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Introduction to Signals

Signals

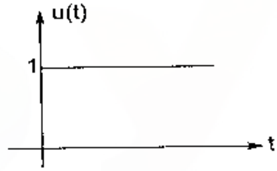
A signal can be defined as a function of one or more independent variable, which conveys information about the behaviour or nature of some phenomenon.

Elementary Signals

1. **Unit step function**

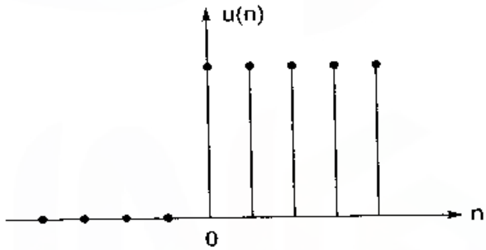
(a) For continuous time

$$u(t) = \begin{cases} 1 & ; \quad t > 0 \\ 0 & ; \quad t < 0 \end{cases}$$



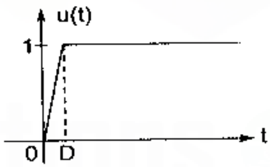
(b) For discrete-time

$$u[n] = \begin{cases} 1 & ; \quad n \geq 0 \\ 0 & ; \quad n < 0 \end{cases}$$



Remember:

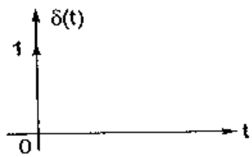
- Extension of wire result into increase in resistance while compression of wire result into decrease in resistance.
- Mathematically $u(0) = \frac{1}{2}$; average value



2. **Unit impulse function**

(a) For continuous time

$$\delta(t) = \begin{cases} \infty & ; \quad t = 0 \\ 0 & ; \quad t \neq 0 \end{cases}$$



Properties of the impulse function

(i) Impulse function is a continuous function and the area under this function is equal to one

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

(ii) Even function of time

$$\delta(-t) = \delta(t)$$

(iii) $$\delta(at) = \frac{1}{|a|}\delta(t)$$

(iv) Product :

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0);\text{where, } t_0 \text{ is time shift}$$

(v) Sampling property :

$$\int_{t_1}^{t_2} x(t)\delta(t-t_0) = x(t_0)\ ; \quad t_1 < t_0 < t_2$$

(vi) $$\int_{-\infty}^{\infty} \frac{d\delta(t)}{dt}x(t)dt = \left.\frac{dx(t)}{dt}\right|_{t=0}$$

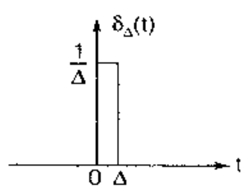
Relationship between unit impulse and unit step function

$$\boxed{u(t) = \int_{-\infty}^{t} \delta(t)\,dt} \text{ and } \boxed{\delta(t) = \frac{du(t)}{dt}}$$

Remember: ....

- As the unit step function is neither continuous nor differentiable at t = 0. The unit impulse function is defined as

$$\boxed{\delta_\Delta(t) = \frac{du_\Delta(t)}{dt}}$$



- $\delta(t)$ is the limit as $\Delta \to 0$ of $\delta_\Delta(t)$

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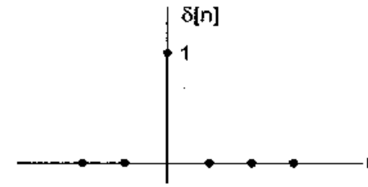
(b) For discrete-time

$$\delta[n] = \begin{cases} 1 & ; \quad n = 0 \\ 0 & ; \quad n \neq 0 \end{cases}$$



Remember: ....

- The discrete-time unit impulse is the first difference of the discrete-time unit step

$$\boxed{\delta[n] = u[n] - u[n-1]}$$

- Relationship between unit impulse and unit step

$$u[n] = \sum_{K=0}^{\infty} \delta[n-k]$$

- Sampling property

$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$

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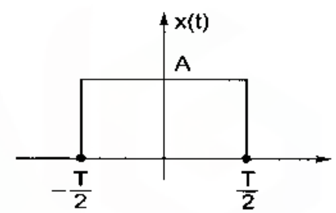
3. Rectangular or Gate function

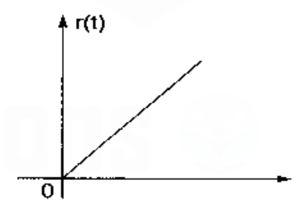
$$x(t) = \begin{cases} A & ; \quad -T/2 < t < T/2 \\ 0 & ; \quad \text{otherwise} \end{cases}$$



4. Unit Ramp function

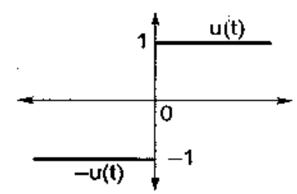
$$\boxed{r(t) = \begin{cases} t\ ; & t > 0 \\ 0\ ; & t < 0 \end{cases}}$$

$$\boxed{\frac{d}{dt}r(t) = u(t)}$$



5. Signum or sgn function

$$\boxed{sgn(t) = 2u(t) - 1}$$



Note:

- $u(-t) = 1 - u(t)$
- Sgn function is not defined at $t = 0$ and is chosen as 0 at $t = 0$

6. Sinc and sinc² function

- The sinc and sinc² functions are defined in terms of an independent variable λ.

$$\text{sinc}(\lambda) = \frac{\sin(\pi\lambda)}{\pi\lambda}$$

- Sinc (λ) is equal to zero for $\lambda = \pm n$ ($n \neq 0$), n an integer.

7. Sine integral function

- Sine integral function is an odd function

$$Si(y) = \int_0^y \frac{\sin(\alpha)}{\alpha} d\alpha \;;\quad Si(y) = \frac{y}{(1)!1} - \frac{y^3}{(3)3!} + \frac{y^5}{(5)5!} - \frac{y^7}{(7)7!} + \cdots$$

Remember:

- $Si(a) = 0,\ Si(\pi) \cong 2.0123,\ Si(\infty) = \left(\frac{\pi}{2}\right)$
- Si function converges fast and only a few terms in above equation are needed for a good approximation

Operators

1. Time scaling

For analog signals

Let x(t) be an arbitrary signal, a time scaled version of x(t) is obtained by replacing 't' by 'at' where 'a' is scaling factor.

$$\varphi(t) = x(at)$$

- $a > 1$ shows compression of x(t).
- $0 < a < 1$ shows expansion of x(t).

For discrete signals

For a discrete time sequence x[n], compression of a signal by factor M is given by

$$\varphi[n] = x[Mn]\ ;\ \text{M and n both are integers}$$

2. Time Shifting

For analog signals

Shifting in time may results in time delay or time advancement.

For a continuous-time signals x(t), time shifting is given as

$\varphi(t) = x(t - t_0)$ ··· delay or shift right by 't_0'.

$\varphi(t) = x(t + t_0)$ ··· advance or shift left by 't_0'.

For discrete signals

For a discrete time sequence x[n] time shifting is given as

$\varphi[n] = x[n - n_0]$ ··· delay or shift right by n_0 samples.

$\varphi[n] = x[n + n_0]$ ··· advance or shift left by n_0 samples.

3. Time Reversal

For analog signals

Time reversal x(t) is achieved by rotation of signal 180° about vertical axis. This operation is also called as folding or reflection about vertical axis.

For a continuous-time signals x(t), time reversal is given as

$$\varphi(t) = x(-t)$$

For discrete signals

For discrete time sequence x[n], time reversal is given as

$$\varphi[n] = x[-n]$$

Note:

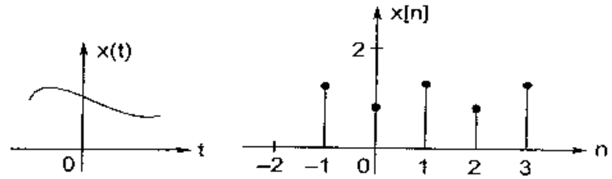
- In priority order: Shifting > Scaling > Reversal
- There is no effect of scaling on unit step signal.
- The time scaling on ramp signal will result into magnitude scaling as

$$r(at) \longrightarrow ar(t)$$

Classification of Signals

1. Continuous time and discrete time signals

x(t) is a continuous-time signal if 't' is a continuous variable. But if 't' is a discrete variable that is x(t) is defined at discrete times, then x(t) is a discrete-time signal.



(a) Continuous-time (b) Discrete-time

Note:

A discrete-time signal x[n] may be obtained by sampling a continuous-time signal x(t).

2. Analog and digital signals

If a continuous-time signal x(t) can take on any value in the continuous interval (a, b) where 'a' may be $-\infty$ and 'b' may be $+\infty$, then the continuous time signal x(t) is called an analog signal. If a discrete-time signal x[n] can take on only a finite number of distinct value, then this signal is called a digital signal.

3. Real and complex signal

A signal x(t) is a real signal if its value is a real number and a signal x(t) is a complex signal if its value is a complex number.

4. Deterministic and Random signals

If x(t) can be perfectly known for any time 't' then it is called deterministic signal. If x(t) can not be exactly determined at any given time then it is called Random signal.

5. Even and Odd signals

A signal x(t) or x[n] is an even signal

if $$x(-t) = x(t)$$
$$x[-n] = x[n]$$

A signal x(t) or x[n] is referred to as an odd signal

if $$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$

Note:

- The product of two even signals or of two odd signals is an even signal.
- The product of an even signal and an odd signal is an odd signal.

6. Energy and Power signals

(a) Energy of the signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

(b) Average power of a signal

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Note:

- An energy signal has always zero average power.
- A power signal has infinite energy
- A signal maintain constant amplitude for all time is a power signal.
- If any signal is power signal for some time and it is energy signal for some other time then resultant signal is power signal.
- As $t \to \pm\infty$, if amplitude tends to ∞, it is neither energy nor power signal.
- All finite duration and bounded signals are energy signal.
- Energy of a signal is only affected by scaling operation as

$$E(at) \longrightarrow \frac{E(t)}{a}$$

7. Periodic and Non Periodic signals

A continuous time signal x(t) is said to be periodic with period T if there is a positive non zero value of T for which

$$x(t + T) = x(t); \quad \forall t$$

In discrete-time signal, a sequence x[n] is periodic with period N, if there is a positive integer N for which

$$x[n + N] = x[n]; \quad \forall n$$

Note:

- The fundamental period for analog signal is

$$T_0 = \frac{2\pi}{\omega_0}$$

- For discrete signal

$$\frac{N}{K} = \frac{2\pi}{\omega_0}; \quad \text{where } K = 0, 1, 2, 3 \cdots$$

- The fundamental period T_0 of x(t) and N of x[n] is the smallest positive integer for which above equation holds good.
- Sum of two continuous-time periodic signals may not be periodic but the sum of two periodic sequences is always periodic.

For periodic signal $x_T(t)$

- Average value of the signal

$$x_{avg} = \frac{1}{T}\int_T x_T(t)dt$$

- Average signal power

$$P_x = \frac{1}{T}\int_T |x_T|^2 dt$$

- Effective or rms value of the signal

$$x_{rms} = \sqrt{P_x}$$

Note:

- If $x_{T_1}(t)$ and $x_{T_2}(t)$ are two periodic functions with periods T_1 and T_2, then

$x(t) = x_{T_1}(t) + x_{T_2}(t)$ is periodic with period T if

$$T = nT_1 = mT_2 \text{ or } \left[\frac{T_1}{T_2}\right] = \left[\frac{m}{n}\right]$$

where m and n are integers and $\left(\frac{T_1}{T_2}\right)$ is a rotational number.

- The period of x(t) is equal the least common multiple (LCM) of T_1 and T_2. The LCM of two integers m and n; is the smallest integer divisible by both m and n.

Symmetries of periodic function

Half-wave symmetry

A periodic function x(t) is half-wave symmetric if

$$x_T(t) = -x_T\left(t - \frac{T}{2}\right)$$

where, T = Period of signal $x_T(t)$

- For even half-wave symmetry

$$x_T(t) = \begin{cases} x_T(-t) = x_T(t) \\ x_T(t) = -x_T\left(t + \frac{T}{2}\right) \end{cases} ; x_T(t+T) = x_T(t)$$

- For odd half-wave symmetry

$$x_T(t) = \begin{cases} x_T(-t) = -x_T(t) \\ x_T(t) = -x_T\left(t + \frac{T}{2}\right) \end{cases} ; x_T(t+T) = x_T(t)$$

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