B–12–Y

Roll No. ....

Total No. of Questions : 29] [Total No. of Printed Pages : 8

XIIARJKUT23
9112-Y
MATHEMATICS

Time : 3 Hours] **[Maximum Marks : 100**

SECTION–A

(MULTIPLE CHOICE QUESTIONS) 1 each

1. Range of the function $f(x) = 4x + 3$, $x > 0$ is :

   (A) $(3, \infty)$

   (B) $[3, \infty)$

   (C) $(-\infty, 3)$

   (D) $(-\infty, 3]$

2. $\tan^{-1} x + \cot^{-1} x$, $x \in \mathbf{R}$ is equal to :

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) π

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then $|\vec{a} \times \vec{b}| =$:

(A) $\cos\theta$

(B) $\sin\theta$

(C) $ab\cos\theta$

(D) $ab\sin\theta$

4. If A is a square matrix of order n, then $|\text{adj}(A)| =$ ....

(A) $|A|^n$

(B) $|A|^{n-1}$

(C) $|A|^{n+1}$

(D) None of these

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

5. Using properties of determinants, prove that :

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

6. Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$.

7. Differentiate $\cos(\sin x)$ with respect to x.

8. Find :

$$\int \frac{(\log x)^2}{x} dx$$

9. A coin is tossed three times. Find P(F/E), where E is the event "at least two heads" and F is the event "at most two heads."

10. Compute P(A/B) if P(B) = 0.5 and P(A ∩ B) = 0.32.

11. Find the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ and having magnitude of 8 units.

12. Define Linear Constraints.

SECTION–C

(SHORT ANSWER TYPE QUESTIONS)

4 each

13. Find gof and fog if $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

14. Prove that :

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$$

15. If :

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

prove that $A^3 - 6A^2 + 7A + 2I = 0$.

16. Find local maxima and local minima if any of the function :

$$f(x) = x^3 - 6x^2 + 9x + 15$$

17. Find general solution of differential equation :

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

18. Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$.

19. Prove that :

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

20. If $y = (\tan^{-1} x)^2$, show that :

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$$

21. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

22. Find the Vector and Cartesian equations of the line that passes through the points (3, −2, −5) and (3, −2, 6).

23. Solve the following graphically :

Minimise :

$$Z = x + 2y$$

Subject to the constraints :

$$2x + y \geq 3$$

$$x + 2y \geq 6$$

$$x, y \geq 0$$

SECTION-D

(LONG ANSWER TYPE QUESTIONS) 6 each

24. Using properties of determinants show that :

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$$

Or

Solve the system of linear equations using matrix method :

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

25. Find $\frac{dy}{dx}$ if $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$.

Or

Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$.

26. Find $\int_0^{\pi/2} \sqrt{\sin\phi} \cos^5\phi \, d\phi$.

Or

Using integration, find the area of region bounded by triangle whose vertices are (–1, 0), (1, 3), (3, 2).

27. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$.

Or

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

28. Find the intervals in which the function $f(x) = (x + 1)^3 (x - 3)^3$ is strictly increasing or decreasing.

Or

Evaluate :

$$\int_0^{\pi/2} \log \sin x \, dx$$

29. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item.

Or

Find the mean and variance of the number obtained on a throw of an unbiased die.