Chapter 1

Fluid Properties and Manometry

CHAPTER HIGHLIGHTS

- *Introduction*
- *Fluid properties*
- *Classification of fluids*
- *Pressure*

Introduction

Fluid Mechanics is defined as the science that deals with a fluid's behaviour, when it is at rest or in motion, and the fluid's interaction with other fluids or solids at the boundaries. *Fluid Statics* deals with fluids at rest while *Fluid Dynamics* deals with fluids in motion. The study of incompressible fluids under static condition is called hydrostatic. The study of compressible static gases is called aeromatics.

Fluid

Matter can be primarily classified as:

1. Solids
2. Liquids
3. Gases

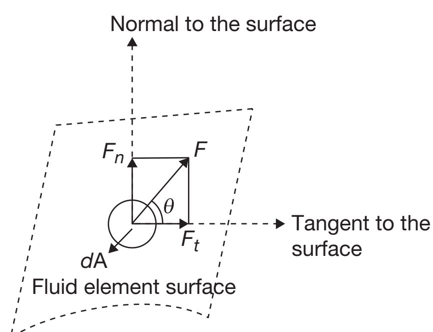
MATTER	Inter-molecular	
	Space	Cohesive Forces
Solids	Small	Large
Liquids	Large	Small
Gases	Very large	Very small

Liquids and gases (including vapours) are commonly referred to as fluids. A fluid is defined as a substance that deforms continuously under the influence of a shear stress of any magnitude, i.e., when subjected to an external shear force, of any magnitude, a fluid will deform continuously as long as the force is applied. A fluid has negligible shear resistance, i.e., it offers negligible resistance towards an applied shear (or tangential) stress that tends to change the shape of the fluid body.

Shear and Normal Stresses

Stress is defined as force per unit area (area upon which the force acts). Let us consider a small area dA, on the surface of a fluid element, on which a force F acts as shown in the figure.

If the tangential and normal components of the force F are respectively F_t and F_n, then



Shear stress (τ) at the surface of the fluid element $= \dfrac{F_t}{dA} = \dfrac{F\cos\theta}{dA}$.

Normal stress at the surface of the fluid element $= \dfrac{F_n}{dA} = \dfrac{F\sin\theta}{dA}$.

Normal stress and shear stress are vector quantities.

For a static fluid body, i.e., a body of fluid that is at rest or has zero velocity, the shear stress is always zero. Also for static fluids, the normal stress is always positive.

SOLVED EXAMPLES

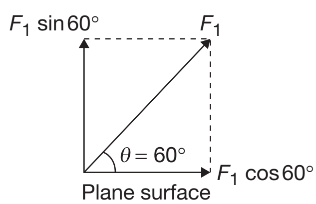
Example 1

A force F_1 (= 20 N) is applied on an area A_1 (= 0.1 cm^2) at the surface of a fluid element in the outward direction. The force F_1 acts at an angle of 60° from the tangential plane at the point of application of the force. Another force F_2 (= 60 N) is applied, in the same manner as the force F_1, on another area, A_2 (= 0.2 cm^2) at the surface of the same fluid element. The ratio of the normal stress at area A_1 to the shear stress at area A_2 is

(A) 2 : 3 (B) $2 : 3\sqrt{3}$
(C) $2 : \sqrt{3}$ (D) $1 : \sqrt{3}$

Solution

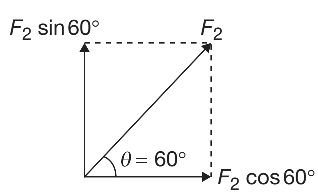
Area A_1:



Normal stress acting on area A_1

$$= \frac{F_1 \sin 60°}{A_1} = \frac{20}{0.1\times10^{-4}} \times \frac{\sqrt{3}}{2}$$
$$= \sqrt{3}\times10^6\ \text{N/m}^2$$

Area A_2:



Shear stress acting on area A_2

$$= \frac{F_2 \cos 60°}{A_2} = \frac{60\times1}{0.2\times10^{-4}\times2}$$
$$= 1.5\times10^6\ \text{N/m}^2$$

Ratio of the normal stress at area A_1 to the shear stress at area A_2

$$= \frac{\sqrt{3}\times10^6}{1.5\times10^6} = \frac{2}{\sqrt{3}} \text{ or } = 2 : \sqrt{3}$$

Hence, the correct answer is option (C).

Example 2

An example for a normal stress is

(A) volume (B) shear stress
(C) pressure (D) temperature

Solution

Pressure is an example for a normal stress. In static fluids, the pressure at a given position is equal to the normal stress at that position.

Hence, the correct answer is option (C).

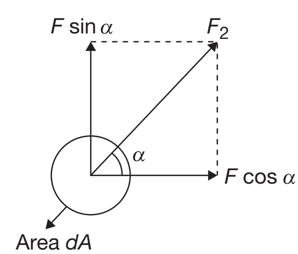
Example 3

On an area of 0.1 cm^2 at the surface of a static fluid element, a force of 40 N is observed to act in the outward direction. If the force acts at an angle α from the tangential plane at the point of application of the force, and the fluid still remains static then the value of α is

(A) 0° (B) 30°
(C) 45° (D) 90°

Solution

Shear stress acting on the given area $= \dfrac{F\cos\alpha}{A}$



For a static fluid element, shear stress = 0

$$\Rightarrow \frac{F\cos\alpha}{A} = 0$$

or $\cos\alpha = 0$ ($\because F \neq 0, A \neq \infty$)

$$\therefore \alpha = 90°$$

Hence, the correct answer is option (D).

FLUID PROPERTIES

1. **Density (mass density or specific mass):** Density is defined as mass per unit volume. If m is the mass of a fluid body having a volume V, then the density of the fluid, denoted by ρ, is $\boxed{\rho = \dfrac{m}{V}}$ The SI unit of density is kg/m^3. For practical calculations, the density of water is taken to be the density of water at 4°C which is 1000 kg/m^3 or 1 g/cm^3 or 1 kg/lit. For most gases, density is inversely proportional to the temperature and proportional to pressure. For liquids, variations in pressure and temperature induce a small (negligible) variation in the density.

Example 4

A gas behaves like a real gas at temperature T_1 and pressure P_1. The gas can be made to behave approximately like an ideal gas by either changing the temperature from T_1 to T_2 or by changing the pressure from P_1 to P_2. One may then conclude that

(A) $T_2 > T_1$ and $P_2 < P_1$ (B) $T_2 < T_1$ and $P_2 < P_1$
(C) $T_2 > T_1$ and $P_2 > P_1$ (D) $T_2 < T_1$ and $P_2 > P_1$

Solution

Real gases have been experimentally observed to behave like ideal gases at low densities.

The density of most gases can be reduced by increasing the temperature $\left(\text{as } \rho \propto \frac{1}{T}\right)$ or by decreasing the pressure (as $\rho \propto P$).

$$\therefore T_2 > T_1 \text{ and } P_2 < P_1.$$

Hence, the correct answer is option (A).

2. **Specific volume:** Specific volume is defined as volume per unit mass. The reciprocal of a fluid's density (ρ) is its specific volume (v), i.e., $\boxed{v = \frac{1}{\rho} = \frac{V}{m}}$. The SI unit of specific volume is m^3/kg.
3. **Specific weight (weight density):** Specific weight is defined as weight per unit volume. The specific weight of a fluid, $\boxed{\omega = \frac{W}{V} = \frac{mg}{V} = \rho g}$, where, g is the acceleration due to gravity and W, V, m and ρ are respectively the weight, volume, mass and density of the fluid. The SI Unit of specific weight is kg/m^2s^2. For practical calculations, the specific weight of water is taken to be 9.81 kN/m^3. Specific weight depends upon temperature, pressure and location.
4. **Specific gravity (relative density):** Specific gravity of a fluid is the ratio of the density of the fluid to the density of a standard fluid. The standard fluid is taken to be pure water at 4°C. Sometimes for gases, the standard fluid is taken to be air at standard temperature and pressure.

   Specific gravity of a fluid,

$$\boxed{SG_{\text{fluid}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{standard fluid}}} = \frac{\omega_{\text{fluid}}}{\omega_{\text{standard fluid}}}}$$

   Where, ω is the specific weight. Specific gravity is a dimensionless quantity, i.e., it has no units. For practical calculations, the specific gravities of water and mercury are taken to be 1 and 13.6 respectively.

Example 5

The specific weight of a body of fluid A is twelve times that of a body of fluid B. The acceleration due to gravity acting on the fluid A is four times that acting on the fluid B. If the specific gravity of fluid B is 1.2, then the density of fluid A (in g/cm^3) is

(A) 57.6 (B) 3.6
(C) 14.4 (D) 0.4

Solution

$$\frac{\text{Specific weight of fluid } A}{\text{Specific weight of fluid } B} = \frac{\omega_A}{\omega_B}$$

$$= \frac{\rho_A g_A}{\rho_B g_B} \qquad (1)$$

It is given that $\frac{\omega_A}{\omega_B} = \frac{12}{1}$ and $\frac{g_A}{g_B} = \frac{4}{1}$

From Eq. (1), we have $\frac{\rho_A}{\rho_B} = \frac{3}{1}$.

Specific gravity of fluid A

$$= \text{Specific gravity of fluid } B \times \left(\frac{\rho_A}{\rho_B}\right)$$

$$= 1.2 \times 3 = 3.6.$$

Density of fluid A = Specific gravity of fluid A × Density of pure water at 4°C

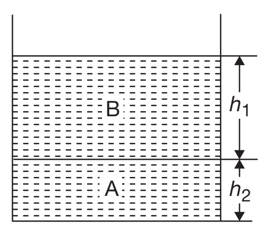
$$= 3.6 \times 1$$

$$= 3.6 \text{ gm/cm}^3.$$

Hence, the correct answer is option (B).

Example 6

Two immiscible liquids A and B, when poured into a cylindrical container, separate out into two distinct layers of different heights as shown in the following figure. The specific gravity of liquid A is thrice that of the liquid B. If the ratio $h_1 : h_2$ is 2 : 1, then the ratio of the mass of the liquid A to the mass of the liquid B in the container is



(A) 1 : 6 (B) 2 : 3
(C) 6 : 1 (D) 3 : 2

Solution

If m_A and m_B are the masses of the liquids A and B respectively in the container, then $\frac{m_A}{m_B} = \frac{SG_A V_A}{SG_B V_B}$, where SG is the fluid's specific gravity and V is the volume of the fluid.

Since the specific gravity of liquid A is greater than that of liquid B ($SG_A = 3 \times SG_B$), liquid A is denser.

Hence, the height h_2 corresponds to the liquid A, i.e., $V_A = h_2 \times a$, where a is the area of the container base and $V_B = h_1 \times a$.

$$\therefore \frac{m_A}{m_B} = \frac{\text{SG}_A\, h_2}{\text{SG}_B\, h_1} = \frac{3}{2}.$$

Hence, the correct answer is option (D).

5. **Viscosity:** Viscosity is the property of the fluid by virtue of which it resists fluid flow, i.e., viscosity represents the internal resistance (fluid friction) of a fluid to motion (or the fluidity) or to shearing stresses. The SI unit of viscosity is kg/ms or Ns/m^2 or Pa/s. Another unit (in CGS units) for viscosity is poise.

$$\boxed{1 \text{ poise} = 0.1 \text{ Ns/m}^2}$$

Viscosity of water, for practical calculations, is taken to be 1 centipoise or 0.01 poise. The device that measures viscosity is called a viscometer.

Variation of Viscosity of Fluids with Temperature

The cohesive forces and molecular momentum transfer result in viscous forces in fluids.

Since temperature affects both the cohesive forces and molecular momentum transfer, viscosity of fluids are affected by variations in temperature.

Liquids

As liquids have a closely packed molecular structure (compared to gases), cohesive forces dominate over the molecular momentum transfer. With increase in temperature, the cohesive forces decrease in liquids, which in turn decreases the viscosity.

Hence, viscosity of liquids decrease with increase in temperature and vice versa.

The relation between viscosity and temperature in liquids is

$$\mu = \mu_0 \left[\frac{1}{1 + \alpha t + \beta t^2}\right],$$

Where

μ = Viscosity of liquid at t°C, in poise
μ_0 = Viscosity of liquid at 0°C, in poise
α, β = Constants for the liquid

The viscosity of water at 1°C is 1 centipoise. Liquids with increasing order of viscosity are gasoline, water, crude oil, castor oil, etc.

Gases

In the case of gases, the molecular momentum transfer dominates over the cohesive forces. As the temperature increases, molecular momentum transfer also increases.

Hence, the viscosity of gases increases with increase in temperature and vice versa.

The relation between viscosity and temperature for gases is

$$\boxed{\mu = \mu_0 + \alpha t - \beta t^2},$$

Where

μ = Viscosity of gas at t°C, in poise
μ_0 = Viscosity of gas at 0°C, in poise
α, β = Constants for the gas

The relation between absolute temperature (T) and dynamic viscosity of an ideal gas is given by **Sutherland equation**, which is

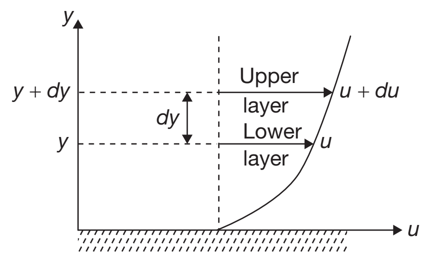
$$\boxed{\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \frac{(T_0 + S)}{(T + S)}}$$

Where

μ = Viscosity at absolute temperature T
μ_0 = Viscosity at absolute temperature T_0
S = Sutherland temperature of the gas (in Kelvin)

Velocity Gradient

Consider the flow of a fluid over a solid surface as shown in the figure below. Consider in this fluid flow, two fluid layers which are at a distance 'dy' apart. The upper fluid layer (at, $y + dy$) is assumed to move at a velocity of $(u + du)$, while the lower fluid layer (at y) is assumed to move at a velocity of u.



Solid surface

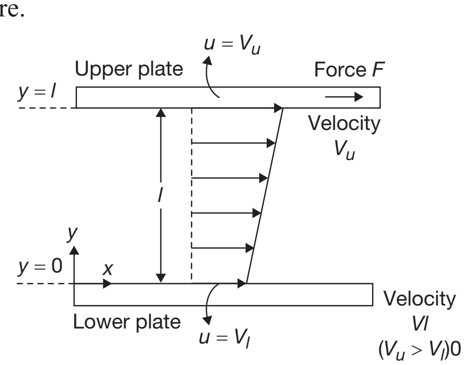
Then, the velocity gradient

$$= \frac{(u + du) - u}{(y + dy) - y} = \frac{du}{dy}$$

$$\boxed{\frac{du}{dy} \approx \frac{\Delta u}{\Delta y} = \frac{u_{y=y_2} - u_{y=y_1}}{y_2 - y_1}}$$

This equation is valid when y_2 is very close to y_1 or for a linear velocity profile.

Now consider a fluid layer between two very large parallel plates, separated by a distance l, as shown in the following figure.



Let a constant parallel force F be applied to the upper plate which would move it at a constant speed V_u, after the initial dynamics. This force would move the fluid layer in contact with the upper plate at the same speed V_u in the direction of motion of the upper plate (due to no-slip condition). Similarly, if the lower plate moves with a velocity V_l the fluid in contact with the lower plate would move with the same velocity V_l in the direction of motion of the lower plate.

If the fluid flow between the plates is steady and laminar, then a linear velocity profile is seen to develop in the fluid layer. That is, the fluid velocity between the plates vary linearly between V_l and V_u.

For the linear velocity profile, the velocity gradient,

$$\boxed{\frac{du}{dy} = \frac{V_u - V_l}{l - 0} = \frac{V_u - V_l}{l}}$$

The linear velocity profile is given by, $\boxed{u(y) = \frac{y}{l}(V_u - V_l)}$

Case 1: When the lower plate is held fixed In this case, $V_l = 0$. Therefore, the velocity gradient,

$$\boxed{\frac{du}{dy} = \frac{V_u}{l}}$$

Case 2: When the lower plate moves in the direction opposite to that of the upper plate motion.

In this case, velocity gradient,

$$\boxed{\frac{du}{dy} = \frac{V_u - (-V_l)}{l} = \frac{V_u + V_l}{l}}$$

For a fluid element, it can be shown that the velocity gradient is equivalent to the rate of deformation or the rate of angular displacement or the rate of shear strain.

Newton's Law of Viscosity

When two fluid layers move relative to each other, the viscosity and the relative velocity causes a shear stress to act between the fluid layers. The top fluid layer causes a shear stress on the adjacent lower layer while the lower fluid layer causes a shear stress on the adjacent top layer. Newton's law of viscosity states that the shear stress acting on a fluid layer is directly proportional to the rate of deformation or the velocity gradient, i.e., $\boxed{\tau \,\alpha\, \frac{du}{dy}}$ or $\boxed{\tau = \mu \frac{du}{dy}}$, where μ is known as the coefficient of viscosity or the dynamic viscosity or the absolute viscosity or simply as viscosity. Fluids which follow this law are generally referred to as *Newtonian fluids.*

For most fluids, shear stress is directly proportional to the velocity gradient or the rate of deformation or the rate of angular displacement or the rate of shear strain.

Direction for solved example 7 and 8:

A fluid flowing over a flat solid surface develops a parabolic velocity distribution. The vertex of the parabolic distribution is situated 10 cm away from the solid surface, where the fluid velocity is 1.5 m/s. The shear stress at a point 5 cm from the solid surface is determined to be 30 N/m^2. The fluid follows Newton's law of viscosity.

Example 7

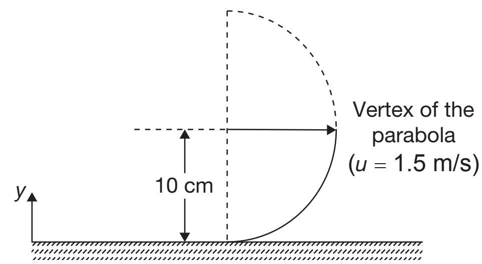
The viscosity of the fluid is

(A) 0.2 poise (B) 2 poise
(C) 0 poise (D) 0.1 poise

Solution

Let the parabolic velocity distribution be

$$u(y) = ay^2 + by + c \quad (1)$$



Solid surface

At $y = 0$, $u = 0$ (no slip condition)

$\therefore$ From Eq. (1), we have $c = 0$.

$$\therefore \quad u(y) = ay^2 + by \quad (2)$$

At $y = 0.1$ m (10 cm), $u = 1.5$ m/s

$\therefore$ From Eq. (2), we have

$$150 = a + 10b \quad (3)$$

At the vertex of the parabolic velocity distribution, i.e., at $y = 0.1$ m (10 cm), we have, $\frac{du}{dy} = 0$

Hence, from Eq. (2), we have,

$$2a + 10b = 0 \quad (4)$$

Solving Eqs. (3) and (4), we get

$a = -150$ and $b = 30$

$$\therefore \quad u(y) = -150y^2 + 30y \quad (5)$$

At $y = 0.05$ m (5 cm),

$\tau = 30$ N/m^2

$$\text{That is, } 30 = \mu \left(\frac{du}{dy}\right)_{y=0.05} \quad (6)$$

($\because$ Fluid follows Newton's law of viscosity).

Inserting the differential of Eq. (5) in Eq. (6) and substituting the value of y by 0.05, we get

$$\mu = 2 \ \text{Ns/m}^2 = 0.2 \ \text{poise}.$$

Hence, the correct answer is option (A).

Example 8

The shear stress at the solid surface is

(A) 30 N/m^2 (B) 10 N/m^2
(C) 60 N/m^2 (D) 0 N/m^2

Solution

Now, shear stress $\tau = \mu \dfrac{du}{dy}$

From Eq. (5), $\dfrac{du}{dy} = -300y + 30$

At the solid surface, $y = 0$

$\therefore$ Shear stress at the wall

$$= \mu \left(\frac{du}{dy}\right)_{y=0}$$

$$= 2 \times 30 = 60 \text{ N/m}^2.$$

Hence, the correct answer is option (C).

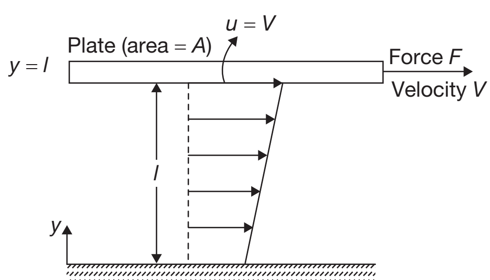
Example 9

A square thin plate, of length 80 cm and mass 30 kg, slides parallel to a solid plane surface inclined at an angle of 60° to the horizontal. A Newtonian fluid layer of thickness 2 mm is present in between the plate and the plane surface. Had the plane been horizontal, a constant force of 192 N would have been required to move the plate at a constant velocity of 3 m/s. If the fluid's velocity profile can be assumed to be linear, then the constant force to be applied, parallel to the inclined plane, on the plate to make it slide at a instant velocity of 6 m/s is

(A) 254.87 N (B) 129.13 N
(C) 384 N (D) 89.7 N

Solution

When the plane is horizontal



Stationary plane

Here, shear stress $\tau = \dfrac{F}{A} = \mu \dfrac{du}{dy}$

($\because$ Fluid is Newtonian)

Since the velocity profile is linear, $\dfrac{du}{dy} = \dfrac{V}{l}$

$$\therefore F = \frac{\mu AV}{l} \quad (1)$$

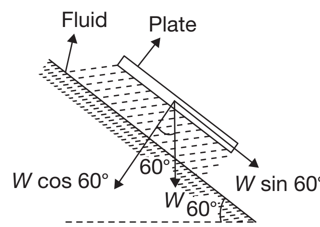
Given, $F = 192$ N, $V = 3$ m/s,

$A = 0.8 \times 0.8$ m^2 and $l = 0.002$ m. Substituting these values in Eq. (1), we get

$$\mu = 0.2 \text{ Ns/m}^2.$$

When the plane is inclined: Constant force to be applied on the plate to make it slide down with a constant velocity of 6 m/s, $F = \dfrac{\mu AV}{l} = \dfrac{0.2 \times 0.8 \times 0.8 \times 6}{0.002} = 384$ N.

Part of this constant force to be applied will be taken care of by the component of the weight of the plate in the downward direction parallel to the inclined plane surface, i.e., by $W\sin 60°$.



$\therefore$ Constant force to be applied

$$= 384 - W\sin 60°$$

$$= 384 - 30 \times 9.81 \times \frac{\sqrt{3}}{2}$$

$$= 129.13 \text{ N}$$

Hence, the correct answer is option (B).

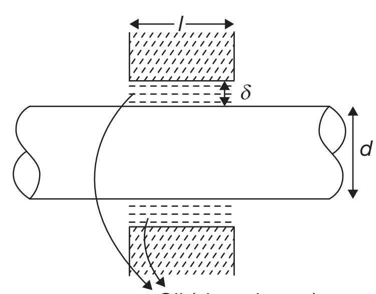
Example 10

In a journal bearing of length 500 mm, a 200 mm diameter shaft is rotating at 1000 rpm. The uniform space between the shaft and the journal bearing is completely filled with an oil (Newtonian fluid) having a viscosity of 900 centipoise. If energy is being dissipated as heat at the rate of 15.5 kJ/s, while overcoming friction, and the velocity profile in the oil is linear, then the thickness of the oil layer between the shaft and the bearing is

(A) 5 mm (B) 1 mm
(C) 2 mm (D) 3 mm

Solution

The rate of energy dissipation as heat, while overcoming friction, can be considered to be the power dissipated as heat or the power utilized (or lost) to overcome the resistance imparted by the fluid viscosity.



If the shaft is rotating at N rpm, then the tangential velocity of the shaft, $u = \dfrac{\pi dN}{60}$, where d is the diameter of the shaft

$$\therefore u = \frac{\pi \times 0.2 \times 1000}{60}$$
$$= 10.472 \text{ m/s}$$

We have, $F = \mu A \dfrac{du}{dy}$

$$0.9 \times 0.2 \times 0.5 \left(\frac{10.472}{\delta}\right) \quad (1)$$

But $F \times u = P = 15500$

$$F = \frac{15500}{10.472} = 1480.14$$

$\therefore$ From Eq. (1) $\delta = 2$ mm.
Hence, the correct answer is option (C).

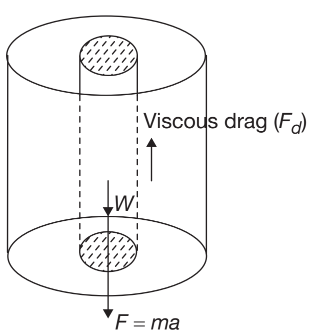
Example 11

A solid cylinder of diameter d, length l and density ρ_c falls due to gravity inside a pipe of diameter D. The clearance between the solid cylinder and the pipe is filled with a Newtonian fluid of density ρ and μ. For this clearance fluid, the terminal velocity of the cylinder is determined to be V, assuming a linear velocity profile. However, if the clearance fluid was changed to a Newtonian fluid of density 2ρ and viscosity 2μ, then for an assumed linear velocity profile, the terminal velocity of the cylinder was determined to be V_1. From the results of these experiments, one may write that

(A) $V_1 = V$ (B) $V = 2V_1$
(C) $2V = V_1$ (D) $V = 4V_1$

Solution

Resolving the forces acting on the cylinder, $F = W - F_d$ or $ma = W - F_d$,



Where m, W and a are the mass, weight and acceleration respectively of the solid cylinder.

When the cylinder attains terminal velocity, $a = 0$

$$\therefore W - F_d = 0 \quad (1)$$

Now $F_d = \tau A$

Since the fluid is Newtonian,

$$F_d = \frac{\mu V}{\frac{D-d}{2}} \times \pi d l \quad \text{(for the first experiment)} \quad (2)$$

Now the weight of the cylinder, $W = \rho_c \times g \times \pi \times \dfrac{d^2}{4} l$ (3)

Substituting Eqs. (2) and (3) in Eq. (1) and rearranging, we get

$$V = \frac{\rho_c g \times d(D-d)}{8\mu}$$

$\therefore$ The terminal velocity of the cylinder does not depend on the density of the fluid.

Hence $\dfrac{V_1}{V} = \dfrac{\mu}{2\mu}$ or $V = 2V_1$.

Hence, the correct answer is option (B).

Alternative solution:
At the condition of terminal velocity force of the drag is the weight. Force of drag

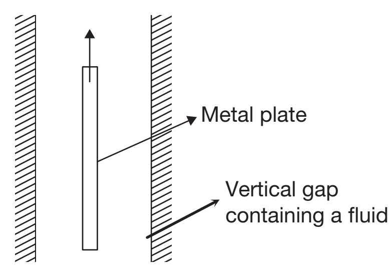
$$F = 6\pi a \mu v$$

Where, μ = the Coefficient of viscosity

$$\therefore F_D \,\alpha\, \mu v$$
$$\therefore \mu v_1 = \mu_2 v_2$$
$$v_2 = \frac{\mu_1 v_1}{\mu_2}$$
$$= \frac{\mu_1 v_1}{2\mu_1} = \frac{v_1}{2}$$
$$v_2 = \frac{v_1}{2}.$$

Example 12

A vertical gap, of width 5 cm and of an infinite extent, contains a Newtonian fluid of viscosity 3 Ns/m^2 and specific gravity 0.5. A metal plate (1.5 m × 1.5 m × 0.5 cm) with a weight of 50 N is to be lifted with a constant velocity of 0.5 m/s as shown in the following figure.



If the plate is lifted such that the plate is parallel apart from the left side of the gap by a distance of 2 cm always, then the force required to pull the plate, neglecting buoyancy effects and assuming linear velocity profiles, is

(A) 468.81 N (B) 929 N
(C) 353.75 N (D) 390.25 N

Solution

The shear force acting on the left side of the metal plate, $F_l = A \times \mu \times \left(\dfrac{V-0}{d_l}\right)$, where A is the surface area of the plate, μ is the fluid viscosity, V is the constant velocity with which the plate moves and dl is the distance of the plate from the left side of the vertical gap.

$$\therefore F_l = 1.5 \times 1.5 \times 3 \times \frac{0.5}{0.02}$$

$$= 168.75 \text{ N}$$

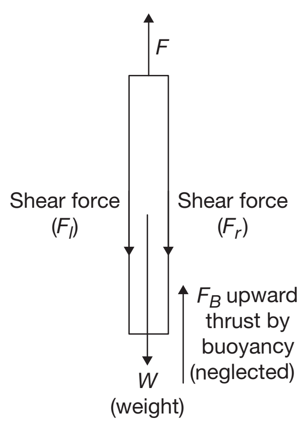
The shear force acting on the right side of the metal plate,

$F_r = A \times \mu \times \left(\frac{V-0}{d_r}\right)$, where d_r is the distance of the plate from the right side of the vertical gap.

Here, $d_r = 0.05 - 0.02 - 0.005$
$= 0.025$ m

$$\therefore F_r = 1.5 \times 1.5 \times 3 \times \frac{0.5}{0.025} = 135 \text{ N}$$

If buoyancy effects were not neglected, then an upward thrust experienced by the metal plate due to buoyancy should be accounted for in the calculations to follow.



$\therefore$ Force required to lift the plate

$$= F_l + F_r + W - F_B$$
$$= 168.75 + 135 + 50$$

($\because$ F_B is neglected)

$= 353.75$ N.

Hence, the correct answer is option (C).

Classification of Fluids

Fluids can be classified into the following types.

1. Ideal fluid (hypothetical fluid) or perfect fluid
2. Real fluid (practical fluid)
3. Newtonian Fluid
4. Non-Newtonian Fluid

These are explained as follows:

Ideal Fluid or Perfect Fluid

These fluids have zero viscosity (i.e., inviscid) and are incompressible (i.e., constant density). These fluids do not offer shear resistance when the fluid is set in motion. Though ideal fluids are hypothetical (i.e., they do not exist in reality), this concept is used in mathematical analysis of flow problems. Surface tension is zero for ideal fluid.

Real Fluid

Real fluids have non-zero viscosity and hence they offer resistance when set in motion. Real fluids have variable density and hence they have some compressibility. Surface tension is not zero for real fluids.

Newtonian Fluid

These are real fluids. These fluids obey Newton's law of viscosity, i.e., the shear stress in the fluid is directly proportional to the rate of shear strain (which is also known as velocity gradient). For such fluids, the graph of shear stress versus velocity gradient is a straight line passing through the origin (point of zero shear stress and zero velocity gradient). The slope of the graph is constant and represents the constant viscosity of the fluid at a given temperature.

Air, water, light oils, gasoline, etc., are examples of Newtonian fluids.

For Newtonian fluids,

$$\boxed{\tau = \mu \frac{du}{dy}}$$

Where

τ = Fluid shear stress

μ = Viscosity of fluid

$\frac{du}{dy}$ = Velocity gradient (or rate of shear strain)

The density of Newtonian fluids can be constant or variable (i.e., they can be compressible or incompressible).

Non-Newtonian Fluid

These are real fluids in which the shear stress is not equal to rate of shear strain. i.e., these fluids do not obey the Newton's law of viscosity.

For non-Newtonian fluids,

$$\tau \neq \mu \frac{du}{dy}$$

The relation between shear stress and velocity gradient for non-Newtonian fluid is $\tau = A\left(\frac{du}{dy}\right)^n + B$

Where, A and B are constants that depend upon type of fluid and condition of flow.

The non-Newtonian fluids can further be classified as shown below.

Time Independent Non-Newtonian Fluids

These are of two types. The first type of fluids start flowing as soon as a shear stress is applied and do not require any minimum shear stress to cause flow. **Dilatant fluids** and **pseudoplastic fluids** belong to this category.

For dilatant fluids, $n > 1$, $A = \mu$ and $B = 0$

Examples: Butter, quick sand.

For pseudo-plastic fluids, $n < 1$, $A = \mu$ and $B = 0$.

Examples: Lipsticks, paints, blood, paper pulp, rubber solution, polymeric solutions, etc.

The second type of time independent non-Newtonian fluids are called **ideal plastics** or **bingham plastic fluids**. For these fluids, the flow occurs only when the shear stress exceeds the yield stress. Once this yield stress is exceeded, increase in shear stresses are proportional to the velocity gradient. Hence for bingham plastic fluids, $n = 1$, $A = \mu$ and $B \neq 0$ but independent of time.

Examples: Tooth paste and gel, drilling mud, sewage sludge, etc.

Time Dependent Non-Newtonian Fluids

For these fluids, flow occurs only when the shear stress exceeds the yield stress.

For **thixotropic fluids**, $n < 1$, $A = \mu$ and $B \neq 0$. Also B is a function of time (t).

Hence, shear stress is of the form, $\tau = \mu\left(\dfrac{du}{dy}\right)^n + f(t)$

Examples: Printer ink, enamels.

Viscosity increases with time for such fluids.

For **rheopectic fluids**, $n > 1$, $A = \mu$ and $B \neq 0$ and B is a function of time (t).

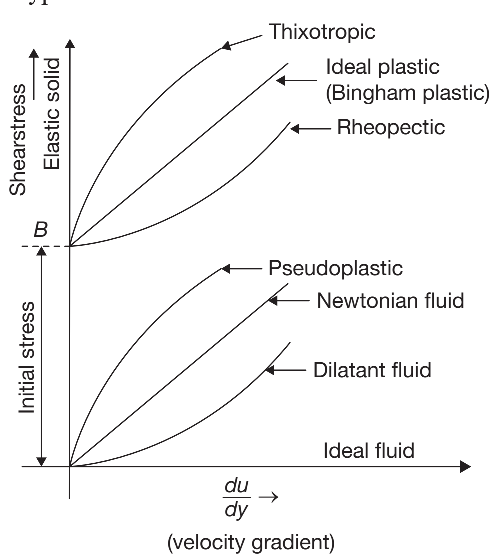
$$\therefore \quad \tau = \mu\left(\frac{du}{dy}\right)^n + f(t)$$

Viscosity decreases with time for such fluids.

Examples: Gypsum solution in water, Bentonite solution.

For non-Newtonian fluids also, the density may be constant or variable, hence non-Newtonian fluids can be incompressible or compressible.

The variation of shear stress with velocity gradient for various types of fluids is shown below.



Apparent Viscosity

The slope of the shear stress versus velocity gradient curve at a point is the apparent viscosity of the respective fluid at that point.

Kinematic Viscosity

Kinematic viscosity (γ) of a fluid is the ratio of the dynamic viscosity (μ) to the density (ρ) of the fluid, i.e., $\boxed{\gamma = \dfrac{\mu}{\rho}}$. The SI unit of kinematic viscosity is m^2/s. Another unit (in CGS units) for kinematic viscosity is stoke.

$$\boxed{1 \text{ stoke} = 1 \text{ cm}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}.}$$

Example 13

The kinematic viscosity of air at 70°C is 2.11×10^{-5} m^2/s. If the Sutherland temperature for air is 110.4 K, then the kinematic viscosity of air at 50°C is

(A) 2.11×10^{-5} m^2/s (B) 1.9×10^{-5} m^2/s
(C) 1.5×10^{-5} m^2/s (D) 3×10^{-5} m^2/s

Solution

Sutherland equation relating absolute temperature and the dynamic viscosity of an ideal gas is,

$$\boxed{\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0 + S}{T + S}\right)}$$

Where

μ = Viscosity at absolute temperature T
μ_0 = Viscosity at absolute temperature T_0
S = Sutherland temperature.

For air, $\dfrac{\rho}{\rho_0} = \dfrac{T_0}{T}$

($\because$ Air is assumed as an ideal gas at constant pressure.)

Now $\quad \dfrac{\gamma}{\gamma_0} = \dfrac{\mu\,\rho_0}{\rho\mu_0} = \dfrac{\mu}{\mu_0}\left(\dfrac{T}{T_0}\right)$

$$\frac{\gamma}{\gamma_0} = \left(\frac{T}{T_0}\right)^{5/2}\left(\frac{T_0 + S}{T + S}\right),$$

Where

S = 110.4 K,
T = 323.15 K,
T_0 = 343.15 K,
$\gamma_0 = 2.11 \times 10^{-5}$ m^2/s
γ = the kinematic viscosity

$\therefore$ Kinematic viscosity of air at 50°C = g = 1.8996×10^{-5} m^2/s.

Hence, the correct answer is option (B).

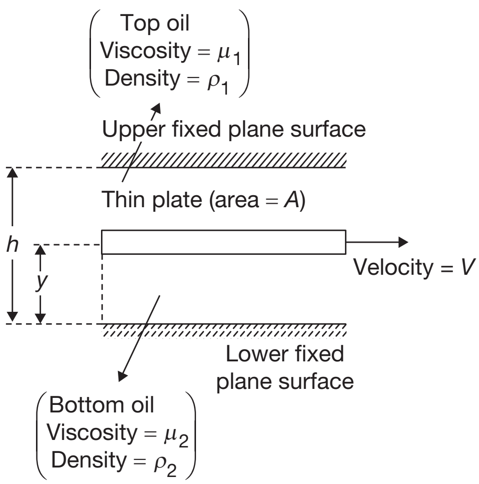
Example 14

Between two large fixed parallel plane surfaces, a thin plate is pulled, parallel to the lower plane surface, with a constant force. The space between the plate and the plane surface is filled with two types of oil where the top oil (oil at the top side of the plate) and the bottom oil (oil at the bottom side of the plate) have different kinematic viscosities. The distance between the plate and the lower plane surface is one fourth the distance between the two plane surfaces. In this horizontal position, the force required to drag the plate is the minimum compared to that required for any other horizontal positions. If the ratio of the specific mass of the top oil to that of the bottom oil is 1 : 3, then the corresponding ratio of their kinematic viscosities, should be

(A) 27 : 1 (B) 9 : 1
(C) 3 : 1 (D) 1 : 3

Solution

For a thin plate, it can be assumed that the plate thickness is negligible



Given, $\dfrac{\rho_1}{\rho_2} = \dfrac{1}{3}$ and $\dfrac{y}{h} = \dfrac{1}{4}$

The oils are assumed to be Newtonian fluids. A linear velocity profile is assumed to be present in the oils.

Shear force on the top side of the plate,

$$F_t = A\mu \frac{du}{dy} = A\mu_1 \frac{V}{h-y}.$$

Similarly shear force on the bottom side of the plate, $F_b = A\mu_2 \dfrac{v}{y}$.

Total force required to drag the plate, $F = F_t + F_b$ $= AV\left[\dfrac{\mu_1}{h-y} + \dfrac{\mu_2}{y}\right]$.

For the required force to be minimum for a given horizontal position of the plate, $\dfrac{\partial F}{\partial y} = 0$

$$\Rightarrow \frac{\mu_1}{(h-y)^2} - \frac{-\mu_2}{y^2} = 0$$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{\left(\frac{3}{4}\right)^2}{\left(\frac{1}{4}\right)^2} = 9.$$

Ratio of kinematic viscocities

$$\frac{\nu_1}{\nu_2} = \frac{\mu_1}{\rho_1}\frac{\rho_2}{\mu_2} = 9 \times 3 = 27 \text{ or } 27{:}1.$$

Hence, the correct answer is option (A).

Vapour Pressure

Vapour pressure of a liquid, at a particular temperature, is the pressure exerted by its vapour in phase equilibrium (when the vapour is saturated) with the liquid at that temperature. As the temperature increases, vapour pressure also increases. When the vapour pressure of a liquid is equal to the external environmental pressure, the liquid will start to boil. Vapour pressure depends upon molecular activity which is function of temperature. Vapour pressure increases with increase in temperature.

This property plays a role in the phenomenon called cavitation. Cavitation, which is highly undesirable due to its destructive properties, is the formation and collapse of vapour bubbles in liquid flow systems. Vapour bubbles are formed at locations where the pressure in the liquid flow system is below the vapour pressure of the liquid. Cavitation usually occurs in hydraulic structures like spillways, sluice gates and hydraulic machinery such as turbine and pumps.

Difference between Vapourisation and Boiling

The translational momentum of some surface molecules of the liquid enable them to overcome the molecular attractive force and these molecules escape into the free space above the liquid surface to become vapour. This process is vapourisation and it can occur at all temperatures. **Vapourisation can be minimized by increasing the pressure over the free surface of liquid**.

When the **pressure above the liquid free surface is less than or equal to the vapour pressure of the liquid at that temperature**, there is **continuous escape of liquid molecules** from the free surface into the space above the liquid surface. This process is called **boiling**.

Bulk Modulus (*K*)

It is also known as bulk modulus of elasticity, coefficient of compressibility or bulk modulus of compressibility.

$$\boxed{K = -V\left(\frac{\partial P}{\partial V}\right) = \rho\left(\frac{\partial P}{\partial \rho}\right)}$$

The SI unit of the bulk modulus is N/m^2 or Pascal. It is also defined as the ratio of the compressive stress to the volumetric strain. Bulk modulus increases for gases as pressure and temperature increases. As temperature increases bulk modulus decreases for liquids.

Lower the value of the bulk modulus of a fluid, more compressible is the fluid considered to be. For a truly incompressible fluid (i.e., fluid whose volume cannot be changed), K = infinity. Liquids are usually considered to be incompressible, i.e., they have a large value of bulk modulus.

The reciprocal of the bulk modulus is called as the compressibility (α), i.e., $\boxed{\alpha = \frac{1}{K}}$

Gases are usually considered to be compressible, i.e., they have a large value of compressibility. Gases compressibility becomes important only when the gas velocity becomes more than 20% of the velocity of sound waves in that gas.

Isothermal bulk modulus,

$$K_T = V\left(\frac{\partial P}{\partial V}\right)_T \text{ (i.e., at constant temperature } T\text{)}$$

Adiabatic bulk modulus,

$$K_S = -V\left(\frac{\partial P}{\partial V}\right)_S \text{ (i.e., at constant entropy } S\text{).}$$

Isothermal compressibility,

$$\alpha_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T \text{ (i.e., at constant temperature } T\text{)}$$

Adiabatic compressibility,

$$\alpha_S = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_S \text{ (i.e., at constant entropy } S\text{)}$$

Example 15

In a piston cylinder arrangement containing gas A, it is found that to reduce isothermally the volume of the gas to 75% of its original volume, an additional pressure of 2 atm is required. In another piston cylinder arrangement containing gas B (density = 1.5 kg/m^3), it is found that the density of the gas can be increased by 1.5 kg/m^3 at a constant temperature, if a pressure change of 6 bar is provided. From these observations, one can state that

(A) gas A and gas B have equal isothermal compressibility.
(B) gas A is 1.2 times more isothermally compressible than gas B.
(C) gas B is 1.35 times more isothermally compressible than gas A.
(D) enough information is not available for the comparison of the isothermal compressibility of the two gases.

Solution

For gas A, let V_1 and V_2 be the original volume and the volume of the gas after compression respectively.

Given, $V_2 = 0.75\ V_1$

$$\Rightarrow \frac{\Delta V}{V} = \frac{V_2 - V_1}{V_1} = -0.25$$

$$\Delta P = 2 \text{ at } m = 2 \times 1.01325 \text{ bar}$$

$$K_{TA} = -V\left(\frac{\partial P}{\partial V}\right)_T$$

$$\cong -\left(\frac{\Delta P}{\Delta \frac{V}{V}}\right)_T$$

$$\cong -\frac{2 \times 1.01325}{-0.25} \cong 8.106 \text{ bar}$$

For gas B, $\rho = 1.5$ kg/m^3

$$\Delta P = 1.5 \text{ kg/m}^3$$

$$\frac{\Delta \rho}{\rho} = 1$$

$$\Delta P = 6 \text{ bar}$$

$$\therefore K_{TB} = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \cong \frac{6}{1} \cong 6 \text{ bar}$$

$$\therefore \frac{K_{TA}}{K_{TB}} = \frac{8.106}{6} = 1.35$$

∴ Gas B is 1.35 times more isothermally compressible than gas A.
Hence, the correct answer is option (C).

Coefficient of Volume Expansion (β)

It is also known as volume expansivity.

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P.$$

The SI unit of the coefficient of volume expansion is 1/K.

Example 16

If the isothermal compressibility and volume expansivity of a fluid are α_T and β respectively, then the fractional change in the volume $\left(\frac{dV}{V}\right)$ of the fluid for a change in temperature (dT) and change in pressure (dP) is equal to.

(A) $\alpha_T dT - \beta dP$
(B) $\beta dT - \alpha_T dP$
(C) $\alpha_T dT + \beta dP$
(D) $\alpha_T dP + \beta dT$

Solution

The volume of the fluid (V) is a function of temperature (T) and pressure (P). This can be written as

$$V = V(T, P)$$

Differentiating, we get

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP \qquad (1)$$

Now, $\alpha_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ and $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$

Substituting the above relations for α_T and β in Eq. (1) and rearranging, we get

$$\frac{dV}{V} = \beta dT - \alpha_T dP$$

Hence, the correct answer is option (B).

Surface Tension

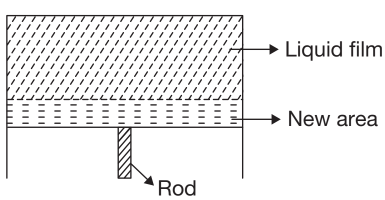
The layer of molecules at the surface of a liquid, in contact with a gas (or another immiscible liquid), tends to behave like a stretched membrane (membrane on which a tensile force is exerted).

This behaviour is a result of the inward pull, arising due to the cohesive forces (intermolecular forces of attraction between molecules of the same liquid), experienced by the liquid's surface molecules.

At the liquid surface, the tensile force dF acting parallely to the plane of the surface (or tangentially to the surface) over a surface length dl is given by the equation:

$\boxed{dF = \sigma dl}$, where σ is called as the coefficient of surface tension of the liquid. Hence, surface tension is equal to the magnitude of the tensile force acting tangentially at the surface per unit length of the surface. The SI unit of surface tension is N/m.

Imagine a metallic frame in which a liquid film is maintained as shown in the following figure.



When the rod is slightly pulled down, the liquid film gets stretched over a larger area. The work done for creating the new area is the surface energy.

$$\frac{\text{Surface energy}}{\text{New area created}} = \text{Surface tension}$$

$\therefore$ Surface energy per unit area = Surface tension

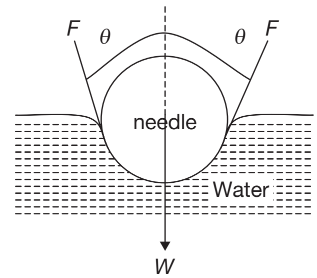
Surface tension (in N/m or J/m^2) thus also represents the amount of stretching work required to increment the surface area by an unit amount. Surface tension of a liquid decreases with temperature and becomes zero at the critical point. The effect of pressure on the surface tension of a liquid can be considered to be negligible. Surface tension of a liquid can be increased or decreased by adding impurities. For example, surface tension of water can be decreased or increased by adding surfactants or NaCl respectively.

Example 17

A solid cylindrical needle (density = 7.8 g/cm^3) of length 5 cm is placed very gently on the surface of a body of water (surface tension = 73 dynes/cm) such that it floats on the water surface. Neglect buoyancy effects and surface tension effects at the circular faces of the needle. The maximum diameter that the needle can have, such that it will still be able to float on the water surface, is

(A) 1.56 mm (B) 4.88 mm
(C) 5.26 mm (D) 1.31 mm

Solution



Let F be the force, due to surface tension of water, acting along the length of the needle on either side as shown in the above figure. Let W be the weight of the needle.

Now, $F = \sigma L$, where σ is the surface tension of water and L is the length of the needle.

If θ is the angle that the force F makes with the vertical, then writing a force balance on the needle gives:

$$W = F\cos\theta + F\cos\theta$$
$$= 2\,\sigma L\cos\theta \qquad (1)$$

If d and ρ are the diameter and density of the needle, then from Eq. (1) we can write

$$\pi\frac{d^2}{4}L\rho g = 2\sigma L\cos\theta$$

$$d = \sqrt{\frac{8\sigma\cos\theta}{\pi\rho g}}$$

The maximum value of d (d_{max}) is obtained when $\theta = 0°$ (provided all other parameters are fixed).

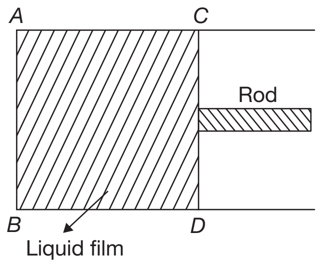
$$\therefore \quad d_{max} = \sqrt{\frac{8\sigma}{\pi\rho g}}$$

$$= \sqrt{\frac{8\times 0.073}{3.14\times 7800\times 9.81}} \quad \boxed{1\text{ dyne} = 10^{-5}\text{ N}} \quad = 1.56\text{ mm.}$$

Hence, the correct answer is option (A).

Example 18

A liquid film, exposed to the atmosphere on both sides, is present in the area $ABCD$ of the metallic frame work shown in the following figure.



The side CD, of length 10 cm, is movable and can be pulled with the help of a rod. The work done to increase the length of side BD by 1 mm, still maintaining the liquid film (Surface tension = 0.073 N/m) in the area $ABCD$, is

(A) 7.3×10^{-6} J (B) 1.46×10^{-5} J
(C) 1.46×10^{-4} J (D) 7.3×10^{-5} J

Solution

Let L be the length of the side CD. Then, $L = 10$ cm = 0.1 m.

At the side CD, there are two lengths on which surface tension acts since the film of liquid is exposed to the atmosphere on both sides. Hence the length along which the surface tension acts at the side $CD = 2L$.

$\therefore$ Work done = $\sigma 2L\, \Delta x$, where $\sigma 2L$ represents the force due to surface tension acting at the side CD.

Here, $\Delta x = 1$ mm $= 1 \times 10^{-3}$ m

$$\sigma = 0.073 \text{ N/m}$$

$$\text{Work done} = 0.073 \times 2 \times 0.1 \times 1 \times 10^{-3}$$
$$= 1.46 \times 10^{-5} \text{ J.}$$

Hence, the correct answer is option (B).

Effects of Surface Tension

1. A falling rain drop attaining a spherical shape.
2. Sap rising in a tree.
3. Birds being able to drink water from ponds.
4. Capillary rise.
5. Dust particles collecting on the surface of a liquid.
6. Liquid jets breaking up.

Excess Pressure

In liquid droplets, gas bubbles, soap bubbles and liquid jets, an amount of pressure in excess to the external pressure is present due to surface tension for maintaining the shape.

1. **Liquid droplet or gas bubble:**

$$\boxed{P_i - P_0 = \Delta P = \frac{4\sigma}{d}},$$

Where, P_i is the pressure inside the liquid droplet or gas bubble, P_0 is the pressure outside the liquid droplet or gas bubble, d is the diameter of the (spherical) liquid droplet or gas bubble and ΔP is the excess pressure.

2. **Soap or liquid bubble:**

A soap or liquid bubble has air both inside and outside it and hence it has two free surfaces on which surface tension acts.

$$\boxed{P_i - P_0 = \Delta P = \frac{8\sigma}{d}}$$

Where, d is the outer diameter of the soap or liquid bubble.

3. **Cylindrical liquid jet:**

$$\boxed{P_i - P_0 = \Delta P = \frac{2\sigma}{d}}$$

Where, d is the diameter of the cylindrical jet.

Example 19

The pressures inside and outside of a water bubble and water drop are found to be the same. If d is the diameter of the water bubble and if the bubble and drop are at the same temperature, then the diameter of the water drop is

(A) d (B) $3d$
(C) $2d$ (D) $d/2$

Solution

Since the inside and outside pressures of the water drop are equal to that of the water bubble, we have

Excess pressure inside the water drop = Excess pressure inside the water bubble, i.e., $\dfrac{4\sigma}{d_d} = \dfrac{8\sigma}{d_b}$,

where d_d and d_b are the diameters of the water drop and water bubble respectively.

$$\therefore d_d = \frac{d_b}{2} = \frac{d}{2}$$

Hence, the correct answer is option (D).

Example 20

Two cylindrical liquid jets A and B have the surface tensions σ_A and σ_B respectively such that $\sigma_A = 2\sigma_B$. The jets A and B are exposed to the respective external pressures P_A and P_B, such that $P_B - P_A = \dfrac{2\sigma_B}{d_B}$, where d_B is the diameter of the cylindrical jet B. If the two jets have the same inside pressure, then the diameter of the cylindrical jet A is

(A) d_B (B) $2d_B$
(C) $0.5\, d_B$ (D) $4d_B$

Solution

Given, $\sigma_A = 2\sigma_B$ and

$$P_A - P_B = \frac{2\sigma_B}{d_B} \tag{1}$$

Jets A and B have the same inside pressure, hence

$$\frac{2\sigma_A}{d_A} + P_A = \frac{2\sigma_B}{d_B} + P_B, \tag{2}$$

Where, d_A is the diameter of the cylindrical jet A.

$$P_B - P_A = \frac{2\sigma_A}{dA} - 2\sigma_B$$

But $P_B - P_A = \dfrac{2\sigma_B}{dB}$

Equating,

$$\therefore \frac{2\sigma_B}{dB} = \frac{2\sigma_A}{dA} - 2\sigma_B/dB$$

$$\frac{4\sigma_B}{dB} = \frac{2\sigma_A}{dA}$$

$$\therefore \quad dA = dB.$$

Hence, the correct answer is option (A).

Capillarity

When a small diameter tube is inserted into a body of liquid, the liquid rises or falls in the tube giving rise to the phenomenon known as capillarity. Capillarity is due to the forces of cohesion (attraction between the same molecules) between the liquid molecules and the forces of adhesion (attraction between different molecules) between the liquid and solid (constituting the tube) molecules.

The rise of the liquid is called as the capillary rise while the fall is called as the capillary drop or capillary depression. Capillarity or capillary effect can be termed to be a consequence of surface tension.

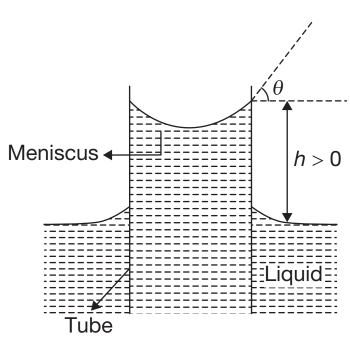
The strength of capillarity (or capillary effect) is quantified by a parameter called as the contact (or wetting) angle (θ). The contact angle is defined as the angle between the solid surface and the tangent to the liquid surface at the point of contact between the two surfaces. The surface tension force acts along the tangent towards the solid surface. The magnitude of the capillary rise of a liquid (surface tension = σ, density = ρ) having a contact angle θ with a tube of constant diameter d is given by

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$$

The contact angle of water with clean glass is nearly zero, i.e., $\theta \approx 0°$. (If 0°, then it is called complete or perfect wetting).

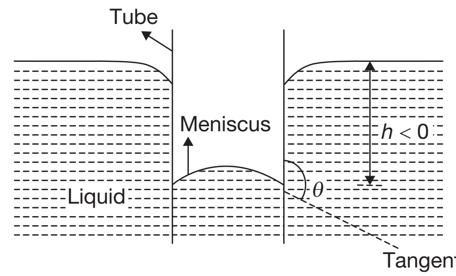
For glass tubes with diameters greater than 1 cm the capillarity effect of water is negligible.

Liquid Wets Solid Surface



1. Contact angle θ is less than 90°.
2. When a small diameter tube made of the solid is dipped in the liquid, capillary rise occurs.
3. Magnitude of cohesive forces < Magnitude of adhesive forces
4. For example, waterglass
5. Capillary drop = h

Liquid does not Wet Solid Surface



1. Contact angle θ is greater than 90°.
2. When a small diameter tube made of the solid is dipped in the liquid, capillary drop occurs.
3. Magnitude of adhesive forces < Magnitude of cohesive forces.
4. Liquid is termed as a non-wetting liquid.
5. For example, mercury-glass
6. Capillary drop = $|h|$

Example 21

When tube A is dipped into the body of a liquid, the liquid makes a contact angle of 30° with the tube. When tube B of different material having twice the diameter of tube A, is dipped into the same liquid body, the liquid makes a contact angle of 120° with the tube. The ratio of the capillary rise seen in one of the tubes to the capillary drop seen in the other is

(A) 0.28 (B) 1.73
(C) 3.46 (D) 0.58

Solution

Let, d_A and θ_A be the diameter and contact angle for tube A.

Let, d_B and θ_B be the diameter and contact angle for tube *B*.

Given $d_B = 2d_A$, $\theta_A = 30°$ and $\theta_B = 120°$.

Since, $\theta_A < 90°$, capillary rise (h_r) will be seen when tube A is dipped.

$$\therefore \ h_r = \frac{4\sigma\cos\theta_A}{\rho g d_A} \qquad (1)$$

Since $\theta_B > 90°$, capillary drop (h_d) will be seen when tube B is dipped.

$$\therefore h_d = \frac{-4\sigma\cos\theta_B}{\rho g d_B} \qquad (2)$$

(Negative sign is introduced since h_d is already referred to as capillary drop)

From Eqs. (1) and (2), we have

$$\therefore \frac{h_r}{h_d} = \frac{-\cos\theta_A \times d_B}{\cos\theta_B \times d_A}$$

$$= \frac{-\cos 30° \times 2d_A}{\cos 120° \times d_A} = 3.46.$$

Hence, the correct answer is option (C).

Example 22

The maximum diameter that a capillary tube can have to ensure that a capillary rise of at least 6 mm is achieved when the tube is dipped into a body of liquid with surface tension = 0.08 N/m and density = 900 kg/m^3, is

(A) 3 mm (B) 6 mm
(C) 5 m (D) 8 mm

Solution

The capillary rise $h = \dfrac{4\sigma\cos\theta}{\rho g d}$, where σ, θ, ρ, g and d have their usual meanings.

$\therefore$ Diameter of the capillary tube $d = \dfrac{4\sigma\cos\theta}{\rho g h}$.

Here θ is taken to be 0°. The diameter d gets the maximum value (d_{max}) when h is minimum (i.e., $h = h_{min}$)

Given, $h_{min} = 6$ mm

$$\therefore d_{max} = \frac{4\sigma}{\rho g h_{min}} = \frac{4 \times 0.08}{900 \times 9.81 \times 0.006}$$

$$= 6 \text{ mm}.$$

Hence, the correct answer is option (B).

Pressure

Pressure is defined as a normal force exerted by a fluid per unit area. The normal stress on any plane through a fluid element at rest is equal to the fluid pressure. The SI unit of pressure is Pascal (Pa) or N/m^2 $\boxed{1 \text{ Pa} = 1 \text{ N/m}^2}$. Other units for pressure are atm (1 atm = 101325 Pa), psi (1 atm = 14.696 psi) and bar (1 bar = 10^5 Pa). Pressure is a scalar quantity. At a point on a surface which is in contact with a fluid, the pressure force exerted by the fluid is normal to the surface.

Atmospheric, Absolute and Gauge Pressure

Atmospheric pressure (P_{atm}) is the pressure exerted on a surface by a planet's atmosphere (Example: the Earth's atmosphere) present above the surface.

Absolute pressure (P_{abs}) is the pressure measured relative to an absolute vacuum (where $P_{abs} = 0$). At any given position, the actual pressure is the absolute pressure.

Gauge pressure (P_{gauge}) is the pressure indicated by a pressure measuring device (or pressure gauge) relative to the local atmospheric pressure. This is stated with the assumption that the pressure gauge is calibrated with the local atmospheric pressure as reference.

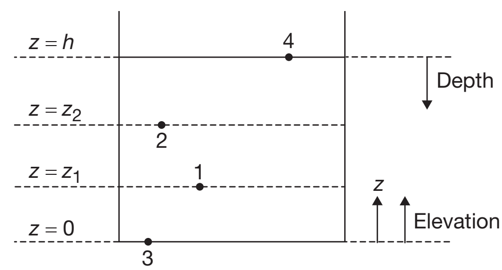
$$\boxed{P_{gauge} = P_{abs} - P_{atm}}$$

If $P_{abs} < P_{atm}$, then P_{gauge} is negative and the negative of the gauge pressure is called as the vacuum pressure (P_{vac}). Pressure gauges measuring vacuum pressures are called as vacuum gauges.

$$P_{vac} = P_{atm} - P_{abs}$$

Pressure Varying with Elevation or Depth (for Static Fluids)

Consider a static body of liquid (density = ρ, specific weight = ω) of height h present in a container as shown in the following figure.



The variation of pressure P in the liquid with respect to the elevation z is given by,

$$\boxed{\frac{dP}{dz} = -\rho g = -\omega} \quad (1)$$

Eq. (1), called as the *hydrostatic* (*differential*) *equation*, corresponds to the *hydrostatic law* which states that 'The rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid'.

Conventionally at $z = 0$, elevation = 0 and depth = h, while at $z = h$, elevation = h and depth = 0. If P_1 and P_2 are the pressures at points 1 ($z = z_1$) and 2 ($z = z_2$), from Eq. (1)' we have

$$\boxed{P_2 - P_1 = \Delta P = -\int_{z=z_1}^{z=z_2} \rho g dz} \quad (2)$$

For liquids, usually the density is considered to be constant upto certain large depths. If the acceleration due to gravity (g) is also constant with respect to the elevation z, then

$$\boxed{P_2 - P_1 = \rho g (z_1 - z_2) = -\rho g \Delta_z} \quad (3)$$

Where, Δz (= $z_2 - z_1$) is sometimes called as the *pressure head* and is interpreted as the height of a column of liquid of density ρ required to provide a pressure difference of $P_1 - P_2$.

If the surface of the liquid in the container is exposed to the atmosphere and ρ and g are assumed to be constant with respect to z, then

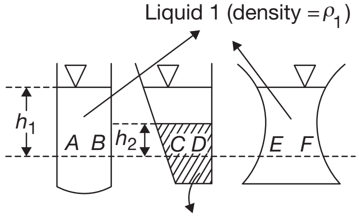
P_{abs} at point 4 = P_{atm}
P_{abs} at point 2 = $P_{atm} + \rho g(h - z_2)$
P_{gauge} at point 1 = $\rho g(h - z_1)$
P_{abs} at point 3 = $P_{atm} + \rho gh$

Eq. (1) is also applicable for gases. However, as gases have a low density, the variation of pressure with height (for small to moderate heights) can be considered to be negligible for a gas.

Pressure Varying Horizontally (for Static Fluids)

For a fluid resting inside a container, pressure does not depend on the shape or cross-section of the container. Also, the pressure is the same at all points on any horizontal plane considered in the fluid present in the container.

Consider three containers, open to the atmosphere, of different shapes where the free surface of the liquids in them are at the same level as shown in the following figure.



The points A, B, C, D, E and F all lie on the same horizontal plane. Here,

$$P_A = P_B = P_E = P_F \text{ and } P_C = P_D$$

Since $\rho_2 > \rho_1$, it can be seen that $P_C > P_B$ and hence $P_C \neq P_B$.

Pascal's Law

Pascal's law states that the pressure at a point in a static fluid has the same magnitude in all directions. This is also true for non-static fluids which have no shear stress, for example, for fluids which move like rigid bodies where there is no relative motion between the fluid elements.

Another version of Pascal's law states that when there is an increase in pressure at any point in a confined fluid, there is an equal increase in the pressure at every other point in the confined fluid. Pascal's law forms the underlying principle of the hydraulic jack and hydraulic press.

Example 23

A hydraulic press has a plunger of 5 cm diameter. If the weight lifted by the hydraulic press is twice the force applied at the plunger, then the diameter of the ram of the hydraulic press is

(A) 5 cm (B) 10 cm
(C) $5\sqrt{2}$ cm (D) $10\sqrt{2}$ cm

Solution

Let the force applied at the plunger be F. Then weight lifted by the hydraulic press, $W = 2F$. (1)

Let d and D be the diameters of the plunger and ram respectively and let a and A be their respective areas.

$$\therefore a = \frac{\pi d^2}{4} \text{ and } A = \frac{\pi D^2}{4} \qquad (2)$$

From Pascal's law, $\dfrac{F}{a} = \dfrac{W}{A}$ (3)

Substituting Eqs. (1) and (2) in Eq. (3), we get

$$D = \sqrt{2}d$$

Given $d = 5$ cm $\therefore D = 5\sqrt{2}$ cm.

Hence, the correct answer is option (C).

NOTE

When the plunger and the Ram are of circular Cross section and 'F' is the load applied at the plunger, load lifted at the ram is,

$$= \frac{F}{\frac{Ad^2}{4}} \times \frac{\pi D^2}{4} = F\frac{D^2}{d^2}$$

Here, $F\dfrac{D^2}{d^2} = 2F;\ \therefore D = \sqrt{2}d$

Example 24

Oil weight density = 8.5 kN/m^3 is present in a tank upto a depth of 6 m. It is observed that an immiscible liquid, with a depth of 2 m, is present in the tank below the oil. The reading on the pressure gauge connected to the tank's bottom is 70 kPa. The specific gravity of the immiscible liquid is

(A) 0.982 (B) 0.968
(C) 0.873 (D) 0.893

Solution

Let the weight density of the immiscible liquid and the oil be ω_L and ω_O respectively.

Pressure at the bottom of the tank,

$$P_b = 6 \times \omega_O + 2 \times \omega_L$$

Given $P_b = 70$ kPa and $\omega_O = 8.5$ kN/m^3

$$\therefore \omega_L = \frac{70\times10^3 - 6\times8.5\times10^3}{2} = 9500 \text{ N/m}^3$$

Specific gravity of the liquid, $SG_L = \dfrac{\omega_L}{\rho_\omega \times g}$,

Where, $\rho_\omega (= 1000 \text{ kg/m}^3)$ is the density of pure water at 4°C.

$$\therefore \quad SG_L = \frac{3500}{1000\times9.81} = 0.968$$

Hence, the correct answer is option (B).

Manometry (Some Cases to Measure the Gauge Pressure)

Manometers are pressure measuring devices which employ liquid columns in vertical or inclined tubes to measure pressure. Manometers are classified as,

1. Simple manometers
2. Differential manometers.

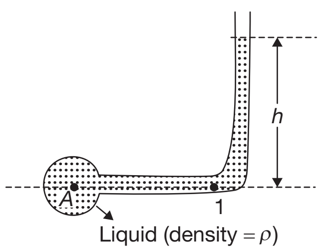
Simple Manometers

A simple manometer consists of a tube whose one end is connected to a point where the pressure is to be measured and the other end is open to the atmosphere. The common types of simple manometers are

1. Piezometer
2. U-tube manometer
3. Single column manometer.

For the following discussion, consider P_1 and P_A to be the pressures at points 1 and A respectively.

Piezometer



Analysis: $P_1 - P_{\text{atm}} + h\rho g$

$P_A = P_1$, since the points A and 1 are at the same elevation and in the same liquid.

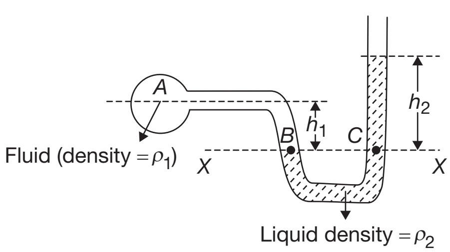
$\therefore$ $$\boxed{P_A = P_{\text{atm}} + h\rho g}$$

NOTE

It is implicitly assumed here that surface tension effects (capillary rise) are negligible.

U-tube Manometer

1.



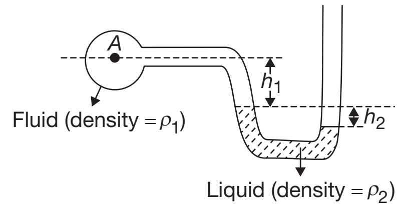
Analysis: Along the section XX, pressure at point B = Pressure at point C Flaid

($\because$ Points B and C are at the same elevation and in the same liquid)

i.e., $$P_A + h_1\rho_1 g = P_{\text{atm}} + h_2\rho_2 g$$

$$\boxed{P_A = P_{\text{atm}} + (h_2\rho_2 - h_1\rho_1)g}$$

2.

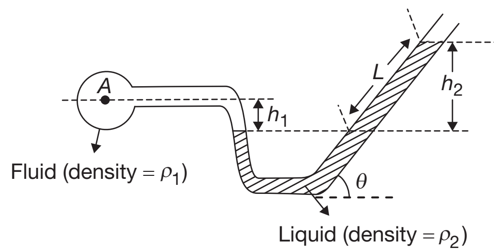


$$\boxed{P_A = P_{\text{atm}} - (h_1\rho_1 + h_2\rho_2)g}$$

NOTE

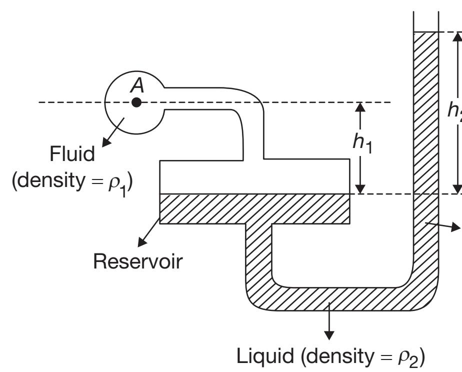
P_A is vacuum pressure

3.



$$\boxed{P_A = P_{\text{atm}} + g(h_2\rho_2 - h_1\rho_1) = P_{\text{atm}} + g(L\sin\theta\rho_2 - h_1\rho_1)}$$

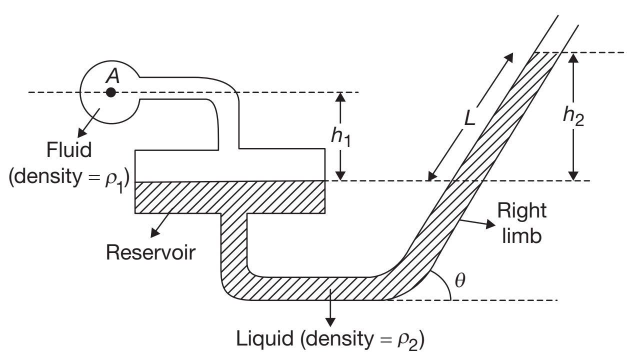
Vertical Single Column Manometer



$$\boxed{P_A = P_{\text{atm}} + \frac{a \times h_2}{A}(\rho_2 g - \rho_1 g) + h_2\,\rho_2 g}$$

Where, A and a are the cross-sectional areas of the reservoir and the right limb respectively.

Inclined Single Column Manometer



$$\boxed{\begin{aligned}P_A &= P_{\text{atm}} + \frac{a \times h_2}{A}(\rho_2 g - \rho_1 g) + h_2\rho_2 g - h_1\rho_1 g \\ P_A &= P_{\text{atm}} + \frac{a \times L\sin\theta}{A}(\rho_2 g - \rho_1 g) + L\sin\theta\rho_2 g - h_1\,\rho_1\,g\end{aligned}}$$

Where, A and a are the cross-sectional areas of the reservoir and the right limb respectively.

Sensitivity of the instrument is inversely proportional to $\sin\theta$.

$$\text{Sensitivity} \propto \frac{1}{\sin\theta}.$$

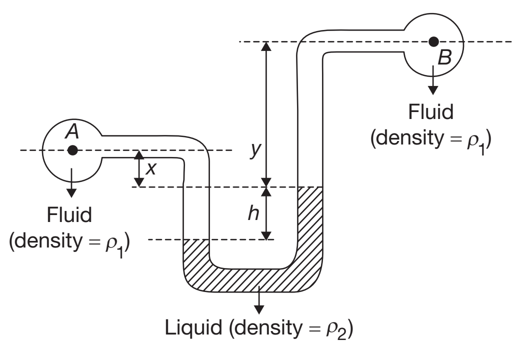
Differential Manometers

Differential manometers are the devices used for measuring the difference between the pressure at a given point in a fluid and the pressure at some other point in the same or different fluid. A differential manometer consists of a U-tube, in which a heavy liquid is present, where two ends are connected to points whose pressure difference is to be measured. Most common types of differential manometers are:

1. U-tube differential manometer
2. Inverted U-tube differential manometer

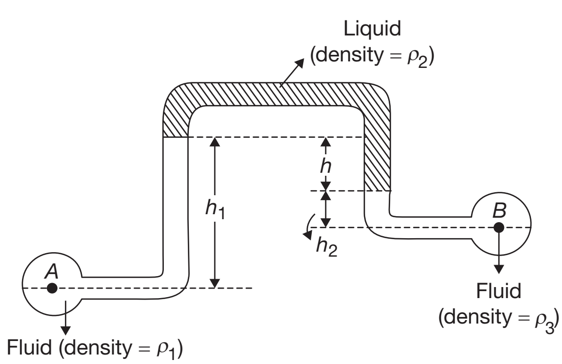
For the following discussion, consider P_A and P_B to be the pressures at the points A and B respectively.

U-tube Differential Manometer



$$\boxed{P_A - P_B = h(\rho_2 - \rho_1)g + y\rho_3 g - x\rho_1 g}$$

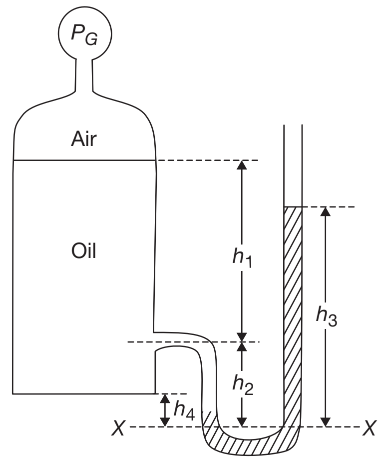
Inverted U-tube Manometer



$$\boxed{P_A - P_B = h_1\,\rho_1\,g - h_2\,\rho_3\,g - h\,\rho_2\,g}$$

Example 25

A closed tank consists of oil (density = ρ_1) and compressed air as shown in the following figure.



A U-tube manometer using a liquid with density = ρ_2, is connected to the tank. The variation of pressure with height is negligible in the tank volume occupied by air. If the pressure reading in the pressure gauge connected to the top of the tank is P_G, then an expression for the height of oil in the tank can be

(A) $h_3\left(\frac{\rho_1}{\rho_2}\right) - \frac{P_G}{\rho_1 g} - h_4$

(B) $h_3\left(\frac{\rho_2}{\rho_1}\right) - \frac{P_G}{\rho_1 g} - h_4$

(C) $h_3\left(\frac{\rho_2}{\rho_1}\right) - \frac{P_G}{\rho_1 g} - h_2$

(D) $h_3\left(\frac{\rho_1}{\rho_2}\right) - \frac{P_G}{\rho_2 g} - h_4$

Solution

Equating pressures at a point in the left limb and at a point in the right limb, where both the points lie on a horizontal plane passing through the meniscus of the liquid (density $= \rho_2$) in the left limb of the U-tube manometer, gives

$$P_{\text{air}} + (h_1 + h_2)\,\rho_1\,g = P_{\text{atm}} + h_3\,\rho_2\,g \qquad (1)$$

Now $$P_G = P_{\text{air}} - P_{\text{atm}} \qquad (2)$$

From the figure in the question it can be shown that the height of the oil in the tank, $h = h_1 + (h_2 - h_4)$ (3)

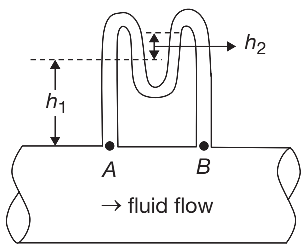
Substituting Eqs. (2) and (3) in Eq. (1) and rearranging, we get

$$h = h_3\left(\frac{\rho_2}{\rho_1}\right) - \frac{P_G}{\rho_1 g} - h_4$$

Hence, the correct answer is option (B).

Example 26

A fluid (weight density = ω_1) flows through a pipe as shown in the following figure. A differential U-tube manometer, with a liquid of weight density = ω_2, is fitted to the pipe in order to determine the pressure difference ($P_A - P_B$), where P_A and P_B are the pressures at the respective points A and B on the pipe.



From the set of variables $\{h_1, h_2, \omega_1, \omega_2\}$, the set of the least number of variables whose values are to be known in order to determine the required pressure difference ($P_A - P_B$) is

(A) $\{h_1, h_2, \omega_1, \omega_2\}$
(B) $\{h_1, \omega_1, \omega_2\}$
(C) $\{h_2, \omega_2\}$
(D) $\{h_2, \omega_1, \omega_2\}$

Solution

Equating pressures at a point in the left limb and at a point in the right limb, where both points lie on a horizontal plane passing through the meniscus of the liquid (weight density = ω_2 in the left limb of the differential U-tube manometer, gives

$$P_A - h_1\,\omega_1 = P_B - (h_1 + h_2)\omega_1 + h_2\omega_2$$

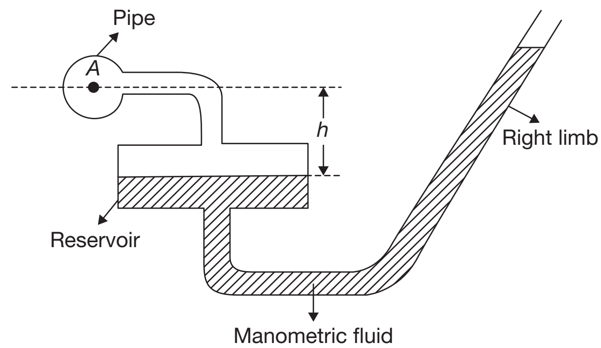
or $P_A - P_B = h_2\,(\omega_2 - \omega_1)$

∴ The set of variables whose values are to be known = $\{h_2, \omega_1, \omega_2\}$.

Hence, the correct answer is option (D).

Example 27

An inclined single column manometer is connected to a pipe transporting a liquid of specific weight $(\omega_1) = 9.81$ kN/m^3, as shown in the following figure. The area of the reservoir is very large compared to the area of the right limb of the manometer. The specific weight (ω_2) of the manometric fluid is 13.6 kN/m^3. The length (L) of the manometric fluid in the right limb, above the manometric fluid's surface in the reservoir, is 100 cm. The gauge pressure (P) at the point A in the pipe is 3.857 kPa. If the value of h is 30 cm, then the right limb of the manometer is inclined to the horizontal at an angle of



(A) 45°
(B) 60°
(C) 30°
(D) 15°

Solution

Let θ be the angle at which the right limb is inclined to the horizontal.

If a and A are the respective cross-sectional areas of the right limb and the reservoir, then p is very small and negligible (∵ $A >>> a$).

For the inclined column manometer, one can write,

$$P = \frac{a}{A} \times L \times \sin\theta(\omega_2 - \omega_1) + L\sin\theta\,\omega_2 - h\omega_1$$

Since $\frac{a}{A}$ is negligible, $P = L\,\sin\theta\,\omega_2 - h\omega_1$

$$\therefore \sin\theta = \frac{P + h\omega_1}{L\omega_2} = \frac{3.857 \times 10^3 + 0.3 \times 9.81 \times 10^3}{1 \times 13.6 \times 10^3}$$

That is, $\theta = 30°$.

Hence, the correct answer is option (C).

Exercises

1. The normal stress is the same in all directions at a point in a fluid only when
   (A) the fluid is frictional.
   (B) the fluid is frictionless and incompressible.
   (C) the fluid has zero viscosity and is at rest .
   (D) one fluid layer has no motion relative to an adjacent layer.

2. An incompressible fluid (kinematic viscosity, 7.4 × 10^{-7} m^2/s, specific gravity 0.88) is held between two parallel plates. If the top plate is moved with a velocity of 0.5 m/s while the bottom one is held stationary, the fluid attains a linear velocity profile in the gap of 0.5 mm between these plates; the shear stress in Pascals on the surface of top plate is

(A) 0.651×10^{-3} (B) 0.651
(C) 6.51 (D) 0.651×10^3

3. For a Newtonian fluid
(A) shear stress is proportional to shear strain.
(B) rate of shear stress is proportional to shear strain.
(C) shear stress is proportional to rate of shear strain.
(D) rate of shear stress is proportional to rate of shear strain.

4. Match List I (Flows over or inside the systems) with List II (Type of flow) and select the correct answer using the codes given below the lists:

	List I		List II
a.	Flow over a sphere	**1.**	Two-dimensional flow
b.	Flow over a long circular cylinder	**2.**	One-dimensional flow
c.	Flow in a pipe bend	**3.**	Axisymmetric flow
d.	Fully developed flow in a pipe at constant flow rate	**4.**	Three-dimensional flow

Codes:

	a	b	c	d		a	b	c	d
(A)	3	1	2	4	(B)	1	4	3	2
(C)	3	1	4	2	(D)	1	4	2	3

5. Consider the following statements:
I. Viscosity
II. Surface tension
III. Capillarity
IV. Vapour pressure

Which of the above properties can be attributed to the flow of jet of oil in an unbroken stream?
(A) I Only (B) II Only
(C) I and III (D) II and IV

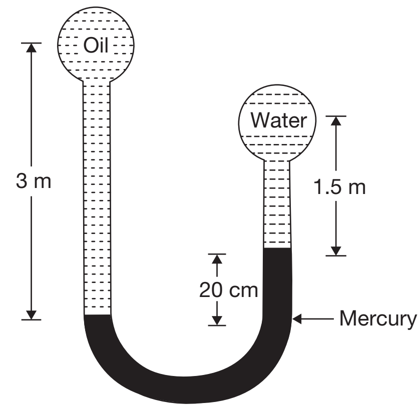
6. The dimensions of a pressure gradient in a fluid flow are
(A) $ML^{-1}T^{2}$ (B) $ML^{-3}T^{-2}$
(C) $ML^{-2}T^{-2}$ (D) $M^{-1}L^{-3}T^{-2}$

7. Shear stress develops on a fluid element, if
(A) the fluid is at rest.
(B) the fluid container is subject to uniform linear acceleration.
(C) the fluid is inviscid.
(D) the fluid is viscous and the flow is non-uniform.

8. If, for a fluid in motion, the pressure at a point is same in all directions, then the fluid is
(A) a real fluid.
(B) a Newtonian fluid.
(C) an ideal fluid.
(D) a Non-Newtonian fluid.

9. The unit of dynamic viscosity of a fluid is
(A) m^2/s (B) Ns/m^2
(C) Pa s/m^2 (D) kg s^2/m^2

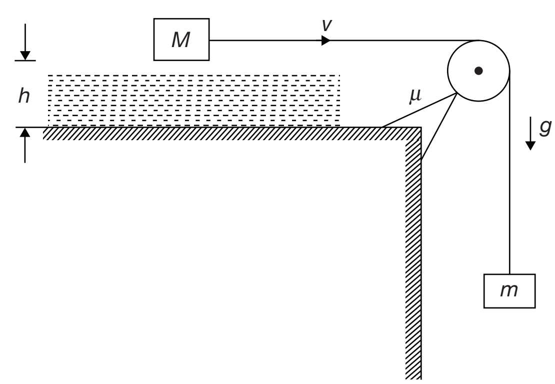
10. Two pipelines, one carrying oil (mass density 900 kg/m^3) and the other water, are connected to a manometer as shown in the figure. By what amount the pressure in the water pipe should be increased so that the mercury levels in both the limbs of the manometer become equal? (Mass density of mercury = 13550 kg/m^3 and g = 9.81 m/s^2)



(A) 24.7 kPa (B) 26.5 kPa
(C) 26.7 kPa (D) 28.9 kPa

11. What is the capillary rise in a narrow two-dimensional slit of width 'w'?
(A) Half of that in a capillary tube of diameter 'w'.
(B) Two-third of that in a capillary tube of diameter 'w'.
(C) One-fourth of that in a capillary tube of diameter 'w'.
(D) One-fourth of that in a capillary tube of diameter 'w'.

12. A cubic block of side 'L' and mass 'M' is dragged over an oil film across table by a string which connects to a hanging block of mass 'm' as shown in the figure. The Newtonian oil film of thickness 'h' has dynamic viscosity 'μ' and the flow condition is laminar. The acceleration due to gravity is 'g'. The steady state velocity 'v' of block is



(A) $\dfrac{Mgh}{\mu L^2}$ (B) $\dfrac{Mgh}{\mu}$

(C) $\dfrac{mgh}{\mu L^2}$ (D) $\dfrac{mgh}{\mu}$

13. Consider the following statements:

I. A small bubble of one fluid immersed in another fluid has a spherical shape.

II. The droplets of a fluid move upward or downward in another fluid due to unbalance between gravitational and buoyant forces.

III. Droplets of bubbles attached to a solid surface can remain stationary in a gravitational fluid if the surface tension exceeds buoyant forces.

IV. Surface tension of a bubble is proportional to its radius while buoyant force is proportional to the cube of its radius.

Which of these statements are correct?

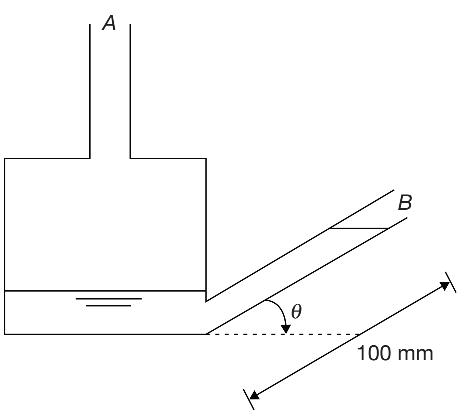
(A) I, II, III and IV (B) I, II and IV only
(C) I and III only (D) II, III and IV only

14. In a quiescent sea, density of water at free surface is ρ_0 and at a point much below the surface density is ρ. Neglecting variation in gravitational acceleration g and assuming a constant value of bulk modulus K, the depth 'h' of the point from the free surface is

(A) $\frac{K}{g}\left(\frac{1}{\rho_0}+\frac{1}{\rho}\right)$ (B) $\frac{K}{g}\frac{(\rho-\rho_0)}{(\rho+\rho_0)^2}$

(C) $\frac{K}{g}\left(\frac{1}{\rho_0}-\frac{1}{\rho}\right)$ (D) $\frac{K}{g}\left(\frac{\rho\rho_0}{\rho+\rho_0}\right)$

15. In the inclined manometer shown in the given figure the reservoir is large. Its surface may be assumed to remain at a fixed elevation. A is connected to a gas pipeline and the deflection noted on the inclined glass tube is 100 mm. Assuming $\theta = 30°$ and the manometric fluid as oil with specific gravity of 0.86, the pressure at A is



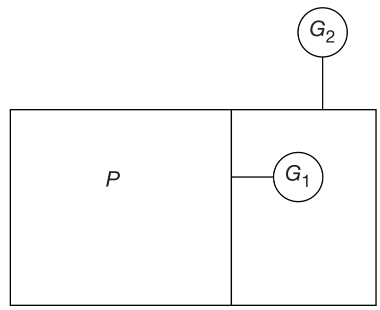
(A) 43 mm water (vacuum)
(B) 43 mm water
(C) 86 mm water
(D) 100 mm water

16. **Assertion (A):** U-tube manometer connected to a venturimeter fitted in a pipeline can measure the velocity through the pipe.

Reason (R): U-tube manometer directly measures dynamic and static heads.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not a correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

17. The pressure gauges G_1 and G_2 installed on the system show pressure of $P_{G1} = 5.00$ bar and $P_{G2} = 1.00$ bar. The value of unknown pressure P is, Given atmospheric pressure 1.01 bar



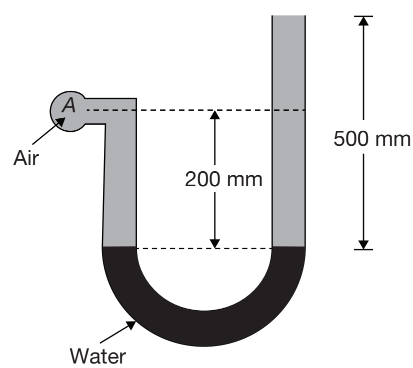
(A) 1.01 bar (B) 2.01 bar
(C) 5.00 bar (D) 7.01 bar

18. Two parallel glass plates, each of width W and negligible thickness, are dipped vertically into a body of liquid (surface tension = σ, density = ρ). If the distance between the plates is t and the contact angle is θ, then the capillary rise of the liquid between the plates is given by

(A) $\frac{2\sigma\cos\theta}{W\rho g}$ (B) $\frac{2\sigma\cos\theta}{t\rho g}$

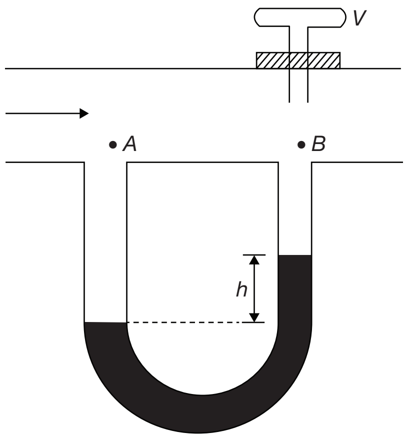
(C) $\frac{4\sigma\cos\theta}{t\rho g}$ (D) $\frac{\sigma\cos\theta}{t\rho g}$

19. In the given figure, air is contained in the pipe and water in the manometer liquid.



The pressure at A is approximately
(A) 10.14 m of water absolute
(B) 0.2 m of water
(C) 1.2 m of water vacuum
(D) 4901 Pa

20. A mercury manometer is fitted to a pipe. It is mounted on the delivery line of a centrifugal pump. One limb of the manometer is connected to the upstream side of the pipe at *A* and the other limb at *B*, just below the valve *V* as shown in the figure.



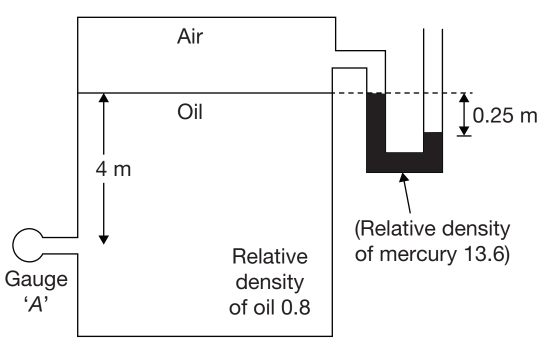
The manometer reading '*h*' varies with different valve positions.

Assertion (A): With gradual closure of the valve, the magnitude of '*h*' will go on increasing and even a situation may arise when mercury will be sucked in by the water flowing around *B*.

Reason (R): With the gradual closure of the valve, the pressure at A will go on increasing.

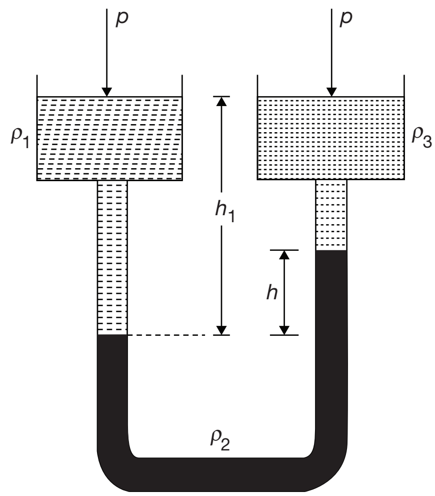
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not a correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

21. The reading of gauge A shown in the figure is



(A) –31.392 kPa (B) –1.962 kPa
(C) 31.392 kPa (D) +19.62 kPa

22. The balancing column shown in the following figure contains 3 liquids of different densities ρ_1, ρ_2 and ρ_3. The liquid level of one limb is h_1 below the top level and there is a difference of '*h*' relative to that in the other limb.



What will be the expression of *h*?

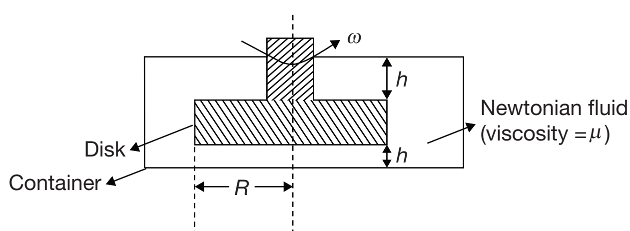
(A) $\left(\dfrac{\rho_1-\rho_2}{\rho_1-\rho_3}\right)h_1$ (B) $\left(\dfrac{\rho_2-\rho_3}{\rho_1-\rho_3}\right)h_1$

(C) $\left(\dfrac{\rho_1-\rho_3}{\rho_2-\rho_3}\right)h_1$ (D) $\left(\dfrac{\rho_1-\rho_2}{\rho_2-\rho_3}\right)h_1$

23. Two spherical soap bubbles, one having a smaller diameter than the other, are present at the two ends of a hollow horizontal cylindrical tube. A restriction at the centre of the tube prevents the flow of air between the two bubbles. If the restriction is removed, then which one of the following is the ONLY possible consequence?

(A) Smaller bubble grows in size.
(B) Both the bubbles do not change in size.
(C) Larger bubble grows in size.
(D) Larger bubble could grow or shrink in size.

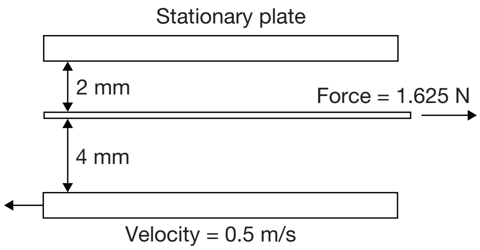
24. The viscous torque on a disk of radius R_1, rotating at an angular velocity of ω_1 inside a container containing a Newtonian fluid of viscosity μ as shown in the figure is determined to be T_1. To determine the viscous torque, a linear velocity profile is assumed and the shear on the outer disk edges is neglected. For another disk of radius R_2 rotating at an angular velocity of ω_2 inside the same container containing the same fluid, the viscous torque on the disk is determined to be T_2. If the clearance of the disk surfaces from the container edges are the same in both cases, $\omega_2 = 8\omega_1$, and $R_2 = 0.5R_1$, then



T_2 is equal to

(A) $2T_1$ (B) $0.25T_1$
(C) $0.5T_1$ (D) $4T_1$

25. A thin square plate (10 cm × 10 cm) is pulled with a force of 1.625 N horizontally through a 6 mm thick layer of Newtonian fluid (viscosity = 1 poise) between two plates, where the top plate is stationary and the bottom plate is moving with a velocity of 0.5 m/s, as shown in the following figure. If a linear velocity profile is assumed, then the minimum distance from the bottom plate, at which the velocity of the fluid is zero, is

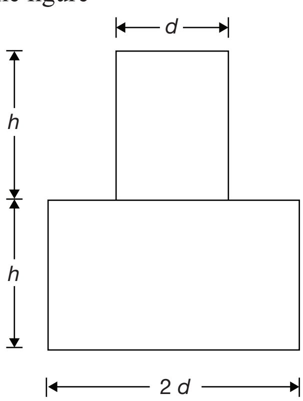


(A) 6 mm (B) 5 mm
(C) 2 mm (D) 0.8 mm

26. A hydraulic jack has a large piston of diameter 15 cm and a small piston of 5 cm diameter. The small piston is above the large piston by a height h. If a force of 100 N applied on the small piston lifts a load of 990 N placed on the large piston, then the value of h (in cm) is
(A) 14 (B) 67
(C) 40 (D) 52

27. What are the forces that influence the problem of fluid statics?
(A) Gravity and viscous forces.
(B) Gravity and pressure forces.
(C) Viscous and surface tension forces.
(D) Gravity and surface tension forces.

28. A stepped cylindrical container is filled with a liquid as shown in the figure



The container with its axis vertical, is first placed with its large diameter downward and then upward. The ratio of the forces at the bottom in the two cases will be
(A) $\frac{1}{2}$ (B) 1
(C) 2 (D) 4

29. When pressure is increased, the bulk modulus of elasticity 'K'+
(A) decreases.
(B) increases.
(C) remains same.
(D) decreases then increase.

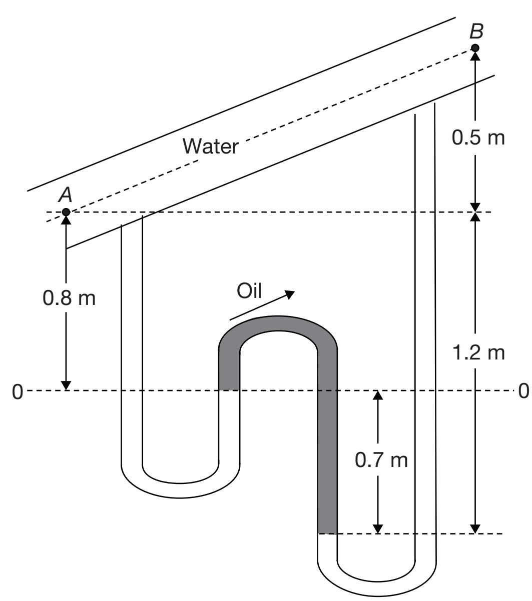
30. The viscosity of water and the viscosity of air with increase in temperature
(A) decrease and increases.
(B) increases and decreases.
(C) decreases and decreases.
(D) decreases and remains same.

31. Pascal's law for a fluid is not valid if
(A) fluid is at rest.
(B) fluid is at constant rotational velocity in a container.
(C) fluid is at constant linear acceleration.
(D) None of these

32. An inverted U-tube manometer is more sensitive than an upright manometric because
(A) the height of levels is greater.
(B) the manometric fluids are heavier than working fluids.
(C) the manometric fluids are lighter than working fluids.
(D) None of these

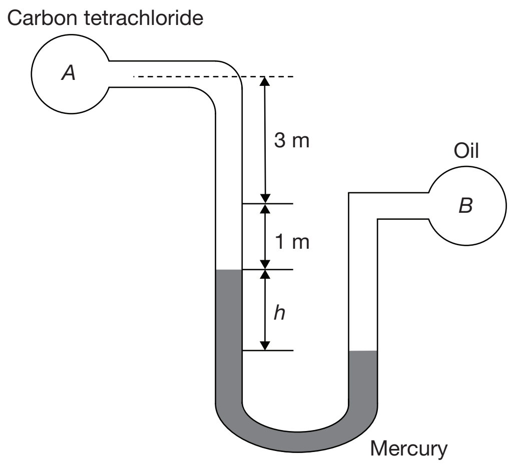
33. Surface Tension is
(A) also known as capillarity.
(B) is a function of curvature of interface.
(C) decreases with fall in temperature.
(D) acts in a plane of interface normal to any line in the surface.

34. An inverted U-tube differential manometer is used to measure pressure difference in an inclined water pipe as shown in the figure. The manometer fluid is oil, of specific gravity 0.75



Pressure difference between points *A* and *B* in N/m^2 is

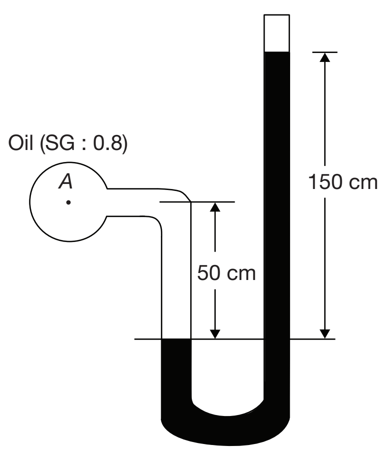
(A) 1792 (B) 2882
(C) 3679 (D) 4216

35. 

Referring to the figure, pipe A contains carbon tetrachloride of specific gravity 1.59 under a pressure of 105 kN/m^2 and pipe B contains oil of specific gravity 0.8 under pressure 170 kN/m^2. Level difference *h* shown by the manometric fluid mercury is

(A) 72 mm (B) 83 mm
(C) 95 mm (D) 115 mm

36. A U-tube mercury manometer is used to measure pressure of oil flowing through a pipe at a point. Specific gravity of oil is 0.8 and the level of mercury is as shown in the figure. The pressure in kPa is



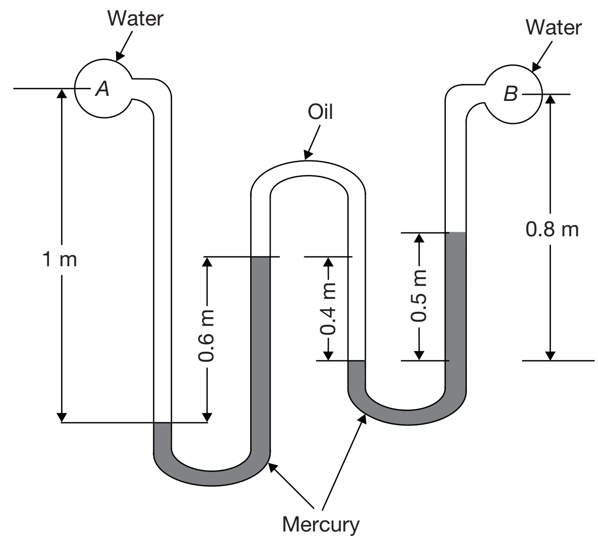
(A) 196.20 (B) 147.15
(C) 110.36 (D) 73.58

37. A glass tube of 3.7 mm diameter is dipped in water. If the contact angle at the meniscus is 0° and surface tension is 0.074 N/m determine the capillary effect in mm. (Take specific weight of water as 10000 N/m^2)

(A) 4 mm (dep) (B) 4 mm (rise)
(C) 8 mm (dep) (D) 8 mm (rise)

38. Differential pressure head measured by a mercury oil differential manometer is 9.5 m of oil. If specific gravity of oil is 0.68, difference in level of mercury is

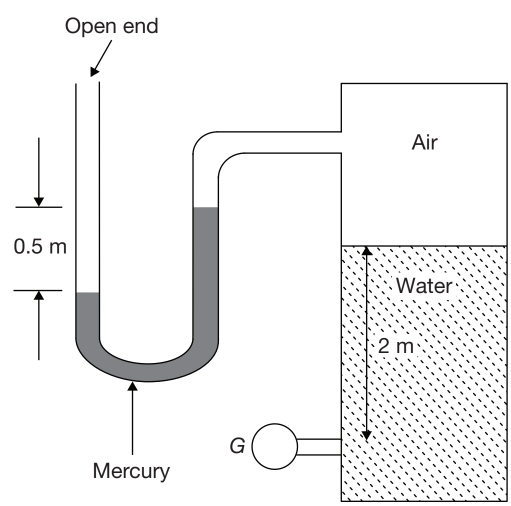
(A) 300 mm (B) 400 mm
(C) 500 mm (D) 600 mm

39. 

For the compound manometer shown in the figure, the pressure difference between points *A* and *B* in kN/m^2 is ______.

(Given that specific gravity of mercury = 13.6 and specific gravity of oil = 0.85)

(A) 115 (B) 125
(C) 135 (D) 150

40. 

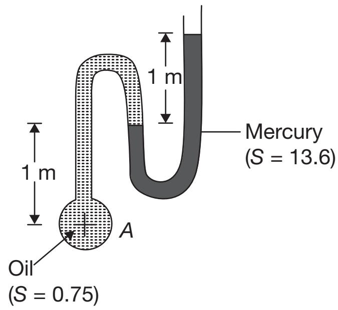
Refer to the figure given above. The tank is filled with water upto 2 m from the gauge *G*. The manometer shows a level difference of 0.5 m as shown. Local atmospheric pressure is 750 mm of mercury.

41. Fluids which follow a linear relationship between shear stress and rate of deformation is known as

(A) ideal fluid.
(B) Newtonian fluid.
(C) non-Newtonian fluid.
(D) dilatant fluid.

42. An example of thyxotropic substance is:
(A) Sewage sludge (B) Milk
(C) Mercury (D) Printer's ink

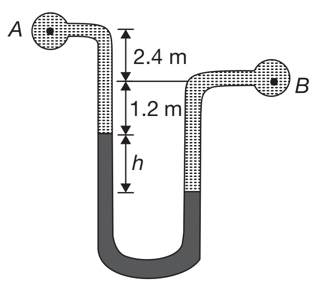
43.



For the U-tube arrangement shown in the figure, pressure at A(in kN/m^2) is______.
(A) 140.23 (B) 140.77
(C) 140.98 (D) 140.62

44. Piezometric head means
(A) Velocity head + Pressure head
(B) Pressure head + Elevation head
(C) Velocity head + Elevation head
(D) None of these

45.



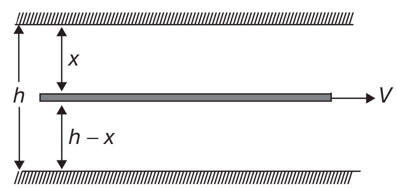
Two pipes A and B containing different liquids are connected by a U-tube manometer containing mercury. Specific gravities of liquids in A and B are 1.6 and 0.8 respectively. If pressure in A and B are 102 kN/m^2 and 170 kN/m^2 respectively, level difference of mercury (in mm) is______.
(A) 112.5 (B) 118.5
(C) 116.5 (D) 110.3

46. A flat thin plate of 0.3 m^2 area is dragged through oil between two large fixed parallel planes, at a velocity of 0.25 m/s. Viscosity of oil is 0.97 Ns/m^2. If the plate is equidistant from both the planes, the drag force (in N) required is______.
(A) 26.4 (B) 29.1
(C) 26.2 (D) 28.3

47. Two vertical parallel glass plates with 1 mm gap between them are immersed in water. If surface tension is 0.073 N/m and angle of contact is zero, rise of water (in mm) in the gap is______.
(A) 0.0128 (B) 0.0149
(C) 0.90 (D) 0.98

48. A 150 mm diameter shaft rotates in a 180 mm long journal bearing at 1450 rpm. Radial clearance in the bearing is 0.25 mm. If the clearance is filled with oil of dynamic viscosity 0.8 poise, power dissipated as heat in the bearing (in kW) is
(A) 3.522 (B) 4.635
(C) 4.842 (D) 5.083

49.



A large thin plate is pulled at a constant velocity V through the gap between two parallel planes as shown in the figure. The upper side of the plate is having oil of viscosity μ and the lower side is having oil of viscosity $\alpha\mu$. The gap width between the planes is h and between upper plane and plate is x. Total drag force to be minimum, value of x is equal to

(A) $\dfrac{h}{1+\alpha}$ (B) $\dfrac{h}{1+\sqrt{\alpha}}$

(C) $\dfrac{h}{1-\alpha}$ (D) $\dfrac{h}{1-\sqrt{\alpha}}$

Previous Years' Questions

1. Oil in a hydraulic cylinder is compressed from an initial volume of 2 m^3 to 1.96 m^3. If the pressure of oil in the cylinder changes from 40 MPa to 80 MPa during compression, the bulk modulus of elasticity of oil is **[GATE, 2007]**
(A) 1000 MPa (B) 2000 MPa
(C) 4000 MPa (D) 8000 MPa

2. A journal bearing has a shaft diameter of 40 mm and a length of 40 mm. The shaft is rotating at 20 rad/s and viscosity of the lubricant is 20 mPa-s. The clearance is 0.020 mm. The loss of torque due to the viscosity of the lubricant is approximately. **[GATE, 2008]**
(A) 0.040 Nm
(B) 0.252 Nm
(C) 0.400 Nm
(D) 0.652 Nm

3. A lightly loaded full journal bearing has journal diameter of 50 mm, bush bore of 50.50 mm and bush

length of 20 mm. If rotational speed of journal is 1200 rpm and average viscosity of liquid lubricant is 0.3 Pa-s, the power loss (in W) will be **[GATE, 2010]**
(A) 37 (B) 74
(C) 118 (D) 237

4. For an incompressible flow field, $\vec{V}$, which one of the following conditions must be satisfied? **[GATE, 2014]**

(A) $\nabla \cdot \vec{V} = 0$ (B) $\nabla \times \vec{V} = 0$

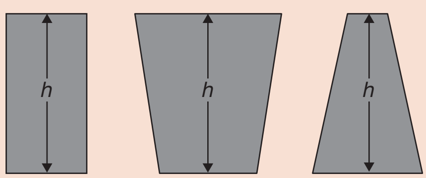
(C) $(\vec{V} \cdot \nabla)\vec{V} = 0$ (D) $\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = 0$

5. The dimension for kinematic viscosity is **[GATE, 2014]**

(A) $\frac{L}{MT}$ (B) $\frac{L}{T^2}$

(C) $\frac{L^2}{T}$ (D) $\frac{ML}{T}$

6. Three rigid buckets, shown as in the figures (1), (2) and (3), are of identical heights and base areas. Further, assume that each of these buckets have negligible mass and are full of water. The weights of water in these buckets are denoted as W_1, W_2, and W_3 respectively. Also, let the force of water on the base of the bucket be denoted as F_1, F_2 and F_3 respectively. The option giving an accurate description of the system physics is **[GATE, 2014]**

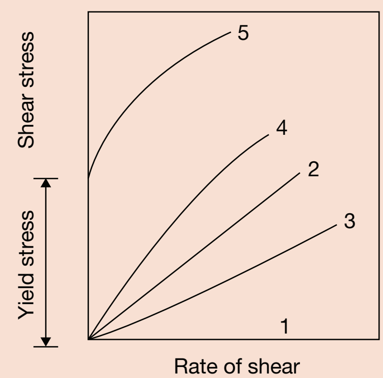


(1) (2) (3)

(A) $W_2 = W_1 = W_3$ and $F_2 > F_1 > F_3$
(B) $W_2 > W_1 > W_3$ and $F_2 > F_1 > F_3$
(C) $W_2 = W_1 = W_3$ and $F_2 = F_1 = F_3$
(D) $W_2 > W_1 > W_3$ and $F_2 = F_1 = F_3$

7. List I contains the types of fluids while List II contains the shear stress—rate of shear relationship of different types of fluids, as shown in the figure. **[GATE, 2016]**

List I	List II
P. Newtonian fluid	**1.** Curve 1
Q. Pseudo plastic fluid	**2.** Curve 2
R. Plastic Fluid	**3.** Curve 3
S. Dilatant fluid	**4.** Curve 4
	5. Curve 5



The correct match between List I and List II is
(A) P-2, Q-4, R-1, S-5 (B) P-2, Q-5, R-4, S-1
(C) P-2, Q-4, R-5, S-3 (D) P-2, Q-1, R-3, S-4

Answer Keys

Exercises

1. D	**2.** B	**3.** C	**4.** C	**5.** A	**6.** C	**7.** D	**8.** C	**9.** B	**10.** A
11. A	**12.** C	**13.** A	**14.** C	**15.** B	**16.** A	**17.** D	**18.** B	**19.** A	**20.** A
21. B	**22.** C	**23.** C	**24.** C	**25.** D	**26.** D	**27.** B	**28.** D	**29.** A	**30.** A
31. D	**32.** C	**33.** C	**34.** C	**35.** B	**36.** A	**37.** D	**38.** C	**39.** C	**40.** B
41. B	**42.** D	**43.** B	**44.** B	**45.** C	**46.** B	**47.** B	**48.** A	**49.** B	

Previous Years' Questions

1. B	**2.** A	**3.** A	**4.** A	**5.** C	**6.** D	**7.** C