UNDERSTANDING QUADRILATERALS

FACTS THAT MATTER

- A simple closed curve made of only line segments is called a polygon.
- If all the sides of a polygon are equal and all angles are also equal, it is said to be a regular polygon.
- An equilateral triangle is a regular polygon of 3 sides and square is a regular polygon of 4 sides.
- The sum of the exterior angles of a polygon is always 360º.
- A quadrilateral in which a pair of opposite sides are parallel is called a trapezium.
- If both the pairs of opposite sides of a quadrilateral are parallel then it is called a paralleogram.
- A parallelogram which has all four sides equal, is called a rhombus.
- A parallelogram which has all its angles at right angles, is called a rectangle.
- A parallelogram with all sides and all angles equal, is a square.
- A kite is a special quadrilateral in which one pair of adjacent sides are equal to each other and other pair of adjacent sides are equal to each other.

REMEMBER

1. A triangle does not have a diagonal as its each vertex is adjacent to the other two.
2. If all sides of a quadrilateral are equal, diagonals are perpendicular.
3. If all angles of a quadrilateral are equal, diagonals are equal.
4. If opposite sides of a quadrilateral are parallel and equal, diagonals bisect each other.
5. In a kite only the obtuse angles opposite to each other are equal not the acute angles.
6. In a trapezium the angels are not equal unless it is an isosceles trapezium.
7. A square is also a trapezium, rectangle and rhombus.

We Know That

A curve which does not cut itself is called an open curve whereas a curve which cuts itself is called a closed curve. The closed curves which do not cross themselves are called simple curves. A curve (in mathematics) can be straight also.

POLYGONS

A simple closed curve made up of only the line segments is called a polygon. The simplest polygon is a

Number of sides or vertices	Number of the polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
.	.
.	.
.	.
n	n-gon

The line connecting any two non-consecutive vertices is called a diagonal. Convex polygons have non portion of their diagonals in their exteriors. Thus a polygon each of whose interior angles is less than 180º, is called a convex polygon, otherwise it is concave polygon.

INTERIOR AND EXTERIOR OF A QUADRILATERAL

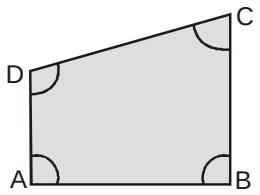
(i) Interior Quadrilateral : The part of the plane made up by all such points as are enclosed by quadrilateral ABCD. This part of the plane is called the interior of the quadrilateral ABCD and any point P of this part is called an interior point of the quadrilateral ABCD.

(ii) Exterior Quadrilateral : The part of the plane made up by all points as are not enclosed by the quadrilateral ABCD. This part of the plane is called the exterior of the quadrilateral ABCD and any point Q of this part is called an exterior point of the quadrilateral ABCD.

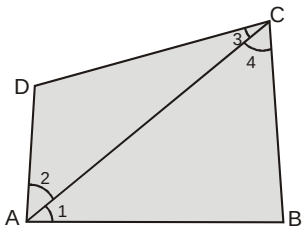
(iii) Quadrilateral Region : The interior of a quadrilateral ABCD, together with quadrilateral ABCD is called the quadrilateral region ABCD.

SUM PROPERTY OF INTERIOR ANGLES AND EXTERIOR ANGLES

(a) Interior Angles : The angles ĎA, ĎB, ĎC and ĎD of quadrilateral ABCD are called its interior angles.



Theorem :– The sum of the angles of a quadrilateral is 360° or 4 right angles.



Given : ABCD is a quadrilateral.

To Prove : $\angle A + \angle B + \angle C + \angle D = 360°$

Construction : Join AC

Proof : Clearly $\angle 1 + \angle 2 = \angle A$ and $\angle 3 + \angle 4 = \angle C$

We known that the sum of the angles of a triangle is 180° therefore,

In ΔABC, we have $\angle 1 + \angle 4 + \angle B = 180°$ ...(i)

and in ΔACD, we have

$\angle 2 + \angle 3 + \angle D = 180°$ ...(ii)

Adding (i) and (ii), we get

$(\angle 1 + \angle 4 + \angle B) + (\angle 2 + \angle 3 + \angle D) = 180° + 180°$

SOLVED EXAMPLES

EX.1 The angles of a quadrilateral are respectively 70º, 92º and 100º. Find the fourth angle.

Sol. Let the fourth angle be xº

∵ The sum of the angles of a quadrilateral = 360º

∴ 70º + 92º + 100º + x = 360º

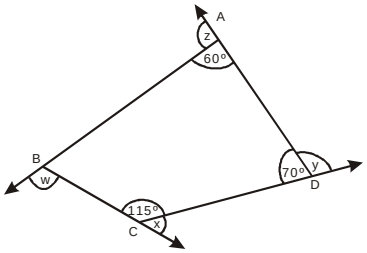
⇒ 262º + x = 360º

⇒ x = 360º – 262º = 98º

Thus, the measure of fourth angle = 98º

Ex.2 In the adjoining figure , find x + y + z + w

Sol. Since, the sum of the measure of interior angles of a quadrilateral is 360º.



Also, 115º + 70º + 60º = 245º

∴ 245º + ∠ABC = 360º

⇒ ∠ABC = 360º – 245º = 115º

Now, x = ext. ∠BCD

= 180º – ∠BCD

= 180º – 115º = 65º

Similarly, y = 180º – 70º = 110º

z = 180º – 60º = 120º

w = 180º – 115º = 65º

∴ x + y + z + w

= 65º +110º +120º +65º = 360º

Ex.3 In a quadrilateral ABCD, the angles A, B, C and d are in the ratio 1 : 2 : 3 : 4. Find the measure of each angle of the quadrilateral.

∵ ∠A : ∠B : ∠C : ∠D = 1 : 2 : 3 : 4

∴ Let us suppose that

∠A = 1xº, ∠B = 2xº

∠C = 3xº, ∠D = 4xº

Since, ∠A + ∠B + ∠C + ∠D = 360º

∴ x + 2x + 3x + 4x = 360º

$\Rightarrow x = \frac{360^\circ}{10} = 36^\circ$

∴ Angles are :

∠A = xº = 36º

∠B = 2xº = 2 × 36º = 72º

∠C = 3xº = 3 × 36º = 108º

∠D = 4xº = 4 × 36º = 144º

Thus, the measure of the angle of the quad. are 36º, 72º, 108º and 144º.

Ex.4 The interior angle of a regular is 108º. Find the number of sides of the polygon.

Sol. Let there are 'n' sides of the regular polygon.

∴ Measure of each exterior angle

$= \left(\frac{360}{n}\right)^\circ$

Since, the measure of each of the interior angle = 108º

∴ Measure of each exterior angle

= 180º – 108º = 72º

$\Rightarrow \frac{360}{n} = 72$

$\Rightarrow n = \frac{360}{n} = 5$

Thus, the reqiured number of sides of the regular polygon is 5.

Ex.5 Two regular polygons are such that the ratio of the measures their interior angles is 4 : 3 and the ratio between their number of sides of the regular polygons.

Sol. Let 2n and n be the number of sides of the regular polygons.

∴ Their interior angles are $\left[\frac{2(2n)-4}{2n}\times 90\right]^\circ$ and $\left[\frac{2n-4}{n}\times 90\right]^\circ$

Since the ratio of the interior angles is 4 : 3

$\therefore \frac{\left[\frac{2(2n)-4}{2n}\times 90\right]}{\left[\frac{(2n-4)}{n}\times 90\right]} = \frac{4}{3}$

$\Rightarrow \frac{n}{2n}\times\frac{[2(2n)-4]}{[2n-4]} = \frac{4}{3}$

$\Rightarrow \frac{1}{2}\times\frac{4n-4}{2n-4} = \frac{4}{3}$

$\Rightarrow \frac{1}{2}\times\frac{4(n-4)}{2(n-2)} = \frac{4}{3}$

$\Rightarrow \frac{n-1}{n-2} = \frac{4}{3}$

⇒ 3(n – 1) = 4 (n – 2)

⇒ 3n – 3 = 4n – 8

⇒ 3n – 4n = –8 + 3

⇒ –n = – 5 ⇒ n = 5

∴ 2n = 2 × 5 = 10

Thus, the number of sides of the polygons are 10 and 5 respectively.

$\angle A = \frac{1}{20} \times 360° = 18°$

$\angle B = \frac{3}{20} \times 360° = 54°$

$\angle C = \frac{7}{20} \times 360° = 126°$

$\angle D = \frac{9}{20} \times 360° = 162°$

(ii) $\angle A : \angle D = 18° + 162° = 180°$

Sum of the interior angles on the same side of the transversal AD is 180°

y AB||DC

Hence, ABCD is a trapezium

(iii) Since the opposite angles of quadrilateral ABCD are not equal

y It is not a parallelogram

Ex.12 The perimeter of a parallelogram is 80 m if the longer side is 10 m greater than the shorter side, find the lengths of the sides of the parallelogram :

Sol. Let the shorter side be xm

Longer side = (x + 10) m

Perimeter = 2(x + 10 + x) m = (4x + 20) m

$\therefore$ 4x + 20 = 80

4x = 80 – 20 = 60

$x = \frac{60}{4}$ or 15

$\therefore$ Length of shorter side = 15 m

Length of longer side = (15 + 10) or 25 m

Ex.13 The angles of a quadrilateral are in the ratio 3:5:7:9. Find the angles :

Sol. Let the angles be 3x, 5x, 7x and 9x

3x + 5x + 7x + 9x = 360°

24x = 360°

$x = \frac{360}{24} = 15°$

3x = 3 × 15° = 45°

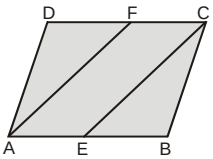
7x = 7 × 15° = 105°

5x = 5 × 15° = 75°

9x = 9 × 15° = 135°

Ex.14 In fig., ABCD is a parallelogram and E and F are the mid points of side AB and CD respectively. Show that quadrilateral AECF is a parallelogram :

Sol. ABCD is a parallelogram



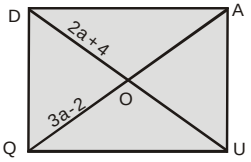
AB = CD and AB||CD

$\frac{1}{2}AB = \frac{1}{2}CD$

AE = FC AE||FC

A quadrilateral is a parallelogram if one pair of the opposite sides is equal and parallel. Hence AECF is a parallelogram

Ex.15 In fig., QUAD is a rectangle. Its diagonals meet at O. Find 'a', if OQ = 3a – 2 and OD = 2a + 4



Sol. Rectangle diagonals are equal

So QA = DU

$\Rightarrow \frac{1}{2}QA = \frac{1}{2}DU$

$\Rightarrow$ OQ = OD

3a – 2 = 2a + 4

3a – 2a = 4 + 2 = 6

a = 6

Ex.16 Three angles of a quadrilateral are in the ratio 2:3:7. The mean of these angles is 64°. Find all the four angles.

Sol. Let the given angles be 2x°, 3x°, 7x°

$\therefore$ Mean of these angles = $\frac{2x° + 3x° + 7x°}{3} = 4x°$

$\therefore$ 4x = 64

or x = 16

$\therefore$ The three given angles are 32°, 48°, 112°

Let the fourth angle be a°

$\therefore$ 32° + 48° + 112° + a = 360°

or 192° + a = 360°

or a = 360° – 192° = 168°

Hence the fourth angle is 168°.

Ex.17 In a quadrilateral ABCD, AO and BO are the bisectors of ĎA and ĎB respectively. Prove that

$\check{D}AOB = \frac{1}{2}(\check{D}C + \check{D}D)$

Sol. In ΔAOB

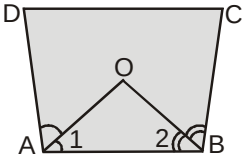
$\angle AOB + \angle 1 + \angle 2 = 180°$

$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

$\angle AOB = 180° - (\angle 1 + \angle 2)$

$\angle AOB = 180° - (\frac{1}{2}\angle A + \frac{1}{2}\angle B)$

$[\angle 1 = \frac{1}{2}\angle A$ and $\angle 2 = \frac{1}{2}\angle B]$



EXERCISE - I UNSOLVED PROBLEMS

Q.1 Find the measure of each exterior angle of a regular polygon having 15 sides.

Q.2 Find the measure of each interior angle of a regular polygon having 12 sides.

Q.3 Find the number of sides of a polygon, the sum of whose interior angles is 1620°.

Q.4 The sides of a pentagon are produced in order and the exterior angles so obtained are of measure x°, (2x)°, (3x+10)°, (4x+5)° and (5x)° respectively. Find the value of x and the measure of each exterior angle of the pentagon.

Q.5 If each interior angle of a regular polygon is 144°, how many sides does this polygon have ?

Q.6 Find the measure of x in the figure(a).

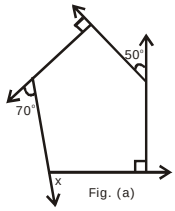


Fig. (a)

Q.7 Find the measure of x in the figure (b).

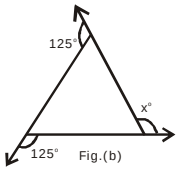


Fig.(b)

Q.8 The ratio of each exterior angle to each interior angle of a regular polygon is 2 : 3. Find the number of sides in the polygon.

Q.9 Each interior angle of a regular polygon is 140°. Find the interior angle of a regular polygon which has double the number of sides as the first polygon.

Q.10 PQRS is a parallelogram. What special name will you give it if the following additional facts are known ?

(i) PQ = PS

(ii) $\angle$SPQ = 90°

(iii) PQ = PS and $\angle$SPQ = 90°

Q.11 PQRS is a trapezium in which PQ|| RS. IF $\angle$P= $\angle$Q = 50°, what are the measures of the other two angles ?

Q.12 ABCD is a trapezium in which AB||CD. If $\angle$A = $\angle$B = 40°, what are the measure of the other two angles ?

Q.13 The angles A, B, C and D of a quadrilateral ABCD are in the ratio 1 : 3 : 7 : 9

(i) Find the measure of each angle.

(ii) Is ABCD a trapezium ? Why ?

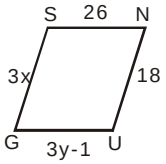
(iii) Is ABCD a parallelogram ? Why ?

Q.14 Two adjacent angles of parallelogram are in the ratio of 1:5. Find all the angles of the parallelogram.

Q.15 One angle of a parallelogram is of measure 70°. Find the measures of the remaining angles of the parallelogram.

Q.16 One angle of a parallelogram is of measure 80°. Find the measures of the remaining angles of the parallelogram.

Q.17 The adjoining figure GUNS is a parallelogram. Find x and y.

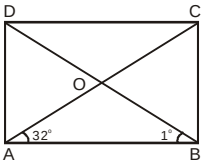


Q.18 Two adjacent angles of a parallelogram are in the ratio 2:3. Find all the angles of the parallelogram.

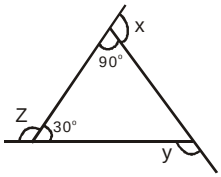
OR

In a parallelogram the ratio of the adjacent angles is 2:3. Find all angles.

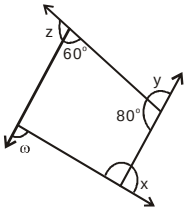
Q.19 ABCD is a rectangle with $\angle$BAC = 32°. Determine $\angle$DBC.



Q.20 Find x + y + z.

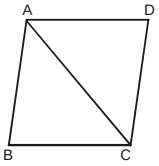


Q.21 Find x + y + z + ω.



(vii) Does diagonal AC of the rhombus bisect ∠A and ∠C ? Why ?

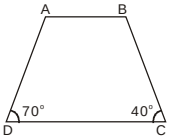
Q.38 Diagonal AC of a rhombus ABCD is equal to one of its side BC. Find all the angles of the rhombus.



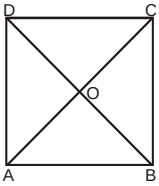
Q.39 In the adjoining figure, ABCD is a rectangle, BM and DN are perpendiculars to AC from B and D respectively.

(i) Is AB = CD ? Why ?
(ii) Is ∠BMA = ∠DNC ? Why ?
(iii) Is ΔBMA = ΔDNC ? Why ?
(iv) Is ΔBMA ≅ ΔDNC ? By which congruence condition
(v) Is BM = DN ? Why ?

Q.40 In figure, ABCD is a trapezium in which AB || DC. If ∠D = 70° ∠C = 40°, find the measure of its remaining two angles.

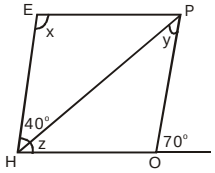


Q.41 In the given figure, ABCD is a rhombus, If AO = 4cm and OB = 3 cm, then write the following measurements :

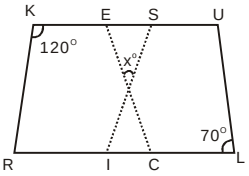


(i) AC = (ii) BD =
(iii) ∠AOB = (iv) AB =

Q.42 The adjacent figure HOPE is a parallelogram. Find the angle measure x, y and z. State the properties you use to find them.



Q.43 In the above figure both RISK and CLUE are parallelograms. Find the value of x.



Q.44 In the adjoining figure ABCD is a trapezium in which $\overline{AB} || \overline{DC}$. If m ∠A = 120° and m ∠B = 130°, what are the measures of the other two angles ?

Q.45 The angle of a quadrilateral are respectively 120°, 73°, 80°. Find the fourth angle :

Q.46 The interior angle of a regular polygon is 156°. Find the number of sides of the polygon.

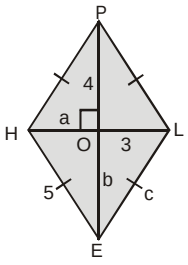
Q.47 What is the sum of the (i) interior angles (ii) exterior angles of a quadrilateral?

Q.48 Name the polygon in which the sum of the interior angles of a quadrilateral is 360°.

Q.49 Find the measure of each exterior angle of regular polygon of 9 sides.

Q.50 How many sides does a regular polygon has if the measure of an exterior angle is 24°?

Q.51 In fig, HELP is a rhombus. Find a, b and c. Give reason in support of your answer.



Q.52 One of the diagonals of a rhombus is equal to one of its sides. Find the angles of the rhombus.

Q.53 The angles of a quadrilateral are 110°, 72°, 55° and x°. Find the value of x.

Q.54 The four angles of a quadrilateral are as 3 : 5 : 7 : 9. Find the angles.

Q.55 Find the number of sides of a regular polygon, when each of its angles has a measure of
(i) 160° (ii) 135° (iii) 175°
(iv) 162° (v) 150°