Circle

Chapter 10 Class 10th

Ex :- 10.1

① How many tangents can a circle have?

Ans. Infinite many

② Fill in the blanks.

(i) A tangent to a circle intersects it in one point.

(ii) A line intersecting a circle in two points is called a secant

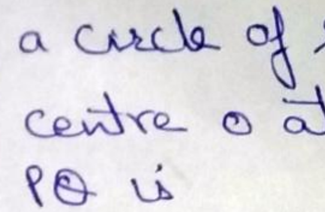
(iii) A circle can have two parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called point of contact.

③ A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. length PQ is

Ⓐ 12 cm. Ⓑ 13 cm Ⓒ 8.5 cm. Ⓓ $\sqrt{119}$ cm.

Ans :- Ⓓ $\sqrt{119}$ cm.



OP = 5 cm.

OQ = 12 cm.

$\because$ OP $\perp$ PQ

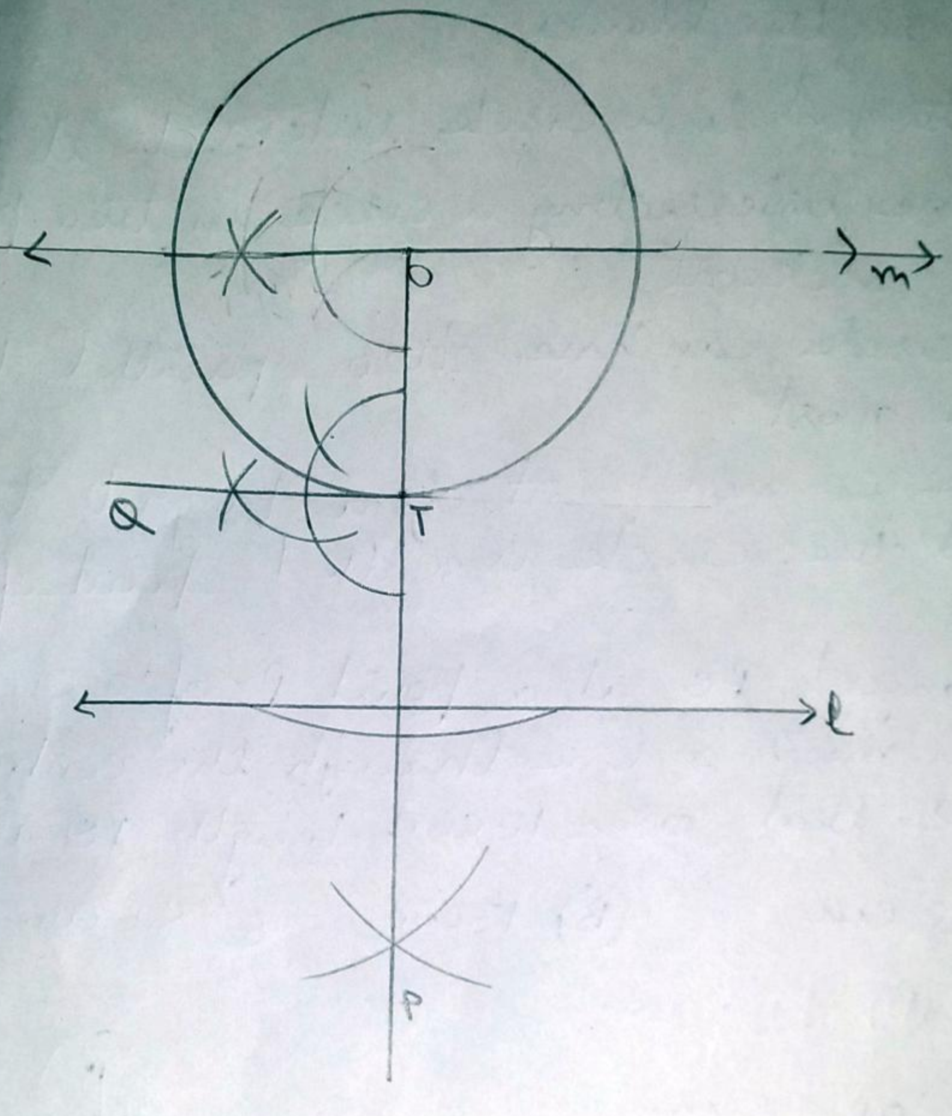
$\angle OPQ = 90^0$

In $\triangle OPQ$

$$OQ^2 = OP^2 + PQ^2$$
$$= (12)^2 = (5)^2 + PQ^2$$
$$PQ^2 = (12)^2 - (5)^2$$
$$PQ^2 = 144 - 25 = 119$$
$$PQ = \sqrt{119} \text{ cm.}$$

Q4 Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Ans:-



Q1 From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is (A) 7cm (B) 12 cm (C) 15 cm (D) 24.5 cm.

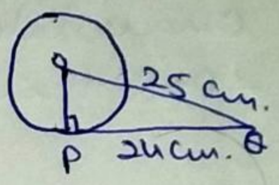
Ans (A) = 7cm.

Sol

In $\triangle OPQ$

$\angle OPQ = 90^0$

$OQ = 2cm.$ $PQ = 24$ cm.



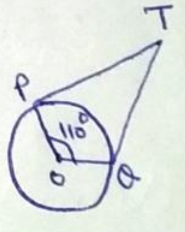
In right angle triangle OPQ

$$OQ^2 = OP^2 + PQ^2$$
$$25^2 = OP^2 + 24^2$$
$$625 = OP^2 + 576$$
$$OP^2 = 625 - 576 = 49$$
$$OP = 7cm. \text{ (radius of circle)}$$

Q2 In fig: TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^0$ then $\angle PTQ$ is equal to

(A) 60^0 (B) 70^0 (C) 80^0 (D) 90^0

Ans (B) $= 70^0$



Sol:- TP and TQ are two tangents to a circle

$\angle POQ = 110^0$

$\angle OPT = \angle OQT = 90^0$ ($\because$ TP and TQ are tangents to a circle)

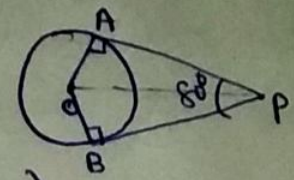
Now in $\square OPTQ$

$\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^0$ ($\because$ Sum of angles of a quadrilateral is 360^0)

$$110^0 + 90^0 + \angle PTQ + 90^0 = 360^0$$
$$290^0 + \angle PTQ = 360^0$$
$$\angle PTQ = 360^0 - 290^0 = 70^0$$
$$\angle PTQ = 70^0$$

Q3 If tangents PA and PB - . . . .

Sol Ans (A) = 50°



Sol:- $\angle OAP = \angle OBP = 90^\circ$

(∵ AP and BP are tangents to circle)

$\angle APB = 80^\circ$ (given)

Now
In Quadrilateral OAPB

$\angle AOB + \angle OAP + \angle APB + \angle OBP = 360^\circ$

(∵ Sum of angles of a quadrilateral ≠ 360°)

$\angle AOB + 90^\circ + 80^\circ + 90^\circ = 360^\circ$

$\angle AOB + 260^\circ = 360^\circ$

$\angle AOB = 360^\circ - 260^\circ = 100^\circ$

Now In Δ AOP and Δ BOP

$\angle PAO = \angle PBO$ (each 90°)

$OA = OB$ (each radius of circle)

$OP = OP$ (common side)

∴ $\Delta AOP \cong \Delta BOP$ (RHS congruency)

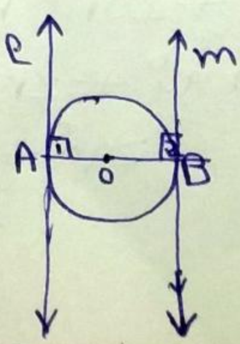
$\angle POA = \angle POB = \frac{\cancel{100}^{50}}{\cancel{2}}$ (C.P.C.T)

∴ $\angle POA = 50^\circ$

Q4 Prove that tangents drawn at the ends of a diameter of a circle are parallel.

Sol:- given:- AB is the diameter of a circle with center O. l and m are the tangents drawn from points A and B respectively.

To prove:- $l \parallel m$.



Proff:- ∵ line l is tangent at point A and AO is the radius of circle

P.T.O.

P.T.O

Q4

$\therefore \angle 1 = 90^0$

Illy $\angle 2 = 90^0$

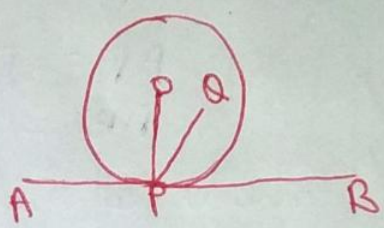
Now $\angle 1 + \angle 2 = 90^0 + 90^0 = 180^0$

$\therefore$ Sum of the interior angles at the same side of transversal line is 180^0

So $l \parallel m$.

Q5 Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Sol Let AB is tangent to circle with centre O at point P.



Now let $PQ \perp AB$ which not passes through point O (centre of circle)

But As we know radius of circle is perpendicular to the tangent of circle

So $\angle OPB = 90^0$

But $\angle QPB = 90^0$ (By construction)
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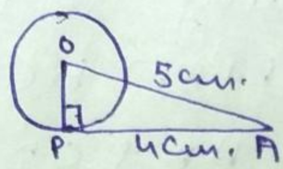
$\therefore \angle OPB = \angle QPB = 90^0$ which is not possible which proves the above result.

Q6 The length of a tangent from a point A at a distance 5cm - - - - -

Sol

$AP = 4cm.$
$OA = 5cm.$
$\angle OPA = 90^0$



In rt angle Δ OAB

$OA^2 = OP^2 + PA^2$
$OP^2 = OA^2 - PA^2$
$OP^2 = 5^2 - 4^2$
$OP^2 = 25 - 16 = 9$

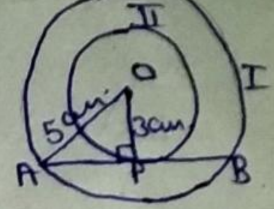
$OP = \sqrt{9} = 3cm.$

Q7 Two concentric circles . - - .

(6)

Sol Let circle I and II are concentric circles with centre O.

Let AB is the chord of circle I



$OP = 3cm$

$OA = 5cm.$

$\angle OPA = 90^0$

In rt $\triangle OAP$

$OA^2 = OP^2 + AP^2$

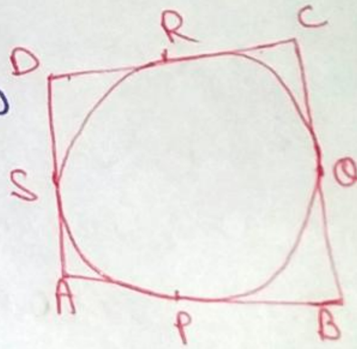
$(5)^2 = (3)^2 + (AP)^2$

$AP^2 = 25 - 9 = 16$

$AP = \sqrt{16} = 4cm.$

$AB = 2 \times 4 = 8cm.$

Q8 A quadrilateral - - ..



Sol :- Given :- A quadrilateral ABCD is drawn to circumscribe a circle.

To prove :- $AB + CD = AD + BC$

Proff :- Point A is exterior from circle.

$\therefore$ $AP = AS$ (1) ($\because$ Length of tangents drawn from exterior point of a circle are equal)

Ily

$BP = BQ$ (2)

$CR = CQ$ (3)

$DR = DS$ (4)

Adding (1), (2), (3) and (4)

$(AP + BP) + (CR + DR) = AS + BQ + CQ + DS$

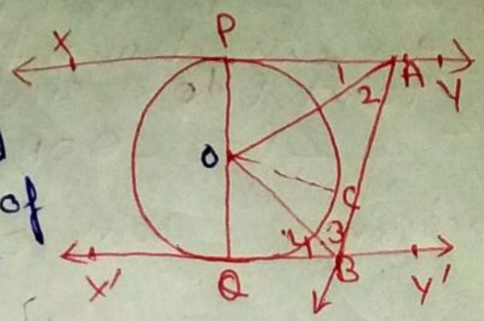
$AB + CD = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC$

Proved.

Q9 In fig XY and X'Y' are two - - - - -

Sol: — Given: XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with pt of contact C intersecting XY at A and X'Y' at B.



To prove :- $\angle AOB = 90^0$

Construction :— Join OC

Proff: — OC is the radius of circle and AB is the tangent at point C.

$\therefore \angle OCA = 90^0$

$\parallel^{ly}$ $\angle OPA = 90^0$

In rt Δ OAC and OAP

$\angle OCA = \angle OPA$ (each 90^0)

$OA = OA$ (common)

$OC = OP$ (each = radius)

$\therefore \Delta OAC \cong \Delta OAP$ (RHS Congruency)

$\therefore \angle 1 = \angle 2$ (C.P.C.T)

$\angle 2 = \frac{1}{2}\angle A$ — (1)

$\parallel^{ly}$ $\angle 3 = \frac{1}{2}\angle B$ — (2)

Adding (1) and (2)

$\angle 2 + \angle 3 = \frac{1}{2}(\angle A + \angle B)$

$= \frac{1}{2} \times 180^0$ ($\because$ XY $\parallel$ X'Y' and AB is a transversal line so $\angle A + \angle B = 180^0$)

$\angle 2 + \angle 3 = 90^0$

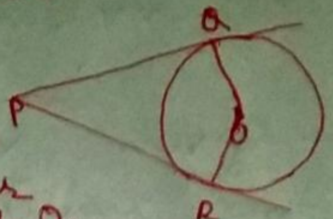
Now In Δ OAB

$\angle AOB + \angle 2 + \angle 3 = 180^0$

$\angle AOB = 180^0 - (\angle 2 + \angle 3)$ $\Rightarrow$ $\angle AOB = 180 - 90^0$

$\angle AOB = 90^0$

Q10 Prove that the angle between two — — .



Sol

Given: — PQ and PQ are two tangents drawn from exterior point P ~~to~~ a circle with centre O.

To prove

$\angle P + \angle QOR = 180^0$

Construction :- Join OQ and OR

Proff :- $\because$ PQ is a tangent and ~~OQ~~ OQ is the radius of circle

$\therefore \quad \angle OQP = 90^0 \quad$ — ① ($\because$ tangent to circle is $\perp$ to the radius of circle)

Ily $\quad \angle ORP = 90^0 \quad$ — ②

In quadrilateral PQOR

$\angle P + \angle OQR + \angle QOR + \angle ORP = 360^0$

$\angle P + 90^0 + \angle QOR + 90^0 = 360^0$ (using ① and ②)

$\angle P + 180^0 + \angle QOR = 360^0$

$\angle P + \angle QOR = 360^0 - 180^0$

$\angle P + \angle QOR = 180^0$ which is proved.

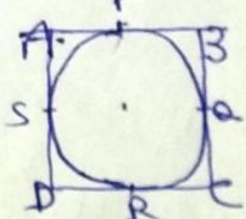
Q11 Prove that the ||gm circumscribing — — —

Given :- ABCD is a parallelogram circumscribing a circle.

To prove : —

→ ABCD is a rhombus.

Proff :- Point A is exterior to circle.



$\therefore \quad AP = AS$ (1)

($\because$ length of tangents drawn from exterior point of a circle are equal.

Ily $\quad BP = BQ$ — ②

$CR = CQ$ — ③

$DR = DS$ — ④

Adding ①, ②, ③, ④

$(AP + BP) + (CR + DR) = AS + BQ + CQ + DS$

$AB + CD = (AS + DS) + BQ + CQ$

$AB + CD = AD + BC$ — ⑤

P.T.O.

P.T.O.   Q11

Q 11

$\because$ ABCD is a parallelogram.

So $AB = CD$ and $AD = BC$

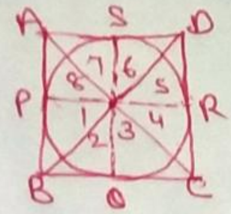
So from equation (5)

$$AB + CD = AD + BC$$
$$AB + AB = BC + BC$$
$$\cancel{2}AB = \cancel{2}BC$$
$$AB = BC \quad \text{proved.}$$

It means ABCD is a rhombus.

Q13 Prove that opposite sides of a — — —.

Sol $\because$ A is exterior point of a circle so



$AP = AS$ $\quad BP = BQ$ $\quad CQ = CR$

and $DR = DS$ ($\because$ length of tangents drawn from exterior point of a circle are equal.

Join OP, OQ, OR and S.

In $\triangle OBP$ and $\triangle OBQ$

$OP = OQ$ (each = radius)

$OB = OB$ (common)

$BP = BQ$

$\therefore \triangle OBP \cong \triangle OBQ$ (S.S.S congruency rule)

$\angle 1 = \angle 2$ (C.P.C.T)

Illy we can prove

$\angle 3 = \angle 4$, $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$ (complete angle)

$\angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^\circ$

$2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$

$\angle 1 + \angle 4 + \angle 5 + \angle 8 = \frac{360}{2} = 180^\circ$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$

we know that Illy we can prove $\angle BOC + \angle AOD = 180^\circ$ (which proves our statement.