CHAPTER 10

Mensuration

MENSURATION

Mensuration is the science of measurement of the lenghts of lines, areas of surfaces and volumes of solids.

Perimeter

Perimeter is sum of all the sides. It is measured in cm, m, etc.

Area

The area of any figure is the amount of surface enclosed within its boundary lines. This is measured in square unit like cm^2, m^2, etc.

Volume

If an object is solid, then the space occupied by such an object is called its volume. This is measured in cubic unit like cm^3, m^3, etc.

Basic Conversions :

I. 1 m = 10 dm
1 dm = 10 cm
1 cm = 10 mm
1 m = 100 cm = 1000 mm
1km = 1000 m

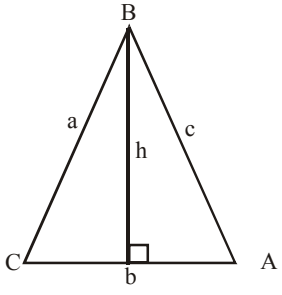
II. 1 km $= \frac{5}{8}$ miles
1 mile = 1.6 km
1 inch = 2.54 cm

III. 100 kg = 1 quintal
10 quintal = 1 tonne
1 kg = 2.2 pounds (approx.)

IV. 1 litre = 1000 cc
1 acre = 100 m^2
1 hectare = 10000 m^2 (100 acre)

PART I : PLANE FIGURES

TRIANGLE



Perimeter (P) = a + b + c

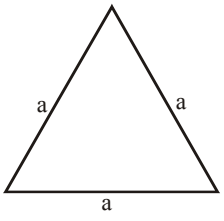
Area $(A) = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$ and a, b and c are three sides of the triangle.

Also, $A = \frac{1}{2} \times bh$; where $b \rightarrow$ base

$h \rightarrow$ altitude

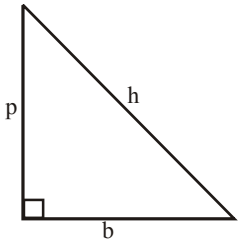
Equilateral triangle



Perimeter = 3a

$A = \frac{\sqrt{3}}{4}a^2$; where $a \rightarrow$ side

Right triangle



$A = \frac{1}{2}pb$ and $h^2 = p^2 + b^2$ (Pythagoras triplet)

where $p \rightarrow$ perpendicular
$b \rightarrow$ base
$h \rightarrow$ hypotenuse

EXAMPLE 1. Find the area of a triangle whose sides are 50 m, 78m, 112m respectively and also find the perpendicular from the opposite angle on the side 112 m.

Sol. Here a = 50 m, b = 78 m, c = 112m

$s = \frac{1}{2}(50 + 78 + 112) = 120\text{m}$

s – a = 120 – 50 = 70 m
s – b = 120 – 78 = 42 m
s – c = 120 – 112 = 8 m

$\therefore$ Area $= \sqrt{120 \times 70 \times 42 \times 8} = 1680$ sq.m.

$\because$ Area $= \frac{1}{2}$ base $\times$ perpendicular

$\therefore$ Perpendicular $= \frac{2\text{Area}}{\text{Base}} = \frac{1680 \times 2}{112} = 30\text{m}.$

EXAMPLE 2. The base of a triangular field is 880 m and its height 550 m. Find the area of the field. Also calculate the charges for supplying water to the field at the rate of ₹24.25 per sq. hectometre.

Sol. Area of the field $= \frac{\text{Base} \times \text{Height}}{2}$

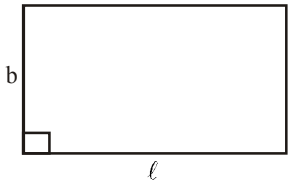
$= \frac{880 \times 550}{2} = 242000$ sq.m. $= 24.20$ sq.hm

Cost of supplying water to 1 sq. hm = ₹ 24.25

$\therefore$ Cost of supplying water to the whole field

$= 24.20 \times 24.25 =$ ₹ 586.85

RECTANGLE



Perimeter = 2 (ℓ + b)

Area = $\ell \times$ b; where $\ell \rightarrow$ length

b $\rightarrow$ breadth

Shortcut Approach

If the length and breadth of a rectangle are increased by a% and b%, respectively, then are will be increased by $\left(a + b + \frac{ab}{100}\right)\%$.

EXAMPLE 3. If the length and breadth of a rectangle are increased by 10% and 8%, respectively, then by what per cent will the area of that rectangle be increased?

Sol. Given that, a = 10, b = 8

According to the formula,

Percentage increase in area $= \left(10 + 8 + \frac{10 \times 8}{100}\right)\%$

$= \left(18 + \frac{80}{100}\right)\% = \left(18 + \frac{4}{5}\right)\% = 18\frac{4}{5}\%$

EXAMPLE 4. If the length of a rectangle is increased by 5% and the breadth of the rectangle is decreased by 6%, then find the percentage change in area.

Sol. Given that a = 5, b = – 6

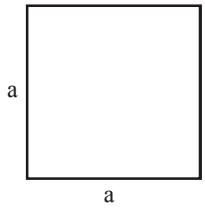
According to the formula,

Percentage change in area $= \left(5 - 6 - \frac{5 \times 6}{100}\right)\%$

$= -1 - \frac{30}{100} = -1 - 0.30 = -1.3\%$ (decrease)

Negative value shown that there is a decrease in area.

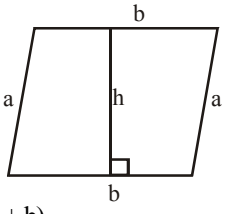
SQUARE



Perimeter = 4 × side = 4a

Area = $(\text{side})^2 = a^2$; where a $\rightarrow$ side

PARALLELOGRAM



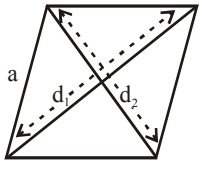
Perimeter = 2 (a + b)

Area = b × h; where a $\rightarrow$ breadth

b $\rightarrow$ base (or length)

h $\rightarrow$ altitude

RHOMBUS



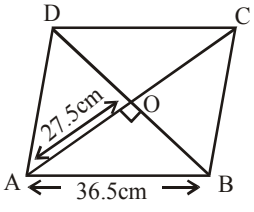
Perimeter = 4 a

Area $= \frac{1}{2} d_1 \times d_2$ where a $\rightarrow$ side and

d_1 and d_2 are diagonals.

EXAMPLE 5. The perimeter of a rhombus is 146 cm and one of its diagonals is 55 cm. Find the other diagonal and the area of the rhombus.

Sol. Let ABCD be the rhombus in which AC = 55 cm.



and AB $= \frac{146}{4} = 36.5$ cm

Also, AO $= \frac{55}{2} = 27.5$m

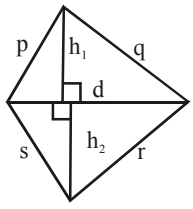
$\therefore$ BO $= \sqrt{(36.5)^2 - (27.5)^2} = 24$ cm

Hence, the other diagonal BD = 48 cm

Now, Area of the rhomhus $= \frac{1}{2}$AC $\times$ BD

$= \frac{1}{2} \times 55 \times 48 = 1320$ sq.cm.

IRREGULAR QUADRILATERAL



Perimeter = p + q + r + s

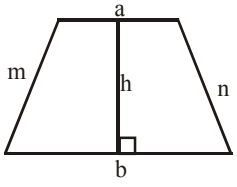
Area $= \frac{1}{2} \times d \times (h_1 + h_2)$

EXAMPLE 6. Find the area of a quadrilateral piece of ground, one of whose diagonals is 60 m long and the perpendicular from the other two vertices are 38 and 22m respectively.

Sol. Area $= \frac{1}{2} \times d \times (h_1 + h_2)$

$= \frac{1}{2} \times 60(38 + 22) = 1800$ sq.m.

TRAPEZIUM



Perimeter = a + b + m + n

Area $= \frac{1}{2}(a + b)h$; where (a) and (b) are two parallel sides;

(m) and (n) are two non-parallel sides;

h → perpendicular distance between two parallel sides.

EXAMPLE 7. A 5100 sq.cm trapezium has the perpendicular distance between the two parallel sides 60 m. If one of the parallel sides be 40m then find the length of the other parallel side.

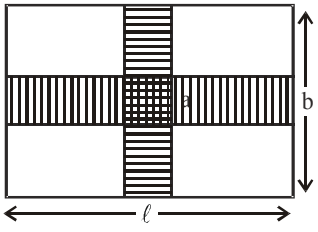
Sol. Since, $A = \frac{1}{2}(a + b)h$

$\Rightarrow \quad 5100 = \frac{1}{2}(40 + x) \times 60$

$\Rightarrow \quad 170 = 40 + x$

$\therefore$ other parallel side = 170 – 40 = 130 m

AREA OF PATHWAYS RUNNING ACROSS THE MIDDLE OF A RECTANGLE

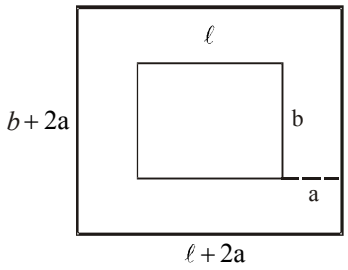


A = a (ℓ + b) – a^2; where ℓ → length

b → breadth,

a → width of the pathway.

Pathways outside

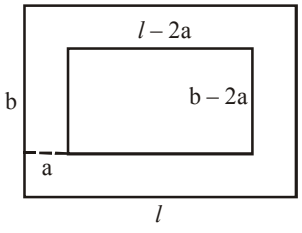


A = (*l* + 2a) (b + 2a) – *l*b; where *l* → length

b → breadth

a → width of the pathway

Pathways inside



A = lb – (l – 2a) (b – 2a); where l → length

b → breadth

a → width of the pathway

☞ Shortcut Approach

If a pathway of width x is made inside or outside a rectangular plot of length *l* and breadth b, then are of pathway is

(i) 2x (*l* + b + 2x), if path is made outside the plot.

(ii) 2x (*l* + b – 2x), if path is made inside the plot.

EXAMPLE 8. There is garden of 140 m × 120 m and a gravel path is to be made of an equal width all around it, so as to take up just one-fourth of the garden. What must be the breadth of the path?

Sol. Since, path covers $\frac{1}{4}$th area of the garden, that means path is inside the garden.

Given, *l* = 140 m, b = 120 m, x = ?

According to the question,

$2x(l + b - 2x) = \frac{1}{4} \times l\ b \Rightarrow 2x(140 + 120 - 2x) = \frac{1}{4} \times 140 \times 120$

$\Rightarrow x(260 - 2x) = 2100 \Rightarrow x^2 - 130x + 1050 = 0 \Rightarrow x = 8.65$ or 121.3

Leaving 121.3, since width of the park cannot be greater than length.

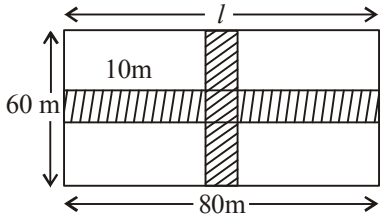
$\therefore$ Width of the park = 8.65 m

☞ Shortcut Approach

If two paths, each of width x are made parallel to length (*l*) and breadth (b) of the rectangular plot in the middle of the plot, then area of the paths is x(*l* + b – x)

EXAMPLE 9. A rectangular grass plot 80 m × 60 m has two roads, each 10 m wide, running in the middle of it, one parallel to length and the other parallel to breadth. Find the area of the roads.

Sol. Then, according to the formula,



Required area = x (l + b –x) = 10 (80 + 60 – 10) = 10 × 130 = 1300 sq m

EXAMPLE 10. A rectangular grassy plot is 112m by 78 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path and the cost of constructing it at ₹ 2 per square metre?

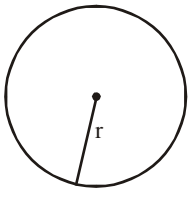
Sol. A= lb – (l – 2a) (b – 2a)

= 112 × 78 – (112 – 5) (78 – 5)

= 112 × 78 – 107 × 73 = 8736 – 7811 = 925 sq.m

∴ Cost of construction = rate × area = 2 × 925 = Rs. 1850

CIRCLE



Perimeter (Circumference) = $2\pi r = \pi d$

Area = πr^2; where r → radius

d → diameter

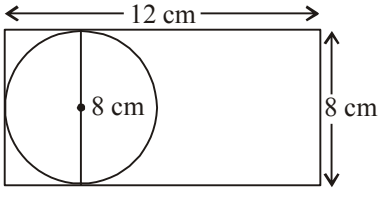
and $\pi = \frac{22}{7}$ or 3.14

Shortcut Approach

The length and breadth of a rectangle are l and b, then are of circle of maximum radius inscribed in that rectangle is $\frac{\pi b^2}{4}$.

EXAMPLE 11. Find the area of circle with maximum radius that can be inscribed in the rectangle of length 12 cm and breadth 8 cm.

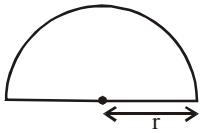
Sol. Here, l = 12 and b = 8



∴ Area of circle wiht maximum radius = $\frac{\pi b^2}{4}$

$= \frac{\pi(8)^2}{4} = \frac{4\pi}{4} = 16\pi \text{ cm}^2$

SEMICIRCLE



Perimeter = $\pi r + 2r$

Area = $\frac{1}{2} \times \pi r^2$

Shortcut Approach

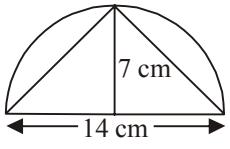
The are a of the largest triangle incribed in a semi-circle of radius r is equal to r^2.

EXAMPLE 12. The largest triangle is incribed in a semi-circle of radius 7 cm. Find the area inside the semi-circle which is not occupied by triangle.

Sol. Given that, radius = 7 cm, diameter = 14 cm

According to the formula,

Area of the triangle = 7^2 = 49 sq cm

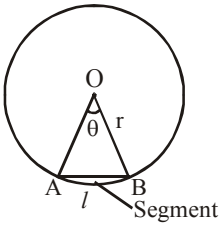


Area of semi-circle = $\frac{\pi r^2}{2} = \frac{\frac{22}{7} \times 7^2}{2} = 11 \times 7 = 77$ sq cm

Required answer = Area of semi-circle – Area of the largest triangle

= (77 – 49) sq cm = 28 sq cm

SECTOR OF A CIRCLE



Area of sector OAB = $\frac{\theta}{360} \times \pi r^2$

Length of an arc (l) = $\frac{\theta}{360} \times 2\pi r$

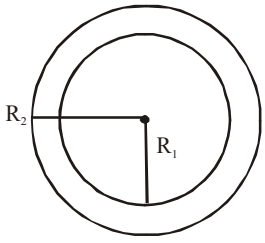
Area of segment = Area of sector – Area of triangle OAB

$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

Perimeter of segment = length of the arc + length of segment

AB = $\frac{\pi r \theta}{180} + 2r \sin\frac{\theta}{2}$

RING



Area of ring $= \pi\left(R_2^2 - R_1^2\right)$

EXAMPLE 13. A wire is looped in the form of a circle of radius 28 cm. It is re-bent into a square form. Determine the length of a side of the square.

(a) 44 cm (b) 45 cm
(c) 46 cm (d) 48 cm

Sol. (a) Length of the wire = Perimeter of the circle
$= 2\pi \times 28$
$= 176\ \text{cm}^2$

Side of the square $= \frac{176}{4} = 44$ cm

EXAMPLE 14. The radius of a circular wheel is $1\frac{3}{4}$ m. How many revolutions will it make in travelling 11 km ?

Sol. Distance to be travelled = 11km = 11000 m

Radius of the wheel $= 1\frac{3}{4}$ m $= \frac{7}{4}$ m

$\therefore$ Circumference of the wheel $= 2 \times \frac{22}{7} \times \frac{7}{4} = 11$ m

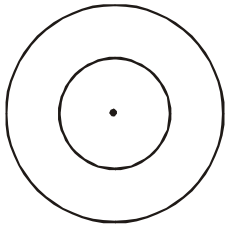
$\therefore$ In travelling 11 m, wheel makes 1 revolution.

$\therefore$ In travelling 11000 m the wheel makes $\frac{1}{11} \times 11000$ revolutions, i.e., 1000 revolutions.

EXAMPLE 15. The circumference of a circular garden is 1012m. Find the area of outsider road of 3.5 m width runs around it. Calculate the area of this road and find the cost of gravelling the road at ₹ 32 per 100 sqm.

Sol:

$A = \pi r^2$, $C = 2\pi r = 1012$



$\Rightarrow r = 1012 \times \frac{1}{2} \times \frac{7}{22} = 161$m

$\therefore$ Area of garden $= \frac{22}{7} \times 161 \times 161 = 81466$ sqm

Area of the road = area of bigger circle – area of the garden

Now, radius of bigger circle = 161 + 3.5 $= \frac{329}{2}$ m

$\therefore$ Area of bigger circle $= \frac{22}{7} \times \frac{329}{2} \times \frac{329}{2} = 85046\frac{1}{2}$ sq.m

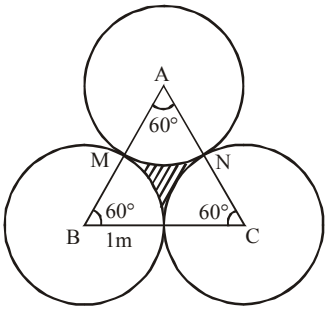
Thus, area of the road $= 85046\frac{1}{2} - 81466 = 3580\frac{1}{2}$ sqm.

Hence, cost $= ₹\frac{7161}{2} \times \frac{32}{100} = ₹1145.76$

EXAMPLE 16. There is an equilateral triangle of which each side is 2m. With all the three corners as centres, circles each of radius1 m are described.

(i) Calculate the area common to all the circles and the triangle.
(ii) Find the area of the remaining portion of the triangle.

Sol.



Area of each sector $= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \frac{22}{7} \times 1 \times 1$

$= \frac{1}{6} \times \frac{22}{7} = \frac{11}{21}\text{m}^2$

Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4} \times 2 \times 2 = \sqrt{3}\text{m}^2$

(i) Common area = 3 × Area of each sector

$= 3 \times \frac{11}{21} = \frac{11}{7} = 1.57\text{m}^2$

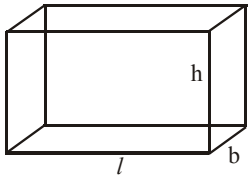
(ii) Area of the remaining portion of the triangle = Ar. of equilateral triangle – 3(Ar. of each sector)

$\sqrt{3} - 1.57 = 1.73 - 1.57 = 0.16\text{m}^2$

PART-II SOLID FIGURE

CUBOID

A cuboid is a three dimensional box.
Total surface area of a cuboid = 2 (lb + bh + lh)
Volume of the cuboid = $l$bh



Area of four walls = 2(l + b) × h

☞ Shortcut Approach

If length, breadth and height of a cuboid are changed by x%, y% and z% respectively, then its volume is increased by

$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2} \right]\%$$

Note: *Increment in the value is taken as positive and decrement in value is taken as negative. Positive result shows total increment and negative result shows total decrement.*

EXAMPLE 17. If all the dimensions of a cuboid are increased by 100%, by what per cent does the volume of cuboid increase?

Sol. Here, $x = y = z = 100$

According to the formula,

Percentage increase in volume

$$= \left[x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{100} \right]\%$$

$$= \left[100 + 100 + 100 + \frac{100 \times 100 + 100 \times 100 + 100 \times 100}{100} + \frac{100 \times 100 \times 100}{(100)^2} \right]\%$$

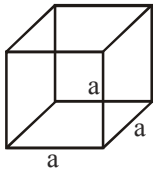
$$= \left[300 + \frac{10000 + 10000 + 10000}{100} + \frac{1000000}{10000} \right]\%$$

$$= \left[300 + \frac{30000}{100} + 100 \right]\%$$

$$= (300 + 300 + 100)\% = 700\%$$

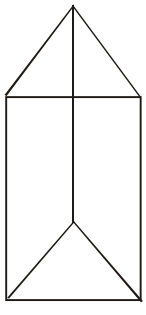
CUBE

A cube is a cuboid which has all its edges equal.
Total surface area of a cube = $6a^2$
Volume of the cube = a^3



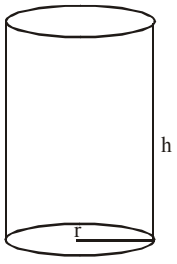
RIGHT PRISM

A prism is a solid which can have any polygon at both its ends.
Lateral or curved surface area = Perimeter of base × height
Total surface area = Lateral surface area + 2 (area of the end)
Volume = Area of base × height



RIGHT CIRCULAR CYLINDER

It is a solid which has both its ends in the form of a circle.
Lateral surface area = $2\pi rh$
Total surface area = $2\pi r(r + h)$
Volume = $\pi r^2 h$; where r is radius of the base and h is the height

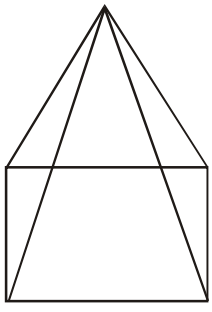


PYRAMID

A pyramid is a solid which can have any polygon at its base and its edges converge to single apex.
Lateral or curved surface area

$$= \frac{1}{2} \text{(perimeter of base)} \times \text{slant height}$$

Total surface area = lateral surface area + area of the base

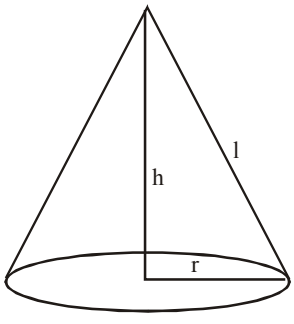


$$\text{Volume} = \frac{1}{3} \text{(area of the base)} \times \text{height}$$

RIGHT CIRCULAR CONE

It is a solid which has a circle as its base and a slanting lateral surface that converges at the apex.
Lateral surface area = πrl
Total surface area = $\pi r(l + r)$

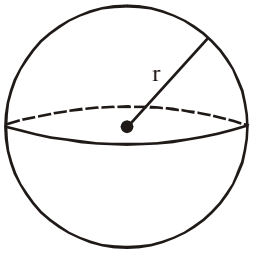
Volume = $\frac{1}{3}\pi r^2 h$; where r : radius of the base
h : height
l : slant height



SPHERE

It is a solid in the form of a ball with radius r.
Lateral surface area = Total surface area = $4\pi r^2$

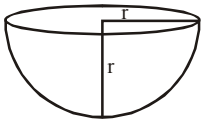
Volume = $\frac{4}{3}\pi r^3$; where r is radius.



HEMISPHERE

It is a solid half of the sphere.
Lateral surface area $= 2\pi r^2$
Total surface area $= 3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$; where r is radius



Shortcut Approach

If side of a cube or radius (or diameter) of sphere is increased by x%, then its volume increases by $\left[\left(1+\frac{x}{100}\right)^3-1\right]\times 100\%$

EXAMPLE 18. If side of a cube is increased by 10%, by how much per cent does its volume increase ?

Sol. Here, x = 10
According to the formula,
Percentage increase in volume

$$=\left[\left(1+\frac{x}{100}\right)^3-1\right]\times 100\%=\left[\left(1+\frac{10}{100}\right)^3-1\right]\times 100\%$$

$$=\left[\left(1+\frac{10}{100}\right)^3-1\right]\times 100\%=\left[\left(\frac{11}{10}\right)^3-1\right]\times 100\%$$

$$=\left[\frac{1331}{1000}-1\right]\times 100\%=(1.331-1)\times 100\%$$

$$=(0.331\times 100)\%=33.1\%$$

Shortcut Approach

If in a cylinder or cone, height and radius both change by x%, then volume changes by $\left[\left(1+\frac{x}{100}\right)^3-1\right]\times 100\%$

EXAMPLE 19. If in a cylinder, both height and radius increase by 100%, by what per cent does its volume increase?

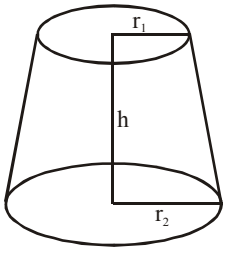
Sol. Here, x = 10
According to the formula,
Percentage increase in volume

$$=\left[\left(1+\frac{x}{100}\right)^3-1\right]\times 100\%=\left[\left(1+\frac{100}{100}\right)^3-1\right]\times 100\%$$

$$=[(2)^3-1]\times 100\%=(8-1)\times 100\%=700\%$$

FRUSTUM OF A CONE

When a cone cut the left over part is called the frustum of the cone.
Curved surface area $= \pi l(r_1+r_2)$
Total surface area $= \pi l(r_1+r_2)+\pi r_1^2+\pi r_2^2$
where $l=\sqrt{h^2+(r_1-r_2)^2}$



Volume $= \frac{1}{3}\pi h\left(r_1^2+r_1r_2+r_2^2\right)$

EXAMPLE 20. The sum of length, breadth and height of a room is 19 m. The length of the diagonal is 11 m. The cost of painting the total surface area of the room at the rate of ₹10 per m^2 is :

(a) ₹ 240 **(b)** ₹2400
(c) ₹ 420 **(d)** ₹ 4200

Sol. (b) Let length, breadth and height of the room be ℓ, b and h, respectively. Then,

$$\ell+b+h=19 \quad ...(i)$$

and $\sqrt{\ell^2+b^2+h^2}=11$

$$\Rightarrow \ell^2+b^2+h^2=121 \quad ...(ii)$$

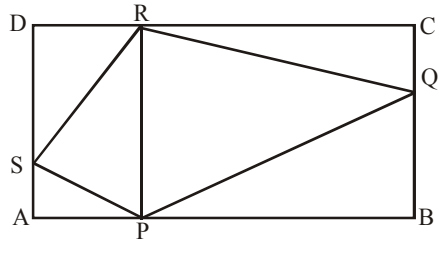
Area of the surface to be painted
$= 2(lb+bh+hl)$

$$(\ell+b+h)^2=\ell^2+b^2+h^2+2(\ell b+\ell h+h\ell)$$

$$\Rightarrow 2(\ell b+bh+h\ell)=(19)^2-121=361-121=240$$

Surface area of the room $=240\ m^2$.
Cost of painting the required area $= 10\times 240 =$ ₹ 2400

EXAMPLE 21. ABCD is a parallelogram. P, Q, R and S are points on sides AB, BC, CD and DA, respectively such that AP = DR. If the area of the rectangle ABCD is 16 cm^2, then the area of the quadrilateral PQRS is :



(a) 6 cm^2 **(b)** 6.4 cm^2
(c) 4 cm^2 **(d)** 8 cm^2

Sol. (d) Area of the quadrilateral PQRS
= Area of ΔSPR + Area of ΔPQR

$$= \frac{1}{2} \times PR \times AP + \frac{1}{2} \times PR \times PB$$

$$= \frac{1}{2} \times PR(AP + PB) = \frac{1}{2} \times AD \times AB$$

(PR = AD and AP + PB = AB)

$$= \frac{1}{2} \times \text{Area of rectangle ABCD} = \frac{1}{2} \times 16 = 8\,cm^2$$

EXAMPLE 22. A road roller of diameter 1.75 m and length 1 m has to press a ground of area 1100 sqm. How many revolutions does it make ?

Sol. Area covered in one revolution = curved surface area

$$\therefore \text{ Number of revolutions} = \frac{\text{Total area to be pressed}}{\text{Curved surface area}}$$

$$= \frac{1100}{2\pi rh} = \frac{1100}{2 \times \frac{22}{7} \times \frac{1.75}{2} \times 1}$$

$$= 200$$

EXAMPLE 23. The annual rainfall at a place is 43 cm. Find the weight in metric tonnes of the annual rain falling there on a hectare of land, taking the weight of water to be 1 metric tonne to the cubic metre.

Sol. Area of land = 10000 sqm

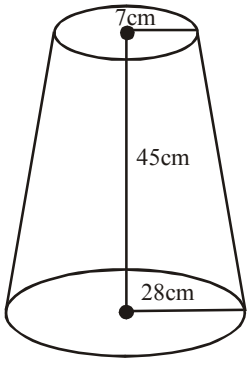
$$\text{Volume of rainfall} = \frac{10000 \times 43}{100} = 4300\,m^3$$

Weight of water = 4300 × 1 m tonnes = 4300 m tonnes

EXAMPLE 24. The height of a bucket is 45 cm. The radii of the two circular ends are 28 cm and 7 cm, respectively. The volume of the bucket is :

(a) 38610 cm^3 **(b) 48600 cm^3**
(c) 48510 cm^3 **(d) None of these**

Sol. (c) Here $r_1 = 7$ cm, $r_2 = 28$ cm and $h = 45$ cm



Volume of the frustum of a cone

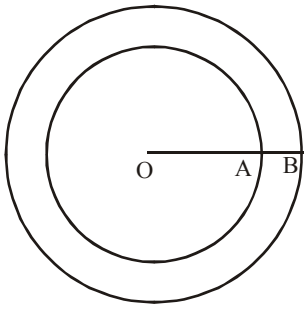
$$\text{Volume of the bucket} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

Hence, the required volume

$$= \frac{1}{3} \times \frac{22}{7} \times 45(28^2 + 7^2 + 28 \times 7) = 48510\,cm^3$$

EXAMPLE 25. A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of the tube be 140 cm, find the number of cubic cm of iron in it.

Sol. Height = 140 cm
External diameter = 50 cm
$\therefore$ External radius = 25 cm



Also, internal radius OA = OB – AB = 25 – 2 = 23 cm
$\therefore$ Volume of iron = $V_{external} - V_{internal}$

$$= \frac{22}{7} \times 140\left(25^2 - 23^2\right) = 42240 \text{ cu. cm.}$$

EXAMPLE 26. A cylindrical bath tub of radius 12 contains water to a depth of 20 cm. A spherical iron ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

(a) 80 cm^2 **(b) 84 cm^2**
(c) 104 cm^2 **(d) 76 cm^2**

Sol. (b) Volume of the spherical ball = volume of the water displaced.

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi(12)^2 \times 6.75$$

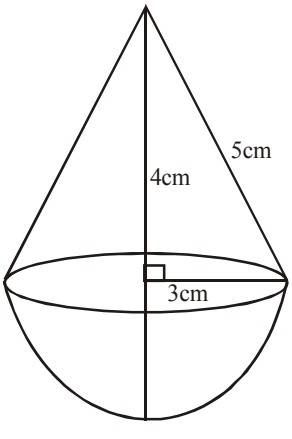
$$\Rightarrow r^3 = \frac{144 \times 6.75 \times 3}{4} = 729$$

or r = 9 cm,

EXAMPLE 27. A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the surface area of the toy. (Use $\pi = 3.14$).

Sol. The radius of the hemisphere $= \frac{1}{2} \times 6 = 3$ cm

Now, slant height of cone $= \sqrt{3^2 + 4^2} = 5$cm



The surface area of the toy

= Curved surface of the conical portion + curved surface of the hemisphere

$= (\pi \times 3 \times 5 + 2\pi \times 3^2)\ cm^2$

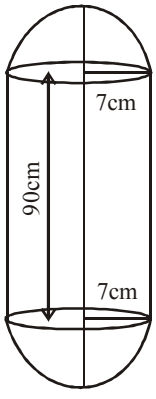
$= 3.14 \times 3\,(5+6)\ cm^2 = 103.62\ cm^2.$

EXAMPLE 28. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Re. 1 per dm^2.

Sol. Let the height of the cylinder be h cm.

Then $h + 7 + 7 = 104$

$\Rightarrow h = 90$



Surface area of the solid

= 2 × curved surface area of hemisphere + curved surface area of the cylinder

$= \left(2 \times 2 \times \frac{22}{7} \times 7 \times 7 + 2 \times \frac{22}{7} \times 7 \times 90\right) cm^2$

$= 616 + 3960\ cm^2 = 4576\ cm^2$

Cost of polishing the surface of the solid

$= ₹.\ \frac{4576 \times 1}{100} = ₹45.76$

EXAMPLE 29. A regular hexagonal prism has perimeter of its base as 600 cm and height equal to 200 cm. How many litres of petrol can it hold ? Find the weight of petrol if density is 0.8 gm/cc.

Sol. $\text{Side of hexagon} = \frac{\text{Perimeter}}{\text{Number of sides}} = \frac{600}{6} = 100\ cm$

$\text{Area of regular hexagon} = \frac{3\sqrt{3}}{2} \times 100 \times 100 = 25950\ \text{sq.cm.}$

Volume = Base area × height

$= 25950 \times 200 = 5190000\ \text{cu.cm.} = 5.19\ \text{cu.m.}$

Weight of petrol = Volume × Density

$= 5190000 \times 0.8\ gm/cc$

$= 4152000\ gm = 4152\ kg.$

EXAMPLE 30. A right pyramid, 12 cm high, has a square base each side of which is 10 cm. Find the volume of the pyramid.

Sol. Area of the base = 10 × 10 = 100 sq.cm.

Height = 12 cm

$\therefore\ \text{Volume of the pyramid} = \frac{1}{3} \times 100 \times 12 = 400\ \text{cu.cm.}$

EXAMPLE 31. Semi-circular lawns are attached to both the edges of a rectangular field measuring 42 m × 35m. The area of the total field is :

(a) 3818.5 m^2 (b) 8318 m^2
(c) 5813 m^2 (d) 1358 m^2

Sol. (a) Area of the field

$= 42 \times 35 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (21)^2 + 2 \times \frac{1}{2} \times \frac{22}{7} \times (17.5)^2$

$= 1470 + 1386 + 962.5 = 3818.5\ m^2$

EXAMPLE 32. A frustum of a right circular cone has a diameter of base 10 cm, of top 6 cm, and a height of 5 cm; find the area of its whole surface and volume.

Sol. Here $r_1 = 5$ cm, $r_2 = 3$ cm and $h = 5$ cm.

$\therefore\ l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{5^2 + (5-3)^2} = \sqrt{29}\ cm = 5.385\ cm$

$\therefore$ Whole surface of the frustum

$= \pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

$= \frac{22}{7} \times 5.385(5+3) + \frac{22}{7} \times 5^2 + \frac{22}{7} \times 3^2 = 242.25\ \text{sq.cm.}$

$\text{Volume} = \frac{\pi h}{3}(r_1^2 + r_1 r_2 + r_2^2)$

$= \frac{22}{7} \times \frac{5}{3}\left[5^2 + 5 \times 3 + 3^2\right] = 256.67\ \text{cu. cm.}$

EXAMPLE 33. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of one end, and the other end as its base. The volume of the cylinder, hemisphere and the cone are, respectively in the ratio :

(a) 2 : 3 : 2 (b) 3 : 2 : 1
(c) 3 : 1 : 2 (d) 1 : 2 : 3

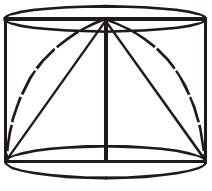
Sol. (b) We have,

radius of the hemisphere = raidus of the cone

= height of the cone

= height of the cylinder = r (say)

Then, ratio of the volumes of cylinder, hemisphere and cone



$= \pi r^3 : \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^3 = 1 : \frac{2}{3} : \frac{1}{3} = 3 : 2 : 1$

EXERCISE

1. The area of rectangular field is 460 square metres. If the length is 15 per cent more than the breadth ,what is the breadth of the rectangular field?
   (a) 15 metres (b) 26 metres
   (c) 34.5 metres (d) Cannot be determined
   (e) None of these
2. What will be the cost of gardening 1-metre – broad boundary around a rectangular plot having perimeter of 340 metres at the rate of ₹ 10 per square metre?
   (a) ₹ 3400 (b) ₹ 1700
   (c) ₹ 3440 (d) Cannot be determined
   (e) None of these
3. The cost of paint is ₹ 60 per kilogram. A kilogram paint covers 20 square feet. How much will it cost to paint the outside of a cube having each side 10 feet?
   (a) ₹ 3000 (b) ₹ 900
   (c) ₹ 1800 (d) ₹ 360
   (e) None of these
4. 20 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be required to fill the same tank if the capacity of each bucket is 9 litres?
   (a) 30 (b) 32
   (c) 60 (d) Data inadequate
   (e) None of these
5. The breadth of a rectangular hall is two-thirds of its length. If the area of the hall is 2400 sq metres, what is the length in metres?
   (a) 120 (b) 80
   (c) 60 (d) 40
   (e) None of these
6. If a pair of opposite sides of a square is increased by 5 cm each, then the ratio of the sides of the new figure is 3 : 2. What is the original area of the square?
   (a) 125 cm^2 (b) 225 cm^2
   (c) 81 cm^2 (d) 100 cm^2
   (e) None of these
7. An equilateral triangle, a square and a circle have equal perimeters. If T denotes the area of the triangle, S, the area of the square and C, the area of the circle, then :
   (a) S > T > C (b) T > C > S
   (c) T > S > C (d) C > S > T
   (e) None of these
8. The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base?
   (a) 1.4 metres (b) 2.8 metres
   (c) 28 metres (d) 14 metres
   (e) None of these
9. The internal measurements of a box with lid are 115 × 75 × 35 cm^3 and the wood of which it is made is 2.5 cm thick. Find the volume of wood.
   (a) 82,125 cm^3 (b) 70,054 cm^3
   (c) 78,514 cm^3 (d) None of these
   (e) None of these
10. The length and the breadth of a rectangle are in the ratio of 3 : 2 respectively. If the sides of the rectangle are extended on each side by 1 metre, the ratio of length to breadth becomes 10 : 7. Find the area of the original rectangle in square metres.
   (a) 56 (b) 50
   (c) 80 (d) Data inadequate
   (e) None of these
11. A right circular cone is exactly fitted inside a cube in such a way that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is on the opposite face of the cube. If the volume of the cube is 343 cc, what **approximately** is the volume of the cone?
   (a) 80 cc (b) 90 cc
   (c) 110 cc (d) 105 cc
   (e) 100 cc
12. If the length of a rectangle is increased by 20% and the breadth is reduced by 20%, what will be the effect on its area?
   (a) 4% increase (b) 6% increase
   (c) 5% decrease (d) 4% decrease
   (e) None of these
13. The ratio between the length and the breadth of a rectangular plot is 7 : 5. If the perimeter of the plot is 144 metres, what is its area?
   (a) 1320 sq. metres (b) 1260 sq. metres
   (c) 1280 sq. metres (d) 1380 sq. metres
   (e) None of these
14. The perimeter of a rectangle is equal to the perimeter of a right-angled triangle of height 12 cm. If the base of the triangle is equal to the breadth of the rectangle, what is the length of the rectangle"
   (a) 18 cm (b) 24
   (c) 22 cm (d) Data inadequate
   (e) None of these
15. The squared value of the diagonal of a rectangle is $(64 + B^2)$ sq cm, where B is less than 8 cm. What is the breadth of that rectangle?
   (a) 6 cm (b) 10 cm
   (c) 8 cm (d) Data inadequate
   (e) None of these
16. If the height of a triangle is decreased by 40%, land its base is increased by 40%, what will be the effect on its area?
   (a) No change (b) 16% increase
   (c) 8% decrease (d) 16% decrease
   (e) None of these

17. A circular ground whose diameter is 35 metres, has a 1.4 metre-broad garden around it. What is the area of the garden in square metres?
(a) 160.16 (b) 6.16
(c) 122.66 (d) Data inadeuate
(e) None of these

18. The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot at the rate of ₹ 26.50 per metre is ₹ 5,300, what is the length of the plot (in metres)?
(a) 40 (b) 120
(c) 50 (d) Data inadequate
(e) None of these

19. A rectangular plate is of 6 in breadth and 12 in length. Two apertures of 2 in diameter each and one aperture of 1 in diameter have been made with the help of a gas cutter. What is the area of the remaining portion of the plate?
(a) 62.5 sq. in (b) 68.5 sq. in
(c) 64.5 sq. in (d) 66.5 sq. in
(e) None of these

20. What would be the length of the diagonal of a square plot whose area is equal to the area of a rectangular plot of 45 m length and 40 m width?
(a) 42.5 m (b) 60 m
(c) 4800 m (d) Data inadequate
(e) None of these

21. What will be the ratio between the area of a rectangle and the area of a triangle with one of the sides of rectangle as base and a vertex on the opposite side of rectangle.
(a) 1 : 2 (b) 2 : 1
(c) 3 : 1 (d) Data inadequate
(e) None of these

22. Two roads XY and YZ of 15 metres and 20 metres length respectively are perpendicular to each other. What is the distance between X & Z by the shortest route?
(a) 35 metres (b) 30 metres
(c) 24 metres (d) 25 metres
(e) None of these

23. What will be the area of a semi-circle of 14 metres diameter?
(a) 154 sq metres (b) 77 sq metres
(c) 308 sq metres (d) 22 sq metres
(e) None of these

24. The area of a right-angled triangle is two-thirds of the area of a rectangle. The base of the triangle is 80 percent of the breadth of the rectangle. If the perimeter of the rectangle is 200 cm, what is the height of the triangle?
(a) 20 cm (b) 30 cm
(c) 15 cm (d) Data inadequate
(e) None of these

25. The area of a rectangular plot is 15 times its breadth. If the difference between the length and the breadth is 10 metres, what is its breadth?
(a) 10 metres (b) 5 metres
(c) 7.5 metres (d) Data inadequate
(e) None of these

26. A rectangular garden has a 5-metre-wide road outside around all the four sides. The area of the road is 600 square metres. What is the ratio between the length and the breadth of that plot?
(a) 3 : 2 (b) 4 : 3
(c) 5 : 4 (d) Data inadequate
(e) None of these

27. Four sheets of 50 cm × 5 cm are to be arranged in such a manner that a square could be formed. What will be the area of inner part of the square so formed?
(a) 2000 cm^2 (b) 1600 cm^2
(c) 1800 cm^2 (d) 2500 cm^2
(e) None of these

28. In order to fence a square Manish fixed 48 poles. If the distance between two poles, is 5 metres then what will be the area of the square so formed?
(a) Cannot be determined (b) 2600 cm^2
(c) 2500 cm^2 (d) 3025 cm^2
(e) None of these

29. The area of a side of a box is 120 sq cm. The area of the other side of the box is 72 sq cm. If the area of the upper surface of the box is 60 sq cm then find the volume of the box.
(a) 259200 cm^3 (b) 86400 cm^3
(c) 720 cm^3 (d) Cannot be determined
(e) None of these

30. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle ?
(a) 88 cm^2 (b) 154 cm^2
(c) 1250 cm^2 (d) 616 cm^2
(e) None of these

31. The cost of carpeting a room 18m long with a carpet 75 cm wide at ₹ 4.50 per metre is ₹810. The breadth of the room is:
(a) 7m (b) 7.5m
(c) 8m (d) 8.5m
(e) None of these

32. If the perimeter and diagonal of a rectangle are 14 and 5 cms respectively, find its area.
(a) 12 cm^2 (b) 16 cm^2
(c) 20 cm^2 (d) 24 cm^2
(e) None of these

33. In an isoscele right angled triangle, the perimeter is 20 metre. Find its area.
(a) 9,320 m^2 (b) 8,750 m^2
(c) 7,980 m^2 (d) 6,890 m^2
(e) None of these

34. When the circumference and area of a circle are numerically equal, then the diameter is numerically equal to
(a) area (b) circumference
(c) 4 (d) 2 π
(e) None of these

35. In a parallelogram, the length of one diagonal and the perpendicular dropped on that diagonal are 30 and 20 metres respectively. Find its area.
(a) 600 m^2 (b) 540 m^2
(c) 680 m^2 (d) 574 m^2
(e) None of these

36. The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions ? $\left(\text{use } \pi = \frac{22}{7}\right)$
(a) $40\,m^2$ (b) $44\,m^2$
(c) $48\,m^2$ (d) $36\,m^2$
(e) None of these

37. The area of a triangle is 615 m^2. If one of its sides is 123 metre, find the length of the perpendicular dropped on that side from opposite vertex.
(a) 15 metres (b) 12 metres
(c) 10 metres (d) 9 metres
(e) None of these

38. A horse is tethered to one corner of a rectangular grassy field 40 m by 24 m with a rope 14 m long. Over how much area of the field can it graze?
(a) $154cm^2$ (b) $308\,m^2$
(c) $150\,m^2$ (d) $407\,m^2$
(e) None of these

39. How many plants will be there in a circular bed whose outer edge measure 30 cms, allowing 4 cm^2 for each plant ?
(a) 18 (b) 750
(c) 24 (d) 120
(e) None of these

40. From a square piece of a paper having each side equal to 10 cm, the largest possible circle is being cut out. The ratio of the area of the circle to the area of the original square is nearly :
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$
(c) $\frac{5}{6}$ (d) $\frac{6}{7}$
(e) None of these

41. A square carpet with an area 169 m^2 must have 2 metres cut-off one of its edges in order to be a perfect fit for a rectangualar room. What is the area of rectangular room?
(a) $180\,m^2$ (b) $164\,m^2$
(c) $152\,m^2$ (d) $143\,m^2$
(e) None of these

42. A picture 30" × 20" has a frame 2½" wide. The area of the picture is approximately how many times the area of the frame?
(a) 4 (b) $2\frac{1}{2}$
(c) 2 (d) 5
(e) None of these

43. A rectangular plot 15 m ×10 m, has a path of grass outside it. If the area of grassy pathway is 54 m^2, find the width of the path.
(a) 4m (b) 3m
(c) 2m (d) 1m
(e) None of these

44. If the area of a circle decreases by 36%, then the radius of a circle decreases by
(a) 20% (b) 18%
(c) 36% (d) 64%
(e) None of these

45. The floor of a rectangular room is 15 m long and 12 m wide. The room is surrounded by a verandah of width 2 m on all its sides. The area of the verandah is :
(a) $124\,m^2$ (b) $120\,m^2$
(c) $108\,m^2$ (d) $58\,m^2$
(e) None of these

46. A typist uses a paper 12" by 5" length wise and leaves a margin of 1" at the top and the bottom and a margin of ½"on either side. What fractional part of the paper is available to him for typing ?
(a) $\frac{2}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) $\frac{5}{7}$
(e) None of these

47. A rectangular lawn 70 m × 30 m has two roads each 5 metres wide, running in the middle of it, one parallel to the length and the other parallel to the breadth. Find the cost of gravelling the road at the rate of ₹ 4 per square metre.
(a) ₹ 2,000 (b) ₹ 1,800
(c) ₹ 1,900 (d) ₹ 1,700
(e) None of these

48. A circular grass lawn of 35 metres in radius has a path 7 metres wide running around it on the outside. Find the area of path.
(a) $1694\,m^2$ (b) $1700\,m^2$
(c) 1598 m^2 (d) $1500\,m^2$
(e) None of these

49. A cylindrical bucket of height 36 cm and radius 21 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed, the height of the heap being 12 cm. The radius of the heap at the base is :
(a) 63 cm (b) 53 cm
(c) 56 cm (d) 66 cm
(e) None of these

50. The radius of the wheel of a bus is 70 cms and the speed of the bus is 66 km/h, then the r.p.m. (revolutions per minutes) of the wheel is
(a) 200 (b) 250
(c) 300 (d) 330
(e) None of these

51. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. The area of the triangle is
(a) $72\,cm^2$ (b) $60\,cm^2$
(c) $66\,cm^2$ (d) $65\,cm^2$
(e) None of these

52. The cross section of a canal is a trapezium in shape. If the canal is 7 metres wide at the top and 9 metres at the bottom and the area of cross-section is 1280 square metres, find the length of the canal.
(a) 160 metres (b) 172 metres
(c) 154 metres (d) 165 metres
(e) None of these

53. It is required to fix a pipe such that water flowing through it at a speed of 7 metres per minute fills a tank of capacity 440 cubic metres in 10 minutes. The inner radius of the pipe should be :
(a) $\sqrt{2}$ m (b) 2m
(c) $\frac{1}{2}$ m (d) $\frac{1}{\sqrt{2}}$ m
(e) None of these

54. The area of a square field is 576 km^2. How long will it take for a horse to run around at the speed of 12 km/h ?
(a) 12 h (b) 10 h
(c) 8 h (d) 6 h
(e) None of these

55. The area of a rectangular field is 144 m^2. If the length had been 6 metres more, the area would have been 54 m^2 more. The original length of the field is
(a) 22 metres (b) 18 metres
(c) 16 metres (d) 24 metres
(e) None of these

56. A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?
(a) 46 (b) 81
(c) 126 (d) 252
(e) None of these

57. A farmer wishes to start a 100 square metres rectangular vegetable garden. Since he has only 30 m barbed wire, he fences three sides of the garden letting his house compound wall act as the fourth side fencing. The dimension of the garden is:
(a) 15 m × 6.67 m (b) 20 m × 5 m
(c) 30 m × 3.33 m (d) 40 m × 2.5 m
(e) None of these

58. A rectangular tank measuring 5m × 4.5 m × 2.1 m is dug in the centre of the field measuring 13.5 m × 2.5. The earth dug out is spread evenly over the remaining portion of a field. How much is the level of the field raised ?
(a) 4.0 m (b) 4.1 m
(c) 4.2 m (d) 4.3 m
(e) None of these

59. A rectangular paper, when folded into two congruent parts had a perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper?
(a) 140 cm^2 (b) 240 cm^2
(c) 560 cm^2 (d) 160 cm^2
(e) None of these

60. The length and breadth of the floor of the room are 20 feet and 10 feet respectively. Square tiles of 2 feet length of different colours are to be laid on the floor. Black tiles are laid in the first row on all sides. If white tiles are laid in the one-third of the remaining and blue tiles in the rest, how many blue tiles will be there?
(a) 16 (b) 24
(c) 32 (d) 48
(e) None of these

61. Four equal circles are described about the four corners of a square so that each touches two of the others. If a side of the square is 14 cm, then the area enclosed between the circumferences of the circles is :
(a) 24 cm^2 (b) 42 cm^2
(c) 154 cm^2 (d) 196 cm^2
(e) None of these

ANSWER KEY

1	(e)	8	(e)	15	(a)	22	(d)	29	(c)	36	(b)	43	(c)	50	(b)	57	(b)
2	(c)	9	(a)	16	(d)	23	(b)	30	(d)	37	(c)	44	(a)	51	(b)	58	(c)
3	(c)	10	(e)	17	(a)	24	(d)	31	(b)	38	(a)	45	(a)	52	(a)	59	(a)
4	(a)	11	(b)	18	(e)	25	(b)	32	(a)	39	(a)	46	(a)	53	(a)	60	(a)
5	(c)	12	(d)	19	(e)	26	(d)	33	(a)	40	(a)	47	(c)	54	(c)	61	(b)
6	(d)	13	(b)	20	(b)	27	(e)	34	(c)	41	(d)	48	(a)	55	(c)		
7	(d)	14	(d)	21	(b)	28	(e)	35	(a)	42	(a)	49	(a)	56	(c)		

Hints & Explanations

1. (e) Let the breadth of the rectangular field be 'x' m. Then, length of the field will be

$$x+\frac{x\times 15}{100}=\frac{23x}{20}$$

Now, $x\times\frac{23x}{20}=460$

or, $23x^2=460\times 20$

or, $x^2=20\times 20$

or, $x=20$ m

2. (c) Let l and b be the length and breadth of rectangular plot respectively.

$\therefore$ According to the question, we have

$2(l+b)=340\Rightarrow l+b=170$

Now, $(l+2)$ and $(b+2)$ be the length and breadth of plot with boundary.

$\therefore$ Required area $=(l+2)(b+2)-lb$

$=lb+2l+2b+4-lb$

$=2(l+b)+4=344$

$\therefore$ Required cost $=344\times 10=3440$

3. (c) Area of cube

$=6\times(\text{side})^2=6\times 10\times 10=600$ square feet.

Cost to paint outside of the cube $=\frac{600}{20}\times 60$

$=₹1800$

4. (a) Capacity of the tank $=20\times 13.5=270$ litres

When the capacity of each bucket = 9 litres, then the required no. of buckets

$=\frac{270}{9}=30$

5. (c) Let the length of the rectangular hall be *'x' m*, then the breadth of the rectangular hall $=\frac{2x}{3}$ m.

Area of hall $=\frac{2x}{3}\times x=\frac{2x^2}{3}$

or, $\frac{2x^2}{3}=2400$ or, $x=60$ m

6. (d) Let the original side of the square $=x$ cm

$\frac{x+5}{x}=\frac{3}{2}$ *or* $2x+10=3x$

$\therefore$ $x=10$ cm

$\therefore$ original area $=(10)^2=100$ cm^2

7. (d) Let the perimeter of each be a.

Then, side of the equilateral triangle $=\frac{a}{3}$; side of the square $=\frac{a}{4}$; radius of the circle $=\frac{a}{2\pi}$.

$\therefore\ T=\frac{\sqrt{3}}{4}\times\left(\frac{a}{3}\right)^2=\frac{\sqrt{3}a^2}{36}$; $S=\left(\frac{a}{4}\right)^2=\frac{a^2}{16}$;

$C=\pi\times\left(\frac{a}{2\pi}\right)^2=\frac{a^2}{4\pi}=\frac{7a^2}{88}$.

So, C > S > T.

8. (e) Capacity (volume) of a cylindrical tank $=\pi r^2h$

(Here r = radius and h = height of the tank)

Now, from the question, $246.4\times 0.001=\frac{22}{7}\times r^2\times 4$

[$\because$ 1 litre $=1000$ cm$^3=0.001$ m^3]

or, $\frac{0.2464\times 7}{22\times 4}=r^2$

or, $r=0.14$ m

or, diameter $=2r=0.28$m

9. (a) Internal volume $=115\times 75\times 35=3{,}01{,}875$ cm^3

External volume

$=(115+2\times 2.5)\times(75+2\times 2.5)\times(35+2\times 2.5)$

$=120\times 80\times 40=3{,}84{,}000$ cm^3

$\therefore$ Volume of wood = External volume – Internal volume

$=3{,}84{,}000-3{,}01{,}875=82{,}125$ cm^3

10. (e) Let the length and breadth be ℓ and b respectively.

$\frac{\ell}{b}=\frac{3}{2}$ or, $\ell=\frac{3}{2}b$ .....(i)

$\frac{\ell+1}{b+1}=\frac{10}{7}$ or, $7\ell-10b=3$ .....(ii)

From eq. (i)

$10.5b-10b=6$ or, $0.5b=3$ or, $b=6$ and $\ell=9$

Area $=\ell\times b=6\times 9=54$m^2

11. (b) Edge of the cube = $\sqrt[3]{343} = 7$ cm

$\therefore$ Radius of cone = 3.5 cm

height = 7 cm

volume of cone = $\frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times(3.5)^2\times 7 = \frac{1}{3}\times 22\times 12.25 \approx 90$cc

12. (d) Percentage change $= x - y - \frac{xy}{100}$

$= 20 - 20 - \frac{20\times 20}{100} = -4\% = 4\%$ decrease

13. (b) Let the length and breadth be $7x$ and $5x$ respectively.

Then, $P = 2(7x + 5x) = 144 \Rightarrow x = 6$

Area = $7 \times 6 \times 5 \times 6 = 1260$sq.m.

14. (d) P = 2 (ℓ + b) = L + B + h = L + b + 12.

Data inadequate.

15. (a) Diagonal^2 = 64 + B^2 or, $10^2 = 64 + 6^2$

16. (d) Regd effect = $\left[+40 - 40 - \frac{40\times 40}{100}\right]\% = -16\%$

i.e., the area will decrease by 16%

17. (a) Req. area = $\pi[(17.5 + 1.4)^2 - (17.5)^2]$

$= \frac{22}{7} \times (36.4 \times 1.4)$ [since $a^2 - b^2 = (a + b)(a - b)$]

$= 22 \times 36.4 \times 0.2 = 160.16$ sq m

18. (e) Perimeter of the rectangular plot = $[(b + 20) + b] \times 2$

$= \frac{5300}{26.5} = 200$

$\therefore (2b + 20)2 = 200$

$\Rightarrow b = 40$

$\Rightarrow l = 40 + 20 = 60$ m

19. (e) Required area = $6 \times 12 - \left\{2 \times \pi\left(\frac{2}{2}\right)^2 + \pi\left(\frac{1}{2}\right)^2\right\}$

$= 72 - \left(2\pi + \frac{\pi}{4}\right) = 72 - \frac{9\pi}{4} = 72 - \frac{9}{4}\times\frac{22}{7}$

$= 72 - \left(\frac{99}{14}\right) = 64.94$ sq in.

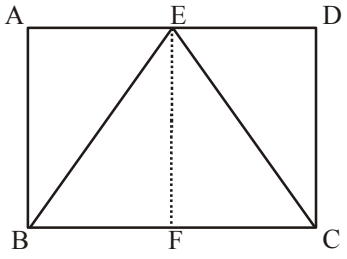
20. (b) $a^2 = 45 \times 40 = 1800$

$\therefore \quad a = \sqrt{1800} = 30\sqrt{2}$

$\therefore$ Diagonal of the square = $\sqrt{2}\,a = \sqrt{2} \times 30\sqrt{2}$

$= 30 \times 2 = 60$ m

21. (b)



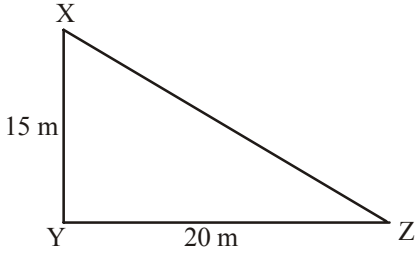
Area of $\Delta EBC = \frac{1}{2} \times BC \times EF$

$= \frac{1}{2} \times BC \times AB$ [Since, $EF = AB$]

Area of $\Delta EBC = \frac{1}{2} \times$ area of $\Delta ABCD$

$\therefore$ Required ratio = 2 : 1.

22. (d) XYZ is a right-angled triangle



XZ = $\sqrt{15^2 + 20^2} = \sqrt{625} = 25$ m

23. (b) Area of semicircle = $\frac{1}{2}\pi r^2$

$= \frac{1}{2}\times\frac{22}{7}\times 7\times 7 = 77\text{m}^2$

24. (d) Let the base and height of triangle, and length and breadth of rectangle be L and h, and L_1 and b_1 respectively. Then $\frac{1}{2}\times L\times h = \frac{2}{3}\times L_1\times b_1$ ...(i)

$L = \frac{4}{5}b_1$ ....(ii) and $L_1 + b_1 = 100$ .....(iii)

In the above we have three equations and four unknowns. Hence the value of 'h' can't be determined.

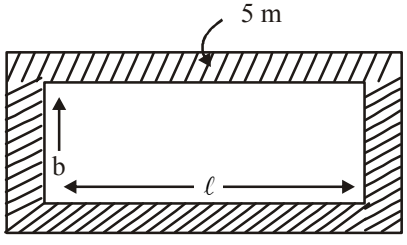
25. (b) $L \times B = 15 \times B$

$\therefore \quad L = 15$ m

and $L - B = 10$

$\therefore \quad B = 15 - 10 = 5$ m

26. (d)



Area of shaded portion = 600 m.

$\therefore$ $(l+10)(b+10)-lb=600$

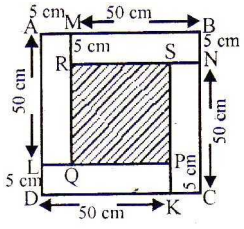
or, $lb+10b+10l+100-lb=600$

or, $10(b+(l)=500$

$\therefore$ $b+l=50$

From this equation we can't get the required ratio.

27. (e)



The four sheets are BMRN, AMQL, NSKC and DLPK

$\therefore$ Side of the new square sheet = 50 + 5 = 55 cm and the side of the inner part of the square (55 – 10) = 45 cm

Hence, area = $(45)^2 = 2025$ sq. cm.

28. (e) Let the side of the square be x *m*.

$\therefore$ Perimeter of the square = $48 \times 5 = 4x$ $\therefore$ $x = 60$ m

$\therefore$ Area = $(60)^2 = 3600\ m^2$

29. (c) Volume of the box $= \sqrt{120 \times 72 \times 60} = 720\ cm^3$

30. (d) Perimeter of the circle = $2\pi r = 2(18+26)$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 14$$

$\therefore$ Area of the circle

$$= \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\ cm^2.$$

31. (b) Length of the carpet $= \left(\frac{\text{Total cost}}{\text{Rate/m}}\right)$

$$= \left(\frac{8100}{45}\right) m = 180\ m.$$

Area of the room = Area of the carpet

$$= \left(180 \times \frac{75}{100}\right) m^2 = 135\ m^2.$$

$$\therefore \text{ Breadth of the room} = \left(\frac{\text{Area}}{\text{Length}}\right) = \left(\frac{135}{18}\right) m$$

$= 7.5$ m.

32. (a) In a rectangle,

$$\frac{(\text{perimeter})^2}{4} = (\text{diagonal})^2 + 2 \times \text{area}$$

$$\Rightarrow \quad \frac{(14)^2}{4} = 5^2 + 2 \times \text{area}$$

$49 = 25 + 2 \times$ area

$$\therefore \text{ Area} = \frac{49-25}{2} = \frac{24}{2} = 12\ cm^2$$

33. (a) In an isoscele right angled triangle,

Area = $23.3 \times \text{perimeter}^2$

$= 23.3 \times 20^2 = 9320\ m^2$

34. (c) According to question, circumference of circle = Area of circle

$$\text{or } \pi d = \pi\left(\frac{d}{2}\right)^2 \qquad [\text{where d = diameter}]$$

$\therefore$ $d = 4$

35. (a) In a parallelogram.

Area = Diagonal × length of perpendicular on it.

$= 30 \times 20 = 600\ m^2$

36. (b) Required area covered in 5 revolutions

$$= 5 \times 2\pi rh = 5 \times 2 \times \frac{22}{7} \times 0.7 \times 2 = 44\ m^2$$

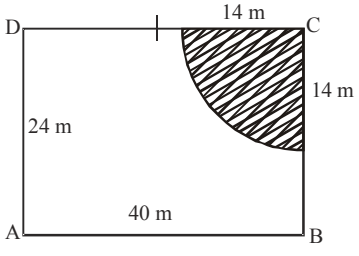
37. (c) In a triangle,

$$\text{Area} = \frac{1}{2} \times \text{length of perpendicular} \times \text{base}$$

$$\text{or } \quad 615 = \frac{1}{2} \times \text{length of perpendicular} \times 123$$

$$\therefore \text{ Length of perpendicular} = \frac{615 \times 2}{123} = 10\ m.$$

38. (a)



Area of the shaded portion

$$= \frac{1}{4} \times \pi(14)^2$$

$= 154\ m^2$

39. (a) Circumference of circular bed = 30 cm

$$\text{Area of circular bed} = \frac{(30)^2}{4\pi}$$

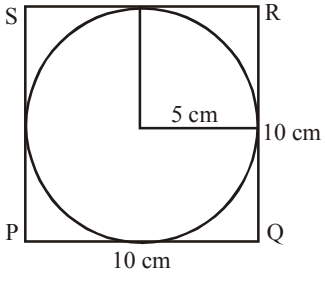
Space for each plant = $4\ cm^2$

$\therefore$ Required number of plants

$$= \frac{(30)^2}{4\pi} \div 4 = 17.89 = 18 \text{ (Approx)}$$

40. (a) Area of the square = $(10)^2 = 100\ cm^2$

The largest possible circle would be as shown in the figure below :



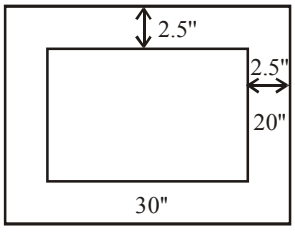
Area of the circle $= \frac{22}{7} \times (5)^2 = \frac{22 \times 25}{7}$

Required ratio $= \frac{22 \times 25}{7 \times 100} = \frac{22}{28} = \frac{11}{14}$

$= 0.785 \approx 0.8 = \frac{4}{5}$

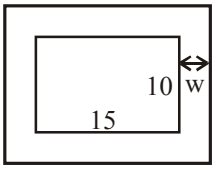
41. (d) Side of square carpet $= \sqrt{\text{Area}} = \sqrt{169} = 13\text{m}$
After cutting of one side,
Measure of one side = 13 – 2 = 11 m
and other side = 13 m (remain same)
$\therefore$ Area of rectangular room = 13 × 11 = 143 m^2

42. (a)



Length of frame = 30 + 2.5 × 2 = 35 inch
Breadth of frame = 20 + 2.5 × 2 = 25 inch
Now, area of picture = 30 × 20 = 600 sq. inch
Area of picture with frame = 35 × 25= 875 sq.inch
$\therefore$ Area of frame = 875 – 600 = 275 sq. inch

43. (c)



Let the width of the path = W m
then, length of plot with path = (15 + 2W) m
and breadth of plot with path = (10 + 2 W) m
Therefore, Area of rectangular plot (wihout path)
$= 15 \times 10 = 150\ m^2$
and Area of rectangular plot (with path)
$= 150 + 54 = 204\ m^2$
Hence, $(15 + 2W) \times (10 + 2W) = 204$
$\Rightarrow 4W^2 + 50\,W - 54 = 0$
$\Rightarrow 2W^2 + 25\,W - 27 = 0$
$\Rightarrow (W - 2)(W + 27) = 0$
Thus W = 2 or –27
$\therefore$ width of the path = 2 m

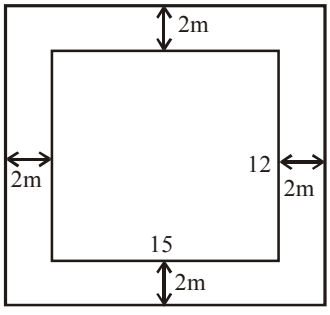
44. (a) If area of a circle decreased by x % then the radius of a circle decreases by

$(100 - 10\sqrt{100 - x})\% = (100 - 10\sqrt{100 - 36})\%$

$= (100 - 10\sqrt{64})\%$

$= 100 - 80 = 20\%$

45. (a) Area of the outer rectangle = 19 × 16 = 304 m^2



Area of the inner rectangle = 15 × 12 = 180 m^2
Required area = (304 – 180) = 124 m^2

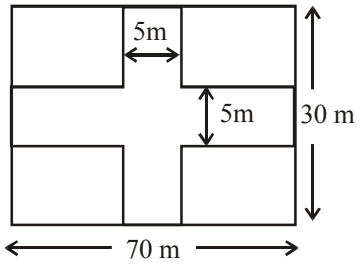
46. (a) Area of paper = 12 × 5 = 60 sq. inch

Area of typing part $= (12 - 1 \times 2) \times (5 - \frac{1}{2} \times 2)$

$= (12 - 2) \times (5 - 1)$
$= (10 \times 4)$ sq. inch

$\therefore$ Required fraction $= \frac{40}{60} = \frac{2}{3}$

47. (c)



Total area of road
= Area of road which parallel to length + Area of road which parallel to breadth – overlapped road
= 70 × 5 + 30 × 5 – 5 × 5
= 350 + 150 – 25
= 500 – 25 = 475 m^2
$\therefore$ Cost of gravelling the road
= 475 × 4 = ₹ 1900

48. (a) Radius of a circular grass lawn (without path) = 35 m
$\therefore$ Area = $\pi r^2 = \pi (35)^2$
Radius of a circular grass lawn (with path)
= 35 + 7 = 42 m
$\therefore$ Area = $\pi r^2 = \pi(42)^2$
$\therefore$ Area of path = $\pi(42)^2 - \pi(35)^2$
$= \pi(42^2 - 35^2)$
$= \pi(42 + 35)(42 - 35)$

$= \pi \times 77 \times 7 = \frac{22}{7} \times 77 \times 7 = 1694\ m^2$

49. (a) Volume of the bucket = volume of the sand emptied
Volume of sand = $\pi (21)^2 \times 36$

Let r be the radius of the conical heap.

Then, $\frac{1}{3}\pi r^2 \times 12 = \pi(21)^2 \times 36$

or $r^2 = (21)^2 \times 9$ or $r = 21 \times 3 = 63$

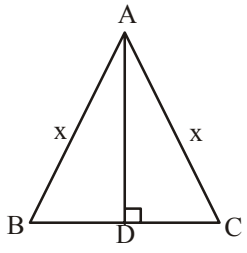
50. (b) Radius of the wheel of bus = 70 cm. Then,
circumference of wheel = $2\pi r = 140\,\pi = 440$ cm
Distance covered by bus in 1 minute

$$= \frac{66}{60} \times 1000 \times 100 \text{ cms}$$

Distance covered by one revolution of wheel
= circumference of wheel
= 440 cm

$$\therefore \text{ Revolutions per minute } = \frac{6600000}{60 \times 440} = 250$$

51. (b) Let ABC be the isosceles triangle and AD be the altitude.
Let AB = AC = x. Then, BC = (32 – 2x).



Since, in an isosceles triangle, the altitude bisects the base. So, BD = DC = (16 – x).
In ΔADC, $AC^2 = AD^2 + DC^2$

$$\Rightarrow x^2 = (8)^2 + (16 - x)^2$$

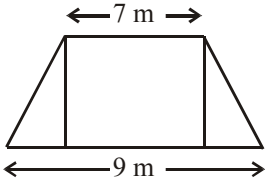
$$\Rightarrow 32x = 320 \Rightarrow x = 10.$$

$\therefore$ BC = (32 – 2x) = (32 – 20) cm = 12 cm.

$$\text{Hence, required area } = \left(\frac{1}{2} \times BC \times AD\right)$$

$$= \left(\frac{1}{2} \times 12 \times 10\right) \text{cm}^2 = 60 \text{ cm}^2.$$

52. (a)



Let the length of canal = h m. Then,

$$\text{area of canal } = \frac{1}{2} \times h(9 + 7)$$

$$\text{or } 1280 = \frac{1}{2} h(16)$$

$$\therefore h = \frac{1280 \times 2}{16} = 160 \text{ m}$$

53. (a) Let inner radius of the pipe be r.

Then, $440 = \frac{22}{7} \times r^2 \times 7 \times 10$

$$\text{or } r^2 = \frac{440}{22 \times 10} = 2$$

$$\text{or } r = \sqrt{2} \text{ m}$$

54. (c) Area of field = 576 km^2. Then,

each side of field = $\sqrt{576} = 24$ km

Distance covered by the horse
= Perimeter of square field
= 24 × 4 = 96 km

$$\therefore \text{ Time taken by horse } = \frac{\text{distance}}{\text{speed}} = \frac{96}{12} = 8 \text{ h}$$

55. (c) Let the length and breadth of the original rectangular field be x m and y m respectively.
Area of the original field = x × y = 144 m^2

$$\therefore x = \frac{144}{y} \quad \ldots(i)$$

If the length had been 6 m more, then area will be

$$(x + 6)\, y = 144 + 54$$

$$\Rightarrow (x + 6)\, y = 198 \quad \ldots(ii)$$

Putting the value of x from eq (i) in eq (ii), we get

$$\left(\frac{144}{y} + 6\right) y = 198$$

$$\Rightarrow 144 + 6y = 198$$

$$\Rightarrow 6y = 54 \Rightarrow y = 9 \text{ m}$$

Putting the value of y in eq (i) we get x = 16 m

56. (c) Clearly, we have : $l = 9$ and $l + 2b = 37$ or b = 14.
$\therefore$ Area = ($l \times b$) = (9 × 14) sq. ft. = 126 sq. ft.

57. (b) We have : $2b + l = 30 \Rightarrow l = 30 - 2b$.
Area = 100 $m^2 \Rightarrow l \times b = 100 \Rightarrow b(30 - 2b) = 100$

$$\Rightarrow b^2 - 15b + 50 = 0 \Rightarrow (b - 10)(b - 5) = 0$$

$\Rightarrow$ b = 10 or b = 5.
When b = 10, $l = 10$ and when b = 5, $l = 20$.
Since the garden is rectangular,
so its dimension is 20 m × 5 m.

58. (c) Area of the field = 13.5 × 2.5 = 33.75 m^2
Area covered by the rectangular tank
= 5 × 4.5 = 22.50 m^2
Area of the field on which the earth dug out is to be spread = 33.75 – 22.50 = 11.25 m^2
Let the required height be h.
Then, 11.25 × h = 5 × 4.5 × 2.1
or h = 4.2 m

59. (a) When folded along breadth, we have :

$$2\left(\frac{l}{2}+b\right)=34 \text{ or } l+2b=34 \quad ...(i)$$

When folded along length, we have :

$$2\left(l+\frac{b}{2}\right)=38 \text{ or } 2l+b=38 \quad ....(ii)$$

Solving (i) and (ii), we get :

$l = 14$ and $b = 10$.

$\therefore$ Area of the paper $= (14 \times 10)\ cm^2 = 140\ cm^2$.

60. (a) Area left after laying black tiles

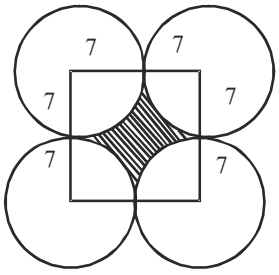
$= [(20-4) \times (10-4)]$ sq. ft. $= 96$ sq. ft.

Area under white tiles $= \left(\frac{1}{3}\times 96\right)$ sq. ft $= 32$ sq. ft.

Area under blue tiles $= (96-32)$ sq. ft $= 64$ sq. ft.

Number of blue tiles $= \frac{64}{(2\times 2)} = 16$.

61. (b)



The shaded area gives the required region.

Area of the shaded region = Area of the square – area of four quadrants of the circles

$$= (14)^2 - 4\times\frac{1}{4}\pi(7)^2$$

$$= 196 - \frac{22}{7}\times 49 = 196 - 154 = 42\ cm^2$$