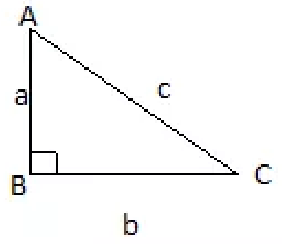
Chapter 11. Radical Expressions and Triangles

Ex. 11.4

Answer 1CU.

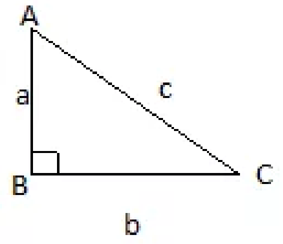
Consider the following right triangle ABC.



Here a, b and c are the lengths of the legs AB, BC and CA respectively. The right angle of the triangle ABC is $\angle ABC$. that is, $\angle ABC = 90^\circ$.

Answer 2CU.

Consider the following right triangle ABC.

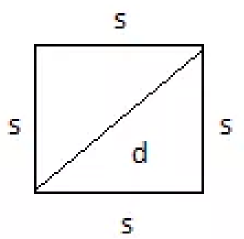


And suppose a, b and c are the lengths of the sides AB, BC and CA respectively. Now, we can use the Pythagorean theorem to find out the right angle of the right triangle. That is, if $c^2 = a^2 + b^2$, then the angle opposite to the side of length c is the right angle and hence $\angle ABC = 90^\circ$.

Answer 3CU.

Step 1

Consider the following square whose side measures s units and join any two opposite vertices to make a diagonal. Let the measure of this diagonal be d units. This diagonal divides the square into two right triangles.



Step 2:

Applying Pythagorean theorem to any one of the two right angled triangle:

$$d^2 = s^2 + s^2$$
$$d^2 = 2s^2$$
$$d = \sqrt{2s^2}$$
$$= s\sqrt{2}$$

Therefore, an equation to find the length of the diagonal d of a square with side length s is $\boxed{d = s\sqrt{2}}$.

Answer 4CU.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$= 14^2 + 12^2$$
$$= 196 + 144$$
$$= 340$$

Therefore, the length of the missing side is $c = \boxed{\sqrt{340}}$ or $c = \boxed{18 \cdot 43}$ to the nearest hundredth.

Answer 5CU.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$41^2 = a^2 + 40^2$$
$$1681 = a^2 + 1600$$
$$a^2 = 81$$
$$a = 9$$

Therefore, the length of the missing side is $a = \boxed{9}$

Answer 6CU.

Here, c is the measure of the hypotenuse of a right triangle. Also, $a = 10, b = 24$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 10^2 + 24^2 \\ &= 100 + 576 \\ &= 676 \end{aligned}$$

Therefore,

$$\begin{aligned} c &= \sqrt{676} \\ &= 26 \end{aligned}$$

Hence, the length of the missing side is $c = \boxed{26}$

Answer 7CU.

Here, c is the measure of the hypotenuse of a right triangle and $a = 11, c = 61$

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$61^2 = 11^2 + b^2$$
$$3721 = 121 + b^2$$
$$b^2 = 3600$$
$$\begin{aligned} b &= \sqrt{3600} \\ &= 60 \end{aligned}$$

Therefore, the length of the missing side is $b = \boxed{60}$

Answer 8CU.

Here, c is the measure of the hypotenuse of a right triangle and $b=13, c=\sqrt{233}$

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$\left(\sqrt{233}\right)^2 = a^2 + 13^2$$
$$233 = a^2 + 169$$
$$a^2 = 64$$
$$a = \sqrt{64}$$
$$= 8$$

Therefore, the length of the missing side is $a = \boxed{8}$

Answer 9CU.

Here, c is the measure of the hypotenuse of a right triangle and $a=7, b=4$

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$= 7^2 + 4^2$$
$$= 49 + 16$$
$$= 65$$
$$c = \sqrt{65}$$
$$= 8.06$$

Therefore, the length of the missing side is $c = \boxed{8 \cdot 06}$ rounded to the nearest hundredth

Answer 10CU.

Here, the measure of the longest side is 9. Therefore, assume the longest side to be hypotenuse, say $c=9$. Also let $a=4, b=6$. Then determine whether $c^2 = a^2 + b^2$ or not.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$9^2 = 4^2 + 6^2$$
$$81 = 16 + 36$$
$$81 = 52$$

This is not possible. That is, $c^2 \neq a^2 + b^2$ and hence the triangle is $\boxed{\text{not a right triangle}}$.

Answer 11CU.

Here, the measure of the longest side is 34. Therefore, assume the longest side to be hypotenuse, say $c = 34$. Also let $a = 30, b = 16$. Then determine whether $c^2 = a^2 + b^2$ or not.

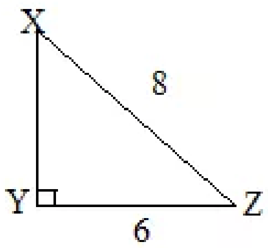
Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 34^2 &= 30^2 + 16^2 \\ 1156 &= 900 + 256 \\ 1156 &= 1156 \end{aligned}$$

That is, $c^2 = a^2 + b^2$ and hence the triangle is $\boxed{\text{a right triangle}}$.

Answer 12CU.

In the right angled triangle XYZ, the length of $\overline{YZ}$ is 6 and the length of the hypotenuse is 8. That is, $\overline{XZ} = 8$



Therefore, applying Pythagorean Theorem, we have:

$$\begin{aligned} XZ^2 &= XY^2 + YZ^2 \\ 8^2 &= XY^2 + 6^2 \\ 64 &= XY^2 + 36 \\ XY^2 &= 28 \\ XY &= \sqrt{28} \\ &= 2\sqrt{7} \end{aligned}$$

Now, Area of a triangle $= \frac{1}{2}bh$, where b is the base and h is the height of the triangle.

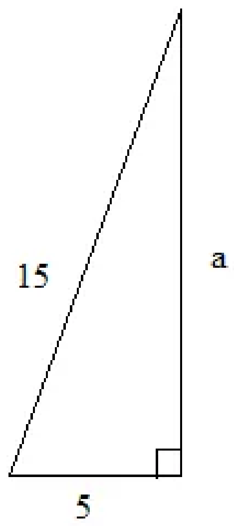
Therefore, area of the triangle $XYZ = \frac{1}{2} \cdot \overline{YZ} \cdot \overline{XY}$

$$\begin{aligned} &= \frac{1}{2} \cdot 2\sqrt{7} \cdot 6 \\ &= 6\sqrt{7} \end{aligned}$$

Thus, the area of the triangle XYZ is $\boxed{6\sqrt{7}\text{ units}^2}$

Answer 13PA.

Let $c=15, b=5,$ and $a=?$



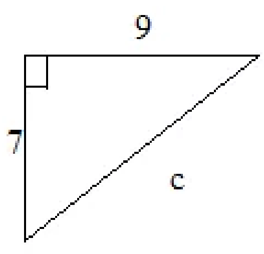
Applying Pythagorean Theorem, we have:

$$c^2=a^2+b^2$$
$$15^2=a^2+5^2$$
$$225=a^2+25$$
$$a^2=200$$
$$a=\sqrt{200}$$
$$=14\cdot 14$$

Therefore, the length of the missing side is $a=\boxed{14.14}$ rounded to the nearest hundredth.

Answer 14PA.

Let $a=7, b=9,$ and $c=?$



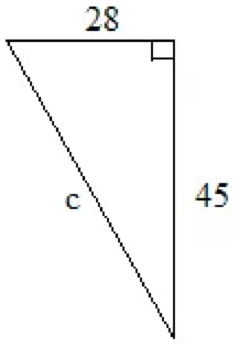
Applying Pythagorean Theorem, we have:

$$c^2=a^2+b^2$$
$$=7^2+9^2$$
$$=49+81$$
$$=130$$
$$c=\sqrt{130}$$
$$=11\cdot 40$$

Therefore the length of the missing side is $c=\boxed{11\cdot 40}$ rounded to the nearest hundredth.

Answer 15PA.

Let $a = 28, b = 45$ and $c = ?$



Applying Pythagorean Theorem, we have:

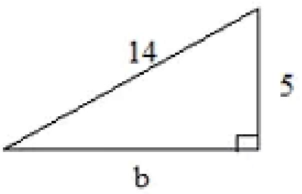
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 28^2 + 45^2 \\ &= 784 + 2025 \\ &= 2809 \end{aligned}$$

$$\begin{aligned} c &= \sqrt{2809} \\ &= 53 \end{aligned}$$

Therefore the length of the missing side is $c = \boxed{53}$ rounded to the nearest hundredth.

Answer 16PA.

Let $c = 14, a = 5$ and $b = ?$



Applying Pythagorean Theorem, we have:

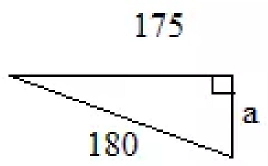
$$\begin{aligned} c^2 &= a^2 + b^2 \\ 14^2 &= 5^2 + b^2 \\ 196 &= 25 + b^2 \\ b^2 &= 171 \end{aligned}$$

$$\begin{aligned} b &= \sqrt{171} \\ &= 13 \cdot 07 \end{aligned}$$

Therefore the length of the missing side is $b = \boxed{13 \cdot 07}$ rounded to the nearest hundredth.

Answer 17PA.

Let $c = 180, b = 175$ and $a = ?$



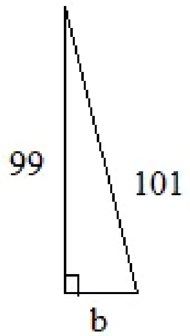
Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$180^2 = a^2 + 175^2$$
$$32400 = a^2 + 30625$$
$$a^2 = 1775$$
$$a = \sqrt{1775}$$
$$= 42 \cdot 13$$

Therefore the length of the missing side is $a = \boxed{42 \cdot 13}$ rounded to the nearest hundredth.

Answer 18PA.

Let $c = 101, a = 99$ and $b = ?$



Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$101^2 = 99^2 + b^2$$
$$10201 = 9801 + b^2$$
$$b^2 = 400$$
$$b = \sqrt{400}$$
$$= 20$$

Therefore the length of the missing side is $b = \boxed{20}$ rounded to the nearest hundredth.

Answer 19PA.

Here c is the measure of the hypotenuse of a right triangle, also $a = 16, b = 63$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 16^2 + 63^2 \\ &= 256 + 3969 \\ &= 4225 \end{aligned}$$

$$\begin{aligned} c &= \sqrt{4225} \\ &= 65 \end{aligned}$$

Therefore, the length of the missing side is $c = \boxed{65}$

Answer 20PA.

Given c is the measure of the hypotenuse of a right triangle, $a = 16, c = 34$ and $b = ?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 34^2 &= 16^2 + b^2 \\ 1156 &= 256 + b^2 \\ b^2 &= 900 \end{aligned}$$

$$\begin{aligned} b &= \sqrt{900} \\ b &= 30 \end{aligned}$$

Therefore, the length of the missing side is $b = \boxed{30}$

Answer 21PA.

Here, c is the measure of the hypotenuse of a right triangle, $b = 3, a = \sqrt{112}$ and $c = ?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= \left(\sqrt{112}\right)^2 + 3^2 \\ &= 112 + 9 \\ &= 121 \end{aligned}$$

$$\begin{aligned} c &= \sqrt{121} \\ c &= 11 \end{aligned}$$

Therefore, the length of the missing side is $c = \boxed{11}$

Answer 22PA.

Here, c is the measure of the hypotenuse of a right triangle, $a=\sqrt{15}, b=\sqrt{10}$ and $c=?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2+b^2 \\ &= \left(\sqrt{15}\right)^2+\left(\sqrt{10}\right)^2 \\ &= 15+10 \\ &= 25 \end{aligned}$$

$$c=\sqrt{25}$$

$$c=5$$

Therefore, the length of the missing side is $c=\boxed{5}$

Answer 23PA.

Here, c is the measure of the hypotenuse of a right triangle, $c=14, a=9$ and $b=?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2+b^2 \\ 14^2 &= 9^2+b^2 \\ 196 &= 81+b^2 \\ b^2 &= 115 \end{aligned}$$

$$b=\sqrt{115}$$

$$b=10\cdot 72$$

Therefore, the length of the missing side is $b=\boxed{10\cdot 72}$ rounded to the nearest hundredth.

Answer 24PA.

Here c is the measure of the hypotenuse of a right triangle, $a=6, b=3$ and $c=?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2+b^2 \\ &= 6^2+3^2 \\ &= 36+9 \\ &= 45 \end{aligned}$$

$$c=\sqrt{45}$$

$$c=6\cdot 70$$

Therefore, the length of the missing side is $c=\boxed{6\cdot 70}$ rounded to the nearest hundredth.

Answer 26PA.

Here, c is the measure of the hypotenuse of a right triangle, $a=4, b=\sqrt{11}$ and $c=?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2+b^2 \\ &= 4^2+11^2 \\ &= 16+121 \\ &= 137 \end{aligned}$$

$$c=\sqrt{137}$$

$$c=11\cdot 70$$

Therefore, the length of the missing side is $c=\boxed{11\cdot 70}$ rounded to the nearest hundredth.

Answer 27PA.

Here, c is the measure of the hypotenuse of a right triangle, $a=\sqrt{225}, b=\sqrt{28}$ and $c=?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2+b^2 \\ &= \left(\sqrt{225}\right)^2+\left(\sqrt{28}\right)^2 \\ &= 225+28 \\ &= 253 \end{aligned}$$

$$c=\sqrt{253}$$

$$c=15\cdot 90$$

Therefore, the length of the missing side is $c=\boxed{15\cdot 90}$ rounded to the nearest hundredth.

Answer 28PA.

Here, c is the measure of the hypotenuse of a right triangle, $a=\sqrt{31}, c=\sqrt{155}$ and $a=?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2+b^2 \\ \left(\sqrt{155}\right)^2 &= \left(\sqrt{31}\right)^2+b^2 \\ 155 &= 31+b^2 \\ b^2 &= 124 \end{aligned}$$

$$b=\sqrt{124}$$

$$b=11\cdot 13$$

Therefore, the length of the missing side is $b=\boxed{11\cdot 13}$ rounded to the nearest hundredth.

Answer 29PA.

Here, c is the measure of the hypotenuse of a right triangle, $a = 8x, b = 15x$ and $c = ?$

Applying Pythagorean Theorem, we have:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= (8x)^2 + (15x)^2 \\ &= 64x^2 + 225x^2 \\ &= 289x^2 \end{aligned}$$

$$c = \sqrt{289x^2}$$

$$c = 17x$$

Therefore, the length of the missing side is $c = \boxed{17x}$ rounded to the nearest hundredth.

Answer 30PA.

Here, c is the measure of the hypotenuse of a right triangle, $b = 3x, c = 7x$ and $a = ?$

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$

$$(7x)^2 = a^2 + (3x)^2$$

$$49x^2 = a^2 + 9x^2$$

$$a^2 = 40x^2$$

$$a = \sqrt{40x^2}$$

$$a = 6 \cdot 32x$$

Therefore, the length of the missing side is $a = \boxed{6 \cdot 32x}$ rounded to the nearest hundredth.

Answer 31PA.

Here, the measure of the longest side is 50. Therefore, assume the longest side to be hypotenuse, say $c = 50$. Also let $a = 30, b = 40$. Then determine whether $c^2 = a^2 + b^2$ or not.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$

$$50^2 = 30^2 + 40^2$$

$$2500 = 900 + 1600$$

$$2500 = 2500$$

Since $c^2 = a^2 + b^2$, therefore the triangle is $\boxed{\text{a right triangle}}$.

Answer 32PA.

Here, the measure of the longest side is 18. Therefore, assume the longest side to be hypotenuse, say $c = 18$. Also let $a = 6, b = 12$. Then determine whether $c^2 = a^2 + b^2$ or not.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$

$$18^2 = 6^2 + 12^2$$

$$324 = 36 + 144$$

$$324 = 180$$

This is not possible. That is, $c^2 \neq a^2 + b^2$, therefore the triangle is $\boxed{\text{not a right triangle}}$.

Answer 33PA.

Here, the measure of the longest side is 36. Therefore, assume the longest side to be hypotenuse, say $c = 36$. Also let $a = 24, b = 30$. Then determine whether $c^2 = a^2 + b^2$ or not.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$36^2 = 24^2 + 30^2$$
$$1296 = 576 + 900$$
$$1296 = 1476$$

This is not possible. That is, $c^2 \neq a^2 + b^2$, therefore the triangle is $\boxed{\text{not a right triangle}}$.

Answer 34PA.

Here, the measure of the longest side is 75. Therefore, assume the longest side to be hypotenuse, say $c = 75$. Also let $a = 45, b = 60$. Then determine whether $c^2 = a^2 + b^2$ or not.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$75^2 = 45^2 + 60^2$$
$$5625 = 2025 + 3600$$
$$5625 = 5625$$

Since $c^2 = a^2 + b^2$, therefore the triangle is $\boxed{\text{a right triangle}}$.

Answer 35PA.

Here, the measure of the longest side is 16. Therefore, assume the longest side to be hypotenuse, say $c = 16$. Also let $a = 15, b = \sqrt{31}$. Then determine whether $c^2 = a^2 + b^2$ or not.

Applying Pythagorean Theorem, we have:

$$c^2 = a^2 + b^2$$
$$16^2 = 15^2 + \left(\sqrt{31}\right)^2$$
$$256 = 225 + 31$$
$$256 = 256$$

Since $c^2 = a^2 + b^2$, therefore the triangle is $\boxed{\text{a right triangle}}$.

Answer 36PA.

Here, the measure of the longest side is $\sqrt{65}$. Therefore, assume the longest side to be hypotenuse, say $c=\sqrt{65}$. Also let $a=4, b=7$. Then determine whether $c^2=a^2+b^2$ or not.

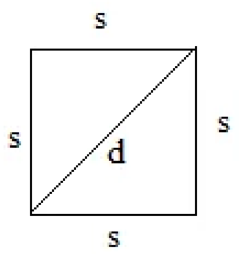
Applying Pythagorean Theorem, we have:

$$c^2=a^2+b^2$$
$$\left(\sqrt{65}\right)^2=4^2+7^2$$
$$65=16+49$$
$$65=65$$

Since $c^2=a^2+b^2$, therefore the triangle is $\boxed{\text{a right triangle}}$.

Answer 37PA.

Consider the following square whose side measures s units and join any two opposite vertices to make a diagonal. Let the measure of this diagonal be $d\ units$. This diagonal divides the square into two right triangles.



Here, the area of the square is 162 square feet

We know that Area of a square $=(\text{side})^2$

$$162=s^2$$
$$s=\sqrt{162}$$
$$=\sqrt{3^2\cdot 3^2\cdot 2}$$
$$=3\cdot 3\sqrt{2}$$
$$=9\sqrt{2}$$

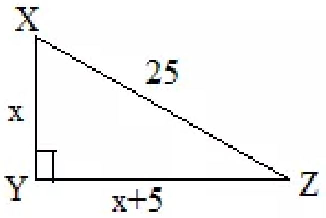
Applying Pythagorean Theorem to any one of the two right angled triangle:

$$d^2=s^2+s^2$$
$$d^2=\left(9\sqrt{2}\right)^2+\left(9\sqrt{2}\right)^2$$
$$d^2=2\left(9\sqrt{2}\right)^2$$
$$d^2=2(81\cdot 2)$$
$$d^2=324$$
$$d=\sqrt{324}$$
$$=18$$

Therefore the length of diagonal of the square $=\boxed{18 \text{ feet}}$

Answer 38PA.

Here, XYZ is a right angled triangle, let the length of one leg be x and the other be $x+5$
The length of the hypotenuse is 25 ,so $\overline{XZ}=25$



Therefore,

Applying Pythagorean Theorem, we have

$$XZ^2 = XY^2 + YZ^2$$
$$25^2 = x^2 + (x+5)^2$$
$$625 = x^2 + x^2 + 10x + 25$$
$$600 = 2x^2 + 10x$$
$$2x^2 + 10x - 600 = 0$$
$$2(x^2 + 5x - 300) = 0$$
$$x^2 + 5x - 300 = 0$$
$$x^2 + 20x - 15x - 300 = 0$$
$$x(x+20) - 15(x+20) = 0$$
$$(x+20)(x-15) = 0$$
$$x = -20, 15$$

The value of $x=-20$ is not possible. Therefore $x=15$.

So, the lengths of one leg $=x$

$=15$

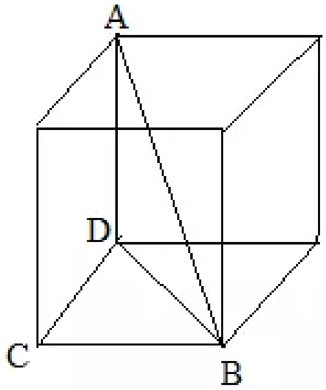
And the length of other leg $=x+5$

$=15+5$
$=20$

Hence the lengths of the legs are $\boxed{15\,\text{cm}, 20\,\text{cm}}$.

Answer 39PA.

Let us consider a cube as shown in the figure and each side is $4\ inches$ long.



The line from D to B completes the triangle DCB, which has a right angle at C.

Thus by applying Pythagorean Theorem, we have:

$$
\begin{aligned}
|DB|^2 &= |DC|^2 + |CB|^2 \\
&= 4^2 + 4^2 \\
&= 16 + 16 \\
&= 32 \\
|DB| &= \sqrt{32} \\
&= 4\sqrt{2}
\end{aligned}
$$

Triangle ABD is also a right triangle with the right angle at D.

Hence by applying Pythagorean theorem, we have

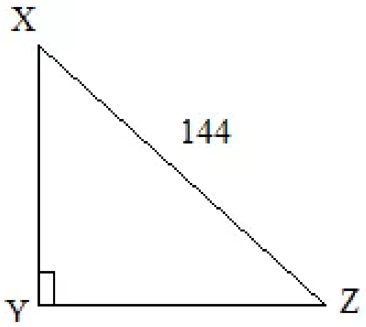
$$
\begin{aligned}
|AB|^2 &= |AD|^2 + |DB|^2 \\
&= 4^2 + \left(4\sqrt{2}\right)^2 \\
&= 16 + (16 \cdot 2) \\
&= 16 + 32 \\
|AB|^2 &= 48 \\
AB &= \sqrt{48} \\
&= 4\sqrt{3}
\end{aligned}
$$

So the length of the diagonal is $\boxed{4\sqrt{3}\text{ units}}$

Answer 40PA.

Here, XYZ is a right angled triangle, the length of the hypotenuse is 144,so $\overline{XZ} = 144$

The ratio of the length of the hypotenuse to the length of the shorter leg is $8:5$



So, consider, the length of the shorter leg is $5x$ and the length of the hypotenuse is $8x$

Now,

$$8x = 144$$
$$x = 18$$

Therefore, the length of shorter leg $XY = 5x$

$$= 5 \cdot 18$$
$$= 90$$

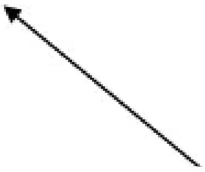
Now, by applying Pythagorean Theorem, we have:

$$XZ^2 = XY^2 + YZ^2$$
$$144^2 = 90^2 + YZ^2$$
$$20736 = 8100 + YZ^2$$
$$YZ^2 = 12636$$
$$YZ = \sqrt{12636}$$
$$= 112 \cdot 40$$

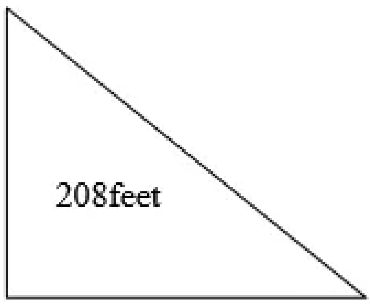
The length of the longer leg is $YZ = \boxed{112 \cdot 40 \text{ meters}}$

Answer 41PA.

Consider a roller coaster which climbs 208 feet higher than its starting point making a horizontal advance of 360 feet .When it comes down ,it makes a horizontal advance of 44 feet.



A



D

B C

44 feet 360feet

In the figure shown above triangle forms a right angled triangle.

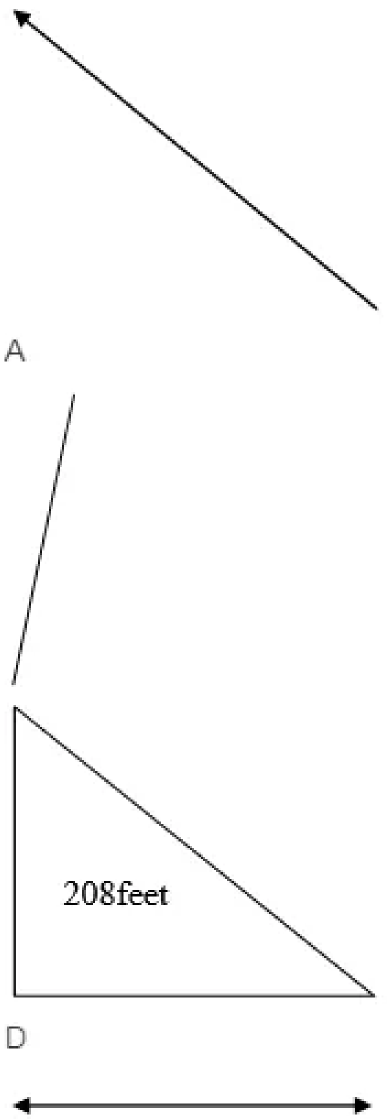
Therefore,by applying Pythagorean theorem we have

$$AC^2 = AD^2 + DC^2$$
$$= 208^2 + 360^2$$
$$= 43264 + 129600$$
$$= 172864$$
$$AC^2 = \sqrt{172864}$$
$$AC = 415 \cdot 76$$
$$= 415 \cdot 8$$

Therefore it will travel about $\boxed{415 \cdot 8 \text{ feet}}$ to get to the top of the ride.

Answer 42PA.

Consider a roller coaster which climbs $208\,\text{feet}$ higher than its starting point making a horizontal advance of $360\,\text{feet}$ .When it comes down ,it makes a horizontal advance of $44\,\text{feet}$.



B C

44 feet 360feet

In the figure shown above triangle forms a right angled triangle.

Therefore,by applying Pythagorean theorem we have

$$\begin{aligned} AB^2 &= BD^2 + AD^2 \\ &= 44^2 + 208^2 \\ &= 1936 + 43264 \\ &= 45200 \end{aligned}$$

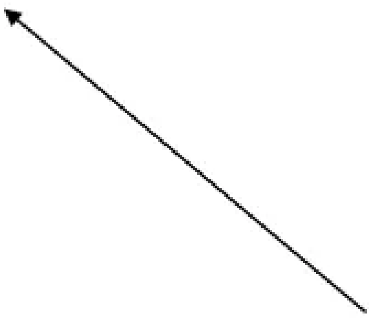
$$AB^2 = \sqrt{45200}$$

$$AB = 212 \cdot 60$$

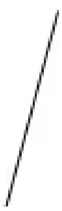
Therefore it will travel about $\boxed{212 \cdot 6 \text{ feet}}$ on the downhill track.

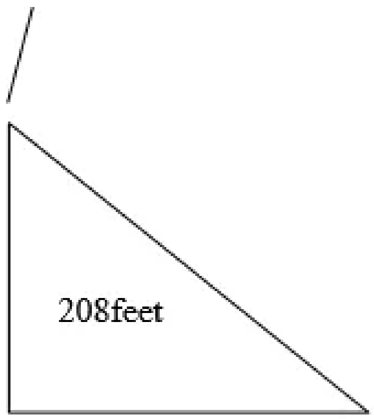
Answer 43PA.

Consider a roller coaster which climbs 208 feet higher than its starting point making a horizontal advance of 360 feet .When it comes down ,it makes a horizontal advance of 44 feet.



A





D B

C

44feet 360feet

Therefore,the roller coaster makes a total horizontal advance of $(DB + BC) = 360 + 44$

that is $\boxed{404\text{ feet}}$.

It reaches a vertical height of $\boxed{208\text{ feet}}$.

In the figure shown above triangle forms a right angled triangle.

Therefore,by applying Pythagorean theorem we have

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 208^2 + 360^2 \\ &= 43264 + 129600 \\ &= 172864 \end{aligned}$$

$$\begin{aligned} AC^2 &= \sqrt{172864} \\ AC &= 415 \cdot 76 \\ &= 415 \cdot 8 \end{aligned}$$

Again ADB also forms a right angled triangle.

Therefore,by applying Pythagorean theorem we have

$$\begin{aligned} AD^2 &= DB^2 + AB^2 \\ &= 44^2 + 208^2 \\ &= 1936 + 43264 \\ &= 45200 \end{aligned}$$

$$\begin{aligned} AD^2 &= \sqrt{45200} \\ AD &= 212 \cdot 60 \end{aligned}$$

Therefore it travels a total track length of

$$\begin{aligned} (AD + AC) &= (212 \cdot 60 + 415 \cdot 8) \\ &= \boxed{628 \cdot 4 \text{ feet}} \end{aligned}$$

Answer 45PA.

Consider a sailboat whose mast and boom form a right angle.The sail itself,called a mainsail is in the shape of a right triangle.

The edge of the mainsail that is attached to the mast is 100 feet long and the edge

of the mainsail that is attached to the boom is 60 feet long .

Therefore the mast and boom form the legs of the right triangle.

Hence $a=100, b=60$

The longest edge of the mainsail form the hypotenuse of the triangle. $c=?$

Applying Pythagorean theorem,we have

$$\begin{aligned} c^2 &= a^2+b^2 \\ &= 100^2+60^2 \\ &= 10000+3600 \\ &= 13600 \end{aligned}$$

$$\begin{aligned} c &= \sqrt{13600} \\ &= 116\cdot 61 \end{aligned}$$

Therefore,the length of the longest edge of the mainsail is $=\boxed{116\cdot 61 \text{ feet}}$

Answer 48PA.

We know that,the area for a semi-circle is $\frac{1}{2}(\pi)(\text{radius}^2)\quad \cdots\cdots(1)$

Now,the radius for each semi-circle is $\frac{1}{2}(\text{side length})$

Therefore,substituting the value of radius in equation (1),

$$\frac{1}{2}(\pi)\left(\frac{1}{2}(\text{side length})\right)^2$$

$$=\frac{1}{2}(\pi)\frac{1}{4}(\text{side length}^2)$$

$$=\frac{1}{8}(\pi)(\text{side length}^2)$$

So if the areas of the two smaller semi-circles is equal to the larger semi-circle,then the

Pythagorean theorem prevails.

$$\frac{1}{8}(\pi)a^2+\frac{1}{8}(\pi)b^2=\frac{1}{8}(\pi)c^2$$

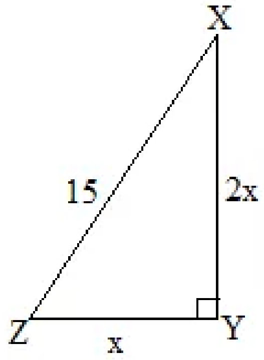
$$\frac{1}{8}(\pi)(a^2+b^2)=\frac{1}{8}(\pi)c^2$$

$$a^2+b^2=c^2$$

Hence $\boxed{a^2+b^2=c^2}$

Answer 51PA.

Here, XYZ is a right angled triangle, the length of $\overline{YZ}$ is x, $\overline{XY}$ is $2x$ and the length of the hypotenuse is 15,so $\overline{XZ} = 15$



Therefore,

Applying Pythagorean Theorem, we have:

$$XZ^2 = XY^2 + YZ^2$$
$$15^2 = (2x)^2 + x^2$$
$$225 = 4x^2 + x^2$$
$$225 = 5x^2$$
$$x^2 = 45$$
$$x = \sqrt{45}$$
$$= 3\sqrt{5}$$

Area of a triangle $= \frac{1}{2}bh$ where b is the base and h is the height of the triangle.

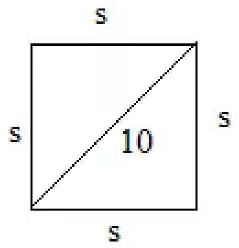
Therefore,

$$\text{area} = \frac{1}{2} \cdot \overline{YZ} \cdot \overline{XY}$$
$$= \frac{1}{2} \cdot 3\sqrt{5} \cdot 2\left(3\sqrt{5}\right)$$
$$= 9\left(\sqrt{5}\right)^2$$
$$= 9 \cdot 5$$
$$= 45$$

Therefore, the area of the triangle is $\boxed{45\,\text{units}^2}$

Answer 52PA.

Consider the following square whose side measures s units and join any two opposite vertices to make a diagonal. The measure of the diagonal is $10\,cm$. This diagonal divides the square into two right triangles.



Applying Pythagorean Theorem to any one of the two right angled triangle:

$$10^2 = s^2 + s^2$$
$$100 = 2s^2$$
$$s^2 = 50$$
$$s = \sqrt{50}$$

Perimeter of a square $= 4s$, where s is the length of any side of the square.

$$= 4\sqrt{50}$$
$$= 4\sqrt{2 \cdot 5^2}$$
$$= 4 \cdot 5\sqrt{2}$$
$$= 20\sqrt{2}$$

Therefore the perimeter of the square $= \boxed{20\sqrt{2}\text{ cm}}$

Answer 53MYS.

Here, the equation is

$$\sqrt{y} = 12$$

Squaring both sides we have:

$$y = 144$$

Therefore the solution is $\boxed{144}$

CHECK:

$$\sqrt{y} = 12 \quad , \quad y = 144$$
$$\sqrt{144} = 12$$
$$12 = 12$$

Since 144 satisfy the original equation, the solution is $\boxed{144}$

Answer 54MYS.

Here the equation is $3\sqrt{s}=126$

Squaring both sides we have,

$$\left(3\sqrt{s}\right)^2=126^2$$
$$9s=15876$$
$$s=1764$$

Therefore the solution is $s=\boxed{1764}$

CHECK:

$$3\sqrt{s}=126, \quad s=1764$$
$$3\sqrt{1764}=126$$
$$3\cdot 42=126$$
$$126=126$$

Since 126 satisfy the original equation, the solution is $\boxed{126}$

Answer 55MYS.

Here the equation is,

$$4\sqrt{2v+1}-3=17$$
$$4\sqrt{2v+1}=20$$

Squaring both sides we have,

$$16\left(\sqrt{2v+1}\right)^2=20^2$$
$$16(2v+1)=400$$
$$32v+16=400$$
$$32v=384$$
$$v=12$$

Therefore the solution is $v=\boxed{12}$

CHECK:

$$4\sqrt{2v+1}-3=17 \quad , \quad v=12$$
$$4\sqrt{24+1}-3=17$$
$$4\sqrt{25}-3=17$$
$$20-3=17$$
$$17=17$$

Since 12 satisfy the original equation, the solution is $\boxed{12}$

Answer 56MYS.

Here,

$$\sqrt{72}=\sqrt{2^2\cdot 3^2\cdot 2}$$
$$=2\cdot 3\sqrt{2}$$
$$=6\sqrt{2}$$

Therefore, the answer is $\boxed{6\sqrt{2}}$

Answer 57MYS.

Here,

$$7\sqrt{z}-10\sqrt{z}$$
$$=(7-10)\sqrt{z} \quad \text{(Distributive property)}$$
$$=-3\sqrt{z}$$

Therefore, the simplified form is $\boxed{-3\sqrt{z}}$

Answer 58MYS.

Here,

$$\sqrt{\frac{3}{7}}+\sqrt{21}$$
$$=\frac{\sqrt{3}}{\sqrt{7}}+\sqrt{3\cdot 7}$$
$$=\frac{\sqrt{3}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}+\sqrt{3}\cdot\sqrt{7} \quad \left(\text{Multiply by }\frac{\sqrt{7}}{\sqrt{7}}\right)$$
$$=\frac{\sqrt{3}\cdot\sqrt{7}}{\left(\sqrt{7}\right)^2}+\sqrt{3}\cdot\sqrt{7}$$
$$=\frac{\sqrt{3}\cdot\sqrt{7}}{7}+\sqrt{3}\cdot\sqrt{7}$$
$$=\sqrt{3}\cdot\sqrt{7}\left(\frac{1}{7}+1\right)$$
$$=\sqrt{21}\left(\frac{1+7}{7}\right)$$
$$=\frac{8\sqrt{21}}{7}$$

Therefore, the simplified form is $\boxed{\frac{8\sqrt{21}}{7}}$

Answer 59MYS.

Here we have to assume that no denominator is equal to zero

$$\frac{5^8}{5^3} = 5^{8-3} \qquad \left(\frac{a^m}{a^n} = a^{m-n}\right)$$

$$= 5^5$$

$$= 3125$$

Therefore,the solution is $\boxed{3125}$.

Answer 60MYS.

Here we have to assume that no denominator is equal to zero

$$d^{-7} = \frac{1}{d^7}$$

Therefore,the solution is $\boxed{\frac{1}{d^7}}$.

Answer 61MYS.

Here we have to assume that no denominator is equal to zero

$$\frac{-26a^4b^7c^{-5}}{-13a^2b^4c^3} = 2a^{4-2}b^{7-4}c^{-5-3}$$

$$= 2a^2b^3c^{-8}$$

$$= \frac{2a^2b^3}{c^8}$$

Therefore,the solution is $\boxed{\frac{2a^2b^3}{c^8}}$.

Answer 62MYS.

Let the speed of the plane be x and the speed of air be y

Total distance covered $= 300 \text{ miles}$

While flying with the wind total speed of plane $= x + y$

Time taken $= 40 \text{ min } or \frac{2}{3} \text{ hour}$ $\quad \left(\begin{aligned} 40 \text{ min} &= \frac{40}{60} \text{ hour} \\ &= \frac{2}{3} \text{ hour}\end{aligned}\right)$

We know that $\text{time} = \frac{\text{distance}}{\text{speed}}$

$$\frac{2}{3}=\frac{300}{x+y}$$

$$2(x+y)=900$$

$$x+y=\frac{900}{2}$$

$$x+y=450 \qquad \cdots\cdots(1)$$

While flying against the wind total speed of plane $=x-y$

Time taken to return $=45 \text{ min or } \frac{3}{4}\text{hour}$ $\qquad \left(\begin{aligned}45\text{ min} &= \frac{45}{60}\text{hour}\\ &= \frac{3}{4}\text{ hour}\end{aligned}\right)$

Distance covered $=300\,\text{miles}$

Now, $\text{time}=\frac{\text{distance}}{\text{speed}}$

$$\frac{3}{4}=\frac{300}{x-y}$$

$$3(x-y)=1200$$

$$x-y=\frac{1200}{3}$$

$$x-y=400 \qquad \cdots\cdots(2)$$

Adding equation (1) and (2), we get

$$(x+y)+(x-y)=450+400$$

$$2x=850$$

$$x=425$$

Substituting this value in equation (1), we get

$$\frac{2}{3}=\frac{300}{(425+y)}$$

$$2(425+y)=900$$

$$850+2y=900$$

$$2y=50$$

$$y=25$$

Hence speed of plane is $\boxed{425\text{ miles/ hour}}$

Answer 63MYS.

The given expression is

$$\sqrt{(6-3)^2+(8-4^2)}$$

$$=\sqrt{3^2+4^2}$$

$$=\sqrt{9+16}$$

$$=\sqrt{25}$$

$$=5$$

Therefore, the simplified form is $\boxed{5}$

Answer 64MYS.

The given expression is

$$\sqrt{(10-4)^2+(13-5)^2}$$

$$=\sqrt{6^2+8^2}$$

$$=\sqrt{36+64}$$

$$=\sqrt{100}$$

$$=10$$

Therefore, the simplified form is $\boxed{10}$

Answer 65MYS.

The given expression is

$$\sqrt{(5-3)^2+(2-9)^2}$$

$$=\sqrt{2^2+(-7)^2}$$

$$=\sqrt{4+49}$$

$$=\sqrt{53}$$

Therefore, the simplified form is $\boxed{\sqrt{53}}$

Answer 66MYS.

The given expression is

$$\sqrt{(-9-5)^2+(7-3)^2}$$
$$=\sqrt{(-14)^2+(4)^2}$$
$$=\sqrt{196+16}$$
$$=\sqrt{212}$$
$$=2\sqrt{53}$$

Therefore, the simplified form is $\boxed{2\sqrt{53}}$

Answer 67MYS.

The given expression is

$$\sqrt{(-4-5)^2+(-4-3)^2}$$
$$=\sqrt{(-9)^2+(-7)^2}$$
$$=\sqrt{81+49}$$
$$=\sqrt{130}$$

Therefore the simplified form is $\boxed{\sqrt{130}}$

Answer 68MYS.

The given expression is

$$\sqrt{(20-5)^2+(-2-6)^2}$$
$$=\sqrt{(15)^2+(-8)^2}$$
$$=\sqrt{225+64}$$
$$=\sqrt{289}$$
$$=17$$

Therefore the simplified form is $\boxed{17}$