9. DEFINITE INTEGRATION

Properties of definite integral

1. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ **2.** $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ **3.** $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

4. $\int_{-a}^{a} f(x)\,dx = \int_0^a (f(x)+f(-x))\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & f(-x)=f(x) \\ 0, & f(-x)=-f(x) \end{cases}$

5. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ **6.** $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

7. $\int_0^{2a} f(x)\,dx = \int_0^a (f(x)+f(2a-x))\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & f(2a-x)=f(x) \\ 0, & f(2a-x)=-f(x) \end{cases}$

8. If f(x) is a periodic function with period T, then

$\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx,\ n \in z,$ $\int_a^{a+nT} f(x)\,dx = n\int_0^T f(x)\,dx,\ n \in z,\ a \in R$

$\int_{mT}^{nT} f(x)\,dx = (n-m)\int_0^T f(x)\,dx,\ m, n \in z,$ $\int_{nT}^{a+nT} f(x)\,dx = \int_0^a f(x)\,dx,\ n \in z,\ a \in R$

$\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^a f(x)\,dx,\ n \in z,\ a, b \in R$

9. If $\psi(x) \le f(x) \le \phi(x)$ for $a \le x \le b$, then $\int_a^b \psi(x)\,dx \le \int_a^b f(x)\,dx \le \int_a^b \phi(x)\,dx$

10. If $m \le f(x) \le M$ for $a \le x \le b$, then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

11. $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$ **12.** If $f(x) \ge 0$ on [a, b] then $\int_a^b f(x)\,dx \ge 0$

Leibnitz Theorem : If $F(x) = \int_{g(x)}^{h(x)} f(t)\,dt$, then $\frac{dF(x)}{dx} = h'(x)\,f(h(x)) - g'(x)\,f(g(x))$