Exercise 3.4

Chapter 3 Applications of Differentiation Exercise 3.4 1E

(A)

$$\lim_{x \to \infty} f(x) = 5$$

It means if we take x, very large then $f(x)$ approaches 5.

(B)

$$\lim_{x \to -\infty} f(x) = 3$$

It means if we take very large negative value of x, $f(x)$ approaches 3

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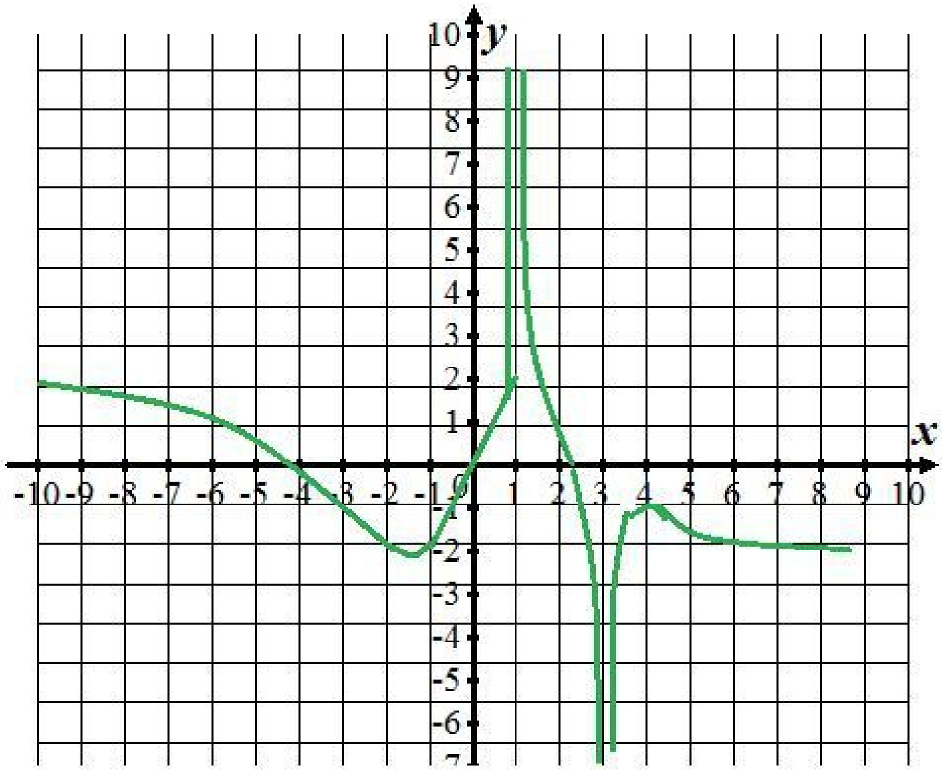
Sketch the following diagram:



(a)

The objective is to find $\lim_{x\to\infty} f(x)$.

From the graph, the values of $f(x)$ becomes -2 as x approaches positive infinite.

Therefore, the result is $\lim_{x\to\infty} f(x) = \boxed{-2}$.

(b)

The objective is to find $\lim_{x\to-\infty} f(x)$.

From the graph, the values of $f(x)$ becomes 2 as x approaches negative infinite.

Therefore, the result is $\lim_{x\to-\infty} f(x) = \boxed{2}$.

(c)

The objective is to find $\lim_{x\to1} f(x)$.

From the graph, the values of $f(x)$ becomes large positive as x tends to 1.

Therefore, the result is $\lim_{x\to1} f(x) = \boxed{\infty}$.

(d)

The objective is to find $\lim_{x\to3} f(x)$.

From the graph, the values of $f(x)$ becomes large negative as x tends to 3.

Therefore, the result is $\lim_{x\to3} f(x) = \boxed{-\infty}$.

(e)

Recollect that the line $y = L$ is called horizontal asymptote of the curve $y = f(x)$ if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.

From the graph, the horizontal asymptotes are $\boxed{y = 2, y = -2}$.

Also, $\lim_{x\to1} f(x) = \infty$ and $\lim_{x\to3} f(x) = -\infty$.

That is vertical asymptotes are $\boxed{x = 1, x = 3}$.

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From the given graph

(a) $\lim_{x\to\infty} g(x) = 2$

(b) $\lim_{x\to-\infty} g(x) = -1$

(c) $\lim_{x\to0} g(x) = -\infty$

(d) $\lim_{x\to2^-} g(x) = -\infty$

(e) $\lim_{x\to2^+} g(x) = \infty$

(e) The equations of asymptotes

$x = 0, x = 1, y = 2, y = -1$

Chapter 3 Applications of Differentiation Exercise 3.4 5E

Consider the following function.

$$f(x)=\frac{x^2}{2^x}$$

Evaluate the given function $f(x)=\frac{x^2}{2^x}$, for the values x=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 50, and 100.

Calculate the value of the function $f(x)=\frac{x^2}{2^x}$ for the given values of x and arrange the results in a table as follows:

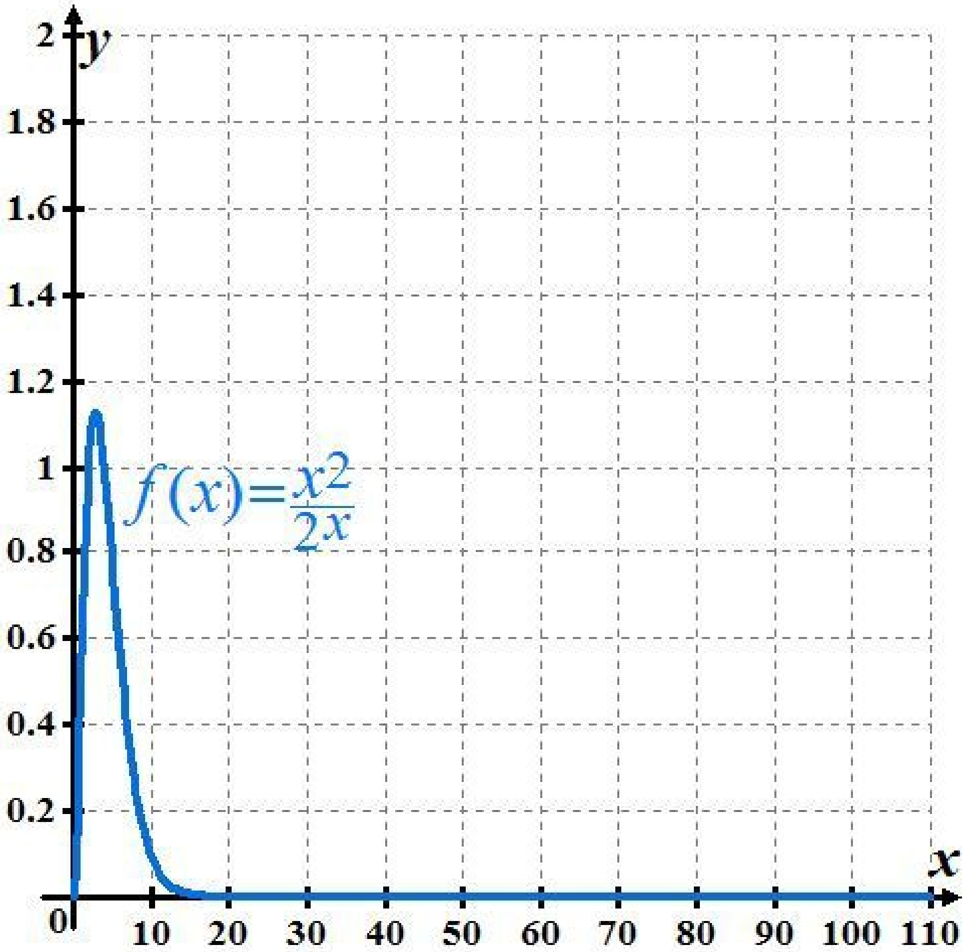
x	$f(x)=\frac{x^2}{2^x}$
0	0
1	0.5
2	1.0
3	1.125
4	1.0
5	0.78125
6	0.5625
7	0.382815
8	0.250
9	0.15820
10	0.09765
20	0.000381
50	2.22×10^{-12}
100	7.89×10^{-27}

Notice that, as the value of x increases, the value of $f(x)$ approaches zero.

Thus, the value of the limit

$$\lim_{x\to\infty} f(x)=0.$$

Sketch the graph of the function $f(x)=\frac{x^2}{2^x}$ as follows:



The graph of the function $f(x)=\frac{x^2}{2^x}$ gives the same result that, the value of $f(x)$ approaches to zero, when the value of *x* increases.

Therefore, the value of the limit

$$\boxed{\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x^2}{2^x}=0}.$$

Chapter 3 Applications of Differentiation Exercise 3.4 6E

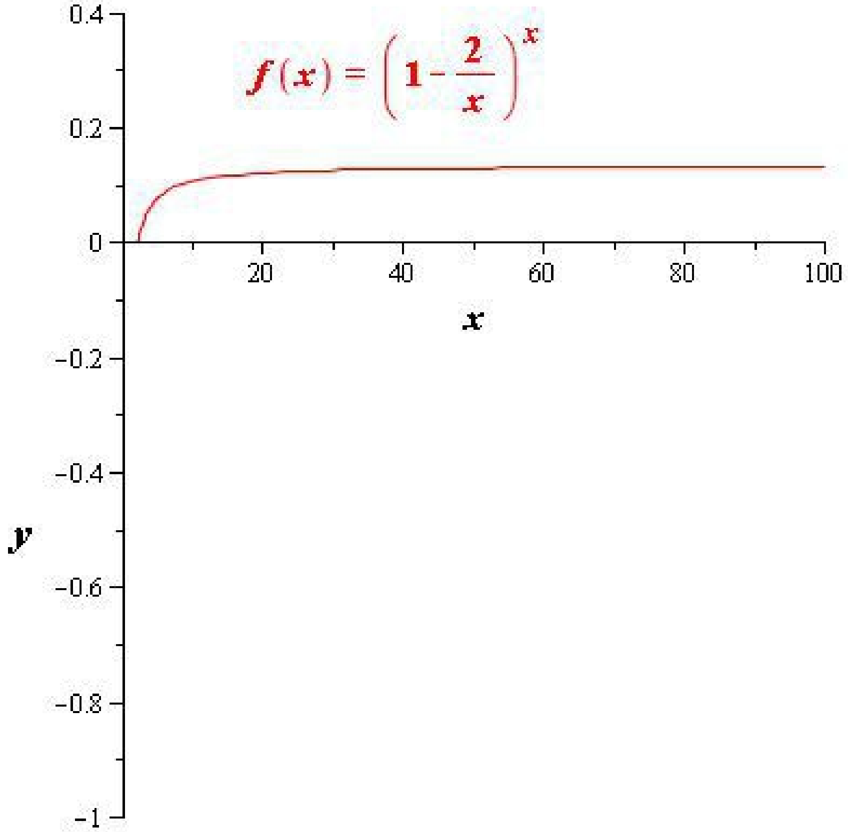
Consider the following function:

$$f(x)=\left(1-\frac{2}{x}\right)^x$$

(a)

To estimate the value of $\lim_{x\to\infty} f(x)$, draw the graph of $f(x)$ for different values of x and see that when x is increasing, where does the value of $f(x)$ approaches.

The maple graph of $f(x)=\left(1-\frac{2}{x}\right)^x$:



From the graph it can be seen that as x is increasing the function gradually approaches to 0.13. Hence the value of limit can be approximated as:

$$\boxed{\lim_{x\to\infty} f(x) = 0.13}$$

(b)

Construct a table for values of the function $f(x)=\left(1-\frac{2}{x}\right)^x$:

x	$f(x)=\left(1-\frac{2}{x}\right)^x$
0	Not defined
1	-1
2	0
3	0.037037
10	0.107374
20	0.121577
40	0.128512
80	0.131938
100	0.13262
1000	0.135065
10000	0.135308
100000	0.135333

The above table suggests that as the value of x is increasing the value of the function $f(x)$ is close to 0.14.

Hence the value of the limit is:

$$\boxed{\lim_{x\to\infty} f(x)=0.14}$$

Chapter 3 Applications of Differentiation Exercise 3.4 7E

We have to evaluate

$$\lim_{x\to\infty}\frac{3x^2-x+4}{2x^2+5x-8}$$

Here as x becomes large both numerator and denominator become large so we have to do some preliminary algebra.

First we divide numerator and denominator by the highest power of x that occurs in the denominator

So we have $$\lim_{x\to\infty}\frac{3x^2-x+4}{2x^2+5x-8}=\lim_{x\to\infty}\frac{\frac{3x^2-x+4}{x^2}}{\frac{2x^2+5x-8}{x^2}}$$

$$=\lim_{x\to\infty}\frac{3-\frac{1}{x}+\frac{4}{x^2}}{2+\frac{5}{x}-\frac{8}{x^2}}$$

$$=\frac{\lim_{x\to\infty}\left(3-\frac{1}{x}+\frac{4}{x^2}\right)}{\lim_{x\to\infty}\left(2+\frac{5}{x}-\frac{8}{x^2}\right)} \qquad \left[\begin{array}{l}\textit{by the law}\\ \lim_{x\to\infty}\frac{f(x)}{g(x)}=\frac{\lim_{x\to\infty}f(x)}{\lim_{x\to\infty}g(x)}\end{array}\right]$$

$$=\frac{\lim_{x\to\infty}3-\lim_{x\to\infty}\frac{1}{x}+4\lim_{x\to\infty}\frac{1}{x^2}}{\lim_{x\to\infty}2+5\lim_{x\to\infty}\frac{1}{x}-8\lim_{x\to\infty}\frac{1}{x^2}} \qquad \left[\begin{array}{l}\textit{by}\\ \lim_{x\to\infty}(f+g)=\lim_{x\to\infty}f+\lim_{x\to\infty}g\\ \textit{and}\\ \lim_{x\to\infty}cf(x)=c\lim_{x\to\infty}f(x)\end{array}\right]$$

$$=\frac{3-0+0}{2+0-0} \qquad \left[\begin{array}{l}\lim_{x\to\infty}c=c \textit{ where } c \textit{ is } \text{constant}\\ \text{if r} > 0,\ \lim_{x\to\infty}\frac{1}{x^r}=0\end{array}\right]$$

$$=\frac{3}{2}$$

So we have $$\boxed{\lim_{x\to\infty}\frac{3x^2-x+4}{2x^2+5x-8}=\frac{3}{2}}$$

202-04.04-8E

We have to evaluate $\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}}$

Divide the numerator and denominator by highest power of x of the denominator that is x^3

So we have

$$\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}}$$

$$=\lim_{x\to\infty}\sqrt{\frac{12-\frac{5}{x^2}+\frac{2}{x^3}}{\frac{1}{x^3}+\frac{4}{x}+3}}$$

$$=\sqrt{\lim_{x\to\infty}\frac{12-\frac{5}{x^2}+\frac{2}{x^3}}{\frac{1}{x^3}+\frac{4}{x}+3}} \qquad \lim_{x\to\infty}\left[f(x)\right]^n=\left[\lim_{x\to\infty}f(x)\right]^n$$

$$=\sqrt{\frac{\lim_{x\to\infty}\left(12-\frac{5}{x^2}+\frac{2}{x^3}\right)}{\lim_{x\to\infty}\left(\frac{1}{x^3}+\frac{4}{x}+3\right)}} \qquad \left[\lim_{x\to\infty}\frac{f(x)}{g(x)}=\frac{\lim_{x\to\infty}f(x)}{\lim_{x\to\infty}g(x)}\right]$$

$$=\sqrt{\frac{\lim_{x\to\infty}12-5\lim_{x\to\infty}\frac{1}{x^2}+2\lim_{x\to\infty}\frac{1}{x^3}}{\lim_{x\to\infty}\frac{1}{x^3}+4\lim_{x\to\infty}\frac{1}{x}+\lim_{x\to\infty}3}} \qquad \left[\begin{array}{l}\lim_{x\to\infty}(f+g)=\lim_{x\to\infty}f+\lim_{x\to\infty}g\\ \lim_{x\to\infty}cf(x)=c.\lim_{x\to\infty}f(x)\end{array}\right]$$

$$=\sqrt{\frac{12-0+0}{0+0+3}}=\sqrt{\frac{12}{3}}=2 \qquad \left[\begin{array}{l}\lim_{x\to\infty}c=c \text{ where c is constant}\\ \text{if r} > 0,\ \lim_{x\to\infty}\frac{1}{x^r}=0\end{array}\right]$$

So $\boxed{\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}}=2}$

Chapter 3 Applications of Differentiation Exercise 3.4 9E

Consider the expression,

$$\lim_{x\to\infty}\frac{3x-2}{2x+1}$$

The objective is to evaluate the limit of the function.

$$\lim_{x\to\infty}\frac{3x-2}{2x+1}=\lim_{x\to\infty}\frac{x\left(3-\frac{2}{x}\right)}{x\left(2+\frac{1}{x}\right)} \quad \text{Factor } x \text{ terms}$$

$$=\lim_{x\to\infty}\frac{\left(3-\frac{2}{x}\right)}{\left(2+\frac{1}{x}\right)}$$

$$=\frac{\lim_{x\to\infty}\left(3-\frac{2}{x}\right)}{\lim_{x\to\infty}\left(2+\frac{1}{x}\right)} \quad \text{Since } \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$$

$$=\frac{\lim_{x\to\infty}3-\lim_{x\to\infty}\frac{2}{x}}{\lim_{x\to\infty}2+\lim_{x\to\infty}\frac{1}{x}} \quad \text{Since } \lim_{x\to a}\left[f(x)\pm g(x)\right]=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$$

$$=\frac{3-0}{2+0}$$

$$=\frac{3}{2}$$

Therefore, the result is $\lim_{x\to\infty}\frac{3x-2}{2x+1}=\boxed{\frac{3}{2}}$.

Chapter 3 Applications of Differentiation Exercise 3.4 10E

Consider the limit of the function $\lim_{x\to\infty}\frac{1-x^2}{x^3-x+1}$.

To evaluate the limit at infinity of any rational function, first divide both numerator and denominator by the highest power of x that occurs in the denominator.

In this case the highest power of x in the denominator is x^3.

$$\lim_{x\to\infty}\frac{1-x^2}{x^3-x+1}=\lim_{x\to\infty}\frac{\frac{1-x^2}{x^3}}{\frac{x^3-x+1}{x^3}}$$

$$=\lim_{x\to\infty}\frac{\frac{1}{x^3}-\frac{1}{x}}{1-\frac{1}{x^2}+\frac{1}{x^3}}$$

$$=\frac{\lim_{x\to\infty}\frac{1}{x^3}-\lim_{x\to\infty}\frac{1}{x}}{\lim_{x\to\infty}1-\lim_{x\to\infty}\frac{1}{x^2}+\lim_{x\to\infty}\frac{1}{x^3}}$$

$$=\frac{0-0}{1-0+0}$$

$$=\frac{0}{1}$$

$$=0$$

Therefore, $\lim_{x\to\infty}\frac{1-x^2}{x^3-x+1}=\boxed{0}$.

Chapter 3 Applications of Differentiation Exercise 3.4 11E

Given that the limit is $\lim_{x\to-\infty}\frac{x-2}{x^2+1}$

We divide the numerator and denominator by the highest power of x in the denominator

$$\lim_{x\to-\infty}\frac{\frac{1}{x}-\frac{2}{x^2}}{1+\frac{1}{x^2}}=\frac{0-0}{1+0}$$

$$=0 \qquad \left[\because \frac{0}{1}=0\right]$$

$$\therefore \lim_{x\to-\infty}\frac{x-2}{x^2+1}=0$$

Chapter 3 Applications of Differentiation Exercise 3.4 12E

Given that the limit is $\lim_{x\to-\infty}\frac{4x^3+6x^2-2}{2x^3-4x+5}$

We divide the numerator and denominator by the highest power of x in the denominator

$$\lim_{x\to-\infty}\frac{4+\frac{6}{x}-\frac{2}{x^2}}{2-\frac{4}{x}+\frac{5}{x^2}}=\frac{4}{2} \qquad \left[\because \frac{a}{\infty}=0,\ \text{where a is any number}\right]$$

$$=2$$

$$\therefore \lim_{x\to-\infty}\frac{4x^3+6x^2-2}{2x^3-4x+5}=2$$

Chapter 3 Applications of Differentiation Exercise 3.4 13E

Given that the limit is $\lim_{t\to\infty}\frac{\sqrt{t}+t^2}{2t-t^2}$

We divide the numerator and denominator by the highest power of t in the denominator

$$\lim_{t\to\infty}\frac{\frac{\sqrt{t}+t^2}{t^2}}{\frac{2t-t^2}{t^2}}=\lim_{t\to\infty}\frac{t^{-3/2}+1}{\frac{2}{t}-1}$$

$$=\frac{0+1}{0-1} \qquad \left[\because \frac{a}{\infty}=0,\ \text{where a is some number}\right]$$

$$=-1$$

$$\therefore \lim_{x\to\infty}\frac{\sqrt{t}+t^2}{2t-t^2}=-1$$

Chapter 3 Applications of Differentiation Exercise 3.4 14E

Take a function f defined on the interval (a,∞).

$$\lim_{x\to a} f(x)=L$$

The limit is defined such that for a number $\varepsilon>0$, there is a number N which satisfies:

if $x>N$, then $|f(x)-L|<\varepsilon$

Consider the limit:

$$\lim_{t\to\infty}\frac{t-t\sqrt{t}}{2t^{3/2}+3t-5}$$

Divide the numerator and denominator by the highest power of t in the denominator and apply the limits:

$$\lim_{t\to\infty}\frac{t-t\sqrt{t}}{2t^{3/2}+3t-5}=\lim_{t\to\infty}\frac{\frac{1}{\sqrt{t}}-1}{2+\frac{1}{\sqrt{t}}-\frac{5}{t^{3/2}}}$$

$$=\frac{0-1}{2+0-0}$$

$$=\frac{-1}{2}$$

Hence, the final value of the limit is $\boxed{\lim_{t\to\infty}\frac{t-t\sqrt{t}}{2t^{3/2}+3t-5}=\frac{-1}{2}}$.

Chapter 3 Applications of Differentiation Exercise 3.4 15E

Given that the limit is $\lim_{x\to\infty}\frac{\left(2x^2+1\right)^2}{(x-1)^2\left(x^2+x\right)}$

$$=\lim_{x\to\infty}\frac{4x^4+4x^2+1}{\left(x^2-2x+1\right)\left(x^2+x\right)}$$

$$=\lim_{x\to\infty}\frac{4x^4+4x^2+1}{x^4-2x^3+x^2+x^3-2x^2+x}$$

$$=\lim_{x\to\infty}\frac{4x^4+4x^2+1}{x^4-x^3-x^2+x}$$

We divide the numerator and denominator by the highest power of in the denominator

$$=\lim_{x\to\infty}\frac{4+\frac{4}{x^2}+\frac{1}{x^4}}{1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}}$$

$$=\frac{4+0+0}{1-0-0+0}$$

$$=4$$

$$\boxed{\therefore\ \lim_{x\to\infty}\frac{\left(2x^2+1\right)^2}{(x-1)^2\left(x^2+x\right)}=4}$$

Chapter 3 Applications of Differentiation Exercise 3.4 16E

Given that the limit is $\lim_{x\to\infty}\frac{x^2}{\sqrt{x^4+1}}$

We divide the numerator and denominator by x^2, we get

$$\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x^4}}}=\frac{1}{\sqrt{1+0}}$$

$$=\frac{1}{1}$$

$$=1$$

$$\boxed{\therefore\ \lim_{x\to\infty}\frac{x^2}{\sqrt{x^4+1}}=1}$$

Chapter 3 Applications of Differentiation Exercise 3.4 17E

We have to evaluate $\lim\limits_{x\to\infty}\dfrac{\sqrt{9x^6-x}}{x^3+1}$

Dividing both numerator and denominator by x^3 and using properties of limits.

$$\lim_{x\to\infty}\frac{\sqrt{9x^6-x}}{x^3+1}=\lim_{x\to\infty}\frac{\sqrt{9-x/x^6}}{1+1/x^3}\qquad \text{since } \sqrt{x^6}=x^3 \quad \text{for } x>0$$

$$=\frac{\lim\limits_{x\to\infty}\sqrt{9-x/x^6}}{\lim\limits_{x\to\infty}\left(1+1/x^3\right)}$$

$$=\frac{\sqrt{\lim\limits_{x\to\infty}9-\lim\limits_{x\to\infty}1/x^5}}{\left(\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}1/x^3\right)}$$

$$=\frac{\sqrt{9-0}}{1+0}\qquad\left[\lim_{x\to\infty}\frac{1}{x^n}=0 \text{ where } n>0\right]$$

$$=\sqrt{9}$$

$$\boxed{\lim_{x\to\infty}\frac{\sqrt{9x^6-x}}{x^3+1}=3}\qquad \textit{for } x>0$$

Chapter 3 Applications of Differentiation Exercise 3.4 18E

We have to evaluate $\lim\limits_{x\to-\infty}\dfrac{\sqrt{9x^6-x}}{x^3+1}$

For computing the limit as $x\to-\infty$ we have $\sqrt{x^6}=\left|x^3\right|=-x^3$

For x < 0

So $\sqrt{9x^6-x}=\sqrt{x^6}\left(\sqrt{9-1/x^5}\right)=-x^3\sqrt{9-1/x^5}$

Then we have

$$\lim_{x\to-\infty}\frac{-x^3\sqrt{9-1/x^5}}{x^3+1}=\lim_{x\to-\infty}\frac{\sqrt{9x^6-x}}{x^3+1}$$

Now divide by x^3, both numerator and denominator

$$\lim_{x\to-\infty}\frac{-x^3\sqrt{9-1/x^5}}{x^3+1}=\lim_{x\to-\infty}\frac{-\sqrt{9-1/x^5}}{1+1/x^3}$$

Now by using limit laws

$$\lim_{x\to-\infty}\frac{-\sqrt{9x^6-1/x^5}}{1+1/x^3}=\frac{-\lim\limits_{x\to-\infty}\left(\sqrt{9-\frac{1}{x^5}}\right)}{\lim\limits_{x\to-\infty}\left(1+\frac{1}{x^3}\right)}$$

$$=\frac{-\sqrt{\lim\limits_{x\to-\infty}9-\lim\limits_{x\to-\infty}\frac{1}{x^5}}}{\lim\limits_{x\to-\infty}1+\lim\limits_{x\to-\infty}\frac{1}{x^3}}$$

$$=\frac{-\sqrt{9-0}}{1+0}\qquad\left[\begin{array}{l}\textit{for, } r>0\\ \lim\limits_{x\to\infty}\frac{1}{x^n}=0\end{array}\right]$$

$$= -3$$

So we have $\boxed{\lim\limits_{x\to-\infty}=\dfrac{\sqrt{9x^6-x}}{x^3+1}=-3}$

Chapter 3 Applications of Differentiation Exercise 3.4 19E

We have to evaluate $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

Multiply the numerator and denominator by the conjugate radical

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)=\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)\times\frac{\left(\sqrt{9x^2+x}+3x\right)}{\left(\sqrt{9x^2+x}+3x\right)}$$

$$=\lim_{x\to\infty}\frac{9x^2+x-9x^2}{\left(\sqrt{9x^2+x}+3x\right)} \qquad \left[(a-b)(a+b)=a^2-b^2\right]$$

$$=\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}$$

Now divide the numerator and denominator by x and using limit laws

$$=\lim_{x\to\infty}\frac{x/x}{\sqrt{9+1/x}+3x/x} \qquad \left[\sqrt{x^2}=x\right]$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{9+1/x}+3}$$

$$=\frac{\lim_{x\to\infty}1}{\sqrt{\lim_{x\to\infty}9+\lim_{x\to\infty}1/x}+\lim_{x\to\infty}3}$$

Now divide the numerator and denominator by x and using limit laws

$$=\lim_{x\to\infty}\frac{x/x}{\sqrt{9+1/x}+3x/x} \qquad \left[\sqrt{x^2}=x\right]$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{9+1/x}+3}$$

$$=\frac{\lim_{x\to\infty}1}{\sqrt{\lim_{x\to\infty}9+\lim_{x\to\infty}1/x}+\lim_{x\to\infty}3}$$

We have

$$\lim_{x\to\infty}\frac{1}{x^n}=0 \textit{ when } n>0 \textit{ so}$$

$$=\frac{1}{\sqrt{9+0}+3}$$

$$=\frac{1}{3+3}$$

$$=\frac{1}{6}$$

So we have

$$\boxed{\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)=\frac{1}{6}}$$

Chapter 3 Applications of Differentiation Exercise 3.4 20E

$\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)$

Multiply and divide by the conjugate radical of the function so

$$\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)=\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)\times\frac{x-\sqrt{x^2+2x}}{x-\sqrt{x^2+2x}}$$

$$=\lim_{x\to-\infty}\frac{x^2-\left(x^2+2x\right)}{x-\sqrt{x^2+2x}}$$

$$=\lim_{x\to-\infty}\frac{-2x}{x-\sqrt{x^2+2x}}$$

For calculating the limit as $x \to -\infty$, we have $\sqrt{x^2} = |x| = -x$ for $x < 0$

So $\sqrt{x^2+2x} = \sqrt{x^2}\left(\sqrt{1+\frac{2}{x}}\right) = -x\sqrt{1+\frac{2}{x}}$

Then we have

$$\lim_{x\to-\infty} \frac{-2x}{(x-\sqrt{x^2+2x})} = \lim_{x\to-\infty} \frac{-2x}{x+x\sqrt{1+\frac{2}{x}}}$$

Divide the numerator and denominator by x

$$= \lim_{x\to-\infty} \frac{-2}{1+\sqrt{1+2/x}}$$

$$= \frac{-\lim\limits_{x\to-\infty} 2}{\left(\lim\limits_{x\to-\infty} 1 + \sqrt{\lim\limits_{x\to-\infty} 1 + 2\lim\limits_{x\to-\infty} (1/x)}\right)}$$

$$= \frac{-2}{1+\sqrt{1+0}}$$

$$= -\frac{2}{2}$$

$$= -1$$

So $\boxed{\lim\limits_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right) = -1}$

Chapter 3 Applications of Differentiation Exercise 3.4 21E

We have to get $\lim\limits_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)$

Multiply the numerator and denominator by the conjugate radical

$$\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right) = \lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right) \times \frac{\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)}{\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)}$$

$$= \lim_{x\to\infty} \frac{\left(x^2+ax\right)-\left(x^2+bx\right)}{\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)} \qquad \left[(a-b)(a+b) = a^2-b^2\right]$$

$$= \lim_{x\to\infty} \frac{(a-b)x}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$= \lim_{x\to\infty} \frac{(a-b)}{\sqrt{1+a/x}+\sqrt{1+b/x}} \qquad \left[\sqrt{x^2} = x\right]$$

$$= \frac{\lim\limits_{x\to\infty}(a-b)}{\sqrt{\lim\limits_{x\to\infty} 1 + a\lim\limits_{x\to\infty} 1/x} + \sqrt{\lim\limits_{x\to\infty} 1 + b\lim\limits_{x\to\infty} 1/x}}$$

$$= \frac{(a-b)}{\sqrt{1+0}+\sqrt{1+0}} = \frac{(a-b)}{2} \qquad \left[\begin{array}{l}\lim\limits_{x\to\infty}\frac{1}{x^n} = 0 \\ \text{Where } n > 0\end{array}\right]$$

So we have

$$\boxed{\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right) = \frac{(a-b)}{2}}$$

Chapter 3 Applications of Differentiation Exercise 3.4 22E

We have to evaluate $\lim\limits_{x\to\infty} \cos x$

As x increases, the value of $\cos x$ oscillate between 1 and -1 infinitely often and so they don't approach any definite number

Thus $\boxed{\lim\limits_{x\to\infty} \cos x \text{ does not exist}}$

Chapter 3 Applications of Differentiation Exercise 3.4 23E

Given that the limit is $\lim_{x\to\infty}\frac{x^4-3x^2+x}{x^3-x+2}$

We divide the numerator and denominator by x^3, we get

$$\lim_{x\to\infty}\frac{x-\frac{3}{x}+\frac{1}{x^2}}{1-\frac{1}{x^2}+\frac{2}{x^3}}=\frac{\infty-0+0}{1-0+0}$$

$$=\infty. \quad \left[\because \ \frac{\infty}{1}=\infty\right]$$

$$\boxed{\therefore \lim_{x\to\infty}\frac{x^4-3x^2+x}{x^3-x+2}=\infty}$$

Chapter 3 Applications of Differentiation Exercise 3.4 24E

Take a function *f* defined on the interval (a,∞).

$$\lim_{x\to a} f(x)=L$$

The limit is defined such that for a number $\varepsilon>0$, there is a number *N* which satisfies:

$$\text{if } x>N, \text{then } \ |f(x)-L|<\varepsilon$$

Consider the limit:

$$\lim_{x\to\infty}\sqrt{x^2+1}$$

Perform the manipulation on the function as shown below and apply the limit:

$$\begin{aligned}\lim_{x\to\infty}\sqrt{x^2+1}&=\lim_{x\to\infty}\sqrt{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}\\&=\lim_{x\to\infty}\frac{x^2+1}{\sqrt{x^2+1}}\\&=\lim_{x\to\infty}\frac{x^2\left(1+\frac{1}{x^2}\right)}{x\sqrt{1+\frac{1}{x^2}}}\\&=\lim_{x\to\infty}\frac{x\left(1+\frac{1}{x^2}\right)}{\sqrt{1+\frac{1}{x^2}}}\end{aligned}$$

Continue further to determine the limit:

$$\begin{aligned}\lim_{x\to\infty}\sqrt{x^2+1}&=\lim_{x\to\infty}\frac{x(1+0)}{\sqrt{1+0}}\\&=\lim_{x\to\infty}x\\&=\text{does not exist}\end{aligned}$$

Hence, the value of the $\lim_{x\to\infty}\sqrt{x^2+1}$ **does not exist**.

Chapter 3 Applications of Differentiation Exercise 3.4 25E

We have to evaluate $\lim_{x\to-\infty}\left(x^4+x^5\right)$

We can not use

$$\begin{aligned}\lim_{x\to-\infty}\left(x^4+x^5\right)&=\lim_{x\to-\infty}x^4+\lim_{x\to-\infty}x^5\\&=\infty+(-\infty)\\&=\infty-\infty\end{aligned}$$

The limit laws can not be applied because $\infty-\infty$ can not be defined

And for $x<-1$, $x^4+x^5\le 0$ because $|x^5|>|x^4|$

So we can make as large negative value of $\left(x^4+x^5\right)$ as we want by taking large enough negative value of x

So we can write $\boxed{\lim_{x\to-\infty}\left(x^4+x^5\right)=-\infty}$

Chapter 3 Applications of Differentiation Exercise 3.4 26

Consider $\lim_{x\to-\infty}\frac{1+x^6}{x^4+1}$

Let $f(x)=\frac{1+x^6}{1+x^4}$.

To find out the limit of the function $f(x)$, we take the common x^6 from the numerator and x^4 from the denominator, obtain that

$$f(x)=\frac{x^6\left(1+\frac{1}{x^6}\right)}{x^4\left(1+\frac{1}{x^4}\right)}$$

$$=\frac{x^2\left(1+\frac{1}{x^6}\right)}{\left(1+\frac{1}{x^4}\right)}$$

Note that,

$$\lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a}f(x)+\lim_{x\to a}g(x)$$

$$\lim_{x\to a}\left[f(x)\cdot g(x)\right]=\left[\lim_{x\to a}f(x)\right]\cdot\left[\lim_{x\to a}g(x)\right]$$

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$$

Therefore,

$$\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}\frac{x^2\left(1+\frac{1}{x^6}\right)}{\left(1+\frac{1}{x^4}\right)}$$

$$=\frac{\lim_{x\to-\infty}x^2\left(\lim_{x\to-\infty}1+\lim_{x\to-\infty}\frac{1}{x^6}\right)}{\left(\lim_{x\to-\infty}1+\lim_{x\to-\infty}\frac{1}{x^4}\right)}$$

$$=\frac{\infty\cdot(1+0)}{1+0}$$

$$=\infty$$

Hence, $\lim_{x\to-\infty}\frac{1+x^6}{x^4+1}=\boxed{\infty}$.

Chapter 3 Applications of Differentiation Exercise 3.4 27E

We have to evaluate $\lim_{x\to\infty}\left(x-\sqrt{x}\right)$

We can not write

$$\lim_{x\to\infty}\left(x-\sqrt{x}\right)=\lim_{x\to\infty}x-\lim_{x\to\infty}\sqrt{x}$$
$$=\infty-\infty$$

The limit laws can not be applied because ∞ is not a number so $\infty-\infty$ can not be defined

So we can write

$\boxed{\lim_{x\to\infty}\left(x-\sqrt{x}\right)=\infty}$ Because $x-\sqrt{x}>0$ for all value of x where x > 0

So if x is very large then $x-\sqrt{x}$ will also very large

Another Method:

$$\lim_{x\to\infty}\left(x-\sqrt{x}\right)=\lim_{x\to\infty}x\left(1-\frac{1}{\sqrt{x}}\right)=\infty(1-0)=\infty$$

Chapter 3 Applications of Differentiation Exercise 3.4 28E

Consider the limit

$$\lim_{x\to\infty}\left(x^2-x^4\right)$$

This would be wrong to write

$$\lim_{x\to\infty}\left(x^2-x^4\right)=\lim_{x\to\infty}x^2-\lim_{x\to\infty}x^4$$

$$=\infty-\infty$$

Limit laws cannot be applied because ∞ is not a number ($\infty-\infty$ is not defined)

Rewrite the function under the limit as $x^2\left(1-x^2\right)$

For large values of x, x^2 is very large and $\left(1-x^2\right)$ is large negative.

For instance consider

$$10^2=100,\left(1-10^2\right)=1-100=-99$$

$$100^2=10000,\left(1-100^2\right)=1-10000=-9999$$

So the product of x^2 and $\left(1-x^2\right)$ becomes large negative for large values of *x*.

Hence, conclude that $\boxed{\lim_{x\to\infty}\left(x^2-x^4\right)=-\infty}$.

Chapter 3 Applications of Differentiation Exercise 3.4 29E

Consider the following expression.

$$\lim_{x\to\infty}x\sin\frac{1}{x}$$

To evaluate the limit $\lim_{x\to\infty}x\sin\frac{1}{x}$, let $\frac{1}{x}=t$.

As $x\to\infty$ then $\frac{1}{x}\to 0$ so, $t\to 0$

Hence, the expression can be written as follows:

$$\lim_{x\to\infty}x\sin\left(\frac{1}{x}\right)=\lim_{x\to 0}\frac{1}{t}\cdot\sin t \quad \text{Substitute } \frac{1}{x}=t$$

$$=\lim_{t\to 0}\frac{\sin t}{t}$$

$$=1$$

Then the value of the limit of the expression is $\boxed{\lim_{x\to\infty}x\sin\frac{1}{x}=1}$

Chapter 3 Applications of Differentiation Exercise 3.4 30E

Consider the limit:

$$\lim_{x\to\infty}\sqrt{x}\sin\frac{1}{x}$$

Apply the limit.

$$\lim_{x\to\infty} \sqrt{x}\sin\frac{1}{x} = \sqrt{\infty}\sin\frac{1}{\infty}$$
$$= \infty \sin 0$$
$$= \infty$$

Thus, **this does not exist.**

Rewrite the given limit as shown below:

$$\lim_{x\to\infty} \sqrt{x}\sin\frac{1}{x} = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{\sqrt{x}}}$$

Now, apply L'Hopital's rule.

$$\lim_{x\to\infty} \frac{\frac{d}{dx}\left(\sin\frac{1}{x}\right)}{\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)} = \lim_{x\to\infty} \frac{-\frac{\cos\left(\frac{1}{x}\right)}{x^2}}{-\frac{1}{2x^{\frac{3}{2}}}}$$
$$= \lim_{x\to\infty} \frac{2\cos\left(\frac{1}{x}\right)}{\sqrt{x}}$$

Now, apply the limit.

$$\lim_{x\to\infty} \frac{2\cos\left(\frac{1}{x}\right)}{\sqrt{x}} = \frac{2\cos\left(\frac{1}{\infty}\right)}{\sqrt{\infty}}$$
$$= \frac{2\cos(0)}{\infty}$$
$$= \frac{2}{\infty}$$
$$= 0$$

Hence,

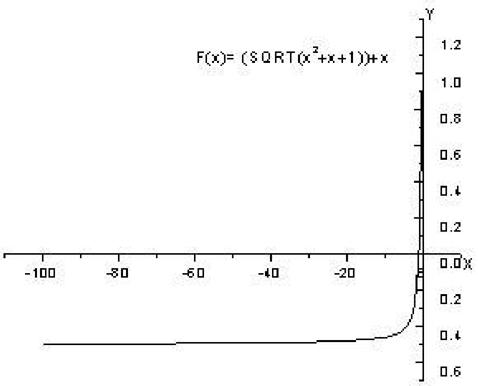
$$\lim_{x\to\infty} \sqrt{x}\sin\frac{1}{x} = \boxed{0}$$

Chapter 3 Applications of Differentiation Exercise 3.4 31E

(A)

By the graph of function $f(x) = \sqrt{x^2+x+1}+x$ we see that as we take large negative value of x then $f(x)$ approaches -0.5

So $\boxed{\lim_{x\to-\infty} \sqrt{x^2+x+1}+x = -0.5}$



(B)

We can estimate the limit by calculating the function $f(x)$ for different negative values of x

x	$f(x)$
0	1
-1	0
-2	-0.26795
-3	-0.35425
-4	-0.39445
-5	-0.41742
-6	-0.43224
-10	-0.46061
-50	-0.49051
-100	-0.49623
-1000	-0.49962
-10000	-0.49996
-100000	-0.5

As we see that by the table that $f(x) \to -0.5$ for large negative value of x

So $\boxed{\lim_{x\to-\infty} \sqrt{x^2+x+1}+x=-0.5}$

(C)

We have to evaluate

$$\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+x\right)$$

Multiply and divide by the conjugate radical

$$\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+x\right)=\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+x\right).\frac{\left(\sqrt{x^2+x+1}-x\right)}{\left(\sqrt{x^2+x+1}-x\right)}$$

$$=\lim_{x\to-\infty}.\frac{x^2+x+1-x^2}{\left(\sqrt{x^2+x+1}-x\right)}=\lim_{x\to-\infty}\frac{x+1}{\sqrt{x^2+x+1}-x}$$

Computing the limit as $x\to-\infty$, we have $\sqrt{x^2}=|x|=-x$ for $x<0$

So $\sqrt{x^2+x+1}=-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}$

Now we have

$$=\lim_{x\to-\infty}\frac{x+1}{-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x}$$

Now divide the numerator and denominator by $-x$ we have

$$\lim_{x\to-\infty}\frac{(x+1)\big/-x}{\left(-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x\right)\Big/-x}=\lim_{x\to-\infty}\frac{-1-\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}$$

$$=\lim_{x\to-\infty}\frac{-\lim_{x\to-\infty}1-\lim_{x\to-\infty}\frac{1}{x}}{\sqrt{\lim_{x\to-\infty}1+\lim_{x\to-\infty}\frac{1}{x}+\lim_{x\to-\infty}\frac{1}{x^2}}+\lim_{x\to-\infty}1}$$

$$=\frac{-1-0}{\sqrt{1+0+0}+1}=\frac{-1}{1+1}=\frac{-1}{2}=-0.5$$

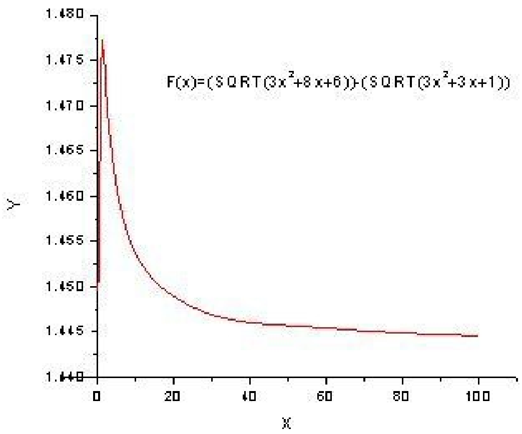
So $\boxed{\lim_{x\to-\infty}\sqrt{x^2+x+1}+x=-0.5=\frac{-1}{2}}$

(A)

By the graph of function $f(x)=\sqrt{3x^2+8x+6}-\sqrt{3x^2+3x+1}$

We see that as we take large value of x then $f(x)$ approaches to

So $\boxed{\lim_{x\to\infty} f(x)=1.44}$



(B)

W can estimate the limit by calculating the function $f(x)$ for different values of x

x	$f(x)$
0	1.44949
1	1.477354
2	1.472053
5	1.460608
10	1.453477
100	1.444557
1000	1.443496
10000	1.443388
100000	1.443377

As we see that by the table that $f(x)\to 1.44$ for large value of x

So $\boxed{\lim_{x\to\infty} f(x)=1.44}$

We have to evaluate $\lim_{x\to\infty}\left(\sqrt{3x^2+8x+6}-\sqrt{3x^2+3x+1}\right)$

Dividing and multiply by the conjugate radical

So

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\sqrt{3x^2+8x+6}-\sqrt{3x^2+3x+1}\times\frac{\left(\sqrt{3x^2+8x+6}+\sqrt{3x^2+3x+1}\right)}{\left(\sqrt{3x^2+8x+6}+\sqrt{3x^2+3x+1}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(3x^2+8x+6\right)-\left(3x^2+3x+1\right)}{\sqrt{3x^2+8x+6}+\sqrt{3x^2+3x+1}} \qquad \left[(a-b).(a+b)=a^2+b^2\right]$$

$$=\lim_{x\to\infty}\frac{5x+5}{\sqrt{3x^2+8x+6}+\sqrt{3x^2+3x+1}}$$

$$=\lim_{x\to\infty}\frac{x\left(5+5/x\right)}{x\sqrt{3+8/x+6/x^2}+x\sqrt{3+3/x+1/x^2}} \qquad \left[\begin{array}{l}\text{divide the numerator}\\ \text{and denominator by x}\end{array}\right]$$

$$=\lim_{x\to\infty}\frac{5+5/x}{\sqrt{3+8/x+6/x^2}+\sqrt{3+3/x+1/x^2}}$$

Apply limit laws

$$= \frac{\lim_{x\to\infty} 5 + \lim_{x\to\infty} 5/x}{\sqrt{\lim_{x\to\infty} 3 + \lim_{x\to\infty} 8/x + \lim_{x\to\infty} 6/x^2} + \sqrt{\lim_{x\to\infty} 3 + \lim_{x\to\infty} 3/x + \lim_{x\to\infty} 1/x^2}}$$

$$= \frac{5+0}{\sqrt{3+0+0}+\sqrt{3+0+0}} = \frac{5}{2\sqrt{3}} \qquad \left[\begin{array}{l}\text{for } r . 0 \\ \lim_{x\to\infty} \frac{1}{x^r} = 0\end{array}\right]$$

So we have $\boxed{\lim_{x\to\infty} f(x) = \frac{5}{2\sqrt{3}}}$

Chapter 3 Applications of Differentiation Exercise 3.4 33E

Consider the curve $y = \frac{2x+1}{x-2}$

Find the horizontal and vertical asymptotes of the curve:

Dividing both numerator and denominator by x and using the properties of limits,

$$\lim_{x\to\infty} \frac{2x+1}{x-2} = \lim_{x\to\infty} \frac{\frac{2x+1}{x}}{\frac{x-2}{x}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{1}{x}}{1-\frac{2}{x}}$$

$$= \frac{2+0}{1-0}$$

$$= 2$$

Therefore, $y = 2$ is a horizontal asymptote of the curve.

Now, find the vertical asymptote:

$$y = \frac{2x+1}{x-2}$$

A vertical asymptote is likely to occur when the denominator, $x-2$ is 0, that is, when $x = 2$.

If x is close to 2 and $x > 2$, then the denominator is close to 0 and $(x-2)$ is positive. The numerator $2x+1$ is positive.

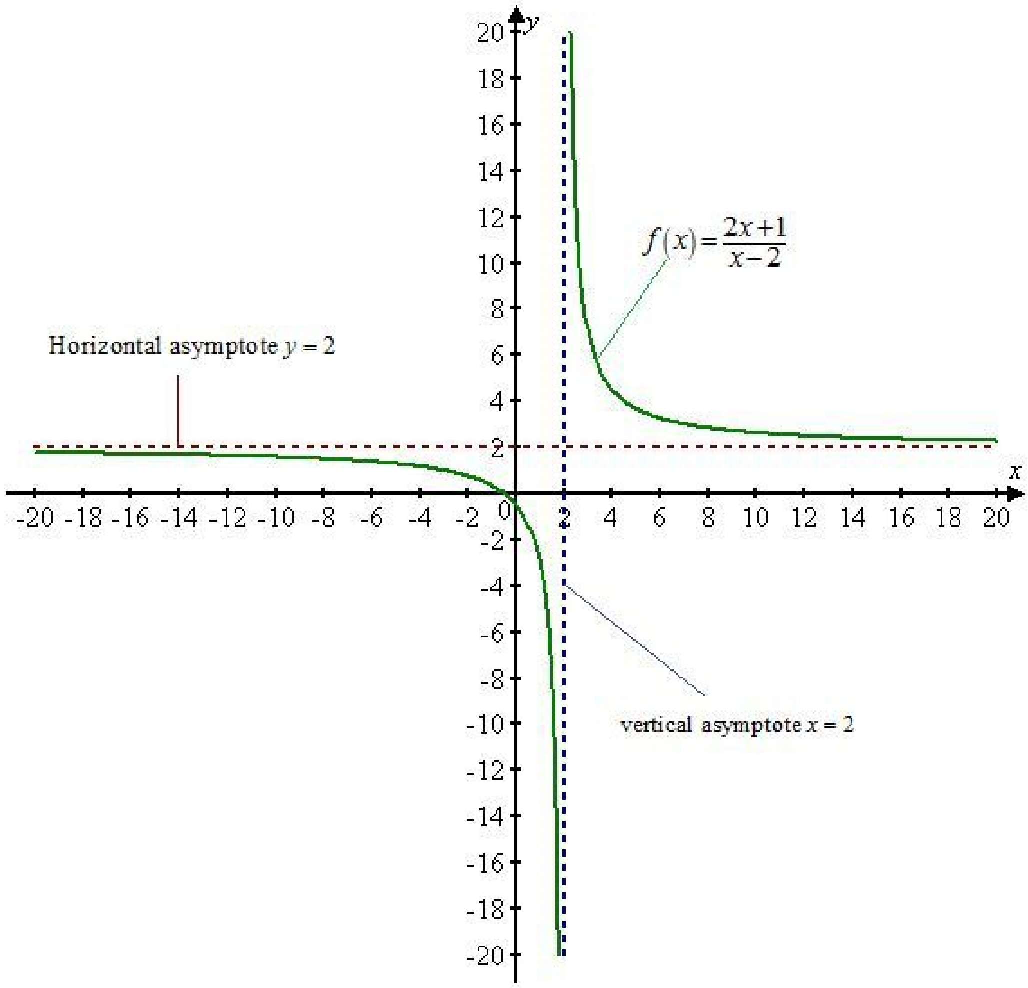
Therefore, $\lim_{x\to 2^+} \frac{2x+1}{x-2} = \infty$

If x is close to 2 and $x < 2$, then the denominator is close to 0 and $(x-2)$ is negative. The numerator $2x+1$ is positive.

Therefore, $\lim_{x\to 2^-} \frac{2x+1}{x-2} = -\infty$

Therefore, $x = 2$ is vertical asymptote.

Sketch the graph of $f(x)=\frac{2x+1}{x-2}$ is as follows:



Chapter 3 Applications of Differentiation Exercise 3.4 34E

Consider the curve $y=\frac{x^2+1}{2x^2-3x-2}$

Find the horizontal and vertical asymptotes of the curve:

Dividing both numerator and denominator by x^2 and using the properties of limits,

$$\lim_{x\to\infty}\frac{x^2+1}{2x^2-3x-2}=\lim_{x\to\infty}\frac{\frac{x^2+1}{x^2}}{\frac{2x^2-3x-2}{x^2}}$$

$$=\frac{\lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)}{\lim_{x\to\infty}\left(2-\frac{3}{x}-\frac{2}{x^2}\right)}$$

$$=\frac{1+0}{2-0-0}$$

$$=\frac{1}{2}$$

Therefore, $y=\frac{1}{2}$ is a horizontal asymptote of the curve.

Now, find the vertical asymptote:

$$y = \frac{x^2+1}{2x^2-3x-2}$$
$$= \frac{x^2+1}{(2x+1)(x-2)}$$

A vertical asymptote is likely to occur when the denominator, $(2x+1)(x-2)$ is 0, that is, when $x = -\frac{1}{2} \text{ or } 2$.

If x is close to $-\frac{1}{2}$ and $x > -\frac{1}{2}$, then the denominator is close to 0 and $(2x+1)(x-2)$ is negative. The numerator x^2+1 is always positive.

Therefore, $\lim_{x \to -\frac{1}{2}^+} \frac{x^2+1}{(2x+1)(x-2)} = -\infty$

If x is close to $-\frac{1}{2}$ and $x < -\frac{1}{2}$, then the denominator is close to 0 and $(2x+1)(x-2)$ is positive. The numerator x^2+1 is always positive.

Therefore, $\lim_{x \to -\frac{1}{2}^-} \frac{x^2+1}{(2x+1)(x-2)} = \infty$

Thus, $x = -\frac{1}{2}$ is vertical asymptote.

If x is close to 2 and $x > 2$, then the denominator is close to 0 and $(2x+1)(x-2)$ is positive. The numerator x^2+1 is positive.

Therefore, $\lim_{x \to 2^+} \frac{x^2+1}{(2x+1)(x-2)} = \infty$

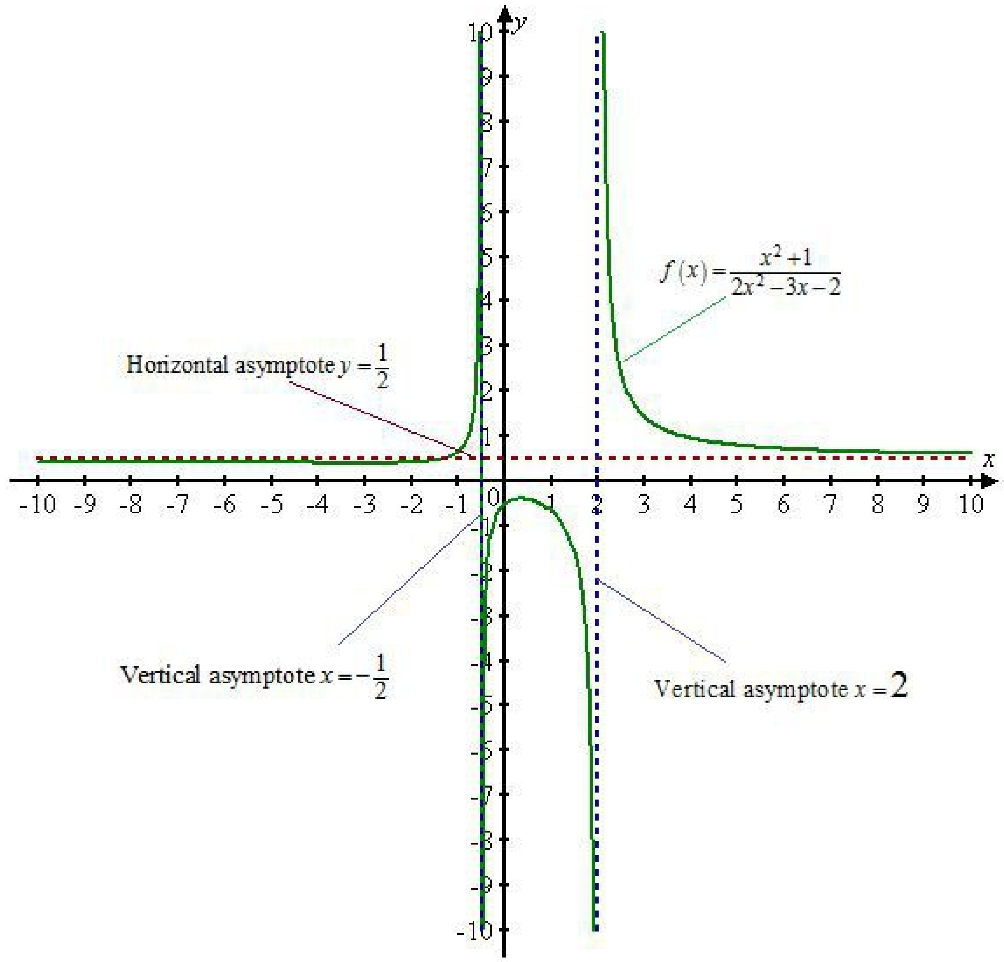
If x is close to 2 and $x < 2$, then the denominator is close to 0 and $(2x+1)(x-2)$ is negative. The numerator x^2+1 is positive.

Therefore, $\lim_{x \to 2^+} \frac{x^2+1}{(2x+1)(x-2)} = -\infty$

Thus, $x = 2$ is vertical asymptote.

Therefore, $x = -\frac{1}{2}$ and $x = 2$ are vertical asymptotes.

Sketch the graph of $f(x)=\dfrac{x^2+1}{2x^2-3x-2}$ is as follows:



Chapter 3 Applications of Differentiation Exercise 3.4 35E

Now, find the vertical asymptote.

$$y=\frac{2x^2+x-1}{x^2+x-2}$$
$$=\frac{2x^2+x-1}{(x+2)(x-1)}$$

A vertical asymptote is likely to occur, when the denominator, $(x+2)(x-1)$ is 0, that is, when,

$x=-2 \text{ or } 1$.

If x is close to -2 and $x>-2$, then the denominator is close to 0 and $(x+2)(x-1)$ is negative. The numerator $2x^2+x-1$ is positive.

Therefore, $\lim\limits_{x\to -2^+}\dfrac{2x^2+x-1}{(x+2)(x-1)}=-\infty$

If x is close to -2 and $x<-2$, then the denominator is close to 0 and $(x+2)(x-1)$ is positive. The numerator $2x^2+x-1$ is positive.

Therefore, $\lim\limits_{x\to -2^-}\dfrac{2x^2+x-1}{(x+2)(x-1)}=\infty$

Thus, $x=-2$ is vertical asymptote.

If x is close to 1 and $x>1$, then the denominator is close to 0 and $(x+2)(x-1)$ is positive.

The numerator $2x^2+x-1$ is positive.

Therefore, $\lim_{x\to 1^+}\frac{2x^2+x-1}{(x+2)(x-1)}=\infty$

If x is close to 1 and $x<1$, then the denominator is close to 0 and $(x+2)(x-1)$ is negative.

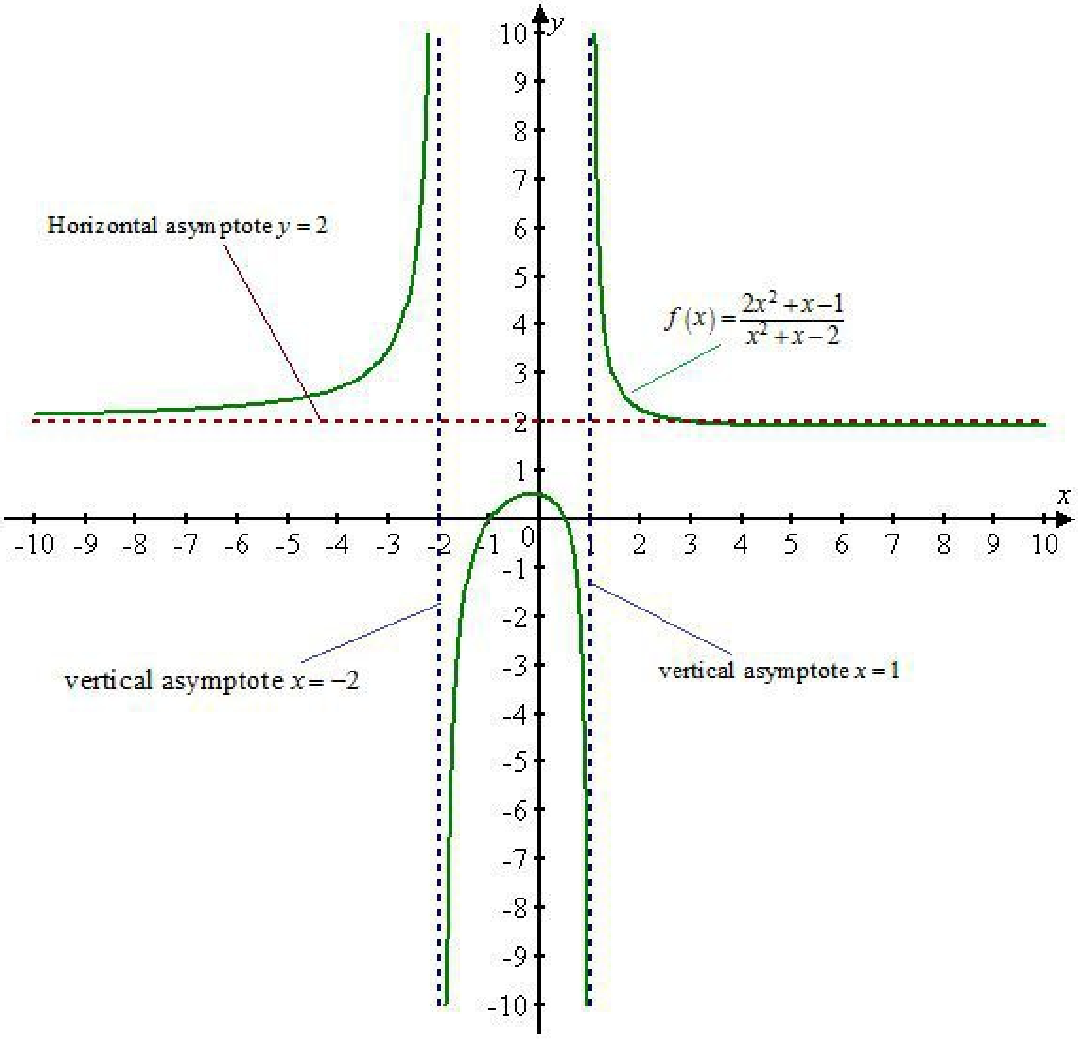
The numerator $2x^2+x-1$ is positive.

Therefore, $\lim_{x\to 1^-}\frac{2x^2+x-1}{(x+2)(x-1)}=-\infty$

Thus, $x=1$ is a vertical asymptote.

Therefore, $x=1$ and $x=-2$ are vertical asymptotes.

Sketch the graph of $f(x)=\frac{2x^2+x-1}{x^2+x-2}$ as follows:



Chapter 3 Applications of Differentiation Exercise 3.4 37E

Consider the function $y=\frac{x^3-x}{x^2-6x+5}$.

The objective is to find the horizontal and vertical asymptotes.

Calculate the limit value of the function $y = f(x)$ as $x \to \infty$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^3 - x}{x^2 - 6x + 5}$$

Divide by x^2 both the numerator and the denominator,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\left(\frac{x^3 - x}{x^2}\right)}{\left(\frac{x^2 - 6x + 5}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{\left(\frac{x^3}{x^2} - \frac{x}{x^2}\right)}{\left(\frac{x^2}{x^2} - \frac{6x}{x^2} + \frac{5}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{\left(x - \frac{1}{x}\right)}{\left(1 - \frac{6}{x} + \frac{5}{x^2}\right)}$$

$$= \frac{\infty}{1}$$

$$= \infty$$

Hence, the value of the limit as $x \to \infty$ is $\boxed{\lim_{x\to\infty} f(x) = \infty}$.

Calculate the limit value of the function $y = f(x)$ as $x \to -\infty$.

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^3 - x}{x^2 - 6x + 5}$$

Divide by x^2 both the numerator and the denominator.

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\left(\frac{x^3 - x}{x^2}\right)}{\left(\frac{x^2 - 6x + 5}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{\left(\frac{x^3}{x^2} - \frac{x}{x^2}\right)}{\left(\frac{x^2}{x^2} - \frac{6x}{x^2} + \frac{5}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{\left(x - \frac{1}{x}\right)}{\left(1 - \frac{6}{x} + \frac{5}{x^2}\right)}$$

$$= \frac{-\infty}{1}$$

$$= -\infty$$

Hence, the value of the limit as $x \to -\infty$ is $\boxed{\lim_{x\to\infty} f(x) = -\infty}$.

Thus, there is no finite number L such that $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.

Therefore, the function $y = \frac{x^3 - x}{x^2 - 6x + 5}$ has no horizontal asymptotes but has slant asymptote.

To find the slant asymptote, divide the numerator with denominator by using long division.

$$\begin{array}{r} x+6 \\ x^2-6x+5 \overline{\big) x^3 \qquad -x} \\ \underline{x^3-6x^2+5x\;(-)} \\ 6x^2-6x \\ \underline{6x^2-36x+30\;(-)} \\ 30x-30 \end{array}$$

Rewrite the function as,

$$\begin{aligned} y &= \frac{x^3-x}{x^2-6x+5} \\ &= (x+6)+\frac{30x-30}{x^2-6x+5} \quad \left(\text{Since, } y = mx+b+\frac{R(x)}{Q(x)} \text{ form}\right) \end{aligned}$$

Hence, the function $y=\frac{x^3-x}{x^2-6x+5}$ has slant asymptote $\boxed{y=x+6}$.

To find the vertical asymptote, first factor the function and then check on what value the function will be infinity.

Rewrite the function $f(x)=\frac{x^3-x}{x^2-6x+5}$ as,

$$\begin{aligned} f(x) &= \frac{x(x^2-1)}{x^2-5x-x+5} \\ &= \frac{x(x^2-1)}{x^2-5x-x+5} \\ &= \frac{x(x^2-1)}{x(x-5)-1(x-5)} \\ &= \frac{x(x^2-1)}{(x-5)(x-1)} \\ &= \frac{x(x-1)(x+1)}{(x-5)(x-1)} \\ &= \frac{x(x+1)}{x-5} \end{aligned}$$

Clearly at $x=5$, the function $f(x)=\frac{x(x+1)}{x-5}$ is undefined.

Observe, $x=1$ is not a vertical asymptote.

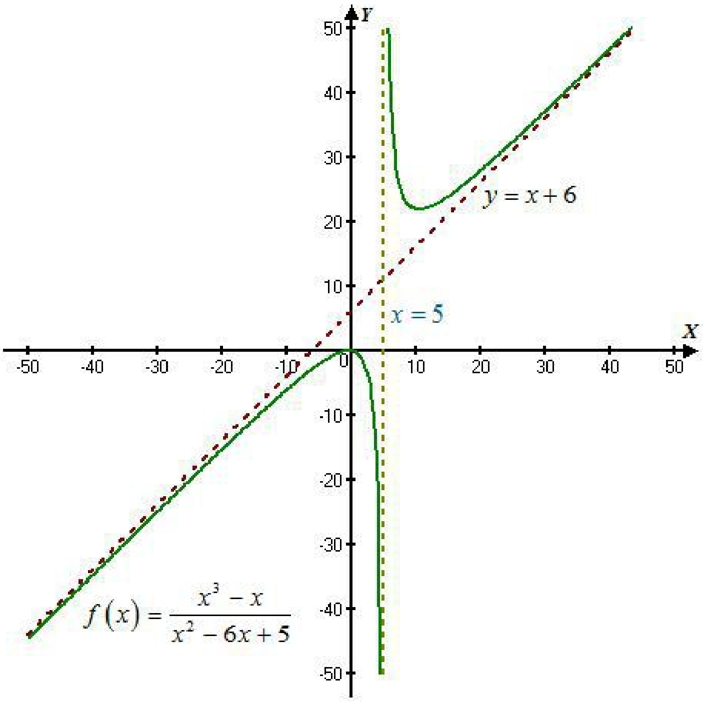
Calculate the limit of the function $f(x)$ as x tends to 5:

$$\begin{aligned} \lim_{x\to5} f(x) &= \lim_{x\to5}\frac{x^3-x}{x^2-6x+5} \\ &= \lim_{x\to5}\frac{x\cancel{(x-1)}(x+1)}{(x-5)\cancel{(x-1)}} \\ &= \lim_{x\to5}\frac{x(x+1)}{(x-5)} \\ &= \frac{5(5+1)}{(5-5)} \\ &= \frac{5\cdot6}{0} \\ &= \infty \end{aligned}$$

Hence, the vertical asymptote of the function $f(x)=\frac{x^3-x}{x^2-6x+5}$ is $\boxed{x=5}$.

Check the horizontal and vertical asymptote of the function using graph of the function.

The graph of the function with asymptotes is shown below.



Chapter 3 Applications of Differentiation Exercise 3.4 38E

We have $F(x) = \dfrac{x-9}{\sqrt{4x^2+3x+2}}$

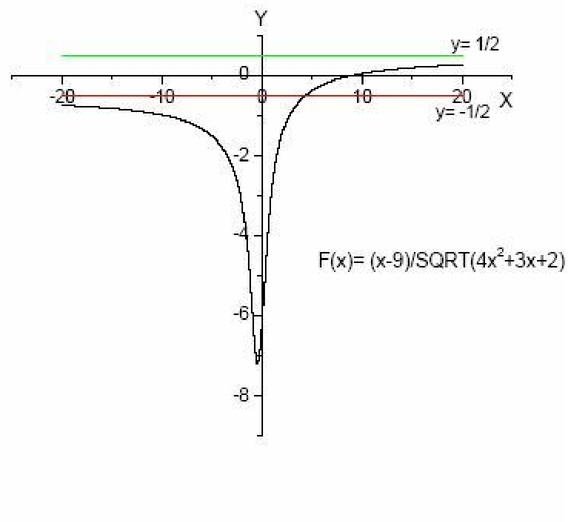


FIGURE - 1

For horizontal tangents we take the limit as $x \to \infty$

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty} \frac{x-9}{\sqrt{4x^2+3x+2}}$$

$$= \lim_{x\to\infty} \frac{x-9}{\sqrt{x^2\left(4+\frac{3}{x}+\frac{2}{x^2}\right)}}$$

$$= \lim_{x\to\infty} \frac{x-9}{x\sqrt{4+\frac{3}{x}+\frac{2}{x^2}}}$$

Now divide the numerator and denominator by x and using limit laws

$$= \lim_{x\to\infty} \frac{1-\frac{9}{x}}{\sqrt{4+\frac{3}{x}+\frac{2}{x^2}}}$$

$$= \frac{\lim_{x\to\infty} 1 - 9\lim_{x\to\infty} \frac{1}{x}}{\sqrt{\lim_{x\to\infty} 4 + 3\lim_{x\to\infty} \frac{1}{x} + 2\lim_{x\to\infty} \frac{1}{x^2}}}$$

$$= \frac{1-0}{\sqrt{4+0+0}} = \frac{1}{2}$$

We get the limit $-\frac{1}{2}$ as $x \to -\infty$

So the horizontal asymptotes are $\boxed{y = \pm\frac{1}{2}}$

$F(x) = \frac{x-9}{\sqrt{4x^2+3x+2}}$ Is not defined when

$4x^2+3x+2=0$ or negative

We take only the inequality $4x^2+3x+2=0$

Or $x = \frac{-3\pm\sqrt{9-32}}{8}$ are not real values.

So there is $\boxed{\text{no any vertical asymptotes}}$

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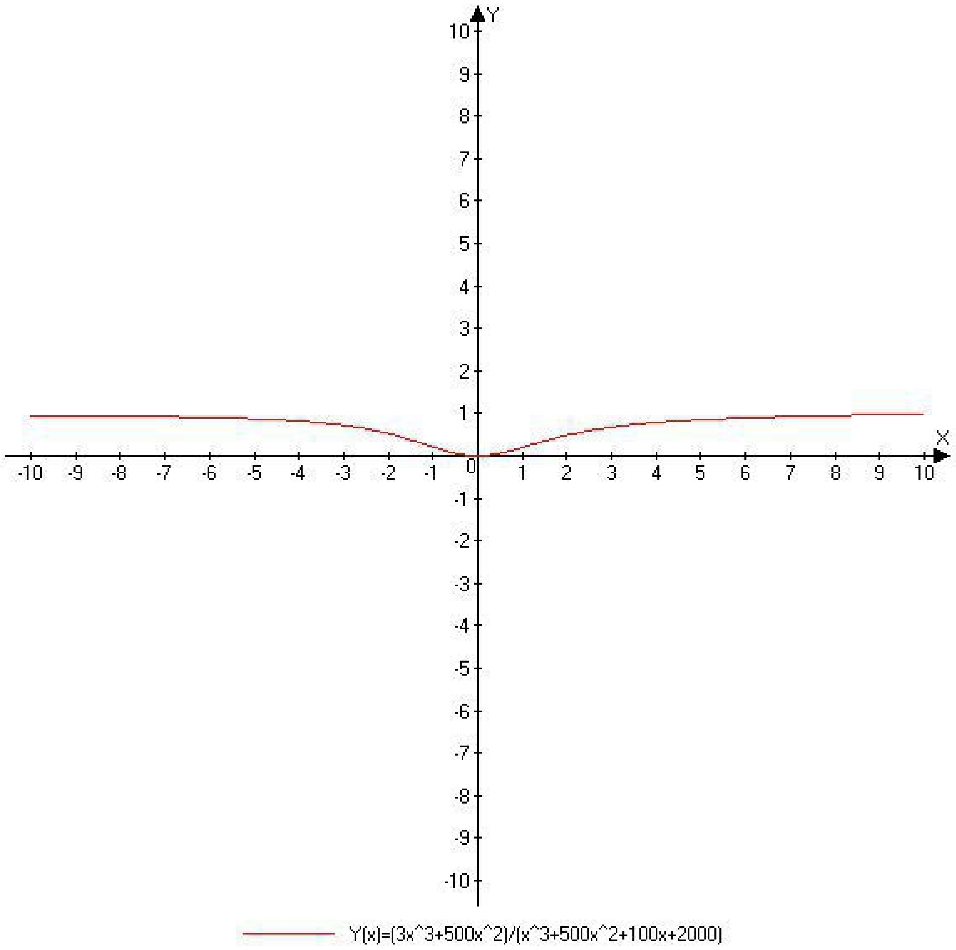
$$f(x) = \frac{3x^3 + 500x^2}{x^3 + 500x^2 + 100x + 2000}$$

we find the assymptotes by evaluating the limits in the interval [-10,10].



observe that the curve has horizontal assymptotes .also the curve is becoming parallel after -9 and 9 on either sides of the origin.

so, we can write $\lim_{x \to -\infty} f(x) = 1, \lim_{x \to \infty} f(x) = 1$

we can either way write that for all x < -8 , | f(x) - 1 | < ε and for all x > 8 , | f(x) - 1| < ε .

∴ y = 1 .

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Given $\lim_{x \to \pm\infty} = 0 \quad \lim_{x \to 0} f(x) = -\infty , f(2) = 0$

$\lim_{x \to 3^-} f(x) = \infty , \lim_{x \to 3^+} f(x) = -\infty$

Conclusions

Here $\lim_{x \to 3^-} f(x) = \infty$ and $\lim_{x \to 3^+} f(x) = -\infty$ and $\lim_{x \to 0} f(x) = -\infty$

So

$x = 0$, $x = 3$ will be the vertical asymptotes

And then x (x - 3) will be the denominator.

$f(2)=0$

Then (2 - x) will be numerator but horizontal asymptotes is y = 0 so the numerator will be in the form of $\frac{2}{x}-\frac{1}{x}$

Numerator $=\frac{(2-x)}{x}$

So the formula of the function is

$$\boxed{f(x)=\frac{(2-x)}{x^2(x-3)}}$$

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Given:

The vertical asymptotes are $x = 1$ and $x = 3$. So $(x-1)$ and $(x-3)$ will be factors of the denominator.

Horizontal asymptotes $y=1$ so the power of x in the numerator will be same as denominator that is x^2

So the formula of $f(x)$ is

$$f(x)=\frac{x^2}{(x-1)(x-3)}$$

Or we can write $f(x)=\frac{x^2}{x^2-x-3x+3}$

So $\boxed{f(x)=\frac{x^2}{x^2-4x+3}}$

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For getting horizontal asymptotes, we find the limit as $x\to\infty$.

So

$$\lim_{x\to\infty} y=\lim_{x\to\infty}\frac{1+2x^2}{1+x^2}$$

Divide the numerator and denominator by x^2

$$\lim_{x\to\infty} y=\lim_{x\to\infty}\frac{\frac{1}{x^2}+2}{\frac{1}{x^2}+1}$$

By using limit laws

$$\lim_{x\to\infty} y=\frac{\lim_{x\to\infty}\frac{1}{x^2}+\lim_{x\to\infty}2}{\lim_{x\to\infty}\frac{1}{x^2}+\lim_{x\to\infty}1}$$

$$=\frac{0+2}{0+1}$$

Or $\lim_{x\to\infty} y=2$ similarly $\lim_{x\to-\infty} y=2$

So the horizontal asymptote $\boxed{y=2}$

Now we differentiate y with respect to x by quotient rule.

$$y'=\frac{(1+x^2)(4x)-(1+2x^2)(2x)}{(1+x^2)^2}$$

$$=\frac{4x+4x^3-2x-4x^3}{(1+x^2)^2}$$

$$=\frac{2x}{(1+x^2)^2}$$

y' is not defined when $1+x^2=0$ or $x^2=-1$
(this is not possible so y' is defined for all x).

$y'=0$ When $x=0$

So we divide the interval in the intervals $(-\infty,0)$ and $(0,\infty)$

We see that

$y'<0$ when $-\infty<x<0$

So **y is decreasing when $-\infty<x<0$** and

$y'>0$ *when* $0<x<\infty$

So **y is increasing when $0<x<\infty$**

y has a local minimum at **$x=0$**

Now differentiate y' with respect to x b quotient rule when

$$y'=\frac{2x}{(1+x^2)^2}$$

$$y''=\frac{(1+x^2)^2.2-2x.2(1+x^2).(2x)}{\left[(1+x^2)^2\right]^2}$$

$$=\frac{2(1+x^2)^2-8x^2(1+x^2)}{(1+x^2)^4}$$

$$=\frac{2(1+x^2)-8x^2}{(1+x^2)^3}$$

$$=\frac{2+2x^2-8x^2}{(1+x^2)^3}$$

$$=\frac{2-6x^2}{(1+x^2)^3}$$

So $\quad y''=\dfrac{2-6x^2}{(1+x^2)^3}$

$y''=0$ When $2-6x^2=0$

$$or\ x^2=\frac{2}{6}$$

$$or\ x=\pm\frac{1}{\sqrt{3}}$$

We check the concavity in the intervals $\left(-\infty,-\frac{1}{\sqrt{3}}\right)$, $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}},\infty\right)$

Intervals	y''	y
$-\infty<x<-\frac{1}{\sqrt{3}}$	-ve	Concave downward on $\left(-\infty,-\frac{1}{\sqrt{3}}\right)$
$-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$	+ve	Concave upward on $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$
$\frac{1}{\sqrt{3}}<x<\infty$	-ve	Concave downward on $\left(\frac{1}{\sqrt{3}},\infty\right)$

So the inflection points has x-co-ordinates $-\frac{1}{\sqrt{3}}\ and\ \frac{1}{\sqrt{3}}$

With the help of the results in step 1, 2 and 3 we can draw the graph of the function

$$y = \frac{1+2x^2}{1+x^2}$$

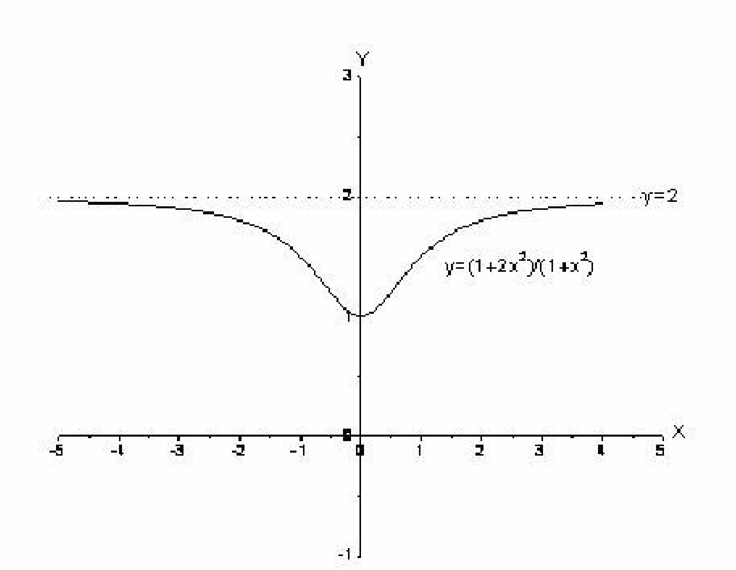


Fig.1

Chapter 3 Applications of Differentiation Exercise 3.4 45E

For finding horizontal asymptotes, we take the limit as $x \to \infty$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{1-x}{1+x}$$

Dividing by x both numerator and denominator

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{\frac{1}{x}-1}{\frac{1}{x}+1}$$

Using limit laws

$$\lim_{x\to\infty} y = \frac{\lim_{x\to\infty}\frac{1}{x} - \lim_{x\to\infty} 1}{\lim_{x\to\infty}\frac{1}{x} + \lim_{x\to\infty} 1}$$

$$= \frac{0-1}{0+1}$$

$$\boxed{\lim_{x\to\infty} y = -1}$$

So the horizontal asymptote is $\boxed{y=-1}$

And denominator $1+x=0 \Rightarrow \boxed{x=-1}$ is the vertical asymptote

Now differentiate y with respect to x by Quotient rule

$$y' = \frac{dy}{dx} = \frac{(1-x)(-1)-(1-x)}{(1+x)^2}$$

$$= \frac{-1-x-1+x}{(1+x)^2}$$

$$\Rightarrow y' = \frac{-2}{(1+x)^2}$$

y' is not defined when $1+x=0$ or $x=-1$ but $y = \frac{(1-x)}{(1+x)}$ is also not defined for $x=-1$.

We consider the intervals $(-\infty,-1)$ and $(-1,\infty)$ and make a chart

Intervals	y'	y
$-\infty < x < -1$	-ve	Decreasing on $(-\infty,-1)$
$-1 < x < \infty$	-ve	Decreasing on $(-1,\infty)$

So the function is decreasing on its entire domain.

Now differentiate y' with respect to x

$$y' = 2(1+x)^{-2}$$

$$y'' = -2(-2)(1+x)^{-3}$$

$$= 4(1+x)^{-3}$$

Or $$y'' = \frac{4}{(1+x)^3}$$

$y'' < 0 \text{ when } -\infty < x < -1$

So y is concave downward when $-\infty < x < -1$

$y'' > 0 \text{ when } -1 < x < \infty$

So y is concave upward when $-1 < x < \infty$

With help of the results in Step 1, 2 and 3 we can draw the graph of the function

$$y = \frac{(1-x)}{(1+x)}$$

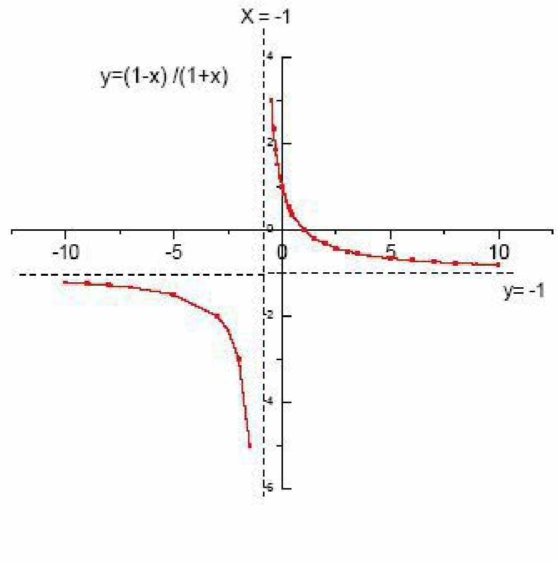


FIGURE - 1

Chapter 3 Applications of Differentiation Exercise 3.4 46E

To find horizontal asymptotes

We have to find the limit as $x \to \infty$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$$

We have $|x| = \sqrt{x^2}$. So we divide the numerator and denominator by x.

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

By using limit laws

$$\lim_{x\to\infty} y = \frac{\lim_{x\to\infty} 1}{\sqrt{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{1}{x^2}}}$$

$$= \frac{1}{\sqrt{1+0}}$$

$$= \frac{1}{\sqrt{1}} = \sqrt{1}$$

As x > 0 so $\sqrt{x^2} = |x| = x$ then

If $x \to \infty$ $\lim_{x\to\infty} y = 1$

And if x < 0 , $\sqrt{x^2} = -x$

Then $\lim_{x\to\infty} y = -1$

So horizontal asymptotes are $\boxed{y=-1}$ *and* $\boxed{y=1}$

Now differentiate y with respect to x

$$y' = \frac{\left(\sqrt{x^2+1}\right).1 - \frac{1}{2}\left(x^2+1\right)^{-1/2}(2x).x}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{\left(\sqrt{x^2+1}\right)}$$

$$= \frac{x^2+1-x^2}{\left(x^2+1\right)\sqrt{x^2+1}}$$

$$= \frac{1}{\left(x^2+1\right)\sqrt{x^2+1}}$$

$y' > 0$ For all x

So y is $\boxed{\text{increasing on its entire domain}}$

$$y' = \frac{1}{\left(x^2+1\right)^{3/2}} = \left(x^2+1\right)^{-3/2}$$

Then

$$y'' = -\frac{3}{2}\left(x^2+1\right)^{-5/2}.(2x)$$

Or $$y'' = -\frac{3x}{\left(x^2+1\right)^{5/2}}$$

$y'' = 0$ When x = 0

So we check the concavity in the intervals $(-\infty, 0)$ *and* $(0,\infty)$

Intervals	y''	y
$-\infty < x < 0$	+ve	Concave upward on $(-\infty, 0)$
$0 < x < \infty$	-ve	Concave downward on $(0,\infty)$

So the inflection point has x-co-ordinates x = 0 with the help of above information's we can draw the graph of y

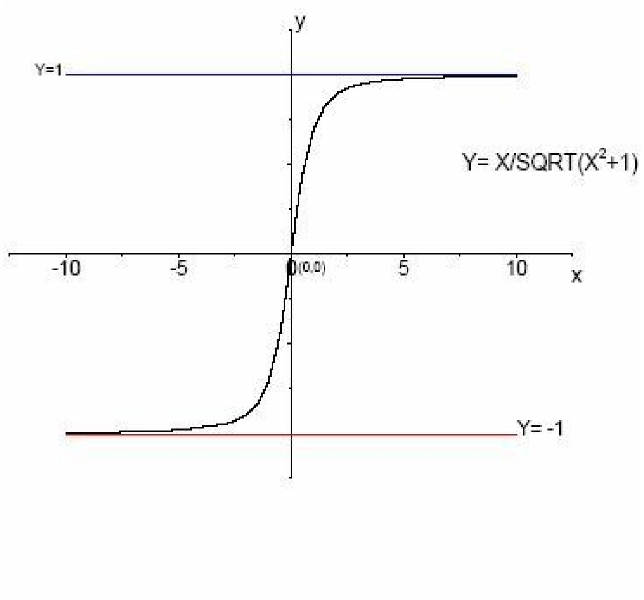


FIGURE - 1

Chapter 3 Applications of Differentiation Exercise 3.4 47E

Getting horizontal asymptotes

We take the limit as $x \to \infty$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{x}{x^2+1}$$

Divide the numerator and denominator by x^2

$$\lim_{x\to\infty} y = \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{1}{x^2}}$$

By using limit laws

$$\lim_{x\to\infty} y = \frac{\lim_{x\to\infty} \frac{1}{x}}{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{1}{x^2}}$$

$$= \frac{0}{1+0} = 0$$

We get the same limit as $x \to -\infty$

So horizontal asymptotes is $\boxed{y=0}$

There is no vertical asymptote (Since $\frac{x}{x^2+1}$ is defined everywhere)

Now we differentiate the function y with respect to x by quotient rule

$$y' = \frac{(x^2+1) - x(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$y' = \frac{(1-x^2)}{(x^2+1)^2}$$

$y' = 0$ When $1-x^2 = 0$ or $x = \pm 1$

So we divide the intervals in the subintervals whose end points are -1 and 1

Intervals	y'	y
$-\infty < x < -1$	-ve	Decreasing on $(-\infty, -1)$
$-1 < x < 1$	+ve	Increasing on $(-1, 1)$
$1 < x < \infty$	-ve	Decreasing on $(1, \infty)$

So y has local maximum at $\boxed{x = 1}$

And local minimum at $\boxed{x = -1}$

Now we differentiate y' with respect to x

$$y'' = \frac{\left(x^2+1\right)^2(-2x) - \left(1-x^2\right)2.\left(x^2+1\right).(2x)}{\left(x^2+1\right)^4}$$

$$= \frac{\left[-2x\left(x^2+1\right) - 4x\left(1-x^2\right)\right]}{\left(x^2+1\right)^3}$$

$$= \frac{-2x^3 - 2x - 4x + 4x^3}{\left(x^2+1\right)^3}$$

$$= \frac{2x^3 - 6x}{\left(x^2+1\right)^3}$$

$y'' = 0$ When $2x^3 - 6x = 0$ or $2x\left(x^2 - 3\right) = 0$

or $x = 0$ *or* $x = \pm\sqrt{3}$

We check the concavity in the intervals $\left(-\infty, -\sqrt{3}\right)$, $\left(-\sqrt{3}, 0\right)$, $\left(0, \sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$

Intervals	y''	y
$-\infty < x < -\sqrt{3}$	-ve	Concave downward on $\left(-\infty, -\sqrt{3}\right)$
$-\sqrt{3} < x < 0$	+ ve	Concave upward on $\left(-\sqrt{3}, 0\right)$
$0 < x < \sqrt{3}$	-ve	Concave downward on ,$\left(0, \sqrt{3}\right)$
And $\sqrt{3} < x < \infty$	+ve	Concave upward on $\left(\sqrt{3}, \infty\right)$

So the inflection points have x-co-ordinates $-\sqrt{3}, 0, \sqrt{3}$

With the help of the results form Step 1, 2 and 3 we can draw the graph of $y = \dfrac{x}{x^2+1}$

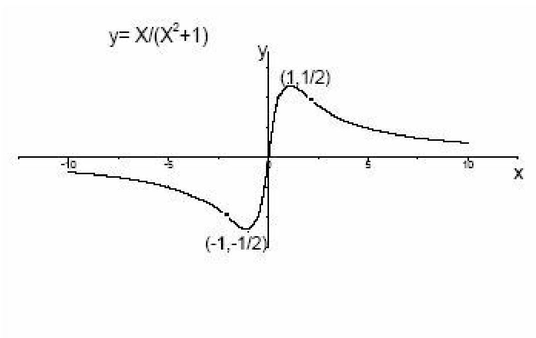


FIGURE - 1

Given that $y = 2x^3 - x^4$

The y-intercept is

$$y(0) = f(0)$$
$$= 0$$

And x-intercepts are found by setting $y = 0$;

$$\Rightarrow \quad 0 = 2x^3 - x^4$$
$$\Rightarrow \quad 0 = x^3(2 - x)$$
$$\Rightarrow \quad x = 0, x = 2$$

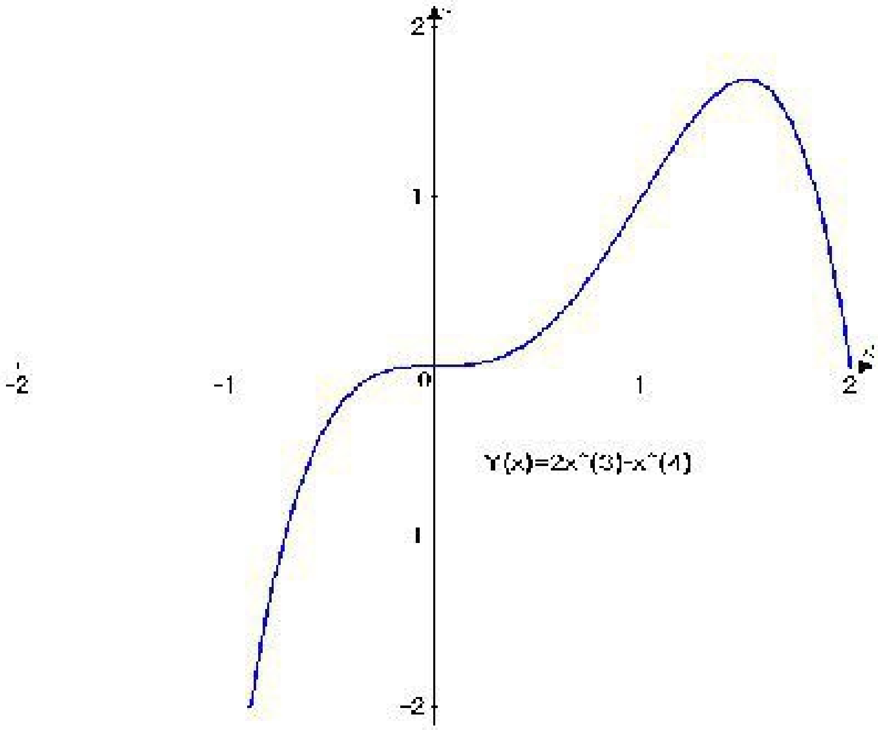
When x is large positive, $2x^3 - x^4$ is large negative.

$$\lim_{x\to\infty}(2x^3 - x^4) = \lim_{x\to\infty} x^3(2 - x)$$
$$= -\infty.$$

When x is large negative, $2x^3 - x^4$ is large negative.

$$\lim_{x\to-\infty}(2x^3 - x^4) = \lim_{x\to-\infty} x^3(2 - x)$$
$$= -\infty.$$

Combining this information, we give a rough sketch of the graph.



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Consider the function,

$y = x^4 - x^6$ ...... (1)

Compute the *y*-intercept of the given function by substituting $x = 0$ and solve for *y*,

$$\begin{aligned} y(0) &= (0)^4 - (0)^6 \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

Compute the *x*-intercept of the given function by substituting $y = 0$ and solve for *x*,

$$\begin{aligned} 0 &= x^4 - x^6 \\ x^4\left(1 - x^2\right) &= 0 \\ x^4(1-x)(1+x) &= 0 \\ x &= 0,\ 1,\ -1 \end{aligned}$$

Thus, *y*-intercept of the given function is at point $(0,0)$ and *x*-intercepts of the given function are at points $(0,0),(1,0)$ and $(-1,0)$

As x^4 cannot be negative, the function does not change sign at $x = 0$. So, the graph does not cross the *x*-axis at $x = 0$.

When *x* is large positive, the factors x^4 and $1+x$ are large positive, and the factor $1-x$ is large negative. So,

$$\begin{aligned} \lim_{x\to\infty} y &= \lim_{x\to\infty} x^4(1-x)(1+x) \\ &= -\infty \end{aligned}$$

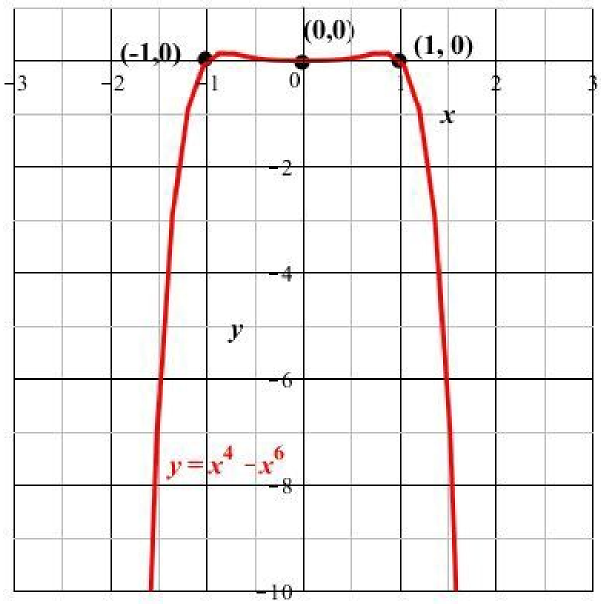
When *x* is large negative, the factors x^4 and $1-x$ are large positive, and the factor $1+x$ is large negative. So,

$$\begin{aligned} \lim_{x\to-\infty} y &= \lim_{x\to-\infty} x^4(1-x)(1+x) \\ &= -\infty \end{aligned}$$

Therefore,

$$\boxed{\begin{aligned} \lim_{x\to\infty} y &= -\infty \\ \text{and, } \lim_{x\to\infty} y &= -\infty \end{aligned}}$$

Combining these information, a rough sketch of the graph of the given function will be as follows:



Take a function *f* defined on the interval (a,∞).

$$\lim_{x\to a} f(x)=L$$

The limit is defined such that for a number $\varepsilon>0$, there is a number *N* which satisfies:

$$\text{if } x>N, \text{then } \left|f(x)-L\right|<\varepsilon$$

Consider the function:

$$y=x^3(x+2)^2(x-1)$$

Determine the limits of the function for $x\to\infty$:

$$\begin{aligned}\lim_{x\to\infty} y &= \lim_{x\to\infty} x^3(x+2)^2(x-1)\\ &=\infty\end{aligned}$$

Determine the limits of the function for $x\to-\infty$:

$$\begin{aligned}\lim_{x\to-\infty} y &= \lim_{x\to-\infty} x^3(x+2)^2(x-1)\\ &= \lim_{x\to-\infty} x^6\left(1+\tfrac{2}{x}\right)^2\left(1-\frac{1}{x}\right)\\ &=\infty\end{aligned}$$

Determine the *x*-intercept of the function.

Substitute $y=0$ and solve for *x*:

$$x^3(x+2)^2(x-1)=0$$

Consider the value of *x* from the first factor:

$$\begin{aligned}x^3&=0\\ x&=0\end{aligned}$$

Consider the value of *x* from the second factor:

$$\begin{aligned}(x+2)^2&=0\\ (x+2)&=0\\ x&=-2\end{aligned}$$

Consider the value of *x* from the third factor:

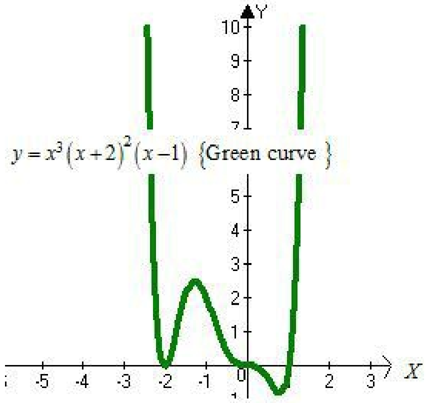
$$\begin{aligned}(x-1)&=0\\ x&=1\end{aligned}$$

Determine the *y*-intercept of the function:

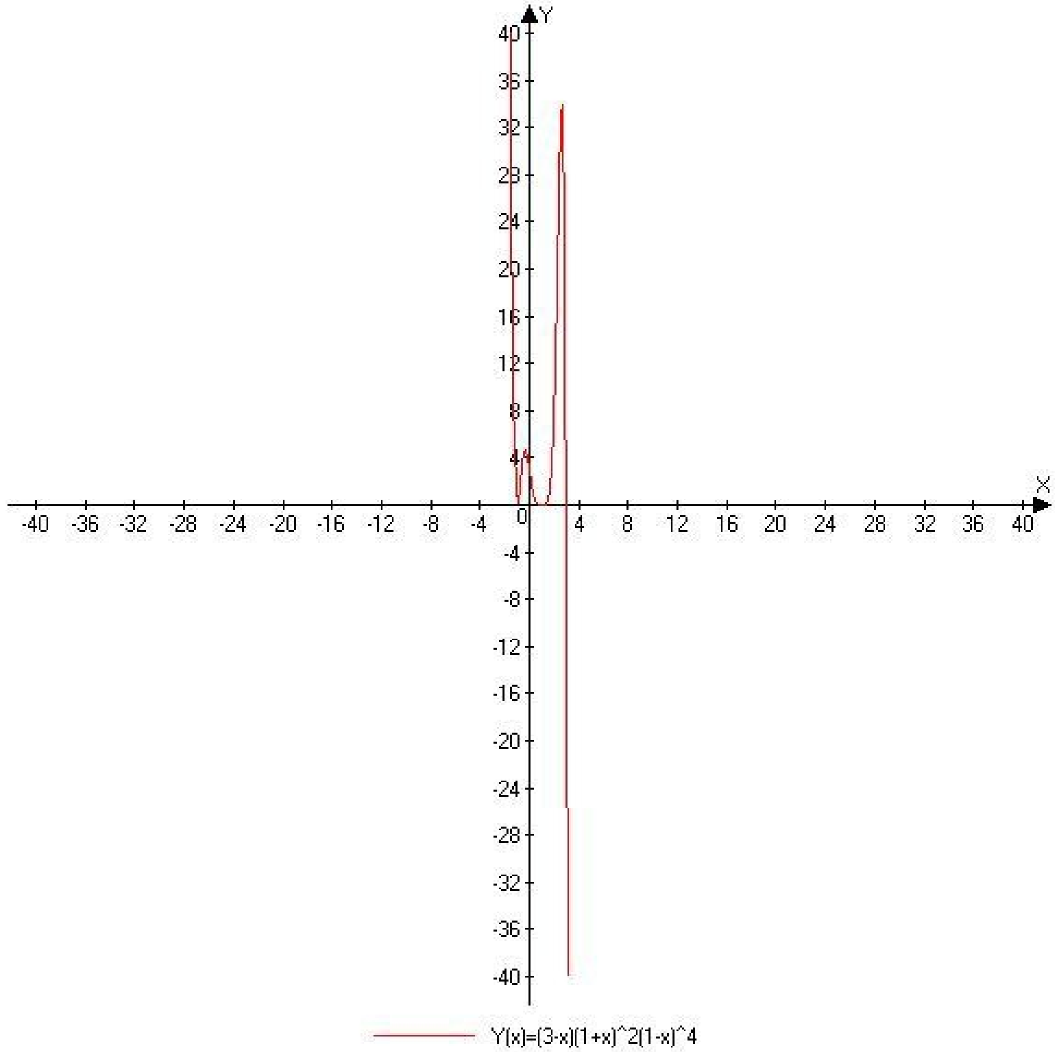
Substitute $x=0$ and solve for *y*:

$$\begin{aligned}y&=(0)^3(0+2)^2(0-1)\\ &=0\times4\times(-1)\\ &=0\end{aligned}$$

Consider the graph of the function as shown below:



f(x) =(3-x)(1+x)2 (1-x)4
we sketch the function first then find the assymptotes :



observe that as x approaches -1 from right side , the curve is becoming parallel to the positive part of y axis and as x approaches 4 from left side , the curve is becoming parallel to the negitive part of y -axis.

thus , we can write $\lim_{x\to-\infty} f(x) = \infty, \lim_{x\to\infty} f(x) = -\infty$

the function has vertical assymptotes.

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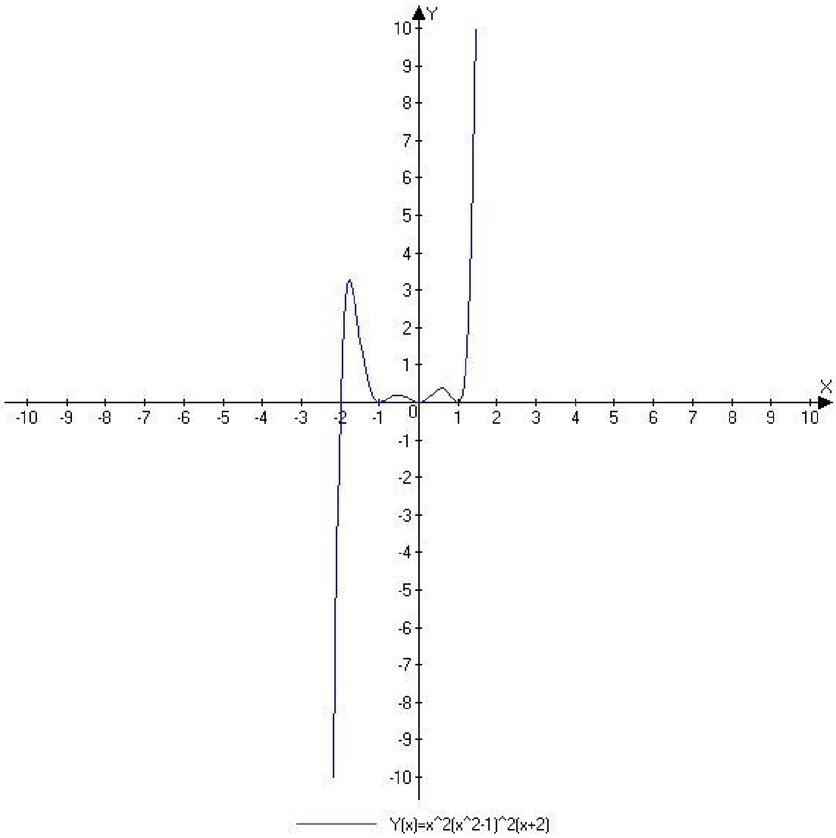
f(x) = x2 (x2-1)2(x+2)

we sketch the function to tell about the assymptotes .



observe that as x approaches -2.5 from right side , the curve is becoming parallel to negitive part of yaxis and as x approaches 2 from leftside, it is becoming parallel to positive part of y axis.

, we write $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

so, the given function has vertical assymptotes .

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First we collect the information's from the given conditions

$f'(2) = 0$ and $f(2) = -1$, $f(0) = 0$

So the graph has horizontal tangent at $x = 2$

$f'(x) < 0$ *if* $0 < x < 2$ Means graph is decreasing on the interval (0, 2) and

$f'(x) > 0$ *if* $x > 0$ means graph is increasing on the interval $(2, \infty)$

So at $x = 2$, $f(x)$ has a local minimum (2, -1)

$f''(x) < 0$ *if* $0 \le x < 1$ or if x > 4 ,means $f(x)$ is concave downward on (0, 1) and $(4, \infty)$

$f''(x) > 0$ *if* $1 < x < 4$ Means $f(x)$ is concave upward when 1 < x < 4 so 1 is the x-co-ordinates of the inflection point

$\lim_{x\to\infty} f(x) = 1$ Means horizontal asymptote is $\boxed{y = 1}$

$f(-x) = f(x)$ For all x means $f(x)$ is even function which is reflected about y axis

With the help of above information's we can draw the graph of the function $f(x)$

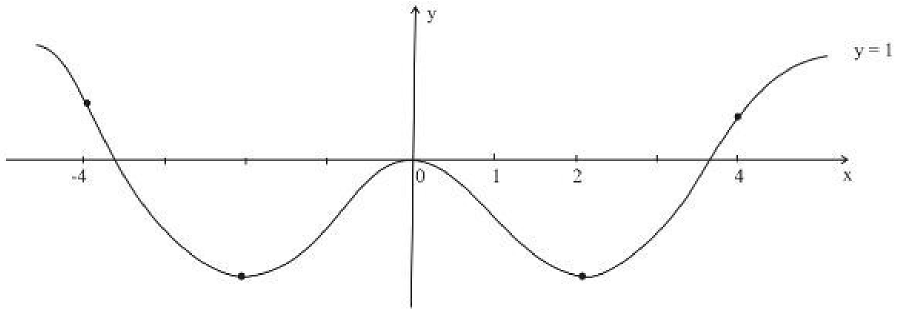


Fig. 1

Chapter 3 Applications of Differentiation Exercise 3.4 54E

First we collect the information's from the given conditions.

$f'(2) = 0$, $f'(0) = 1$ Means $f(x)$ has horizontal tangent at $x = 2$
and at $x = 0$, $f(x)$ has slope 1
$f'(x) > 0$ *if* $0 < x < 2$ Means $f(x)$ is increasing on (0, 2)
$f'(x) < 0$ *if* $x > 2$ Means $f(x)$ is decreasing on $(2, \infty)$
So $f(x)$ has local maximum at $x = 2$.

$f''(x) < 0$ *if* $0 < x < 4$ Means $f(x)$ is concave downward on the interval (0, 4)
$f''(x) > 0$ *if* $x > 4$ Means $f(x)$ is concave upward on $(4, \infty)$
So, at $x = 4$, $f(x)$ has an inflection point

$\lim_{x \to \infty} f(x) = 0$, means horizontal asymptote is $\boxed{y = 0}$

$f(-x) = -f(x)$ for all x, so $f(x)$ is an odd function and symmetric about the origin.

By these information's we draw the graph of $f(x)$

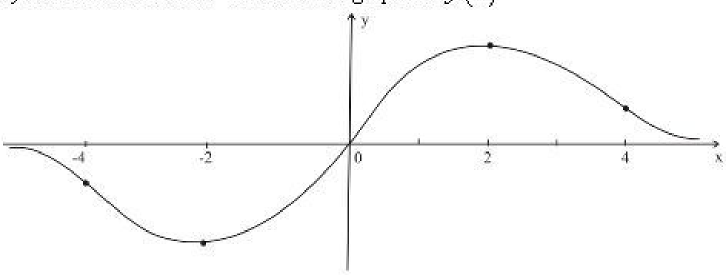


Fig. 1

Chapter 3 Applications of Differentiation Exercise 3.4 55E

First we collect the information's
$f(1) = f'(1) = 0$
$f(x)$ Has horizontal tangent at $x = 1$

$\lim_{x \to 2^-} f(x) = -\infty$ And $\lim_{x \to 2^+} f(x) = \infty$ means $f(x)$ has a vertical asymptote at $x = 2$

$\lim_{x \to 0} f(x) = -\infty$ Means $f(x)$ has a vertical asymptote at $x = 0$

$\lim_{x \to -\infty} f(x) = \infty$ Means for negative large value of x $f(x)$ is large positive

$\lim_{x \to \infty} f(x) = 0$ Means, $f(x)$ has a horizontal asymptote $y = 0$

$f''(x) > 0$ for $x > 2$ Means $f(x)$ is concave upward at $(2, \infty)$

$f''(x) < 0$ for $x < 0$ And $0 < x < 2$ means $f(x)$ is concave downward on $(-\infty, 0)$ and $(0, 2)$

We draw the graph of $f(x)$ with the help of above information's

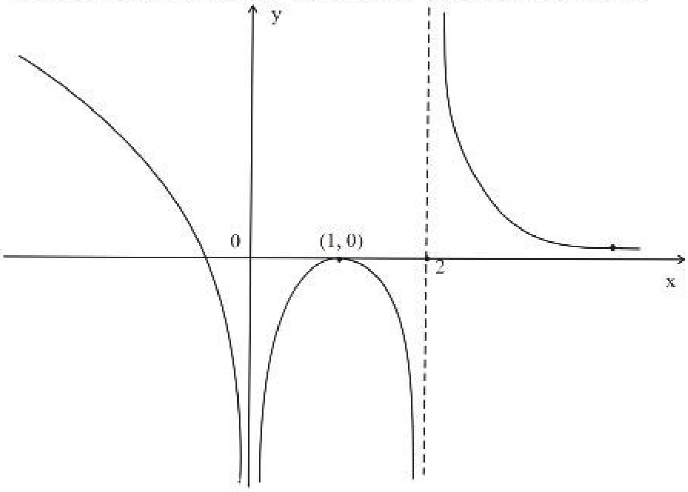


Fig. 1

Chapter 3 Applications of Differentiation Exercise 3.4 56E

First we collect the information's

$g(0) = 0$

$g''(x) < 0$ *for* $x \neq 0$ Means $g(x)$ is concave downward for all x except 0

$\lim_{x \to \infty} g(x) = -\infty$ Means $g(x)$ is large negative when x is large positive

$\lim_{x \to -\infty} g(x) = \infty$ Means $g(x)$ is large positive when x is large negative

$\lim_{x \to 0^-} g'(x) = -\infty$ And $\lim_{x \to 0^+} g'(x) = \infty$ means $g'(x)$ has a vertical asymptote at x = 0.

So $g(x)$ is not defined at x = 0 so $g(x)$ has a corner at x = 0

With the help of above information's we draw the rough sketch of graph of $g(x)$

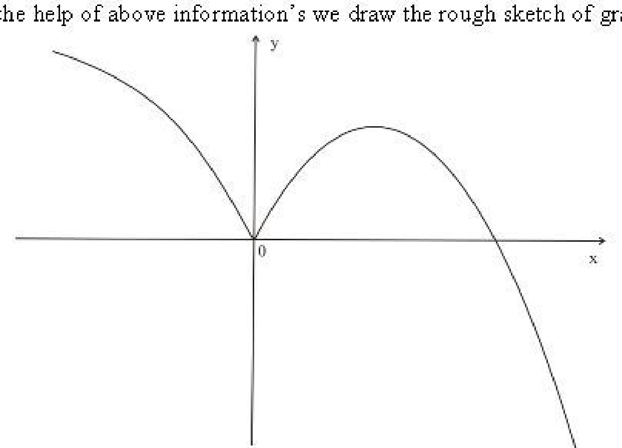


Fig. 1

Chapter 3 Applications of Differentiation Exercise 3.4 57

(A)

We can not use $\lim_{x\to\infty}\frac{\sin x}{x}=\lim_{x\to\infty}\frac{1}{x}.\lim_{x\to\infty}\sin x$

Because $\lim_{x\to\infty}\sin x$ does not exist

Since $-1\le\sin\le1$

Then $\frac{-1}{x}\le\frac{1}{x}\sin x\le\frac{1}{x}$

We know that $\lim_{x\to\infty}\left(-\frac{1}{x}\right)=\lim_{x\to\infty}\left(\frac{1}{x}\right)=0$

So by squeeze theorem if $f(x)\le g(x)\le h(x)$ when x is near ∞ and

$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}h(x)=L$ *then* $\lim_{x\to\infty}g(x)=L$

So here $\boxed{\lim_{x\to\infty}\frac{\sin x}{x}=0}$ so $\boxed{y=0}$ or x-axis is horizontal asymptote

(B) We draw the graph of $f(x)=\frac{\sin x}{x}$ and see that the graph crosses, so many times or an infinite number of times, the asymptote $\boxed{y=0}$

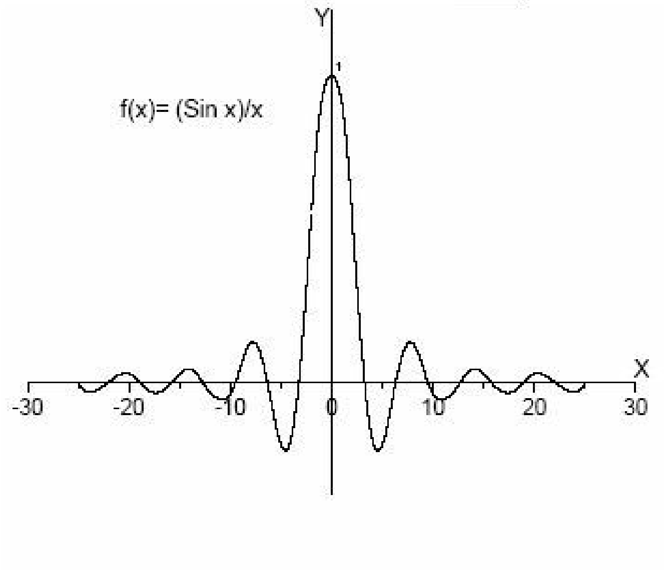


FIGURE - 1

Chapter 3 Applications of Differentiation Exercise 3.4 58E

(A)

By graphing the functions $P(x)=3x^5-5x^3+2x$ and $Q(x)=3x^5$ in the viewing rectangles [-2, 2] by [-2, 2] and [-10, 10] by [-10000, 10000], we see that end behavior of the functions are same because

$\lim_{x\to-\infty}P(x)=-\infty$ And $\lim_{x\to-\infty}Q(x)=-\infty$

And $\lim_{x\to\infty}P(x)=\infty$ and $\lim_{x\to\infty}Q(x)=\infty$

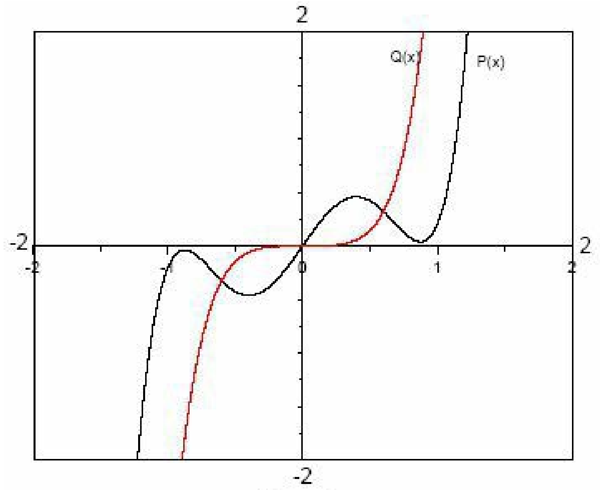


figure 1

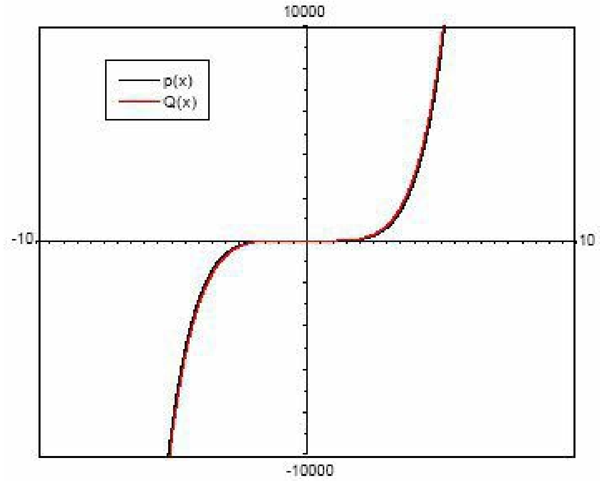


FIGURE - 2

(B) Ratio of the function is $\frac{P(x)}{Q(x)} = \frac{3x^5 - 5x^3 + 2x}{3x^5}$

Taking the limit as $x \to \infty$

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \lim_{x\to\infty} \frac{3x^5 - 5x^3 + 2x}{3x^5}$$

Divide the numerator and denominator by x^5
We have

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \lim_{x\to\infty} \frac{3 - \frac{5}{x^2} + \frac{2}{x^4}}{3}$$

By using limit laws

$$= \frac{\lim_{x\to\infty} 3 - 5\lim_{x\to\infty} \frac{1}{x^2} + 2\lim_{x\to\infty} \frac{1}{x^4}}{\lim_{x\to\infty} 3}$$

$$= \frac{3 - 0 + 0}{3} = 1$$

Thus $\lim_{x\to\infty} \frac{P(x)}{Q(x)} = 1$ So both the functions have same end behavior

Chapter 3 Applications of Differentiation Exercise 3.4 59E

(a) Let $P(x) = \pm x^n$ then $Q(x) = \pm x^{n+1}$

Since degree of P < degree of Q

Then

$$\frac{P(x)}{Q(x)} = \pm\frac{x^n}{x^{n+1}}$$

Taking the limit as $x \to \infty$

$$\lim_{x\to\infty}\frac{P(x)}{Q(x)} = \lim_{x\to\infty}\pm\frac{x^n}{x^{n+1}}$$

(a) Let $P(x) = \pm x^n$ then $Q(x) = \pm x^{n+1}$

Since degree of P < degree of Q

Then

$$\frac{P(x)}{Q(x)} = \pm\frac{x^n}{x^{n+1}}$$

Taking the limit as $x \to \infty$

$$\lim_{x\to\infty}\frac{P(x)}{Q(x)} = \lim_{x\to\infty}\pm\frac{x^n}{x^{n+1}}$$

(B)

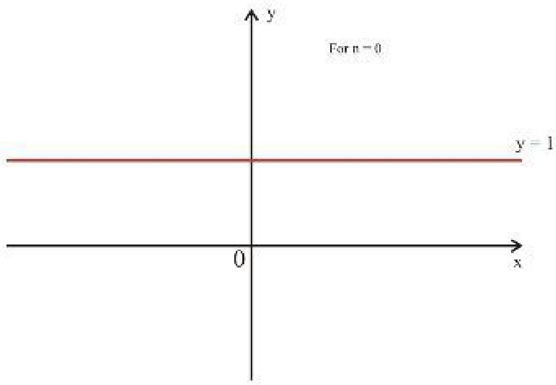
Chapter 3 Applications of Differentiation Exercise 3.4 60E

Given $y = x^n$

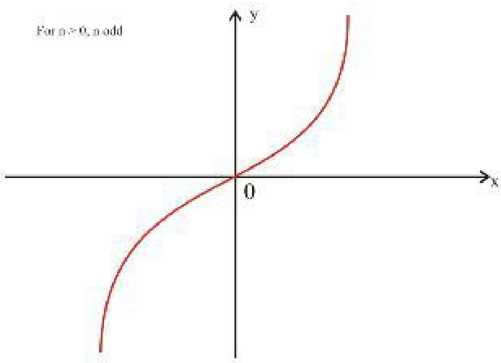
(I)

For n =0 $y = x^0 \Rightarrow y = 1$

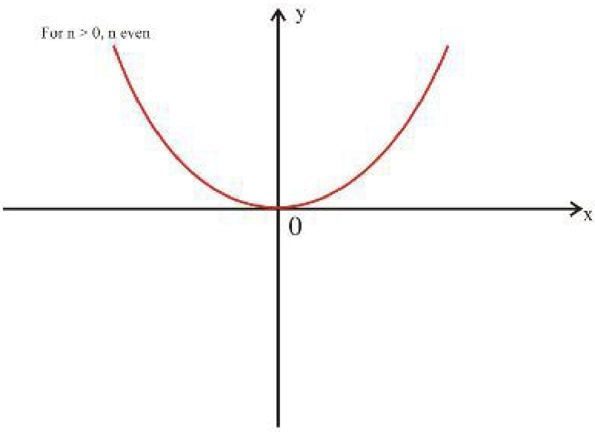
The graph will be



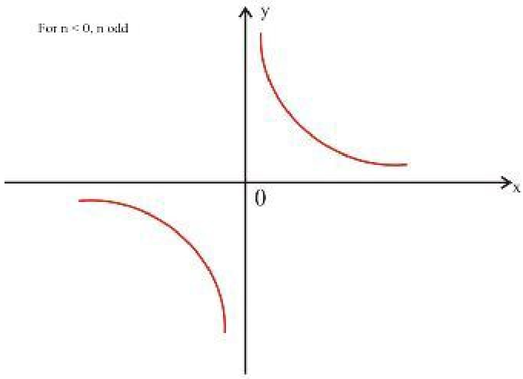
(II) For n>0, n odd



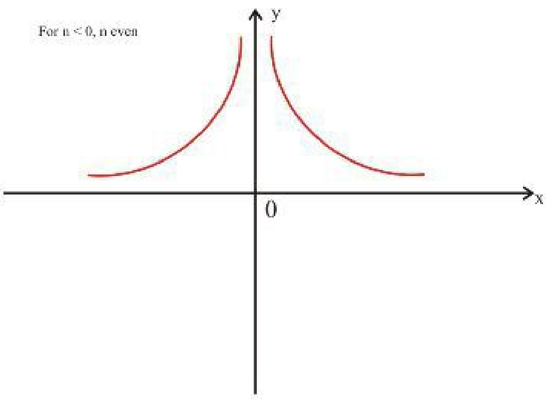
(III) For $n > 0$, n *even*



(IV) $n < 0$, n *odd*



(V) For n<0, n even



(A) (I) For $n = 0$, $\lim_{x \to 0^+} x^0 = 1$

(II) For n>0, n odd

(IV) For n<0, n odd

$$\lim_{x \to 0^+} x^n = +\infty$$

(V) For n<0, n even

$$\lim_{x \to 0^+} x^n = +\infty$$

(B) I) For $n = 0$, $\lim_{x \to 0^-} x^0 = 1$

(II) For n>0, n odd

$$\lim_{x \to 0^-} x^n = 0$$

(III) For n >0, n even

$$\lim_{x \to 0^-} x^n = 0$$

(IV) For n<0, n odd

$$\lim_{x \to 0^-} x^n = -\infty$$

(V) For n<0, n even

$$\lim_{x \to 0^-} x^n = +\infty$$

(C) I) For $n = 0$, $\lim_{x \to \infty} x^0 = 1$

Step 17 of

(II) For n>0, n odd

$$\lim x^n = \infty$$

(III) For n >0, n even

$$\lim_{x \to \infty} x^n = \infty$$

(III) For n >0, n even

$$\lim_{x \to \infty} x^n = \infty$$

(V) For n<0, n even

$$\lim_{x \to \infty} x^n = 0$$

(V) For n<0, n even

$$\lim_{x \to \infty} x^n = 0$$

(II) For n>0, n odd

$$\lim_{x \to -\infty} x^n = -\infty$$

(III) For n >0, n even

$$\lim_{x \to -\infty} x^n = \infty$$

(IV) For n<0, n odd

$$\lim_{x \to -\infty} x^n = 0$$

(V) For n<0, n even

$$\lim_{x \to -\infty} x^n = 0$$

Chapter 3 Applications of Differentiation Exercise 3.4 61E

We have $\frac{4x-1}{x} < f(x) < \frac{4x^2+3}{x^2}$

For all x > 5

We can use Squeeze theorem to get $\lim_{x \to \infty} f(x)$

$$\lim_{x\to\infty}\frac{4x-1}{x}=\lim_{x\to\infty}\frac{4-\frac{1}{x}}{1}$$
$$=\frac{\lim_{x\to\infty}4-\lim_{x\to\infty}\frac{1}{x}}{\lim_{x\to\infty}1}$$
$$=\frac{4-0}{1}=4$$

$$\lim_{x\to\infty}\frac{4x^2-3x}{x^2}=\lim_{x\to\infty}\frac{4-\frac{3}{x}}{1}$$
$$=\frac{\left(\lim_{x\to\infty}4-3\lim_{x\to\infty}\frac{1}{x}\right)}{\lim_{x\to\infty}1}$$
$$=\frac{(4-0)}{4}=4$$

We have $\lim_{x\to\infty}\frac{4x-1}{x}=\lim_{x\to\infty}\frac{4x^2-3x}{x^2}=4$

And $\frac{4x-1}{x}<f(x)<\frac{4x^2-3x}{x^2}$

So by the Squeeze theorem we can say that

$\boxed{\lim_{x\to\infty}f(x)=4}$ For all x > 5

Chapter 3 Applications of Differentiation Exercise 3.4 62E

(A) Initially tank contains 5000 L of pure water.
Brine is pumped into the tank at the rate of 25 L / min.
Brine pumped in t minutes, is 25t L
And amount of salt that contains 25t L of brine is = $30\times 25t$ grams
Now, total amount of liquid in the tank after t minutes is = $(5000+25t)$ L.
Thus, the concentration of salt after t minutes is

$$C(t)=\frac{\text{Amount of salt after t minutes}}{\text{Total liquid in the tank after t minutes}}$$
$$=\frac{30\times 25t}{5000+25t}$$
$$=\frac{25(30t)}{25(200+t)}$$
$$\boxed{C(t)=\frac{30t}{200+t}}\quad \text{Grams /Liter}$$

(B) Now, we have to calculate $\lim_{t\to\infty}C(t)$

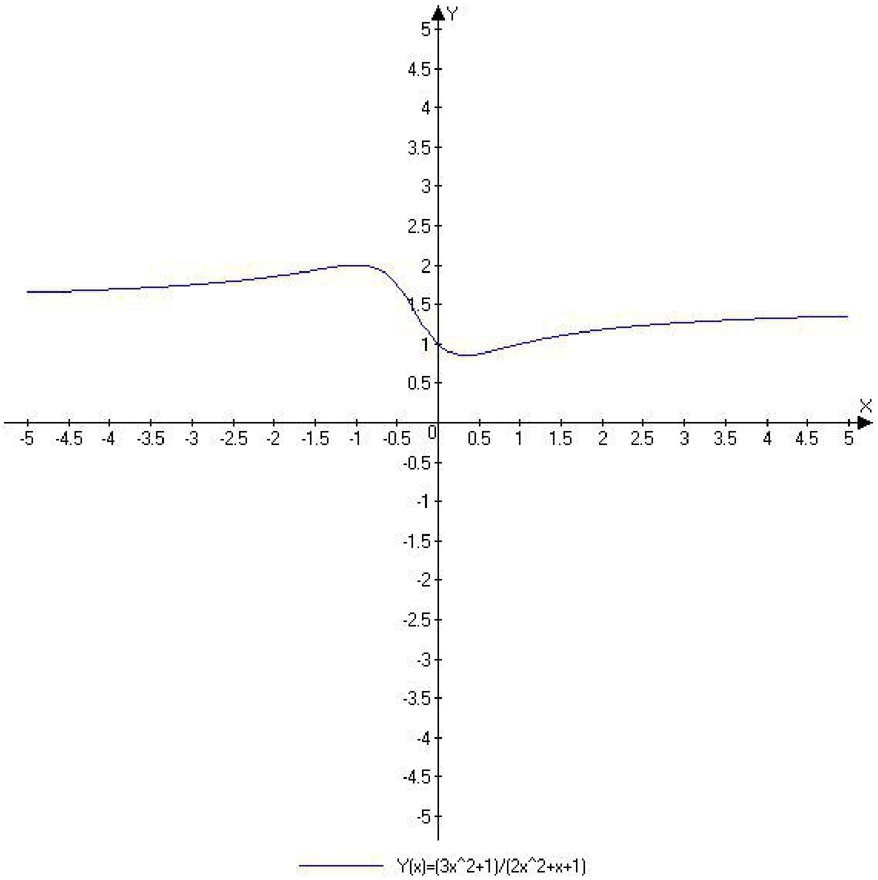
$$\lim_{t\to\infty}C(t)=\lim_{t\to\infty}\frac{30t}{200+t}$$
$$=\lim_{t\to\infty}\frac{30t}{t(200/t+1)}$$
$$=\lim_{t\to\infty}\frac{30}{\left(\frac{200}{t}+1\right)}$$
$$=\frac{30}{(0+1)}=30 \qquad \text{Since } \lim_{t\to\infty}\frac{200}{t}=0$$

Thus $C(t)\to 30$ g/L as $t\to\infty$.

Chapter 3 Applications of Differentiation Exercise 3.4 63E

$$f(x) = \frac{3x^2+1}{2x^2+x+1}$$

we sketch the graph and tell the number N such that for all x > N, | f(x) - 1.5| < 0.05.



observe that the curve reaches y = 0.7 and y = 2 but later it becomes parallel at y = 1.5.

so, the curve reaches y = 1.5 almost by a variation of 0.05 after x = 15.

but as x is tending to -∞ , we cannot justify after which stage the curve is becoming close

to y = 1.5.

∴ we write | f(x) - 1.5| < 0.05 for all x > N where N = 15.

Chapter 3 Applications of Differentiation Exercise 3.4 64E

We have $\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{x+1} = 2$

By the definition 5 we have, Let f be a function defined on some interval (a,∞)

then $\lim_{x\to\infty} f(x) = L$

Means for every $\in > 0$ there is a number N such that

$|f(x) - L| < \in$ Whenever x > N

So here we have

$\left|\frac{\sqrt{4x^2+1}}{x+1} - 2\right| < \in$ Whenever x > N

Now for $\in = 0.5$

We can rewrite the inequality

$$1.5 < \frac{\sqrt{4x^2+1}}{x+1} < 2.5 \text{ Whenever } x > N$$

We have to let the value of x for which the given curve lies between the horizontal lines y = 1.5 and y = 2.5. So we draw the curve and these lines, and use the cursor to estimate that the curve crosses the line y = 1.5 when $x \approx 2.8$. To the right of this number the curve lies between the lines y = 1.5 and y = 2.5

We have, $\left|\frac{\sqrt{4x^2+1}}{x+1} - 2\right| < 0.5$ Whenever x > 3

In other words for $\in < 0.5$ we can choose $\boxed{N \geq 3}$

For $\in = 0.1$

We can rewrite the inequality

$$1.9 < \frac{\sqrt{4x^2+1}}{x+1} < 2.1 \text{ Whenever } x > N$$

We have to get the value of x for which the given curve lies between the horizontal lines y = 1.9 and y = 2.1.So we draw the curve and these lines and use the cursor to estimate that the curve crosses the line y =1.9 when $x \approx 18.8$. To the right of this number the curve lies between the lines y = 1.9 and y = 2.1

We have, $\left|\frac{\sqrt{4x^2+1}}{x+1}\right| < 0.1$ Whenever x > 19

In other words for $\in = 0.1$ we can choose $\boxed{N \geq 19}$

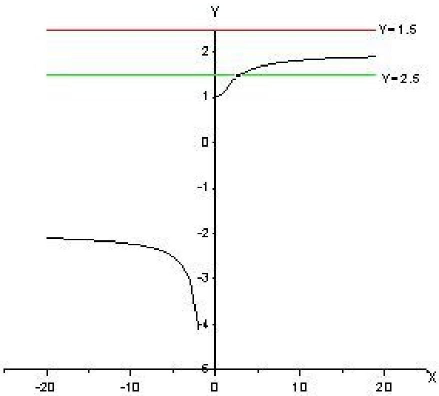


Figure -1

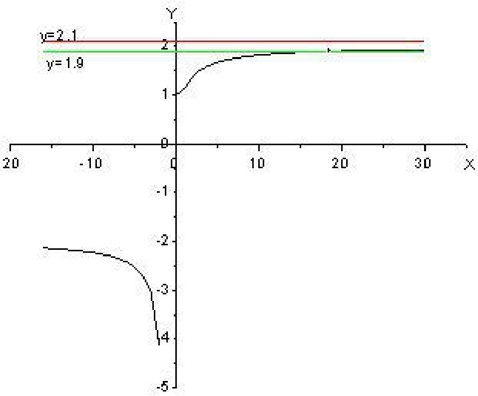


Figure -2

We have $\lim_{x\to-\infty} \frac{\sqrt{4x^2+1}}{x+1} = -2$

By the definition 6 we have, let f be a function defined on some interval $(-\infty, a)$

then $\lim_{x\to-\infty} f(x) = L$

Means for every $\in > 0$, there is a number N such that

$|f(x) - L| < \in$ Whenever x < N

So we have

$\left|\frac{\sqrt{4x^2+1}}{x+1} - (-2)\right| < \in$ Whenever x < N

We have $\in = 0.5$

We can rewrite the inequality

$$-2.5 < \frac{\sqrt{4x^2+1}}{x+1} < -1.5$$

We have to get the value of x for which the given curve lies between the horizontal lines y = -2.5 and y = -1.5. So we draw the curve and these lines .We use the cursor to estimate that the curve crosses the line y = -2.5 when $x \approx -5.2$. To the left of this point the curve lies between the lines y = -2.5 and y = -1.5.

We have, $\left|\frac{\sqrt{4x^2+1}}{x+1} + 2\right| < 0.5$ Whenever x < -6

So for $\in = 0.5$ we can choose $\boxed{N \le -6}$

For $\in = 0.1$

We can rewrite the inequality

$$-2.1 < \frac{\sqrt{4x^2+1}}{x+1} < -1.9 \text{ Whenever } x < N$$

We have to let the value of x for which the given curve lies between the horizontal lines y = -2.1 and y = -1.9. So we draw the curve and these lines and use the cursor to estimate that the curve crosses the line y = -2.1 when $x \approx -21.11$ .To the left of this number the curve lies between the lines y = -2.1 and y = -1.9

We have $\left|\frac{\sqrt{4x^2+1}}{x+1} + 2\right| < 0.1$ Whenever x < -22

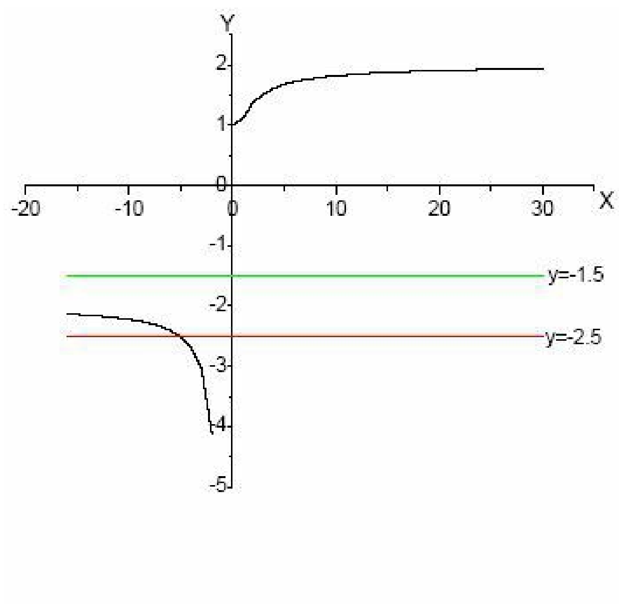


FIGURE – 1

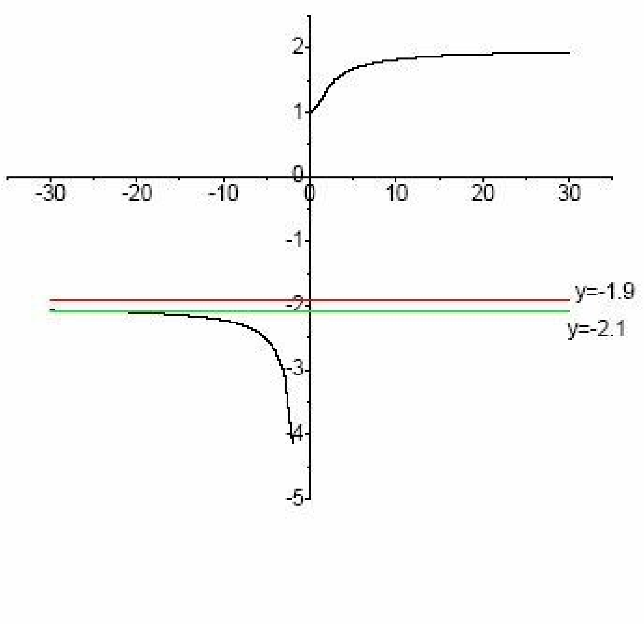


FIGURE - 2

Chapter 3 Applications of Differentiation Exercise 3.4 66E

We have $\lim_{x\to\infty} \frac{2x+1}{\sqrt{x+1}} = \infty$

By the definition 7 :-

Let f be a function defined on some interval (a,∞) then

$$\lim_{x\to\infty} f(x) = \infty$$

Means that for every positive number M there is a corresponding positive number N such that

$$f(x) > M \text{ Whenever } x > N$$

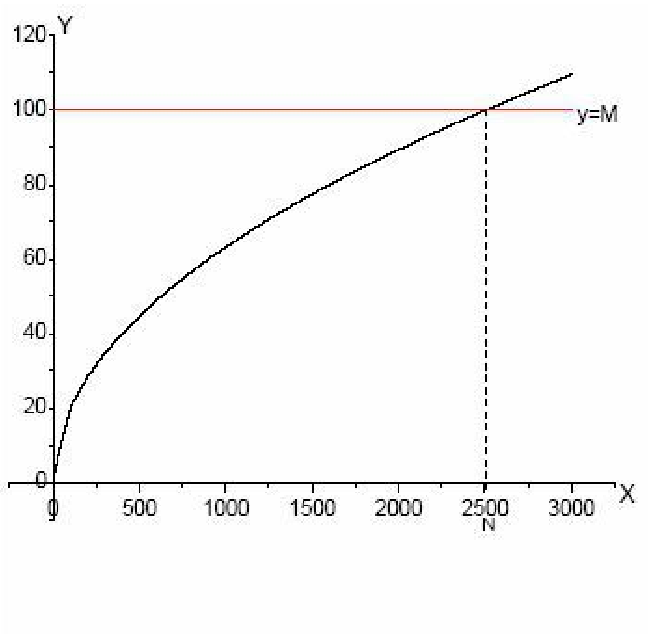


FIGURE - 1

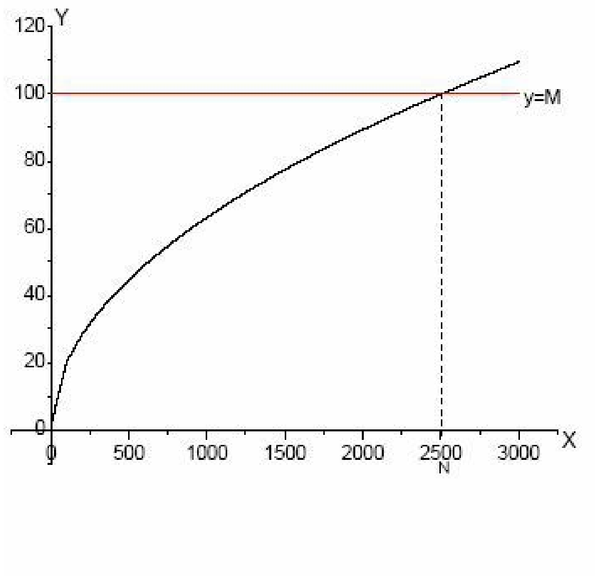


FIGURE - 1

Chapter 3 Applications of Differentiation Exercise 3.4 67E

(A) We have $\frac{1}{x^2} < 0.0001$

So $x^2 > \frac{1}{0.0001}$

Or $x^2 > 10000$

Or $|x| > |\sqrt{10000}|$

Or $|x| > |100|$

So $\boxed{x > 100}$

So we have to take $x > 100$ so that $\frac{1}{x^2} < 0.0001$

(B) Guessing a value for N: -

Given $\in > 0$

We have to find N such that $\left|\frac{1}{x^2} - 0\right| < \varepsilon$ whenever $x < N$

Let x > 0

In this case $\left|\frac{1}{x^2} - 0\right| = \left|\frac{1}{x^2}\right| = \frac{1}{x^2}$

So $\frac{1}{x^2} < \in$ Whenever $x > N$

Or $x^2 > \frac{1}{\in}$ Whenever $x > N$

That is $x > \frac{1}{\sqrt{\in}}$ whenever $x > N$

So we should take $N = \frac{1}{\sqrt{\in}}$

Showing that this N works: -

Given $\in > 0$,

We take $N = \frac{1}{\sqrt{\in}}$

Let $x > N$

Then $x^2 > N^2$

Or $\frac{1}{x^2} < \frac{1}{N^2}$

Now $\left|\frac{1}{x^2}-0\right|=\frac{1}{|x^2|}$

$=\frac{1}{x^2}<\frac{1}{N^2}=\in$

So $\left|\frac{1}{x^2}-0\right|<\frac{1}{N^2}$

Or $\left|\frac{1}{x^2}-0\right|<\frac{1}{\left(1/\sqrt{\in}\right)^2}$

Or $\left|\frac{1}{x^2}-0\right|<\in$ Whenever $x>$ N

Therefore by the definition 5

$$\boxed{\lim_{x\to\infty}\frac{1}{x^2}=0}$$

Chapter 3 Applications of Differentiation Exercise 3.4 68E

(A)

We have $\frac{1}{\sqrt{x}}<0.0001$

So $\sqrt{x}>\frac{1}{0.0001}$

Or $\sqrt{x}>10000$

Taking square of both sides

$$\boxed{x>100000000}$$

So we have to take $\boxed{x>100000000}$ so that $\frac{1}{\sqrt{x}}<0.0001$

(B) Guessing a value of N:

Given $\in>0$

We have to find N such that

$\left|\frac{1}{\sqrt{x}}-0\right|<\in$ Whenever x > N

Let x > 0, in which case

$\left|\frac{1}{\sqrt{x}}-0\right|=\left|\frac{1}{\sqrt{x}}\right|=\frac{1}{\sqrt{x}}<\in$

So $\frac{1}{\sqrt{x}}<\in$ Whenever x > N

Or $\sqrt{x}<\frac{1}{\in}$ Whenever x > N

That is $x<\frac{1}{\in^2}$ whenever x > N

So we should take $N=\frac{1}{\in^2}$

Showing that this N works

Given $\in>0$, we take $N=\frac{1}{\in^2}$ Let x > N

Then

$\sqrt{x}>\sqrt{N}$

Or $\frac{1}{\sqrt{x}}<\frac{1}{\sqrt{N}}$

Now $\left|\frac{1}{\sqrt{x}}-0\right|=\left|\frac{1}{\sqrt{x}}\right|=\frac{1}{\sqrt{x}}<\frac{1}{\sqrt{N}}$ Where x > 0

So $\left|\frac{1}{\sqrt{x}}-0\right|<\frac{1}{\sqrt{N}}$ whenever x > N

Or $\left|\frac{1}{\sqrt{x}}-0\right|<\frac{1}{\sqrt{1/\epsilon^2}}$ Whenever x > N

That is $\left|\frac{1}{\sqrt{x}}-0\right|<\epsilon$ whenever x > N

So by the definition

$$\boxed{\lim_{x\to\infty}\frac{1}{\sqrt{x}}=0}$$

Chapter 3 Applications of Differentiation Exercise 3.4 69E

Guessing the value of N: -
Given $\epsilon>0$
We have to find a number N such that

$$\left|\frac{1}{x}-0\right|<\epsilon \qquad \text{Whenever } x<N$$

Let x < 0 in this case

$$\left|\frac{1}{x}-0\right|=\left|\frac{1}{x}\right|$$
$$=-\frac{1}{x}$$

So we have

$$-\frac{1}{x}<\epsilon \qquad \text{Whenever } x<N$$

Or $\frac{1}{x}>-\epsilon$ Whenever $x<N$

That is $x<-\frac{1}{\epsilon}$ whenever $x<N$

So we must choose $N=-\frac{1}{\epsilon}$

Showing that this N works
Given $\epsilon>0$
We take $N=-\frac{1}{\epsilon}$

Let $x<N$ or $-x>-N$ that is $-\frac{1}{x}<-\frac{1}{N}$

If x < 0 then $\left|\frac{1}{x}-0\right|=\left|\frac{1}{x}\right|$

$$=-\frac{1}{x}<-\frac{1}{N}$$

So $\left|\frac{1}{x}-0\right|<-\frac{1}{N}$

Or $\left|\frac{1}{x}-0\right|<-\frac{1}{\left(-1/\epsilon\right)}=\epsilon$

So $\left|\frac{1}{x}-0\right|<\epsilon$

Then by the definition 6 we have

$$\boxed{\lim_{x\to-\infty}\frac{1}{x}=0}$$

Chapter 3 Applications of Differentiation Exercise 3.4 70E

We have to prove $\lim_{x\to\infty} x^3 = \infty$

Let $f(x) = x^3$

Let M be any positive number such that

$f(x) > M$

i.e. $x^3 > M$

We choose $N = \sqrt[3]{M}$ (N > 0)

whenever x > N

i.e. $x > \sqrt[3]{M}$

i.e. $x^3 > M$

i.e. $f(x) > M$

Hence we find that for every positive number M, there corresponds a positive number N such that $f(x) > M$ Whenever x > N

Hence using the definition (7) we find that

$\lim_{x\to\infty} f(x) = \infty$

i.e. $\lim_{x\to\infty} x^3 = \infty$

Chapter 3 Applications of Differentiation Exercise 3.4 71E

Let $\lim_{t\to 0^+} f\left(\frac{1}{t}\right) = L$ (1)

Then by the definition of limit for every $\in > 0$, there is a positive number δ such that

$\left|f\left(\frac{1}{t}\right) - L\right| < \in$ Whenever $0 < t < 0 + \delta$

Or $\left|f\left(\frac{1}{t}\right) - L\right| < \in$ Whenever $0 < t < \delta$

Let $\frac{1}{t} = x$ and $\frac{1}{\delta} = N$

Then $|f(x) - L| < \in$ whenever $0 < \frac{1}{x} < \delta$

Or $|f(x) - L| < \in$ whenever $0 > \frac{1}{\delta} = N$

That is $|f(x) - L| < \in$ whenever x > N

So by the definition 5 we have

$\lim_{x\to\infty} f(x) = L$ (2)

From (1) and (2) we have

$$\boxed{\lim_{x\to\infty} f(x) = \lim_{t\to 0^+} f\left(\frac{1}{t}\right)}$$

Let $\lim_{t\to 0^-} f\left(\frac{1}{t}\right) = L$ (1)

Then by the definition of limit for $\in > 0$, there is a number $\delta > 0$ such that

$\left|f\left(\frac{1}{t}\right) - L\right| < \in$ *whenever* $0 - \delta < t < 0$

Or $\left|f\left(\frac{1}{t}\right) - L\right| < \in$ *whenever* $-\delta < t < 0$

Let $x = \frac{1}{t}$ and $-\frac{1}{\delta} = N$

Then $|f(x) - L| < \in$ whenever $-\delta < \frac{1}{x} < 0$

Or $|f(x) - L| < \in$ whenever $\frac{1}{x} < -\delta$

Or $|f(x) - L| < \in$ whenever $x < -\frac{1}{\delta}$

That is $|f(x) - L| < \in$ whenever $x < N$

So by the definition 6 we have

$$\lim_{x \to -\infty} f(x) = L \qquad (2)$$

From (1) and (2) we have

$$\boxed{\lim_{x \to -\infty} f(x) = \lim_{t \to 0^-} f\left(\frac{1}{t}\right)}$$

Chapter 3 Applications of Differentiation Exercise 3.4 72E

Let $f(x)$ be a function defined on some interval $(-\infty, a)$ then

$$\lim_{x \to -\infty} f(x) = -\infty$$

Means for every negative number M there is a corresponding negative number N such that

$$f(x) < M \text{ when ever } x < N$$

For every negative number M there is corresponding number N such that

$$(1 + x^3) < M \text{ when ever } x < N$$

Now, to guess a value of N: -

We have

$$1 + x^3 < M \text{ when ever } x < N$$

$$\Rightarrow x^3 < M - 1 \text{ when ever } x < N$$

Or $x < \sqrt[3]{M - 1}$ when ever $x < N$

So we should choose $N = \sqrt[3]{M - 1}$