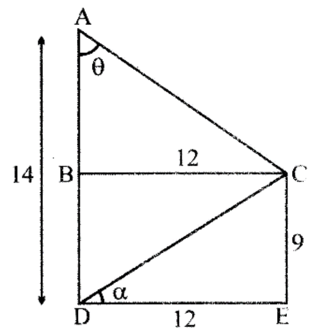
Introduction to Trigonometry

OLYMPIAD EXCELLENCE BOOK

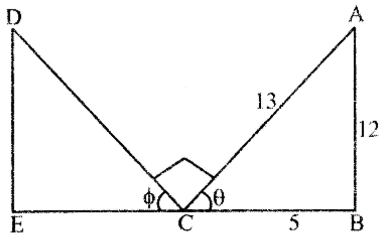
QUESTIONS

1. **The value of** $\sin\theta$ **in the adjoining figure is**



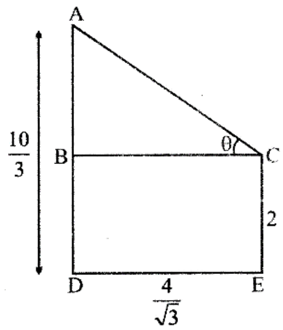
(a) $\frac{12}{5}$ (b) $\frac{12}{13}$ (c) $\frac{13}{12}$ (d) $\frac{5}{12}$

2. **In the given figure the value of** $\cos\phi$ **is**



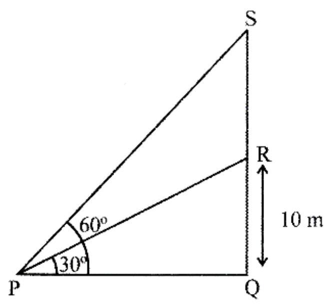
(a) $\frac{12}{5}$ (b) $\frac{5}{12}$ (c) $\frac{12}{5}$ (d) $\frac{12}{13}$

3. **In the given figure below, the value of** θ **is**



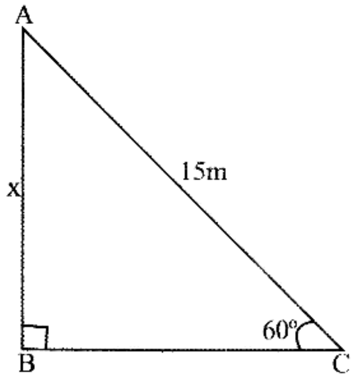
(a) 60° (b) 45° (c) 90° (d) 30°

4. **In the given figure below, what is the value of SQ?**



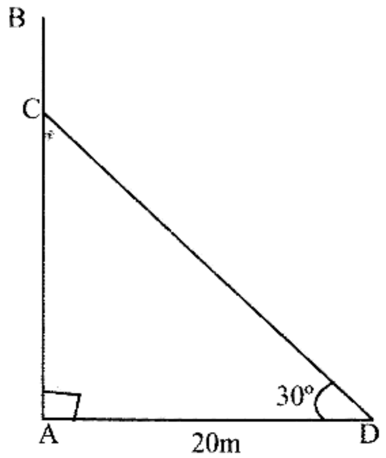
(a) $\frac{10}{\sqrt{3}}$ m (b) $10\sqrt{3}$ m (c) $\frac{20}{\sqrt{3}}$ m (d) 30 m

5. In the given figure below the value of AB is



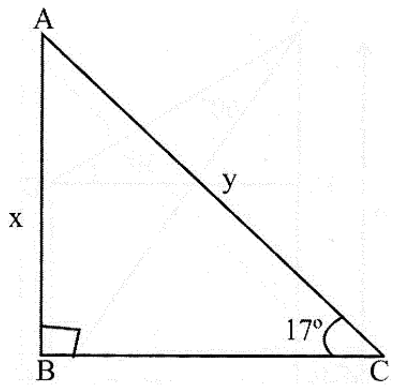
(a) 10.8 (b) 8.6 (c) 6.8 (d) 7.8

6. In the given figure $BC = CD$. Then, AB is equal to



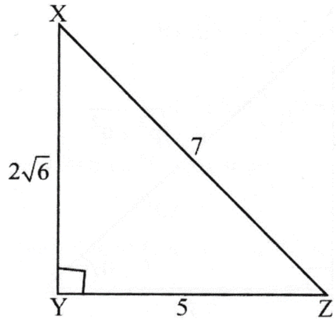
(a) $20\sqrt{3}$ (b) $15\sqrt{3}$ (c) $18\sqrt{2}$ (d) $20\sqrt{2}$

7. In the adjoining figure the value of $\sec 17^{\circ} - \text{sm} 73^{\circ}$ is



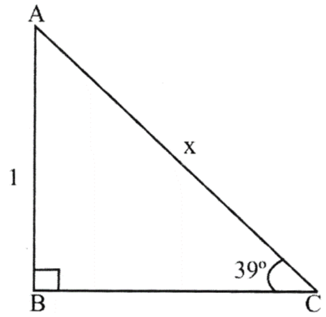
(a) $\frac{y}{x\sqrt{y^2 - x^2}}$ (b) $\frac{x^2}{y\sqrt{y^2 - x^2}}$ (c) $\frac{x^2}{y\sqrt{x^2 - y^2}}$ (d) $\frac{y^2}{x\sqrt{x^2 - y^2}}$

8. In the given right angles triangle XYZ. What is the value of $\sec x + \tan x$?



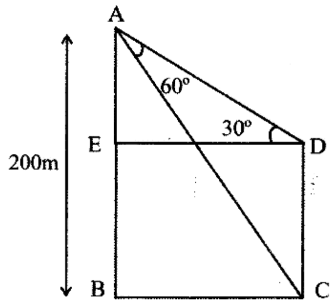
(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $2\sqrt{6}$ (d) $\frac{\sqrt{6}}{2}$

9. In the given figure, the value of $\frac{1}{\cos es^2}51^\circ + \sin^2 90^\circ + \tan^2 51^\circ - \frac{1}{\sin^2 51^\circ \sec^2 39^\circ}$ **is**



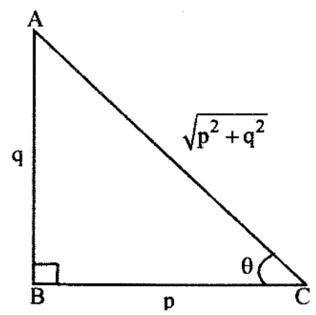
(a) $\sqrt{x^2-1}$ (b) $\sqrt{1-x^2}$ (c) x^2-1 (d) $1-x^2$

10. In the adjoining figure, the value of CD is



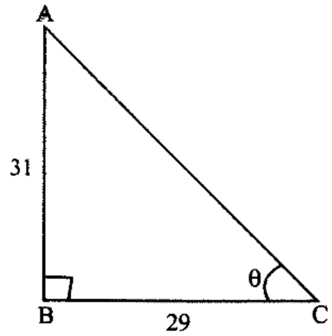
(a) $\frac{200}{3}$ (b) $200\sqrt{3}$ (c) $\frac{400}{3}$ (d) $400\sqrt{3}$

11. In the given figure what is the value of $\tan\theta$?



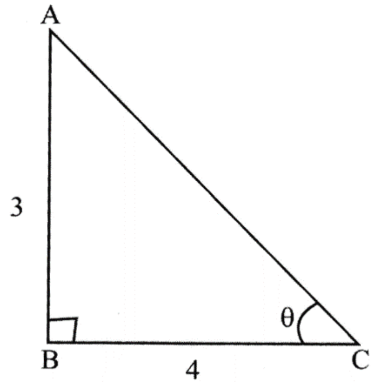
(a) $\frac{p}{q}$ (b) $\frac{q}{p}$ (c) pq (d) p^2q^2

12. In the given figure the value of $\frac{1+2\sin\theta.\cos\theta}{1-2\sin\theta.\cos\theta}$ is



(a) 810 (b) 540 (c) 900 (d) 490

13. In the given figure what is the value of $3\sin\theta+4\cos\theta$?



(a) 3 (b) 4 (c) 5 (d) 1

14. If $sec\theta+tan\theta=x,(x\neq 0)$ then $\sec\theta$ is equal to

(a) $\left(x-\frac{1}{x}\right),x\neq 0$

(b) $2\left(x-\frac{1}{x}\right),x\neq 0$

(c) $\left(x+\frac{1}{x}\right),x\neq 0$

(d) $\frac{1}{2}\left(x+\frac{1}{x}\right),x\neq 0$

15. If $\sin\theta-\cos\theta=\sqrt{2}\sin\left(90^\circ-\theta\right)$ then $\cot\theta$ is equal to:

(a) $\sqrt{2}$ (b) 0 (c) $\sqrt{2}-1$ (d) $\sqrt{2}+1$

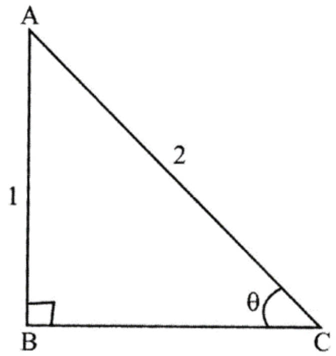
16. If $sec^2\theta+tan^2\theta=\sqrt{2}$, then the value of $\left(\sec^4\theta-\tan^4\theta\right)$ is

(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{2}$ (d) 0

17. If $2\beta\sin\theta=\alpha\cos\theta$ and $2\alpha\,\mathrm{cosec}\,\theta-\beta\sec\theta=3$ then what is the value of $\left(\alpha^2+4\beta^2\right)$?

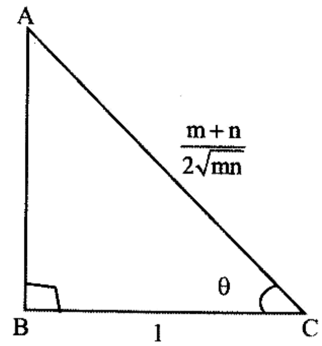
(a) 4 (b) 1 (c) 2 (d) 5

18. **In the given figure** $\frac{2\tan\theta}{1-\tan^2\theta}$ **is equal to**



(a) $\cos 60^\circ$ (b) $\sin 60^\circ$ (c) $\tan 60^\circ$ (d) $\sin 30^\circ$

19. **In the given figure** $\sin\theta = ?$



(a) $\frac{m+n}{m-n}$ (b) $\frac{m}{m+n}$ (c) $\frac{m-n}{m+n}$ (d) $\frac{n}{m+n}$

20. **If** $\sin A = \frac{5}{13}$, **then match the column.**

Column I		**Column II**	
A	Cos A	P	12/13
B	Tan A	Q	5/12
C	Cosec A	R	13/5
D	Sec A	S	13/12

(a) (A-P, B-Q, C-R, D-S) (b) (A-Q, B-R, C-S, D-P)

(c) (A-R, B-Q, C-S, D-P) (d) (A-P, B-Q, C-S, D-R)

21. **If cosec** $A = \sqrt{5}$ **then match the column**

Column I		**Column II**	
A	Tan A	P	$2/\sqrt{5}$
B	Sin A	Q	$\frac{3}{2}$
C	Cos A	R	$\frac{1}{\sqrt{5}}$

D	$\frac{1}{\tan A}+\frac{\sin A}{1+\cot A}$	S	$\frac{1}{2}$

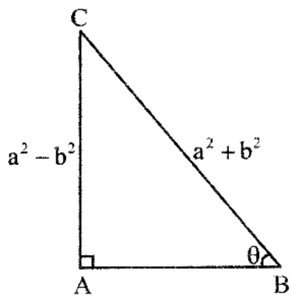
(a) (A-P, B-Q, C-R, D-S)

(b) (A-S, B-R, C-P, D-Q)

(c) (A-P, B-Q, C-S, D-R)

(d) (A-P, B-S, C-Q, D-R)

22. In the given figure the value of $\cot\theta$ is



(a) $\frac{2ab}{a^2-b^2}$ (b) $\frac{2ab}{a^2+b^2}$ (c) $\frac{a^2-b^2}{a^2+b^2}$ (d) $\frac{a^2+b^2}{2ab}$

23. Match the column

Column I		Column II	
A	$\operatorname{cosec}(90^\circ-A)$	P	Tan A
B	$\cot(90^\circ-A)$	Q	Sec A
C	$\tan(90^\circ-A)$	R	Cosec A
D	$\sec(90^\circ-A)$	S	Cot A

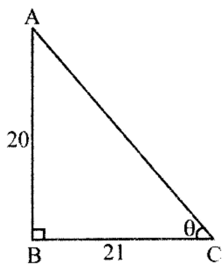
(a) (A-Q, B-P, C-S, D-R)

(b) (A-P, B-Q, C-S, D-R)

(c) (A-P, B-Q, C-R, D-S)

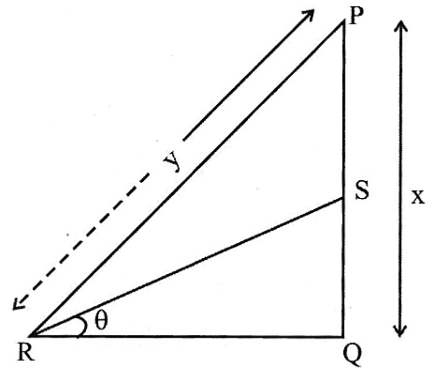
(d) (A-R, B-S, C-P, D-Q)

24. In given figure what is the value of $\cos^2\theta-\sin^2\theta$?



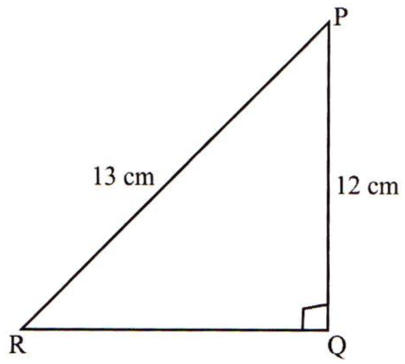
(a) $\frac{841}{41}$ (b) 0 (c) 1 (d) $\frac{41}{841}$

25. **In the given figure, $PS = SQ$, then $\sin\theta = ?$**



(a) $\frac{4Q^2 - 3p^2}{p}$ (b) $\frac{p}{4Q^2 - 3p^2}$ (c) $\frac{\sqrt{4Q^2 - 3p^2}}{p}$ (d) 1

26. **In the given figure, tan P. cot R =?**



(a) 1 (b) 0 (c) $\frac{25}{144}$ (d) $\frac{144}{25}$

27. **The value of** $\tan 4°.\tan 43°.\tan 47°.\tan 86°$ **is**

(a) 2 (b) 3 (c) 1 (d) 4

28. $\mathbf{\sin^2 6^\circ + \sin^2 6^\circ + \ldots\ldots + \sin^2 84^\circ + \sin^2 85^\circ = ?}$

(a) $39\frac{1}{2}$ (b) $40\frac{1}{2}$ (c) 40 (d) $49\frac{1}{\sqrt{2}}$

29. **If $\tan\alpha = y\tan\beta$ and $\sin\alpha = x\sin\beta$, then $\cos^2\alpha$ is**

(a) $\frac{x^2}{y^2+1}$ (b) $\frac{x^2}{y^2}$ (c) $\frac{x^2-1}{y^2-1}$ (d) $\frac{x^2+1}{y^2+1}$

30. **The product $\cos 1^\circ \cos 2^\circ \cos 3^\circ \cos 4^\circ \ldots\ldots \cos 180^\circ$ is equal to**

(a) -1 (b) $\frac{1}{4}$ (c) 1 (d) 0

31. **The simplified value of $(\sec x \sec y + \tan x \tan y) - (\sec x \tan y + \tan x \sec y)$ is**

(a) -1 (b) 0 (c) sec^2x (d) 1

32. **If $(1+\sin x)(1+\sin y)(1+\sin z)=(1-\sin x)(1-\sin y)(1-\sin z)$ then each side is equal to**

(a) $\pm \cos x \cos y \cos z$ (b) $\pm \sin x \sin y \sin z$

(c) $\pm \sin x \cos y \cos z$ (d) $\pm \sin x \sin y \cos z$

33. **The numerical value of** $\frac{1}{1+\cot^2\theta}+\frac{4}{1+\tan^2\theta}+3\sin^2\theta$ **will be**

(a) 2 (b) 5 (c) 6 (d) 4

34. **The value of** $\frac{\sin\theta}{1+\cos\theta}+\frac{\sin\theta}{1-\cos^2\theta}$ **is** $\left(0^\circ<\theta<90^\circ\right)$

(a) $2cosec\theta$ (b) $2\sec\theta$ (c) $2\sin\theta$ (d) $2\cos\theta$

35. **Lf $x=60^\circ$ then** $\frac{1}{2}\sqrt{1+\cos x}+\frac{1}{2}\sqrt{1-\cos x}$ **is equal to**

(a) $\cot\frac{\theta}{2}$ (b) $\sec\frac{\theta}{2}$ (c) $\sin\frac{\theta}{2}$ (d) $\cos\frac{\theta}{2}$

36. **If $\tan^2 x=1-e^2$, then the value of $\sec x+\tan^3 x \ cosec x$ is**

(a) $(2+e^2)^{\frac{3}{2}}$ (b) $\left(2-e^2\right)^{\frac{1}{2}}$ (c) $\left(2+e^2\right)^{\frac{1}{2}}$ (d) $(2-e^2)^{\frac{3}{2}}$

37. **For any real values** of $\theta, \frac{\sqrt{\sec\theta-1}}{\sqrt{\sec\theta+1}}=?$

(a) $\cot\theta-cosec\theta$ (b) $\sec\theta-\tan\theta$ (c) $cosec\theta+\cot\theta$ (d) $\tan\theta-\sec\theta$

38. **Value of** $\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}-\sec^2\theta$ **is**

(a) 1 (b) 2 (c) -1 (d) 0

39. **If** $\tan(x+y)=\sqrt{3}$ **and** $\tan(x-y)=\frac{1}{\sqrt{3}}$, $\angle x+\angle y<90^\circ, x\ge y$, **then** $\angle x$ **is**

(a) 90° (b) 30° (c) 45° (d) 60°

40. **The value of** $\left(1+\sec 40^\circ-\cot 50^\circ\right)\left(1-cosec 40^\circ+\tan 50^\circ\right)$ **is equal to**

(a) 0 (b) 1 (c) 2 (d) -1

ANSWER KEY & HINTS

1. (b) $AB = 14 - 9 = 5$

$BC = 12; AC = \sqrt{5^2 + 12^2} = 13 \Rightarrow \sin\theta = \frac{BC}{AC} = \frac{12}{13}$

2. (d) $\phi + e = 90^\circ$

$\phi = \left(90^\circ - \theta\right)$

$cos\phi = cos\left(90 - \theta\right)$

$= sin\theta$

$sin\theta = \frac{12}{13}$

3. (d) $AB = \frac{10}{3} - 2 = \frac{4}{3}$

$BC = \frac{4}{\sqrt{3}}$

$\tan\theta = \frac{4}{3} \times \frac{\sqrt{3}}{4} = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$

4. (d) $tan30^\circ = \frac{10}{PQ}$

$\frac{1}{\sqrt{3}} = \frac{10}{PQ}, PQ = 10\sqrt{3}$

$\tan 60^\circ = \frac{SQ}{10\sqrt{3}} = \frac{SR + 10}{10\sqrt{3}}$

$\sqrt{3} = \frac{SR + 10}{10\sqrt{3}}$

$SR = 20$

$SQ = 20 + 10 = 30$

5. (b)

6. (a) $\tan 30^\circ = \frac{AC}{AD} = \frac{AC}{20}$

$\frac{1}{\sqrt{3}} = \frac{AC}{20}, \qquad AC = \frac{20}{\sqrt{3}}$

$$cos30^{\circ} = \frac{20}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{20}{BC}, BC = \frac{40}{\sqrt{3}}$$

$$AC + BC = \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = 20\sqrt{3}$$

7. (b) $\sin 17^{\circ} = \frac{x}{y}$

$$cos17^{\circ} = \sqrt{1 - sin^2 17^{\circ}}$$

$$= \sqrt{1 - \frac{x^2}{y^2}} = \sqrt{\frac{y^2 - x^2}{y^2}}$$

$$\sqrt{\frac{y^2 - x^2}{y^2}}$$

$$\therefore sec17^{\circ} = \frac{y}{\sqrt{y^2 - x^2}}$$

$$sin73^{\circ} = sin\left(90^{\circ} - 17^{\circ}\right) = cos17^{\circ}$$

$$\therefore sec17^{\circ} - sin73^{\circ}$$

$$= \frac{y}{\sqrt{y^2 - x^2}} - \frac{\sqrt{y^2 - x^2}}{y}$$

$$= \frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$$

8. (b)

$$sec\,X = \frac{7}{2\sqrt{6}}$$

$$\&\, tanX = \frac{5}{2\sqrt{6}}$$

$$\therefore sec\,X + tan\,X = \frac{7}{2\sqrt{6}} + \frac{5}{2\sqrt{6}} = \frac{12}{2\sqrt{6}} = \sqrt{6}$$

9. (c) $\frac{1}{\cos ec^2 51^{\circ}} + sin^2 39^{\circ} + tan^2 51^{\circ} - \frac{1}{\sin^2 51^{\circ} \sec^2 39^{\circ}}$

$= \sin 25^{\circ} sin^2 39^{\circ} tan^2(90^{\circ} - 39^{\circ}) - \frac{1}{\sin^2(90^{\circ} - 39^{\circ}).\sec^2 39^{\circ}} = cos^2 39^{\circ} + \sin^2 39^{\circ} + cot^2 39^{\circ} - \frac{1}{\cos^2 39^{\circ} \sec^2 39^{\circ}}$

$\left[\therefore sin(90^{\circ} - \theta) = cos\theta \; tan(90^{\circ} - \theta) = cot\theta\right]$

$= 1 + cot^2 39^{\circ} - 1 = cot^2 39^{\circ}$

$= cosec 39^{\circ} - 1 = x^2 - 1$

10. (c) $\angle BCA = 60^{\circ}$

$tan 60^{\circ} = \frac{200}{BC}$

$\sqrt{3} = \frac{200}{BC}, BC = \frac{200}{\sqrt{3}}$

$ED = BC = \frac{200}{\sqrt{3}}$

$tan 30^{\circ} = \frac{AE}{ED}$

$\frac{1}{\sqrt{3}} = \frac{AE}{\frac{200}{\sqrt{3}}}$

$\therefore AE = \frac{200}{3}$

$EB = 200 - \frac{200}{3} = \frac{400}{3}$

$\therefore CD = \frac{400}{3}$

11. (b) $AB = \sqrt{\left(\sqrt{p^2 + q^2}\right)^2 - p^2}$

$= p^2 + q^2 - p^2 = \sqrt{q^2} = q$

$tan\theta = \frac{q}{p}$

12. (c) $29 tan\theta = 31 \Rightarrow tan\theta = \frac{31}{29}$

Expression= $\frac{1 + 2 sin\theta.cos\theta}{1 - 2 \sin cos\theta}$

$$= \frac{sin^2\theta + cos^2\theta + 2\sin\theta.cos\theta}{sin^2\theta + cos^2\theta - 2sin.cos\theta}$$

$$= \frac{(sin\theta + cos\theta)^2}{(sin\theta - cos\theta)^2}$$

$$= \left(\frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}\right) = \left(\frac{\tan\theta + 1}{\tan\theta - 1}\right)^2$$

$$= \left(\frac{\frac{31}{29} + 1}{\frac{31}{29} - 1}\right)^2 = \left(\frac{\frac{31+29}{29}}{\frac{31-29}{29}}\right)^2$$

$$= \left(\frac{60}{2}\right)^2 (30)^2 = 900$$

13. (c) $AC = \sqrt{3^2 + 4^2} = 5$

$\therefore 3.sin\theta + 4cos\theta$

$$= 3 \times \frac{3}{5} + 4 \times \frac{4}{5}$$

$$= \frac{9}{5} + \frac{16}{5} = \frac{25}{5} = 5$$

14. (d): $sec\theta + tan\theta = x$ ...(i)

$\therefore sec^2\theta - tan^2\theta = 1$

$\Rightarrow (sec\theta + tan\theta)(sec\theta - tan\theta) = 1$

$\Rightarrow sec\theta - tan\theta = \frac{1}{x}$ ...(ii)

On adding both the equations

$$2sec\theta = x + \frac{1}{x}$$

$$\Rightarrow sec\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

15. (c): $sin\theta - cos\theta = \sqrt{2}sin(90^\circ - \theta)$

$sin\theta - cos\theta = \sqrt{2}cos\theta$

$\Rightarrow \sqrt{2}\cos\theta + cos\theta = sin\theta$

$$\Rightarrow cos\theta(\sqrt{2}+1) = sin\theta$$

$$\Rightarrow \frac{cos\theta}{sin\theta} = (\sqrt{2}+1)$$

$$\Rightarrow cot\theta = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$\Rightarrow \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

16. (c):- $sec^2\theta + tan^2\theta = \sqrt{2}$

And $sec^2\theta - tan^2\theta = 1$

$$\therefore sec^4\theta - tan^4\theta$$

$$= \left(sec^2\theta + tan^2\theta\right)\left(sec^2\theta - tan^2\theta\right)$$

$$= \sqrt{2} \times 1 = \sqrt{2}$$

17. (a): $2\beta sin\theta = \alpha cos\theta$

$$\Rightarrow \alpha = \frac{2\beta sin\theta}{cos\theta} \qquad \text{........(i)}$$

$$\therefore 2\alpha cosec\theta - \beta sec\theta = 3$$

$$\Rightarrow \frac{2 \times 2 \times \beta sin\theta . cosec\theta}{\cos\theta} - \beta sec\theta = 3$$

$$\Rightarrow 4\beta sec\theta - \beta sec\theta = 3$$

$$\Rightarrow 3\beta sec\theta = 3$$

From equation (i)

$$\alpha = \frac{2 \times cos\theta . sin\theta}{\cos\theta} = 2\sin\theta$$

$$\therefore \alpha^2 + 4\beta^2 = \left(2\sin\theta\right)^2 + 4\cos^2\theta$$

$$= 4\left(sin^2\theta + cos^2\theta\right) = 4$$

18. (c) $\frac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3} = \tan 60^\circ$$

19. (c) $AB=\sqrt{\left(\frac{m+n}{2\sqrt{mn}}\right)^2-1}$

$=\frac{m-n}{2\sqrt{mn}}$

$\sin\theta=\frac{\frac{m-n}{2\sqrt{mn}}}{\frac{m+n}{2\sqrt{mn}}}$

$=\frac{m-n}{m+n}$

20. (a) Not Available

21. (b) Not Available

22. (a) Not Available

23. (a) Not Available

24. (d) $AC=\sqrt{20^2+21^2}=\sqrt{841}$

$=\frac{(21)^2}{841}-\frac{(20)^2}{841}$

$=\frac{41}{841}$

25. (b) Not Available

26. (c) $RQ=\sqrt{13^2-12^2}=5$

Tan P. cot $R=\frac{5}{12}.\frac{5}{12}=\frac{25}{144}$

27. (c): $tan4^{\circ}.\ tan43^{\circ}.tan47^{\circ}.tan85^{\circ}$

$=tan4^{\circ}.tan43^{\circ}.tan\left(90^{\circ}-43^{\circ}\right).tan\left(90^{\circ}-4^{\circ}\right)$

$=tan4^{\circ}\times tan43^{\circ}\times cot43^{\circ}\times cot4^{\circ}=1$

$\left[tan\left(90^{\circ}-\theta\right)=cot\theta; tan\theta.cot\theta-1\right]$

28. (b): $(sin^2 5^{\circ}+cos^2 5^{\circ})+(sin^2 6^{\circ}\ \ cos^2 6^{\circ})+........+\text{ to 40 terms}+\sin^2 45^{\circ}=40+\left(\frac{1}{\sqrt{2}}\right)^2=40+\frac{1}{2}=40\frac{1}{2}$

29. (c): $tan\alpha=ytan\beta$

$\Rightarrow tan\beta = \frac{1}{y} tan\alpha$

$\Rightarrow cot\beta = \frac{y}{\tan\alpha}$ and $sin\alpha = xsin\beta \Rightarrow sin\beta = \frac{1}{x} sin\alpha$

$\Rightarrow cosec\beta = \frac{x}{\tan^2\alpha}$

$[\therefore cosec^2\beta - cot^2\beta = 1]$

$\Rightarrow \frac{x^2}{\sin^2\alpha} - \frac{y^2}{\tan^2\alpha} = 1$

$\Rightarrow \frac{x^2}{\sin^2\alpha} - \frac{y^2\cos^2\alpha}{\sin^2\alpha} = 1$

$x^2 - \ y^2cos^2\alpha = sin^2\alpha \ = 1 - cos^2\alpha$

$\Rightarrow x^2 - 1 = y^2cos^2\alpha - cos^2a = \left(y^2 - 1\right)\cos^2\alpha$

$\Rightarrow cos^2\alpha = \frac{x^2 - 1}{y^2 - 1}$

30. (d): $cos1^\circ cos2^o cos3^o cos4^o .....cos180^o = 0\left[cos90^\circ = 9\right]$

31. (d): $\left(secx.secy + tanx.tany\right)^2 - \left(secx.tany + tanx.secy\right)^2$

$= sec^2x.sec^2y + tan^2x.tan^2y + \ 2sec\,x\,.\,sec\,y\,.\,tan\,x\,.\,tan\,y - sec^2x\ .$

$tan^2y - tan^2x\,.\,sec^2y - 2\,sec\,x\,.\ secy\,.\ tanx\,.\ tany$

$= sec^2x\,.\,sec^2y - sec^2x\,.\,tan^2y - tan^2.sec^2y + tan^2x\,.\,tan^2y = sec^2x\left(sec^2y - tan^2y\right) - tan^2x\left(sec^2y - tan^2y\right)$

$= sec^2x - tan^2x = 1$

32. (a):

$\left(1+\sin x\right)\left(1+\sin y\right)\left(1+sinz\right) \ = \left(1-\sin x\right)\left(1-\sin y\right)\left(1-\sin z\right) = x$

$\therefore x.x = \left(1+\sin x\right)\left(1-\sin x\right)\left(1+\sin y\right)\left(1-\sin y\right)\left(1+\sin z\right)\left(1-sinz\right) = \left(1-sin^2x\right)\left(1-sin^2y\right)\left(1-sin^2z\right)$

$= cos^2x.cos^2y.cos^2z$

$\therefore x = \pm\ cosx.cosy.cosz$

33. (d): $\frac{1}{1+\cot^2\theta} + \frac{4}{1+\tan^2\theta} + 3\sin^2\theta$

$$= \frac{1}{\operatorname{cosec}^2\theta} + \frac{4}{\sec^2\theta} + 3\sin^2\theta$$

$$= \sin^2\theta + 4\cos^2\theta + 3\sin^2\theta$$

$$= 4\left(\sin^2\theta + \cos^2\theta\right) = 4$$

34. (a): $\frac{\sin\theta}{1+\cos\theta} + \frac{\sin\theta}{1-\cos\theta}$

$$= \frac{\sin\theta(1-\cos\theta) + \sin\theta(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{\sin\theta - \sin\theta\cos\theta + \sin\theta + \sin\theta.\cos\theta}{1-\cos^2\theta}$$

$$= \frac{2\sin\theta}{\sin^2\theta} = 2\operatorname{cosec}\theta$$

35. (d): $\frac{1}{2}\sqrt{1+\cos\theta} + \frac{1}{2}\sqrt{1-\cos\theta}$

$$= \frac{1}{2}\left(1+\cos 60^\circ + 1 - \cos 60^\circ\right)$$

$$= \frac{1}{2}\left(\sqrt{1+\frac{\sqrt{3}}{2}} + \sqrt{1-\frac{\sqrt{3}}{2}}\right)$$

$$= \frac{1}{2\sqrt{2}}\left(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}\right)$$

$$= \frac{1}{2\sqrt{2}} \times \frac{1}{2}\left(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}}\right)$$

$$= \frac{1}{4}\sqrt{\left(\sqrt{3}+1\right)^2} + \sqrt{\left(\sqrt{3}-1\right)^2}$$

$$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30^\circ$$

$$= \cos\frac{\theta}{2}$$

36. (d): $\tan^2 x = 1 - e^2$

$$\therefore \sec x + \tan^3 x.\operatorname{cosec} x$$

$$= \sec x + \tan^2 x.\tan x.\ \operatorname{cosec} x$$

$$= \sec x + \tan^2 x.\frac{\sin x}{\cos x}.\frac{1}{\sin x}$$

$= secx + tan^2x.\ secx$

$= secx.\left(1 + tan^2x\right)$

$= \left(1 + \tan^2 x\right)^{\frac{1}{2}}.\left(1 + \tan^2 x\right)$

$= (1 + \tan^2 x)^{\frac{3}{2}} = (1 + 1 - e^2)^{\frac{3}{2}}$

$= \left(2 - e^2\right)^{\frac{3}{2}}$

37. (c) $\sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = \sqrt{\dfrac{\dfrac{1}{\cos\theta} + 1}{\dfrac{1}{\cos\theta} - 1}}$

$= \sqrt{\dfrac{\dfrac{1 + \cos\theta}{\cos\theta}}{\dfrac{1 - \cos\theta}{\cos\theta}}}$

$= \sqrt{\dfrac{1 + \cos\theta}{1 - \cos\theta}} = \sqrt{\dfrac{(1 + \cos\theta)(1 + \cos\theta)}{(1 - \cos\theta)(1 - \cos\theta)}}$

(Rationalizing the numerator and the denominator)

$= \sqrt{\dfrac{(1 + \cos\theta)^2}{1 - \cos^2\theta}} = \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= cosec\theta + cot\theta.$

38. (c): $\dfrac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta} - \sec^2\theta$

$\dfrac{\sin^2\left(1 - 2\sin^2\theta\right)}{2\cos^2\theta\left(2\cos^2\theta - 1\right)} - \sec^2\theta$

$\dfrac{\sin^2\left(1 - 2\left(1 - \cos^2\theta\right)\right)}{\cos^2\theta\left(2\cos^2\theta - 1\right)} - \sec^2\theta$

$= \tan^2\theta \dfrac{\left(2\cos^2\theta - 1\right)}{\left(2\cos^2\theta - 1\right)} - \sec^2\theta$

$= tan^2\theta - sec^2\theta = -[sec^2\theta - tan\theta] = -1$

39. (c): $tan(x + y) = \sqrt{3} = tan60^\circ$

$\Rightarrow x + y = 60^\circ$ ....(i)

$tan(x - y) = \frac{1}{\sqrt{3}} = tan30^\circ$

$\Rightarrow x - y = 30^\circ$ .....(ii)

$\therefore x + y + x - y = 60^\circ + 30^\circ$

$\Rightarrow 2x = 90^\circ$

$\Rightarrow x = \frac{90^\circ}{2} = 45^\circ$

40. (c): $\left(1 + sec40^\circ - cot50^\circ\right)\left(1 - cosec40^\circ + tan50^\circ\right)$

$= \left(1 + sec40^\circ + tan40^\circ\right)\left(1 - cosec40^\circ + cot40^\circ\right)$ $\left[\therefore tan\left(90^\circ - \theta\right) = cot\theta; cot\left(90^\circ - \theta\right) = tan\theta\right]$

$= \left(1 + \frac{1}{\cos 40^\circ} + \frac{\sin 40^\circ}{\cos 40^\circ}\right)$

$= \left(1 - \frac{1}{\sin 40^\circ} + \frac{\cos 40^\circ}{\sin 40^\circ}\right)$

$= \frac{\cos 40^\circ + 1 + \sin 40^\circ}{\cos 40^\circ}$

$= \frac{\left(\cos 40^\circ + \sin 40^\circ\right)^2 - 1}{\sin 40^\circ . \cos 40^\circ}$

$= \frac{\cos^2 40^\circ + \sin^2 40^\circ + 2\sin 40^\circ . \cos 40^\circ - 1}{\sin 40^\circ . \cos 40^\circ}$

$= \frac{1 + 2\sin 40^\circ . \cos 40^\circ - 1}{\sin 40^\circ . \cos 40^\circ} = 2$