing Eqs. (ii) and (iii), we get

$$\Rightarrow \quad f = \frac{1}{3}$$

11. For possible interference maxima on the screen, the condition is

$$d \sin \theta = n\lambda \quad \text{...(i)}$$

Given d = slit-width = 2λ

$$\therefore \quad 2\lambda \sin \theta = n\lambda$$
$$\Rightarrow \quad 2 \sin \theta = n$$

The maximum value of $\sin\theta$ is 1, hence,

$$n = 2 \times 1 = 2$$

Thus, Eq. (i) must be satisfied by 5 integer values *ie*, −2, −1, 0, 1, 2. Hence, the maximum number of possible interference maxima is 5.

12. Let the spherical conductors B and C have same charge as q. The electric force between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

where r, being the distance between them. When third uncharged conductor A is brought in contact with B, then charge on each conductor

$$q_A = q_B = \frac{q_A + q_B}{2} = \frac{0+q}{2} = \frac{q}{2}$$

When this conductor A is now brought in contact with C, then charge on each conductor

$$q_A = q_C = \frac{q_A + q_C}{2} = \frac{(q/2)+q}{2} = \frac{3q}{4}$$

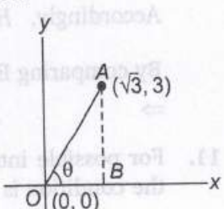
Hence, electric force acting between B and C is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q_B q_C}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(3q/4)}{r^2} = \frac{3}{8}\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right] = \frac{3F}{8}$$

13. In gamma ray emission the energy is released from nucleus, so that nucleus get stablished.

14. Key Idea *Slope of the path of the particle gives the measure of angle required.*

Draw the situation as shown. OA represents the path of the particle starting from origin $O(0, 0)$. Draw a perpendicular from point A to x-axis. Let path of the particle makes an angle θ with the x-axis, then



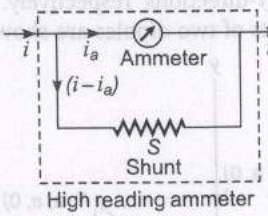
$$\tan\theta = \text{slope of line } OA$$

$$= \frac{AB}{OB} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

or $\theta = 60°$

15. Key Idea *The potential difference across ammeter and shunt is same.*

Let i_a is the current flowing through ammeter and i is the total current. So, a current $i - i_a$ will flow through shunt resistance.



Potential difference across ammeter and shunt resistance is same.

ie, $$i_a \times R = (i - i_a) \times S$$

or $$S = \frac{i_a R}{i - i_a} \quad \ldots\text{(i)}$$

Given, $i_a = 100$ A, $i = 750$ A, $R = 13\,\Omega$

Hence, $$S = \frac{100 \times 13}{750 - 100} = 2\,\Omega$$

16. The magnetic flux linked with the primary coil is given by

$$\phi = \phi_0 + 4t$$

So, voltage across primary

$$V_p = \frac{d\phi}{dt} = \frac{d}{dt}(\phi_0 + 4t)$$

$$= 4\text{ V} \qquad (\text{as } \phi_0 = \text{constant})$$

Also, we have

$$N_p = 50 \text{ and } N_s = 1500$$

From relation,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

or $$V_s = \frac{V_p N_s}{N_p} = 4\left(\frac{1500}{50}\right) = 120\text{ V}$$

Note As in case of given transformer, voltage in secondary is increased, hence it is a step-up transformer.

17. The moment of inertia of the uniform rod about an axis through one end and perpendicular to its length is

$$I = \frac{ml^2}{3}$$

where m is mass of rod and l is length.

Torque ($\tau = I\alpha$) acting on centre of gravity of rod is given by

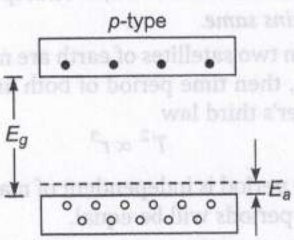
$$\tau = mg\,\frac{l}{2}$$

or $$I\alpha = mg\,\frac{l}{2}$$

or $$\frac{ml^2}{3}\,\alpha = mg\,\frac{l}{2}$$

or $$\alpha = \frac{3g}{2l}$$

18. The given figure represents p-type semiconductor as described below



When one of the silicon atoms (valence = 4) has been replaced by an atom of aluminium (valence = 3), the aluminium atom can bond covalently with only three silicon atoms, so there is now a "missing" electron (a hole) in one aluminium-silicon bond with a small expanditure of energy, an electron can be torn from a neighbouring silicon-silicon bond to fill this hole, thereby creating a hole in that bond. Similarly, an electron from some other bond can be moved to fill the second hole. In this way, the hole can migrate through the lattice.

The aluminium atom is called an acceptor atom because it readily accepts an electron from a neighbouring bond that is from the valence band of silicon. As figure suggests, this electron occupies a localized acceptor state that lies within the energy gap, at an average energy interval E_a above the top of the valence band.

By adding acceptor atoms, it is possible to increase very greatly the number of holes in the valence band.

19. Let displacement equation of particle executing SHM is

$$y = a \sin \omega t$$

As particle travels half of the amplitude from the equilibrium position, so

$$y = \frac{a}{2}$$

Therefore, $$\frac{a}{2} = a \sin \omega t$$

or $$\sin \omega t = \frac{1}{2} = \sin \frac{\pi}{6}$$

or $$\omega t = \frac{\pi}{6}$$

or $$t = \frac{\pi}{6\omega}$$

or $$t = \frac{\pi}{6\left(\frac{2\pi}{T}\right)} \quad \left(\text{as } \omega = \frac{2\pi}{T}\right)$$

or $$t = \frac{T}{12}$$

Hence, the particle travels half of the amplitude from the equilibrium in $\frac{T}{12}$ sec.

20. Block B will come to rest, if force applied to it will vanish due to frictional force acting between block B and surface, *ie*,

force applied = frictional force

ie, $$\mu mg = ma$$

or $$\mu mg = m\left(\frac{v}{t}\right)$$

or $$t = \frac{v}{\mu g}$$

21. The efficiency of transformer

$$= \frac{\text{Energy obtained from the secondary coil}}{\text{Energy given to the primary coil}}$$

or $$\eta = \frac{\text{Output power}}{\text{Input power}}$$

or $$\eta = \frac{V_s I_s}{V_p I_p}$$

Given, $V_s I_s = 100$ W, $V_p = 220$ V, $I_p = 0.5$ A

Hence, $$\eta = \frac{100}{220 \times 0.5} = 0.90 = 90\%$$

Note The efficiency of an ideal transformer is 1 (or 100%). But in practice due to loss in energy, the efficiency of transformer is always less than 1 (or less than 100%).

22. **Key Idea** *Apply first law of electrolysis during deposition of charge on an electrode through electrolysis.*

If m is the mass of a substance deposited or liberated at an electrode during electrolysis when a charge q passes through electrolyte, then according to Faraday's first law of electrolysis,

$$m \propto q$$

or $$m = zq$$

where z is a constant of proportionality and is called electrochemical equivalent (ECE) of the the substance.

If an electric current I flows through the electrolyte, then

$$q = It$$

$\therefore$ $$m = zit$$

Here, $i = 1.5\text{ A}$, $t = 10\text{ min} = 10 \times 60\text{ s}$,

$$z = 30 \times 10^{-5}\text{ gC}^{-1}$$

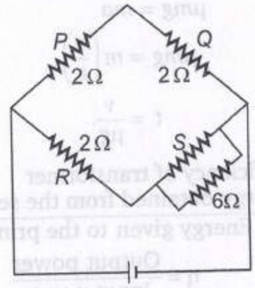
Hence, mass of copper deposited on the electrode

$$m = 30 \times 10^{-5} \times 1.5 \times 10 \times 60$$

$$= 27 \times 10^{-2} = 0.27\text{ g}$$

23. Key Idea *The bridge formed with given resistances is a balanced Wheatstone's bridge.*

The situation can be depicted as shown in figure.



As resistances S and $6\,\Omega$ are in parallel their effective resistance is

$$\frac{6S}{6+S}\,\Omega$$

As the bridge is balanced, hence it is balanced Wheatstone's bridge.

For balancing condition,

$$\frac{P}{Q} = \frac{R}{\left(\frac{6S}{6+S}\right)}$$

or $$\frac{2}{2} = \frac{2(6+S)}{6S}$$

or $$3S = 6 + S$$

or $$S = 3\,\Omega$$

24. Let the minimum amplitude of SHM is a.

Restoring force on spring

$$F = ka$$

Restoring force is balanced by weight mg of block. For mass to execute simple harmonic motion of amplitude a.

$\therefore$ $$ka = mg$$

or $$a = \frac{mg}{k}$$

Here, $m = 2\text{ kg}$, $k = 200\text{ N/m}$, $g = 10\text{ m/s}^2$

$\therefore$ $$a = \frac{2 \times 10}{200} = \frac{10}{100}\text{ m}$$

$$= \frac{10}{100} \times 100\text{ cm} = 10\text{ cm}$$

Hence, minimum amplitude of the motion should be 10 cm, so that the mass gets detached from the pan.

25. Key Idea *In same orbit, orbital speed of satellites remains same.*

When two satellites of earth are moving in same orbit, then time period of both are equal. From Kepler's third law

$$T^2 \propto r^3$$

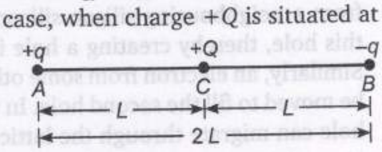
Time period is independent of mass, hence their time periods will be equal.

The potential energy and kinetic energy are mass dependent, hence the PE and KE of satellites are not equal.

But, if they are orbiting in a same orbit, then they have equal orbital speed.

26. Key Idea *Work done is equal to change in potential energy.*

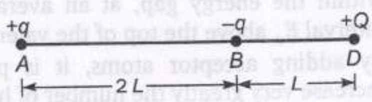
In Ist case, when charge $+Q$ is situated at C.



Electric potential energy of system

$$U_1 = \frac{1}{4\pi\varepsilon_0}\frac{(q)(-q)}{2L} + \frac{(-q)Q}{4\pi\varepsilon_0 L} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{L}$$

In IInd case, when charge $+Q$ is moved from C to D.



Electric potential energy of system in that case

$$U_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(q)(-q)}{2L} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{3L} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)(Q)}{L}$$

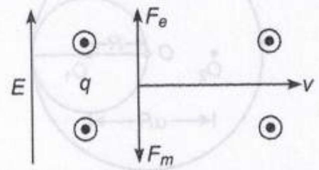
$\therefore$ Work done $= \Delta U = U_2 - U_1$

$$= \left(-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2L} + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{3L} - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\right) - \left(-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2L} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{L} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{L}\right)$$

$$= \frac{qQ}{4\pi\varepsilon_0}\cdot\left[\frac{1}{3L} - \frac{1}{L}\right] = \frac{qQ}{4\pi\varepsilon_0}\frac{(1-3)}{3L}$$

$$= \frac{-2qQ}{12\pi\varepsilon_0 L} = -\frac{qQ}{6\pi\varepsilon_0 L}$$

27. If both electric and magnetic fields are present and perpendicular to each other and the particle is moving perpendicular to both of them with $F_e = F_m$. In this situation $\vec{E} \neq 0$ and $\vec{B} \neq 0$.



But if electric field becomes zero, then only force due to magnetic field exists. Under this force, the charge moves along a circle.

28. Key Idea *Photons are the packets of energy.*

Power emitted,

$$P = 2 \times 10^{-3}\text{ W}$$

Energy of photon,

$$E = h\nu = 6.6 \times 10^{-34} \times 6 \times 10^{14}\text{ J}$$

where h being Planck's constant.

Number of photons emitted per second

$$n = \frac{P}{E}$$

$$= \frac{2 \times 10^{-3}}{6.6 \times 10^{-34} \times 6 \times 10^{14}} = 5 \times 10^{15}$$

29. $E = Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$E_{(4\to3)} = Rhc\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$= Rhc\left[\frac{7}{9\times16}\right] = 0.05\,Rhc$$

$$E_{(4\to2)} = Rhc\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$= Rhc\left[\frac{3}{16}\right] = 0.2\,Rhc$$

$$E_{(2\to1)} = Rhc\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$$

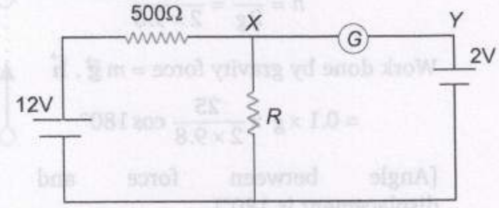
$$= Rhc\left[\frac{3}{4}\right] = 0.75\,Rhc$$

$$E_{(1\to3)} = Rhc\left[\frac{1}{(3)^2} - \frac{1}{(1)^2}\right]$$

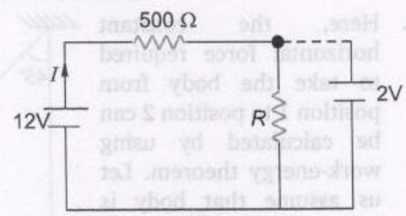
$$= -\frac{8}{9}Rhc = -0.9\,Rhc$$

Thus, transition III gives most energy. Transition I represents the absorption of energy.

30. The galvanometer shows zero deflection *ie*, current through XY is zero.



As a result potential drop across R is 2 V, circuit can be redrawn as



$$I = \frac{12}{500 + R}$$

Voltage across R, $V = IR$

$$\Rightarrow \quad 2 = \frac{12}{500 + R} \times R$$

$$\Rightarrow 1000 + 2R = 12R \Rightarrow R = 100\,\Omega$$

31. $I = I_0\cos^2\theta$

Intensity of polarized light

$$= \frac{I_0}{2}$$

$\therefore$ Intensity of untransmitted light

$$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

32. Given $v_o = \frac{v}{5} \Rightarrow v_o = \frac{320}{5} = 64\text{ m/s}$

When observer moves towards the stationary source, then

$$n' = \left(\frac{v + v_o}{v}\right)n$$

$$n' = \left(\frac{320 + 64}{320}\right)n$$

$$n' = \left(\frac{384}{320}\right) n$$

$$\frac{n'}{n} = \frac{384}{320}$$

Hence, percentage increase

$$\left(\frac{n'-n}{n}\right) = \left(\frac{384-320}{320} \times 100\right) \%$$

$$= \left(\frac{64}{320} \times 100\right) \% = 20 \%$$

33. The height (h) traversed by particle while going up is

$$h = \frac{u^2}{2g} = \frac{25}{2 \times 9.8}$$

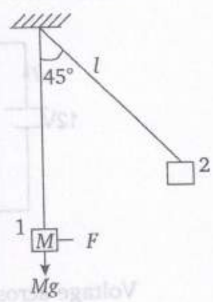
Work done by gravity force $= m\vec{\mathbf{g}} \cdot \vec{\mathbf{h}}$

$$= 0.1 \times g \times \frac{25}{2 \times 9.8} \cos 180°$$

[Angle between force and displacement is 180°]

$\therefore \; W = -0.1 \times \frac{25}{2} = -1.25 \text{ J}$

34. Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem. Let us assume that body is taken slowly so that its speed doesn't change, then



$$\Delta K = 0$$

$$= W_F + W_{Mg} + W_{\text{tension}}$$

[symbols have their usual meanings]

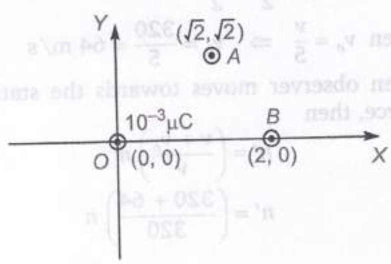
$$W_F = F \times l \sin 45°,$$

$$W_{Mg} = M_g (l - l \cos 45°),$$

$$W_{\text{tension}} = 0$$

$$\therefore \; F = Mg(\sqrt{2} - 1)$$

35. Potential at A due to charge at O



$$V_A = \frac{1}{4\pi\varepsilon_0} \frac{(10^{-3})}{OA}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10^{-3})}{\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{10^{-3}}{2}$$

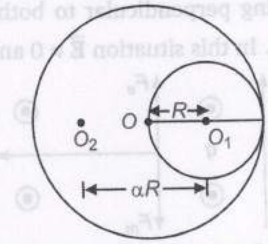
Potential at B due to charge at O

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10^{-3})}{OB}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10^{-3})}{2}$$

So, $V_A - V_B = 0$

36. The distance of centre of mass of new disc from the centre of mass of remaining disc is αR.



Mass of remaining disc $= M - \frac{M}{4} = \frac{3M}{4}$

$$\therefore -\frac{3M}{4} \alpha R + \frac{M}{4} R = 0$$

$$\Rightarrow \qquad \alpha = \frac{1}{3}$$

37. **Key Idea** *AC power gain is ratio of change in output power to the change in input power.*

AC power gain

$$= \frac{\text{Change in output power}}{\text{Change in input power}}$$

$$= \frac{\Delta V_c \times \Delta i_c}{\Delta V_i \times \Delta i_b}$$

$$= \left(\frac{\Delta V_c}{\Delta V_i}\right) \times \left(\frac{\Delta i_c}{\Delta i_b}\right) = A_V \times \beta_{AC}$$

where A_V is voltage gain and $(\beta)_{AC}$ is AC current gain. Also,

$$A_V = \beta_{AC} \times \text{resistance gain} \left(= \frac{R_o}{R_i}\right)$$

Given, $A_V = 50$, $R_o = 200\,\Omega$, $R_i = 100\,\Omega$

Hence, $50 = \beta_{AC} \times \frac{200}{100}$

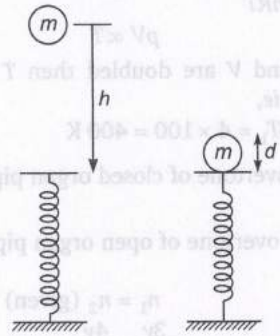
$\therefore \qquad \beta_{AC} = 25$

Now, AC power gain $= A_V \times \beta_{AC}$

$= 50 \times 25 = 1250$

38. **Key Idea** *Work done is equal to change in energy of body.*

Situation is shown in figure. When mass m falls vertically on spring, then spring is compressed by distance d.



Hence, net work done in the process is

W = Potential energy stored in the spring + Loss of potential energy of mass

$$= mg(h+d) - \frac{1}{2}kd^2$$

39. Resistance

$$R = \frac{\text{potential difference}}{\text{current}} = \frac{V}{i} = \frac{W}{qi}$$

($\because$ Potential difference is equal to work done per unit charge)

So, Dimensions of R

$$= \frac{[\text{Dimensions of work}]}{[\text{Dimensions of charge}][\text{Dimensions of current}]}$$

$$= \frac{[ML^2T^{-2}]}{[IT][I]} = [ML^2T^{-3}I^{-2}]$$

40. For adiabatic process,

$$dQ = 0$$

So, $dU = -\Delta W$

$\Rightarrow$ $nC_V dT = +146 \times 10^3$ J

$\Rightarrow$ $\frac{nfR}{2} \times 7 = 146 \times 10^3$

[$f \to$ Degree of freedom]

$\Rightarrow$ $\frac{10^3 \times f \times 8.3 \times 7}{2} = 146 \times 10^3$

$$f = 5.02 \approx 5$$

So, it is a diatomic gas.

41. The accelerating force on the rocket

$$= \text{upward thrust} = \frac{\Delta m}{\Delta t} \cdot u$$

Given, $\frac{\Delta m}{\Delta t} = 50 \times 10^{-3}$ kg/s, $v = 400$ m/s

So, accelerating force $= 50 \times 10^{-3} \times 400$

$$= 20 \text{ N}$$

42. Surface energy = surface tension × surface area

$$E = T \times 2A$$

New surface energy,

$$E_1 = T \times 2\left(\frac{A}{2}\right) = T \times A$$

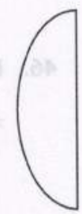
% decrease in surface energy

$$= \frac{E - E_1}{E} \times 100$$

$$= \frac{2TA - TA}{2TA} \times 100$$

$$= 50\%$$

43. Biconvex lens is cut perpendicularly to the principal axis, it will become a plano-convex lens.



Focal length of biconvex lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\frac{2}{R} \quad (\because R_1 = R, R_2 = -R)$$

$\Rightarrow$ $f = \frac{R}{2(n-1)}$ ...(i)

For plano-convex lens

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$f_1 = \frac{R}{(n-1)}$ ...(ii)

Comparing Eqs. (i) and (ii), we see that focal length becomes double.

As power of lens $P \propto \frac{1}{\text{focal length}}$

Hence, power will become half.

$$\text{New power} = \frac{4}{2} = 2 \text{ D}$$

44. Let emf of both cells are E_1 and E_2 and internal resistances are r_1 and r_2. In parallel order, we have

$$E = E_1 = E_2$$

Effective internal resistance of both cells

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r}$$

$\Rightarrow$ $R = \frac{r}{2}$

So, emf is equal to the emf of any of the cell and internal resistance is less than the resistance of any of cell.

Hence, (ii) is right and (i) is wrong.

45. $T = 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow \quad T^2 = 4\pi^2 \frac{l}{g}$

$\Rightarrow \quad g = \frac{4\pi^2 l}{T^2}$

Here % error in

$l = \frac{1 \text{ mm}}{100 \text{ cm}} \times 100 = \frac{0.1}{100} \times 100 = 0.1\%$

and % error in $T = \frac{0.1}{2 \times 100} \times 100 = 0.05\%$

$\therefore$ % error in g = % error in l + 2 (% error in T)

$= 0.1 + 2 \times 0.05$

$= 0.2\%$

46. $U = \frac{GMm}{r}$

$\Rightarrow 7.79 \times 10^{28}$

$= \frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22} \times 6 \times 10^{24}}{r}$

$\Rightarrow \quad r = 3.8 \times 10^8$ m

47. $K = \frac{\Delta p}{\Delta V/V} = \frac{\rho h g}{\Delta V/V}$

$= \frac{200 \times 10^3 \times 10}{0.1/100}$

$= 2 \times 10^9$

48. $pV = nRT$

$\Rightarrow \quad pV \propto T$

If p and V are doubled then T becomes four times *ie*,

$T_2 = 4T_1 = 4 \times 100 = 400$ K

49. First overtone of closed organ pipe $n_1 = \frac{3v}{4l_1}$

Third overtone of open organ pipe $n_2 = \frac{4v}{2l_2}$

$n_1 = n_2$ (given)

$\Rightarrow \quad \frac{3v}{4l_1} = \frac{4v}{2l_2}$

$\frac{l_1}{l_2} = \frac{3}{8}$

50. The atoms in the chromosphere absorb certain wavelengths of light coming from photosphere. This gives rise to absorption lines.

Chemistry

1. Mass of one unit cell (m)

$= \text{volume} \times \text{density}$

$= a^3 \times d = a^3 \times \frac{Mz}{N_0 a^3} = \frac{Mz}{N_0}$

$m = \frac{58.5 \times 4}{6.02 \times 10^{23}}$ g

$\therefore$ Number of unit cells in 1 g $= \frac{1}{m}$

$= \frac{6.02 \times 10^{23}}{58.5 \times 4}$

$= 2.57 \times 10^{21}$

2. $Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$

Gram eq. wt. $= \frac{\text{Gram molecular weight}}{\text{Change in oxidation number per atom}}$

Gram eq. wt. $= \frac{294}{6}$

$= 49$ g

3. $LiAlH_4$ reduces —COOH group to —CH_2OH group without affecting C=C bond.

4.

Molecule	Hybridisation	Repulsion	Bond angle
SO_2	sp^2	$lp - bp, bp - bp$	119°
OH_2	sp^3	$lp - lp, bp - lp$ $bp - bp$	104.5°
SH_2	sp^3	–do–	90°
NH_3	sp^3	$lp - bp, bp - bp$	107°

5. $E_{cell} = E^\circ_{cell} - \frac{0.0591}{n} \log Q$

$\underset{0.1\text{ M}}{Cu^{2+}} + Zn \longrightarrow \underset{1\text{ M}}{Zn^{2+}} + Cu$

$Q = \frac{[Zn^{2+}]}{[Cu^{2+}]} = \frac{1}{0.1} = 10$

$E_{cell} = 1.10 - \frac{0.0591}{2} \log 10$

$= 1.10 - 0.0295 = 1.0705$ V

6. Rate becomes x^y times if concentration is made x times of a reactant giving y^{th} order reaction.

Rate $= k[A]^n[B]^m$

Concentration of A is doubled, hence $x = 2$, $y = n$ and rate becomes $= 2^n$ times

Concentration of B is halved, hence $x = \frac{1}{2}$ and

$y = m$ and rate becomes $= \left(\frac{1}{2}\right)^m$ times

$$\text{Net rate becomes} = (2)^n \left(\frac{1}{2}\right)^m \text{ times}$$

$$= (2)^{n-m} \text{ times}$$

7. $CH_2{=}CH_2 + H_2 \longrightarrow CH_3{-}CH_3$

$\Delta H = (BE)_{reactants} - (BE)_{products}$

$= 4(BE)_{C-H} + (BE)_{C=C} + (BE)_{H-H} - [6(BE)_{C-H} + (BE)_{C-C}]$

$= -125$ kJ

8. As temperature increases desorption increases.

Adsorbent + adsorbate $\rightleftharpoons$ Adsorbed state + ΔE

Adsorption is exothermic process (forward direction), desorption is endothermic process (backward direction).

According to Le-Chatelier's principle increase in temperature favours endothermic process.

9. NH_3, CH_3NH_2, $(CH_3)_2NH$

$$(CH_3)_2NH + H_2O \overset{K_1}{\rightleftharpoons} (CH_3)_2\overset{+}{N}H_2 + OH^-$$

I^+ effect maximum, stabilization more, H-bond stability high, hence most basic

$$CH_3NH_2 + H_2O \overset{K_2}{\rightleftharpoons} CH_3\overset{+}{N}H_3 + OH^-$$

$$NH_3 + H_2O \overset{K_3}{\rightleftharpoons} NH_4^+ + OH^-$$

$$K_1 > K_2 > K_3$$

Hence, basicity order is

$$(CH_3)_2NH > CH_3NH_2 > NH_3$$

10. Nylon threads are made up of polyamides. Some common are

Nylon-6

$H_2N{-}(CH_2)_5{-}\overset{O}{\overset{\|}{C}}{-}OH$ Monomer

$-HN{-}(CH_2)_5{-}\overset{O}{\overset{\|}{C}}{-}NH{-}(CH_2)_5{-}\overset{O}{\overset{\|}{C}}{-}NH-$

amide linkage

Nylon-66

$HO{-}\overset{O}{\overset{\|}{C}}{-}(CH_2)_4{-}COOH$ and $H_2N{-}(CH_2)_6{-}NH_2$

Monomers

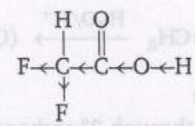
$-HN{-}\overset{O}{\overset{\|}{C}}{-}(CH_2)_4{-}\overset{O}{\overset{\|}{C}}{-}NH{-}(CH_2)_6{-}NH{-}\overset{O}{\overset{\|}{C}}{-}(CH_2)_4{-}\overset{O}{\overset{\|}{C}}-$

amide linkage

11. Free radicals have unpaired electrons, so are neutrals and reactive.

$$\dot{C}H_3 + \dot{C}H_3 \longrightarrow CH_3{-}CH_3$$

12. Fluoro group causes negative inductive effect, thus increasing ionisation, hence 0.1 M difluoroacetic acid has highest electrical conductivity.



13. $Al^{3+} + 3e^- \longrightarrow Al$

$$w = zQ$$

where, w = amount of metal

$w = 5.12 \text{ kg} = 5.12 \times 10^3$ g

z = electrochemical equivalent

$$z = \frac{\text{Equivalent weight}}{96500} = \frac{\text{Atomic mass}}{\text{Electrons} \times 96500}$$

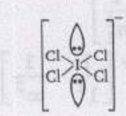
$$z = \frac{27}{3 \times 96500}$$

$$5.12 \times 10^3 = \frac{27}{3 \times 96500} \times Q$$

$$Q = \frac{5.12 \times 10^3 \times 3 \times 96500}{27} \text{ C}$$

$$= 5.49 \times 10^7 \text{ C}$$

14.

Molecule	Structure	Hybridisation of central atom	Lone pair
SF_4		sp^3d	one
SiF_4		sp^3	zero
ICl_4^-		sp^3d^2	two

15. $Hg_2Cl_2 + 2NH_3 \longrightarrow \underset{\text{white}}{HgNH_2Cl} + \underset{\text{black}}{Hg} + NH_4Cl$

16. It is Corey-House synthesis of alkanes.

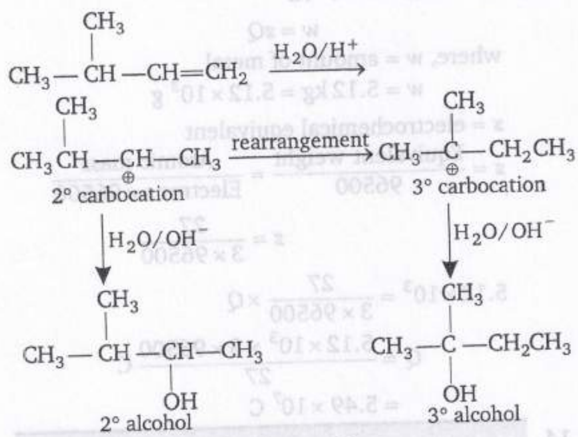
17. $CH_2{=}CH_2 \xrightarrow{H_2O/H^+} \underset{1^\circ \text{ alcohol}}{CH_3CH_2OH}$

$CH_3{-}CH{=}CH_2 \xrightarrow{H_2O/H^+} CH_3\underset{\displaystyle OH}{\underset{|}{C}}HCH_3$

2° alcohol through 2° carbocation $(CH_3\overset{\oplus}{C}HCH_3)$

$CH_3{-}\underset{\displaystyle CH_3}{\underset{|}{C}}{=}CH_2 \xrightarrow{H_2O/H^+} (CH_3)_3COH$

3° alcohol through 3° carbocation $[(CH_3)_3\overset{\oplus}{C}]$



Thus, best alternate is (c)

18. Equivalent weight of oxalic acid = 126/2

($\because$ Basicity of oxalic acid = 2)

Normality of oxalic acid solution $= \dfrac{6.3 \times 1000}{63 \times 250}$

$= 0.4\ N$

$\underset{(\text{Acid})}{N_1V_1} = \underset{(\text{Base})}{N_2V_2}$

$0.4 \times 10\ \text{mL} = 0.1 \times V_2$

$V_2 = 40\ \text{mL}$

19. $\dfrac{1}{\lambda} = \bar{v}_H = \bar{R}_H \left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$= 1.097 \times 10^7 \left[\dfrac{1}{1^2} - \dfrac{1}{\infty^2}\right]$

$\therefore \quad \lambda = \dfrac{1}{1.097 \times 10^7}\ \text{m}$

$= 9.11 \times 10^{-8}\ \text{m}$

$= 91.1 \times 10^{-9}\ \text{m}$

$= 91.1\ \text{nm}$ $\quad (1\ \text{nm} = 10^{-9}\ \text{m})$

20. Boiling point

$= T_0\,(\text{Solvent}) + \Delta T_b\,(\text{Elevation in b. p.})$

$\Delta T_b = mik_b$

where, m is the molality

ie, the van't Hoff factor (i)

$= [1 + (y-1)x]$

k_b = molal elevation constant.

Thus, $\quad \Delta T_b \propto im$

Assume 100% ionisation

(a) $mi\ (Na_2SO_4) = 0.01 \times 3 = 0.03$

(b) $mi\ (KNO_3) = 0.01 \times 2 = 0.02$

(c) mi (urea) = 0.015

(d) mi (glucose) = 0.015

21. F_2 has the most negative ΔG° value which is dependent on hydration enthalpy.

22. In the expression for equilibrium constant (K_p or K_c) species in solid state are not written (*ie*, their molar concentrations are taken as 1)

$P_4(s) + 5O_2(g) \rightleftharpoons P_4O_{10}(s)$

Thus, $\quad K_c = \dfrac{1}{[O_2]^5}$

23. $\text{I}: C(s) + O_2(g) \longrightarrow CO_2(g);$ $\quad \Delta H = -393.5\ \text{kJ}$

$\text{II}: CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g);$ $\quad \Delta H = -283.0\ \text{kJ}$

I and II gives

$\text{III}: C(s) + \dfrac{1}{2}O_2(g) \longrightarrow CO(g);$ $\quad \Delta H = -110.5\ \text{kJ}$

Equation III also represents the enthalpy of formation of 1 mole of CO and thus, enthalpy change is the heat of formation of CO(g).

24. Froth-floatation method is used to concentrate sulphide ores [Galena (PbS)].

25. $CuSO_4 + 2KI \longrightarrow \underset{\text{unstable}}{CuI_2} + K_2SO_4$

$2CuI_2 \longrightarrow Cu_2I_2 + I_2$

Thus, CuI_2 is not formed.

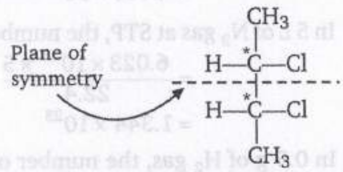
26. (a) $\underset{sp^3}{CH_3}{-}\underset{sp^2}{\overset{\overset{\displaystyle O}{||}}{C}}{-}\underset{sp^3}{CH_3}$ (b) $\underset{sp^3}{CH_3}{-}\underset{sp^2}{\overset{\overset{\displaystyle O}{||}}{C}}{-}OH$

(c) $\underset{sp^3}{CH_3}{-}\underset{sp}{C}{\equiv}N$ (d) $\underset{sp^3}{CH_3}{-}\underset{sp^2}{\overset{\overset{\displaystyle O}{||}}{C}}{-}NH_2$

Acetonitrile does not contain sp^2-hybridised carbon.

27. (a) $CH_3-\overset{*}{C}H(Cl)-CH_2CH_3$

One asymmetric carbon atom, forms *d*- and *l*-optical isomers.



Meso due to internal compensation

(b) Two asymmetric carbon atoms, forms *d*-, *l*- and *meso* forms.

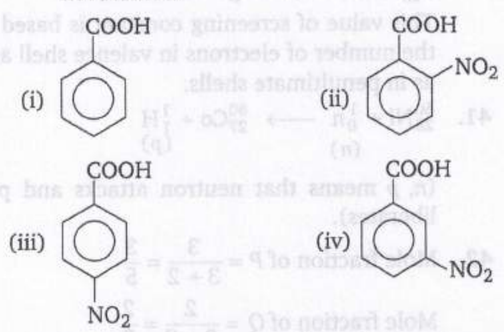
(c) $CH_3-CH(Cl)-CH(Cl)-CH_2CH_3$

Two asymmetric carbon atoms but does not have symmetry. Hence, *meso* form is not formed.

(d) $CH_3-\overset{*}{C}H(OH)-COOH$

One asymmetric carbon atom, *meso* form is not formed.

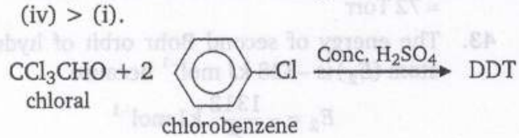
28.



$-NO_2$ group at any position shows electron withdrawing effect, thus acid strength is increased. But *o*-nitro benzoate ion is stabilised by intramolecular H-bonding like forces, hence its acid strength is maximum.

Thus, the order of acid strength is (ii) > (iii) > (iv) > (i).

29.



30. NO, NO_2, SO_2 and SO_3 are responsible for smog (environmental pollution).

31. Half-life $(t_{1/2}) = 12.3$ yr.

Initial amount $(N_0) = 32$ mg

Amount left $(N) = ?$

Total time $(T) = 49.2$ yr

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

where n = total number of half-life

$$n = \frac{\text{Total time}}{\text{Half-life}}$$

$$\frac{49.2}{12.3} = 4$$

So,
$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{N}{32} = \left(\frac{1}{2}\right)^4$$

$$\frac{N}{32} = \frac{1}{16}$$

$$N = \frac{32}{16} = 2 \text{ mg}$$

32. Lactic acid obtained in the given reaction is an optically active compound due to the presence of chiral C-atom. It exists as *d* and *l* forms whose ratio is 1 : 1.

$$\underset{H}{\overset{CH_3}{>}}C{=}O + HCN \longrightarrow$$

$$H-\underset{CN}{\overset{CH_3}{C}}-OH \;+\; HO-\underset{CN}{\overset{CH_3}{C}}-H$$

50% L-isomer 50% D-isomer

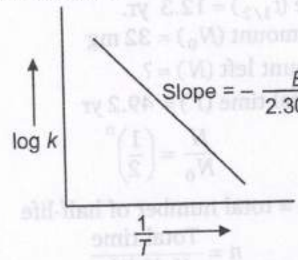
33. The method of zone refining of metals is based on the principle of greater solubility of the impurity in the molten state than in the solid. Elements which are used as semiconductors like Si, Ge, Ga etc are refined by this method.

34. Arrhenius equation, $k = Ae^{-E/RT}$

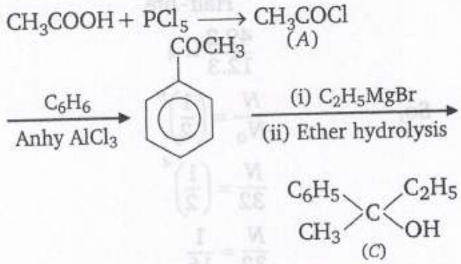
$$\ln k = \ln A - \frac{E}{RT}$$

or
$$\log k = \log A - \frac{E}{2.303\,RT}$$

Hence, E is calculated with the help of slope of following graph



35. $CH_3COOH + PCl_5 \longrightarrow \underset{(A)}{CH_3COCl}$



36. $B(OH)_3$ is not a protonic acid because it does not give proton on ionisation directly while it acts as Lewis acid due to acceptance of OH^- from water and forms a hydrated species.

$$B(OH)_3 + H_2O \longrightarrow [B(OH)_4]^- + H^+$$

37. $NaCl + H_2SO_4 \xrightarrow{\Delta} NaHSO_4 + HCl\uparrow$

$HCl + H_2SO_4 \xrightarrow{\Delta}$ No reaction

$NaBr + H_2SO_4 \xrightarrow{\Delta} NaHSO_4 + HBr$

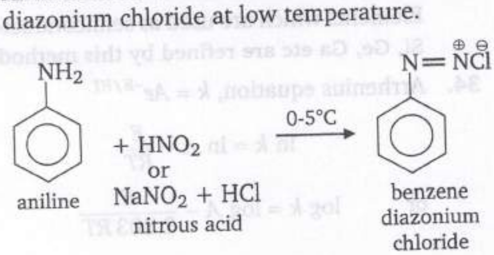
$2HBr + H_2SO_4 \xrightarrow{\Delta} Br_2\uparrow + SO_2 + 2H_2O$

$NaI + H_2SO_4 \xrightarrow{\Delta} NaHSO_4 + HI$

$2HI + H_2SO_4 \xrightarrow{\Delta} I_2\uparrow + SO_2 + 2H_2O$

Hence, the gases released are HCl, Br_2 and I_2.

38. Aniline reacts with nitrous acid to give benzene diazonium chloride at low temperature.



39. In 15 L of H_2 gas at STP, the number of molecules

$$= \frac{6.023 \times 10^{23}}{22.4} \times 15$$

$$= 4.033 \times 10^{23}$$

In 5 L of N_2 gas at STP, the number of molecules

$$= \frac{6.023 \times 10^{23} \times 5}{22.4}$$

$$= 1.344 \times 10^{23}$$

In 0.5 g of H_2 gas, the number of molecules

$$= \frac{6.023 \times 10^{23} \times 0.5}{2}$$

$$= 1.505 \times 10^{23}$$

In 10 g of O_2 gas, the number of molecules

$$= \frac{6.023 \times 10^{23} \times 10}{32}$$

$$= 1.882 \times 10^{23}$$

Hence, maximum molecules are present in 15 L of H_2 at STP.

40. Ionic radii $\propto \frac{1}{Z_{eff}}$

Z_{eff} = Effective nuclear charge

This Z_{eff} is calculated as follows:

$Z_{eff} = Z -$ screening constant (σ)

This value of screening constant is based upon the number of electrons in valence shell as well as in penultimate shells.

41. $^{60}_{28}Ni + \underset{(n)}{^{1}_{0}n} \longrightarrow {}^{60}_{27}Co + \underset{(p)}{^{1}_{1}H}$

(n, p means that neutron attacks and proton liberates).

42. Mole fraction of $P = \frac{3}{3+2} = \frac{3}{5}$

Mole fraction of $Q = \frac{2}{3+2} = \frac{2}{5}$

Hence, total vapour pressure

= Mole fraction of $P \times$ V. P. of P

+ mole fraction of $Q \times$ V. P. of Q

$$= \left(\frac{3}{5} \times 80 + \frac{2}{5} \times 60\right) = 48 + 24$$

$$= 72 \text{ Torr}$$

43. The energy of second Bohr orbit of hydrogen atom (E_2) is -328 kJ mol^{-1} because

$$E_2 = -\frac{1312}{2^2} \text{ kJ mol}^{-1}$$

$\therefore \quad E_n = -\dfrac{1312}{n^2}\ \text{kJ mol}^{-1}$

If $n = 4$

$\therefore \quad E_4 = -\dfrac{1312}{4^2}\ \text{kJ mol}^{-1}$

$= -82\ \text{kJ mol}^{-1}$

44. van der Waals' equation for one mole of a gas is

$$\left[p + \frac{a}{V^2}\right](V - b) = RT$$

b is the volume correction. It arises due to finite size of molecules.

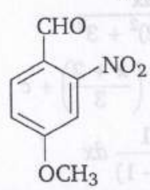
45. Freons or chlorofluoro carbons are responsible for depletion of ozone layer in the upper strata of the atmosphere. They are used as propellants, aerosol spray caps, refrigerents, fire fighting reagents etc. They are stable and chemically inert compounds. They absorb UV-radiation and break down liberating free atomic chlorine which causes decomposition of ozone through free radical reaction. This results in the depletion of the ozone layer.

Freons are mainly freon-1 ($CFCl_3$) and freon-12 (CF_2Cl_2). They form free radical of chlorine in presence of UV-radiation. Such free radical decomposes O_3 as follows :

$$Cl^{\bullet} + O_3 \longrightarrow ClO^{\bullet} + O_2$$

$$ClO^{\bullet} + O_3 \longrightarrow \underset{\text{chlorine free radical}}{Cl^{\bullet}} + 2O_2$$

46.



4-methoxy-2-nitrobenzaldehyde

47. Chlorine heptachloride (Cl_2O_7) is the anhydride of perchloric acid.

$$2HClO_4 \xrightarrow{\Delta} Cl_2O_7 + H_2O$$

48. "10 volume H_2O_2" means 1 mL of its solution on decomposition at NTP, give 10 mL oxygen gas. Volume of oxygen formed from 100 mL of solution at NTP = 1000 mL

$$\underset{\substack{2\text{ mol} \\ 2 \times 34\text{ g}}}{2H_2O_2} \longrightarrow 2H_2O + \underset{\substack{1\text{ mol} \\ 22400\text{ mL}}}{O_2}$$

$\because$ 22400 mL O_2 formed at NTP by decomposition of 68 g H_2O_2.

$\therefore$ 1 mL O_2 formed at NTP from $\dfrac{68}{22400}$ g of H_2O_2

$\therefore$ 1000 mL O_2 formed at NTP from

$\dfrac{68 \times 1000}{22400}$ g H_2O_2 = 3.035 g H_2O_2

So, concentration of "10 volume H_2O_2"

= 3.0% approximately

49. Charge of electron $= 1.6 \times 10^{-19}$ C

Dipole moment of HBr $= 1.6 \times 10^{-30}$

Inter-atomic spacing = 1 Å

$= 1 \times 10^{-10}$ m

% of ionic character in HBr

$$= \frac{\text{dipole moment of HBr} \times 100}{\text{inter spacing distance} \times q}$$

$$= \frac{1.6 \times 10^{-30}}{1.6 \times 10^{-19} \times 10^{-10}} \times 100$$

$= 10^{-30} \times 10^{29} \times 100$

$= 10^{-1} \times 100$

$= 0.1 \times 100$

$= 10\%$

50. When calcium carbide reacts with nitrogen at 1000°C, calcium cyanamide and carbon is formed.

$$CaC_2 + N_2 \xrightarrow{1000°C} \underset{\text{calcium cyanamide}}{CaCN_2} + C$$

Mathematics

1. **Key Idea**

$$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{a}\cdot(|\vec{b}||\vec{c}|\sin\theta\,\hat{n})$$

Given that, $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 4$

$$\therefore \quad [\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot\left(|\vec{b}||\vec{c}|\sin\frac{2\pi}{3}\,\hat{n}\right)$$

$$= |\vec{a}||\vec{b}||\vec{c}|\left(\sin\frac{2\pi}{3}\right)$$

$$[\because \vec{a}\cdot\hat{n} = |\vec{a}||\hat{n}|\cos 0^\circ = |\vec{a}|]$$

$$= 2\times 3\times 4\times\frac{\sqrt{3}}{2}$$

$$= 12\sqrt{3}$$

2. Given that, $y^2dx + (x^2 - xy + y^2)\,dy = 0$

$$\Rightarrow \quad dx + \frac{x^2 - xy + y^2}{y^2}\,dy = 0$$

$$\Rightarrow \quad \frac{dx}{dy} + \left(\frac{x}{y}\right)^2 - \left(\frac{x}{y}\right) + 1 = 0$$

Let $v = \frac{x}{y} \Rightarrow x = vy$

$$\Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\therefore \quad v + y\frac{dv}{dy} + v^2 - v + 1 = 0$$

$$\Rightarrow \quad y\frac{dv}{dy} = -(v^2 + 1)$$

$$\Rightarrow \quad \frac{dv}{v^2 + 1} + \frac{dy}{y} = 0$$

On integrating, we get

$$\tan^{-1} v + \log y + c = 0$$

$$\Rightarrow \quad \tan^{-1}\frac{x}{y} + \log y + c = 0$$

3. We have, $2|\hat{i} + x\hat{j} + 3\hat{k}|$

$$= |4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}|$$

$$\Rightarrow 2\sqrt{1 + x^2 + 9} = \sqrt{4^2 + (4x - 2)^2 + 2^2}$$

$$\Rightarrow \quad 4(x^2 + 10) = 16 + 16x^2 + 4 - 16x + 4$$

$$\Rightarrow \quad 12x^2 - 16x - 16 = 0$$

$$\Rightarrow \quad 3x^2 - 4x - 4 = 0$$

$$\Rightarrow \quad (3x + 2)(x - 2) = 0$$

$$\Rightarrow \quad x = 2, -\frac{2}{3}$$

4. **Key Idea** *Resultant of two forces P and Q making an angle α with each other, is*

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

Since, forces of magnitude 30, 60 and P are in equilibrium, then magnitude of P is equal to the magnitude of resultant of forces of magnitude 30 and 60.

$$\therefore \quad P = \sqrt{(30)^2 + (60)^2 + 2(30)(60)\cos 60^\circ}$$

$$= \sqrt{900 + 3600 + 3600\times\frac{1}{2}}$$

$$= \sqrt{4500 + 1800} = \sqrt{6300} = 30\sqrt{7}$$

5. Given equation $\frac{d^2y}{dx^2} = e^{-2x}$

On integrating both sides

$$\int\frac{d^2y}{dx^2}\,dx = \int e^{-2x}\,dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$$

Again integrating, we get

$$y = \frac{e^{-2x}}{4} + cx + d$$

6. Let $I = \int\frac{dx}{x^2 + 4x + 13}$

$$= \int\frac{dx}{x^2 + 4x + 4 + 9}$$

$$= \int\frac{dx}{(x + 2)^2 + 3^2}$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x + 2}{3}\right) + c$$

7. Let $I = \int_2^3\frac{x + 1}{x^2(x - 1)}\,dx$

$$= \int_2^3\left(\frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x - 1}\right)dx$$

$$= \left[-2\log x + \frac{1}{x} + 2\log(x - 1)\right]_2^3$$

$$= \left[2\log\left(\frac{x - 1}{x}\right) + \frac{1}{x}\right]_2^3$$

$$= \left[2\left(\log\frac{2}{3} - \log\frac{1}{2}\right) + \frac{1}{3} - \frac{1}{2}\right]$$

$$= 2\log\frac{4}{3} - \frac{1}{6} = \log\frac{16}{9} - \frac{1}{6}$$

8. Let $I = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x)\,dx$

$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4} + [\sin x + \cos x]_{2\pi}^{\pi/4}$

$= \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - (\sin 0 + \cos 0)\right] - \left[\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4} - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right] + \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - (\sin 2\pi + \cos 2\pi)\right]$

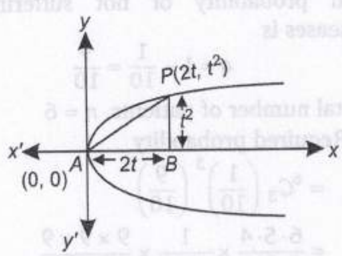
$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] - \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] + \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$

$= \left[\frac{2}{\sqrt{2}} - 1\right] - \left[-\frac{4}{\sqrt{2}}\right] + \left[\frac{2}{\sqrt{2}} - 1\right]$

$= (\sqrt{2} - 1) + 2\sqrt{2} + (\sqrt{2} - 1)$

$= 4\sqrt{2} - 2.$

9. Let A be the vertex of the parabola and AP is chord of parabola such that slope of AP is $\cot\alpha$.



Let coordinates of P be $(2t, t^2)$ which is a point on the parabola.

$\therefore$ Slope of $AP = \frac{t}{2}$

$\Rightarrow \quad \cot\alpha = \frac{t}{2}$

$\Rightarrow \quad t = 2\cot\alpha$

In ΔAPB, $\quad AP = \sqrt{4t^2 + t^4}$

$= t\sqrt{4 + t^2}$

$\therefore AP = 2\cot\alpha\sqrt{4(1 + \cot^2\alpha)}$

$= 2\cot\alpha\sqrt{4\operatorname{cosec}^2\alpha} = 4\cot\alpha\operatorname{cosec}\alpha$

$= 4\frac{\cos\alpha}{\sin\alpha}\operatorname{cosec}\alpha = 4\cos\alpha\operatorname{cosec}^2\alpha$

10. Let the equation of tangent, which is perpendicular to the line $3x + 4y = 7$, is $4x - 3y = \lambda$.

Since, it is a tangent to the ellipse.

$\therefore \quad \lambda^2 = a^2m^2 + b^2$

Here, $a^2 = 9$, $b^2 = 4$ and $m = \frac{4}{3}$.

$\therefore \quad \lambda^2 = 9 \times \left(\frac{4}{3}\right)^2 + 4$

$= 16 + 4$

$\Rightarrow \quad \lambda^2 = 20$

$\Rightarrow \quad \lambda = \pm\sqrt{20}$

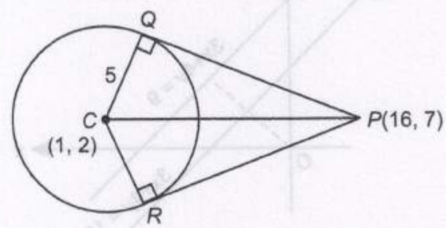
$\therefore$ Equation is $4x - 3y = \pm\sqrt{20}$

11. **Key Idea** *If tangents are drawn from a point to the circle, then quadrilateral PQCR makes two equal right triangles.*

Given, equation of circle is

$$x^2 + y^2 - 2x - 4y - 20 = 0$$

$\therefore$ Centre is $(1, 2)$ and radius,



$r = \sqrt{1^2 + 2^2 + 20}$

$= \sqrt{25} = 5$

Now, $\quad PC = \sqrt{(16 - 1)^2 + (7 - 2)^2}$

$= \sqrt{225 + 25}$

$= \sqrt{250}$

In ΔPCQ,

$PQ = \sqrt{PC^2 - QC^2}$

$= \sqrt{(\sqrt{250})^2 - (5)^2}$

$= \sqrt{250 - 25}$

$= \sqrt{225}$

$= 15$

$\therefore$ Area of quadrilateral $PQCR$

$= 2 \cdot$ area of ΔPCQ

$= 2 \cdot \frac{1}{2} PQ \cdot QC$

$= 1 \cdot 15 \cdot 5$

$= 75$ sq unit

12. **Key Idea** *If coefficient of x and y of both the lines are same, then the lines are parallel.*

Given, equation of lines are

$$3x + 4y = 9 \quad \ldots(i)$$

and $6x + 8y = 15$

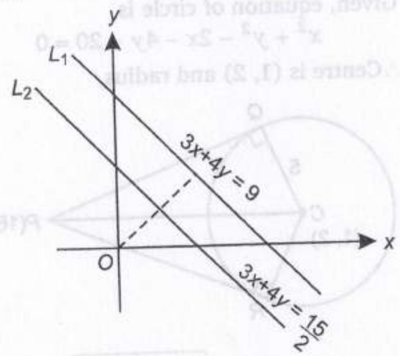
$\Rightarrow$ $3x + 4y = \frac{15}{2} \quad \ldots(ii)$

$\therefore$ Both lines are parallel, therefore the distance between two lines

$$= \frac{\left|\frac{15}{2} - 9\right|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{|15 - 18|}{2\sqrt{25}} = \frac{3}{2 \cdot 5} = \frac{3}{10}$$

Alternative The perpendicular distance from origin to the line L_1 is



$$d_1 = \frac{9}{\sqrt{3^2 + 4^2}} = \frac{9}{5}$$

and $d_2 = \frac{\frac{15}{2}}{\sqrt{3^2 + 4^2}}$

$$= \frac{15}{2 \cdot 5} = \frac{15}{10}$$

$\therefore$ Distance between L_1 and L_2 is

$$d = d_1 - d_2$$

$$= \frac{9}{5} - \frac{15}{10} = \frac{18 - 15}{10}$$

$$= \frac{3}{10}$$

13. $\cot A + \cot B$

$$= \frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}$$

$$= \frac{\cos A \sin B + \cos B \sin A}{\sin A \sin B}$$

$$= \frac{\sin (A + B)}{\sin A \sin B}$$

$$= \frac{\sin (\pi - C)}{\sin A \sin B} \quad (\because A + B + C = \pi)$$

$$= \frac{\sin C}{\sin A \sin B}$$

$$= \frac{1}{\sin A \sin B} \quad (\because C = 90°) \quad \ldots(i)$$

Applying sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$\Rightarrow$ $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{1}{c} \quad (\because \sin 90° = 1)$

$\Rightarrow$ $\sin A = \frac{a}{c}, \quad \sin B = \frac{b}{c}$

From Eq. (i),

$$\cot A + \cot B = \frac{1}{\frac{a}{c} \cdot \frac{b}{c}}$$

$$= \frac{c^2}{ab}$$

14. The probability of suffering from a diseases is 10%.

ie, $p = \frac{10}{100} = \frac{1}{10}$

and probability of not suffering from a diseases is

$$q = 1 - \frac{1}{10} = \frac{9}{10}$$

Total number of patients, $n = 6$

$\therefore$ Required probability

$$= {}^6C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^3$$

$$= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \times \frac{1}{1000} \times \frac{9 \times 9 \times 9}{1000}$$

$$= \frac{20}{10^6} \times 729 = \frac{2}{10^5} \times 729 = 1458 \times 10^{-5}$$

15. **Key Idea** *In out of 40 consecutive numbers, 20 are odd and 20 are even numbers.*

Now, the sum of two numbers is odd only when one is odd and other is even.

$\therefore$Required probability $= \frac{{}^{20}C_1 \cdot {}^{20}C_1}{{}^{40}C_2}$

$$= \frac{20 \times 20}{\frac{40 \times 39}{2 \times 1}} = \frac{20 \times 20}{20 \times 39}$$

$$= \frac{20}{39}$$

16. Key Idea *If z is purely imaginary, then real coefficient will be zero.*

Let $z = x + iy$

$$\therefore \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$$

$$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{(x-1)(x+1) - iy(x-1) + iy(x+1) - i^2y^2}{(x+1)^2 - i^2y^2}$$

$$= \frac{x^2 - 1 - iyx + iy + iyx + iy + y^2}{(x+1)^2 + y^2}$$

$$= \frac{x^2 - 1 + y^2 + 2iy}{(x+1)^2 + y^2}$$

$$= \frac{x^2 - 1 + y^2}{(x+1)^2 + y^2} + \frac{2y}{(x+1)^2 + y^2} i$$

Since, it is purely imaginary, then

$$\frac{x^2 - 1 + y^2}{(x+1)^2 + y^2} = 0$$

$$\Rightarrow \quad x^2 + y^2 - 1 = 0$$

$$\Rightarrow \quad |z| = 1 \quad (\because |z| = x^2 + y^2)$$

17. Since, α, β are the roots of the equation $lx^2 + mx + n = 0$.

$$\therefore \quad \alpha + \beta = -\frac{m}{l}, \quad \alpha\beta = \frac{n}{l}$$

Now, $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$

$$= \alpha\beta[(\alpha+\beta)^2 - 2\alpha\beta]$$

$$= \frac{n}{l}\left[\left(\frac{-m}{l}\right)^2 - \frac{2n}{l}\right] = \frac{n}{l}\left[\frac{m^2}{l^2} - \frac{2n}{l}\right]$$

and $\alpha^3\beta \cdot \alpha\beta^3 = (\alpha\beta)^4 = \frac{n^4}{l^4}$

$\therefore$ Required quadratic equation is

$$x^2 - (\alpha^3\beta + \alpha\beta^3)x + \alpha^3\beta \cdot \alpha\beta^3 = 0$$

$$\Rightarrow \quad x^2 - \frac{n}{l}\left[\frac{m^2}{l^2} - \frac{2n}{l}\right]x + \frac{n^4}{l^4} = 0$$

$$\Rightarrow \quad l^4x^2 - nl(m^2 - 2nl)x + n^4 = 0$$

18. Let $S = 2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \ldots$

$$= 2^{1/4} \cdot 2^{2/8} \cdot 2^{3/16} \cdot 2^{4/32} \ldots$$

$$= 2^{\left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \ldots\right)}$$

$$= 2^{\frac{1}{4}\left(1 + 2\frac{1}{2} + 3\frac{1}{2^2} + 4\frac{1}{2^3} + \ldots\right)}$$

Let $S_1 = 1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + 4 \cdot \frac{1}{2^3} + \ldots$ ...(i)

$$\therefore \quad \frac{1}{2}S_1 = \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \cdot \frac{1}{2^3} + \ldots \quad \text{...(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$\frac{1}{2}S_1 = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots$$

$$= \frac{1}{1 - \frac{1}{2}}$$

$$\Rightarrow \quad \frac{1}{2}S_1 = 2$$

$$\Rightarrow \quad S_1 = 4$$

$$\therefore \quad S = 2^{\frac{1}{4}(4)} = 2$$

19. Key Idea *The boys in majority, if boys are more than girls.*

The boys are in majority, if the groups are

4B 3G, 5B 2G, 6B 1G.

Total number of combinations

$$= {}^6C_4 \times {}^4C_3 + {}^6C_5 \times {}^4C_2 + {}^6C_6 \times {}^4C_1$$

$$= 15 \times 4 + 6 \times 6 + 1 \times 4$$

$$= 60 + 36 + 4 = 100$$

20. Key Idea *The domain of* $\sin^{-1} x$ *is* $[-1, 1]$.

Since, domain of $\sin^{-1} x$ is $[-1, 1]$.

$$\therefore \quad \sin^{-1}\left[\log_2 \frac{x^2}{2}\right]$$

$$\Rightarrow \quad -1 \le \log_2 \frac{x^2}{2} \le 1$$

$$\Rightarrow \quad 2^{-1} \le \frac{x^2}{2} \le 2$$

$$\Rightarrow \quad \frac{1}{2} \le \frac{x^2}{2} \le 2$$

$$\Rightarrow \quad 1 \le x^2 \le 4$$

$$\Rightarrow \quad |x| \le 2 \text{ and } |x| \ge 1$$

$$\Rightarrow \quad x \in \{[-2, 2] - (-1, 1)\}.$$

21. Key Idea *Every polynomial function is continuous.*

Given, function is

$$f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \ge 2 \end{cases}$$

Since, it is a polynomial function, so it is continuous for every value of x except at $x = 2$.

At $x = 2$, LHL $= \lim_{x \to 2^-} x - 1$

$$= \lim_{h \to 0} 2 - h - 1 = 1$$

$$\text{RHL} = \lim_{x \to 2^+} 2x - 3$$
$$= \lim_{h \to 0} 2(2 + h) - 3 = 1$$

and $f(2) = 2(2) - 3 = 1$

$\therefore$ LHL = RHL = $f(2)$

Thus, $f(x)$ is continuous for all real values of x.

22. Let $y = \sqrt{\sec \sqrt{x}}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2}(\sec \sqrt{x})^{-1/2} \cdot \frac{d}{dx}(\sec \sqrt{x})$$
$$= \frac{1}{2\sqrt{\sec \sqrt{x}}} \cdot \sec\sqrt{x} \cdot \tan \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$
$$= \frac{1}{4\sqrt{x}}(\sec \sqrt{x})^{1/2} \frac{\sin \sqrt{x}}{\cos \sqrt{x}}$$
$$= \frac{1}{4\sqrt{x}}(\sec \sqrt{x})^{1/2} \cdot \sin \sqrt{x} \cdot \sec \sqrt{x}$$
$$= \frac{1}{4\sqrt{x}}(\sec \sqrt{x})^{3/2} \cdot \sin \sqrt{x}$$

23. Let $f(x) = x^5 - 5x^4 + 5x^3 - 1$

On differentiating w.r.t. x, we get

$$f'(x) = 5x^4 - 20x^3 + 15x^2$$

For maximum or minimum, put $f'(x) = 0$

$\Rightarrow \quad 5x^4 - 20x^3 + 15x^2 = 0$

$\Rightarrow \quad x^2(5x^2 - 20x + 15) = 0$

$\Rightarrow \quad 5x^2(x - 1)(x - 3) = 0$

$\Rightarrow \quad x = 0, 1, 3$

Again, differentiating w.r.t. x, we get

$$f''(x) = 5(4x^3 - 12x^2 + 6x)$$

At $x = 1$, $f''(1) = 5(4 - 12 + 6)$
$= -10 < 0$, maximum.

At $x = 3$, $f''(3) = 5(4 \times 27 - 12 \times 9 + 6 \times 3)$
$= 90 > 0$, minimum.

At $x = 0$, $f''(0) = 5(0 - 12 \times 0 + 6 \times 0) = 0$, we have further check

$$f'''(x) = 5(12x^2 - 24x + 6)$$

$\Rightarrow \quad f'''(0) = 30 \neq 0 \quad$ (Inflexion)

$\therefore f(x)$ is maximum at $x = 1$ and minimum at $x = 3$.

24. Key Idea *If two planes are perpendicular, then the sum of the product of two direction ratios will be zero.*

Given two planes are

$$2x + y - 2z = 0$$

and $$x + 2y + kz = 0$$

Since, two planes are perpendicular, then

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\Rightarrow \quad 2 \cdot 1 + 1 \cdot 2 - 2(k) = 0$

$\Rightarrow \quad -2k + 4 = 0$

$\Rightarrow \quad k = 2$

25. Key Idea *The coordinate of x in yz-plane is 0.*

Let the point R divides the line joining the points $P(2, 4, 5)$ and $Q(3, 5, -4)$ in the ratio $m : n$.

$P(2, 4, 5)$ —— m —— R —— n —— $Q(3, 5, -4)$

$\therefore$ The coordinate of R is

$$\left(\frac{3m + 2n}{m + n}, \frac{5m + 4n}{m + n}, \frac{-4m + 5n}{m + n}\right)$$

Since, the point R is on yz-plane, therefore x-coordinate will be zero.

$\therefore \quad \frac{3m + 2n}{m + n} = 0$

$\Rightarrow \quad 3m + 2n = 0 \Rightarrow 3m = 2n$

$\Rightarrow \quad \frac{m}{n} = -\frac{2}{3}$

26. Given, equation of the circles are

$S_1 \equiv x^2 + y^2 - 16x + 60 = 0$ ...(i)

$S_2 \equiv x^2 + y^2 - 12x + 27 = 0$ ...(ii)

and $S_3 \equiv x^2 + y^2 - 12y + 8 = 0$ ...(iii)

The radical axis of circles (i) and (ii) is

$$S_1 - S_2 = 0$$

$\Rightarrow \quad (x^2 + y^2 - 16x + 60) - (x^2 + y^2 - 12x + 27) = 0$

$\Rightarrow \quad -4x + 33 = 0$

$\Rightarrow \quad x = \frac{33}{4}$ ...(iv)

The radical axis of circles (ii) and (iii) is

$$S_2 - S_3 = 0$$

$\Rightarrow \quad (x^2 + y^2 - 12x + 27) - (x^2 + y^2 - 12y + 8) = 0$

$\Rightarrow \quad -12x + 12y + 19 = 0$ ...(v)

On solving Eqs. (iv) and (v), we get radical centre $\left(\frac{33}{4}, \frac{20}{3}\right)$.

27. Key Idea *If three lines are concurrent, then the intersecting point of two lines lies on the third line.*

Given equation of lines are

$3x + 4y + 1 = 0$ ...(i)

$5x + \lambda y + 3 = 0$ ...(ii)

and $2x + y - 1 = 0$ ...(iii)

The intersecting point of lines (i) and (iii) is (1, – 1).

Since, the lines are concurrent, therefore the intersecting point (1, – 1) lies on line (ii).

$\therefore \quad 5(1) + \lambda(-1) + 3 = 0$

$\Rightarrow \quad \lambda = 8$

Alternative Since, the given lines are concurrent.

$$\therefore \begin{vmatrix} 3 & 4 & 1 \\ 5 & \lambda & 3 \\ 2 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow 3(-\lambda - 3) - 4(-5 - 6) + 1(5 - 2\lambda) = 0$

$\Rightarrow \quad -3\lambda - 9 + 20 + 24 + 5 - 2\lambda = 0$

$\Rightarrow \quad -5\lambda + 40 = 0$

$\Rightarrow \quad \lambda = 8$

28. We know that, $h = ut + \frac{1}{2} gt^2$

$\because \quad u = 0$

$\therefore \quad h = \frac{1}{2} gt^2$

$= \frac{1}{2} \times 10 \times (3)^2$

$= 5 \times 9$

$= 45 \text{ m}$

29. **Key Idea** *In a given series there is a factorial in denominator, therefore it may be in the form of exponential series.*

$\therefore 1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 2)^3}{3!} + \dots$

$= e^{-\log 2}$

$= e^{\log 2^{-1}}$

$= 2^{-1} = \frac{1}{2}$

30. Given, $f(x) = \frac{x}{4 + x + x^2}$

Let $f(x) = \frac{1}{u}$

$\therefore \quad u = \frac{4 + x + x^2}{x}$

$= \frac{4}{x} + 1 + x$

$\frac{du}{dx} = -\frac{4}{x^2} + 1, \frac{d^2u}{dx^2} = \frac{8}{x^3}$

For maximum or minimum, put $\frac{du}{dx} = 0$

$\Rightarrow \quad 1 - \frac{4}{x^2} = 0$

$\Rightarrow \quad x^2 - 4 = 0$

$\Rightarrow \quad x = \pm 2$

$\therefore$ At $x = \pm 2$, $\frac{d^2u}{dx^2} = \frac{8}{(\pm 2)^2} < 0$, maximum

It is a decreasing function.

$\Rightarrow f(x)$ is an increasing function $[\because f(x) = \frac{1}{u}]$

$\Rightarrow f(x)$ is an increasing function in $[-1, 1]$.

$\therefore$ The maximum value at $x = 1$ is

$$f(x) = \frac{1}{4 + 1 + 1} = \frac{1}{6}$$

Note If u is increasing, then reciprocal of u is decreasing.

31. Let $I = \int \frac{e^x}{(2 + e^x)(e^x + 1)} dx$

Put $\quad e^x = t$

$\Rightarrow \quad e^x dx = dt$

$\therefore \quad I = \int \frac{dt}{(2 + t)(t + 1)}$

$= \int \left[\frac{1}{(1 + t)} - \frac{1}{(2 + t)} \right] dt$

$= \log(1 + t) - \log(2 + t) + c$

$= \log(1 + e^x) - \log(2 + e^x) + c$

$= \log\left(\frac{1 + e^x}{2 + e^x} \right) + c$

32. Given, $\frac{dr}{dt} = 2 \text{ cm/s}$, where r be radius of circle and t be the time.

Now, area of circle is given by $A = \pi r^2$.

On differentiating w.r.t. t, we get

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\Rightarrow \quad \frac{dA}{dt} = 2\pi \cdot 20 \cdot 2$

$\Rightarrow \quad \frac{dA}{dt} = 80 \pi \text{ cm}^2/\text{s}$

Thus, the rate of change of area of circle with respect to time is $80 \pi \text{ cm}^2/\text{s}$.

33. The given equation of parabola is

$y^2 - x - 2y + 2 = 0$

$\Rightarrow \quad y^2 - 2y + 1 - x + 1 = 0$

$\Rightarrow \quad (y - 1)^2 = x - 1$

$\Rightarrow \quad Y^2 = X$, where $Y = y - 1$, $X = x - 1$

Here, $a = \frac{1}{4}$

$\therefore$ Focus is $(a, 0)$ *ie*, $\left(\frac{1}{4}, 0\right)$

$\Rightarrow \quad X = \frac{1}{4}, Y = 0$

$\Rightarrow \quad x - 1 = \frac{1}{4}, \ y - 1 = 0$

$\Rightarrow \quad x = \frac{5}{4}, y = 1$

$\therefore$ Required focus is $\left(\frac{5}{4}, 1\right)$.

34. **Key Idea** *The equation of the normal at the point* (x_1, y_1) *to the ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ *is*

$$\frac{x - x_1}{x_1 / a^2} = \frac{y - y_1}{y_1 / b^2}.$$

The equation of ellipse is

$$9x^2 + 5y^2 = 45$$

or $\quad \frac{x^2}{5} + \frac{y^2}{9} = 1$

Here, $a^2 = 5, \ b^2 = 9$

The equation of normal to the ellipse at the point (0, 3) is

$$\frac{x - 0}{0/5} = \frac{y - 3}{3/9}$$

$\Rightarrow \quad x - 0 = 0$

$\Rightarrow \quad x = 0$

Which is the equation of y-axis.

Alternative Given equation is

$$9x^2 + 5y^2 = 45$$

On differentiating, we get

$$18x + 10y\frac{dy}{dx} = 0$$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{18x}{10y}$

At (0, 3), $\left(\frac{dy}{dx}\right) = \frac{-18(0)}{10(3)} = 0$

$\therefore$ Equation of normal is

$$y - 3 = -\frac{1}{0}(x - 0)$$

$\Rightarrow \quad x = 0$

$\Rightarrow \quad y$-axis.

35. The equation of given straight line is

$$y - x + 5 = 0$$

$\Rightarrow \quad y = x - 5 \quad \ldots(i)$

The equation of any straight line parallel to the given straight line will be

$$y = x + c \qquad (\because m = 1) \quad \ldots(ii)$$

This straight line will be tangent to the given hyperbola

$$\frac{x^2}{3} - \frac{y^2}{2} = 1 \quad \ldots(iii)$$

Here, $a^2 = 3, b^2 = 2$,

$$c^2 = a^2m^2 - b^2$$

$\Rightarrow \quad c^2 = 3 \cdot 1 - 2$

$\Rightarrow \quad c^2 = 1 \Rightarrow c = \pm 1$

Hence, the equation of the required tangent will be

$$y = x \pm 1$$

$\Rightarrow \ y - x - 1 = 0$ or $y - x + 1 = 0$

$\Rightarrow \ x - y + 1 = 0$ or $x - y - 1 = 0$

36. Given that, $f(x) = x^2 - 1, g(x) = \sqrt{(x^2 + 1)}$

and $\quad h(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

$\therefore \quad ho(fog)(x) = hof\{g(x)\}$

$= hof\{\sqrt{(x^2 + 1)}\}$

$= h\{(\sqrt{x^2 + 1})^2 - 1\}$

$= h\{x^2 + 1 - 1\}$

$= h\{x^2\}$

$= x^2$

37. **Key Idea** *The argument of z is* $\tan^{-1}\frac{y}{x}$.

Let $\quad z = \frac{13 - 5i}{4 - 9i} \times \frac{4 + 9i}{4 + 9i}$

$= \frac{52 + 117i - 20i - 45i^2}{(4)^2 - (9i)^2}$

$= \frac{52 + 97i + 45}{16 + 81}$

$= \frac{97 + 97i}{97}$

$\Rightarrow \quad z = 1 + i$

$\therefore$ arg $(z) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$.

38. Since, $\sin \alpha$ and $\cos \alpha$ are the roots of the equation $px^2 + qx + r = 0$

$\therefore \quad \sin \alpha + \cos \alpha = -\frac{q}{p} \quad \ldots(i)$

and $\quad \sin \alpha \cdot \cos \alpha = \frac{r}{p} \quad \ldots(ii)$

On squaring Eq. (i), we get

$$(\sin \alpha + \cos \alpha)^2 = \left(-\frac{q}{p}\right)^2$$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \dfrac{q^2}{p^2}$

$\Rightarrow \quad 1 + 2\cdot\dfrac{r}{p} = \dfrac{q^2}{p^2}$

$\Rightarrow \quad \dfrac{(p+2r)}{p} = \dfrac{q^2}{p^2}$

$\Rightarrow \quad p(p+2r) = q^2$

$\Rightarrow \quad p^2 + 2rp = q^2$

$\Rightarrow \quad p^2 - q^2 + 2rp = 0$

39. The general term in the expansion of $\left(2x^2 - \dfrac{1}{x}\right)^{12}$ is

$$T_{r+1} = {}^{12}C_r\,(2x^2)^{12-r}\cdot\left(-\frac{1}{x}\right)^r$$

$$= (-1)^r\,{}^{12}C_r\,2^{12-r}\cdot x^{24-2r-r}$$

$$= (-1)^r\,{}^{12}C_r\cdot 2^{12-r}\cdot x^{24-3r}$$

The term independent of x, if

$24 - 3r = 0$

$\Rightarrow \quad 24 = 3r$

$\Rightarrow \quad r = 8$

Now, $\quad r + 1 = 8 + 1$

$= 9$th term

40. Key Idea *If* $\cos\theta$ *is positive and* $\tan\theta$ *is negative, then the angle lies in the IVth quadrant.*

Here, we have $\cos\theta = \dfrac{1}{\sqrt{2}}, \quad \tan\theta = -1$

$\therefore$ It lies in the IVth quadrant

$\Rightarrow \quad \theta = 315° = \dfrac{7\pi}{4}$

$\therefore$ The general value of θ is

$$2n\pi + \frac{7\pi}{4}, n \in I$$

41. We have $r_1 = 2r_2 = 3r_3$

$\Rightarrow \quad \dfrac{\Delta}{s-a} = \dfrac{2\Delta}{(s-b)} = \dfrac{3\Delta}{(s-c)}$

$\Rightarrow s - b = 2(s-a)$ and $(s-c) = 3(s-a)$

Taking $s - b = 2(s-a)$

$\Rightarrow \dfrac{a+b+c}{2} - b = 2\left(\dfrac{a+b+c}{2} - a\right)$

$\left(\because\ s = \dfrac{a+b+c}{2}\right)$

$\Rightarrow \quad a + c - b = 2(-a + b + c)$

$\Rightarrow \quad 3a - c - 3b = 0$

$\Rightarrow \quad 3a = 3b + c \quad \ldots$(i)

Now, taking $(s-c) = 3(s-a)$

$\Rightarrow \quad \dfrac{a+b+c}{2} - c = 3\left(\dfrac{a+b+c}{2} - a\right)$

$\Rightarrow \quad a + b - c = 3(-a + b + c)$

$\Rightarrow \quad 4a - 2b - 4c = 0$

$\Rightarrow \quad 4a = 2b + 4c$

$\Rightarrow \quad 2a = b + 2c \quad \ldots$(ii)

From Eqs. (i) and (ii), we get

$6a = 6b + 2a - b$

$\Rightarrow \quad 4a = 5b$

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{5}{4}$

42. $\displaystyle\lim_{x\to\infty} \frac{x^2 + bx + 4}{x^2 + ax + 5}$

$$= \lim_{x\to\infty} \frac{\left(1 + \frac{b}{x} + \frac{4}{x^2}\right)x^2}{\left(1 + \frac{a}{x} + \frac{5}{x^2}\right)x^2}$$

$= 1$

Alternative $\displaystyle\lim_{x\to\infty} \frac{x^2 + bx + 4}{x^2 + ax + 5}$

$= \displaystyle\lim_{x\to\infty} \frac{2x + b}{2x + a}$ (using L' Hospital's rule)

$= \displaystyle\lim_{x\to\infty} \frac{2}{2}$

$= 1$

43. Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad \displaystyle\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \quad \displaystyle\lim_{x\to 0} \frac{\sin \pi x}{5x} = k$

$\Rightarrow \quad \displaystyle\lim_{x\to 0} \left(\frac{\sin \pi x}{\pi x}\right)\frac{\pi}{5} = k$

$\Rightarrow \quad (1)\dfrac{\pi}{5} = k \quad \left(\because \displaystyle\lim_{x\to 0}\frac{\sin x}{x} = 1\right)$

$\Rightarrow \quad k = \dfrac{\pi}{5}$

44. Given that, $|\vec{\mathbf{a}}| = 3$, $|\vec{\mathbf{b}}| = 4$ and $|\vec{\mathbf{c}}| = 5$

and $\quad \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

On squaring both sides, we get

$|\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}}) = 0$

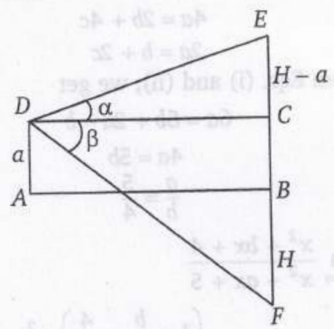
$\Rightarrow \quad 3^2 + 4^2 + 5^2 + 2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}}) = 0$

$$\Rightarrow \quad 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = -(9+16+25)$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -\frac{50}{2}$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -25$$

45. In ΔCDE,



$$\cot\alpha = \frac{DC}{H-a} \qquad \ldots\text{(i)}$$

And in ΔCDF

$$\cot\beta = \frac{DC}{H+a} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$(H+a)\cot\beta = (H-a)\cot\alpha$$

$$\Rightarrow \quad H = \frac{a(-\cot\beta - \cot\alpha)}{\cot\beta - \cot\alpha}$$

$$H = \frac{a(\cot\alpha + \cot\beta)}{\cot\alpha - \cot\beta}$$

$$= \frac{(a\cos\alpha\sin\beta + \cos\beta\sin\alpha)}{\cos\alpha\sin\beta - \cos\beta\sin\alpha}$$

$$= a\frac{\sin(\alpha+\beta)}{\sin(\beta-\alpha)}\text{ m}$$

46. **Key Idea** *If $A = diag\,(a_1, a_2, a_3)$ and $B = diag\,(b_1, b_2, b_3)$ then $AB = diag\,(a_1 b_1, a_2 b_2, a_3 b_3)$*

Given matrix can be rewritten as

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\text{Now, } A^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow A^2B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$= \text{diag}\,(-4, 3, 18)$$

Alternative

$\therefore\ A^2B = \text{diag}\,(2, -1, 3)\ \text{diag}\,(2, -1, 3)\ \text{diag}\,(-1, 3, 2)$

$= \text{diag}\,(4, 1, 9),\ \text{diag}\,(-1, 3, 2)$

$= \text{diag}\,(-4, 3, 18)$

47. The negation of $(p \wedge \sim q) \Rightarrow r$ is

$\sim[(p \wedge \sim q) \Rightarrow r]$

$\equiv (\sim p \vee q) \Rightarrow \sim r$

48. Let $p: 2+3=5,\ q: 8<10$

$\therefore$ Given proposition becomes $p \wedge q$

Its negation is

$\sim(p \wedge q) = \sim p \vee \sim q$

$\therefore$ We have $2+3 \neq 5$ or $8 \nless 10$

49. Given, parabola is $y = x^2$ ...(i)

and straight line $y = 2x - 4$ ...(ii)

From Eqs. (i) and (ii), $x^2 - 2x + 4 = 0$

Let $f(x) = x^2 - 2x + 4$

On differentiating w.r.t. x, we get

$f'(x) = 2x - 2$

For least distance, $f'(x) = 0$

$\Rightarrow 2x - 2 = 0$

$\Rightarrow x = 1$

On putting $x = 1$ in Eq. (i), we get

$$y = 1$$

Hence, the point least distance from the line is (1, 1).

50. The general term in the expansion of $(y^{1/5} + x^{1/10})^{55}$ is $T_{r+1} = {}^{55}C_r (y^{1/5})^{55-r}(x^{1/10})^r$

$$= {}^{55}C_r\, y^{11-r/5} x^{r/10}$$

Since, T_{r+1} will be independent of radicals, so the exponents $r/5$ and $r/10$ are integers for $0 \le r \le 55$, which is possible only when $r = 0, 10, 20, 30, 40, 50$.

$\therefore$ There are six terms viz $T_1, T_{11}, T_{21}, T_{31}, T_{41}, T_{51}$ Which are independent of radicals.