CBSE Class 11 Mathematics

Important Questions

Chapter 15

Statistics

1 Marks Questions

1. In a test with a maximum marks 25, eleven students scored 3,9,5,3,12,10,17,4,7,19,21 marks respectively. Calculate the range.

Ans. The marks can be arranged in ascending order as 3,3,4,5,7,9,10,12,17,19,21.

Range = maximum value – minimum value

=21-3

= 18

2. Coefficient of variation of two distributions is 70 and 75, and their standard deviations are 28 and 27 respectively what are their arithmetic mean?

Ans. Given C.V (first distribution) = 70

Standard deviation = σ_1 = 28

C.V $\frac{\sigma 1}{\overline{x}1}\times 100$

= $70=\frac{28}{\overline{x}1}\times 100$

$\overline{x}=\frac{28}{70}\times 100$

$\overline{x}=40$

Similarly for second distribution

$$C.V = \frac{\sigma_2}{x_2} \times 100$$

$$75 = \frac{27}{\bar{x}_2} \times 100$$

$$\bar{x}_2 = \frac{27}{75} \times 100$$

$$\bar{x}_2 = 36$$

3. Write the formula for mean deviation.

Ans. MD $\left(\bar{x}\right) = \frac{\sum f_i \,|\, x_i - \bar{x} \,|}{\sum f_i} = \frac{1}{x} \sum f_i \,|\, x_i - \bar{x} \,|$

4. Write the formula for variance

Ans. Variance $\sigma^2 = \frac{1}{n} \sum f_i \left(x_i - \bar{x}\right)^2$

5. Find the median for the following data.

x_i **579101215**

f_i **862226**

Ans.

x_i	**5**	**7**	**9**	**10**	**12**	**15**
f_i	8	6	2	2	2	6
$c.f$	8	14	16	18	20	26

$n = 26$. Median is the average of 13^{th} and 14^{th} item, both of which lie in the c.f 14

$\therefore x_i = 7$

$$\therefore \text{ median} = \frac{13\text{observation} + 14\text{th observation}}{2}$$

$$= \frac{7+7}{2} = 7$$

6. Write the formula of mean deviation about the median

Ans. $MD.(M) = \frac{\sum f_i | x_i M |}{\sum f_i} = \frac{1}{n} \sum f_i | x_i - M |$

7. Find the rang of the following series 6,7,10,12,13,4,8,12

Ans. Range = maximum value – minimum value

= 113-4

=9

8. Find the mean of the following data 3,6,11,12,18

Ans. Mean = $\frac{\text{sun of observation}}{\text{Total no of observation}}$

$$= \frac{50}{5} = 10$$

9. Express in the form of a + ib (3i-7) + (7-4i) – (6+3i) + i^{23}

Ans. Let

Z = $\cancel{3i} - \cancel{7} + \cancel{7} - 4i - 6 - \cancel{3i} + (i^4)^5 \, i^3$

$$= -4i - 6 - i \left[\begin{array}{l} \because i^4 = 1 \\ i^3 = -i \end{array} \right.$$

$$= -5i - 6$$

$$= -6 + (-5i)$$

10. Find the conjugate of $\sqrt{-3} + 4i^2$

Ans. Let $z = \sqrt{-3} + 4i^2$

$$= \sqrt{3}\ i - 4$$

$$\bar{z} = -\sqrt{3}\ i - 4$$

11. Solve for x and y, 3x + (2x-y) i= 6 – 3i

Ans.3x = 6

x = 2

2x – y = - 3

2 × 2 – y = - 3

- y = - 3 – 4

y = 7

12. Find the value of 1+i^2 + i^4 + i^6 + i^8 + ---- + i^{20}

Ans. $1 + i^2 + (i^2)^2 + (i^2)^3 + (i^2)^4 + ----- + (i^2)^{10} = 1 \left[\because i^2 = -1 \right.$

13. Multiply 3-2i by its conjugate.

Ans. Let z = 3 – 2i

$$\bar{z} = 3+2i$$

$$z\,\bar{z} = (3-2i)(3+2i)$$

$$= 9 + \cancel{6i} - \cancel{6i} - 4i^2$$

$$= 9 - 4(-1)$$

$$= 13$$

14. Find the multiplicative inverse 4 – 3i.

Ans. Let z = 4 – 3i

$$\bar{z} = 4+3i$$

$$|z| = \sqrt{16+9} = 5$$

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$= \frac{4+3i}{25}$$

$$= \frac{4}{25} + \frac{3}{25}i$$

15. Express in term of a + ib $\dfrac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$

Ans. $= \dfrac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{\cancel{3}}+\sqrt{2}i-\sqrt{\cancel{3}}+i\sqrt{2}}$

$$= \frac{9+5}{2\sqrt{2}i} = \frac{\cancel{14}\ 7}{\cancel{2}\sqrt{2}i}$$

$$= \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i} = \frac{7\sqrt{2}i}{-2}$$

16. Evaluate $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

Ans. $= i^n + i^n.i^1 + i^n.i^2 + i^n.i^3$

$= i^n + i^n.i - i^n + i^n.(-i) \quad \left[\begin{array}{l} i^3 = -i \\ i^2 = -1 \end{array}\right.$

$= 0$

17. If 1, w, w^2 are three cube root of unity, show that (1 – w + w^2) (1 + w – w^2) = 4

Ans. (1 – w + w^2) (1 + w – w^2)

(1 + w^2 - w) (1 + w – w^2)

$(-w-w)(-w^2-w^2)\left[\begin{array}{l} \because 1+w=-w^2 \\ 1+w^2=-w \end{array}\right.$

$(-2w)(-2w^2)$

$4w^3\left[w^3=1\right.$

4×1

$=4$

18. Find that sum product of the complex number $-\sqrt{3}+\sqrt{-2}$ and $2\sqrt{3}-i$

Ans. $z_1+z_2=-\sqrt{3}+\sqrt{2}i+2\sqrt{3}-i$

$=\sqrt{3}+\left(\sqrt{2}-1\right)i$

$z_1z_2=\left(-\sqrt{3}+\sqrt{2}i\right)\left(2\sqrt{3}-i\right)$

$=-6+\sqrt{3}i+2\sqrt{6}i-\sqrt{2}\ i^2$

$=-6+\sqrt{3}\ i+2\sqrt{6}\ i+\sqrt{2}$

$= \left(-6+\sqrt{2}\right)+\left(\sqrt{3}+2\sqrt{6}\right)i$

19. Write the real and imaginary part 1 – 2i^2

Ans. Let z = 1 – 2i^2

=1 – 2 (-1)

= 1 + 2

= 3

= 3 + 0.i

Re (z) = 3, Im (z) = 0

20. If two complex number z_1, z_2 are such that $|z_1| = |z_2|$, is it then necessary that $z_1 = z_2$

Ans. Let z_1 = a + ib

$|z_1| = \sqrt{a^2+b^2}$

$z_2 = b + ia$

$|z_2| = \sqrt{b^2+a^2}$

Hence $|z_1|=|z_2|$ but $z_1 \neq z_2$

21. Find the conjugate and modulus of $\overline{9-i}+\overline{6+i^3}-\overline{9+i^2}$

Ans. Let $z=\overline{9-i}+\overline{6-i}-\overline{9-1}$

$=9+i+6+i-0$

$=5+2i$

$\bar{z}=5-2i$

$$|z| = \sqrt{(5)^2 + (-2)^2}$$

$$= \sqrt{25 + 4}$$

$$= \sqrt{29}$$

22. Find the number of non zero integral solution of the equation |1-i|^{x} = 2^{x}

Ans. $|1 - i|^x = 2^x$

$$\left(\sqrt{(1)^2 + (-1)^2}\right)^x = 2^x$$

$$\left(\sqrt{2}\right)^x = 2^x$$

$$(2)^{\frac{1}{2}x} = 2^x$$

$$\frac{1}{2}x = x$$

$$\frac{1}{2} = 1$$

$$1 = 2$$

Which is false no value of x satisfies.

23. If (a + ib) (c + id) (e + if) (g + ih) = A + iB then show that

$$\left(a^2 + b^2\right)\left(c^2 + d^2\right)\left(e^2 + f^2\right)\left(g^2 + h^2\right) = A^2 + B^2$$

Ans. $(a + ib)(c + id)(e + if)(g + ih) = A + iB$

$$\Rightarrow |(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$$

$$|a + ib||c + id||e + if||g + ih| = |A + iB|$$

$$\left(\sqrt{a^2+b^2}\right)\left(\sqrt{c^2+d^2}\right)\left(\sqrt{e^2+f^2}\right)\left(\sqrt{g^2+h^2}\right)=\sqrt{A^2+B^2}$$

sq. both side

$$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$$

CBSE Class 12 Mathematics
Important Questions
Chapter
Statistics

4 Marks Questions

1.The mean of 2,7,4,6,8 and p is 7. Find the mean deviation about the median of these observations.

Ans. Observations are 2, 7, 4, 6, 8 and p which are 6 in numbers $\therefore n=6$

The near of these observations is 7

$$\frac{2+7+4+6+8+p}{6}=7$$

$$=27+p=42$$

$$=p=15$$

Arrange the observations in ascending order 2,4,6,7,8,15

$\therefore$ Medias (M) = $\dfrac{\frac{n}{2}\text{ th observation }+\left(\frac{n}{2}+1\right)\text{ th observation}}{2}$

$$=\frac{3\text{rd observation}+4\text{th observation}}{2}$$

$$=\frac{6+7}{2}=\frac{13}{2}$$

$$=6.5$$

Calculation of mean deviation about Median.

xi	**xi-M**	**|xi-M|**

2	-4.5	4.5
4	-2.5	2.5
6	-0.5	0.5
7	0.5	0.5
8	1.5	1.5
15	8.5	8.5
Total		18

∴ Media's deviation about median $= \frac{{}_{3}\cancel{18}}{\cancel{6}} = 3.$

2.Find the mean deviation about the mean for the following data!

x_i **1030507090**

f_i **42428168**

Ans. To calculate mean, we require $f_i xi$ values then for mean deviation, we require $| xi - \bar{x} |$ values and $f_i | xi - \bar{x} |$ values.

xi	f_i	$f_i xi$	$\vert xi - \bar{x} \vert$	$fi \vert xi - \bar{x} \vert$
10	4	4	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$n = \sum f_i = 80 \quad \sigma d \sum f_i xi = 4000$

$$\bar{x} = \frac{\sum f_i xi}{n} = \frac{4000}{80} = 50$$

Mean deviation about the mean

MD $\left(\bar{x}\right)=\frac{\sum f_i\,|xi-\bar{x}|}{n}=\frac{1280}{80}=16$

3.Find the mean, standard deviation and variance of the first n natural numbers.

Ans. The given numbers are 1, 2, 3, ……, n

Mean

$$\bar{x}=\frac{\sum n}{n}=\frac{\frac{n(n+1)}{2}}{n}=\frac{n+1}{2}$$

Variance

$$\sigma 2=\frac{\sum xi^2}{n}-\bar{x}$$

$$=\frac{\sum n^2}{n}-\left(\frac{n+1}{2}\right)^2$$

$$=\frac{n(n+1)(2n+1)}{6n}-\frac{(n+1)^2}{4}$$

$$=(n+1)\left[\frac{2n+1}{6}-\frac{n+1}{4}\right]$$

$$=(n+1)\left(\frac{n-1}{12}\right)=\frac{n^2-1}{12}$$

$\therefore$ Standard deviation $\sigma=\frac{\sqrt{n^2-1}}{12}$

4.Find the mean variance and standard deviation for following data

Ans.

x_i	**4**	**8**	**11**	**17**	**20**	**24**	**32**
f_i	3	5	9	5	4	3	1

Note: - 4th, 5th and 6th columns are filled in after calculating the mean.

xi	f_i	$f_i x_i$	$xi - \bar{x}$	$\left(xi - \bar{x}\right)^2$	$f_i x_i \left(xi - \bar{x}\right.$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	402			1374

Here $n = \sum f_i = 30$, $\sum f_i x_i = 420$

$\therefore$ Mean $\bar{x} = \dfrac{\sum f_i x_i}{n} = \dfrac{420}{30} = 14$

$\therefore$ Variance $\sigma^2 = \dfrac{1}{n} \sum f_i \left(x_i - \bar{x}\right)^2$

$= \dfrac{1}{30} \times 1374$

$= 45.8$

$\therefore$ Standard deviation $\sigma = \sqrt{45.8}$

= 6.77

5.The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Ans. Let $x_i, x_2 ......x_6$ be the six given observations

Then $\bar{x} = 8$ and $\sigma = 4$

$$\bar{x} = \frac{\sum x_i}{n} = 8 = \frac{x_1 + x_2 + ... + x_6}{6}$$

$$x_1 + x_2 + ...x_6 = 48$$

Also $\sigma^2 \frac{\sum x_1^2}{n} - \left(\bar{x}\right)^2$

$$= 4^2 = \frac{x_1^2 + x_2^2 ...... + x_6^2}{6} - (8)^2$$

$$= x_1^2 + x_2^2 + ...x_6^2$$

$$= 6 \times (16 + 64) = 480$$

As each observation is multiplied by 3, new observations are

$$3x_1, 3x_2, ...3x_6$$

New near $\overline{X} = \frac{3x_1 + 3x_2 + ...3x_6}{6}$

$$= \frac{3(x_1 + x_2 + ...x_6)}{6}$$

$$= \frac{3 \times 48}{6}$$

$$= 24$$

Let σ_1 be the new standard deviation, then

$$\sigma_1^2 = \frac{(3x_1)^2 + (3x_2)^2 + ... + (3x_6)^2}{6} - \left(\overline{X}\right)^2$$

$$=\frac{9\left(x_1^2+x_2^2+......x_6^2\right)}{6}-(24)^2$$

$$=\frac{9\times 480}{6}-576$$

$$=720-576$$

$$=144$$

$$\sigma_1=12$$

6.Prove that the standard deviation is independent of any change of origin, but is dependent on the change of scale.

Ans. Let us use the transformation $u=ax+b$ to change the scale and origin

Now $u=ax+b$

$$=\sum u=\sum(ax+b)=a\sum x+b.n$$

Also $\sigma u^2=\frac{\sum\left(u-\bar{u}\right)^2}{n}=\frac{\sum\left(ax+b-a\bar{x}-b\right)^2}{n}$

$$=\frac{\sum a^2\left(x-\bar{x}\right)^2}{n}=\frac{a^2\sum\left(x-\bar{x}\right)^2}{n}$$

$$=a^2\sigma x^2$$

$$\therefore\quad \sigma^2 u=a2\sigma^2 u$$

$$=\sigma u=|a|\,\sigma x$$

Both σu, σx are positive which shows that standard deviation is independent of choice of origin, but depends on the scale.

7.Calculate the mean deviation about the mean for the following data

Expenditure0-100100-200200-300300-400400-500500-600600-700700-800

persons 489107543

Ans.

Expenditure	**No. of persons** f_i	**Mid point** x_i	$f_i x_i$	$\lvert x_i - \overline{x} \rvert$	$f_i \lvert x_i - \overline{x} \rvert$
0-100	4	50	200	308	1232
100-200	8	150	1200	208	1664
200-300	9	250	2250	108	972
300-400	10	350	3500	8	80
400-500	7	450	3150	92	644
500-600	5	550	2750	192	960
600-700	4	650	2600	292	1168
700-800	3	750	2250	392	1176
	50		17900		7896

$n = \sum f_i = 50$

$\sum f_i x_i = 17900$

$\therefore \text{mean} = \frac{1}{n}\sum f_i x_i = \frac{17900}{50} = 358$

$MD\left(\overline{x}\right) = \frac{1}{n}\sum fi \lvert x_i - \overline{x} \rvert$

$= \frac{7896}{50} = 157.92$

8.Find the mean deviation about the median for the following data

Marks 0-1010-2020-3030-4040-5050-60

No. of boys 810101642

Ans.

Marks	No. of boys	Cumulative Frequency	Mid points	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
0-10	8	8	5	22	176
10-20	10	18	15	12	120
20-30	10	28	25	2	20
30-40	16	44	35	8	128
40-50	4	48	45	18	72
50-60	2	50	55	28	56
total	50				572

$\frac{n}{2}^{th}$ or 25^{th} item $= 20-30$, which is the median class.

Median $= l + \frac{\frac{n}{2} - c}{f} \times c = 20 + \frac{25-18}{10} \times 10$

= 27

$$MD(M) = \frac{1}{n} \sum f_i \lvert x_i - M \rvert = \frac{572}{50} = 11.44$$

9.An analysis of monthly wages point to workers in two firms A and B, belonging to the same industry, given the following result. Find mean deviation about median.

Firm AFirm B

No of wages earns586648

Average monthly wagesRs 5253Rs 5253

Ans. For firm A, number of workers = 586

Average monthly wage is Rs 5253

Total wages = Rs 5253 $\times$ 586

= Rs 3078258

For firm B, total wages = Rs 253 $\times$ 648

=Rs 3403944

Hence firm B pays out amount of monthly wages.

10.Find the mean deviation about the median of the following frequency distribution

Class 0-66-1212-1818-2424-30

Frequency8101295

Ans.

Class	**Mid value**	**Frequency**	$C.f$	$\lvert x_i - 14 \rvert$	$f_i \lvert x_i - 14 \rvert$
0-6	3	8	8	11	88
6-12	9	10	18	5	50
12-18	15	12	30	1	12
18-24	21	9	39	7	63
21-30	27	5	44	13	65
			$N = \sum f_i = 44$		$\sum f_i \lvert x_i - 14 \rvert = 278$

$$N = 44 = \frac{N}{2}$$

12-18 is the medias class

Medias = $l + \dfrac{\frac{N}{2} - F}{f} \times h$

$$h = 6,\ l = 12,\ f = 12,\ F = 18$$

Medias

$$=12+\frac{22-18}{12}\times 6$$

$$=12+\frac{4\times 6}{12}$$

$$=14$$

Mean deviation about median = $\frac{1}{N}\sum f_i\,|x_i-14|$

$$=\frac{278}{74}=6.318$$

11.Calculate the mean deviation from the median from the following data

Salary per week(in Rs) 10-2020-3030-4040-5050-6060-70

no. of workers 461020106

Ans.

Salary per Week (in Rs)	**Mid value** x_i	**Frequency** f_i	Cf	$\|d_i\|=x_i-45$	$f\,\|d_i\|$
10-20	15	4	4	30	120
20-30	25	6	10	20	120
30-40	35	10	20	10	100
40-50	45	20	40	0	0
50-60	55	10	50	10	100
60-70	65	6	56	20	120
70-80	75	4	60	30	120
		$N=\sum f_i=60$			$\sum f_i\,\|d_i\|=680$

$N=60 \qquad =\frac{N}{2}=30$

40-50 is the median class

$l = 40,\ f = 20,\ h = 10,\ F = 20$

Medias = $\dfrac{l - \frac{N}{2} - F}{f} \times h$

$= \dfrac{40 + 30 - 20}{20} \times 10 = 45$

Mean deviation = $\dfrac{\sum f_i \mid d_i \mid}{N}$ = $\dfrac{680}{60} = 11.33$

12.Let $x_1, x_2 \ldots\ldots x_n$ values of a variable Y and let 'a' be a non zero real number. Then prove that the variance of the observations $ay_1, ay_2 \ldots\ldots ay_n$ is $a^2 \operatorname{var}(Y)$. also, find their standard deviation.

Ans. Let $v_1, v_2 \ldots\ldots v_n$ value of variables v such that $v_1 = ay_i, 1, 2 \ldots\ldots n$, then

$$\overline{V} = \frac{1}{n}\sum_{i=1}^{n} v_i = \frac{1}{n}\sum_{i=1}^{n}(ayi) = a\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right) = a\overline{y}$$

$$v_i - \overline{V} = ay_i - a\overline{y}$$

$$v_i - \overline{V} = a\left(y_i - \overline{Y}\right)$$

$$\left(v_i - \overline{V}\right)^2 = a^2\left(y_i - \overline{Y}\right)^2$$

$$\sum_{i=1}^{n}\left(v_i - \overline{V}\right)^2 = a^2 \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \overline{Y}\right)^2$$

$$\operatorname{Var}\ (V) = a^2\ \operatorname{Var}\ (Y)$$

$$\sigma_u = \sqrt{\operatorname{var}(v)} = \sqrt{a^2 \operatorname{var}(Y)} = \mid a \mid \sqrt{\operatorname{var}(Y)}$$

$$= \mid a \mid \sigma_y$$

13.If $a+ib=\frac{(x+i)^2}{2x^2+1}$ Prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Ans. $a+ib=\frac{(x+i)^2}{2x^2+1}$ (i) (Given)

Taking conjugate both side

$$a-ib=\frac{(x-i)^2}{2x^2+1} \quad \text{(ii)}$$

(i) $\times$ (ii)

$$(a+ib)(a-ib)=\left(\frac{(x+i)^2}{2x^2+1}\right)\times\left(\frac{(x-i)^2}{2x^2+1}\right)$$

$$(a)^2-(ib)^2=\frac{(x^2-i^2)^2}{(2x^2+1)^2}$$

$$a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2} \quad \text{proved.}$$

14.If $(x+iy)^3=u+iv$ then show that $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$

Ans. $(x+iy)^3=4+iv$

$$x^3+(iy)^3+3x^2(iy)+3.x(iy)^2=u+iv$$

$$x^3-iy^3+3x^2yi-3xy^2=u+iv$$

$$x^3-3xy^2+(3x^2y-y^3)i=u+iv$$

$$x\left(x^2-3y^2\right)+y\left(3x^2-y^2\right)i=u+iv$$

$$x\left(x^2-3y^2\right)=u,\ \text{y}\left(3x^2-y^2\right)=v$$

$$x^2-3y^2=\frac{u}{x}\ \text{(i)}\ \Big|\ 3x^2-y^2=\frac{v}{y}\ \text{(ii)}$$

(i) + (ii)

$$4\text{x}^2\text{-}4\text{y}^2=\frac{u}{x}+\frac{v}{y}$$

$$4\left(x^2-y^2\right)=\frac{u}{x}+\frac{v}{y}$$

15.Solve $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

Ans. $\sqrt{3}\ x^2-\sqrt{2}\ \text{x}+3\ \sqrt{3}=0$

$$\text{a}=\sqrt{3},b=-\sqrt{2},c=3\sqrt{3}$$

$$D=b^2-4ac$$

$$=\left(-\sqrt{2}\right)^2-4\times\sqrt{3}\left(3\sqrt{3}\right)$$

$$=2-36$$

$$=-34$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$=\frac{-\left(-\sqrt{2}\right)\pm\sqrt{-34}}{2\times\sqrt{3}}$$

$$=\frac{\sqrt{2}\pm\sqrt{34}\ i}{2\sqrt{3}}$$

16.Find the modulus $i^{25}+(1+3i)^3$

Ans.$i^{25}+(1+3i)^3$

$$=\left(i^4\right)^6.i+1+27i^3+3(1)(3i)(1+3i)$$

$$=i+\left(1-27i+9i+27i^2\right)$$

$$=i+1-18i-27$$

$$=-26-17i$$

$$\left|i^{25}+(1+3i)^3\right|=\left|-26-17i\right|$$

$$=\sqrt{(-26)^2+(-17)^2}$$

$$=\sqrt{676+289}$$

$$=\sqrt{965}$$

17.If $a+ib=\frac{(x+i)^2}{2x-i}$ prove that $a^2+b^2=\frac{\left(x^2+1\right)^2}{4x^2+1}$

Ans. $a+ib=\frac{(x+i)^2}{2x-i}$ (i) (Given)

$a-ib=\frac{(x-i)^2}{2x+i}$ (ii) [taking conjugate both side

(i) × (ii)

$$(a+ib)(a-ib)=\frac{(x+i)^2}{(2x-i)}\times\frac{(x-i)^2}{(2x+i)}$$

$$a^2+b^2=\frac{(x^2+1)^2}{4x^2+1}\quad \text{proved.}$$

18.Evaluate $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Ans. $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

$$\left[\left(i^4\right)^4.i^2+\frac{1}{i^{25}}\right]^3$$

$$\left[i^2+\frac{1}{\left(i^4\right)^6.i}\right]^3$$

$$\left[-1+\frac{1}{i}\right]^3$$

$$\left[-1+\frac{i^3}{i^4}\right]^3$$

$$[-1-i]^3=-(1+i)^3$$

$$=-\left[1^3+i^3+3.1.i(1+i)\right]$$

$$=-\left[1-i+3i+3i^2\right]$$

$$=-[1-i+3i-3]$$

$= -[-2+2i] = 2-2i$

19.Find that modulus and argument $\frac{1+i}{1-i}$

Ans. $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{(1+i)^2}{1^2 - i^2}$

$= \frac{1+i^2+2i}{1+1}$

$= \frac{2i}{2}$

$= i$

$z = 0 + i$

$r = |z| = \sqrt{(0)^2 + (1)^2} = 1$

Let α be the acute $\angle$s

$\tan \alpha = \left|\frac{1}{0}\right|$

$\alpha = \pi/2$

$\arg(z) = \pi/2$

$r = 1$

20.For what real value of x and y are numbers equal (1+i) y^2 + (6+i) and (2+i) x

Ans.(1+i) y^2 + (6 + i) = (2 + i) x

$y^2 + iy^2 + 6 + i = 2x + xi$

$(y^2 + 6) + (y^2 + 1)\, i = 2x + xi$

$y^2 + 6 = 2x$

$y^2 + 1 = x$

$y^2 = x - 1$

$x - 1 + 6 = 2x$

$5 = x$

$y = \pm 2$

21.If x + iy = $\sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

Ans. $x + iy = \sqrt{\frac{1+i}{1-i}}$ (i) (Given)

taking conjugate both side

$x - iy = \sqrt{\frac{1-i}{1+i}}$ (ii)

(i) $\times$ (ii)

$(x+iy)(x-iy) = \sqrt{\frac{1+i}{1-i}} \times \sqrt{\frac{1-i}{1+i}}$

$(x)^2 - (iy)^2 = 1$

$x^2 + y^2 = 1$

Proved.

22.Convert in the polar form $\dfrac{1+7i}{(2-i)^2}$

Ans. $\dfrac{1+7i}{(2-i)^2}=\dfrac{1+7i}{4+i^2-4i}=\dfrac{1+7i}{3-4i}$

$=\dfrac{1+7i}{3-4i}\times\dfrac{3+4i}{3+4i}$

$=\dfrac{3+4i+21i+28i^2}{9+16}$

$=\dfrac{25i-25}{25}=i-1$

$=-1+i$

$r=|z|=\sqrt{(-1)^2+1^2}=\sqrt{2}$

Let α be the acute $\angle$s

ten $\alpha=\left|\dfrac{1}{-1}\right|$

$\alpha=\pi/4$

since Re (z) < 0, Im (z) > 0

$\theta=\pi-\alpha$

$=\pi-\dfrac{\pi}{4}=3\pi/4$

$z=r(\text{Cos}\theta+\text{i Sin}\theta)$

$=\sqrt{2}\left(\text{Cos}\dfrac{3\pi}{4}+i\,Sin\dfrac{3\pi}{4}\right)$

23.Find the real values of x and y if (x - iy) (3 + 5i) is the conjugate of – 6 – 24i

Ans.

(x – iy) (3 + 5i) = - 6 + 24i

3x + 5xi – 3yi – 5yi^2 = - 6 + 24i

$(3x+5y)+(5x-3y)i=-6+24i$

$3x+5y=-6$

$5x-3y=24$

$x=3$

$y=-3$

24.If $|z_1|=|z_2|=1$, prove that $\left|\frac{1}{z_1}+\frac{1}{z_2}\right|=|z_1+z_2|$

Ans. If $|z_1|=|z_2|=1$ (Given)

$\Rightarrow |z_1|^2=|z_2|^2=1$

$\Rightarrow z_1\ \overline{z_1}=1$

$\overline{z_1}=\frac{1}{z_1}$ (1)

$z_2\ \overline{z_2}=1$

$\overline{z_2}=\frac{1}{z_2}$ (2)

$\left[\ \because z\ \overline{z}=|z|^2\right.$

$$\left|\frac{1}{z_1}+\frac{1}{z_2}\right| = \left|\overline{z_1}+\overline{z_2}\right|$$

$$= \left|\overline{z_1+z_2}\right|$$

$$= \left|z_1+z_2\right|$$

$\left[\because \left|\overline{z}\right| = \left|z\right|\right.$ proved.

CBSE Class 12 Mathematics

Important Questions

Chapter

Statistics

6 Marks Questions

1.Calculate the mean, variance and standard deviation of the following data:

Classes	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
Frequency	**3**	**7**	**12**	**15**	**8**	**3**	**2**

Ans.

Classes	**Frequency**	**Mid Point**	$f_i xi$	$\left(x_i - \overline{x}\right)^2$	$f_i\left(x_i - \overline{x}\right)^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
Total	50		3100		10050

Here $n = \sum f_i = 50, \sum f_i x_i = 3100$

$\therefore$ Mean $\overline{x} = \frac{\sum f_i x_i}{n} = \frac{3100}{50} = 62$

Variance $\sigma^2 = \frac{1}{n}\sum f_i\left(xi - \overline{x}\right)^2$

$= \frac{1}{50} \times 10050$

$= 201$

Standard deviation $\sigma = \sqrt{201} = 14.18$

2.The mean and the standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who mistook one observation as 50 instead of 40. What are the correct mean and standard deviation?

Ans. Given that $n = 100$

Incorrect mean $\bar{x} = 40$,

Incorrect S.D $(\sigma) = 5.1$

As $\bar{x} = \frac{\sum x_i}{n}$

$$40 = \frac{\sum x_i}{100} = \sum x_i = 4000$$

= incorrect sum of observation =4000

= correct sum of observations = 4000-50+40

= 3990

So correct mean = $\frac{3990}{100} = 39.9$

Also $\sigma = \sqrt{\frac{1}{n}\sum x_i^2 - (\bar{x})^2}$

Using incorrect values,

$$5.1 = \sqrt{\frac{1}{100}\sum x_i^2 - (40)^2}$$

$$= 26.01 = \left[\frac{1}{100}\sum x_i^2 - 1600\right]$$

$= \sum x_i^2 = 2601 + 160000$

= 162601

= incorrect $\sum x_i^2 = 162601$

= correct $\sum x_i^2 = 162601 - (50)^2 + (40)^2$

= 162601-2500+1600=161701

$\therefore$ Correct $\sigma = \sqrt{\frac{1}{100} \text{correct} \sum x_i^2 - \left(\text{correct } \bar{x}\right)^2}$

$= \sqrt{\frac{1}{100}(161701) - (39.9)^2} \quad = \sqrt{1617.01 - 1592.01}$

$= \sqrt{25} = 5$

Hence, correct mean is 39.9 and correct standard deviation is 5.

3.200 candidates the mean and standard deviation was found to be 10 and 15 respectively. After that if was found that the scale 43 was misread as 34. Find the correct mean and correct S.D

Ans. $n = 200, \overline{X} = 40, \sigma = \overline{15}$

$\overline{X} = \frac{1}{n} \sum x_i = \sum x_i = n\overline{X} = 200 \times 40 = 8000$

Corrected $\sum x_i$ = Incorrect $\sum x_i -$ (sum of incorrect +sum of correct value)

= 8000-34+43= 8009

$\therefore$ Corrected mean = $\frac{\text{corrected} \sum x_i}{n} = \frac{8009}{200} = 40.045$

$\sigma = 15$

$$15^2 = \frac{1}{200}\left(\sum x_i^{\ 2}\right) - \left(\frac{1}{200}\sum x_i\right)^2$$

$$225 = \frac{1}{200}\left(\sum x_i^{\ 2}\right) - \left(\frac{8000}{200}\right)^2$$

$$225 = \frac{1}{200} \times 1825 = 365000$$

Incorrect $\sum x_i^{\ 2} = 365000$

Corrected $\sum x_i^{\ 2}$ = (incorrect $\sum x_i^{\ 2}$) – (sum of squares of incorrect values) + (sum of square of correct values)

$= 365000 - (34)^2 + (43)^2 = 365693$

Corrected σ = $\sqrt{\frac{1}{n}\sum x_i^{\ 2} - \left(\frac{1}{n}\sum x_i\right)^2} = \sqrt{\frac{365693}{200} - \left(\frac{8009}{200}\right)^2}$

$\sqrt{1828.465 - 1603.602} = 14.995$

4.Find the mean deviation from the mean 6,7,10,12,13,4,8,20

Ans. Let $\overline{X}$ be the mean

$$\overline{X} = \frac{6+7+10+12+13+4+8+20}{8} = 10$$

x_i	$\lvert d_i \rvert = \lvert x_i - \overline{X} \rvert = \lvert x_i - 10 \rvert$
6	4
7	3
10	0

12	2
13	3
4	6
8	2
20	10
Total	$\sum d_i$ = 30

$\sum d_i$ = 30 and n = 8

$$\therefore MD = \frac{1}{n}\sum |d_i| = \frac{30}{8} = 3.75$$
$$\therefore MD = 3.75$$

5.Find two numbers such that their sum is 6 and the product is 14.

Ans. Let x and y be the no.

x + y = 6

xy = 14

$$x^2-6x+14=0$$

$$D=-20$$

$$x=\frac{-(-6)\pm\sqrt{-20}}{2\times 1}$$

$$=\frac{6\pm 2\sqrt{5}\ \mathrm{i}}{2}$$

$$=3\pm\sqrt{5}\ \mathrm{i}$$

$$\mathrm{x}=3+\sqrt{5}\ \mathrm{i}$$

$$y=6-\left(3+\sqrt{5}\ \mathrm{i}\right)$$

$$=3-\sqrt{5}\ \mathrm{i}$$

when $\mathrm{x}=3-\sqrt{5}\ \mathrm{i}$

$$y=6-\left(3-\sqrt{5}\ \mathrm{i}\right)$$

$$=3+\sqrt{5}\ \mathrm{i}$$

6.Convert into polar form $z=\dfrac{i-1}{\cos\dfrac{\pi}{3}+i\ \mathrm{Sin}\ \dfrac{\pi}{3}}$

Ans. $z=\dfrac{i-1}{\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i}$

$$=\frac{2(i-1)}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$z=\frac{\sqrt{3}-1}{2}+\frac{\sqrt{3}+1}{2}i$$

$$r=|z|=\left(\frac{\sqrt{3}-1}{2}\right)^2+\left(\frac{\sqrt{3}+1}{2}\right)^2$$

$r = 2$

Let α be the acule $\angle$s

$$\tan\alpha = \left|\frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}\right|$$

$$= \left|\frac{\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\left(1-\frac{1}{\sqrt{3}}\right)}\right|$$

$$= \left|\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right|$$

$$\tan\alpha = \left|\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right|$$

$$\alpha = \frac{\pi}{4}+\frac{\pi}{6} = \frac{5\pi}{12}$$

$$z = 2\left(Cos\,\frac{5\pi}{12} + i\ \text{Sin}\ \frac{5\pi}{12}\right)$$

7.If α and β are different complex number with |β| = 1 Then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Ans. $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\bar{\alpha}\beta}}\right) \qquad \left[\because |z|^2 = z\bar{z}\right.$

$$= \left(\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right)\left(\frac{\overline{\beta} - \overline{\alpha}}{1 - \alpha\overline{\beta}}\right)$$

$$= \left(\frac{\beta\overline{\beta} - \beta\overline{\alpha} - \alpha\overline{\beta} + \alpha\overline{\alpha}}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + \alpha\overline{\alpha}\beta\overline{\beta}}\right)$$

$$= \left(\frac{|\beta|^2 - \beta\overline{\alpha} - \alpha\overline{\beta} + |\alpha|^2}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + |\alpha|^2|\beta|^2}\right)$$

$$= \left(\frac{1 - \beta\overline{\alpha} - \alpha\overline{\beta} + |\alpha|^2}{1 - \alpha\overline{\overline{\beta}} - \overline{\overline{\alpha}}\beta + |\alpha|^2}\right) \quad [\because |\beta| = 1$$

$$= 1$$

$$\left|\frac{\beta - \alpha}{1 - \alpha\beta}\right| = \sqrt{1}$$

$$\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right| = 1$$