JCECE
ENGINEERING ENTRANCE EXAM.
SOLVED PAPER 2006

Physics

1. Which of the following rays can be polarised?
(a) Water wave and sound wave
(b) Sound wave and radio wave
(c) *X*-rays and water wave
(d) Light wave and *X*-ray

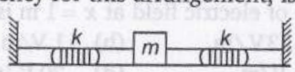
2. Quantum nature can prove :
(a) interference (b) photoelectric effect
(c) diffraction (d) polarisation

3. Which one has highest binding energy per nucleon?
(a) Fe^{56} (b) Li^{6}
(c) U^{235} (d) Ca^{40}

4. Huygen's wave theory can't explain :
(a) interference (b) photoelectric effect
(c) diffraction (d) all of these

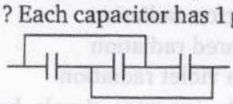
5. If the refractive index of a glass prism is $\cot(A/2)$ and A is angle of prism, then angle of minimum deviation is :
(a) $\left(\frac{\pi}{2} A\right)$ (b) $\left(2\pi \frac{A}{2}\right)$
(c) $\left(\frac{\pi - A}{2}\right)$ (d) $(\pi - 2A)$

6. If f is the frequency when mass m is attached to a spring of spring constant k, then new frequency for this arrangement, is :



(a) $f/2$ (b) $\sqrt{2f}$
(c) $f/\sqrt{2}$ (d) $2\sqrt{2f}$

7. What is equivalent capacitance of the network ? Each capacitor has $1\,\mu F$ capacitance :



(a) $\frac{1}{3}\mu F$ (b) $2\mu F$
(c) $\frac{3}{2}\mu F$ (d) $3\mu F$

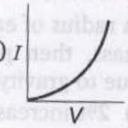
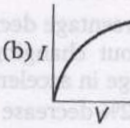
8. The two capacitors C_1 and C_2 are charged to potentials V_1 and V_2 and then connected in parallel. There will be no flow of energy , if :
(a) $C_1V_1 = C_2V_2$ (b) $V_1 = V_2$
(c) $C_1 = C_2$ (d) $\frac{C_1}{V_1} = \frac{C_2}{V_2}$

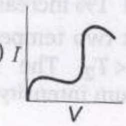
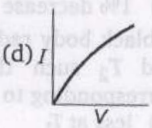
9. Which is not the unit of electric field?
(a) $\frac{N}{C}$ (b) $\frac{N \cdot m}{C}$
(c) $\frac{V}{m}$ (d) $\frac{J}{C \cdot m}$

10. If a body moves for 2 s with 15 m/s velocity towards east and then moves with 5 m/s velocity for 8 s towards north, then average velocity is :
(a) 5 m/s (b) 15 m/s
(c) 30 m/s (d) 7.5 m/s

11. For monoatomic gas which is correct?
(a) $C_V = \frac{3}{5}R$ (b) $C_P = \frac{5}{2}R$
(c) $C_P - C_V = 2R$ (d) $\frac{C_P}{C_V} = \frac{3}{5}$

12. Which is correct relationship for diode?

(a)  (b) 

(c)  (d) 

13. In a properly biased transistor :
(a) both depletion layers are equally large
(b) both depletion layers are equally small
(c) emitter-base depletion layer is large but base-collector depletion layer is small
(d) emitter-base depletion layer is small but base-collector depletion layer is large

14. A wire has resistance 20 Ω. If its length is increased three times its initial length, then new resistance is :
(a) 40Ω (b) 80Ω
(c) 60Ω (d) 180Ω

15. A circular coil of diameter d is rotated in electric field such that electric flux is changed from zero to maximum value ϕ then, electric field is :
(a) $\frac{\phi}{\pi d^2}$ (b) $\frac{2\phi}{\pi d^2}$
(c) $\frac{4d^2}{\pi\phi^2}$ (d) $\frac{4\phi}{\pi d^2}$

16. Work done in rotating a bar magnet from 0 to angle θ is :
(a) $MH(1-\cos\theta)$ (b) $\frac{M}{H}(1-\cos\theta)$
(c) $\frac{M}{H}(\cos\theta-1)$ (d) $MH(\cos\theta-1)$

17. If a convex lens of refractive index 1.44 is dipped in liquid of refractive index 1.49, then it behaves as :
(a) concave lens (b) convex lens
(c) mirror (d) none of these

18. If a source approaches and recedes from observer with same velocity, the ratio of frequencies (apparent) is 6 : 5, then velocity of source is : (v_s = 330 m/s)
(a) 20 m/s (b) 10 m/s
(c) 30 m/s (d) 33 m/s

19. If a boy swings in a circle so the minimum and maximum height from ground is 3 m and 6 m, then, its maximum velocity is :
(a) $5\sqrt{2}$ m/s (b) $2\sqrt{5}$ m/s
(c) $3\sqrt{5}$ m/s (d) $5\sqrt{3}$ m/s

20. If percentage decrease in radius of earth is 1% without changing its mass, then percentage change in acceleration due to gravity is :
(a) 2% decrease (b) 2% increase
(c) 1% decrease (d) 1% increase

21. A black body radiates at two temperatures T_1 and T_2 such that $T_1 < T_2$. The frequency corresponding to maximum intensity is :
(a) less at T_1
(b) more at T_1
(c) equally in the two cases
(d) cannot say

22. If temperature is increased by 1 K at constant volume, then work done on the gas is :
(a) $\frac{5}{2}R$ (b) $\frac{3}{2}R$
(c) zero (d) $\frac{1}{2}R$

23. A body cools from 75°C to 70°C in time t_1, from 70°C to 65°C in time t_2 and from 65°C to 60°C in time t_3, then :
(a) $t_3 > t_2 > t_1$ (b) $t_1 > t_2 > t_3$
(c) $t_2 > t_1 = t_3$ (d) $t_1 > t_2 > t_3$

24. A gas is at 27°C. Its volume is doubled keeping pressure constant, then final temperature is :
(a) 600°C (b) 327 K
(c) 327°C (d) 273°C

25. If the volume of gas is changed from V_1 to V_2 isothermally, then work done is :
(a) $RT\ln\frac{1}{V_2}$ (b) $RT\ln\frac{V_2}{V_1}$
(c) $R(T_2-T_1)\ln\frac{V_2}{V_1}$ (d) $R(V_2-V_1)\ln\frac{T_2}{T_1}$

26. If energy is supplied to a gas isochorically, increase in internal energy is dU then :
(a) $dQ = dU + dW$ (b) $dQ = dU - dW$
(c) $dQ = dU$ (d) $dQ = -dU$

27. A nucleus ${}^A_Z X$ emits one α and 2β particles, then final nucleus is :
(a) Y^A_{Z-2} (b) Y^{A-4}_{Z-4}
(c) Y^{A-4}_{Z} (d) X^A_Z

28. The fringe width for red light is approximately how many times that for violet light in Young's slit experiment?
(a) 2 times (b) 3 times
(c) Equal (d) 1/2 times

29. A person sees clearly at a distance of 100 cm, then power of lens used to see object at 40 cm is :
(a) 3 D (b) −3 D
(c) −1.5 D (d) +1.5 D

30. The electric potential V is given as a function of distance x (metre) by $V = (5x^2 + 10x - 4)$V. Value of electric field at $x = 1$ m is :
(a) −23 V/m (b) 11 V/m
(c) 6 V/m (d) −20 V/m

31. In an image convertor tube fluorescent material is bombarded by :
(a) visible radiation
(b) electron radiation
(c) infrared radiation
(d) ultra violet radiation

32. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is :
(a) $f/2$ (b) f
(c) $2f$ (d) $4f$

33. The work done by the centripetal force F when the body completes one rotation around the circle of radius R is :
(a) $2\pi RF$ (b) $2RF$
(c) RF (d) zero

34. The unit mass having $\vec{r} = 8\hat{i} - 4\hat{j}$ and $\vec{v} = 8\hat{i} + 4\hat{j}$ in its angular momentum is :
(a) 64 unit in $-\hat{k}$ direction
(b) 64 unit in $+\hat{k}$ direction
(c) 64 unit in $+\hat{j}$ direction
(d) 64 unit in $+\hat{i}$ direction

35. Which is nuclear fusion direction?
(a) Hydrogen to helium
(b) Uranium to krypton
(c) Hydrogen to water
(d) Neutron to proton

36. If an AC produces same heat as that produced by a steady current of 4 A, then peak value of current is :
(a) 4 A (b) 1.56 A
(c) 5.6 A (d) 1.41 A

37. In LCR circuit $f = \frac{50}{\pi}$ Hz, $V = 50$ volt, $R = 300\Omega$. If $L = 1$ H and $C = 20\mu C$, then voltage across capacitor is :
(a) zero (b) 20 V
(c) 30 V (d) 50 V

38. If two forces each of 2 N are inclined at 60°, then resultant force is :
(a) 2 N (b) $2\sqrt{5}$ N
(c) $3\sqrt{2}$ N (d) $4\sqrt{2}$ N

39. A block of mass 10 kg is placed on a rough horizontal surface whose coefficient of friction is 0.5. If a horizontal force of 100 N is applied on it, then acceleration of block will be :
(a) $10\,m/s^2$ (b) $5\,m/s^2$
(c) $15\,m/s^2$ (d) $0.5\,m/s^2$

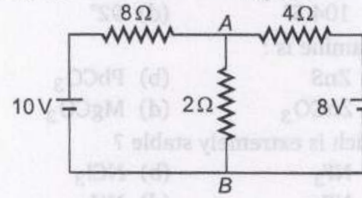
40. The potential difference across an instrument in an AC circuit of frequency f is V and the current through it is I such that $V = 5\cos 2\pi ft$ volt and $I = 2\sin 2\pi ft$ amp. The power dissipated in the instrument is :
(a) zero (b) 10 W
(c) 5 W (d) 2.5 W

41. If ratio of intensities of interfering waves is 16 : 9, then ratio of maximum to minimum intensity is :
(a) 49 : 1 (b) 225 : 81
(c) 3 : 1 (d) 9 : 1

42. The power of a lens, a short sighted person uses is −2 D. Find the maximum distance of an object which he can see without spectacles :
(a) 25 cm (b) 50 cm
(c) 100 cm (d) 10 cm

43. The first overtone frequency of a wave on string of length 2 m is 250 Hz. Then, its velocity is :
(a) 1000 m/s (b) 25 m/s
(c) 500 m/s (d) 10 m/s

44. What is the current flowing in arm AB ?



(a) $\frac{35}{4}$ A (b) $\frac{13}{7}$ A
(c) $\frac{5}{7}$ A (d) $\frac{7}{5}$ A

45. A projectile is fired making an angle 2θ with horizontal with velocity 4 m/s. At any instant it makes an angle θ, then its velocity is :
(a) $4\cos\theta$
(b) $4(2\cos\theta - \sec\theta)$
(c) $2(\sec\theta + 4\cos\theta)$
(d) $4(\sec\theta + \cos\theta)$

46. If path difference becomes $(2n-1)\frac{\lambda}{2}$ then :
(a) white fringe is formed
(b) bright fringe is formed
(c) uniform illumination is obtained
(d) dark fringe is formed

47. If the intensity of fringe at wavelength λ is K, then its intensity at wavelength $\lambda/2$ is :
(a) $\frac{K}{2}$ (b) K
(c) zero (d) $\sqrt{2K}$

48. A positively charged particle moving with velocity v enters a region of space having a uniform magnetic field B. The particle will experience the large deflecting force, when the angle between v and B is :
(a) 0° (b) 45°
(c) 90° (d) 180°

49. In a step-up transformer the turn ratio is 1 : 8. A lead accumulator (emf = 6 V) is connected across the primary coil of the transformer. The voltage across the secondary coil is :
(a) 48 V (b) 0.75 V
(c) 14 V (d) zero

50. The mass of a lift is 500 kg. When it ascends with an acceleration of $2 m/s^2$, the tension in the cable will be : ($g = 10 m/s^2$)
(a) 6000 N (b) 5000 N
(c) 4000 N (d) 1000 N

Chemistry

1. A water molecule can form maximum number of H-bond which is equal to :
(a) 1 (b) 2
(c) 3 (d) 4

2. Bond angle in H_2O is :
(a) 109°28′ (b) 107°10
(c) 104.5° (d) 92°

3. Calamine is :
(a) ZnS (b) $PbCO_3$
(c) $ZnCO_3$ (d) $MgCO_3$

4. Which is extremely stable ?
(a) NF_3 (b) NCl_3
(c) NBr_3 (d) NH_3

5. CH_3COCl does not react with :
(a) diethyl ether (b) phenol
(c) ethanol (d) aniline

6. In SO_2 hybridisation is :
(a) sp (b) sp^3
(c) dsp^2 (d) sp^2

7. Lowest melting point chloride is :
(a) LiCl (b) NaCl (c) KCl (d) CsCl

8. Half-life of a substance is 6 min. If its initial amount is 32 g, then amount present after 18 min is:
(a) 4 g (b) 8 g (c) 16 g (d) 2 g

9. If calcium acetate and calcium formate react, then product formed is :
(a) acetaldehyde (b) acetic acid
(c) formic acid (d) ethyl formate

10. Reduction with aluminium isopropoxide in excess of isopropyl alcohol is called Meerwein Pondorf Verley reduction (MPV). What will be the firal product when cyclohex-2-enone is selectively reduced in MPV reaction?
(a) Cyclohexanol (b) Cyclohex-2-enol
(c) Cyclohexanone (d) Benzene

11. If pH of a solution is 4, then H^+ is :
(a) 10^4 (b) 10^{10}
(c) 10^{-4} (d) 10^{-10}

12. When sodium nitrate is heated above 600°C, then :
(a) only Na_2O is formed
(b) only N_2 is formed
(c) only O_2 is formed
(d) all are formed

13. Which of the following produces Cl_2 gas?
(a) $NaCl + HNO_3$ (b) $MnO_2 + HCl$
(c) $KMnO_4 + HCl$ (d) $HCl + HNO_3$

14. Which is correctly arranged as increasing size?
(a) $F < O < C < Cl < Br$
(b) $C < O < F < Cl < Br$
(c) $Cl < Br < F < C < O$
(d) $O < F < C < Cl < Br$

15. NH_3 is absorbed by :
(a) ozone (b) CaO
(c) pyrargallol (d) $CaCl_2$

16. 1.25 g NH_3 contains how many atoms?
(a) 10^{23} (b) 2×10^{23}
(c) 6×10^{13} (d) 4×10^{23}

17. Which of the following has smallest bond angle?
(a) Ethane (b) Ethene
(c) Ethyne (d) Ethanol

18. Chloroform in air is oxidised to :
(a) CCl_4 (b) dichloromethane
(c) phosgene (d) oxygen

19. Gypsum is :
(a) $CaSO_4 \cdot 2H_2O$
(b) $CaSO_4 \cdot \frac{1}{2} H_2O$
(c) $MgSO_4 \cdot 2H_2O$
(d) $CuSO_4$

20. Which is not soluble in water?
(a) $PbSO_4$ (b) $CdSO_4$
(c) $Bi(SO_4)_2$ (d) $CuSO_4$

21. Which of the following is colour red?
(a) Cu_2O (b) CuF
(c) ZnF_2 (d) $ZnCl_2$

22. How many unpaired electrons are present in $[Cr(NH_3)_5]Br_3$?
(a) 1 (b) 2
(c) 3 (d) 4

23. $S + \frac{3}{2}O_2 \longrightarrow SO_3\ \Delta H = 2x,$

$SO_2 + \frac{1}{2}O_2 \longrightarrow SO_3\ \Delta H = y,$

then heat of formation of SO_2 is :
(a) $2x - y$ (b) $2x + y$
(c) $x + y$ (d) $\frac{2x - y}{2}$

24. Reagent (catalyst) used in Friedel-Craft's alkylation reaction is :
(a) $AlCl_3$ (b) anhyd. $AlCl_3$
(c) N_2 (d) He

25. Catalyst used in making H_2SO_4 in contact process is :
(a) V_2O_5 (b) Fe_2O_3
(c) Cr_2O_3 (d) CrO_3

26. When acetamide is reacted with NaOBr, then product formed is :
(a) ethanamine (b) methanamine
(c) methanamide (d) ethanenitrile

27. Isocyanide is prepared by :
(a) Friedel Craft's reaction
(b) Wurtz's reaction
(c) Williamson synthesis
(d) Carbylamine reaction

28. If rate of diffusion of CH_4 is twice than that of a gas x, then its molecular mass is :
(a) 64 g (b) 16 g
(c) 32 g (d) 8 g

29. Which one is not Lewis acid?
(a) BeF_2 (b) $SnCl_4$
(c) $AlCl_3$ (d) BF_3

30. Natural gas mainly consists of:
(a) methane (b) butane
(c) propane (d) ethane + octane

31. Phenol is treated with Zn to form:
(a) benzoic acid (b) benzyl alcohol
(c) benzene (d) benzoquinone

32. Which one is isoelectronic with CO?
(a) N_2^- (b) N_2^+
(c) CN^- (d) NO

33. How much volume of 1M H_2SO_4 is required to neutralize 20 mL of 1M NaOH?
(a) 10 mL (b) 20 mL
(c) 5 mL (d) 15 mL

34. Fenton's reagent is:
(a) $SnCl_2 + HCl$ (b) $AgNO_3 + NH_4OH$
(c) $CuSO_4 + NaOH$ (d) $FeSO_4 + H_2O_2$

35. $Cu^{2+} + Ag \longrightarrow Cu + Ag^+$ oxidation half reaction is :
(a) $Cu^{2+} \rightarrow Cu$ (b) $Ag \rightarrow Ag^+$
(c) $Cu \rightarrow Cu^{2+}$ (d) all of these

36. $C_4H_{10}O$ has how many isomeric alcohols?
(a) 1 (b) 2
(c) 3 (d) 4

37. IUPAC name of $CH_3-C\equiv C-HC-(CH_3)_2$ is :
(a) 4-methyl-2-pentyne
(b) 1,1-dimethyl-2-butyne
(c) 2-methyl-4-pentyne
(d) 4,4-dimethyl-2-butyne

38. Fehling test is given by:
(a) glucose (b) fructose
(c) sucrose (d) all of these

39. Number of isomeric primary amine of molecular formula $C_4H_{11}N$ is :
(a) 1 (b) 2
(c) 3 (d) 4

40. For reaction $3X + Y \rightleftharpoons X_3Y$ $\Delta H = +$ ve, amount of X_3Y can be changed by:
(a) changing temperature
(b) changing pressure
(c) changing temperature, pressure,
(d) changing temperature, pressure, adding catalyst

41. Which one is most ionic?
(a) P_2O_5 (b) MnO_2
(c) Mn_2O_7 (d) P_2O_3

42. Which one gives I_2 on reaction with KI?
(a) Ag_2SO_4 (b) $CuSO_4$
(c) $PbSO_4$ (d) $CdSO_4$

43. Colour of the solution when KI reacts with Br_2 is :
(a) blue (b) black
(c) red (d) no change

44. Finely divided iron combines with CO to give:
(a) $Fe(CO)_5$ (b) $Fe_2(CO)_9$
(c) $Fe_3(CO)_{12}$ (d) $Fe(CO)_6$

45. Which of the following is pyramidal?
(a) PCl_3 (b) CO_3^{2-}
(c) SO_2 (d) NO_3^-

46. Which of the following conducts electricity?
(a) Crystal NaCl (b) Diamond
(c) Molten KBr (d) Sulphur

47. Ratio of kinetic energy of hydrogen and helium gas at 300 K is :

(a) 2 : 1 (b) 4 : 5
(c) 1 : 1 (d) 1 : 2

48. Which of the following has highest energy?

(a) $n = 2,\ l = 1$
(b) $n = 3,\ l = 2$
(c) $n = 3,\ l = 1$
(d) $n = 2,\ l = 0$

49. Oxidation state of phosphorus in pyrophosphoric acid is :

(a) + 5 (b) + 3 (c) + 4 (d) + 1

50. If the 75% of a first order reaction is complete in 8 min, then time taken to decompose 50% of its initial amount is :

(a) 2 min (b) 4 min
(c) 12 min (d) 1 min

Mathematics

1. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having α/β and β/α as its roots is :

(a) $3x^2 + 19x + 3 = 0$
(b) $3x^2 - 19x + 3 = 0$
(c) $3x^2 - 19x - 3 = 0$
(d) $x^2 - 16x + 1 = 0$

2. If $y = (x + \sqrt{1 + x^2})^n$, then $(1 + x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is :

(a) n^2y (b) $-n^2y$
(c) $-y$ (d) $2x^2y$

3. If 1, $\log_3\sqrt{(3^{1-x} + 2)}$, $\log_3(4 \cdot 3^x - 1)$ are in AP, then x equals :

(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

4. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved, is :

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

5. The angle of elevation of a tower at a point distant d metres from its base is 30°. If the tower is 20 m high, then the value of d is :

(a) $10\sqrt{3}$ m (b) $\frac{20}{\sqrt{3}}$ m
(c) $20\sqrt{3}$ m (d) 10 m

6. l, m, n are the pth, qth and rth terms of a GP and all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals :

(a) 3 (b) 2
(c) 1 (d) zero

7. $\displaystyle\lim_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2}\,x}$ is equal to :

(a) λ (b) -1
(c) zero (d) does not exist

8. A triangle with vertices (4, 0), (–1, –1), (3, 5) is :

(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles

9. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to :

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$
(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

10. A plane which passes through the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is :

(a) $x - y + z = 1$ (b) $x + y + z = 5$
(c) $x + 2y - z = 1$ (d) $2x - y + z = 5$

11. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is :

(a) $\frac{e^{-2x}}{4}$ (b) $\frac{e^{-2x}}{4} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-2x} + c + d$

12. $\displaystyle\lim_{x \to \infty}\left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$ is equal to :

(a) e^4 (b) e^2 (c) e^3 (d) e

13. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is :

(a) [1, 9] (b) [–1, 9]
(c) [–9, 1] (d) [–9, –1]

14. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \ldots \infty$ is :

(a) 1 (b) 2 (c) 3/2 (d) 4

15. Fifth term of a GP is 2, then the product of its 9 terms is :

(a) 256 (b) 512
(c) 1024 (d) none of these

16. $\int_0^{10\pi} |\sin x|\, dx$ is :

(a) 20 (b) 8
(c) 10 (d) 18

17. $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\lim_{n \to \infty} n[I_n + I_{n+2}]$ is equal to :

(a) $\frac{1}{2}$ (b) 1
(c) ∞ (d) zero

18. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is :

(a) $\frac{\pi^2}{4}$ (b) π^2
(c) zero (d) $\frac{\pi}{2}$

19. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is :

(a) π (b) $\frac{\pi}{2}$
(c) 2π (d) none of these

20. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is :

(a) $\frac{1+x}{1+\log x}$ (b) $\frac{1-\log x}{1+\log x}$
(c) not defined (d) $\frac{\log x}{(1+\log x)^2}$

21. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$:

(a) cut at right angles
(b) touch each other
(c) cut at an angle $\frac{\pi}{3}$
(d) cut at an angle $\frac{\pi}{4}$

22. The function $f(x) = \cot^{-1} x + x$ increases in the interval :

(a) $(1, \infty)$
(b) $(-1, \infty)$
(c) $(-\infty, \infty)$
(d) $(0, \infty)$

23. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on $[0, 1]$ is :

(a) 1 (b) 2
(c) 3 (d) 1/3

24. $\int \frac{dx}{x(x^n+1)}$ is equal to :

(a) $\frac{1}{n} \log\left(\frac{x^n}{x^n+1}\right) + c$
(b) $\frac{1}{n} \log\left(\frac{x^n+1}{x^n}\right) + c$
(c) $\log\left(\frac{x^n}{x^n+1}\right) + c$
(d) none of the above

25. The area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$ is given by :

(a) $\frac{9}{2}$ sq unit
(b) $\frac{43}{6}$ sq unit
(c) $\frac{35}{6}$ sq unit
(d) none of these

26. The differential equation of all non-vertical lines in a plane is :

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{d^2x}{dy^2} = 0$
(c) $\frac{dy}{dx} = 0$ (d) $\frac{dx}{dy} = 0$

27. Given two vectors $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ the unit vector coplanar with the two vectors and perpendicular to first is :

(a) $\pm\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (b) $\frac{1}{\sqrt{5}}(2\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(c) $\pm\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ (d) none of these

28. The vector $\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{\mathbf{i}} + (4x-2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The value of x is :

(a) $\left\{-\frac{2}{3}, 2\right\}$ (b) $\left\{\frac{1}{3}, 2\right\}$
(c) $\left\{\frac{2}{3}, 0\right\}$ (d) $\{2, 7\}$

29. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelopiped to piped is :

(a) 7 (b) $\sqrt{38}$
(c) $\sqrt{155}$ (d) none of these

30. The equation of the plane containing the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where :
(a) $ax_1+by_1+cz_1=0$
(b) $al+bm+cn=0$
(c) $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$
(d) $lx_1+my_1+nz_1=0$

31. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is :
(a) $\frac{1}{25}$ (b) $\frac{24}{25}$
(c) $\frac{2}{25}$ (d) none of these

32. The equation of the directrix of the parabola $y^2+4y+4x+2=0$ is :
(a) $x=-1$ (b) $x=1$
(c) $x=-\frac{3}{2}$ (d) $x=\frac{3}{2}$

33. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals :
(a) 5 (b) 7 (c) 6 (d) 4

34. In a triangle ABC, $2ca \sin\frac{A-B+C}{2}$ is equal to :
(a) $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$
(c) $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

35. For $x \in R$, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to :
(a) e (b) e^{-1} (c) e^{-5} (d) e^5

36. The incentre of the triangle with vertices $(1,\sqrt{3})$, $(0, 0)$ and $(2, 0)$ is :
(a) $\left(1,\frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$ (d) $\left(1,\frac{1}{\sqrt{3}}\right)$

37. If the vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ from the sides BC, CA and AB respectively, of a triangle ABC, then :
(a) $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{b}}=0$
(b) $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{c}}\times\vec{\mathbf{a}}=\vec{\mathbf{0}}$
(c) $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}=0$
(d) $\vec{\mathbf{a}}\times\vec{\mathbf{a}}+\vec{\mathbf{a}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}=\vec{\mathbf{0}}$

38. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals :
(a) $128\,\omega$ (b) $-128\,\omega$
(c) $128\,\omega^2$ (d) $-128\,\omega^2$

39. $\sin^2\theta=\frac{4xy}{(x+y)^2}$ is true, if and only if :
(a) $x+y\neq 0$ (b) $x=y, x\neq 0, y\neq 0$
(c) $x=y$ (d) $x\neq 0, y\neq 0$

40. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre at $(0, 3)$ is :
(a) 4 (b) 3 (c) $\sqrt{12}$ (d) $\frac{7}{2}$

41. If $(\omega\neq 1)$ is a cubic root of unity, then
$$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$$ equals :
(a) zero (b) 1
(c) i (d) ω

42. Let $f(2)=4$ and $f'(2)=4$. Then $\lim_{x\to 2}\frac{x\,f(2)-2f(x)}{x-2}$ is given by :
(a) 2 (b) -2
(c) -4 (d) 3

43. Three straight lines $2x+11y-5=0$, $24x+7y-20=0$ and $4x-3y-2=0$:
(a) form a triangle
(b) are only concurrent
(c) are concurrent with on line bisecting the angle between the other two
(d) none of the above

44. A straight line through the point $(2, 2)$ intersects the lines $\sqrt{3}x+y=0$ and $\sqrt{3}x-y=0$ at the points A and B. The equation to the line AB, so that triangle OAB is equilateral is :
(a) $x-2=0$ (b) $y-2=0$
(c) $x+y-4=0$ (d) none of these

45. The greatest distance of the point $P(10, 7)$ from the circle $x^2+y^2-4x-2y-20=0$ is :
(a) 10 (b) 15
(c) 5 (d) none of these

46. The equation of the ellipse whose foci are $(\pm 2, 0)$ and eccentricity $\frac{1}{2}$ is :

(a) $\frac{x^2}{12}+\frac{y^2}{16}=1$ (b) $\frac{x^2}{16}+\frac{y^2}{12}=1$

(c) $\frac{x^2}{16}+\frac{y^2}{8}=1$ (d) none of these

47. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is :

(a) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$

(b) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$

(c) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$

(d) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

48. Let R be the resultant of P and Q and if $\frac{P}{3}=\frac{Q}{7}=\frac{R}{5}$, the the angle between P and R is :

(a) $\cos^{-1}\left(\frac{11}{14}\right)$ (b) $\cos^{-1}\left(-\frac{11}{14}\right)$

(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

49. Two bodies of different masses m_1 and m_2 are dropped from different heights h_1 and h_2. The ratio of the time taken by the two bodies to fall through these distances is :

(a) $h_1 : h_2$ (b) $\sqrt{h_1} : \sqrt{h_2}$

(c) $h_1^2 : h_2^2$ (d) $h_2 : h_1$

50. If var $(x) = 8.25$, var $(y) = 33.96$ and cov $(x, y) = 10.2$, then the correlation coefficient is :

(a) 0.89 (b) −0.98

(c) 0.61 (d) −0.16

ANSWERS

PHYSICS

1. (d)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (a)
11. (b)	**12.** (a)	**13.** (d)	**14.** (d)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (b)
21. (a)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (d)
31. (b)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (a)	**36.** (c)	**37.** (d)	**38.** (b)	**39.** (b)	**40.** (a)
41. (a)	**42.** (b)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (d)	**47.** (c)	**48.** (c)	**49.** (d)	**50.** (a)

CHEMISTRY

1. (d)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (b)
11. (c)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (a)	**20.** (a)
21. (a)	**22.** (c)	**23.** (a)	**24.** (b)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (a)
31. (c)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (b)	**36.** (d)	**37.** (a)	**38.** (a)	**39.** (d)	**40.** (c)
41. (d)	**42.** (b)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (b)	**49.** (a)	**50.** (b)

MATHEMATICS

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (a)
11. (b)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (d)
21. (a)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (a)	**30.** (b)
31. (b)	**32.** (d)	**33.** (b)	**34.** (b)	**35.** (c)	**36.** (d)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (a)
41. (a)	**42.** (c)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (b)	**50.** (c)

HINTS & SOLUTIONS

Physics

1. Key Idea : *Only transverse waves can be polarised.*

Light waves and X-rays are transverse, hence the vibrations of the electric vector must occur in a plane perpendicular to the direction of propagation of light and hence, give rise to polarised light.

Note : Longitudinal waves cannot be polarised.

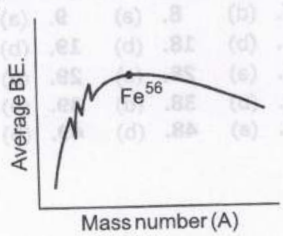
2. According to quantum theory light travels in small bundles or packets of energy known as photons. The energy of photon in such case is $h\nu$, where h is Planck's constant and ν the frequency of light. The intensity of light depends on the number of these photons. Einstein explained the phenomenon of photoelectric effect on the basic of Planck's quantum theory.

Note : Interference, diffraction and polarisation proved the wave nature of light.

3. Key Idea : *Binding energy per nucleon measures the stability of nucleus.*

In order to compare the stability of the nuclei of different atoms, binding energy per nucleon is determined. Higher the binding energy per nucleon more stable is the nucleus. The curve has almost a flat maximum roughly from $A = 50$ to $A = 80$ corresponding to an averge BE per nucleon of about 8.5 MeV.

The nuclei having mass number between 50 and 80 are most stable. Iron $(Fe^{56})(A = 56)$ having BE per nucleon of about 8.8 MeV is most stable.

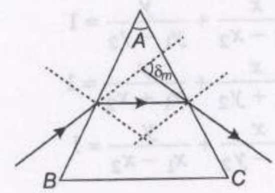


4. Light shows dual character at times it behaves as a wave and at times as particle. While interference, diffraction show the wave nature of light, particle nature is supported by photoelectric effect.

5. Key Idea : $\sin(90° - \theta) = \cos\theta$

The refractive index (μ) of a prism of angle A, and minimum deviation δ_m is given by

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A/2}$$



Given, $\mu = \cot\frac{A}{2}$

$$\therefore \quad \cot\frac{A}{2} = \frac{\sin\frac{(A + \delta_m)}{2}}{\sin(A/2)}$$

$$\Rightarrow \quad \frac{\cos A/2}{\sin A/2} = \frac{\sin\frac{(A + \delta_m)}{2}}{\sin(A/2)}$$

$$\Rightarrow \quad \cos\frac{A}{2} = \sin\left(\frac{A + \delta_m}{2}\right)$$

$$\therefore \sin\left(90° - \frac{A}{2}\right) = \sin\left(\frac{A + \delta_m}{2}\right)$$

$$\Rightarrow \quad 90° - \frac{A}{2} = \frac{A + \delta_m}{2}$$

$$\Rightarrow \quad 180° - A = A + \delta_m$$

$$\Rightarrow \quad \delta_m = 180° - 2A = \pi - 2A$$

6. Key Idea : *The system executes SHM.*

The frequency of oscillation of spring mass system is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

For the given arrangement

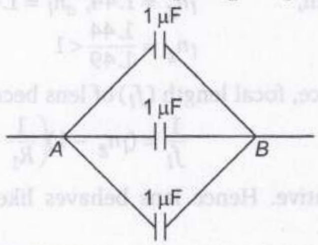
$$k' = k_1 + k_2 = k + k = 2k$$

Hence, frequency of oscillation is

$$f' = \frac{1}{2\pi}\sqrt{\frac{k'}{m}}$$

$$\Rightarrow \quad f' = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \sqrt{2}f$$

7. The given circuit can be rearranged as follows, it consists of three capacitors connected in parallel, hence equivalent resistance of the combination is $C' = C_1 + C_2 + C_3$



Given, $C' = 1 + 1 + 1 = 3\mu F$

8. When a conductor of capacitance C is charged, it acquires a potential

$$V = \frac{q}{C}$$

When connected in parallel, equivalent capacitance of combination is

$$C' = C_1 + C_2$$

Also, energy flow $\left(E = \frac{1}{2}C'V^2\right)$ will not take place, if potential across capacitors is same.

9. The electric field (E) intensity is given by

$$E = -\frac{dV}{dx}$$

Its unit is V/m.

Also, $\quad F = qE$

$\Rightarrow \quad E = \frac{F}{q}$

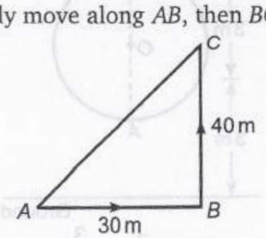
Hence, unit of E is $\frac{\text{N}}{\text{C}}$

Also $\quad E = \frac{W}{qx}$

Hence unit is $\frac{\text{J}}{\text{C-m}}$

10. Key Idea : *Average velocity* $= \frac{\textit{Displacement}}{\textit{Time taken}}$

Let body move along AB, then BC.



Distance travelled in 2s.

$$S_1 = 15 \times 2 = 30 \text{ m}$$

Distance travelled in 8s

$$S_2 = 5 \times 8 = 40 \text{ m}$$

$\therefore$ Displacement $AC = \sqrt{(30)^2 + (40)^2}$

$$= \sqrt{900 + 1600} = \sqrt{2500}$$

$$= 50 \text{ m}$$

Hence average velocity $= \frac{\text{displacement}}{\text{time taken}}$

$$= \frac{50}{8+2} = 5 \text{ m/s}$$

11. Key Idea : *Monoatomic gas has three translational degrees of freedom.*

Specific heat at constant volume $(C_V) = \frac{f}{2}R$

and specific heat at constant pressure (C_P)

$$C_P = \left(\frac{f}{2} + 1\right)R$$

For monoatomic gas $f = 3$

$\therefore \quad C_V = \frac{3}{2}R, C_P = \left(\frac{3}{2} + 1\right)R = \frac{5}{2}R$

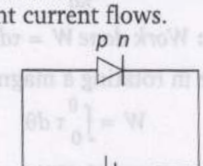
From Mayor's formula

$$C_P - C_V = R$$

$\therefore \quad \frac{5}{2}R - \frac{3}{2}R = R$

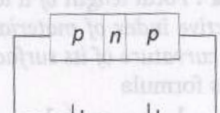
and $\quad \gamma = \frac{C_P}{C_V} = \frac{5/2}{3/2} = \frac{5}{3}$

12. When for a p-n junction diode when p-type is connected to positive of battery and n-type to negative then the diode is forward biased, and a constant current flows.



Hence, correct variation is shown by graph (a).

13. When a transistor is properly biased emitter base junction is forward biased so the size of depletion layer becomes small due to which majority becomes small due to which majority carriers move from emitter to base. The collector-base junction is reverse biased, due to which the collecter-base depletion layer becomes large.



14. Key Idea : *When length is increased volume remians same and specific resistance constant.*

The resistance of a wire of length l is given by

$$R = \rho \frac{l}{A} \quad \text{...(i)}$$

When wire is stretched, volume remains constant, hence,

$$V = Al = \text{constant} \quad \text{...(ii)}$$

Multiply and divide Eq. (i) by l, we get

$$R = \frac{\rho l^2}{Al}$$

$$\therefore \quad \frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2$$

Given, $\quad l_2 = 3l_1$

$$\frac{R_2}{20} = \left(\frac{3l_1}{l_1}\right)^2 = 9$$

$$\Rightarrow \quad R_2 = 20 \times 9 = 180\,\Omega$$

15. From Gauss's theorem

$$\int E \cdot ds = \phi$$

where E is electric field intensity, s the surface area, ϕ the flux.

Given, $\quad s = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{2}$

where r is radius and d the diameter.

$$\phi = E \times \frac{\pi d^2}{4}$$

$$\Rightarrow \quad E = \frac{4\phi}{\pi d^2}$$

16. Key Idea : *Work done $W = \tau d\theta$*

Work done in rotating a magnet is given by

$$W = \int_0^\theta \tau\, d\theta$$

where τ is torque and $d\theta$ angular charge

Also, $\quad \tau = MH \sin\theta$

$$\therefore \quad W = \int_0^\theta MH \sin\theta\, d\theta$$

$$\Rightarrow \quad W = MH \int_0^\theta \sin\theta d\theta$$

$$\Rightarrow \quad W = MH[1 - \cos\theta]_0^\theta$$

$$\Rightarrow \quad W = MH[1 - \cos\theta + \cos 0]$$

$$\Rightarrow \quad W = MH[1 - \cos\theta]$$

17. Key Idea : *Focal length of a lens depends upon the refractive index of material of lens and the radius of curvature of its surfaces.*

From lens formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When lens is dipped in liquid, its focal length is

$${}_l n_g = \frac{{}_a n_g}{{}_a n_l}$$

Given, $\quad {}_l n_g = 1.44,\ {}_a n_l = 1.49$

$$\therefore \quad {}_l n_g = \frac{1.44}{1.49} < 1$$

Hence, focal length (f_l) of lens becomes

$$\frac{1}{f_l} = ({}_l n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

negative. Hence lens behaves like a concave lens.

Note : If focal length was positive it will behave like a convex lens, when infinite it behaves like a plane transparent plate.

18. From Doppler's effect, the perceived frequency when source approaches observer is

$$n' = n\left(\frac{v}{v - v_s}\right) \quad \text{...(i)}$$

When source recedes the observer

$$n'' = n\left(\frac{v}{v + v_s}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{n'}{n''} = \frac{v + v_s}{v - v_s}$$

$$\frac{6}{5} = \frac{330 + v_s}{330 - v_s}$$

$$\Rightarrow \quad 1980 - 6v_s = 1650 + 5v_s$$

$$\Rightarrow \quad 11v_s = 1980 - 1650 = 330$$

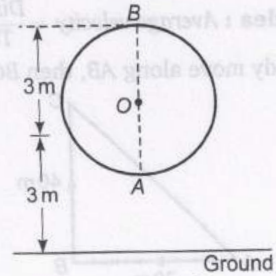
$$v_s = \frac{330}{11} = 30 \text{ m/s}$$

19. Key Idea : *Maximum velocity at lowest point is $\sqrt{5}$ times maximum velocity at highest point.*

Diameter AB of circle is 3m

Therefore, radius $r = \frac{3}{2}$ m

velocity at $A(v) = \sqrt{5gr}$



Given, $\quad g = 10 \text{ m/s}^2,\ r = \frac{3}{2}$ m

$$\therefore \quad v = \sqrt{5 \times 10 \times \frac{3}{2}} = 5\sqrt{3} \text{ m/s}$$

20. The acceleration due to gravity g is given by

$$g = \frac{GM}{R^2} \quad \ldots(i)$$

where G is gravitational constant, M the mass, R the radius of earth.

Taking log on both sides, we get

$$\log g = \log G + \log M - 2 \log R$$

Differentiating, we get

$$\frac{\Delta g}{g} = 0 + 0 - \frac{2\Delta R}{R}$$

Given, $\frac{\Delta R}{R} = -1\%$

$\therefore$ $\frac{\Delta g}{g} = -2(-1\%) = 2\%$ increase

21. From Wein's displacement law

$$\lambda_m T = \text{constant} \quad \ldots(i)$$

where λ_m corresponds to maximum wavelength, T is absolute temperature.

Also, $v = n\lambda \quad \ldots(ii)$

where v is velocity, n the frequency, λ the wavelength.

From Eqs. (i) and (ii), we get

$$\frac{v}{n} \cdot T = \text{constant}$$

Given $T_1 < T_2$, hence frequency corresponding to maximum intensity is less at T_1.

22. Work done on a gas is given by

$$W = P\Delta V$$

where P is pressure, ΔV the change in volume.

Given $\Delta V = 0$, therefore

$$W = P \times 0 = 0$$

23. From Newton's law of cooling

$$\frac{dH}{dt} = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

where θ_0 is temperature of surrounding, $\frac{\theta_1 + \theta_2}{2}$ the temperature of body.

Hence, $t_3 > t_2 > t_1$.

24. Key Idea : *Since pressure is constant, gas obeys Charle's law.*

From Charle's law

$$\frac{V}{T} = \text{constant}$$

where V is volume, T the temperature.

$$\therefore \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

Given $T_1 = 27°\text{C} = 300 \text{ K}$, $V_2 = 2V_1$

$\therefore$ $2 \times 300 = T_2$

$\Rightarrow$ $T_2 = 600 \text{ K}$

In centigrade $T_2 = 600 - 273 = 327°\text{C}$.

25. Key Idea : *Ideal gas equation is*

$$PV = RT$$

Work done in isothermal process is

$$W = \int_{V_1}^{V_2} p\, dV$$

From ideal gas law $PV = RT$

$$\therefore \quad W = \int_{V_1}^{V_2} \left(\frac{RT}{V}\right) dV = RT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\Rightarrow \quad W = RT \ln\left(\frac{V_2}{V_1}\right)$$

26. Key Idea : *In isochoric process, volume is constant.*

Work done $= P\Delta V$,

where P is pressure, ΔV the change in volume.

For an isochoric process $\Delta V = 0$,

Therefore $dW = PdV = 0$

From first law of thermodynamics

$$dQ = dU + dW$$

where $dW = 0$

$\therefore$ $dQ = dU$

27. Key Idea : *β-particles are fast moving electrons.*

When α-particle is emitted by an atom, its atomic number decreases by 2 and mass number by 4.

$$\therefore \quad {}^{A}_{Z}X \xrightarrow{\alpha} {}^{A-4}_{Z-2}M$$

When β-particle is emitted, the atomic number increases by 1, while mass number remains unchanged, therefore

$${}^{A-4}_{Z-2}M \xrightarrow{2\beta} {}^{A-4}_{Z}Y$$

28. The fringe width (W) is given by

$$W = \frac{D\lambda}{d}$$

where λ is wavelength, d the distance between coherent sources, D the distance between source and screen.

$$\therefore \quad \frac{W_R}{W_V} = \frac{\lambda_R}{\lambda_V}$$

λ_R (red light) is 6400-7900 Å,

λ_V (violet light) is 4000-4500 Å

$\therefore$ $W_R \approx 2W_V$

29. Key Idea : *Power* $= \dfrac{1}{\textit{focal length (m)}}$

From lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Given, $v = -100$ cm, $u = -40$ cm

$$\therefore \quad \frac{1}{f} = \frac{1}{100} - \frac{1}{-40}$$

$$\Rightarrow \quad \frac{1}{f} = -\frac{1}{100} + \frac{1}{40}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{-1 + 2.5}{100} = \frac{1.5}{100}$$

Hence, power of lens is

$$P = \frac{100}{f(\text{cm})} = 100 \times \frac{1.5}{100} = 1.5 \text{ D}$$

30. Key Idea : $E = -\dfrac{dV}{dx}$

Electric field E is given by

$$E = -\frac{dV}{dx}$$

Given, $V = 5x^2 + 10x - 4$

$$\therefore \quad E = \frac{dV}{dx} = 10x + 10$$

At $x = 1$, $E = -(10 \times 1 + 10) = -20$ V/m

31. The bombardment of fluorescent material with electron radiation cause fluorescence in image convertor tube.

32. For a particle executing SHM the displacement equation is given by

$$y = a \sin \omega t = a \sin 2\pi ft.$$

kinetic energy $\text{KE} = \dfrac{1}{2} mv^2 = \dfrac{1}{2} m\left(\dfrac{dy}{dt}\right)^2$

where $\dfrac{dy}{dt} = 2\pi\, fa \cos 2\pi\, ft$

$$\therefore \quad \text{KE} = \frac{1}{2} m\{(2\pi fa)^2 \cos^2 2\pi ft\}$$

$$\text{KE} \propto (1 + \cos 4\pi ft)$$

$\cos 4\pi ft$ changes periodically with frequency $2f$.

33. Key Idea : *Work done* (W) = *Force*(F) × *displacement* (d)

For a body rotating in circular motion, centripetal force is directed towards the centre of the circle when body completes one rotation, its displacement is zero, hence

Work done = force × displacement

$$W = F \times 0 = 0$$

34. Key Idea : Angular momentum $\vec{\mathbf{L}} = m(\vec{\mathbf{r}} \times \vec{\mathbf{v}})$

For a body of mass m rotating with velocity v in a circle of radius r, the angular momentum is given by

$$\vec{\mathbf{L}} = m(\vec{\mathbf{r}} + \vec{\mathbf{v}})$$

For unit mass $m = 1$

$$\therefore \quad |\vec{\mathbf{L}}| = (8\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}) \times (8\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$$

$$|\vec{\mathbf{L}}| = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 8 & -4 & 0 \\ 8 & 4 & 0 \end{vmatrix}$$

$$|\vec{\mathbf{L}}| = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(32+32)$$

$$\Rightarrow \quad |\vec{\mathbf{L}}| = 64\,\hat{\mathbf{k}} \text{ unit.}$$

35. Hydrogen bomb is also known as nuclear fusion bomb based upon the fusion of heavy hydrogen nuclei since fusion takes place under the extreme conditions of high pressure and high temperature, a fission bomb must be used as igniter of a fusion bomb.

36. The peak value of current (I_0) is given by

$$I_0 = \sqrt{2}\, I_{\text{rms}}$$

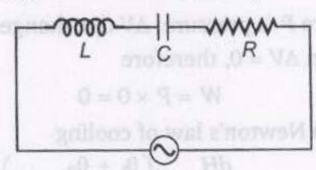
where I_{rms} is known as root mean square value of current.

Given, $I_{\text{rms}} = 4$ A

$$\therefore \quad I_0 = \sqrt{2} \times 4 = 5.6 \text{ A}$$

37. For an LCR circuit

the impedance (Z) is given by



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where $X_L = \omega L = 2\pi fL$

and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$

Given, $f = \dfrac{50}{\pi}$ Hz, $R = 300\,\Omega$, $L = 1$H,

$$C = 20\,\mu\text{C} = 20 \times 10^{-6}\text{C}.$$

$$\therefore \quad Z = \sqrt{(300)^2 + \left(2\pi \times \frac{50}{\pi} \times 1 - \frac{1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}}\right)}$$

$$Z = \sqrt{90{,}000 + (100 - 500)^2}$$

$$Z = \sqrt{90{,}000 + 16{,}0000} = 500\,\Omega$$

Hence, current in circuit is given by

$$i = \frac{V}{Z} = \frac{50}{500} = 0.1\text{ A}$$

Voltage across capacitor is,

$$V_C = iX_C = \frac{i}{2\pi fC} = \frac{0.1}{2\pi \times \frac{50}{\pi} \times 20 \times 10^{-6}}$$

$$= \frac{0.1 \times 10^6}{100 \times 20}$$

$\Rightarrow \quad V_C = 50\text{ V}$

38. The resultant force is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Given, $A = 2\text{ N}$, $B = 2\text{ N}$, $\theta = 60°$

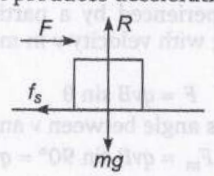
$\therefore \quad R = \sqrt{2^2 + 2^2 + 2 \times 2 \times 2\cos 60°}$

$\Rightarrow \quad R = \sqrt{16 + 4}$

$\Rightarrow \quad R = \sqrt{20}$

$\Rightarrow \quad R = 2\sqrt{5}\text{ N}$

39. Force that produces acceleration is



$$F' = F - \mu R$$

Given, $F = 100\text{ N}$,

$F = \mu R = 0.5 \times 10 \times 10$

$\therefore \quad F' = 100 - 50 = 50\text{ N}$

Also, acceleration $= \frac{F'}{m} = \frac{50}{10} = 5\text{ m/s}^2$

40. Key Idea : *Average power dissipated depends on phase difference between voltage and current.*

Average power dissipated in an AC circuit is given by.

$$P = VI\cos\phi$$

where V is voltage, I the current, ϕ the phase difference

Given, $\quad V = 5\cos 2\pi ft$

Using $\sin(90° + \theta) = \cos\theta$

we have

$$V = 5\sin\left(2\pi ft + \frac{\pi}{2}\right)$$

and $\quad I = 2\sin 2\pi ft$

Hence, phase difference between V and I is

$$\phi = \frac{\pi}{2}$$

$$p = VI\cos\phi = VI\cos\frac{\pi}{2} = 0$$

Note : The given circuit is a wattless circuit, because average power dissipated is zero.

41. Key Idea : *Intensity* $\propto$ *(amplitude)*^{2}.

We know that

$$I = ka^2$$

where a is amplitude and I the intensity

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

Given $\quad \frac{I_1}{I_2} = \frac{16}{9} = \frac{a_1^2}{a_2^2}$

$\Rightarrow \quad \frac{a_1}{a_2} = \frac{4}{3}$

$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{(4+3)^2}{(4-3)^2} = \frac{7^2}{1} = \frac{49}{1}$

42. Key Idea : *Image is formed at focus.*

We know that

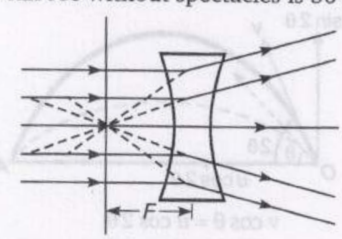
Power $\quad (P) = \frac{1}{\text{focal length}(m)}$

Given $\quad P = -2\text{D}$

$\therefore$ Focal length $= -\frac{1}{2} = -0.5\text{ m}$

$\Rightarrow \quad F = -50\text{ cm}$

Hence, maximum distance of an object, which he can see without spectacles is 50 cm.



43. The first overtone of a wave on string is given by

$$n = \frac{v}{\lambda}$$

where λ is wavelength, v the velocity and n the frequency.

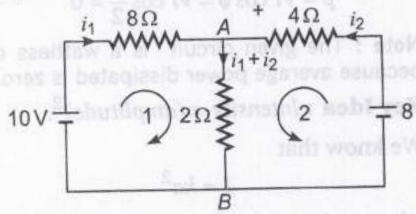
Given, $\lambda = l$ (in first overtone)

$\therefore \quad n = \frac{v}{l}$

$\Rightarrow \quad v = nl = 250 \times 2 = 500\text{ m/s}.$

44. Key Idea : *As per Kirchhoff's law $\Sigma iR = V$*

Let i_1, i_2 be current in the two loops of the given circuit then



Applying Kirchhoff's law to loop 1, we get

$$(\Sigma iR = V)$$

$$8i_1 + 2(i_1 + i_2) = 10$$

$$\Rightarrow \quad 10i_1 + 2i_2 = 10$$

$$\Rightarrow \quad 5i_1 + i_2 = 5 \quad \ldots\text{(i)}$$

Applying Kirchhoff's law to loop 2, we get

$$4i_2 + 2(i_1 + i_2) = 8$$

$$\Rightarrow \quad 6i_2 + 2i_1 = 8$$

$$\Rightarrow \quad i_1 + 3i_2 = 4 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

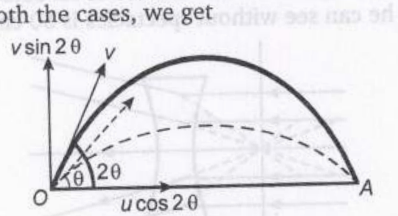
$$i_1 = \frac{11}{14}\text{ A}, i_2 = \frac{15}{14}\text{ A}$$

Hence current flowing is arm AB is

$$i_1 + i_2 = \frac{11}{14} + \frac{15}{14} = \frac{26}{14} = \frac{13}{7}\text{ A}$$

45. Key Idea : *Horizontal component of velocity remains same.*

Let v be the velocity, when projected with angle θ then equating the horizontal velocities in both the cases, we get



$$v\cos\theta = u\cos 2\theta$$

$$\Rightarrow \quad v = \frac{u\cos 2\theta}{\cos\theta}$$

where $\quad \sec\theta = \dfrac{1}{\cos\theta}$

$$\therefore \quad v = u\cos 2\theta \sec\theta$$

Using $\quad \cos 2\theta = 2\cos^2\theta - 1$, we get

Given, $\quad u = 4$ m/s, we get

$$v = 4(2\cos^2\theta - 1)\sec\theta$$

$$\Rightarrow \quad v = 4(2\cos\theta - \sec\theta)$$

46. Key Idea : *Path difference is odd multiple of $\frac{\lambda}{2}$.*

Given path difference $= (2n-1)\frac{\lambda}{2}$

which is an odd multiple of $\frac{\lambda}{2}$.

Hence, destructive interference takes place and dark fringe is formed.

47. Key Idea : *Resultant intensity is given by*

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Resultant intensity due to two waves of intensities I_1, I_2 is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

when path difference is λ, $\phi = 2\pi$

when path difference is $\frac{\lambda}{2}$, $\phi = \pi$

$$\therefore \quad I_{R_1} = I + I + 2I\cos 2\pi$$

$$I_{R_1} = 4I = K$$

$$I_{R_2} = I + I + 2I\cos\pi$$

$$I_{R_2} = 0$$

48. Key Idea : *Maximum force is experienced when angle between v and B is 90°.*

Force experienced by a particle of change q, travelling with velocity v in magnetic field B is given by

$$F = qvB\sin\theta$$

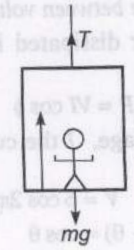
where θ is angle between v and B

$$\therefore \quad F_m = qvB\sin 90° = qvB$$

49. Key Idea : *Lead accumulator battery is a DC power supply source.*

Transformer is based on the principle of mutual induction and works on AC power supply since lead accumulator is a DC source of supply, hence voltage across secondary is zero.

50. The resultant acceleration (a) is provided by resultant of tension (T) and weight (mg). Therefore,



$$T - mg = ma$$

$$\Rightarrow \quad T = m(g + a)$$

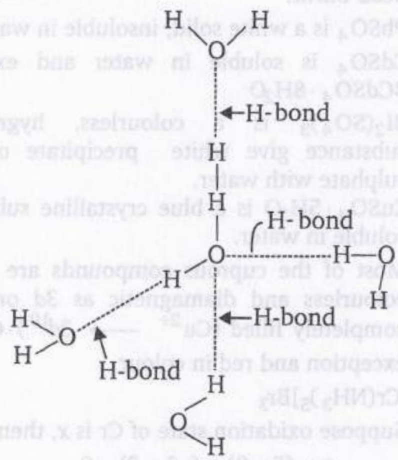
Given $m = 500$ kg, $g = 10$ m/s^2, $a = 2$ m/s^2

$$\therefore \quad T = 500\,(10 + 2)$$

$$\Rightarrow \quad T = 6000\text{ N}$$

Chemistry

1. A water molecule can form maximum four hydrogen bonds which is clear from the given structure

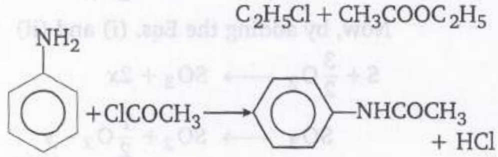


2. The H—O—H bond angle in water is 104°28′, in accordance with the VSEPR prediction of slightly less than tetrahedral due to the presence of lone pairs of electrons on oxygen atom. Thus the orbitals used for bonding by O are close to sp^3 hybrids.

3. $ZnCO_3$ is known as calamine. It is an important ore of zinc. It is also called smitsonite in the USA.

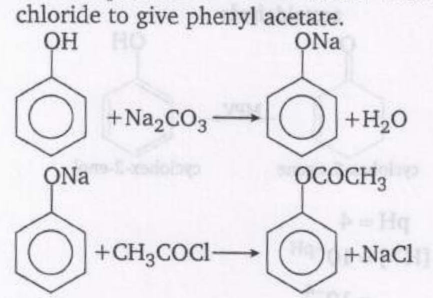
4. Nitrogen form trihalides of the formula NX_3. Among halides of nitrogen, NF_3 is extremely stable due to strong N—F bond in NF_3. Small size of both nitrogen and fluorine atom causes strongness of N—F bond. NCl_3 is explosive yellow oil. NBr_3 and NI_3 are unstable compound and known only as ammonia complex *e.q.*, $NBr_3 \cdot 6NH_3$.

5. $\underset{\text{diethyl ether}}{C_2H_5OC_2H_5} + CH_3\overset{O}{\overset{\|}{C}}Cl \xrightarrow{ZnCl_2} C_2H_5Cl + CH_3COOC_2H_5$

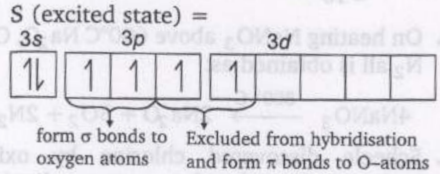


$CH_3CO\boxed{Cl + H}\underset{\text{ethanol}}{OC_2H_5} \longrightarrow CH_3COOC_2H_5 + HCl$

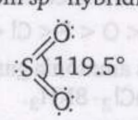
Acetyl chloride does not react direct with phenol. Firstly phenol reacts with alkali to form sodium phenoxide which reacts with acetyl chloride to give phenyl acetate.



6. S (excited state) =



It has two bonds and one lone pair The S—O) double bond arise from $p\pi - d\pi$ bonding due to lateral overlap of *p*-orbitals of oxygen with *d*-orbitals of sulphur. The bond between S and O results from sp^2 hybridization in S-atom.



7. Alkali metal halides have high melting point. The melting points of alkali metal chloride decreases from NaCl to CsCl. (except Li).

LiCl	NaCl	KCl	RbCl	CsCl
833 K	1081 K	1043 K	990 K	918 K

So, NaCl has highest melting point and CsCl has lowest melting point.

8. $t_{1/2} = 6$ min

$N_0 = 32$ g

Total time = 18 min

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$n = \frac{\text{Total time}}{\text{Half-life}} = \frac{18}{6} = 3$$

$$\frac{N}{32} = \left(\frac{1}{2}\right)^3$$

$$\frac{N}{32} = \frac{1}{8}$$

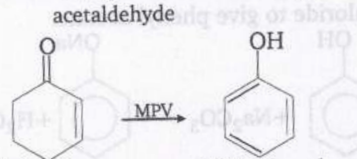
$$N = \frac{32}{8} = 4\text{ g}$$

9. $\begin{array}{l} CH_3COO \\ CH_3COO \end{array} > Ca + Ca < \begin{array}{l} O\,OCH \\ O\,OCH \end{array}$

calcium acetate calcium formate

$\xrightarrow{\Delta} 2CH_3CHO + 2CaCO_3$

acetaldehyde

10. 

cyclohex-2-enone cyclohex-2-enol

11. $pH = 4$

$[H^+] = 10^{-pH}$

$= 10^{-4}$

12. On heating $NaNO_3$ above 600°C Na_2O, O_2 and N_2 all is obtained as:

$$4NaNO_3 \xrightarrow{800°\,C} 2Na_2O + 5O_2 + 2N_2$$

13. Scheele discovered chlorine by oxidising hydrochloric acid with manganese dioxide

$$4HCl + MnO_2 \longrightarrow 2Cl_2 + MnCl_2 + 2H_2O$$

14. Atomic radius decreases in period from left to right and increase in a group from top to bottom. So, the correct answer is:

$$C < O < F < Cl < Br$$

15. $CaCl_2$ absorbs ammonia gas to form an addition compound $CaCl_2 \cdot 8NH_3$.

$$CaCl_2 + 8NH_3 \longrightarrow CaCl_2 \cdot 8NH_3$$

16. No. of atoms = No. of gram molecular weight $\times 6.02 \times 10^{23} \times$ atomicity

No. of atoms in 1.25 g of NH_3

$$= \frac{1.25}{17} \times 6.02 \times 10^{23} \times 4$$

$$= 2.2 \times 10^{23}$$

17. The bond length of

Ethane C—C is 1.54 $sp^3 - sp^3$

Ethene C=C is 1.34 $sp^2 - sp^2$

Ethyne C≡C is 1.20 $sp - sp$

Ethanol C—O is 1.42 $sp^3 - sp$

So, the smallest bond angle is of ethyne.

18. Chloroform when exposed to air and sunlight, oxidised to phosgene, a highly poisonous substance.

$$CHCl_3 + O_2 \xrightarrow[\text{air}]{\text{sunlight}} \underset{\text{phosgene}}{COCl_2} + HCl$$

19. Gypsum is hydrated calcium sulphate.

$$CaSO_4 \cdot 2H_2O$$

Gypsum when heated strongly loses whole of its water of hydration. It does not set like plaster of Paris and so, gypsum is said to be dead burnt.

20. $PbSO_4$ is a white solid, insoluble in water.

$CdSO_4$ is soluble in water and exists as $3CdSO_4 \cdot 8H_2O$

$Bi_2(SO_4)_3$ is a colourless, hygroscopic substance give white precipitate of basic sulphate with water.

$CuSO_4 \cdot 5H_2O$ is a blue crystalline substance, soluble in water.

21. Most of the cuprous compounds are usually colourless and diamagnetic as $3d$ orbital is completely filled ($Cu^{2+} \longrightarrow 3d^{10}$). Cu_2O is exception and red in colour.

22. $[Cr(NH_3)_5]Br_3$

Suppose oxidation state of Cr is x, then.

$$x + (5 \times 0) + (-1 \times 3) = 0$$

$$x - 3 = 0$$

$$x = +3$$

$_{24}Cr = 1s^2\, 2s^2\, 2p^6, 3s^2 3p^6 3d^5, 4s^1$

$Cr^{3+} = 1s^2\, 2s^2\, 2p^6, 3s^2\, 3p^6\, 3d^3$

$3d^3$

1	1	1		

So, three unpaired electrons are present in $[Cr(NH_3)_5]Br_3$.

23. Given:

$$S + \frac{3}{2}O_2 \longrightarrow SO_3 \quad \Delta n = 2x \quad \ldots(i)$$

$$SO_2 + \frac{1}{2}O_2 \longrightarrow SO_3 \quad \Delta n = y \quad \ldots(ii)$$

Heat of formation of SO_2 can be calculated by reversing the Eq. (ii) and then adding the Eqs. (i) and (iii).

$$SO_3 \longrightarrow SO_2 + \frac{1}{2}O_2 - y \quad \ldots(iii)$$

Now, by adding the Eqs. (i) and (ii)

$$S + \frac{3}{2}O_2 \longrightarrow SO_3 + 2x \quad \ldots(i)$$

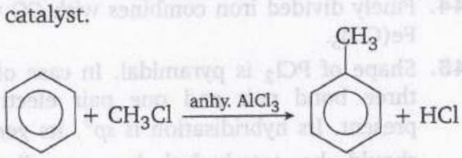
$$SO_3 \longrightarrow SO_2 + \frac{1}{2}O_2 - y \quad \ldots(ii)$$

$$S + O_2 \longrightarrow SO_2 + (2x - y) \quad \ldots(iii)$$

So, heat of formation of SO_2 is $2x - y$.

24. Friedel-Craft's reaction: In this reaction alkyl group is introduced in the benzene ring in presence of anhy. $AlCl_3$

25. Even at 400-500°C the rate of reaction is very low. Therefore, to increase the reaction velocity a suitable catalyst is used. Now a days most of the sulphuric acid plants use V_2O_5 as a catalyst.



26. When acetamide is treated with NaBr or $Br_2/NaOH$, Hofmann degradation reaction takes place and a primary amine is obtained as:

$$CH_3CONH_2 + NaOBr \text{ or } Br_2/NaOH \longrightarrow \underset{\substack{\text{methyl amine}\\ \text{or}\\ \text{methanamine}}}{CH_3NH_2} + 2NaBr + NaHCO_3 + H_2O$$

27. Isocyanides are prepared by carbylamine reaction. In this treated with chloroform, and NaOH or KOH to give isocyanide. (unpleasant smelling compound)

$$CHCl_3 + 3KOH + \underset{\substack{\text{primary}\\ \text{amine}}}{R\text{—}NH_2} \longrightarrow \underset{\text{isocyanide}}{R\text{—}NC} + 3H_2O + 3KCl$$

28. If $r_x = r$ then $r_{CH_4} = 2r$

$M_{CH_4} = 12 + 4 = 16$

$M_x = ?$

We know that,

$$\frac{r_{CH_4}}{r_x} = \sqrt{\frac{M_x}{M_{CH_4}}}$$

$$\frac{2r}{r} = \sqrt{\frac{M_x}{16}}$$

$$(2)^2 = \frac{M_x}{16}$$

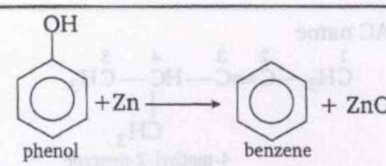
$$4 = \frac{M_x}{16}$$

or $\quad M_x = 64$

29. Lewis acids are the compounds having a tendency to accept a pair of electron. *e.g.*, BF_3, $AlCl_3$, BeF_2, etc.So, $SnCl_4$ is not a Lewis acid.

30. Natural gas mainly consists of CH_4 (methane)

31. Phenol when treated with Zn powder, benzene is obtained as :



32. Isoelectronic species are the species having same number of electrons.

Number of electrons in CO $= 6 + 8 = 14$

Number of electrons in $N_2^- = 14 + 1 = 15$

Number of electrons in $N_2^+ = 14 - 1 = 13$

Number of electrons in $CN^- = 6 + 7 + 1 = 14$

Number of electrons in NO $= 7 + 8 = 15$

So, CN^- and CO are isoelectronic species.

33. Firstly molarity is converted into normality as

Molarity × mol. wt. = Normality × Eq. wt.

For H_2SO_4

$$1 \times 98 = N \times 49$$

$$N = \frac{98}{49} = 2$$

For NaOH,

$$1M = 1N$$

Now from normality equation

$$H_2SO_4 = NaOH$$

$$N_1V_1 = N_2V_2$$

$$2 \times V_1 = 1 \times 20$$

$$V_1 = \frac{20}{2} = 10 \text{ mL of } H_2SO_4$$

34. A solution of H_2O_2 with Fe^{2+} (Ferrous) salt is known as Fenton's reagent. ($FeSO_4 + H_2O_2$)

35.

$$\overset{+2}{Cu^{2+}} + \overset{0}{Ag} \longrightarrow \overset{0}{Cu} + \overset{+1}{Ag^+}$$

Reduction: $Cu^{2+} \rightarrow Cu$; Oxidation: $Ag \rightarrow Ag^+$

So, oxidation half reaction is $Ag \longrightarrow Ag^+$

36. $C_4H_{10}O$ form total seven isomers (four alcohol) + three ether)

(1) $CH_3CH_2CH_2CH_2OH$

(2) $CH_3\text{—}\underset{\substack{|\\ OH}}{CH}\text{—}CH_2\text{—}CH_3$

(3) $(H_3C)_2\rangle CH\text{—}CH_2OH$

(4) $(H_3C)_3 \rangle C\text{—}OH$

37. IUPAC name

$$\overset{1}{C}H_3-\overset{2}{C}\equiv\overset{3}{C}-\overset{4}{H}C(CH_3)-\overset{5}{C}H_3$$

4-methyl-2-pentyne

38. Glucose reduces Fehling solution to give red ppt. of Cu_2O as:

$$CHO(CHOH)_4CH_2OH + \underset{\text{Fehling solution}}{2Cu(OH)_2} + NaOH \longrightarrow \underset{\text{red ppt.}}{Cu_2O} + COONa(CHOH)_4CH_2OH + 3H_2O$$

Fructose and sucrose does not respond to Fehling solution.

39. Number of primary amines formed from $C_4H_{11}N$ is four:

as

(1) $CH_3CH_2CH_2CH_2NH_2$

(2) $CH_3-CH(NH_2)-CH_2-CH_3$

(3) $(H_3C)_2CH-CH_2NH_2$

(4) $(H_3C)_3C-NH_2$

40. Given

$$3X + Y \rightleftharpoons X_3Y - Q \text{ cal.}$$

According to Le-Chatelier principle amount of X_3Y can be changed by changing temperature because reaction is endothermic and it can be changed by changing in pressure because number of moles of reactant and products are different. But can never change by adding catalyst because the above example is (a) reversible reaction.

41. According to Fajan's rule compounds having large charge on cation are more covalent or least electrovalent. So, compounds having low charge on cation will be more electrovalent or ionic.

$\overset{+4}{Mn}O_2$ $\quad \overset{+7}{Mn_2}O_7$

$\overset{+5}{P_2}O_5$ $\quad \overset{+3}{P_2}O_3$

So, P_2O_3 will be most ionic.

42. $CuSO_4$ reacts with KI to give I_2 as :

$$2CuSO_4 + 4KI \longrightarrow 2CuI + K_2SO_4 + I_2$$

43. Bromine liberates iodine from KI solution because of its oxidising property. Colour of iodine is grey black so, black colour is obtained.

$$2KI + Br_2 \longrightarrow 2KBr + I_2 \text{ (Black or grey)}$$

44. Finely divided iron combines with CO to give $Fe(CO)_5$.

45. Shape of PCl_3 is pyramidal. In case of PCl_3, three bond pair and one pair electrons is present. Its hybridisation is sp^3. Its geometry should be tetrahedral but actually it is **pyramidal** due to the presence of **lone pair of electron.**

46. Solid NaCl contains immobile ions. Diamond and sulphur are non ionic substances. Molten KBr contains free ions. So, it conducts electricity.

47. $KE = \frac{3}{2}nRT$

$KE \propto n \qquad KE \propto \frac{w}{M}$

$$\frac{K_{H_2}}{K_{He}} = \frac{M_{He}}{M_{H_2}} = \frac{2}{2} = \frac{1}{1}$$

$$KE_{H_2} : KE_{He} = -1 : 1$$

48. Energy is highest in the outermost shell

$n = 2, l = 1, = 2p$

$n = 3, l = 2 = 3d$

$n = 3, l = 1 = 3p$

$n = 2, l = 0 = 2s$

According to aufbau's principle $3d$ has highest energy

49. Pyrophosphoric acid is

$$\overset{+1}{H_4}\overset{x}{P_2}\overset{-2}{O_7}$$

$$(+1\times 4) + 2x + (-2\times 7) = 0$$

$$+4 + 2x - 14 = 0$$

$$2x = 14 - 4$$

$$2x = 10$$

$$x = +5$$

50. First order kinetic equation is :

$$k = \frac{2.303}{t}\log\frac{a}{(a-x)}$$

or $$t = \frac{2.303}{k}\log\frac{a}{(a-x)}$$

According to question

$$8 = \frac{2.303}{k}\log\frac{100}{100-75}$$

$$8 = \frac{2.303}{k} \log \frac{100}{25}$$

$$8 = \frac{2.303}{k} \log 4 \quad \ldots(i)$$

And

$$t = \frac{2.303}{k} \log \frac{100}{100-50}$$

$$= \frac{2.303}{k} \log 2 \quad \ldots(ii)$$

By dividing Eq. (ii) by Eq. (i), we get

$$\frac{t}{8} = \frac{\frac{2.303}{k}\log 2}{\frac{2.303}{k} \times 2\log 2}$$

$$\frac{t}{8} = \frac{\log 2}{2\log 2}$$

$$\frac{t}{8} = \frac{1}{2}$$

$$2t = 8$$

$$t = \frac{8}{2} = 4 \text{ min}$$

Mathematics

1. We have, $\alpha^2 = 5\alpha - 3$

$\Rightarrow \quad \alpha^2 - 5\alpha + 3 = 0$

$\Rightarrow \quad \alpha = \dfrac{5 \pm \sqrt{13}}{2}$

Similarly, $\beta^2 = 5\beta - 3$

$\Rightarrow \quad \beta = \dfrac{5 \pm \sqrt{13}}{2}$

Since, $\quad \alpha \neq \beta$

$\therefore \quad \alpha = \dfrac{5+\sqrt{13}}{2}$ and $\beta = \dfrac{5-\sqrt{13}}{2}$

or $\quad \alpha = \dfrac{5-\sqrt{13}}{2}$

and $\quad \beta = \dfrac{5+\sqrt{13}}{2}$

$\therefore \quad \alpha^2 + \beta^2 = \dfrac{50+26}{4} = 19$

and $\quad \alpha\beta = \dfrac{1}{4}(25-13) = 3$

Thus, the equation having $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ as its roots is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow \quad x^2 - x\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right) + 1 = 0$$

$$\Rightarrow \quad 3x^2 - 19x + 3 = 0$$

2. We have, $y = (x + \sqrt{1+x^2})^n \quad \ldots(i)$

Let $\quad \dfrac{d^2y}{dx^2} = y_2$

and $\quad \dfrac{dy}{dx} = y_1$

On differentiating Eq. (i), we get

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{n(x+\sqrt{1+x^2})^n}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{ny}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad y_1^2(1+x^2) = n^2y^2$$

Again differentiating, we get

$$2y_1y_2(1+x^2) + 2xy_1^2 = 2n^2yy_1$$

On dividing by $2y_1$, we get

$$y_2(1+x^2) + xy_1 = n^2y$$

$$\Rightarrow \frac{d^2y}{dx^2}(1+x^2) + x\frac{dy}{dx} = n^2y$$

3. $1, \log_3\sqrt{3^{1-x}+2}, \log_3(4\cdot 3^x - 1)$ are in AP

$\therefore 2\log_3(3^{1-x}+2)^{1/2} = 1 + \log_3(4\cdot 3^x - 1)$

$\log_3(3^{1-x}+2) = \log_3 3 + \log_3(4\cdot 3^x - 1)$

$\log_3(3^{1-x}+2) = \log_3[3(4\cdot 3^x - 1)]$

$$3^{1-x} + 2 = 3(4\cdot 3^x - 1)$$

$$3\cdot 3^{-x} + 2 = 12\cdot 3^x - 3$$

Let $3^x = t$

$$\therefore \quad \frac{3}{t} + 2 = 12t - 3$$

$$3 + 2t = 12t^2 - 3t$$

$$12t^2 - 5t - 3 = 0$$

$$(12t^2 - 9t + 4t - 3) = 0$$

$$3t(4t-3) + 1(4t-3) = 0$$

$$\therefore \quad t = -\frac{1}{3}, \frac{3}{4}$$

$$3^x = \frac{3}{4}$$

$$\Rightarrow \quad x = \log_3 \left(\frac{3}{4}\right) = \log_3 3 - \log_3 4$$

$$\Rightarrow \quad x = 1 - \log_3 4$$

4. $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}$ and $P(E_3) = \frac{1}{4}$

$$P(E_1 \cup E_2 \cup E_3) = 1 - P(\bar{E}_1) P(\bar{E}_2) P(\bar{E}_3)$$

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)$$

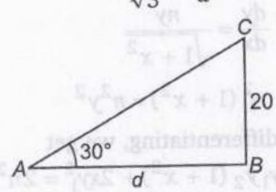
$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$$

Note : If the events E_1, E_2 and E_3 are independent, then $\bar{E}_1, \bar{E}_2$ and $\bar{E}_3$ are also independent.

5. In ΔABC,

$$\tan 30° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{20}{d}$$



$$\Rightarrow \quad d = 20\sqrt{3} \text{ m}$$

6. Let the first term and common ratio of a GP be A and R respectively.

$$l = AR^{p-1} \Rightarrow \log l = \log A + (p-1) \log R$$

$$m = AR^{q-1} \Rightarrow \log m = \log A + (q-1) \log R$$

$$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1) \log R$$

Now,
$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix} = 0$$

7. $\lim\limits_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2}\,x}$

$$= \lim_{x \to 0} \frac{\sqrt{1 - (1 - 2\sin^2 x)}}{\sqrt{2}\,x}$$

$$= \lim_{x \to 0} \frac{\sqrt{2\sin^2 x}}{\sqrt{2}\,x} = \lim_{x \to 0} \frac{|\sin x|}{x}$$

Let $\quad f(x) = \frac{|\sin x|}{x}$

Now, $f(0+0) = \lim\limits_{h \to 0} \frac{|\sin(0+h)|}{0+h}$

$$= \lim_{h \to 0} \frac{\sin h}{h} = 1$$

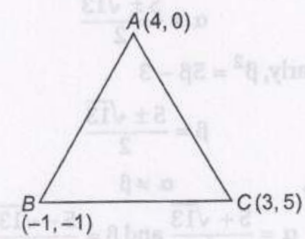
and $\quad f(0-0) = \lim\limits_{h \to 0} \frac{|\sin(0-h)|}{-h} = -1$

$\therefore$ The limit of function does not exist.

8. $AB = \sqrt{(4+1)^2 + (0+1)^2} = \sqrt{26}$

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

$CA = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{26}$



So, in isosceles triangles side $AB = CA$

For right angled triangle

$$BC^2 = AB^2 + AC^2$$

So, here $\quad BC = \sqrt{52} = BC^2 = 52$

or $\quad (\sqrt{26})^2 + (\sqrt{26})^2 = 52$

So, given vertices is right angled and also is isosceles triangle.

9. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$$

$$\Rightarrow \quad \tan^{-1} \frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \left(\frac{1}{\sqrt{\cos\alpha}}\right)\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \quad \tan^{-1} \frac{1 - \cos\alpha}{2\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \quad \tan x = \frac{1 - \cos\alpha}{2\sqrt{\cos\alpha}}$$

$$\Rightarrow \quad \cot x = \frac{2\sqrt{\cos\alpha}}{1 - \cos\alpha}$$

$$\Rightarrow \quad \operatorname{cosec} x = \frac{1+\cos\alpha}{1-\cos\alpha}$$

$$\therefore \quad \sin x = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$= \frac{1-\left(1-2\sin^2\frac{\alpha}{2}\right)}{1+2\cos^2\frac{\alpha}{2}-1}$$

$$\Rightarrow \quad \sin x = \tan^2\frac{\alpha}{2}$$

10. **Key Idea :** *If a plane contains a line, then the normal to the plane is perpendicular to the line.*

Any plane passing through (3, 2, 0) is

$$a(x-3)+b(y-2)+c(z-0)=0 \quad \text{...(i)}$$

It passes through (3, 6, 4)

$$\therefore \quad 0.a+4b+4c=0 \quad \text{...(ii)}$$

Normal to plane (i) is perpendicular to given line

$$\therefore a+5b+4c=0 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$\frac{a}{1}=\frac{b}{-1}=\frac{c}{1}=k$$

So, $\quad a=k,\ b=-k,\ c=k$

Putting the value of a, b, c in Eq. (i), we get

$$x-y+z=1$$

Note : In any line of a plane it has only one direction cosines and direction ratios may be more than one.

11. $\dfrac{d^2y}{dx^2}=e^{-2x}$

On integrating, we get

$$\frac{dy}{dx}=\frac{e^{-2x}}{-2}+c$$

Again integrating, we get

$$y=\frac{e^{-2x}}{4}+cx+d$$

12. $\lim\limits_{x\to\infty}\left[1+\dfrac{4x+1}{x^2+x+2}\right]^x = \lim\limits_{x\to\infty}(1+\alpha)^{1/\alpha]\alpha x}$

where $\alpha=\dfrac{4x+1}{x^2+x+2}$

$$=\frac{4+\frac{1}{x}}{x\left(1+\frac{1}{x}+\frac{2}{x^2}\right)}\to 0 \text{ as } x\to\infty$$

and $\alpha x = \dfrac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{2}{x^2}} \to 4 \text{ as } x\to\infty$

Given limit $= e^4$

13. **Key Idea :** *The domain of* $\sin^{-1}x$ *is* $[-1, 1]$.

$$f(x)=\sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right) \text{ exists, if}$$

$$-1\le\log_3\left(\frac{x}{3}\right)\le 1$$

$$\Rightarrow \quad 3^{-1}\le\frac{x}{3}\le 3^1$$

$$\Rightarrow \quad 1\le x\le 9 \Rightarrow \text{domain is } [1, 9].$$

14. $2^{1/4}.4^{1/8}.8^{1/16}\dots\infty$

$$=2^{\frac14+\frac28+\frac3{16}+\dots\infty}=2^{\frac{1}{2^2}\left(1+\frac22+\frac3{2^2}+\frac4{2^3}+\dots\right)}$$

$$=2^{\frac{1}{2^2}\left(\frac{1}{1-\frac12}+\frac{1\cdot\frac12}{\left(1-\frac12\right)^2}\right)}=2^{\frac{1}{2^2}|2+2|}=2^1=2$$

15. $$ar^4=2 \quad \text{...(i)}$$

Let a be the first term and r be the common ratio, then according to the given condition.

Since, $\quad a\times ar\times ar^2\times ar^3\times ar^4\times ar^5 \times ar^6\times ar^7\times ar^8$

$$=a^9r^{36}=(ar^4)^9=2^9=512$$

16. **Key Idea :** *The period of* $|\sin x|$ *is* π.

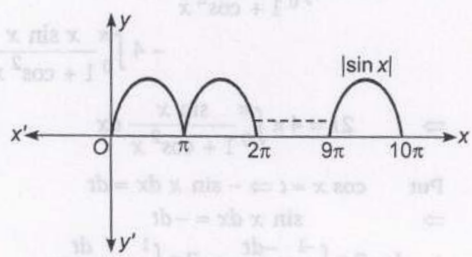
$$\int_0^{10\pi}|\sin x|\,dx$$

$$=10\left[\int_0^{\pi/2}\sin x\,dx+\int_{\pi/2}^{\pi}\sin x\,dx\right]$$

$$=10[-\cos x]_0^{\pi/2}+[-\cos x]_{\pi/2}^{\pi}$$

$$=10[1+1]=10\times 2=20$$

Alternative Solution :



$\therefore$ Required area

$$=10\int_0^{\pi}\sin x\,dx$$

$$=10[-\cos x]_0^{\pi}=-10(\cos\pi-\cos 0)=20$$

17. $I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x\,(1+\tan^2 x)\,dx$

$$= \int_0^{\pi/4} \tan^n x \sec^2 x dx$$

$$= \int_0^1 t^n\,dt \quad \text{where } t = \tan x$$

$$\Rightarrow \quad I_n + I_{n+2} = \frac{1}{n+1}$$

$$\therefore \quad \lim_{n\to\infty} n[I_n + I_{n+2}]$$

$$= \lim_{n\to\infty} n\cdot\frac{1}{n+1} = \lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)} = 1$$

18. Key Idea : $\int_{-\pi}^{\pi} f(x)\,dx$ *{= 2f(x), if f(x) is an even function.*

= 0, if f(x) is an odd function.

Let $$I = \int_{-\pi}^{\pi}\frac{2x\,(1+\sin x)}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad I = \int_{-\pi}^{\pi}\frac{2x}{1+\cos^2 x}\,dx + 2\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad I = 0 + 4\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}\,dx$$

$$\left(\because \frac{2x}{1+\cos^2 x}\,dx \text{ is an odd function}\right)$$

$$\Rightarrow \quad I = 4\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$$

$$\Rightarrow \quad I = 4\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad I = 4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\,dx - 4\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}\,dx$$

$$\Rightarrow \quad 2I = 4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\,dx$$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

$\Rightarrow \quad \sin x\,dx = -dt$

$$\therefore \quad I = 2\pi\int_1^{-1}\frac{-dt}{1+t^2} = 2\pi\int_{-1}^{1}\frac{dt}{1+t^2}$$

$$= 2\pi[\tan^{-1} t]_{-1}^{1}$$

$$= 2\pi[\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$= 2\pi\left[\frac{\pi}{4}+\frac{\pi}{4}\right] = 2\pi\times\frac{\pi}{2} = \pi^2$$

19. Key Idea : *The period of* $\sin^4 x$ *or* $\cos^4 x$ *is* $\frac{\pi}{2}$.

$$f(x) = \sin^4 x + \cos^4 x$$

$$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$$

$$= 1 - \frac{1}{2}(\sin 2x)^2$$

$$= 1 - \frac{1}{2}\left[\frac{1-\cos 4x}{2}\right]$$

$$= \frac{3}{4} + \frac{1}{4}\cos 4x$$

Since, $\cos x$ is periodic with period 2π.

$\therefore$ The period of $f(x) = \frac{2\pi}{4} = \frac{\pi}{2}$.

Alternative Solution :

Since the period of $\sin^4 x$ and $\cos^4 x$ is $\frac{\pi}{2}$.

$\therefore$ Period of $f(x) = \sin^4 x + \cos^4 x$ is the LCM of the period of $\sin^4 x$ and $\cos^4 x$.

$\therefore$ Required period is $\frac{\pi}{2}$.

20. $x^y = e^{x-y}$

Taking log on both sides, we get

$$y\log x = x - y$$

$$\Rightarrow \quad y(\log x + 1) = x$$

$$\Rightarrow \quad y = \frac{x}{1+\log x}$$

On differentiating w.r.t x, we get

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(1+\log x) - x\times\frac{1}{x}}{(1+\log x)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

21. Key Idea : *The angle between the curves is the angle between the tangents to the curve.*

We have, $x^3 - 3xy^2 + 2 = 0$ ...(i)

and $3x^2y - y^3 - 2 = 0$ ...(ii)

On differentiating Eqs. (i) and (ii) with respect to x, we get

$$\left(\frac{dy}{dx}\right)_{c_1} = m_1 = \frac{x^2-y^2}{2xy}$$

and $$\left(\frac{dy}{dx}\right)_{c_2} = m_2 = \frac{-2xy}{x^2-y^2}$$

Since $m_1 \times m_2 = -1$

Hence, the two curves cut at right angles.

22. Key Idea : *A function is said to be increasing, if $f'(x) > 0$ and it said be decreasing if $f'(x) < 0$.*

Given, $f(x) = \cot^{-1} x + x$

$$\Rightarrow \quad f'(x) = -\frac{1}{1+x^2} + 1$$

$$= \frac{x^2}{1+x^2}, \text{ clearly, } f'(x) > 0 \text{ for all } x.$$

So, $f(x)$ increases in $(-\infty, \infty)$.

23. We have,

$$f(x) = (x+1)^{1/3} - (x-1)^{1/3}$$

$$\therefore \quad f'(x) = \frac{1}{3}\left[\frac{1}{(x+1)^{2/3}} - \frac{1}{(x-1)^{2/3}}\right]$$

$$= \frac{(x-1)^{2/3} - (x+1)^{2/3}}{3(x^2-1)^{2/3}}$$

Clearly, $f'(x)$ does not exists at $x = \pm 1$ for maxima or minima, put

$f'(x) = 0$, then $(x-1)^{2/3} = (x+1)^{2/3}$

$$\Rightarrow \quad x = 0$$

Clearly, $f'(x) \neq 0$ for any other value of $x \in [0, 1]$. The value of $f(x)$ at $x = 0$ is 2.

Hence, the greatest value of $f(x)$ is 2.

Note : If any function has no critical point in the given interval $[a, b]$, then we check the maximum value of $x = a, b$.

24. Let $I = \int \frac{dx}{x(x^n+1)}$

Putting $\quad x^n + 1 = t$

$$\Rightarrow \quad n x^{n-1} dx = dt$$

$$\therefore \quad I = \frac{1}{n}\int \frac{dt}{t(t-1)}$$

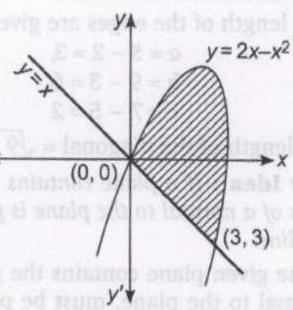
$$= \frac{1}{n}\int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt$$

$$= \frac{1}{n}\log\left(\frac{t-1}{t}\right) + c$$

$$= \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + c$$

25. Required area $= \int_0^3 (y_1 - y_2)\,dx$

$$= \int_0^3 (2x - x^2) - (-x)\,dx$$



$$= \int_0^3 (3x - x^2)\,dx$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{27}{2} - 9$$

$$= \frac{9}{2} \text{ sq unit}$$

26. The general equation of all non-vertical lines in a plane is $ax + hy = 1$, where $h \neq 0$

On differentiating w.r.t x, we get

$$a + h\frac{dy}{dx} = 0$$

Again differentiating, we get

$$h\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 0$$

27. The required vector is along the vector

$$\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\,\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\,\vec{\mathbf{a}}$$

$$= -(\hat{\mathbf{i}} - \hat{\mathbf{j}}) - 2(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$$

$$= -3\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$$

Hence, required vector are given by

$$\pm\frac{(-3\hat{\mathbf{i}} - 3\hat{\mathbf{j}})}{\sqrt{9+9}} = \pm\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

28. We have, $2|\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}|$

$$= |4\hat{\mathbf{i}} + (4x-2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}|$$

$$\Rightarrow \quad 4(x^2+10) = 20 + (4x-2)^2$$

$$\Rightarrow \quad 3x^2 - 4x - 4 = 0$$

$$\Rightarrow \quad x = 2, -\frac{2}{3}$$

29. The length of the edges are given by

$$a = 5 - 2 = 3,$$
$$b = 9 - 3 = 6$$
and $$c = 7 - 5 = 2$$

So, length of the diagonal $= \sqrt{9 + 36 + 4} = 7$

30. Key Idea : *If a plane contains a line, then the DR's of a normal to the plane is perpendicular to the line.*

If the given plane contains the given line, then normal to the plane, must be perpendicular to the line and the condition for the same is

$$al + bm + cn = 0.$$

31. The number of ways in which either player can choose a number from 1 to 25 is 25, so the total number of ways a choosing numbers is $25 \times 25 = 625$. So the probability that they will not win a prize in a single trial

$$= 1 - \frac{1}{25} = \frac{24}{25}$$

32. The equation of parabola is

$$(y + 2)^2 = -4\left(x - \frac{1}{2}\right)$$

Shifting the origin $\left(\frac{1}{2}, -2\right)$, the equation of parabola becomes $Y^2 = -4X$

where $X = x - \frac{1}{2}, Y = y + 2$.

An equation of its directrix is $X = 1$.

$\therefore$ Required directrix, $x = \frac{3}{2}$.

33. Key Idea : *For making a triangle, we needs a three vertices of a triangle.*

$$\Rightarrow \quad T_{n+1} - T_n = 21$$
$$\Rightarrow \quad {}^{n+1}C_3 - {}^nC_3 = 21 \qquad (\because T_n = {}^nC_3)$$
$$\Rightarrow \quad {}^nC_2 + {}^nC_3 - {}^nC_3 = 21 \Rightarrow {}^nC_2 = 21$$
$$\Rightarrow \quad \frac{n(n-1)}{2} = 21$$
$$\Rightarrow \quad n^2 - n - 42 = 0$$
$$\therefore \quad n = 7$$

34. We have, $A + C = \pi - B$ and

$$\frac{A - B + C}{2} = \frac{\pi}{2} - B$$

$$\therefore 2ca \sin\left(\frac{\pi}{2} - B\right) =$$

$$2ca \cos B = 2ca \frac{c^2 + a^2 - b^2}{2ca} = a^2 + c^2 - b^2$$

35. $$\lim_{x \to \infty}\left(\frac{x - 3}{x + 2}\right)^x = \lim_{x \to \infty}\left(\frac{1 - \frac{3}{x}}{1 + \frac{2}{x}}\right)^x$$

$$= \lim_{x \to \infty}\frac{\left(1 - \frac{3}{x}\right)^x}{\left(1 + \frac{2}{x}\right)^x}$$

$$= \frac{e^{-3}}{e^2} = e^{-5}$$

36. Key Idea : *If triangle is an equilateral, then orthocentre, incentre and centroid will be same.*

Let $A(1, \sqrt{3}), B(0, 0), C(2, 0)$ be the given points.

$$\therefore \quad a = BC$$
$$= \sqrt{(2-0)^2 + (0-0)^2} = 2$$
$$\therefore \quad b = CA$$
$$= \sqrt{(2-1)^2 + (0-\sqrt{3})^2} = 2$$
$$c = AB$$
$$= \sqrt{(0-1)^2 + (0-\sqrt{3})^2} = 2$$

Triangle is equilateral.

$\therefore$ Incentre is the same as centroid of the triangle.

$\therefore$ Incentre is $\left(\frac{1 + 0 + 2}{3}, \frac{\sqrt{3} + 0 + 0}{3}\right)$

i.e., $$\left(1, \frac{1}{\sqrt{3}}\right)$$

37. We have, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$$\therefore \quad \vec{\mathbf{a}} + \vec{\mathbf{b}} = -\vec{\mathbf{c}}$$
$$\Rightarrow \quad \vec{\mathbf{a}} \times \vec{\mathbf{c}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} = 0$$
$$\Rightarrow \quad \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$$

Similarly, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

38. Key Idea : *The cubic roots of unity are* $1, \omega, \omega^2$ *and* $1 + \omega + \omega^2 = 0$.

We have, $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$

$$= (-2)^7 (\omega^2)^7$$
$$= -128\,\omega^2$$

39. Key Idea : $\sin^2\theta$ *is lies between 0 to 1.*

We know that $\sin^2\theta \geq 1$

$$\Rightarrow \quad \frac{4xy}{(x+y)^2} \geq 1 \quad \Rightarrow \quad 4xy \geq (x+y)^2$$

$$\Rightarrow \quad (x-y)^2 \leq 0$$

$$\Rightarrow \quad x - y = 0$$

$$\Rightarrow y = x \quad \text{and} \quad x \neq 0, y \neq 0$$

40. We have, $a^2 = 16$

and $\quad 9 = b^2 = a^2(1-e^2)$

$$\Rightarrow \quad 9 = 16(1-e^2)$$

$$\therefore \quad e = \frac{\sqrt{7}}{4}$$

Thus, the foci are $(\pm\sqrt{7}, 0)$.

The radius of required circle

$$= \sqrt{(\sqrt{7}-0)^2 + 3^2}$$

$$= \sqrt{7+9}$$

$$= 4$$

41. Applying $R_1 \to R_1 + R_3$, we obtain

$$\begin{vmatrix} 1-i & \omega^2+\omega & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix} = 0$$

$\because \ \omega^2 + \omega = -1$ which R_1 and R_2 become identical.

42. $\lim\limits_{x \to 2} \dfrac{x f(2) - 2f(x)}{x-2}$

$$= \lim_{x \to 2} \frac{x f(2) - 2f(2) + 2f(2) - 2f(x)}{x-2}$$

$$= \lim_{x \to 2} \frac{(x-2) f(2)}{x-2} - 2 \lim_{x \to 2} \frac{f(x) - f(2)}{x-2}$$

$$= f(2) - 2f(2)$$

$$= 4 - 2 \times 4 = -4$$

43. For the two lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$, the angle bisects are given by

$$\frac{24x + 7y - 20}{25} = \pm \frac{4x - 3y - 2}{5}$$

Taking positive sign, we get

$$2x + 11y - 5 = 0$$

$\therefore$ The given three lines are concurrent with on line bisecting the angle between the other two.

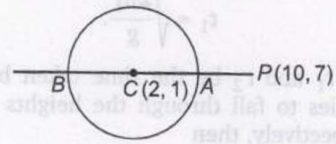
44. $\sqrt{3}x + y = 0$ makes an angle of 120° with OX and $\sqrt{3}x - y = 0$ makes an angle 60° with OX.

So, the required line is $y - 2 = 0$

45. Key Idea : *If a point is outside the circle, then the greatest distance is equal to the length of diameter and lowest distance between the circle and a point.*

Since, $S_1 = 10^2 + 7^2 - 4 \times 10 - 2 \times 7 - 20 > 0$.

So, P lies outside the circle. Join P with the centre $C(2, 1)$ of the given circle. Suppose PC cuts the circle at A and B, then PB is the greatest distance of P from the circle.



Now, $PC = \sqrt{(10-2)^2 + (7-1)^2} = 10$

$BC = \sqrt{4 + 1 + 20} = 5$

$\therefore \quad PB = PC + CB$

$= 10 + 5 = 15$

46. Let the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

It is given that $e = \dfrac{1}{2}$ and $ae = 2$

Therefore, $a = 4$

Now, $b^2 = a^2(1-e^2)$

$b^2 = 12$

Thus, the required ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1$

47. The mid point of the chord is $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$. The equation of the chord in terms of its mid point is $T = S$.

or $\quad x\left(\dfrac{y_1+y_2}{2}\right) + y\left(\dfrac{x_1+x_2}{2}\right)$

$$= 2\left(\frac{x_1+x_2}{2}\right)\left(\frac{y_1+y_2}{2}\right)$$

$$x(y_1+y_2) + y(x_1+x_2) = (x_1+x_2)(y_1+y_2)$$

$$\frac{x}{x_1+x_2} + \frac{y}{y_1+y_2} = 1$$

48. We have,

$$\frac{P}{3} = \frac{Q}{7} = \frac{R}{5} = \lambda \quad \text{(say)}$$

$$\Rightarrow \quad P = 3\lambda, Q = 7\lambda, R = 5\lambda$$

$$\therefore \quad R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$\Rightarrow \quad 25\lambda^2 = 9\lambda^2 + 49\lambda^2 + 42\lambda^2 \cos\theta$$

$$\Rightarrow \quad -33 = 42\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{11}{14}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(-\frac{11}{14}\right)$$

49. Key Idea : *Time taken by the body to fall through the height h_1 is*

$$t_1 = \sqrt{\frac{2h_1}{g}}.$$

Let t_1 and t_2 be the time taken by the two bodies to fall through the heights h_1 and h_2 respectively, then

$$h_1 = \frac{1}{2}gt_1^2 \text{ and } h_2 = \frac{1}{2}gt_2^2$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{2h_1}{g}}, t_2 = \sqrt{\frac{2h_2}{g}}$$

$$\therefore \quad t_1 : t_2 = \sqrt{\frac{2h_1}{g}} : \sqrt{\frac{2h_2}{g}}$$

$$\Rightarrow \quad t_1 : t_2 = \sqrt{h_1} : \sqrt{h_2}$$

50. Key Idea : *If the variance of x and y is var (x) and var (y) and covariance of x and y is cov (x, y), then the correlation coefficient is*

$$r_{xy} = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x) \cdot \text{var}(y)}}$$

We know,

$$r_{xy} = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x) \cdot \text{var}(y)}} = \frac{10.2}{\sqrt{(8.25) \cdot (33.96)}}$$

$$= \frac{10.2}{16.74}$$

$$= 0.61$$

Note : If two variables are independent, then correlation coefficient is zero.