Electromagnetics Test I

Number of Questions: 35 **Time: 90 min.**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. Find the Laplacian of V where $V = \rho^2\cos 2\phi$

(A) $4\cos 2\phi$ (B) $-4\rho\cos 2\phi$
(C) $8\cos 2\phi$ (D) 0

2. An electric potential field is produced in air by point charges $2\mu C$ and $3\mu C$ which are located at (–1, 1, 3) and (2, –1, 4) respectively. The energy stored in the field is

(A) 14mJ (B) 4mJ
(C) 12mJ (D) 3.2mJ

3. If electric flux density is $3y^2z\hat{a}_x + 3x^2yz\hat{a}_y + y^2\hat{a}_z C/m^2$, then find the total charge enclosed with in the region $0 < x, y, z < 2$.

(A) 24 (B) 64
(C) $\frac{64}{3}$ (D) 32

4. For a conductor conductivity (σ) is 5×10^6s/m, and relative permittivity is 3 then Relaxation time is

(A) 1.76 psec (B) 5.3×10^{-18} sec
(C) 5.65 psec (D) 6 psec

5. A parallel plate capacitor has 2pF capacitance and voltage $70\sin 10^4 t$ V applied to plates. Then the displacement current is

(A) 70 mA (B) $1.4\cos 10^4 t$ μ A
(C) $1.4\sin 10^4 t$ nA (D) $\sin 10^4 t$ mA

6. In a nonmagnetic medium $E = 5\cos(2\pi \times 10^8 t - 10x)$ $\hat{a}_z$ V/m then time average power carried by the wave is

(A) 147 mW/ m^2 (B) 158 mW/m^2
(C) 166 mW/m^2 (D) Zero

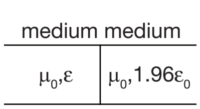
7. An electromagnetic wave is travelling in free space with electric field component $E_s = 5e^{j(0.95x+0.8z)}\hat{a}_y$ V/m, then determine ω?

(A) 3.72 GHz (B) 273 MHz
(C) 723 GHz (D) 372 MHz

8. A distortionless line has $Z_0 = 50\Omega$, $\alpha = 15mN_p/m$ then find resistance of line.

(A) 0.75 Ω/m (B) 3.33 Ω/m
(C) 0.5 Ω/m (D) 1.33 Ω/m

9. A wave is travelling from medium 1 to medium 2 as shown in below figure. Then the value of standing wave ratio is



(A) 7/6 (B) 1.4
(C) 6 (D) 2.4

10. A Quarter wave transformer can be used for matching accurately

(A) inductive loads only
(B) capacitive loads only
(C) loads with any nature of reactance
(D) purely resistive loads only

11. A copper wire carries a conduction current $2A$ at 60 KHz and copper wire has $\varepsilon = 2\varepsilon_0$, $\mu = \mu_0$, $\sigma = 10^{-2}S/m$. What is the displacement current in the wire?

(A) 1.33 mA (B) 3.7 mA
(C) 2.6 μA (D) 2 mA

12. The magnetic flux density B and the vector magnetic potential A are related as

(A) $B = \nabla \times A$ (B) $A = \nabla \times B$
(C) $B = \nabla . A$ (D) $A = \nabla . B$

13. If the $\vec{E}$ field of a plane polarized electromagnetic wave travelling in the y-direction is $\vec{E} = \vec{a}_x E_x + \vec{a}_z E_z$, then its H field is [Z_0 is intrinsic impedance]

(A) $\frac{E_x}{Z_0}\hat{a}_x + \frac{E_z}{Z_0}\hat{a}_z$ (B) $\frac{E_x}{Z_0}\hat{a}_x - \frac{E_z}{Z_0}\hat{a}_z$
(C) $\frac{E_z}{Z_0}\hat{a}_x - \frac{E_x}{Z_0}\hat{a}_z$ (D) $\frac{E_z}{Z_0}\hat{a}_x + \frac{E_x}{Z_0}\hat{a}_z$

14. A parabolic antenna is operating at mid frequency of X – band, if frequency is now shifted to S – band mid frequency. Determine decrease in gain.

(A) $\frac{3}{10}$ (B) $\frac{10}{3}$
(C) $\frac{100}{9}$ (D) 9/100

15. A 50Ω line is terminated with a load impedance of 75Ω. Find the percentage of transmitted power.

(A) 20% (B) 4%
(C) 96% (D) 80%

16. If $S(r) = 10z(\rho\,\hat{a}\rho + \hat{a}_\varphi)$, determine the flux of S out of the entire surface of cylinder $\rho = 2, 0 \le Z \le 2$

(A) 80π (B) 40
(C) 80 (D) 160π

17. Point charges Q_1 and Q_2 are located at (3, 0, –2) and (4, 0, –3) respectively. If Q_2 is 5nC. Find the value of Q_1 when the field E at (5, 0, 6) has no 'X' component.

(A) 2 μ C (B) –1.9 nC
(C) –3.3 μ (D) 3.3 nC

18. In a one – dimensional device change density is given by $\rho_v = \frac{5x^2}{a}$. If $E = 0$ at $x = 0$ and $V = 0$ at $x = a$ then E is

(A) $\frac{+5}{12a\varepsilon_0}\left(a^4 - x^4\right)$ (B) $\frac{5}{12\varepsilon_0}\left(a^3 - x^3\right)$

(C) $\frac{12}{5\varepsilon_0 a}\left(a^4 - x^4\right)$ (D) $\frac{12}{5\varepsilon_0}\left(a^3 - x^3\right)$

19. A lossy dielectric has an intrinsic impedance of $70\angle 60°\Omega$ at a particular ω. If at that frequency, the electric field component of a wave is $20.e^{-\alpha x}\cos(\omega t - 3x)\,\hat{a}_y$ V/m then find skin depth?

(A) $-\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$ (D) $\frac{-1}{\sqrt{3}}$

20. In a free space, a plane wave with $E_i = 8\cos(10^7 t - \beta z)\,\hat{a}_x$ V/m is incident normally on a loss less medium ($\varepsilon = 2\varepsilon_0$, $\mu = 8\mu_0$) in region $Z \geq 0$. Find transmitted wave H_t.

(A) $14.14\cos\left(10^7 t - \beta z\right)\hat{a}_y$ mA/m

(B) $7\cos\left(10^7 t + \beta z\right)\hat{a}_y$ mA/m

(C) $14\cos\left(10^7 t - \beta z\right)\hat{a}_z$ A/m

(D) $7\cos\left(10^7 t - \beta z\right)\hat{a}_z$ A/m

21. In an air filled rectangular wave guide is operating at 6GHz and it's electric field component is in the form of, $E_x = 4\sin\left(\frac{3\pi x}{a}\right)\cos\left(\frac{2\pi y}{b}\right)\sin\left(\omega t - 10z\right)$ V/m, then find the cut – off frequency by

(A) 5.98 GHz (B) 5 GHz

(C) 7GHz (D) data is insufficient

22. A rectangular wave guide 1.5 cm × 0.75 cm is placed in air and is being operated at 25 GHz in its dominant mode of propagation. Total power fed into wave guide is 150 watts. Determine the maximum value of electric field Intensity?

(A) 3.3 KV/m (B) 3.17 KV/m

(C) 3.5 KV/m (D) 3.4 KV/m

23. The homogenous material inside a co-axial capacitor has the parameter $\varepsilon_r = 2.5$, $\mu_r = 10$ $\sigma = 10^{-6}$S/m. Given electric field intensity $E = \frac{10^6}{\rho}\sin\left(10^4 t\right)\hat{a}_\rho$ V/m. Then find Quality factor of the capacitor?

(A) 0.1 (B) 1.1

(C) 0.22 (D) 0.8

24. Consider the region defined by $|x|,|y|,|z| < 1$. Let $\varepsilon = 3\varepsilon_0$, $\mu = 4\mu_0$ and $\sigma = 0$, the displacement current density J_d equals to $15\cos(2 \times 10^8 t - bx)\,\hat{a}_y$ μA/m². Assume no *D*. *C* fields are present. Calculate value of electric field intensity (*E*)

(A) $8.47\sin(2 \times 10^8 t - bx)$ mV/m $\hat{a}_z$

(B) $3\cos(2 \times 10^8 t - bx)$ mV/m $\hat{a}_x$

(C) $6.2\text{Cos}(2 \times 10^8 t - bx)$ mV/m $\hat{a}_z$

(D) $2.82\sin(2 \times 10^8 t - bx)$ mV/m $\hat{a}_y$

25. Magnetic field component of a plane wave in a lossless dielectric ($\mu_r = 1$) is $H = 15\sin(2\pi \times 10^8 t - 3x)\,\hat{a}_z$ mA/m, then find maximum magnitude of displacement of current density.

(A) 0.023 (B) 0.032

(C) 0.141 (D) 0.047

26. For a given two dielectric media, medium 1 is free space and medium 2 has $\varepsilon_2 = 9\varepsilon_0$. Find the reflection co-efficient for oblique incidence $\theta = 45°$ for parallel polarization.

(A) 0.73 (B) 0.37

(C) 0.5 (D) 0.25

27. For taking antenna far field pattern, what must be the distance *R*, between transmitting and receiving antennas?

(A) $R > \frac{2D^2}{\lambda^2}$ (B) $R > \frac{4D^2\lambda^2}{3}$

(C) $R > \frac{2D^2}{\lambda}$ (D) $R > \frac{D^2}{2\lambda^2}$

28. Inside a waveguide with perfectly conducting walls, any current is in the form of

(A) conduction current only

(B) displacement current only

(C) sometimes displacement current and some times conduction current

(D) partially displacement current and partially conduction current

29. A plane wave with $E = 20e^{-\alpha z}\sin(\omega t - z)\,\hat{a}_x$ V/m is propagating through a lossy dielectric medium having an internsic impedance of $277\angle 30°\Omega$. Determine the depth at which the amplitude of the field is 12% of the value at $Z = 0$.

(A) 1.732 m (B) 2.4 m

(C) 3 m (D) 3.67 m

30. A uniform transmission line of characteristic impedance 200Ω and feeding purely with resistive load of 100Ω by using single stub. The stub is placed at a distance of '*r*' from the load. The VSWR at '*r*' and on the stub itself will be

(A) 0.5 and 0 respectively (B) 3 and 1 respectively

(C) 2 and ∞ respectively (D) 2 and 1 respectively

31. Find the electric field intensity at a distance of 6 km from an antenna having directive gain of 6 dB and radiating power of 10 kW.

(A) 0.4 V/m (B) 0.26 V/m

(C) 0.024 V/m (D) 0.9 V/m

Common data for Questions 32 and 33:

Region $Y < 0$ consists of a perfect conductor while region $Y > 0$ is a dielectric medium ($\varepsilon_{r1} = 1.5$). If there is a surface charge of $5nC/m^2$ on conductor

32. Determine E at (5, –1, 1).

(A) 2 V/m (B) 3V/m
(C) 4/3 V/m (D) Zero

33. Find E at point (–2, ,3, 7).

(A) $5\hat{a}_x$ V/m (B) $2\hat{a}_y$ V/m
(C) $376.5\hat{a}_y$ V/m (D) $\left(-2\hat{a}_x + 3\hat{a}_y\right)$ V/m

Statement for Liked answer Questions: 34 and 35:

The observed standing wave ratio on a 150 Ω lossless line is equal to 10. If the first maximum voltage occurs at 0.4λ from the load.

34. Calculate load impedance.

(A) $256.42\angle 7.5°\Omega$ (B) $33.5\angle 90°\Omega$
(C) $15\angle -90°\Omega$ (D) $256.42\angle 82.5°\Omega$

35. Find voltage reflection constant.

(A) $0.81\angle -80°$ (B) $0.89\angle 60.42°$
(C) $0.634\angle 0°$ (D) $1\angle 0°$

Answer Keys

1. D	**2.** A	**3.** D	**4.** B	**5.** B	**6.** B	**7.** D	**8.** A	**9.** B	**10.** D
11. A	**12.** A	**13.** C	**14.** D	**15.** C	**16.** D	**17.** B	**18.** A	**19.** B	**20.** A
21. A	**22.** A	**23.** C	**24.** D	**25.** A	**26.** B	**27.** C	**28.** B	**29.** D	**30.** C
31. B	**32.** D	**33.** C	**34.** D	**35.** B					

Hints and Explanations

1. $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\varphi^2} + \frac{\partial^2 V}{\partial Z^2}$

$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(2\rho^2\cos 2\varphi\right) - \frac{1}{\rho^2}4\rho^2\cos 2\varphi + 0$

$= 4\cos 2\phi - 4\cos 2\phi = 0$ Choice (D)

2. $W_E = \frac{Q_1Q_2}{4\pi\varepsilon_0 r}$

$= \frac{2\times 3\times 10^{-12}\times 9\times 10^9}{\sqrt{9+4+1}} = \frac{54}{\sqrt{14}}\times 10^{-3} \approx 14\text{mJ}$

Choice (A)

3. $D = 3y^2z\hat{a}_x + 3x^2yz\hat{a}_y + y^2\hat{a}_z$

$\nabla . D = \rho_v = 3x^2z$

then volume integral $\Rightarrow \int_0^2\int_0^2\int_0^2 3x^2z\,dx\,dy\,dz$

$= 3\frac{x^3}{3}\frac{z^2y}{2} = \frac{8\times 4\times 2}{2} = 32$ Choice (D)

4. T_r (Relaxation time) $= \frac{\varepsilon_r\ \varepsilon_0}{\sigma}$

$= \frac{3\times 10^{-9}}{5\times 10^6\times 36\pi} = 5.3\times 10^{-18}$sec Choice (B)

5. $J_d = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d}\frac{\partial V}{\partial t}$ [∵ D = εE and E = V/d]

$\Rightarrow I_d = \frac{\varepsilon A}{d}\frac{\partial V}{\partial t} = C\frac{\partial V}{\partial t}$

$= 2\times 10^{-12}\times 70\times 10^4\cos 10^4 t$

$= 1.4\cos 10^4 t\ \mu A$ Choice (B)

6. $P_{avg} = \frac{E^2}{2\eta} = \left[but\ \eta = \frac{120\pi}{\sqrt{\varepsilon_r}}\right]$

$\sqrt{\varepsilon_r} = \frac{10\times 3\times 10^8}{2\pi\times 10^8} = \frac{15}{\pi} = \varepsilon_r = \frac{225}{\pi^2}$

$P_{avg} = \frac{25\times 15}{2\times 120\pi\times\pi}$

$= 0.158\text{W/m}^2 \simeq 158\text{mW/m}^2$ Choice (B)

7. $E_s = E_0 e^{jk.r} = E_0\, e^{j\left(k_x\hat{a}_x + k_y\hat{a}_y + k_z\hat{a}_z\right)}$

$[K_x = 0.95,\ k_z = 0.8]$

$K = \sqrt{k_x^2 + k_x^2 + k_z^2}$

$= \sqrt{(0.95)^2 + (0.8)^2}$ [∵ $k_x = 0.95$

$K_z = 0.8$]

$K = 1.242$

$\Rightarrow$ In free space $\beta = 1.242 = \omega\sqrt{\mu_0\,\varepsilon_0}$

$\Rightarrow \omega = \frac{1.242}{\sqrt{\mu_0\,\varepsilon_0}} = 1.242\times 3\times 10^8$

$= 3.72\times 10^8 = 372$MHz Choice (D)

8. distortionless line $RC = GL$

$Z_0 = \sqrt{L/C}$

$\alpha = \sqrt{RG} = \sqrt{R\times\frac{RC}{L}} = R\sqrt{C/L}$

$\Rightarrow R = \alpha Z_0 = 0.75\Omega/m.$ Choice (A)

9. $\Box = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_2} + \sqrt{\varepsilon_1}} = \frac{1-1.4}{1.4+1} = \frac{-0.4}{2.4}$

$|\Gamma| = 1/6$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{7}{5} = 1.4$$

Choice (B)

10. In a Quarter wave transformer

$Z_{in} = \frac{Z_0^2}{Z_L}$ so, it is used for accurate matching of purely resistive loads only. Choice (D)

11. $\frac{\text{Conduction current density}}{\text{displacement current density}} = \frac{I_C A}{I_D A} = \frac{\sigma E}{\omega\varepsilon E}$

$$\frac{I_C}{I_D} = \frac{\sigma}{\omega\varepsilon}$$

$$\Rightarrow \quad I_D = \frac{I_c\,\omega\varepsilon}{\sigma} = \frac{2\times 2\pi\times 60\times 2\varepsilon_0\times 10^3}{10^{-2}}$$

$= 1.33$ mA Choice (A)

12. From max well equations $\nabla \,.\, B = 0$
and from vector identify $\nabla \,.\, (\nabla\times A) = 0$
$\Rightarrow \quad B = \nabla\times A$ Choice (A)

13. $\vec{H} = \frac{1}{Z_0}\left(\hat{a}_y\, X\vec{E}\right) = \frac{1}{Z_0}\left[\hat{a}_y\times\hat{a}_x E_x + \hat{a}_y\times\hat{a}_z\; E_z\right]$

$= \frac{-E_x}{Z_0}\hat{a}_z + \hat{a}_x\frac{E_z}{z_0} = \frac{E_z}{Z_0}\hat{a}_x - \frac{E_x}{Z_0}\hat{a}_z$ Choice (C)

14. X band range → 8 – 12GHz → mid frequency 10GHz
S band range → 2 – 4GHz → mid frequency 3GHz
Gain of parabolic antenna (G) = $6(D/\lambda)^2$

$$G\alpha\frac{1}{\lambda^2}$$

$G\,\alpha\, f^2$

$$\frac{G_s}{G_x} = \frac{9}{100}$$

Choice (D)

15. $\Gamma = \frac{75-50}{75+50} = \frac{25}{125} = 1/5$

$\Rightarrow \quad P_t = P_{in}(1-\Gamma^2)$
$= P_{in}(1 - 1/25) = 0.96P_{in}$ Choice (C)

16. $\Psi = \oint_S S.\,ds = \int_V \nabla.S\,dV$

$$\nabla\,.\,S = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho S\rho) + \frac{1}{\rho}\frac{\partial}{\partial\varphi}S_\varphi + \frac{\partial}{\partial Z}(S_Z)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(10Z\rho^2\right) + \frac{1}{\rho}\frac{\partial}{\partial\varphi}(10Z) = 20Z$$

$$\Psi = \int_V \nabla.SdV = \int_0^{2\pi}\int_0^2\int_0^2 20Z\rho d\rho d\varphi dz$$

$$= 20\times\left.\frac{z^2}{2}\right|_0^2\left.\frac{\rho^2}{2}\right|_0^2\times 2\pi = 160\pi$$

Choice (D)

17. $E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1\left(-2\hat{a}_x - 8\hat{a}_z\right)}{(68)^{3/2}} + \frac{5\times10^{-9}\left(-\hat{a}_x - 9\hat{a}_z\right)}{(82)^{3/2}}\right]$

Q_1 at E has no x component

$$\frac{-2Q_1}{(68)^{\frac{3}{2}}} = \frac{5\times10^{-9}}{(82)^{\frac{3}{2}}} \Rightarrow Q_1 = \frac{5\times10^{-9}}{2}\left[\frac{68}{82}\right]^{\frac{3}{2}}$$

$\Rightarrow \quad Q_1 = -1.9\times10^{-9}C$ Choice (B)

18. Poisson's equation $\nabla^2 V = \frac{-\rho_v}{\varepsilon_0} = \frac{-5x^2}{a\varepsilon_0}$

Integrating on both sides $\nabla V = \frac{-5x^3}{3a\varepsilon_0} + A$

And $E = -\nabla V = 0$ at $x = 0$ so $A = 0$

$\Rightarrow \quad \nabla V = \frac{-5x^3}{3a\varepsilon_0}$ integrating both sides $V = \frac{-5x^4}{12\varepsilon_0} + B$

$V = 0$ at $x = \text{a}$

$$\Rightarrow \quad B = \frac{5a^3}{12\varepsilon_0}$$

$\Rightarrow \quad V = \frac{+5}{12a\varepsilon_0}\left(a^4 - x^4\right)$ Choice (A)

19. Given that $E = 20e^{-\alpha x}\cos(\omega t - 3x)\ \frac{V}{m}$

β = 3

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}\right]^{1/2}$$

but $\frac{\sigma}{\omega\varepsilon} = \tan 2\theta_\eta = -\sqrt{3}$

$$\frac{\alpha}{\beta} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = \sqrt{3}$$

Skin depth $\delta = 1/\alpha = \frac{1}{\sqrt{3}}$ Choice (B)

20. $\frac{E_i}{H_i} = \eta_i$

$$\Rightarrow \quad H_i = \frac{+8}{120\pi}\cos\left(10^7 t - \beta z\right)\hat{a}_y$$

$$\eta_2 = 120\pi\sqrt{\frac{8}{2}} = 240\pi$$

$$\text{Reflection co-efficient } \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(2-1)\eta_0}{(2+1)\eta_0} = 1/3$$

$$H_t = \frac{2}{3}H_i \Rightarrow H_t = \frac{16}{360\pi}\cos\left(10^7 t - \beta z\right)\hat{a}_y$$

$$= 14.14\cos\left(10^7 t - \beta z\right)\hat{a}_y \text{ mA/m}$$

Choice (A)

21. Intrinsic impedance $(Z_i) = \frac{\omega\mu_0}{\beta} = \sqrt{\frac{\mu}{\varepsilon}}$

$$= \frac{2\pi \times 6 \times 10^9 \times 4\pi \times 10^{-7}}{10} = 4.737\text{k}\Omega$$

We know that $Z_i = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{6}\right)^2}} = 4737$

$\Rightarrow$ $f_c = 5.98$ GHz

Choice (A)

22. Poynting vector $= \frac{1}{2}E_m H_m = \frac{1}{2}\frac{E_m^2}{\eta}$

T_{E10} is dominant mode for a rectangular wave guide

$$|P| = \frac{\text{Power flow}}{\text{area}} = \frac{150}{1.5 \times 0.75} = 133.33 \times 10^4 \text{ W/m}^2$$

$$\Rightarrow Z_{TE} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}}$$

$$f_c = \frac{C}{2a} = \frac{3\times10^{10}}{2\times1.5} = 10\text{ GHz}$$

$$Z_{TE} = \frac{377}{\sqrt{1-\left(\frac{10}{25}\right)^2}} = 411.34\Omega$$

$$E_{max} = \sqrt{2\times|\vec{P}| X Z_{TE}}$$

$$= \sqrt{2\times411.34\times133.33\times10^4}\text{ V/m}$$

$E_{max} = 3.3$ kV/m

Choice (A)

23. Quality factor of capacitor

$$Q = \frac{|\text{displacement current}|}{|\text{Conduction current}|} = \frac{|I_d|}{|I_c|}$$

Conduction current $(I_c) = \iint J.ds$

$$J = \sigma E = 10^{-6}\frac{10^6}{\rho}\sin\left(10^4 t\right) = \frac{\sin\left(10^4 t\right)}{\rho}$$

$$I_C = \iint \frac{\sin\left(10^4 t\right)}{\rho} ds = 2\pi\rho\ell\frac{\left(\sin 10^4 t\right)}{\rho}$$

$$\Rightarrow |I_C| = 2\pi l$$

$$J_d = \varepsilon\frac{\partial E}{\partial t} = \varepsilon\times\frac{10^6\times10^4}{\rho}\cos\left(10^4 t\right)$$

$$\Rightarrow I_D = \iint\frac{10^{10}\varepsilon}{\rho}\cos 10^4 t.ds$$

$$= 2\pi\rho l\left(\frac{10^{10}\times\varepsilon}{\rho}\cos 10^4 t\right) = \frac{25}{8}\ell\cos(10^4 t)$$

$$|I_d| = \frac{25}{8}\ell$$

$$\text{Quality factor } Q = \frac{|I_d|}{|I_c|} = \frac{25\ell}{8\times2\Pi\ell} \simeq 0.22$$

Choice (C)

24. $J_d = \frac{\partial D}{\partial t}$

$$\Rightarrow D = \int J_d\, dy = \int 15\cos\left(2\times10^8 t - bx\right)10^{-6}\,dt$$

$$= \frac{15\times10^{-6}}{2\times10^8}\sin\left(2\times10^8 t - bx\right) + C_1$$

[$c_1 = 0$ because No DC fields]

$$\Rightarrow E = \frac{D}{\varepsilon} = \frac{7.5\times10^{-14}}{\varepsilon}\sin\left(2\times10^8 t - bx\right)$$

$= 2.82\sin(2\times10^8 t - \text{bx})\,\hat{a}_y$ mV/m

Choice (D)

25. $H = 15\sin(2\pi\times10^8 t - 3x)\ \hat{a}_z$ mA/m

$\Rightarrow$ $\beta = 3$, $\omega = 2\pi\times10^8$

$$\beta = \frac{\omega}{C}\sqrt{\varepsilon_r}$$

$$\Rightarrow \frac{3\times3\times10^8}{2\pi\times10^8} = \sqrt{\varepsilon_r} = 1.43$$

$\varepsilon_r = 2.05$

$$E = \eta H = \frac{377}{\sqrt{\varepsilon_r}}\times15\times10^{-3}\sin\left(2\pi\times10^8 t - 3x\right)\hat{a}_y$$

$$= 3.947\sin\left(2\pi\times10^8 t - 3x\right)\hat{a}_y$$

$$\Rightarrow D = \varepsilon E = 3.49\times10^{-11}\sin\left(2\pi\times10^8 t - 3x\right)\hat{a}_y$$

$$\Rightarrow J_D = \frac{\partial D}{\partial t}$$

$$= 3.49\times10^{-11}\times2\pi\times10^8\cos\left(2\pi\times10^8 t - 3x\right)\hat{a}_y$$

$= 22.9\cos(2\pi\times10^8 t - 3x)$ mA/m^2

$(J_D)_{max} = 22.9\times10^{-3} = 23\times10^{-3} \simeq 0.023$

Choice (A)

26. For parallel polarization

$$\text{Reflection coefficient} = \Gamma = \frac{\sqrt{\varepsilon_{r_2}}\cos\theta_i - \sqrt{\varepsilon_{r_1}}\cos\theta_t}{\sqrt{\varepsilon_{r_2}}\cos\theta_i + \sqrt{\varepsilon_{r_1}}\cos\theta_t}$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{9\varepsilon_0}{\varepsilon_0}} = 3$$

$\Rightarrow$ $\text{Sin}\theta_t = \frac{1}{3\sqrt{2}} \Rightarrow \theta_t = 13.63$

$\cos\theta t = 0.97$

$$\Gamma = \frac{3\cos 45° - 0.97}{3\cos 45° + 0.97}$$

$$= \frac{2.121 - 0.97}{2.121 + 0.97} = \frac{1.15}{3.09}$$

$\Gamma = 0.37$ Choice (B)

27. For far field pattern $R > \frac{2D^2}{\lambda}$ Choice (C)

28. Choice (B)

29. We know that $\tan 2\theta\eta = \frac{\sigma}{\omega\varepsilon} = \tan 60° = \sqrt{3}$

From equation we get $\beta = 1$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}\right]^{\frac{1}{2}}$$

$$\frac{\alpha}{\beta} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\alpha = \frac{1}{\sqrt{3}}$

$\Rightarrow$ depth at which amplitude is 12%

$20e^{-\alpha z} = 20 \times 0.12 = 2.4$

$\Rightarrow$ $z = 3.67\text{m}$ Choice (D)

30. VSWR on the length r is $= \frac{Z_0}{Z_L}$ $[\because Z_0 > Z_L)$

$$= \frac{200}{100} = 2$$

On stub $Z_0 = 200$, $Z_L = 0$

VSWR on stub $= \frac{Z_0}{Z_L} = \frac{200}{0} = \infty$ Choice (C)

31. $G_d(\text{dB}) = 6 = 10\log_{10} G_d \Rightarrow G_d = 4$

$\Rightarrow$ We know that $|E_s|^2 = \frac{2\eta G_d P_{rad}}{4\pi r^2}$

$$= \frac{2\times 120\pi \times 4 \times 10^4}{4\times\pi\times\left(6\times 10^3\right)^2}$$

$$|E_s|^2 = 666\times 10^{-4}$$

$|E_s| = 0.26 \text{ V/m}$ Choice (B)

32. Inside conductors $E = 0$ Choice (D)

33. Point (–2, 3, 7) is in directive medium since $Y = 3 > 0$

$D_n = \rho_s = 5\text{nC/m}^2$

Hence $D_n = 5\hat{a}_y nC/m^2$

$$E = \frac{D}{\varepsilon_0\ \varepsilon_r} = \frac{5\times 10^{-9}}{1.5}\times 36\Pi\times 10^9$$

$= 376.5\hat{a}_y \text{V/m}$ Choice (C)

34. First voltage maxima occurs at 0.4λ from the load

$$\beta l = \frac{2\pi}{\lambda}\times 0.4\lambda$$

$= 2.513\text{rad}$

Input impedance seen from voltage maximum point is

$$Z_{in} = \frac{Z_0}{S} = \frac{150}{10} = 15\Omega$$

We know that

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta\ell}{Z_0 + jZ_L\tan\beta\ell}\right]$$

$$15 = 150\left[\frac{Z_L + j150\tan(2.513)}{150 + jZ_L\tan(2.513)}\right]$$

$$\Rightarrow \frac{1}{10} = \frac{Z_L - j109}{150 - j0.726Z_L}$$

[$\therefore$ Consider Z_L as complex number $Z_L = R_L + jX_L$]

$\Rightarrow$ $15 - j\,R_L(0.0726) + 0.0726x = R_L + j(X_L - 109)$

By comparing real and imaginary parts

$R_L = 15 + 0.0726X_L$ and $R_L\,(0.0726) = X_L - 109$

$\Rightarrow$ $X_L = 254.23$

$R_L = 33.45$

$\Rightarrow$ $Z_L = 33.45 + j254.23 = 256.4\angle 82.5°$ Choice (D)

35. $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\Gamma = \frac{33.45 + j254.23 - 150}{33.45 + j254.23 + 150}$$

$$= \frac{-116.55 + j254.23}{183.45 + j254.23}$$

$$= \frac{279.67\angle 114.6°}{313.5\angle 54.18}$$

$\Gamma = 0.89\angle 60.42°$ Choice (B)