Chapter 18

trigonometric Ratios and standard Angles

Exercise 18.1

1. find the values of

(i) 7 sin 30° cos 60°
(ii) 3 sin² 45° + 2 cos² 60°
(iii) cos² 45° + sin² 60° + sin² 30°
(iv) cos 90° + cos² 45° sin 30° tan 45°

Solution

(i) 7 sin 30° cos 60°

Substituting the values

$= 7 \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{7\times1\times1}{2\times2}$

$= \frac{7}{4}$

(ii) $3 \sin^2 45^\circ + 2 \cos^2 60^\circ$

Substituting the values

$= 3\times \left(\frac{1}{\sqrt{2}}\right)^2 + 2 \times \left(\frac{1}{2}\right)^2$

By further calculation

$= 3 \times \frac{1}{2} + 2 \times \frac{1}{4}$

$= \frac{3}{2} + \frac{1}{2}$

So we get

$= \frac{3+1}{2}$

$= \frac{4}{2}$

$= 2$

(iii) $\cos^2 45^o + \sin^2 60^o + \sin^2 30^o$

Substituting the values

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

By further calculation

$= \frac{1}{2} + \frac{3}{4} + \frac{1}{4}$

Taking LCM

$\frac{2+3+1}{4}$

$\frac{6}{4}$

$\frac{3}{2}$

(iv) $\cos 90^o + \cos^2 45^o \sin 30^o \tan 45^o$

Substituting the values

$0 + \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2} \times 1$

By further calculation

$\frac{1}{2} \times \frac{1}{2} \times 1$

$\frac{1}{4}$

2. find the values of

(i) $\frac{\sin^2 45° + \cos^2 45°}{\tan^2 60°}$

(ii) $\frac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \times \tan 60°}$

(iii) $\frac{4}{3}\tan^2 30° + \sin^2 60° - 3\cos^2 60° + \frac{3}{4}\tan^2 60° - 2\tan^2 45°$

Solution

(i) $\frac{\sin^2 45° + \cos^2 45°}{\tan^2 60°}$

Substituting the values

$= \frac{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}{\sqrt{3}^2}$

By further calculation

$= \frac{\frac{1}{2}+\frac{1}{2}}{3}$

$= \frac{1}{3}$

(ii) $\frac{sin30°-sin90°+2cos0°}{tan30°\times tan60°}$

Substituting the values

$=\frac{\frac{1}{2}-1+2\times1}{\frac{1}{\sqrt{3}}\times\sqrt{3}}$

$=\frac{\frac{1}{2}-1+2}{1}$

So we get

$=\frac{1}{2}-\ 1+2$

$=\frac{1}{2}+1$

$=\frac{1+2}{2}$

$=\frac{3}{2}$

(iii) $\frac{4}{3}tan^2 30°+sin^2 60°-3cos^2 60°+\frac{3}{4}tan^2 60°-2tan^2 45°$

Substituting the values

$=\frac{4}{3}\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2-3\left(\frac{1}{2}\right)^2+\frac{3}{4}\times\left(\sqrt{3}\right)^2-2\times1^2$

By further calculation

$=\frac{4}{3}\ \times\frac{1}{3}+\frac{3}{4}-\ 3\ \times\frac{1}{4}+\frac{3}{4}\ \times3-2\times1$

$= \frac{4}{9} + \frac{3}{4} - \frac{3}{4} + \frac{9}{4} - 2$

So we get

$= \frac{4}{9} + \frac{9}{4} - 2$

Taking LCM

$= \frac{16 + 81 - 72}{36}$

$= \frac{97-72}{36}$

$= \frac{25}{36}$

3. find the values of

(i) $\frac{\boldsymbol{sin60°}}{\boldsymbol{cos^2 45°}} - \boldsymbol{3tan30° + 5cos90°}$

(ii) $\boldsymbol{2\sqrt{2}cos45°\, cos60° + 2\sqrt{3}sin30° tan60° - cos0°}$

(iii) $\boldsymbol{\frac{4}{5} tan^2 60° - \frac{2}{sin^2 30°} - \frac{3}{4} tan^2 30°}$

Solution

(i) $\frac{sin60°}{cos^2 45°} - 3tan30° + 5cos90°$

Substituting the values

$= \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)^2} - 3 \times \frac{1}{\sqrt{3}} + 5 \times 0$

$= \frac{\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} - \sqrt{3} - 0$

So we get

$\frac{\sqrt{3}}{2} \times \frac{2}{1} - \sqrt{3}$

$= \sqrt{3}$ - $\sqrt{3}$

= 0

(ii) $2\sqrt{2}cos45°\, cos60° + 2\sqrt{3}sin30°tan60° - cos0°$

Substituting the values

$=2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{1}{2} \times \sqrt{3} - 1$

By further calculation

$= 2 \times \frac{1}{1} \times \frac{1}{2} + 2 \times 3 \times \frac{1}{2} - 1$

= 1 + 3-1

= 3

(iii) $\frac{4}{5}tan^2 60° - \frac{2}{sin^2 30°} - \frac{3}{4}tan^2 30°$

Substituting the values

$= \frac{4}{5} \times \sqrt{3}^2 - \frac{2}{\left(\frac{1}{2}\right)^2} - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2$

By further calculation

$= \frac{4}{5} \times 3 - \frac{\frac{2}{1}}{4} - \frac{3}{4} \times \frac{1}{3}$

So we get

$= \frac{12}{5} - \frac{2 \times 4}{1} - \frac{1}{4}$

$= \frac{12}{5} - \frac{8}{1} - \frac{1}{4}$

Taking LCM

$= \frac{48-160-5}{20}$

$= \frac{43-160}{20}$

$= -\frac{117}{20}$

$= -5\frac{17}{20}$

4. prove that

(i) $\cos^2 30^o + \sin 30^o + \tan^2 45^o = 2\frac{1}{4}$

(ii) $4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45^o - \sin^2 90^o) = 2$

(iii) $\cos 60^o = \cos^2 30^o - \sin^2 30^o$

Solution

(i) $\cos^2 30^o + \sin 30^o + \tan^2 45^o = 2\frac{1}{4}$

Consider

LHS = $\cos^2 30^o + \sin 30^o + \tan^2 45^o$

Substituting the values

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} + 1^2$

By further calculation

$= \frac{3}{4} + \frac{1}{2} + 1$

Taking LCM

$= \frac{3+2+4}{4}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

= RHS

Therefore LHS =RHS

(ii) $4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45^o - \sin^2 90^o) = 2$

Consider

LHS = $4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45^o - \sin^2 90^o)$

Substituting the values

$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - 1^2\right]$

It can be written as

$= 4\left[\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right] - 3\left[\frac{1}{3} - 1\right]$

By further calculation

$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left(-\frac{1}{2}\right)$

$= 4\left[\frac{1+1}{16}\right] + \frac{3}{2}$

So we get

$= \frac{4\times 3}{16} + \frac{3}{2}$

$= \frac{8}{16} + \frac{3}{2}$

$= \frac{1}{2} + \frac{3}{2}$

$= \frac{1+3}{2}$

$= \frac{4}{2}$

= 2

= RHS

Therefore LHS=RHS

(iii) $\cos 60^{o} = \cos^2 30^{o} - \sin^2 30^{o}$

Consider

LHS = $\cos 60^{o} = \frac{1}{2}$

RHS = $\cos^2 30^{o} - \sin^2 30^{o}$

Substituting the values

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

By further calculation

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

= RHS

Therefore , LHS = RHS

5. (i) if x = 30°, verify that tan 2x = $\frac{2tanx}{1-tan^2x}$

(ii) if x = 15°, verify that 4 sin 2x cos 4x sin 6x =1

Solution

(i) it is given that

X = 30 °

Consider LHS = tan2x

Substituting the value of x

= tan 60°

= $\sqrt{3}$

RHS = $\frac{2tanx}{1-tan^2x}$

Substituting the value of x

= $\frac{2tan30°}{1-tan^2 30°}$

By further calculation

= $\frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

So we get

$= \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

$= \frac{\frac{2}{\sqrt{3}}}{3-\frac{1}{3}}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{2}$

$= \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

Therefore , LHS = RHS

(ii) if $x = 15^o$, verify that 4 sin 2x cos 4x sin 6x =1

It is given that

$x = 15^o$

$2x = 15\times 2 = 30^o$

$2x = 15\times 4 = 60^o$

$6x = 15\times 6 = 90^o$

Here

LHS = 4sin 2x cos 4x sin 6x

It can be written as

$= 4 \sin 30^\circ \cos 60^\circ \sin 90^\circ$

So we get

$= 4 \times \frac{1}{2} \times \frac{1}{2} \times 1$

$=1$

= RHS

Therefore , LHS =RHS

6. find the values of

(i) $\sqrt{\frac{1-\cos^2 30^\circ}{1-\sin^2 30^\circ}}$

(ii) $\frac{\sin 45^\circ \cos 45^\circ \cos 60^\circ}{\sin 60^\circ \cos 30^\circ \tan 45^\circ}$

Solution

(i) $\sqrt{\frac{1-\cos^2 30^\circ}{1-\sin^2 30^\circ}}$

Substituting the values

$$= \sqrt{\frac{1-\left(\frac{\sqrt{3}}{2}\right)^2}{1-\left(\frac{1}{2}\right)^2}}$$

By further calculation

$$= \sqrt{\frac{1-\frac{3}{4}}{1-\frac{1}{4}}}$$

Taking LCM

$$= \sqrt{\frac{4-\frac{3}{4}}{4-\frac{1}{4}}}$$

$$= \sqrt{\frac{\frac{1}{4}}{\frac{3}{4}}}$$

$$= \sqrt{\frac{1}{4} \times \frac{4}{3}}$$

So we get

$$= \sqrt{\frac{1}{3}}$$

$$= \frac{1}{\sqrt{3}}$$

(ii) $\frac{sin45°cos45°cos60°}{sin60°cos30°tan45°}$

Substituting the values

$$=\frac{\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times\frac{1}{2}}{\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\times1}$$

By further calculation

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{3}{4}\times1}$$

$$=\frac{\frac{1}{4}}{\frac{3}{4}}$$

$$=\frac{1}{3}$$

7. if θ = 30º , verify that

(i) sin 2 θ = 2sin θ cos θ

(ii) cos 2 θ= 2cos^2 θ-1

(iii) sin 3 θ= 3sin θ – 4sin^3 θ

(iv) cos 3 θ = 4 cos^3 θ - 3 cos θ.

Solution

It is given that $\theta = 30^o$

(i) $\sin 2\theta = 2\sin\theta\cos\theta$

Consider

LHS = $\sin 2\theta$

Substituting the value of θ

$= \sin 2 \times 30^o$

$= \sin 60^o$

$= \frac{\sqrt{3}}{2}$

Therefore LHS = RHS

(ii) $\cos 2\theta = 2\cos^2\theta - 1$

Consider

LHS = $\cos 2\theta$

Substituting the value of θ

$= \cos 2 \times 30^o$

$= \cos 60^o$

$= \frac{1}{2}$

LHS $= 2\cos^2\theta - 1$

Substituting the value of θ

$= 2\cos^2 30^o - 1$

So we get

$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2 \times \frac{3}{4} - 1$

$= \frac{3}{2} - 1$

$= \frac{3-2}{2}$

$= \frac{1}{2}$

Therefore , LHS = RHS

(iii) sin 3 θ= 3sin θ – 4sin^3 θ

Consider

LHS = sin 3 θ

Substituting the value of θ

Sin 3 × 30^o

Sin 90^o

1

RHS = 3 sin θ - 4sin^3 θ

Substituting the value of θ

= 3 sin 30^o – 4 sin^3 30^o

So we get

$= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^2$

$= \frac{3}{2} - 4 \times \frac{1}{8}$

$= \frac{3}{2} - \frac{1}{2}$

Taking LCM

$= \frac{3-1}{2}$

$= \frac{2}{2}$

= 1

Therefore , LHS = RHS

(iv) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Consider

LHS = $\cos 3\theta$

Substituting the value of θ

$= \cos 3 \times 30^o$

$= \cos 90^o$

= 0

RHS = $4\cos^3\theta - 3\cos\theta$

Substituting the value of θ

$= 4\cos^3 30^o - 3\cos 30^o$

So we get

$= 4 \times \left(\frac{\sqrt{3}}{2}\right)^3 - 3 \times \left(\frac{\sqrt{3}}{2}\right)$

By further calculation

$= 4 \times \frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

$= 0$

Therefore LHS = RHS

8. if $\theta = 30^o$, find the ratio 2 sin θ: sin 2 θ

Solution

It is given that $\theta = 30^o$

We know that

$2 \sin \theta : \sin 2\theta = 2\sin 30^o : \sin 2 \times 30^o$

So we get

$= 2\sin 30^o : \sin 60^o$

$= \frac{2 \sin 30°}{\sin 60°}$

Substituting the values

$= \frac{\frac{2 \times \frac{1}{2}}{\sqrt{3}}}{2}$

By further simplification

$= \frac{\frac{1}{\sqrt{3}}}{2}$

$= \frac{1 \times 2}{\sqrt{3}}$

$= \frac{2}{\sqrt{3}}$

$= 2 : \sqrt{3}$

Therefore 2 sin : sin 2 θ= 2 : $\sqrt{3}$

9. by means of an example , show that sin (A+B) ≠ sinA + sin B.

Solution

Consider A = 30^o and B = 60^0

LHS = sin(A+B)

Substituting the values of A and B

= $\sin(30^o + 60^o)$

= $\sin 90^0$

= 1

RHS = sin A + sin B

Substituting the values

= sin 30^o + sin 60^o

So we get

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1+\sqrt{3}}{2}$

Therefore LHS ≠ RHS i.e. sin (A+B) ≠ sin A + sin B .

10. if A = 60^o and B = 30^o , verify that

(i) sin(A+B) = sin A cos B + cos A sin B

(ii) cos (A+B) = cos A cos B – sin A sin B

(iii) sin(A-B) = sin A cos B – cos A sin B

(iv) tan (A-B) = $\frac{\tan A - \tan B}{1+\tan A \tan B}$

Solution

It is given that A = 60^o and B = 30^o

(i) sin(A+B) = sin A cos B + cos A sin B

Here

LHS = sin(A+B)

Substituting the values of A and B

= sin (60^o + 30^o)

$= \sin 90^o$

$= 1$

RHS = sin A cos B + cos A sin B

Substituting the values of A and B

$= \sin 60^o \cos 30^o + \cos 60^o \sin 30^o$

So we get

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

By further calculation

$= \frac{3}{4} + \frac{1}{4}$

$= \frac{4}{4}$

$=1$

Therefore , LHS = RHS

(ii) cos (A+B) = cos A cos B – sin A sin B

Here

LHS = cos(A+B)

Substituting the value of A and B

$= \cos(60^o + 30^o)$

$= \cos 90^o$

$= 0$

RHS = cos A cos B – sin A sin B

Substituting the value of A and B

$= \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

So we get

$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= 0$

Therefore, LHS = RHS

(iii) sin(A-B) = sin A cos B – cos A sin B

Here

LHS = sin(A-B)

Substituting the values of A and B

$= \sin(60^\circ - 30^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$

RHS = sin A cos B – cos A sin B

Substituting the values of A and B.

$= \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

So we get

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

Therefore, LHS = RHS

(iv) tan (A-B) = $\frac{\tan A - \tan B}{1+\tan A \tan B}$

Here

LHS = tan (A-B)

Substituting the values of A and B

= tan(60^o -30^o)

= tan 30^o

$= \frac{1}{\sqrt{3}}$

RHS = $\frac{\tan A - \tan B}{1+\tan A \tan B}$

Substituting the values of A and B

$= \frac{\tan 60° - \tan 30°}{1+\tan 60° \tan 30°}$

So we get

$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+\sqrt{3} \times \frac{1}{\sqrt{3}}}$

By further simplification

$= \frac{\frac{3-1}{\sqrt{3}}}{1+1}$

$= \frac{\frac{2}{\sqrt{3}}}{2}$

We get

$= \frac{2}{\sqrt{3}} \times \frac{1}{2}$

$\frac{1}{\sqrt{3}}$

Therefore , LHS = RHS

11. (i) if 2θ is an acute angle and 2 sin 2 $\theta = \sqrt{3}$, find the value of θ.

(ii) if $20^o + x$ is an acute angle and $\cos(20^o + x) = \sin 60^o$, then find the value of x .

(iii) if $3\sin^2\theta = 2\frac{1}{4}$ and θ is less than 90^o, find the value of θ.

Solution

(i) it is given that

2θ is an acute angle

$2\sin 2\theta = \sqrt{3}$

It can be written as

$\text{Sin } 2\theta = \frac{\sqrt{3}}{2} = \sin 60^o$

By comparing

$2\theta = 60^o$

So we get

$\theta = \frac{60^\circ}{2} = 30^o$

Therefore θ= 30^o

(ii) it is given that

20^o+x is an acute angle

Cos(20^o + x) = sin 60^o = cos (90^o -60^o)

= cos 30^o

By comparing

20^o + x = 30^o

X = 30^o -20^o = 10^o

Therefore, x = 10^o

(iii) it is given that

$3\sin^2 - \theta = 2\frac{1}{4}$

θ is less than 90^o

We can write it as

$\text{Sin}^2 - \theta = \frac{9}{4\times 3} = \frac{3}{4}$

So we get

$\text{Sin}\ \theta = \frac{\sqrt{3}}{2}$ = sin 60^o

By comparing

$\theta = 60^o$

Therefore, $\theta = 60^o$

12. if θ is an acute angle and sin θ= cos θ, find the value of θ and hence , find the value of $2\tan^2\theta + \sin^2\theta - 1$

Solution

It is given that

Sin θ= cos θ

We can write it as

$$\frac{\text{Sin}\theta}{\cos\theta} = 1$$

$\tan\theta = 1$

we know that $\tan 45^o = 1$

$\tan\theta = \tan 45^o$

so we get

$\theta = 45^o$

We know that

$2\tan^2\theta + \sin^2\theta - 1 = 2\tan^2 45^o + \sin^2 45^o - 1$

Substituting the values

$$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$$

By further calculation

$= 2\times 1 \times 1 + \frac{1}{2} - 1$

$= 2 + \frac{1}{2} - 1$

$= \frac{5}{2} - 1$

Taking LCM

$= \frac{5-2}{2}$

$= \frac{3}{2}$

Therefore, $2\tan^2\theta + \sin^2\theta - 1 = \frac{3}{2}$

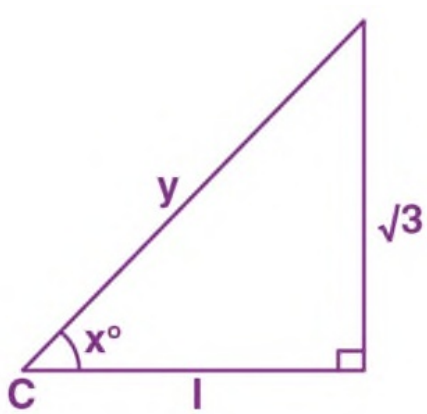
13. from the adjoining figure, find

(i) tan x°

(ii) x

(iii) cos x°

(iv) use sin x° to find y .



Solution

(i) $\tan x^{o} = \frac{perpendicular}{base}$

It can be written as

$= \frac{AB}{BC}$

$= \frac{\sqrt{3}}{1}$

$= \sqrt{3}$

(ii) $\tan x^{o} = \sqrt{3}$

We know that $\tan 60^{o} = \sqrt{3}$

$\tan x^{o} = \tan 60^{o}$

$x = 60$

(iii) we know that

$\text{Cos } x^{o} = \cos 60^{o}$

So we get

$\text{Cos } x^{o} = \frac{1}{2}$

(iv) $\sin x^{o} = \frac{perendicular}{hypotenuse} = \frac{AB}{AC}$

Substituting x = 60 from (ii)

$\text{Sin } 60^{o} = \frac{\sqrt{3}}{y}$

We know that $\sin 60^{o} = \frac{\sqrt{3}}{2}$

$\text{Sin } 60^{o} = \frac{\sqrt{3}}{y}$

We know that $\sin 60^{o} = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{y}$

By further calculation

$Y = \frac{\sqrt{3}\times 2}{\sqrt{3}}$

$Y = 2 \times \frac{1}{1} = 2$

Therefore, y =2

14. if 3θ is an acute angle , solve the following equation for θ :

(i) $2 \sin 3\theta = \sqrt{3}$

(ii) $\tan 3\,\theta = 1$

Solution

(i) 2 sin 3θ = $\sqrt{3}$

It can be written as

Sin 3θ = $\sqrt{3}$

We know that sin $60^o = \frac{\sqrt{3}}{2}$

Sin 3 θ = sin 60^o

3 θ = 60^o

So we get

$\theta = \frac{60}{3} = 20^o$

(ii) tan 3 θ =1

We know that tan 45^o = 1

tan 3 θ = tan 45^o

so we get

3 θ= 45^o

θ = 15^o

15. if tan 3x = sin 45º cos 45º + sin 30º , find the value of x.

Solution

We know that

tan 3x = sin 45º cos 45º + sin 30º

substituting the values

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$

$= \frac{1}{2} + \frac{1}{2}$

= 1

We know that

Tan 3x = tan 45º

By comparing

3x = 45º

$X = \frac{45}{3} = 15^{o}$

Therefore, the value of x is 15º

16. if 4 $\cos^2 x^o$ -1 = 0 and $0 \leq x \leq 90$, find

(i) x

(ii) $\sin^2 x^o + \cos^2 x^o$

(iii) $\cos^2 x^o - \sin^2 x^o$

Solution :

It is given that

$4\cos^2 x^o - 1 = 0$

$4\cos^2 x^o = 1$

It can be written as

$\text{Cos}^2 x^o = \frac{1}{4}$

$\text{Cos } x^o = \pm \frac{\sqrt{1}}{4}$

$\text{Cos } x^o = +\frac{\sqrt{1}}{4}$ $[\, 0 \leq x \leq 90°\, , then \cos x° \; is\; positive\,]$

$\text{Cos } x^o = \frac{1}{2}$

We know that $\cos 60^o = \frac{1}{2}$

$\text{Cos } x^o = \cos 60^o$

By comparing

X = 60

(ii) $\sin^2 x^o + \cos^2 x^o = \sin^2 60^o + \cos^2 60^o$

Substituting the values

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

By further calculation

$= \frac{3}{4} + \frac{1}{4}$

$= 3 + \frac{1}{4}$

$= \frac{4}{4}$

= 1

Therefore , $\sin^2 x^o + \cos^2 x^o = 1$

(iii) $\cos^2 x^o – \sin^2 x^o = \cos^2 60^o - \sin^2 60^o$

Substituting the values

$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

By further calculation

$= \frac{1}{4} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} - \frac{3}{4}$

$= \frac{1-3}{4}$

$= -\frac{2}{4}$

$= -\frac{1}{2}$

Therefore , $\cos^2 x^o - \sin^2 x^o = -\frac{1}{2}$

17 . (i) if sec θ = cosec θ and $0^o \leq \theta \leq 90°$, find the value of θ

(ii) if tan θ = cot θ and $0^o \leq \theta \leq 90°$, find the value of θ

Solution

(i) it is given that

Sec θ = cosec θ

We know that

Sec $\theta = \frac{1}{cos\theta}$

Cosec $\theta = \frac{1}{sin\theta}$

So we get

$\frac{1}{cos\theta} = \frac{1}{sin\theta}$

$\frac{sin\theta}{cos\theta} = 1$

tan $\theta = 1$

here tan $45^o = 1$

$\tan \theta = \tan 45^o$

$\theta = 45^o$

(ii) it is given that

$\tan \theta = \cot \theta$

we know that $\cot \theta = \frac{1}{tan\theta}$

$\tan \theta = \frac{1}{tan\theta}$

so we get

$\tan^2 \theta = 1$

$\tan \theta = \pm\sqrt{1}$

$\tan \theta = +1$ [$0^o \leq \theta \leq 90°$, tan θ is positive]

$\tan \theta = \tan 45^o$

by comparing

$\theta = 45^o$

18. if sin 3x = 1 and $0^o \leq 3x \leq 90°$, find the values of

(i) sin x

(ii) cos 2x

(iii) $\tan^2 x - \sec^2 x$.

Solution

It is given that

Sin 3x = 1

We know that sin 90^o =1

Sin 3x = sin 90^o

By comparing

3x = 90

$X = \frac{90}{3}$

$X = 30^o$

(i) sin x = sin $30^o = \frac{1}{2}$

(ii) $\cos 2x = \cos 2 \times 30 = \cos 60^o = \frac{1}{2}$

(iii) $\tan^2 x - \sec^2 x = \tan^2 30^o - \sec^2 30^o$

Substituting the values

$= \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2$

By further calculation

$= \frac{1}{3} - \frac{4}{3}$

$= \frac{1-4}{3}$

$= - \frac{3}{3}$

$= -1$

Therefore, $\tan^2 x - \sec^2 x = -1$

19 . if $3\tan^2 \theta - 1 = 0$, find $\cos 2\theta$, given that θ is acute .

Solution

It is given that

$3\tan^2 \theta - 1 = 0$

We can write it as

$3 \tan^2 \theta = 1$

$\tan^2 \theta = \frac{1}{3}$

$\tan \theta = \frac{1}{\sqrt{3}}$ [θ is acute so $\tan \theta$ is positive]

$\theta = 30^o$

So we get

$\text{Cos } 2\theta = \cos 2 \times 30^o = \cos 60^o = \frac{1}{2}$

20 . if sin x + cos y = 1 , x = 30^o and y is acute angle , find the value of y .

Solution

It is given that

Sin x + cos y = 1

X = 30^o

Substituting the values

Sin 30^o + cos y = 1

$\frac{1}{2}$ + cos y = 1

It can be written as

Cos y =1 - $\frac{1}{2}$

Taking LCM

Cos y = $\frac{2-1}{2} = \frac{1}{2}$

We know that cos 60^o = $\frac{1}{2}$

Cos y = cos 60^o

So we get

Y = 60^o

21. if sin(A+B) = $\frac{\sqrt{3}}{2}$ = cos (A –B) , 0° $< A + B \leq 90° \ (A > B)$, find the values of A and B.

Solution

It is given that

Sin(A+B) = $\frac{\sqrt{3}}{2} = \cos(A - B)$

Consider

Sin (A+B) = $\frac{\sqrt{3}}{2}$

We know that sin 60° = $\frac{\sqrt{3}}{2}$

Sin(A+B) = sin60°

A+B = 60° ......(1)

Similarly

Cos (A-B) = $\frac{\sqrt{3}}{2}$

We know that cos 30° = $\frac{\sqrt{3}}{2}$

Cos(A-B) = cos 30°

A – B = 30°......(2)

By adding both the equations

A+ B + A – B = 60° + 30°

So we get

$2A = 90^o$

$A = \frac{90°}{2} = 45^o$

Now substitute the value of A in equation (1)

$45^o + B = 60^o$

By further calculation

$B = 60^o – 45^o = 15^o$

Therefore , $A = 45^o$ and $B = 15^o$

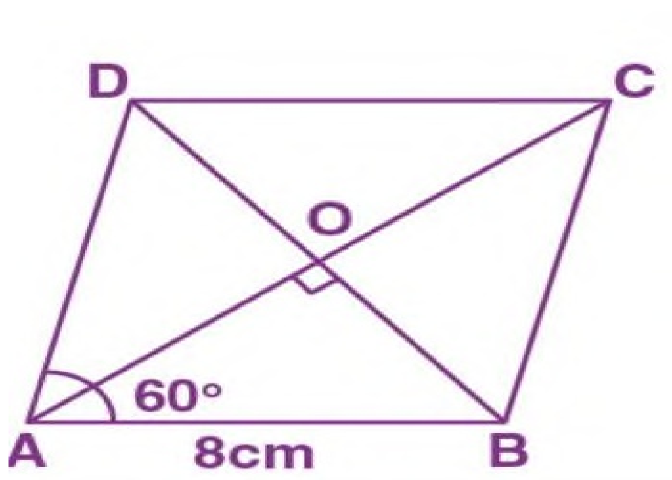
22. if the length of each side of a rhombus is 8 cm and its one angle is 60^0 , then find the lengths of the diagonals of the rhombus.

Solution

It is given that

Each side of a rhombus = 8 cm

One angle = 60^o



We know that the diagonals bisect the opposite angles

$\angle OAB = \frac{60°}{2} = 30°$

In right ∠AOB

Sin $30^{o} = \frac{OB}{AB}$

So we get

$\frac{1}{2} = \frac{OB}{8}$

By further calculation

OB = $\frac{8}{2} = 4$ cm

BD = 2OB = 2×4 = 8 cm

Cos $30^{0} = \frac{AO}{AB}$

Substituting the values

$\frac{\sqrt{3}}{2} = \frac{AO}{8}$

By further calculation

AO = $8\frac{\sqrt{3}}{2} = 4\sqrt{3}$

Here

AC = $4\sqrt{3} \times 2 = 8\sqrt{3}$ cm

Therefore , the length of the diagonals of the rhombus are 8cm and $8\sqrt{3}$ cm .

23. in the right angled triangle ABC , $\angle c = 90^o$ and $\angle B = 60^o$. if AC = 6cm , find the lengths of the sides BC and AB

Solution

In the right – angled triangle ABC , $\angle C = 90^o$ and $\angle B = 60^o$

AC = 6 cm

We know that

tan B = $\frac{AC}{BC}$

substituting the values

tan $60^o = \frac{6}{bc}$

so we get

$\sqrt{3} = \frac{6}{BC}$

BC $= \frac{6}{\sqrt{3}}$

It can be written as

$= \frac{6\sqrt{3}}{\sqrt{3}+\sqrt{3}}$

$= \frac{6\sqrt{3}}{3}$

$= 2\sqrt{3}$ cm

Sin 60º = $\frac{AC}{AB}$

Substituting the values

$\frac{\sqrt{3}}{2} = \frac{6}{AB}$

By further calculation

AB = $\frac{6\times 2}{\sqrt{3}}$

So we get

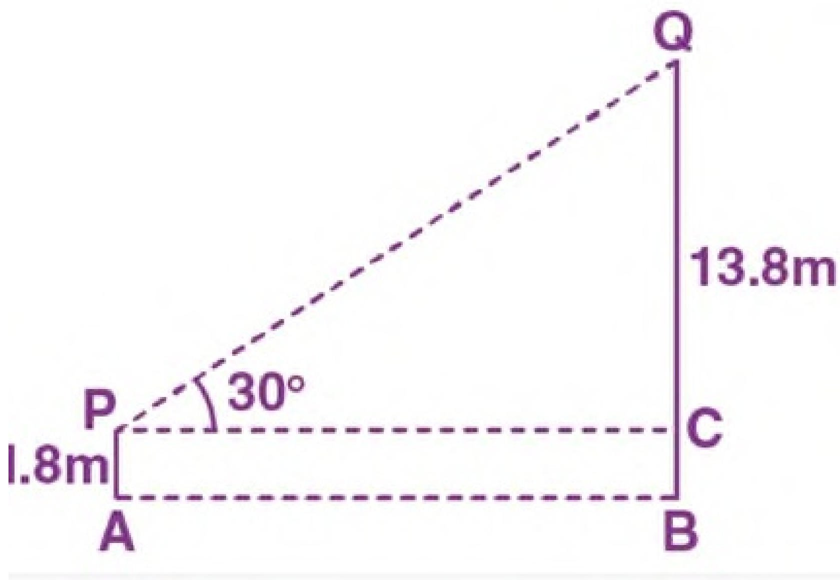
AB = $\frac{12\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

= $12\frac{\sqrt{3}}{3}$

= $4\sqrt{3}$ cm

Therefore the lengths of the sides BC = $2\sqrt{3}$ cm and AB = $4\sqrt{3}$ cm.

24. in the adjoining figure , AP is a mean of height 1.8 m and BQ is a building 13.8 m high. If the mean sees the top of building by focusing his binoculars at an angle of 30º to the horizontal, find the distance of the man from the building.



Solution

It is given that

Height of man AP = 1.8 m

Height of building BQ = 13. 8 m

Angle of elevation from the top of the building to the man = 30 °

Consider AB as the distance of the man from the building

AB = x then PC = x

QC = 13.8 - 1.8 = 12 m

In right ⊿ PQC

$\tan\theta = \frac{QC}{PC}$

substituting the values

$\tan 30° = \frac{12}{x}$

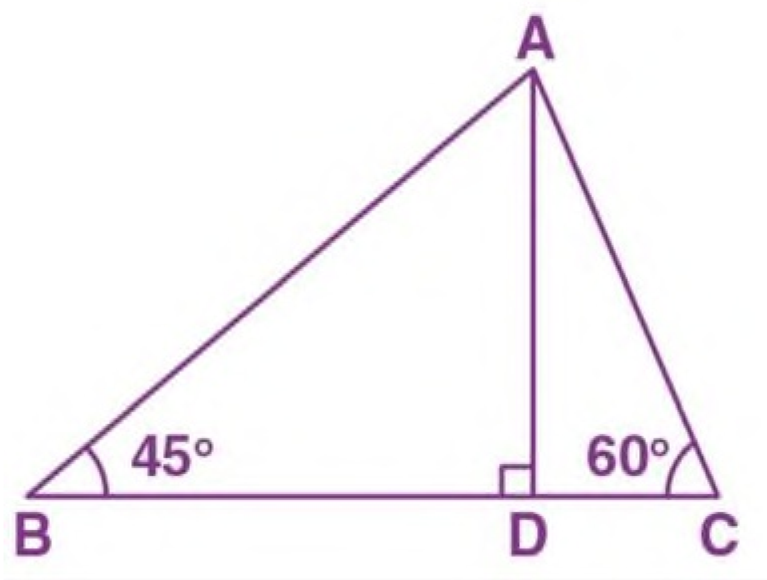
by further calculation

$\frac{1}{\sqrt{3}} = \frac{12}{x}$

x = $12\sqrt{3}$ m

therefore , the distance of the man from the building is $12\sqrt{3}$ m.

25. In the adjoining figure, ABC is a triangle in which ∠B = 45° and ∠c = 60° . if AD ⊥ BC and BC = 8 m , find the length of the altitude AD .



Solution

In triangle ABC

∠B = 45° and ∠c = 60°

AD ⊥ BC and BC = 8m

In right △ ABC

tan 45° = $\frac{AD}{BD}$

so we get

$1 = \frac{AD}{BD}$

AD = BD

In right $\triangle$ ACD

$\tan 60^{o} = \frac{AD}{DC}$

so we get

$\sqrt{3} = \frac{AD}{DC}$

$DC = \frac{AD}{\sqrt{3}}$

Here

$BD + DC = AD + \frac{AD}{\sqrt{3}}$

Taking LCM

$BC = \frac{\sqrt{3}AD + AD}{\sqrt{3}}$

$8 = \left[AD\,\frac{\sqrt{3}+1}{\sqrt{3}}\right]$

By further calculation

$AD = \frac{8\sqrt{3}}{\sqrt{3}+1}$

It can be written as

$= \frac{8\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

So we get

$= \frac{8(3-\sqrt{3})}{3-1}$

$= \frac{8(3-\sqrt{3})}{2}$

$= 4(3-\sqrt{3})$m

Therefore , the length of the altitude AD is (3 - $\sqrt{3}$) m

Exercise 18.2

Without using trigonometric tables , evaluate the following (1 to 5):

1. (i) $\frac{\cos 18°}{\sin 72°}$

(ii) $\frac{tan 41°}{cot 49°}$

(iii) $\frac{\text{cosec } 17° \ 30`}{\sec 7° \ 30`}$

Solution

(i) $\frac{\cos 18°}{\sin 72°}$

It can be written as

$= \frac{\cos 18°}{\cos 18°}$

= 1

(ii) $\frac{tan 41°}{cot 49°}$

It can be written as

$= \frac{tan 41°}{\cot(90°-41°)}$

So we get

$= \frac{tan 41°}{tan 41°}$

$= 1$

(iii) $\frac{\text{cosec } 17° \, 30`}{\text{sec} 7° \, 30`}$

It can be written as

$= \frac{\text{cosec } 17° \, 30`}{\text{sec}(90° - 17° \, 30`)}$

So we get

$= \frac{\text{cosec } 17° \, 30`}{\text{cosec}(17° \, 30`)}$

$= 1$

2. (i) $\frac{cot40°}{tan50°} - \frac{1}{2}\left(\frac{cos35°}{sin55°}\right)$

(ii) $\left(\frac{sin49°}{cos41°}\right)^2 + \left(\frac{cos41°}{sin49°}\right)^2$

(iii) $\frac{sin72°}{cos18°} - \frac{sec32°}{cosec58°}$

(iv) $\frac{cos75°}{sin15°} + \frac{sin12°}{cos78°} - \frac{cos18°}{sin72°}$

(v) $\frac{sin25°}{sec65°} + \frac{cos25°}{cosec65°}$

Solution

(i) $\frac{cot40°}{tan50°} - \frac{1}{2}\left(\frac{cos35°}{sin55°}\right)$

It can be written as

$= \frac{cot40°}{\tan(90°-40°)} - \frac{1}{2}\left(\frac{cos35°}{sin90°-35°}\right)$

By further calculation

$= \frac{cot40°}{cot40°} - \frac{1}{2}\,\frac{cos35°}{cos35°}$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

(ii) $\left(\frac{sin49°}{cos41°}\right)^2 + \left(\frac{cos41°}{sin49°}\right)^2$

It can be written as

$= \left(\frac{sin49°}{cos90°-49°}\right)^2 + \left(\frac{cos41°}{sin90°-41°}\right)^2$

By further calculation

$= \left(\frac{sin49°}{sin49°}\right)^2 + \left(\frac{cos41°}{cos41°}\right)^2$

So we get

$= 1^2 + 1^2$

$= 1 + 1$

$= 2$

(iii) $\frac{sin72°}{cos18°} - \frac{sec32°}{cosec58°}$

It can be written as

$= \frac{sin72°}{\cos(90°-72°)} - \frac{sec32°}{cosec(90°-32°)}$

By further calculation

$= \frac{sin72°}{\sin(72°)} - \frac{sec32°}{sec(32°)}$

$= 1 - 1$

$= 0$

(iv) $\frac{cos75°}{sin15°} + \frac{sin12°}{cos78°} - \frac{cos18°}{sin72°}$

It can be written as

$= \frac{cos75°}{\sin(90°-75°)} + \frac{sin12°}{sin12°} - \frac{cos18°}{cos18°}$

So we get

$= 1 + 1 - 1$

$=1$

(v) $\frac{sin25°}{sec65°} + \frac{cos25°}{cosec65°}$

It can be written as

$= (\sin 25^o \times \cos 65^o) + (\cos 25^o \times \sin 65^o)$

By further calculation

$= [\sin 25^o \times \cos(90^o - 25^o)] + [\cos 25^o \times \sin(90^o - 25^o)]$

So we get

$= (\sin 25^o \times \sin 25^o) + (\cos 25^o \times \cos 25^o)$

$= \sin^2 25^o + \cos^2 25^o$

$= 1$

3. (i) $\sin 62^o - \cos 28^o$

(ii) $\text{cosec } 35^o - \sec 55^o$

Solution

(i) $\sin 62^o - \cos 28^o$

It can be written as

$= \sin(90^o - 28^o) - \cos 28^o$

So we get

$= \cos 28^o - \cos 28^o$

$= 0$

(ii) cosec 35° – sec55°

It can be written as

= cosec 35° – sec(90° – 35°)

= 0

4. (i) $\dfrac{\boldsymbol{cos^2 26° + cos64° sin26° + tan36°}}{\boldsymbol{cot54°}}$

(ii) $\dfrac{\boldsymbol{sec17°}}{\boldsymbol{cosec73°}} + \dfrac{\boldsymbol{tan68°}}{\boldsymbol{cot22° + cos^2 44° + cos^2 46°}}$

Solution

(i) $\dfrac{cos^2 26° + cos64° sin26° + tan36°}{cot54°}$

It can be written as

$= \dfrac{cos^2 26° + \cos(90° - 26°) sin26° + tan36°}{\cot(90° - 36°)}$

We know that

Cos(90° – θ) = sin θ

Cot(90° – θ) = tan θ

Sin^2 θ + cos^2 θ =1

So we get

$= \frac{cos^2 26° + sin^2 26° + tan36°}{tan36°}$

= 1+1

= 2

(ii) $\frac{sec17°}{cosec73°} + \frac{tan68°}{cot22° + cos^2 44° + cos^2 46°}$

It can be written as

$= \frac{\sec(90° - 73°)}{cosec73°} + \frac{\tan(90° - 22°)}{cot22° + cos^2(90° - 46°) + cos^2 46°}$

We know that $\sin^2\theta + \cos^2\theta = 1$

So we get

= 1 +1+1

= 3

5. (i) $\frac{cos65°}{sin25°} + \frac{cos32°}{sin58°} - sin28° sec62° + cosec^2 30°$

(ii) $\frac{\sec 29°}{\mathrm{cosec}\, 61°} + 2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°)$

Solution

(i)$\frac{cos65°}{sin25°} + \frac{cos32°}{sin58°} - \ sin28°sec62° + cosec^2 30°$

It can be written as

$= \frac{cos65°}{\sin(90°-65°)} + \frac{cos32°}{\sin(90°-32°)} - \ sin28° \sec(90° - 28°) +$

$cosec^2 30°$

By further calculation

$= \frac{cos65°}{cos65°} + \frac{cos32°}{cos32°} - \ sin28° \ cosec28° + cosec^2 30°$

We know that cosec30° = 2

= 1 +1 -1 + 4

= 5

(ii) $\frac{\sec 29°}{\mathrm{cosec} 61°} + 2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82° -$

$3(\sin^2 38° + \sin^2 52°)$

It can be written as

$= \frac{\sec 29°}{\mathrm{cosec}(90°-29°)} + 2\cot 8° \cot 17° \cot 45° \cot(90° -$

$17°) \cot(90° - 8°) - 3(\sin^2 38° + \sin^2(90° - 38°))$

By Further calculation

$= \frac{\sec 29°}{\sec 29°} + 2\cot 8° \cot 17° \times 1 \times \tan 17° \cot 45° \cot(90° -$

$17°) \cot(90° - 8°) - 3(\sin^2 38° + \sin^2(90° - 38°)$

By further calculation

$= \frac{\sec 29°}{\sec 29°} + 2\cot 8° \cot 17° \times 1 \times \tan 17° \tan 8° - 3(\sin^2 38° +$

$\cos^2 38°)$

So we get

$= 1+2 \cot 8^{o} \tan 8^{o} \cot 17^{o} \tan 17^{o} \times 1 - 3\times 1$

We know that

$\text{Cosec}(90^{o} - \theta) = \sec \theta$

$\text{Cot}(90^{o} - \theta) = \tan \theta$

$\text{Sin}^2 \theta + \cos^2 \theta = 1$

Here

$= 1 + 2\times 1\times 1\times 1 - 3$

$= 1+ 2 - 3$

$= 0$

6. express each of the following in terms of trigonometric ratios of angle between 0^{o} to 45^{o}

(i) $\tan 81^{o} + \cos 72^{o}$

(ii) $\cot 49^{o} + \text{cosec } 87^{o}$

Solution

(i) $\tan 81^o + \cos 72^o$

It can be written as

$= \tan(90^o - 9^o) + \cos(90^o - 18^o)$

So we get

$= \cot 9^o + \sin 18^o$

(ii) $\cot 49^o + \operatorname{cosec} 87^o$

It can be written as

$= \cot(90^o - 41^o) + \operatorname{cosec}(90^o - 3^o)$

So we get

$= \tan 41^o + \sec 3^o$

Without using trigonometric table , prove that(7 to 11) :

7. (i) $\sin^2 28^o - \cos^2 62^o = 0$

(ii) $\cos^2 25^o + \cos^2 65^o = 1$

(iii) $\operatorname{cosec}^2 67^o - \tan^2 23^o = 1$

(iv) $\sec^2 22^o - \cot^2 68^o = 1$

Solution

(i) $\sin^2 28^\circ - \cos^2 62^\circ = 0$

Consider

LHS = $\sin^2 28^\circ - \cos^2 62^\circ$

It can be written as

$= \sin^2 28^\circ - \cos^2(90^\circ - 28^\circ)$

So we get

$= \sin^2 28^\circ - \sin^2 28^\circ$

= 0

= RHS

(ii) $\cos^2 25^\circ + \cos^2 65^\circ = 1$

Consider

LHS = $\cos^2 25^\circ + \cos^2 65^\circ$

It can be written as

$= \cos^2 25^\circ + \cos^2(90^\circ - 25^\circ)$

We know that $\sin^2 \theta + \cos^2 \theta = 1$

So we get

$= \cos^2 25^\circ + \sin^2 25^\circ$

= 1

(iii) $\text{cosec}^2\, 67^o - \tan^2 23^o = 1$

Consider

LHS = $\text{cosec}^2\, 67^o - \tan^2 23^o$

It can be written as

$= \text{cosec}^2 67^o - \tan^2(90^o - 67^o)$

We know that $\text{cosec}^2\,\theta - \cot^2\theta = 1$

So we get

$= \text{cosec}^2\, 67^o - \cot^2 67^o$

$= 1$

(iv) $\sec^2 22^o - \cot^2 68^o = 1$

Consider

LHS = $\sec^2 22^o - \cot^2 68^o$

It can be written as

$= \sec^2 22^o - \cot^2(90^o - 22^o)$

We know that $\sec^2\theta - \tan^2\theta = 1$

So we get

$= \sec^2 22^o - \tan^2 22^o$

$= 1$

8. (i) $\sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = 1$

(ii) $\sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ = 2$

Solution

(i) $\sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = 1$

Consider

LHS = $\sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ$

It can be written as

$= \sin 63^\circ \cos(90^\circ - 63^\circ) + \cos 63^\circ \sin(90^\circ - 63^\circ)$

$= \sin 63^\circ \sin 63^\circ + \cos 63^\circ \cos 63^\circ$

We know that $\sin^2\theta + \cos^2\theta = 1$

So we get

$= \sin^2 63^\circ + \cos^2 63^\circ$

$= 1$

(ii) $\sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ = 2$

Consider

LHS = $\sec 31^\circ \sin 59^\circ + \cos 31^\circ \operatorname{cosec} 59^\circ$

It can be written as

$$= \frac{1}{cos31^\circ} \times sin59^\circ + cos31^\circ \times \frac{1}{sin59^\circ}$$

By further calculation

$$= \frac{sin59°}{\cos(90°-59°)} + \frac{cos31°}{\sin(90°-31°)}$$

So we get

$$= \frac{sin59°}{sin59°} + \frac{cos31°}{cos31°}$$

= 1+1

= 2

= RHS

9. (i) sec70^o sin20^o – cos20^o cosec 70^o = 0

(ii) sin^2 20^o + sin^2 70^o – tan^2 45^o = 0

Solution

(i) sec70^o sin20^o – cos20^o cosec 70^o = 0

Consider

LHS = sec70^o sin20^o – cos 20^o cosec70^o

By further simplification

$$= \frac{sin20°}{\cos(90°-20°)} - \frac{cos20°}{\sin(90°-20°)}$$

So we get

$$= \frac{sin20°}{sin20°} - \frac{cos20°}{cos20°}$$

= 1 -1

= 0

= RHS

(ii) $\sin^2 20^o + \sin^2 70^o - \tan^2 45^o = 0$

Consider

LHS = $\sin^2 20^o + \sin^2 70^o - \tan^2 45^o$

It can be written as

= $\sin^2 20^o + \sin^2 (90^o - 20^o) - \tan^2 45^o$

By further calculation

= $\sin^2 20^o + \cos^2 20^o - \tan^2 45^o$

We know that $\sin^2 \theta + \cos^2 \theta = 1$ and $\tan 45^o = 1$

So we get

= 1 -1

= 0

= RHS

10. (i) $\boldsymbol{\frac{cot54^\circ}{tan36^\circ} + \frac{tan20^\circ}{cot70^\circ} - 2 = 0}$

(ii) $\boldsymbol{\frac{sin50^\circ}{cos40^\circ} + \frac{cosec40^\circ}{\sec 50^\circ} - 4\cos 50^\circ\, cosec\, 40^\circ + 2 = 0}$

Solution

(i) $\frac{cot54^\circ}{tan36^\circ} + \frac{tan20^\circ}{cot70^\circ} - 2 = 0$

Consider

LHS = $\frac{cot54^\circ}{\tan(90^\circ - 54^\circ)} + \frac{tan20^\circ}{\cot(90^\circ - 20^\circ)} - 2$

By further calculation

$$\frac{cot54°}{cot54°} + \frac{tan20°}{tan20°} - 2$$

So we get

= 1 + 1 -2

= 0

= RHS

(ii) $\frac{sin50°}{cos40°} + \frac{cosec40°}{\sec 50°} - 4\cos 50°\, cosec\ 40° + 2 = 0$

Consider

LHS = $\frac{sin50°}{cos40°} + \frac{cosec40°}{\sec 50°} - 4\cos 50°\, cosec\ 40° + 2$

It can be written as

$$= \frac{sin50°}{\cos(90°-50°)} + \frac{cosec40°}{\sec(90°-40°)} - 4\cos 50°\, cosec\ (90° - 50°) + 2$$

By further calculation

$$= \frac{sin50°}{sin50°} + \frac{cosec40°}{cosec40°} - \ cos50°\, sec50° + 2$$

So we get

$$= \frac{1+1 - 4\cos 50°}{\cos 50° + 2}$$

= 1 + 1 – 4 + 2

= 4 – 4

= 0

= RHS

11. (i) $\boldsymbol{\frac{cos70°}{sin20°} + \frac{cos59°}{\sin 31°} - 8sin^2 30° = 0}$

(ii) $\boldsymbol{\frac{cos80°}{sin10°} + cos59°\, cosec31° = 2}$

Solution

(i) $\frac{cos70°}{sin20°} + \frac{cos59°}{\sin 31°} - 8sin^2 30° = 0$

Consider

LHS = $\frac{cos70°}{\sin(90°-70°)} + \frac{cos59°}{\sin(90°-59°)} - 8sin^2 30° = 0$

We know that sin30° = $\frac{1}{2}$

= $\frac{cos70°}{cos70°} + \frac{cos59°}{cos59°} - 8\left(\frac{1}{2}\right)^2$

By further calculation

= 1 + 1 – 8 $\times \frac{1}{4}$

So we get

= 2 – 2

= 0

= RHS

(ii) $\frac{cos80°}{sin10°} + cos59°\ cosec31° = 2$

Consider

LHS = $\frac{cos80°}{sin10°} + cos59°\ cosec31°$

It can written as

$= \frac{cos80°}{\sin(90°-80°)} + \frac{cos59°}{sin31°}$

By further simplification

$= \frac{cos80°}{cos80°} + \frac{cos59°}{\sin(90°-59°)}$

So we get

$= 1 + \frac{cos59°}{cos59°}$

= 1 +1

= 2

= RHS

12. without using Trigonometrical tables , evaluate :

(i) $\mathbf{2\left(\frac{tan35°}{cot55°}\right)^2 + \left(\frac{cot55°}{tan35°}\right) - 3\left(\frac{sec40°}{cosec50°}\right)}$

(ii) $\mathbf{\frac{sin35°\ cos55° + cos35°\ sin55°}{cosec^2 10° - tan^2 80°}}$

(iii) $\mathbf{\sin^2 34^o + \sin^2 56^o + 2\tan 18^o\ 72^o - \cot^2 30^o}$ **.**

Solution

(i) $2\left(\frac{tan35°}{cot55°}\right)^2 + \left(\frac{cot55°}{tan35°}\right) - 3\left(\frac{sec40°}{cosec50°}\right)$

It can be written as

$= 2\left(\frac{\tan(90°-55°)}{cot55°}\right)^2 + \left(\frac{\cot(90°-35°)}{tan35°}\right) - 3\left(\frac{\sec(90°-50°)}{cosec50°}\right)$

By further calculation

$= 2\left(\frac{\cot 55°}{cot55°}\right)^2 + \left(\frac{\tan 35°}{tan35°}\right) - 3\left(\frac{cosec50°}{cosec50°}\right)$

So we get

$= 2 + 1 - 3$

$= 0$

(ii) $\frac{sin35°\ cos55° + cos35°\ sin55°}{cosec^2 10° - tan^2 80°}$

It can be written as

$= \frac{sin35° \cos(90°-35°) + cos35° \sin(90°-35°)}{cosec^2 10° - tan^2(90°-10°)}$

By further calculation

$= \frac{sin35° .\ sin35° + cos35° .\cos 35°}{cosec^2 10° - cot^2 10°}$

So we get

$= \frac{sin^2 35° + cos^2 35°}{cosec^2 10° - cot^2 10°}$

We know that $\sin^2\theta + \cos^2\theta = 1$ and $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$= \frac{1}{1}$

$=1$

(iii) $\sin^2 34^\circ + \sin^2 56^\circ + 2\tan 18^\circ\, 72^\circ - \cot^2 30^\circ$.

It can be written as

$= \sin^2 34^\circ + [\sin(90^\circ - 34^\circ)]^2 + 2\tan 18^\circ \tan(90^\circ - 18^\circ) - \cot^2 30^\circ$

By further simplification

$= \sin^2 34^\circ + \cos^2 34^\circ + 2\tan 18^\circ \cot 18^\circ - (\sqrt{3})^2$

So we get

$= 1 + 2\tan 18^\circ \times \frac{1}{\tan 18^\circ} - 3$

$= 1 + 2 - 3$

$= 0$

13 . prove the following

(i) $\frac{\cos\theta}{\sin(90^\circ-\theta)} + \frac{\sin\theta}{\cos(90^\circ-\theta)} = 2$

(ii) $\cos\theta \sin(90^\circ - \theta) + \sin\theta \cos(90^\circ - \theta) = 1$

(iii) $\frac{\tan\theta}{\tan(90^\circ-\theta)} + \frac{\sin(90^\circ-\theta)}{\cos\theta} = \sec^2\theta$

Solution

(i) $\frac{cos\theta}{\sin(90°-\theta)} + \frac{sin\theta}{\cos(90°-\theta)} = 2$

We know that

LHS = $\frac{cos\theta}{\sin(90°-\theta)} + \frac{sin\theta}{\cos(90°-\theta)}$

So we get

= $\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\sin\theta}$

= 1 + 1

= 2

= RHS

(ii) $\cos\theta \sin(90^o - \theta) + \sin\theta \cos(90^o - \theta) = 1$

Consider

LHS = $\cos\theta \sin(90^o - \theta) + \sin\theta \cos(90^o - \theta)$

It can be written as

= $\cos\theta . \text{Cos}\, \theta + \sin\theta . \text{Sin}\, \theta$

So we get

= $\cos^2\theta + \sin^2\theta$

= 1

= RHS

(iii) $\frac{tan\theta}{\tan(90°-\theta)} + \frac{\sin(90°-\theta)}{cos\theta} = sec^2\theta$

Consider

LHS = $\frac{tan\theta}{\tan(90°-\theta)} + \frac{\sin(90°-\theta)}{cos\theta}$

By further calculation

= $\frac{\tan\theta}{\cot\theta} + \frac{\cos\theta}{\cos\theta}$

So we get

= $\tan\theta \times \tan\theta + 1$

= $\tan^2\theta$ +1

= $\sec^2\theta$

= RHS

14. prove the following

(i) $\frac{\cos(90°-A)\sin(90°-A)}{\tan(90°-A)} = 1 - cos^2A$

(ii) $\frac{\sin(90°-A)}{cosec(90°-A)} + \frac{\cos(90°-A)}{\sec(90°-A)} = 1$

Solution

(i) $\frac{\cos(90°-A)\sin(90°-A)}{\tan(90°-A)} = 1 - cos^2 A$

Consider

LHS = $\frac{\cos(90°-A)\sin(90°-A)}{\tan(90°-A)}$

It can be written as

$= \frac{\sin A\; cosA}{\cot A}$

So we get

$= \frac{(\sin A\; cosA \times \sin A)}{\cos A}$

$= \sin^2 A$

$= 1 - \cos^2 A$

= RHS

(ii) $\frac{\sin(90°-A)}{cosec(90°-A)} + \frac{\cos(90°-A)}{\sec(90°-A)} = 1$

Consider

LHS = $\frac{\sin(90°-A)}{cosec(90°-A)} + \frac{\cos(90°-A)}{\sec(90°-A)}$

It can be written as

$= \frac{\cos A}{\sec A} + \frac{\sin A}{cosec\; A}$

So we get

$= \cos A \times \cos A + \sin A \times \sin A$

$= \cos^2 A + \sin^2 A$

$= 1$

= RHS

15. simply the following

(i) $\frac{\cos\theta}{\sin(90°-\theta)} + \frac{\cos(90°-\theta)}{\sec(90°-\theta)} - 3tan^2 30°$

(ii) $\frac{cosec\,(90°-\theta)\sin(90°-\theta)\cot(90°-\theta)}{\cos(90°-\theta)\sec(90°-\theta)tan\theta} + \frac{\cot\theta}{\tan(90°-\theta)}$

Solution :

(i) $\frac{\cos\theta}{\sin(90°-\theta)} + \frac{\cos(90°-\theta)}{\sec(90°-\theta)} - 3tan^2 30°$

It can be written as

$= \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{cosec\theta} - 3tan^2 30°$

By further calculation

$= 1 + \sin\theta \times \sin\theta - 3\left(\frac{1}{\sqrt{3}}\right)^2$

So we get

$= \sin^2\theta + 1 - 3 \times \frac{1}{3}$

$= \sin^2\theta + 1 - 1$

$= \sin^2\theta$

(ii) $\frac{cosec\,(90°-\theta)\sin(90°-\theta)\cot(90°-\theta)}{\cos(90°-\theta)\sec(90°-\theta)tan\theta} + \frac{\cot\theta}{\tan(90°-\theta)}$

It can be written as

$= \frac{\sec\theta\cos\theta\tan\theta}{\sin\theta\ cosec\ \theta\tan\theta} + \frac{\cot\theta}{\cot\theta}$

So we get

$= \frac{\sec\theta\cos\theta}{\sin\theta\ cosec\ \theta+1}$

$= \frac{1}{1} + 1$

$= 1+1$

$= 2$

16. show that

$$\frac{\boldsymbol{cos^2(45°+\theta)+cos^2(45°-\theta)}}{\mathbf{tan(60°+\theta)\,tan(30°-\theta)}} = \mathbf{1}$$

Solution

LHS = $\frac{cos^2(45°+\theta)+cos^2(45°-\theta)}{\tan(60°+\theta)\tan(30°-\theta)}$

It can be written as

$= \frac{cos^2(45°+\theta)+cos^2[90°(45°-\theta)]}{\tan(60°+\theta)\tan[90°(30°-\theta)]}$

By further calculation

$= \frac{cos^2(45°+\theta)+sin^2(45°-\theta)}{\tan(60°+\theta)\cot(30°-\theta)}$

We know that cos (90° –) = sin θ, tan(90° – θ) = cot θ and tan θ cot θ = 1

So we get

$= \frac{1}{1}$

= 1

= RHS

17. find the value of A if

(i) sin 3A = cos (A – 6°), where 3A and A – 6 ° are acute angles

(ii) tan2A= cot(A – 18°), where 2A and A – 18° are acute angles

(iii) if sec 2A = cosec (A – 27°) where 2A is an acute angle , find the measure of ∠A .

Solution

(i) sin 3A = cos (A – 6°), where 3A and A – 6 ° are acute angles

It is given that

Sin 3A = cos(A – 6°)

We know that cos (90° – θ) = sin θ

Cos (90° – 3A) = cos(A – 6°)

By comparing both

90° – 3A = A – 6°

By further calculation

90° + 6° = A +3A

96° = 4A

So we get

A = $\frac{96°}{4} = 24°$

Hence the value of A is 24°.

(ii) $\tan 2A = \cot(A - 18^\circ)$,

We know that $\cot(90^\circ - \ ) = \tan\theta$

$\cot(90^\circ - 2A) = \cot(A - 18^\circ)$

By comparing both

$90^\circ - 2A = A - 18^\circ$

By further calculation

$90^\circ + 18^\circ = A + 2A$

So we get

$3A = 108^\circ$

$A = \frac{108^\circ}{3} = 36^\circ$

Hence , the value of A is 36°

(iii) if $\sec 2A = \operatorname{cosec}(A - 27^\circ)$

We know that $\operatorname{cosec}(90^\circ - \ ) = \sec\theta$

$\operatorname{Cosec}(90^\circ - 2A) = \cos(A - 27^\circ)$

By comparing both

$90^\circ - 2A = A - 27^\circ$

By further calculation

$90^\circ + 27^\circ = A + 2A$

So we get

$3A = 117^\circ$

$A = \frac{117°}{3} = 39°$

Hence the value of A = 39°.

18 . find the value of $\theta(0° < \theta < 90°)$if

(i) cos 63° sec(90° – θ) = 1

(ii) tan35° cot(90° – θ) = 1

Solution

(i) cos 63° sec(90° – θ) = 1

It can be written as

$\text{Cos } 63° = \frac{1}{\sec(90°-\theta)} = 1$

We know that $\frac{1}{\sec\theta} = \cos\theta$

Cos 63° = cos (90° – θ)

By comparing both

90° – θ = 63°

By further calculation

θ = 90° – 63° = 27°

(ii) tan35° cot(90° – θ) = 1

It can be written as

$\tan 35° = \frac{1}{\cot(90°-\theta)}$

we know that $\frac{1}{\cot\theta} = cos\theta$

tan 35° = tan(90° – θ)

by comparing both

35° = 90° – θ

By further calculation

$\theta = 90° - 35° = 55°$

19. If A,B and C are the interior angles of a Δ ABC , show that

(i) cos $\frac{A+B}{2} = sin\frac{C}{2}$

(ii) tan $\frac{c+A}{2} = \cot\frac{B}{2}$

Solution :

A,B and C are the interior angles of a Δ ABC

It can be written as

$\angle A + \angle B + \angle C = 180°$

Dividing both sides by 2

$\frac{\angle A+\angle B+\angle C}{2} = \frac{180°}{2}$

$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90°$

(i) cos $\frac{A+B}{2} = sin\frac{C}{2}$

We can write it as

$\frac{A+B}{2} = 90° - \frac{c}{2}$

We know that

Cos $\left(90° - \frac{c}{2}\right) = \sin\frac{c}{2}$

Here cos (90° –) = sin θ

Sin $\frac{c}{2} = \sin\frac{c}{2}$

(ii) tan$\frac{c+A}{2} = \cot\frac{B}{2}$

We know that $\frac{A+C}{2} = 90° - \frac{B}{2}$

= tan (90° – $\frac{B}{2}$)

So we get

= cot $\frac{B}{2}$

= RHS

Chapter test

1. find the values of :

(i) $\sin^2 60^\circ - \cos^2 45^\circ + 3\tan^2 30^\circ$

(ii) $\frac{2\cos^2 45^\circ + 3\tan^2 30^\circ}{\sqrt{3}\cos 30^\circ + \sin 30^\circ}$

(iii) $\sec 30^\circ \tan 60^\circ + \sin 45^\circ \operatorname{cosec} 45^\circ + \cos 30^\circ \cot 60^\circ$

Solution

(i) $\sin^2 60^\circ - \cos^2 45^\circ + 3\tan^2 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{3}{4} - \frac{1}{2} + 3 \times \frac{1}{3} = \frac{3}{4} - \frac{1}{2} + \frac{1}{1}$

$= \frac{3-2+4}{4} = \frac{7-2}{4} = \frac{5}{4} = 1\frac{1}{4}$

Therefore, $\sin^2 60^\circ - \cos^2 45^\circ + 3\tan^2 30^\circ = 1\frac{1}{4}$

(ii) $\frac{2cos^2 45° + 3tan^2 30°}{\sqrt{3}cos30° + sin30°}$

$= \frac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3\times\left(\frac{1}{\sqrt{3}}\right)^2}{\sqrt{3}\times\left(\sqrt{\frac{3}{2}}\right) + \frac{1}{2}}$

$= \frac{2\times\frac{1}{2} + 3\times\frac{1}{3}}{\frac{3}{2} + \frac{1}{2}} = \frac{1+1}{\frac{3+1}{2}} = \frac{2}{\frac{4}{2}} = \frac{2}{2} = 1$

Hence ,

$\frac{2cos^2 45° + 3tan^2 30°}{\sqrt{3}cos30° + sin30°} = 1$

(iii) sec30° tan60° + sin45° cosec45° + cos30° cot60°

$= \frac{2}{\sqrt{3}} \times \sqrt{3} + \frac{1}{\sqrt{2}} \times \sqrt{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{2}{1} + \frac{1}{1} + \frac{1}{2}$

$= 2 + 1 + \frac{1}{2} = 3 + \frac{1}{2} = \frac{6+1}{2}$

$= \frac{7}{2} = 3\frac{1}{2}$

Thus , sec 30° tan60° + sin45° cosec 45° + cos30° cot60° = $3\frac{1}{2}$

2. taking A = 30^o , verify that

(i) $\cos^4 A - \sin^4 A = \cos 2A$

(ii) $4\cos A \cos(60^o - A)\cos(60^o + A) = \cos 3A$.

Solution

(i) $\cos^4 A - \sin^4 A = \cos 2A$

Let's take A = 30^o

So, we have

LHS = $\cos^4 A - \sin^4 A = \cos^4 30^o - \sin^4 30^o$

$= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4$

$= \frac{\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}}{2\times2\times2\times2} - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{9}{16} - \frac{1}{16}$

$= \frac{9-1}{16} = \frac{8}{16} = \frac{1}{2}$

Now,

R.H.S $= \cos 2A = \cos 2(30^o) = \frac{1}{2}$

Therefore, LHS = RHS hence verified

(ii) $4 \cos A \cos(60^o - A) \cos(60^o + A) = \cos 3A$

Let's take $A = 30^o$

LHS $= 4 \cos A \cos(60^o - A) \cos(60^o + A)$

$= 4 \cos 30^o \cos(60^o - 30^o) \cos(60^o + 30^o)$

$= 4 \cos 30^o \cos 30^o \cos 90^o$

$= 4 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times 0$

$= 0$

Now

LHS = cos3A

R.H.S $= \cos 3A$

$= \cos(3 \times 30^o) = \cos 90^o = 0$

Hence, L.H.S = R.H.S hence verified

3. if $A = 45^o$ and $B = 30^o$, verify that $\frac{\sin A}{(\cos A + \sin A + \sin B)} = \frac{2}{3}$

Solution

Taking

L.H.S $\frac{\sin A}{(\cos A + \sin A + \sin B)}$

$= \frac{\sin 45°}{(\cos 45° + \sin 45° + \sin 30°)}$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\times\frac{1}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{4}}$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{2\sqrt{2}+\sqrt{2}}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{3\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2}}{2} \times \frac{4}{3\sqrt{2}} = \frac{2}{3} = R.H.S$$

Hence verified

4. taking A = 60º and B = 30º , verify that

(i) $\boldsymbol{\frac{\sin a+b}{\cos A \ \cos B} = \tan a + \tan b}$

(ii) $\boldsymbol{\frac{\sin(A-B)}{\sin A \ \sin B} = \cot B - \cot A}$

Solution

(i) here A = 60º and B = 30º

L.H.S = $\frac{\sin a+b}{\cos A \ \cos B} = \frac{\sin(60°+30°)}{cos 60° \cos 30°}$

$$= \frac{\sin 90°}{\cos 60° \cos 30°} = \frac{\frac{1}{1}}{2} \times \frac{\sqrt{3}}{2}$$

$= \frac{1}{\frac{\sqrt{3}}{4}} = \frac{4}{\sqrt{3}}$

Now,

R.H.S $= \tan A + \tan B$

$= \tan 60^{o} + \tan 30^{o}$

$= \sqrt{3} + \frac{1}{\sqrt{3}}$

$= \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

∴ L.H.S = R.H.S

(ii) $A = 60^{o}$, $B = 30^{o}$

L.H.S $= \frac{\sin(A-B)}{\sin A \ \sin B} = \frac{\sin(60°-30°)}{sin60° \sin 30°}$

$= \frac{sin30°}{sin60° \ sin30°} = \frac{1}{\sin 60°} = \frac{\frac{1}{\sqrt{3}}}{2}$

$= \frac{2}{\sqrt{3}}$

R.H.S = cot B – cot A

Cot 30° – cot 60°

$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

L.H.S = R.H.S

5. if $\sqrt{2}\tan 2\theta = \sqrt{6}$ *and* $\theta° < 2\theta < 90°$, find the value of $\sin\theta + \sqrt{3}\cos\theta - 2\tan^2\theta$.

Solution

Given ,

$\sqrt{2}\tan 2\theta = \sqrt{6}$

$\tan 2\theta = \frac{\sqrt{6}}{\sqrt{2}}$

$= \sqrt{3}$

= tan 60°

$= 2\theta = 60°$

$= \theta = 30°$

Now ,

$\text{Sin}\theta + \sqrt{3}\cos\theta - 2\,tan^2\theta$

$= \text{Sin } 30° + \sqrt{3}\cos 30° - 2\,tan^2 30°$

$= \frac{1}{2} + \sqrt{3} \times \frac{\sqrt{3}}{2} - 2\left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{1}{2} + \frac{3}{2} - \frac{2}{3}$

$= \frac{4}{2} - \frac{2}{3}$

$= \frac{12-4}{6}$

$= \frac{8}{6}$

$= \frac{4}{3}$

6. if 3θ is an acute angle , solve the following equation for θ :

(i) (cosec 3θ - 2) (cot 2θ -1) = 0

(ii) (tan θ -1) (cosec 3θ -1) = 0

Solution

(i) (cosec 3θ - 2) (cot 2θ -1) = 0

Now , either

Cosec 3θ- 2 or cot 2θ - 1 = 0

= cosec 3θ = 2 or cot 2θ = 1

So ,

Cosec 3θ = cosec $3\theta°$ or cot 2θ = cot 45°

= 3θ = 30° or 2 θ = 45°

Thus, $\theta = 30° \, or \, 45°$

(ii) $(\tan\theta - 1)(cosec\ 3\theta - 1) = 0$

Now, either

$\tan\theta$ -1 = 0 or cosec 3 θ- 1 = 0

= $\tan\theta$ = 1 or cosec 3 θ = 1

So ,

$\tan\theta = \tan 45^{o}$ or cosec 3θ = cosec 90^{o}

= $\theta = 45^{o}$ or 3 $\theta = 90^{o}$ i.e. $\theta = 30^{o}$

Thus $\theta = 45^{o}$ or 30^{o} .

7. if tan (A+B) = $\sqrt{3}$ and tan (A- B) =1 and A, B (B< A) are acute angles, find the values of A and B.

Solution

Given , tan (A+B) = $\sqrt{3}$

So, tan (A +B) = tan 60^{o} [since , tan 60^{o} = $\sqrt{3}$]

= A + B = 60^{o} ....(i)

Also given

tan (A – B) = 1

so , tan (A – B) = tan 45^{o} [tan45^{o} =1]

= A – B = 45^{o} ......(ii)

From equation (1) and (2) we get

$A + B = 60^o$

$A – B = 45^o$

$2A = 105^o$

$A = 52\frac{1}{2}\,^\circ$

Now on substituting the value of A in equation (i) , we get

$52\frac{1}{2}\,^\circ + B = 60^o$

$B = 60^o - 52\frac{1}{2}\,^\circ = 7\frac{1}{2}\,^\circ$

Therefore , the value of $A = 52\frac{1}{2}\,^\circ$ and $B = 7\frac{1}{2}\,^\circ$

8. without using Trigonometrical tables , evaluate the following :

(i) $\sin^2 28^o + \sin^2 62^o – \tan^2 45^o$

(ii) $2\frac{cos27^\circ}{sin63^\circ} + \frac{tan27^\circ}{cot63^\circ} + \cos 0^\circ$

(iii) $\cos 18^o \sin 72^o + \sin 18^o \cos 72^o$

(iv) $5 \sin 50^o \sec 40^o – 3\cos 59^o \text{cosec } 31^o$

Solution

(i) $\sin^2 28^o + \sin^2 62^o - \tan^2 45^o$

$= \sin^2 28^o + \sin^2 (90^o - 28^o) - \tan^2 45^o$

$= \sin^2 28^o + \cos^2 28^o - \tan^2 45^o$

$= 1 - (1)^2$ ($\therefore \sin^2\theta + \cos^2\theta = 1$ and $\tan 45^o = 1$)

$= 1 - 1$

$= 0$

(ii) $2\frac{cos27°}{sin63°} + \frac{tan27°}{cot63°} + \cos 0°$

$= 2\frac{cos27°}{\sin(90°-27°)} + \frac{tan27°}{\cot(90°-27°)} + \cos 0°$

$= 2\frac{cos27°}{cos27°} + \frac{tan27°}{\tan 27°} + 1 \quad (\therefore \cos 0° = 1)$

$= 2\times 1 + 1 + 1$

$= 2 + 1 + 1$

$= 4$

(iii) $\cos 18^\circ \sin 72^\circ + \sin 18^\circ \cos 72^\circ$

$= \cos(90^\circ - 12^\circ) \sin 72^\circ + \sin(90^\circ - 12^\circ) \cos 12^\circ$

$= \sin 72^\circ . \sin 12^\circ + \cos 12^\circ \cos 12^\circ$

$= \sin^2 12^\circ + \cos^2 12^\circ$

$= 1$ ($\because \sin^2\theta + \cos^2\theta = 1$)

(iv) $5 \sin 50^\circ \sec 40^\circ - 3\cos 59^\circ \operatorname{cosec} 31^\circ$

$$= 5\,\frac{\sin 50^\circ}{\cos 40^\circ} - 3\,\frac{\cos 59^\circ}{\sin 31^\circ}$$

$$= 5\,\frac{\sin 50^\circ}{\cos(90^\circ - 50^\circ)} - 3\,\frac{\cos 59^\circ}{\sin(90^\circ - 59^\circ)}$$

$$= 5\,\frac{\sin 50^\circ}{\sin 50^\circ} - 3\,\frac{\cos 59^\circ}{\cos 59^\circ} = 5 \times 1 - 3 \times 1$$

$= 5 - 3$

$= 2$

9. prove that :

$$\frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\ \tan\theta}{\operatorname{cosec}(90^\circ-\theta)\ \sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta} = 2$$

Solution

$$\text{LHS} = \frac{\cos(90^\circ-\theta)\sec(90^\circ-\theta)\ \tan\theta}{\operatorname{cosec}(90^\circ-\theta)\ \sin(90^\circ-\theta)\cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot\theta}$$

$= \frac{sin\theta\ cosec\ \theta\ tan\theta}{sec\theta \cos\theta\ tan\theta} + \frac{cot\theta}{cot\theta}$

$= \frac{1\times\tan\theta}{1\times\tan\theta} + 1 = 1 + 1 = 2$ = R.H.S

Thus , L.H.S = R.H.S

Hence proved

10. when $0^o < A < 90°$, solve the following equations:

(i) sin 3A = cos 2A

(ii) tan 5A = cot A

Solution :

(i) sin 3A = cos 2A

= sin 3A = sin(90^o – 2A)

So,

3A = 90^o – 2A

3A + 2A = 90^o

5A = 90^o

∴ A = $\frac{90°}{5}$ = 18^o

(ii) tan 5A = cot A

$= \tan 5A = \tan (90^\circ - A)$

So ,

$5A = 90^\circ - A$

$5A + A = 90^\circ$

$6A = 90^\circ$

$\therefore A = \frac{90^\circ}{6} = 15^\circ$

11. find the value of θ if

(i) sin (θ + 36°) = cos θ , where θ and θ + 36° are acute angles.

(ii) sec4 θ = cosec (θ- 20°) , where 4 θ and θ – 20° are acute angles.

Solution

(i) given , θ and ($\theta + 36^\circ$) are acute angles.

And,

$\sin(\theta + 36^\circ) = \cos\theta = \sin(90^\circ - \theta)$ [As, $\sin(90^\circ - \theta) = \cos\theta$]

On comparing ,we get

$\theta + 36^\circ = 90^\circ - \theta$

$\theta + \theta = 90^\circ - 36^\circ$

$2\theta = 54^\circ$

$\theta = \frac{54^\circ}{2}$

$\theta = 27^o$

(ii) given θ and (θ- 20^o) are acute angles

And ,

Sec 4 θ = cosec(θ- 20^o)

Cosec(90^o - 4 θ) = cosec(θ- 20^o) [since , cosec (90^o – θ) = sec θ]

On comparing , we get

$90^o - 4\theta = \theta - 20^o$

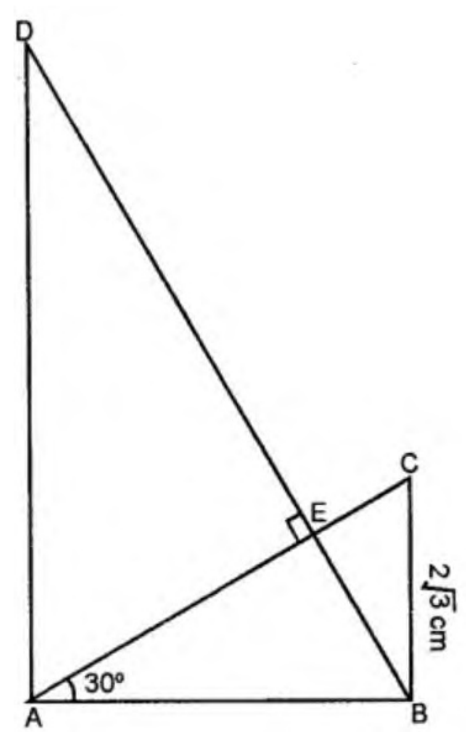
$90^o + 20^o = \theta + 4\theta$

$5\theta = 110^o$

$\theta = \frac{110^\circ}{5}$

$\therefore \ \theta = 22^\circ$

12. in the adjoining figure , ABC is right angled triangle at B and ABD is right angled triangle at A. If BD ⊥ AC and BC = $2\sqrt{3}$ cm , find the length of AD .



Solution

Given ΔABC and ΔABD are right angled triangles in which ∠A = 90° and ∠B = 90°

And ,

BC = $2\sqrt{3}$ cm . AC and BD intersect each other at E at right angle and ∠CAB = 30°

Now in right Δ ABC we have

$\tan\theta = \frac{BC}{AB}$

$= \tan 30^o = \frac{2\sqrt{3}}{AB}$

$= \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}}{AB}$

AB = $2\sqrt{3} \times \sqrt{3} = 2\times 3 = 6$ cm

In Δ ABE , ∠EAB = 30^o and ∠EAB = 90^o

Hence ,

∠ ABE or ∠ABD = $180^o - 90^o - 30^o$

= 60^o

Now in right Δ ABD , we have

$\tan 60^o = \frac{AD}{AB}$

$= \sqrt{3} = \frac{AD}{6}$

Thus AD = $6\sqrt{3}$ cm.