Matrices

EXERCISE 2.1 [PAGES 39 - 40]

Exercise 2.1 | Q 1.1 | Page 39

Construct a matrix A = $[a_{\text{ij}}]_{3\times2}$ whose element a_{ij} is given by

$a_{ij} = \dfrac{(i-j)^2}{5-i}$

Solution:

A = $[a_{\text{ij}}]_{3\times2}$

$\therefore$ A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$a_{ij} = \dfrac{(i-j)^2}{5-i}$

$\therefore$ $a_{11} = \dfrac{(1-1)^2}{5-1} = 0, a_{12} = \dfrac{(1-2)^2}{5-1} = \dfrac{(-1)^2}{4} = \dfrac{1}{4}$

$a_{21} = \dfrac{(2-1)^2}{5-2} = \dfrac{1}{3}, a_{22} = \dfrac{(2-2)^2}{5-2}$ = 0,

$a_{31} = \dfrac{(3-1)^2}{5-3} = \dfrac{2^2}{2} = 2, a_{32} = \dfrac{(3-2)^2}{5-3} = \dfrac{1}{2}$

$\therefore$ A = $\begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{3} & 0 \\ 2 & \frac{1}{2} \end{bmatrix}$.

Exercise 2.1 | Q 1.2 | Page 39

Construct a matrix A = [aij]$_{3\times2}$ whose element a_{ij} is given by

$a_{ij} = i - 3j$

Solution: $a_{ij} = i - 3j$

$\therefore a_{11} = 1 - 3(1) = 1 - 3 = -2$

$a_{12} = 1 - 3(2) = 1 - 6 = -5$

$a_{21} = 2 - 3(1 = 2 - 3 = -1$

$a_{22} = 2 - 3(2) = 2 - 6 = -4$

$a_{31} = 3 - 3(1) = 3 - 3 = 0,$

$a_{32} = 3 - 3(2) = 3 - 6 = -3$

$$\therefore A = \begin{bmatrix} -2 & -5 \\ -1 & -4 \\ 0 & -3 \end{bmatrix}$$

Exercise 2.1 | Q 1.3 | Page 39

Construct a matrix A = $[a_{ij}]_{3\times2}$ whose element a_{ij} is given by

$$a_{ij} = \frac{(i+j)^3}{5}$$

Solution:

$$a_{ij} = \frac{(i+j)^3}{5}$$

$$\therefore a11 = \frac{(1+1)^3}{5} = \frac{2^3}{5} = \frac{8}{5}, a_{12} = \frac{(1+2)^3}{5} = \frac{3^3}{5} = \frac{27}{5}$$

$$a_{21} = \frac{(2+1)^3}{5} = \frac{3^3}{5} = \frac{27}{5}, a_{22} = \frac{(2+2)^2}{5} = \frac{4^3}{5} = \frac{64}{5}$$

$$a_{31} = \frac{(3+1)^3}{5} = \frac{4^3}{5} = \frac{64}{5}, a_{32} = \frac{(3+2)^2}{5} = \frac{5^3}{5} = \frac{125}{5}$$

$$\therefore A = \begin{bmatrix} \frac{8}{5} & \frac{27}{5} \\ \frac{27}{5} & \frac{64}{5} \\ \frac{64}{5} & \frac{125}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 27 \\ 27 & 64 \\ 64 & 125 \end{bmatrix}.$$

Exercise 2.1 | Q 2.1 | Page 39

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper traingular, a lower triangular matrix.

$$\begin{bmatrix} 3 & -2 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 3 & -2 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

As every element below the diagonal is zero in matrix A.

∴ A is an upper triangular matrix.

Exercise 2.1 | Q 2.2 | Page 39

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper traingular, a lower triangular matrix.

$$\begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$

As matrix A has only one column.

∴ A is a column matrix.

Exercise 2.1 | Q 2.3 | Page 39

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper traingular, a lower triangular matrix.

[9 √2 -3]

Solution:

Let A = $\begin{bmatrix} 9 & \sqrt{2} & -3 \end{bmatrix}$

As matrix A has only one row.

∴ A is a row matrix.

Exercise 2.1 | Q 2.4 | Page 39

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper traingular, a lower triangular matrix.

$\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

As matrix A has all its non-diagonal elements zero and diagonal elements same.

∴ A is a scalar matrix.

Exercise 2.1 | Q 2.5 | Page 39

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper traingular, a lower triangular matrix.

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$$

As every element above the diagonal is zero in matrix A.

∴ A is a lower triangular matrix.

Exercise 2.1 | Q 2.6 | Page 39

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper traingular, a lower triangular matrix.

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

As matrix A has all its non-diagonal elements zero.

∴ A is a diagonal martix.

Exercise 2.1 | Q 2.7 | Page 39

Classify each of the following matrices as a row, a column, a square, a diagonal, a scalar, a unit, an upper traingular, a lower triangular matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

In matrix A, all the non-diagonal elements are zero and diagonal elements are one.

∴ A is a unit (identity) matrix.

Exercise 2.1 | Q 3.1 | Page 39

Which of the following matrices are singular or non singular?

$$\begin{bmatrix} a & b & c \\ p & q & r \\ 2a - p & 2b - q & 2c - r \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} a & b & c \\ p & q & r \\ 2a - p & 2b - q & 2c - r \end{bmatrix}$

∴ |A| = $\begin{vmatrix} a & b & c \\ p & q & r \\ 2a - p & 2b - q & 2c - r \end{vmatrix}$

Applying $R_3 \to R_3 + R_2$, we get

$$|A| = \begin{vmatrix} a & b & c \\ p & q & r \\ 2a & 2b & 2c \end{vmatrix}$$

Taking 2 common from R_3, we get

$$|A| = 2\begin{vmatrix} a & b & c \\ p & q & r \\ a & b & c \end{vmatrix}$$

= 2(0) ...[∵ R1 and R3 are identical]

= 0

∴ A is a singular martix.

Exercise 2.1 | Q 3.2 | Page 39

Which of the following matrices are singular or non singular?

$$\begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 5 & 0 & 5 \\ 1 & 99 & 100 \\ 6 & 99 & 105 \end{vmatrix}$

Applying $C_2 \to C_2 + C_1$, we get

$$|A| = \begin{vmatrix} 5 & 5 & 5 \\ 1 & 100 & 100 \\ 6 & 105 & 105 \end{vmatrix}$$

$= 0$ ....[$\because C_2$ aand C_3 are identical]

$\therefore$ A is a singular matrix.

Exercise 2.1 | Q 3.3 | Page 40

Which of the following matrices are singular or non singular?

$$\begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 3 & 5 & 7 \\ -2 & 1 & 4 \\ 3 & 2 & 5 \end{vmatrix}$$

= 3(5 – 8) – 5(–10 – 12) + 7(– 4 – 3)

= – 9 + 110 – 49

= 52 ≠ 0

$\therefore$ A is a non-singular martix.

Exercise 2.1 | Q 3.4 | Page 40

Which of the following matrices are singular or non singular?

$$\begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 7 & 5 \\ -4 & 7 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 7 & 5 \\ -4 & 7 \end{vmatrix}$

= 49 + 20

= 69 ≠ 0

$\therefore$ A is a non-singular martix.

Exercise 2.1 | Q 4.1 | Page 40

Find K if the following matrices are singular.

$\begin{bmatrix} 7 & 3 \\ -2 & \text{K} \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 7 & 3 \\ -2 & \text{K} \end{bmatrix}$

Since A is a singular matrix,

|A| = 0

$\therefore \begin{vmatrix} 7 & 3 \\ -2 & \text{K} \end{vmatrix} = 0$

$\therefore$ 7K + 6 = 0

$\therefore$ 7K = – 6

$\therefore$ K = $\frac{-6}{7}$.

Exercise 2.1 | Q 4.2 | Page 40

Find K if the following matrices are singular.

$$\begin{bmatrix} 4 & 3 & 1 \\ 7 & K & 1 \\ 10 & 9 & 1 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 4 & 3 & 1 \\ 7 & K & 1 \\ 10 & 9 & 1 \end{bmatrix}$

Since A is a singular matrix,

$|A| = 0$

$\therefore \begin{vmatrix} 4 & 3 & 1 \\ 7 & K & 1 \\ 10 & 9 & 1 \end{vmatrix} = 0$

$\therefore 4(K - 9) - (7 - 10) + 1(63 - 10K) = 0$

$\therefore 4K - 36 + 9 + 63 - 10K = 0$

$\therefore -6K + 36 = 0$

$\therefore 6K = 36$

$\therefore K = 6$

Exercise 2.1 | Q 4.3 | Page 40

Find K if the following matrices are singular.

$$\begin{bmatrix} K-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} K-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$

Since A is a singular matrix,

|A| = 0

$\therefore \begin{vmatrix} K-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{vmatrix} = 0$

$\therefore$ (K – 1)(4 + 4) –2(12 –2) + 3(–6 – 1) = 0

$\therefore$ 8K – 8 – 20 – 21 = 0

$\therefore$ 8K = 49

$\therefore$ K = $\dfrac{49}{8}$.

EXERCISE 2.2 [PAGES 46 - 47]

Exercise 2.2 | Q 1.1 | Page 46

If A = $\begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$, Show that A + B = B + A

Solution:

A + B = $\begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$

= $\begin{bmatrix} 2-1 & -3+2 \\ 5+2 & -4+2 \\ -6+0 & 1+3 \end{bmatrix}$

$$\therefore A + B = \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} \quad \text{....(i)}$$

$$B + A = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 2-3 \\ 2+5 & 2-4 \\ 0-6 & 3+1 \end{bmatrix}$$

$$\therefore B + A = \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} \quad \text{....(ii)}$$

From (i) and (ii), we get

A + B = B + A.

Exercise 2.2 | Q 1.2 | Page 46

If $A = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$, Show that

(A + B) + C = A + (B + C)

Solution:

$$(A + B) + C = \left\{ \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix} \right\} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & -3+2 \\ 5+2 & -4+2 \\ -6+0 & 1+3 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 7 & -2 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & -1+3 \\ 7-1 & -2+4 \\ -6-2 & 4+1 \end{bmatrix}$$

$$\therefore (A+B)+C = \begin{bmatrix} 5 & 2 \\ 6 & 2 \\ -8 & 5 \end{bmatrix} \qquad \text{....(i)}$$

$$A+(B+C) = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \left\{ \begin{bmatrix} -1 & 2 \\ 2 & 2 \\ 0 & \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 4 \\ -2 & 1 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} -1+4 & 2+3 \\ 2-1 & 2+4 \\ 0-2 & 3+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 6 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & -3+5 \\ 5+1 & -4+6 \\ -6-2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 6 & 2 \\ -8 & 5 \end{bmatrix} \qquad \text{....(ii)}$$

From (i) and (ii), we get

(A + B) + C = A + (B + C).

Exercise 2.2 | Q 2 | Page 46

If A = $\begin{bmatrix}1 & -2\\5 & 3\end{bmatrix}, B = \begin{bmatrix}1 & -3\\4 & -7\end{bmatrix}$, then find the matrix A – 2B + 6I, where I is the unit matrix of order 2.

Solution:

$$A - 2B + 6I = \begin{bmatrix}1 & -2\\5 & 3\end{bmatrix} 2\begin{bmatrix}1 & -3\\4 & -7\end{bmatrix} + 6\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & -2\\5 & 3\end{bmatrix} - \begin{bmatrix}2 & -6\\8 & -14\end{bmatrix} + \begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$$

$$= \begin{bmatrix}1 - 2 + 6 & -2 + 6 + 0\\5 - 8 + 0 & +14 + 6\end{bmatrix}$$

$$= \begin{bmatrix}5 & 4\\-3 & 23\end{bmatrix}.$$

Exercise 2.2 | Q 3 | Page 46

If A = $\begin{bmatrix}1 & 2 & -3\\-3 & 7 & -8\\0 & -6 & 1\end{bmatrix}, B = \begin{bmatrix}9 & -1 & 2\\-4 & 2 & 5\\4 & 0 & -3\end{bmatrix}$ then find the matrix C such that A + B + C is a zero matrix.

Solution: A + B + C is a zero martix.

∴ A + B + C = 0

∴ C = – (A + B)

$$= -\left\{\begin{bmatrix} 1 & 2 & 3 \\ -3 & 7 & -8 \\ 0 & -6 & 1 \end{bmatrix} + \begin{bmatrix} 9 & -1 & 2 \\ -4 & 2 & 5 \\ 4 & 0 & -3 \end{bmatrix}\right\}$$

$$= -\begin{bmatrix} 1+9 & 2-1 & -3+2 \\ -3-4 & 7+2 & -8+5 \\ 0+4 & -6+0 & 1-3 \end{bmatrix}$$

$$= -\begin{bmatrix} 10 & 1 & -1 \\ -7 & 9 & -3 \\ 4 & -6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -1 & 1 \\ -7 & -9 & 3 \\ -4 & 6 & -2 \end{bmatrix}.$$

Exercise 2.2 | Q 4 | Page 46

If $A = \begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix}$, find the matrix X such that 3A – 4B + 5X = C.

Solution:

3A – 4B + 5X = C

∴ 5X =C + 4B – 3A

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + 4\begin{bmatrix} -1 & -2 \\ 4 & 2 \\ 1 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ -1 & -4 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ 16 & 8 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 3 & -6 \\ 9 & -15 \\ -18 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4-3 & 4-8+6 \\ -1+16-9 & -4+8+5 \\ -3+4+18 & 6+20-0 \end{bmatrix}$$

$$= 5X = \begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix}$$

$$\therefore X = \frac{1}{5}\begin{bmatrix} -5 & 2 \\ 6 & 19 \\ 19 & 26 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & \frac{2}{5} \\ \frac{6}{5} & \frac{19}{5} \\ \frac{19}{5} & \frac{26}{5} \end{bmatrix}.$$

Exercise 2.2 | Q 5 | Page 46

If A = $\begin{bmatrix} 5 & 1 & -4 \\ 3 & 2 & 0 \end{bmatrix}$, find $(A^T)^T$.

Solution:

$$A = \begin{bmatrix} 5 & 1 & -4 \\ 3 & 2 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 5 & 3 \\ 1 & 2 \\ -4 & 0 \end{bmatrix}$$

$$\therefore (A^T)^T = \begin{bmatrix} 5 & 1 & -4 \\ 3 & 2 & 0 \end{bmatrix} = A.$$

Exercise 2.2 | Q 6 | Page 46

If A = $\begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$, find $(A^T)^T$.

Solution:

A = $\begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 7 & -2 & 5 \\ 3 & -4 & 9 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore (A^T)^T = \begin{bmatrix} 7 & 3 & 1 \\ -2 & -4 & 1 \\ 5 & 9 & 1 \end{bmatrix}$ = A.

Exercise 2.2 | Q 7 | Page 47

Find a, b, c, if $\begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$ is a symmetric matrix.

Solution:

Let A = $\begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 1 & b & 4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$

Since A is a symmetric matrix,

A = A^T

$$\therefore \begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & -7 \\ -4 & c & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & b & -4 \\ \frac{3}{5} & -5 & c \\ a & -7 & 0 \end{bmatrix}$$

∴ By equality of matrices, we get

a = – 4, b = $\frac{3}{5}$, c = – 7.

Exercise 2.2 | Q 8 | Page 47

Find x, y, z if $\begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$ is a skew symmetric matrix.

Solution:

Let A = $\begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$

$\therefore$ A^T = $\begin{bmatrix} 0 & -5\mathrm{i} & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$

Since A is a skew-symmetric matrix,

A = A^T

$$\therefore \begin{bmatrix} 0 & -5i & x \\ y & 0 & z \\ \frac{3}{2} & -\sqrt{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -y & \frac{-3}{2} \\ 5i & 0 & \sqrt{2} \\ -x & -z & 0 \end{bmatrix}$$

∴ By equality of matrices, we get

$$x = \frac{-3}{2}, y = 5i, z = \sqrt{2}$$

Exercise 2.2 | Q 9.1 | Page 47

For each of the following matrices, find its transpose and state whether it is symmetric, skew- symmetric or neither.

$$\begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$$

$\therefore A^T = A$ i.e., $A = A^T$

∴ A is a symmetric matrix.

Exercise 2.2 | Q 9.2 | Page 47

For each of the following matrices, find its transpose and state whether it is symmetric, skew- symmetric or neither.

$$\begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 5 & 1 \\ -5 & 4 & 6 \\ -1 & -6 & 3 \end{bmatrix}$

$\therefore$ A^T = $\begin{bmatrix} 2 & -5 & -1 \\ -5 & 4 & -6 \\ 1 & 6 & 3 \end{bmatrix}$

$\therefore$ A^T = $\begin{bmatrix} -2 & 5 & 1 \\ -5 & -4 & 6 \\ -1 & -6 & -3 \end{bmatrix}$

$\therefore$ $A \neq A^T$ and $A \neq -A^T$

$\therefore$ A is neither symmetric nor skew – symmetric matrix.

Exercise 2.2 | Q 9.3 | Page 47

For each of the following matrices, find its transpose and state whether it is symmetric, skew- symmetric or neither.

$$\begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 0 & 1-2i & 2-i \\ -1+2i & 0 & 7 \\ i-2 & -7 & 0 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 0 & 1+2i & i-2 \\ -1-2i & 0 & -7 \\ 2-i & 7 & 0 \end{bmatrix}$

$\therefore A^T = -A$ i.e. $A = -A^T$

$\therefore$ A is a skew-symmetric matrix.

Exercise 2.2 | Q 10 | Page 47

Construct the matrix $A = [a_{ij}]_{3\times3}$ where $a_{ij} = i - j$. State whether A is symmetric or skew-symmetric.

Solution: $A = [a_{ij}]_{3x3}$

$\therefore A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Given, $a_{ij} = i - j$

$\therefore a_{11} = 1 - 1 = 0$, $a_{12} = 1 - 2 = -1$, $a_{13} = 1 - 3 = -2$

$a_{21} = 2 - 1 = 1$, $a_{22} = 2 - 2 = 0$, $a_{23} = 2 - 3 = -1$,

$a_{31} = 3 - 1 = 2$, $a_{32} = 3 - 2 = 1$, $a_{33} = 3 - 3 = 0$

$\therefore$ A = $\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore$ A^T = $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$

= $-\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore$ $A^T = -A$ i.e., $A = -A^T$

$\therefore$ A is a skew-symmetric matrix.

Exercise 2.2 | Q 11 | Page 47

Solve the following equations for X and Y, if 3X – Y = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and X – 3Y = $\begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$.

Solution:

Given equations are

3X – Y = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ ...(i)

and X – 3Y = $\begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$ ...(ii)

By (i) x 3 – (ii), we get

8X = $3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

$= 3\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3-0 & -3+1 \\ -3-0 & 3+1 \end{bmatrix}$

$\therefore$ 8X = $\begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix}$

$\therefore$ X = $\frac{1}{8}\begin{bmatrix} 3 & -2 \\ -3 & 4 \end{bmatrix}$

$\therefore$ X = $\begin{bmatrix} \frac{3}{8} & \frac{-2}{8} \\ \frac{-3}{8} & \frac{4}{8} \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{8} & \frac{-1}{4} \\ \frac{-3}{8} & \frac{1}{2} \end{bmatrix}$

By (i) – (ii) x 3, we get

8Y = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ 0 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1-0 & -1+3 \\ -1-0 & 1+3 \end{bmatrix}$

$\therefore$ 8Y = $\begin{bmatrix} 1 & 2 \\ -11 & 4 \end{bmatrix}$

$\therefore$ Y = $\frac{1}{8}\begin{bmatrix} 1 & 2 \\ -11 & 4 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{8} & \frac{2}{8} \\ \frac{-1}{8} & \frac{4}{8} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{-1}{8} & \frac{1}{2} \end{bmatrix}.$

Exercise 2.2 | Q 12 | Page 47

Find matrices A and B, if 2A – B = $\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and A – 2B = $\begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$.

Solution:

Given equations are

2A – B = $\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ ...(i)

and A – 2B = $\begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$ ...(ii)

By (i) – (ii) x 2, we get

3B = $\begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & -7 \end{bmatrix}$

$= \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 16 \\ -4 & 2 & -14 \end{bmatrix}$

$= \begin{bmatrix} 6-6 & -6-4 & 0-16 \\ -4+4 & 2-2 & 1+14 \end{bmatrix}$

$\therefore$ 3B = $\begin{bmatrix} 0 & -10 & -16 \\ 0 & 0 & 15 \end{bmatrix}$

$\therefore$ B = $\frac{1}{3}\begin{bmatrix}0 & -10 & -16\\0 & 0 & 15\end{bmatrix}$

$\therefore$ B = $\begin{bmatrix}0 & \frac{-10}{3} & \frac{-16}{3}\\0 & 0 & 5\end{bmatrix}$

By (i) x 2 – (ii), we get

3A = $2\begin{bmatrix}6 & -6 & 0\\-4 & 2 & 1\end{bmatrix} - \begin{bmatrix}3 & 2 & 8\\-2 & 1 & -7\end{bmatrix}$

= $\begin{bmatrix}12 & -12 & 0\\-8 & 4 & 2\end{bmatrix} - \begin{bmatrix}3 & 2 & 8\\-2 & 1 & -7\end{bmatrix}$

= $\begin{bmatrix}12-3 & -12-2 & 0-8\\-8+2 & 4-1 & 2+7\end{bmatrix}$

$\therefore$ 3A = $\begin{bmatrix}9 & -14 & -8\\-6 & 3 & 9\end{bmatrix}$

$\therefore$ A = $\frac{1}{3}\begin{bmatrix}9 & 14 & -8\\-6 & 3 & 9\end{bmatrix}$

$\therefore$ A = $\begin{bmatrix}3 & \frac{-14}{3} & \frac{-8}{3}\\-2 & 1 & 3\end{bmatrix}$.

Exercise 2.2 | Q 13 | Page 47

Find x and y, if $\begin{bmatrix}2x+y & -1 & 1\\3 & 4y & 4\end{bmatrix}\begin{bmatrix}-1 & 6 & 4\\3 & 0 & 3\end{bmatrix} = \begin{bmatrix}3 & 5 & 5\\6 & 18 & 7\end{bmatrix}$

Solution:

$$\begin{bmatrix}2x+y & -1 & 1\\3 & 4y & 4\end{bmatrix}\begin{bmatrix}-1 & 6 & 4\\3 & 0 & 3\end{bmatrix} = \begin{bmatrix}3 & 5 & 5\\6 & 18 & 7\end{bmatrix}$$

$\therefore \begin{bmatrix} 2x+y-1 & -1+6 & 1+4 \\ 3+3 & 4y+0 & 4+3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$

$\therefore \begin{bmatrix} x+y-1 & 5 & 5 \\ 6 & 4y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 6 & 18 & 7 \end{bmatrix}$

∴ By equality of matrices, we get

2x + y – 1 = 3 and 4y = 18

∴ 2x + y = 4 and y = $\frac{18}{4} = \frac{9}{2}$

∴ $2 + \frac{9}{2}$ = 4

∴ 2x = $4 - \frac{9}{2}$

∴ 2x = $-\frac{1}{2}$

∴ x = $-\frac{1}{4}$ and y = $\frac{9}{2}$.

Exercise 2.2 | Q 14 | Page 47

If $\begin{bmatrix} 2a+b & 3a-b \\ c+2d & 2c-d \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$, find a, b, c and d.

Solution:

$$\begin{bmatrix} 2a+b & 3a-b \\ c+2d & 2c-d \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

∴ By equality of matrices, we get
2a + b = 2 ....(i)
3a – b = 3 ....(ii)
c + 2d = 4 ....(iii)
2c –d = – 1 ....(iv)
Adding (i) and (ii), we get
5a = 5

∴ a = 1
Substituting a = 1 in (i), we get
2(1) + b = 2
∴ b = 0
By (iii) + (iv) x 2, we get
5c = 2

∴ c = 2/5
Substituting c = 2/5 i (iii), we get

2/5 + 2d = 4

∴ 2d = 4 – 2/5

∴ 2d = 18/5

∴ d = 9/5.

Exercise 2.2 | Q 15.1 | Page 47

There are two book shops own by Suresh and Ganesh. Their sales (in Rupees) for books in three subject - Physics, Chemistry and Mathematics for two months, July and August 2017 are given by two matrices A and B. July sales (in Rupees) :

Physics Chemistry Mathematics

$$A = \begin{bmatrix} 5600 & 6750 & 8500 \\ 6650 & 7055 & 8905 \end{bmatrix} \begin{bmatrix} \text{Suresh} \\ \text{Ganesh} \end{bmatrix}$$

August Sales (in Rupees :

$$B = \begin{bmatrix} 6650 & 7055 & 8905 \\ 7000 & 7500 & 10200 \end{bmatrix} \begin{bmatrix} \text{Suresh} \\ \text{Ganesh} \end{bmatrix}$$

Find the increase in sales in Rupees from July to August 2017.

Solution: Increase sales rrupees from july to August 2017.

For Suresh :

Increase in sales for Physics books

= 6650 – 5600 = ₹ 1050

Increase in sales for Chemistry books

= 7055 – 6750 = ₹ 305

Increase in sales for Mathematics books

= 8905 – 8500 = ₹ 405

For Ganesh :

Increase in sales for Physics books

= 7000 – 6650 = ₹ 350

Increase in sales for Chemistry books

= 7500 – 7055 = ₹ 455

Increase in sales for Mathematics books

= 10200 – 8905 = ₹ 1295.

Exercise 2.2 | Q 15.2 | Page 47

There are two book shops own by Suresh and Ganesh. Their sales (in Rupees) for books in three subject - Physics, Chemistry and Mathematics for two months, July and August 2017 are given by two matrices A and B. July sales (in Rupees) :

Physics Chemistry Mathematics

$$A = \begin{bmatrix} 5600 & 6750 & 8500 \\ 6650 & 7055 & 8905 \end{bmatrix} \begin{bmatrix} \text{Suresh} \\ \text{Ganesh} \end{bmatrix}$$

August Sales (in Rupees :

$$B = \begin{bmatrix} 6650 & 7055 & 8905 \\ 7000 & 7500 & 10200 \end{bmatrix} \begin{bmatrix} \text{Suresh} \\ \text{Ganesh} \end{bmatrix}$$

If both book shops get 10% profit in the month of August 2017, find the profit for each book seller in each subject in that month.

Solution: Both book shops got 10% profit in the month of August 2017.

For Suresh :

Profit for Physics books = $\frac{6650 \times 10}{100}$ = ₹ 665

Profit for Chemistry books = $\frac{7055 \times 10}{100}$ = ₹ 705.50

Profit for Mathematics books = $\frac{8905 \times 10}{100}$ = ₹ 890.50

For Ganesh :

Profit for Physics books = $\frac{7000 \times 10}{100}$ = ₹ 700

Profit for Chemistry books = $\frac{7500 \times 10}{100}$ = ₹ 750

Profit for Mathematics books = $\frac{10200 \times 10}{100}$ = ₹ 1020

EXERCISE 2.3 [PAGES 55 - 56]

Exercise 2.3 | Q 1.1 | Page 55

Evaluate : $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3(2) & 3(-4) & 3(3) \\ 2(2) & 2(-4) & 2(3) \\ 1(2) & 1(-4) & 1(3) \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 & 9 \\ 4 & 8 & 6 \\ 2 & -4 & 3 \end{bmatrix}.$$

Exercise 2.3 | Q 1.2 | Page 55

Evaluate : $[2 - 1 \;\; 3]\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$

Solution:

$$[2 - 1 \;\; 3]\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

$= [2(4) -1(3) + 3(1)]$

$= [8 - 3 + 3]$

$= [8].$

Exercise 2.3 | Q 2 | Page 55

If A = $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, state whether AB = BA?

Justify your answer.

Solution:

AB = $\begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

= $\begin{bmatrix} -2+3+1 & -1+0+2 & -4+2+1 \\ 4+9+0 & 2+0+0 & 8+6+0 \\ 2-9+1 & 1+0+2 & 4-6+1 \end{bmatrix}$

∴ AB = $\begin{bmatrix} 2 & 1 & -1 \\ 13 & 2 & 14 \\ -6 & 3 & -1 \end{bmatrix}$ ...(i)

BA = $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & -3 & 1 \end{bmatrix}$

= $\begin{bmatrix} -2+2+4 & 2+3-12 & 2+0+4 \\ -3+0+2 & 3+0-6 & 3+0+2 \\ -1+4+1 & 1+6-3 & 1+0+1 \end{bmatrix}$

∴ BA = $\begin{bmatrix} 4 & -7 & 6 \\ -1 & -3 & 5 \\ 4 & 4 & 2 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get

AB ≠ BA.

Exercise 2.3 | Q 3 | Page 55

Show that AB = BA, where A = $\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

Solution:

AB = $\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}\begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

= $\begin{bmatrix} -2+6-3 & -6+6-0 & 2-3+1 \\ -1+4-3 & -3+4-0 & 1-2+1 \\ -6+18-12 & -18+18+0 & 6-9+4 \end{bmatrix}$

∴ AB = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ...(i)

BA = $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

= $\begin{bmatrix} 2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6+0+6 & 9+0-9 & 3+0+4 \end{bmatrix}$

∴ BA = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get

AB = BA.

Exercise 2.3 | Q 4 | Page 55

Verify A(BC) = (AB)C, if A = $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix}$

Solution:

$$BC = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-4 & 4-0 & -2+4 \\ -3+2 & -2+0 & 1-2 \\ 0+6 & 0+0 & 0-6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$\therefore A(BC) = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 6 & 0 & -6 \end{bmatrix}$$

$$\therefore A(BC) = \begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & -34 \end{bmatrix} \quad \text{...(i)}$$

$$AB = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 & -2+0+3 \\ 4-3+0 & -4+3+0 \\ 0-4+0 & 0+4+15 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix}$$

$\therefore$ (AB)C = $\begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix}$

= $\begin{bmatrix} 6+2 & 4+0 & -2-2 \\ 3-2 & 2-0 & -1+2 \\ -12+38 & -8+0 & 4-38 \end{bmatrix}$

$\therefore$ (AB)C = $\begin{bmatrix} 8 & 4 & -4 \\ 1 & 2 & 1 \\ 26 & -8 & 34 \end{bmatrix}$ ...(ii)

From (i) and (ii), we get

A(BC) = (AB)C.

Exercise 2.3 | Q 5 | Page 55

Verify that A(B + C) = AB + AC, if A =
$\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$ and $\mathrm{C} = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$

Solution:

A(B + C) = $\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix} \right\}$

= $\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1+4 & 1+1 \\ 3+2 & -2-1 \end{bmatrix}$

= $\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$

= $\begin{bmatrix} 12-10 & 8+6 \\ 6+15 & 4-9 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \quad \text{...(i)}$$

$$AB + AC = \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -4-6 & 4+4 \\ -2+9 & 2-6 \end{bmatrix} + \begin{bmatrix} 16-4 & 4+2 \\ 8+6 & 2-3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 8 \\ 7 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 6 \\ 14 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -10+12 & 8+6 \\ 7+14 & -4-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 14 \\ 21 & -5 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

A(B + C) = AB + AC.

Exercise 2.3 | Q 6 | Page 56

If $A = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -2 \end{bmatrix}$ show that matrix AB is non singular.

Solution:

$$AB = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3+2 & 8+0-4 \\ -1-2+0 & -2+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ -3 & -2 \end{bmatrix}$$

$$\therefore |AB| = \begin{vmatrix} 3 & 4 \\ -3 & -2 \end{vmatrix}$$

= – 6 + 12

= 6 ≠ 0

∴ AB is a non-singular matrix.

Exercise 2.3 | Q 7 | Page 56

If A + I = $\begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix}$, find the product (A + I)(A – I).

Solution:

A – I = A I – 2I

$$= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-0 & 0-0 \\ 5-0 & 4-2 & 2-0 \\ 0-0 & 7-0 & -3-2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 0 \\ 5 & 2 & 2 \\ 0 & 7 & -5 \end{bmatrix}$$

$$(A + I)(A - I) = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 4 & 2 \\ 0 & 7 & -3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 0 \\ 5 & 2 & 2 \\ 0 & 7 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -1+10+0 & 2+4+0 & 0+4+0 \\ -5+20+0 & 10+8+14 & 0+8-10 \\ 0+35-0 & 0+14-21 & 0+14+15 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 6 & 4 \\ 15 & 32 & 2 \\ 35 & -7 & 29 \end{bmatrix}.$$

Exercise 2.3 | Q 8 | Page 56

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A$ is a scalar matrix.

Solution:

A2 – 4A = A.A – 4A

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4 & 8-8 & 8-8 \\ 8-8 & 9-4 & 8-8 \\ 8-8 & 8-8 & 9-4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$, which is a scalar martix.

Exercise 2.3 | Q 9 | Page 56

If A = $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k so that $A^2 - 8A - kI = O$, where I is a 2 × 2 unit and O is null matrix of order 2.

Solution: $A^2 - 8A - kI = O$

∴ A.A 8A – kI = O

$$\therefore \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1+0 & 0+0 \\ -1\,7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1-8-k & 0-0-0 \\ -8+8-0 & 4-6-k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

∴ By equality of matrices, we get

1 – 8 – k = 0

∴ k = – 7.

Exercise 2.3 | Q 10 | Page 56

If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, prove that $A^2 - 5A + 7I = 0$, where I is a 2 x 2 unit matrix.

Solution:

$A^2 - 5A + 7I = A.A - 5A + 7I$

$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

= 0.

Exercise 2.3 | Q 11 | Page 56

If A = $\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix}$ and $(A + B)^2 \;\; A^2 + B^2$, find the values of a and b.

Solution: Given $(A + B)^2 \;\; A^2 + B^2$

$\therefore A^2 + AB + BA + B^2 = A^2 + B^2$

$\therefore AB + BA = 0$

$\therefore AB = -BA$

$$\therefore \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} = -\begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2-2 & a+2b \\ -2+2 & -a-2b \end{bmatrix} = -\begin{bmatrix} 2-a & 4-2a \\ -1-b & -2-2b \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & a+2b \\ 0 & -a-2b \end{bmatrix} = \begin{bmatrix} -2+a & -4+2a \\ 1+b & 2+2b \end{bmatrix}$$

∴ By equality of matrices, we get

– 2 + a = 0 and 1 + b = 0

∴ a = 2 and b = – 1.

Exercise 2.3 | Q 12 | Page 56

Find k, if A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $A^2 = kA - 2I$.

Solution: $A^2 = kA - 2I$

∴ A.A + 2I = kA

$$\therefore \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1+2 & -2+0 \\ 4+0 & -4+2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

$$\therefore \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$$

∴ By equality of matrices, we get

3k = 3

∴ k = 1.

Exercise 2.3 | Q 13 | Page 56

Find x and y, if $\left\{4\begin{bmatrix}2 & -1 & 3\\1 & 0 & 2\end{bmatrix}-\begin{bmatrix}3 & -3 & 4\\2 & 1 & 1\end{bmatrix}\right\}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

Solution:

$$\left\{4\begin{bmatrix}2 & -1 & 3\\1 & 0 & 2\end{bmatrix}-\begin{bmatrix}3 & -3 & 4\\2 & 1 & 1\end{bmatrix}\right\}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \left\{\begin{bmatrix}8 & -4 & 12\\4 & 0 & 8\end{bmatrix}-\begin{bmatrix}3 & -3 & 4\\2 & 1 & 1\end{bmatrix}\right\}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \begin{bmatrix}8-3 & -4+3 & 12-4\\4-2 & 0-1 & 8-1\end{bmatrix}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \begin{bmatrix}5 & -1 & 8\\2 & -1 & 7\end{bmatrix}\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \begin{bmatrix}10+1+8\\4+1+7\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$$\therefore \begin{bmatrix}19\\12\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$$

$\therefore$ By equality of matrices, we get

x = 19 and y = 12.

Exercise 2.3 | Q 14 | Page 56

Find x, y, x, if $\left\{3\begin{bmatrix}2 & 0\\0 & 2\\2 & 2\end{bmatrix} - 4\begin{bmatrix}1 & 1\\-1 & 2\\3 & 1\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$

Solution:

$$\left\{3\begin{bmatrix}2 & 0\\0 & 2\\2 & 2\end{bmatrix} - 4\begin{bmatrix}1 & 1\\-1 & 2\\3 & 1\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \left\{\begin{bmatrix}6 & 0\\0 & 6\\6 & 6\end{bmatrix} - \begin{bmatrix}4 & 4\\-4 & 8\\12 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \begin{bmatrix}6-4 & 0-4\\0+4 & 6-8\\6-12 & 6-4\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \begin{bmatrix}2 & -4\\4 & -2\\-6 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \begin{bmatrix}2-8\\4-4\\-6+4\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

$$\therefore \begin{bmatrix}-6\\0\\-2\end{bmatrix} = \begin{bmatrix}x-3\\y-1\\2z\end{bmatrix}$$

∴ By equality of martices, we get

x – 3 = – 6, y – 1 = 0, 2z = – 2

∴ x = – 3, y = 1, z = – 1.

Exercise 2.3 | Q 15 | Page 56

Jay and Ram are two friends. Jay wants to buy 4 pens and 8 notebooks, Ram wants to buy 5 pens and 12 notebooks. The price of one pen and one notebook was ₹ 6 and ₹ 10 respectively. Using matrix multiplication, find the amount each one of them requires for buying the pens and notebooks.

Solution: Let A be the matrix of pens and notebooks and B be the matrix od prices of one pen and one notebook.

Pens Notebooks

$$\therefore A = \begin{bmatrix} 4 & 8 \\ 5 & 12 \end{bmatrix} \frac{\text{jay}}{\text{Ram}}$$

$$\text{and } B = \begin{bmatrix} 6 \\ 10 \end{bmatrix} \frac{\text{Pen}}{\text{Notebook}}$$

The total amount required for each one of them is obtained by matrix AB.

$$\therefore AB = \begin{bmatrix} 4 & 8 \\ 5 & 12 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 24 + 80 \\ 30 + 120 \end{bmatrix}$$

$$= \begin{bmatrix} 104 \\ 150 \end{bmatrix}$$

∴ Jay needs ₹ 104 and Ram needs ₹ 150.

EXERCISE 2.4 [PAGES 59 - 60]

Exercise 2.4 | Q 1.1 | Page 59

Find A^T, if $A = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}$$

Exercise 2.4 | Q 1.2 | Page 59

Find A^T, if $A = \begin{bmatrix} 2 & -6 & 1 \\ -4 & 0 & 5 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 2 & -6 & 1 \\ -4 & 0 & 5 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 2 & -4 \\ -6 & 0 \\ 1 & 5 \end{bmatrix}.$$

Exercise 2.4 | Q 2 | Page 59

If $[a_{ij}]_{3\times3}$, where $a_{ij} = 2(i -- j)$, find A and A^T. State whether A and A^T both are symmetric or skew-symmetric matrices?

Solution:

$$A = [a_{ij}]_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Given, $a_{ij} = 2\ (i - j)$

$\therefore a_{11} = 2(1 - 1) = 0$, $a_{12} = 2(1 - 2) = -2$

$a_{13} = 2(1 - 3) = -4$, $a_{21} = 2(2 - 1) = 2$,

$a_{22} = 2(2 - 2) = 0$, $a_{23} = 2(2 - 3) = -2$,

$a_{31} = 2(3 – 1) = 4$, $a_{32} = 2(3 – 2) = 2$,

$a_{33} = 2(3 – 3) = 0$

$$\therefore A = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & -2 \\ -4 & -2 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix} = -\mathbf{A}$$

$\therefore A^T = – A$ and $A = – A^T$

$\therefore$ A and A^T both are skew-symmetric matrices.

Exercise 2.4 | Q 3 | Page 59

If $A = \begin{bmatrix} 5 & -3 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}$, prove that $(A^T)^T = A$.

Solution:

$$A = \begin{bmatrix} 5 & -3 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$\therefore A^T = \begin{bmatrix} 5 & 4 & -2 \\ -3 & -3 & 1 \end{bmatrix}$

$\therefore (A^T)^T = \begin{bmatrix} 5 & -3 \\ 4 & -3 \\ -2 & 1 \end{bmatrix}$

A.

Exercise 2.4 | Q 4 | Page 59

If A = $\begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$, prove that $A^T = A$.

Solution:

A = $\begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$

=A.

Exercise 2.4 | Q 5.1 | Page 59

If A = $\begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ -2 & 3 \end{bmatrix}$, then show that $(A + B)^T = A^T + B^T$.

Solution:

A = $\begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}$

= $\begin{bmatrix} 2+2 & -3+1 \\ 5+4 & -4-1 \\ -6-3 & 1+3 \end{bmatrix}$

= $\begin{bmatrix} 4 & -2 \\ 9 & -5 \\ -9 & 4 \end{bmatrix}$

∴ $(A + B)^T$ = $\begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix}$ ...(i)

Now, AT = $\begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix}$ and $B^T = \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 3 \end{bmatrix}$

∴ AT + BT = $\begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -3 \\ 1 & -1 & 3 \end{bmatrix}$

= $\begin{bmatrix} 4 & 9 & -9 \\ -2 & -5 & 4 \end{bmatrix}$ ...(ii)

From (i) and (ii, we get

$(A + B)^T = A^T + B^T$.

Exercise 2.4 | Q 5.2 | Page 59

If A = $\begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & -1 \\ -3 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ -2 & 3 \end{bmatrix}$, then show that $(A - C)^T = A^T - C^T$.

Solution:

$$A - C = \begin{bmatrix} 2 & -3 \\ 5 & -4 \\ -6 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & -3-2 \\ 5+1 & -4-4 \\ -6+2 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -5 \\ 6 & -8 \\ -4 & -2 \end{bmatrix}$$

$$\therefore (A - C)^T = \begin{bmatrix} 1 & 6 & -4 \\ -5 & -8 & -2 \end{bmatrix} \quad \text{...(i)}$$

Now, $A^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix}$ and

$$C^T = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\therefore A^T - C^T = \begin{bmatrix} 2 & 5 & -6 \\ -3 & -4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & -2 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5+1 & -6+2 \\ -3-2 & -4-4 & 1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & -4 \\ -5 & -8 & -2 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

$(A - C)^T = A^T - C^T$.

Exercise 2.4 | Q 6 | Page 59

If A = $\begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 3 \\ 4 & -1 \end{bmatrix}$, then find C^T, such that 3A – 2B + C = I, whre I is e unit matrix of order 2.

Solution:

3A – 2B + C = I

∴ C = I + 2B – 3A

∴ C = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} -1 & 3 \\ 4 & -1 \end{bmatrix} - 3\begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$

= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 8 & -2 \end{bmatrix} - \begin{bmatrix} 15 & 12 \\ -6 & 9 \end{bmatrix}$

= $\begin{bmatrix} 1-2-15 & 0+6-12 \\ 0+8+6 & 1-2-9 \end{bmatrix}$

∴ C = $\begin{bmatrix} -16 & -6 \\ 14 & -10 \end{bmatrix}$

∴ C = $\begin{bmatrix} -16 & 14 \\ -6 & -10 \end{bmatrix}$.

Exercise 2.4 | Q 7.1 | Page 59

If A = $\begin{bmatrix} 7 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix}$, then find $A^T + 4B^T$.

Solution:

A = $\begin{bmatrix} 7 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix}$

∴ A^T = $\begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix}$ and $B^T = \begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$

$A^T + 4B^T = \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} + 4\begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 8 \\ -8 & 4 \\ 12 & -16 \end{bmatrix}$

$= \begin{bmatrix} 7+0 & 0+8 \\ 3-8 & 4+4 \\ 0+12 & -2-16 \end{bmatrix}$

$= \begin{bmatrix} 7 & 8 \\ -5 & 8 \\ 12 & -18 \end{bmatrix}.$

Exercise 2.4 | Q 7.2 | Page 59

If $A = \begin{bmatrix} 7 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix}$, then find $5A^T - 5B^T$.

Solution:

$A = \begin{bmatrix} 7 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix} \text{and} B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix}$

$\therefore AT = \begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} \text{and} B^T = \begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}$

$5A^T - 5B^T = 5(A^T - B^T)$

$= 5\left(\begin{bmatrix} 7 & 0 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ -2 & 1 \\ 3 & -4 \end{bmatrix}\right)$

$$= 5\begin{bmatrix} 7-0 & 0-2 \\ 3+2 & 4-1 \\ 0-3 & -2+4 \end{bmatrix}$$

$$= 5\begin{bmatrix} 7 & -2 \\ 5 & 3 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & -10 \\ 25 & 15 \\ -15 & 10 \end{bmatrix}.$$

Exercise 2.4 | Q 8 | Page 59

If A = $\begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 2 & 3 \\ -1 & -1 & 0 \end{bmatrix}$,

verify that $(A + 2B + 3C)^T = A^T + 2B^T + C^T$.

Solution:

A + 2B + 3C

$$= \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -2 \end{bmatrix} + 3\begin{bmatrix} 0 & 2 & 3 \\ -1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 2 & -8 \\ 6 & 1 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 6 & 9 \\ -3 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 0+2+6 & 1-8+9 \\ 3+6-3 & 1+10-3 & 2-4+0 \end{bmatrix}$$

$$\therefore A + 2B + 3C = \begin{bmatrix} 5 & 8 & 2 \\ 6 & 8 & -2 \end{bmatrix}$$

$\therefore [A + 2B + 3C]^T = \begin{bmatrix} 5 & 6 \\ 8 & 8 \\ 2 & -2 \end{bmatrix}$ ...(i)

Now, $A^T = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}, B^T = \begin{bmatrix} 2 & 3 \\ 1 & 5 \\ -4 & -2 \end{bmatrix}$

and $C^T = \begin{bmatrix} 0 & -1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$

$\therefore A^T + 2B^T + 3C^T$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 3 \\ 1 & 5 \\ -4 & -2 \end{bmatrix} + 3\begin{bmatrix} 0 & -1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 2 & 10 \\ -8 & -4 \end{bmatrix} + \begin{bmatrix} 0 & -3 \\ 6 & -3 \\ 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 3+6+3 \\ 0+2+6 & 1+10-3 \\ 1-8+9 & 2-4+0 \end{bmatrix}$$

$\therefore A^T + 2B^T + 3C^T = \begin{bmatrix} 5 & 6 \\ 8 & 8 \\ 2 & -2 \end{bmatrix}$ ...(iii)

From (i) and (ii), we get

$[A + 2B + 3C]^T = A^T + 2B^T + 3C^T$.

Exercise 2.4 | Q 9 | Page 59

If A = $\begin{bmatrix}-1 & 2 & 1\\-3 & 2 & -3\end{bmatrix}$ and B = $\begin{bmatrix}2 & 1\\-3 & 2\\-1 & 3\end{bmatrix}$, prove that $(A + B^T)^T = A^T + B$.

Solution:

A = $\begin{bmatrix}-1 & 2 & 1\\-3 & 2 & -3\end{bmatrix}$ and B = $\begin{bmatrix}2 & 1\\-3 & 2\\-1 & 3\end{bmatrix}$

$\therefore A^T = \begin{bmatrix}-1 & -3\\2 & 2\\1 & -3\end{bmatrix}$ and $B^T = \begin{bmatrix}2 & -3 & -1\\1 & 2 & 3\end{bmatrix}$

$\therefore A + B^T = \begin{bmatrix}-1 & 2 & 1\\-3 & 2 & -3\end{bmatrix} + \begin{bmatrix}2 & -3 & 1\\1 & 2 & 3\end{bmatrix}$

$= \begin{bmatrix}-1+2 & 2-3 & 1-1\\-3+1 & 2+2 & -3+3\end{bmatrix}$

$= \begin{bmatrix}1 & -1 & 0\\-2 & 4 & 0\end{bmatrix}$

$\therefore (A + B^T)^T = \begin{bmatrix}1 & -2\\-1 & 4\\0 & 0\end{bmatrix}$ ...(i)

Now, $A^T + B = \begin{bmatrix}-1 & -3\\2 & 2\\1 & -3\end{bmatrix} + \begin{bmatrix}2 & 1\\-3 & 2\\-1 & 3\end{bmatrix}$

$= \begin{bmatrix}-1+2 & -3+1\\2-3 & 2+2\\1-1 & -3+3\end{bmatrix}$

$$= \begin{bmatrix} 1 & -2 \\ -1 & 4 \\ 0 & 0 \end{bmatrix} \quad \ldots\text{(ii)}$$

From (i) and (ii), we get

$(A + B^T)^T = A^T + B$.

Exercise 2.4 | Q 10.1 | Page 59

Prove that $A + A^T$ is a symmetric and $A - A^T$ is a skew symmetric matrix, where $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\therefore A + A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2+3 & 4-2 \\ 3+2 & 2+2 & 1-3 \\ -2+4 & -3+1 & 2+2 \end{bmatrix}$$

$$\therefore A + A^T = \begin{bmatrix} 2 & 5 & 2 \\ 5 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

$$\therefore (A + A^T)^T = \begin{bmatrix} 2 & 5 & 2 \\ 5 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}$$

$\therefore (A + A^T)^T = A + AT$ i.e., $A + A^T = (A + A^T)^T$

$\therefore A + A^T$ is a symmetric matrix.

$$A - A^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & -3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2-3 & 4+2 \\ 3-2 & 2-2 & 1+3 \\ -2-4 & -3-1 & 2-2 \end{bmatrix}$$

$$\therefore A - A^T = \begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & 4 \\ -6 & -4 & 0 \end{bmatrix}$$

$$\therefore (A - A^T)^T = \begin{bmatrix} 0 & 1 & -6 \\ -1 & 0 & -4 \\ 6 & 4 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & -1 & 6 \\ 1 & 0 & 4 \\ -6 & -4 & 0 \end{bmatrix}$$

∴ $(A - A^T)^T = -\ (A - A^T)$

i.e., $A - A^T = -\ (A - A^T)^T$

∴ $A - A^T$ is a skew symmetric matrix.

Exercise 2.4 | Q 10.2 | Page 59

Prove that $A + A^T$ is a symmetric and $A - A^T$ is a skew symmetric matrix, where $A = \begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$$

$$\therefore A + A^T = \begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+5 & 2+3 & -4+4 \\ 3+2 & -7-7 & 2-5 \\ 4-4 & -5+2 & -3-3 \end{bmatrix}$$

$$\therefore A + A^T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & -14 & -3 \\ 0 & -3 & -6 \end{bmatrix}$$

∴ $(A + AT)^T = A + A^T$ i.e., $A + A^T = (A + AT)^T$

∴ $A + A^T$ = is a symmetric matrix.

$$A - A^T = \begin{bmatrix} 5 & 2 & -4 \\ 3 & -7 & 2 \\ 4 & -5 & -3 \end{bmatrix} - \begin{bmatrix} 5 & 3 & 4 \\ 2 & -7 & -5 \\ -4 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-5 & 2-3 & -4-4 \\ 3-2 & -7+7 & 2+5 \\ 4+4 & -5-2 & -3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & 7 \\ 8 & -7 & 0 \end{bmatrix}$$

$$\therefore (A - A^T)^T = \begin{bmatrix} 0 & 1 & 8 \\ -1 & 0 & -7 \\ -8 & 7 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & 7 \\ 8 & -7 & 0 \end{bmatrix}$$

$\therefore (A - A^T)^T = -(A - A^T)$

i.e., $A - A^T = -(A - A^T)^T$

$\therefore A - A^T$ is a skew symmetric matrix.

Exercise 2.4 | Q 11.1 | Page 59

Express each of the following matrix as the sum of a symmetric and a skew symmetric matrix $\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$.

Solution: A square matrix A can be expressed as the sum of a symmetric and a skew-symmetric matrix as

$$A = \frac{1}{2}\left(A + A^T\right) + \frac{1}{2}\left(A - A^T\right)$$

Let A = $\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$

$\therefore$ A^T = $\begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

$\therefore$ A + A^T = $\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

= $\begin{bmatrix} 4+4 & -2+3 \\ 3-2 & -5-5 \end{bmatrix}$

= $\begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix}$

Also, A – A^T = $\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ -2 & -5 \end{bmatrix}$

= $\begin{bmatrix} 4-4 & -2-3 \\ 3+2 & -5+5 \end{bmatrix}$

= $\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

Let P = $\frac{1}{2}\left(A + A^T\right)$

= $\frac{1}{2}\begin{bmatrix} 8 & 1 \\ 1 & -10 \end{bmatrix}$

= $\begin{bmatrix} 4 & \frac{1}{2} \\ \frac{1}{2} & -5 \end{bmatrix}$

and

Q = $\frac{1}{2}\left(A - A^T\right)$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

∴ P is a symmetric matrix ...[∵ $a_{ij} = a_{ij}$]

and Q is a skew-symmetric matrix. ...[∵ $a_{ij} = -a_{ij}$]
∴ A = P + Q

$$\therefore A = \begin{bmatrix} 4 & \frac{1}{2} \\ \frac{1}{2} & -5 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

Exercise 2.4 | Q 11.2 | Page 59

Express each of the following matrix as the sum of a symmetric and a skew symmetric matrix $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$.

Solution: A square matrix A can be expressed as the sum of a symmetric and a skew-symmetric matrix as

$$A = \frac{1}{2}\left(A + A^T\right) + \frac{1}{2}\left(A - A^T\right)$$

$$\text{Let } A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$\therefore$ A + A^T = $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

= $\begin{bmatrix} 3+3 & 3-2 & -1-4 \\ -2+3 & -2-2 & 1-5 \\ -4-1 & -5+1 & 2+2 \end{bmatrix}$

= $\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

Also, A – A^T = $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

= $\begin{bmatrix} 3-3 & 3+2 & -1+4 \\ -2-3 & -2+2 & 1+5 \\ -4+1 & -5-1 & 2-2 \end{bmatrix}$

= $\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

Let P = $\frac{1}{2}\left(\mathbf{A}+\mathbf{A}^{\mathrm{T}}\right)$

= $\frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

and Q = $\frac{1}{2}\left(\mathbf{A}-\mathbf{A}^{\mathrm{T}}\right)$

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

∴ P is a symmetric matrix ...[∵ $a_{ij} = a_{ij}$]

and Q is a skew symmetric matrix. ...[∵ $a_{ij} = -\ a_{ij}$]
∴ A = P + Q

$$\therefore A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

Exercise 2.4 | Q 12.1 | Page 60

If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

Solution:

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\therefore AT = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix} \text{ and } B^T = \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 6+1 & -8-1 \\ 0-4 & 9+2 & -12-2 \\ 0+2 & 12-1 & -16+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 7 & -9 \\ -4 & 11 & -14 \\ 2 & 11 & -15 \end{bmatrix}$$

$$\therefore (AB)^T = \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix} \quad \text{...(i)}$$

$$B^TA^T = \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0-2 & 0-4 & 0+2 \\ 6+1 & 9+2 & 12-1 \\ -8-1 & -12-2 & -16+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 & 2 \\ 7 & 11 & 11 \\ -9 & -14 & -15 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

$(AB)^T = B^TA^T$.

Exercise 2.4 | Q 12.2 | Page 60

If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix}$, verify that $(BA)^T = A^TB^T$.

Solution:

$$BA = \begin{bmatrix} 0 & 3 & -4 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9-16 & 0-6-4 \\ 4-3+4 & -2+2+1 \end{bmatrix}$$

$$\therefore BA = \begin{bmatrix} -7 & -10 \\ 5 & 1 \end{bmatrix}$$

$$\therefore (BA)^T = \begin{bmatrix} -7 & 5 \\ -10 & 1 \end{bmatrix} \quad \text{...(i)}$$

$$A^T B^T = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 3 & -1 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9-16 & 4-3+4 \\ 0-6-4 & -2+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 5 \\ -10 & 1 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii)

$(BA)^T = A^T B^T$.

EXERCISE 2.5 [PAGES 71 - 72]

Exercise 2.5 | Q 1.1 | Page 71

Apply the given elementary transformation on each of the following matrices $\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$, $R_1 \leftrightarrow R_2$.

Solution:

$$\text{Let } A = \begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$, we get

$$A \sim \begin{bmatrix} 2 & 2 \\ 3 & -4 \end{bmatrix}.$$

Exercise 2.5 | Q 1.2 | Page 71

Apply the given elementary transformation on each of the following matrices $\begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$, $C_1 \leftrightarrow C_2$.

Solution:

Let $A = \begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get

$$A \sim \begin{bmatrix} 2 & 4 \\ -5 & 1 \end{bmatrix}.$$

Exercise 2.5 | Q 1.3 | Page 71

Apply the given elementary transformation on each of the following matrices $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, $3R_2$ and $C_2 \leftrightarrow C_2 - 4C_1$.

Solution:

Let $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Applying $R_2 \leftrightarrow 3R_2$, we get

$$A \sim \begin{bmatrix} 3 & 1 & -1 \\ 3 & 9 & 3 \\ -1 & 1 & 3 \end{bmatrix}$$

Applying $C_2 \rightarrow C_2 - 4C_1$ on A, we get

$$A \sim \begin{bmatrix} 3 & 1-4(3) & -1 \\ 1 & 3-4(1) & 1 \\ -1 & 1-4(-1) & 3 \end{bmatrix}$$

$$\therefore A \sim \begin{bmatrix} 3 & 1-12 & -1 \\ 1 & 3-4 & 1 \\ -1 & 1+4 & 3 \end{bmatrix}$$

$$\therefore A \sim \begin{bmatrix} 3 & -11 & -1 \\ 1 & -1 & 1 \\ -1 & 5 & 3 \end{bmatrix}.$$

Exercise 2.5 | Q 2 | Page 71

Transform $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ into an upper traingular matrix by suitable row transformations.

Solution:

Let A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$

Applying $R_2 \rightarrow R_2 - 2R_1$
and $R_3 \rightarrow R_3 - 3R_1$, we get

$$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 5 & -2 \end{bmatrix}$$

Applying $R_3 \to R_3 - \left(\frac{5}{3}\right)R_2$, we get

$$A \sim \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix},$$

which is an upper triangular matrix.

Exercise 2.5 | Q 3.1 | Page 72

Find the cofactor matrix, of the following matrices : $\begin{bmatrix} 1 & 2 \\ 5 & -8 \end{bmatrix}$

Solution: The co-factor A_{ij} of a_{ij} is equal to $(-1)^{i+j}\ M_{ij}$.

Here,

$a_{11} = 1$

$\therefore M_{11} = -8$

and $A_{11} = (-1)^{1+2}\ M_{11} = (1)(-8) = -8$

$a_{12} = 2$

$\therefore M_{11} = 5$

and $A_{12} = (-1)^{1+2}\ M_{12} = (-1)(5) = -5$

$a_{21} = 5$

$\therefore M_{21} = 2$

and $A_{21} = (-1)^{2+1}\ M_{21} = (-1)(2) = -2$

$a_{22} = -8$

$\therefore M_{22} = 1$

and $A_{22} = (-1)^{2+2}\ M_{22} = (1)(1) = 1$

$\therefore$ The matrix of the co-factors is

$[A_{ij}]_{2x2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$= \begin{bmatrix} -8 & -5 \\ -2 & 1 \end{bmatrix}$.

Exercise 2.5 | Q 3.2 | Page 72

Find the cofactor matrix, of the following matrices: $\begin{bmatrix} 5 & 8 & 7 \\ -1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix}$

Solution: The co-factor Aij of aij is equal to (– 1)i+j Mij.

Here, $a_{11} = 5$

$\therefore M_{11} = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = -2 - 1 = -3$

and $A_{11} = (-1)^{1+1} M_{11} = (1)(-3) = -3$

$a_{12} = 8$

$\therefore M_{12} = \begin{vmatrix} -1 & 1 \\ -2 & 1 \end{vmatrix} = -1 + 2 = 1$

and $A_{12} = (-1)^{1+2} M_{11} = (-1)(1) = -1$

$a_{13} = 7$

$\therefore M_{13} = \begin{vmatrix} -1 & -2 \\ -2 & 1 \end{vmatrix} = -1 - 4 = -5$

and $A_{13} = (-1)^{1+3} M_{13} = (1)(-5) = -5$

$a_{21} = -1$

$\therefore M_{21} = \begin{vmatrix} 8 & 7 \\ 1 & 1 \end{vmatrix} = 8 - 7 = 1$

and $A_{21} = (-1)^{2+1} M_{21} = (-1)(1) = -1$

$a_{22} = -2$

$\therefore M_{22} = \begin{vmatrix} 5 & 7 \\ -2 & 1 \end{vmatrix} = 5 + 14 = 19$

and $A_{22} = (-1)^{2+2} M_{22} = (-1)(19) = 19$

$a_{23} = 1$

$\therefore M_{23} = \begin{vmatrix} 5 & 8 \\ -2 & 1 \end{vmatrix} = 5 + 16 = 21$

and $A_{23} = (-1)^{2+3} M_{23} = (-1)(21) = -21$

$a_{31} = -2$

$\therefore M_{31} = \begin{vmatrix} 8 & 7 \\ -2 & 1 \end{vmatrix} = 8 + 14 = 22$

and $A_{31} = (-1)^{3+2} M_{31} = (1)(22) = 22$

$a_{32} = 1$

$\therefore M_{32} = \begin{vmatrix} 5 & 7 \\ -1 & 1 \end{vmatrix} = 5 + 7 = 12$

and $A_{32} = (-1)^{3+2} M_{32} = (-1)(12) = -12$

$a_{33} = 1$

$\therefore M_{33} = \begin{vmatrix} 5 & 8 \\ -1 & -2 \end{vmatrix} = -10 + 8 = -2$

and $A_{33} = (-1)^{3+3} M_{33} = (1)(-2) = -2$

$\therefore$ The matrix of the co-factors is

$$[A_{ij}]_{3x3} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 & -5 \\ -1 & 19 & -21 \\ 22 & -12 & -2 \end{bmatrix}.$$

Exercise 2.5 | Q 4.1 | Page 72

Find the adjoint of the following matrices : $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

Here,
$a_{11} = 2$
∴ $M_{11} = 5$ and $A_{11} = (-1)^{1+1}(5) = 5$
$a_{12} = -3$

∴ $M_{12} = 3$ and $A_{12} = (-1)^{1+2}(3) = -3$
$a_{21} = 3$

∴ $M_{21} = -3$ and $A_{21} = (-1)^{2+1}(-3) = 3$
$a_{22} = 5$
∴ $M_{22} = 2$ and $A_{22} = (-1)^{2+2}(2) = 2$
∴ The matrix of the co-factors is

$$[A_{ij}]_{2x2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$$

Now, adj A = $[A_{ij}]^{T}_{2\times 2} = \begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$.

Exercise 2.5 | Q 4.2 | Page 72

Fid the adjoint of the following matrices : $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

Here,

$a_{11} = 1$

$\therefore M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = -3 - 0 = -3$

and $A_{11} = (-1)^{1+1}(-3) = -3$

$a_{12} = -1$

$\therefore M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = 2 + 10 = 12$

and $A_{12} = (-1)^{1+2}(12) = -12$

$a_{13} = 2$

$\therefore M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$

and $A_{13} = (-1)^{1+3}(6) = 6$

$a_{21} = -2$

$\therefore M_{21} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1$

and $A_{21} = (-1)^{2+1}(1) = -1$

$a_{22} = 3$

$\therefore M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$

and $A_{22} = (-1)^{2+2}(3) = 3$

$a_{23} = 5$

Exercise 2.5 | Q 5.1 | Page 72

Find the inverse of the following matrices by the adjoint method

$\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$.

Solution:

Let $A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} = -3 + 2 = -1 \neq 0$

$\therefore A^{-1}$ exists.

$A_{11} = (-1)^{1+1} M_{11} = (1)(-1) = -1$

$A_{12} = (-1)^{1+2} M_{12} = (-1)(2) = -2$

$A_{21} = (-1)^{2+1} M_{21} = (-1)(-1) = 1$

$A_{22} = (-1)^{2+2} M_{22} = (1)(3) = 3$

$\therefore$ The matrix of the co-factors is

$[A_{ij}]_{2x2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix}$

Now adj A = $[A_{ij}]^T_{2\times2} = \begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$

∴ A–1 = $\frac{1}{|A|}(\text{adj A})$

= $\frac{1}{-1}\begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$

= $\begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$.

Exercise 2.5 | Q 5.2 | Page 72

Find the inverse of the following matrices by the adjoint method

$\begin{bmatrix} 2 & -2 \\ 4 & 5 \end{bmatrix}$.

Solution:

Let A = $\begin{bmatrix} 2 & -2 \\ 4 & 5 \end{bmatrix}$

∴ |A| = $\begin{bmatrix} 2 & -2 \\ 4 & 5 \end{bmatrix}$ = 10 + 8 = 18 ≠ 0

∴ A^{-1} exists.

$A_{11} = (-1)^{1+1} M_{11} = (1)(5) = 5$

$A_{12} = (-1)^{1+2} M_{12} = (-1)(4) = -4$

$A_{21} = (-1)^{2+1} M_{21} = (-1)(-2) = 2$

$A_{22} = (-1)^{2+2} M_{22} = (1)(2) = 2$

∴ The matrix of the co-factors is

$[A_{ij}]_{2x2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ 2 & 2 \end{bmatrix}$

Now adj A = $[A_{ij}]^T_{2\times 2} = \begin{bmatrix} 5 & 2 \\ -4 & 2 \end{bmatrix}$

∴ A–1 = $\dfrac{1}{|A|}(\text{adj } A)$

= $\dfrac{1}{18}\begin{bmatrix} 5 & 2 \\ -4 & 2 \end{bmatrix}$.

Exercise 2.5 | Q 5.3 | Page 72

Find the inverse of the following matrices by the adjoint method

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$.

Solution:

Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

∴ |A| = $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$

= 1(10 – 0) –2(0 –0) + 3(0 –0) = 10 ≠ 0

∴ A^{-1} exists.

$A_{11} = (-1)^{1+1} M_{11} = 1\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix}$ = 1(10 – 0) = 10

$A_{12} = (-1)^{1+2} M_{12} = -1\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix}$ = 1(0 – 0) = 0

$A_{13} = (-1)^{1+3} M_{13} = 1\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$ = 1(0 – 0) = 0

$A_{21} = (-1)^{2+1} M_{21} = -1\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -1(10-0) = -10$

$A_{22} = (-1)^{2+2} M_{22} = 1\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 1(5-0) = 5$

$A_{23} = (-1)^{2+3} M_{23} = -1\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -1(0-0) = 0$

$A_{31} = (-1)^{3+1} M_{31} = 1\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 1(8-6) = 2$

$A_{32} = (-1)^{3+2} M_{32} = -1\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -1(4-0) = -4$

$A_{33} = (-1)^{3+3} M_{33} = 1\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 1(2-0) = 2$

∴ The matrix of the co-factor is

$$[A_{ij}]_{3x3} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

$$\text{Now, adj } A = [A_{ij}]^T_{3\times 3} = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}.$$

Exercise 2.5 | Q 6.1 | Page 72

Find the inverse of the following matrices by transformation method: $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$ = – 1 – 4 = –5 ≠ 0

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Appying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Applying $R_2 \to \left(-\frac{1}{5}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix}$$

Appying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}.$$

Exercise 2.5 | Q 6.2 | Page 72

Find the inverse of the following matrices by transformation method: $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Solution:

Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$

= 2(3 – 0) –0(15 –0) –1(5 – 0)

= 6 – 0 – 5

= 1 ≠ 0

$\therefore$ A^{-1} exists.

Consider AA–1 = I

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

= 2(3 – 0) –0(15 –0) –1(5 – 0)

= 6 – 0 – 5

= 1 ≠ 0

$\therefore$ A^{-1} exists.

Consider AA–1 = I

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \rightarrow R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 3R_3$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 - R_2$,we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

Applying $R_1 \to (-1) R_3$, we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

Applying $R_1 \to R_1 + 2R_3$, $R_2 \to R_2 - 4R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

Exercise 2.5 | Q 7 | Page 72

Find the inverse of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by elementary column transformations.

Solution:

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}$$

$= 1(2 - 6) - 0 + 1)0 - 2)$

$= -4 - 2$

$= -6 \neq 0$

$\therefore A^{-1}$ exists.

Consider $A^{-1}A = I$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $C_2 \leftrightarrow C_3$, we get

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Applying $C_2 \to C_2 - C_3$, we get

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Applying $C_3 \to C_3 - 2C_2$, we get

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

Applying $C_3 \to \left(\frac{1}{6}\right) C_3$, we get

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & \frac{1}{3} \\ 0 & -1 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{3} \end{bmatrix}$$

Applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 + 2C_3$, we get

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 2 & 2 & -2 \end{bmatrix}.$$

Exercise 2.5 | Q 8 | Page 72

Find the inverse $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ of the elementary row tranformation.

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$$

$= 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$

$= -13 + 6 + 6$

$= -1 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to (-1)\, R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 - 7R_3$ aand $R_2 \to R_2 + 2R_1$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$.

Exercise 2.5 | Q 9 | Page 72

If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then find a matrix X such that XA = B.

Solution:

A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

XA = B

Post multiplying by A^{-1}, we get

$XAA^{-1} = BA^{-1}$

$\therefore X = BA^{-1}$ ...(i)

$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}$

$= 1(2 - 6) - 0 + 1(0 - 2)$

$= -4 - 2$

$= -6 \neq 0$

$\therefore A^{-1}$ exists.

$A_{11} = (-1)^{1+1} M_{11} = 1\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 1(2 - 6) = -4$

$A_{12} = (-1)^{1+2} M_{12} = -1\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = -1(0 - 3) = 3$

$A_{13} = (-1)^{1+3} M_{13} = 1\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = 1(0 - 2) = -2$

$A_{21} = (-1)^{2+1} M_{21} = -1\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -1(0 - 2) = 2$

$A_{22} = (-1)^{2+2} M_{22} = 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1(1 - 1) = 0$

$A_{23} = (-1)^{2+3} M_{23} = -1\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = -1(2 - 0) = -2$

$A_{31} = (-1)^{3+1} M_{31} = 1\begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = 1(0 - 2) = -2$

$A_{32} = (-1)^{3+2} M_{32} = -1\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -1(3 - 0) = -3$

$A_{33} = (-1)^{3+3} M_{33} = 1\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 1(2 - 0) = 2$

∴ The matrix of the co-factors is

$$[A_{ij}]_{3\times3} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -4 & 3 & -2 \\ 2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$$

Now, $\text{adj } A = [A_{ij}]^T_{3\times3} = \begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

$X = BA^{-1}$ ...[From (i)]

$$\therefore X = \begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}\left\{\left(\frac{1}{6}\right)\begin{bmatrix}-4 & 2 & -2\\3 & 0 & -3\\-2 & -2 & 2\end{bmatrix}\right\}$$

$$= \frac{1}{6}\begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}\begin{bmatrix}4 & -2 & 2\\-3 & 0 & 3\\2 & 2 & -2\end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix}4-6+6 & -2+0+6 & 2+6-6\\4-3+10 & -2+0+10 & 2+3-10\\8-12+14 & -4+0+14 & 4+12-14\end{bmatrix}$$

$$\therefore X = \frac{1}{6}\begin{bmatrix}4 & 4 & 2\\11 & 8 & -5\\10 & 10 & 2\end{bmatrix}.$$

Exercise 2.5 | Q 10 | Page 72

Find matrix X, if AX = B, where A = $\begin{bmatrix}1 & 2 & 3\\-1 & 1 & 2\\1 & 2 & 4\end{bmatrix}$ and $B = \begin{bmatrix}1\\2\\3\end{bmatrix}$.

Solution: Given,AX = B

$$\therefore \begin{bmatrix}1 & 2 & 3\\-1 & 1 & 2\\1 & 2 & 4\end{bmatrix}X = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

Applying $R_2 \to R_2 + R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{bmatrix}1 & 2 & 3\\0 & 3 & 5\\0 & 0 & 1\end{bmatrix}X = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

Applying $R_2 \to \left(\frac{1}{3}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Applying $R_1 \to R_1 + \left(\frac{1}{3}\right) R_3$ and $R_2 \to R_2 - \left(\frac{5}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

EXERCISE 2.6 [PAGES 79 - 80]

Exercise 2.6 | Q 1.1 | Page 79

Solve the following equations by method of inversion.

x + 2y = 2, 2x + 3y = 3

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

This is of the form AX = B.

where A = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \end{bmatrix} \text{and } \mathrm{B} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

To determine X, we have to find A^{-1}.

|A| = $\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$

= 3 – 4

= – 1 ≠ 0

∴ A^{-1} exists.

Consider AA^{-1} = I

∴ $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Applying $R_2 \to (-1)R_2$, we get

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

∴ A^{-1} = $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Pre-multiplying AX = B by A– 1, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = A^{-1}B$

$$\therefore X = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6+6 \\ 4-3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

∴ By equality of matrices, we get

x = 0 and y = 1.

Exercise 2.6 | Q 1.2 | Page 79

Solve the following equations by method of inversion.

2x + y = 5, 3x + 5y = – 3

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

This is of the form AX = B,

where A = $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \end{bmatrix} \text{and } \mathrm{B} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

To deterrmine X, we have to find A^{-1}.

$|A| = \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix}$

= 10 – 3

= 7 ≠ 0

$\therefore$ A^{-1} exissts.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & 4 \\ 0 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

Applying $R_2 \to \left(-\frac{1}{7}\right) R_2$, we get

$$\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

Applying $R_1 \to R_1 - 4R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{5}{7} & -\frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix} \frac{5}{7} & -\frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix}$

Pre-multiplying AX = B by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1} A)X = A^{-1} B$

$\therefore Ix = A^{-1} B$

$\therefore X = A^{-1} B$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{7} & -\frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{25}{7} & \frac{+3}{7} \\ -\frac{15}{7} & -\frac{6}{7} \end{bmatrix} = \begin{bmatrix} \frac{28}{7} \\ -\frac{21}{7} \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

x = 4 and y = – 3.

Exercise 2.6 | Q 1.3 | Page 79

Solve the following equations by method of inversion.

2x – y + z = 1, x + 2y + 3z = 8 and 3x + y – 4z = 1

Solution: Matrix m the given system of equations is

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

This is of the form AX = B

where A = $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \\ y \\ z \end{bmatrix}$

and B = $\begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$

To determine X , we have to find A^{-1}

Now, |A| = $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{vmatrix}$

= 2(– 8 – 3) + 1(–4 – 9) + 1(1 – 6)

= – 22 – 13 – 5

= – 40 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & -4 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -5 \\ 0 & -5 & -13 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

Applying $R_2 \to \left(\dfrac{-1}{5}\right) R_2$, we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -5 & -13 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-1}{5} & \frac{2}{5} & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

Appying $R_1 \to R_1 - 2R_2$, $R_3 \to R_3 - 5R_2$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -8 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ \frac{-1}{5} & \frac{2}{5} & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

Applying $R_3 \to \left(\dfrac{-1}{8}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ \frac{-1}{5} & \frac{2}{5} & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{-1}{8} \end{bmatrix}$$

Appying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{11}{40} & \frac{3}{40} & \frac{1}{8} \\ \frac{-13}{40} & \frac{11}{40} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{-1}{8} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{11}{40} & \frac{3}{40} & \frac{1}{8} \\ \frac{-13}{40} & \frac{11}{40} & \frac{1}{8} \\ \frac{1}{8} & \frac{1}{8} & \frac{-1}{8} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{40}\begin{bmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{bmatrix}$$

Pre-mutiplying AX = B by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1} A)X = A^{-1} B$

$\therefore Ix = A^{-1} B$

$\therefore X = A^{-1} B$ ...(i)

$$\therefore X = \frac{1}{40} = \begin{bmatrix} 11 & 3 & 5 \\ -13 & 11 & 5 \\ 5 & 5 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} \quad \text{...[From (i)]}$$

$$\therefore \frac{1}{40}\begin{bmatrix} 11+4+5 \\ -13+88+5 \\ 5+40-5 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ 40 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

x = 1, y = 2, z = 1.

Exercise 2.6 | Q 1.4 | Page 79

Solve the following equations by method of inversion.

x + y + z = 1, x – y + z = 2 and x + y – z = 3

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

This is of the form AX = B,

where A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Appying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Appying $R_2 \to \left(-\frac{1}{2}\right) R_2$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Appying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Appying $R_3 \to \left(-\frac{1}{2}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

Appying $R_1 \to R_1 - R_3$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

pre-multiplying $AX = B$ by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0+1+\frac{3}{2} \\ \frac{1}{2}-1+0 \\ \frac{1}{2}+0-\frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ -\frac{1}{2} \\ -1 \end{bmatrix}$$

$\therefore$ By equality of matrices, we get

x = $\frac{5}{2}, y = -\frac{1}{2}$ and $z = -1$.

Exercise 2.6 | Q 2.1 | Page 80

Express the following equations in matrix form and solve them by method of reduction.

x + 3y = 2, 3x + 5y = 4

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

This is of the form AX = B,

where A = $\begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Applying $R_2 \to R_2 - 3R_1$, we get

$$\begin{bmatrix} 1 & 3 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

Hence, the original matrix A is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix} x+3y \\ 0-4y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

∴ By euality of matrices, we get

$x + 3y = 2$ ...(i)

$-4y = -2$ ...(ii)

From equation (ii),

$y = \frac{1}{2}$

Sustituting $y = \frac{1}{2}$ in equation (i), we get

$x + \frac{3}{2} = 2$

$\therefore x = 2 - \frac{3}{2} = \frac{1}{2}$

$\therefore x = \frac{1}{2}$ and $y = \frac{1}{2}$ is the required soution.

Exercise 2.6 | Q 2.2 | Page 80

Express the following equations in matrix form and solve them by method of reduction.

$3x - y = 1$, $4x + y = 6$

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

This is of the form AX = B

where A = $\begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$

Applying R1 → R1 + R2, we get

$$\begin{bmatrix} 7 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

Hence, the original matrix A is reduced to a lower triangular matrix.

$$\therefore \begin{bmatrix} 7x + 0 \\ 4x + y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

∴ By equality of matrices, we get

7x = 7 ...(i)

4x + y = 6 ...(ii)

From equation (i), x = 1

Substittutig x = 1 in equation (ii), we get

4 + y = 6

∴ y = 6 - 4 = 2

∴ x = 1 and y = 2 is the required solution.

Exercise 2.6 | Q 2.3 | Page 80

Express the following equations in matrix form and solve them by method of reduction.

x + 2y + z = 8, 2x + 3y – z = 11 and 3x – y – 2z = 5

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

This is of the form AX = B,

where A = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix}$,

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

Applying $R_3 \rightarrow R_3 - 7R_2$, we get

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -16 \end{bmatrix}$$

Hence, the original matrix A is reduced to an upper triangular matrix.

$$\begin{bmatrix} x + 2y + z \\ 0 - y - 3z \\ 0 + 0 + 16z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

∴ By equality of martices, we get

$x + 2y + z = 8$ ....(i)

$-y - 3z = -5$ ....(ii)

$16z = 16$ ....(iii)

From equation (iii), $z = 1$

Substituting $z = 1$ in equation (ii), we get

$-y - 3 = -5$

∴ $y = 2$

Substituting $y = 2$ and $z = 1$ in equation (i), we get

$x + 4 + 1 = 8$

∴ $x = 3$

∴ $x = 3$, $y = 2$, $z = 1$ is the require solution.

Exercise 2.6 | Q 2.4 | Page 80

Express the following equations in matrix form and solve them by method of reduction.

x + y + z = 1, 2x + 3y + 2z = 2 and x + y + 2z = 4

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix}1 & 1 & 1\\2 & 3 & 2\\1 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

This is of the for AX = B,

where A = $\begin{bmatrix}1 & 1 & 1\\2 & 3 & 2\\1 & 1 & 2\end{bmatrix}, \mathrm{X} = \begin{bmatrix}x\\y\\z\end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix}1\\2\\3\end{bmatrix}$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 - R_1$, we get

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\3\end{bmatrix}$$

Hence, the original matrix A is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix}x+y+z\\0+y+0\\0+0+z\end{bmatrix} = \begin{bmatrix}1\\0\\3\end{bmatrix}$$

∴ By equality of martices, we get

x + y + z = 1 ...(i)

y = 0

z = 3

Substituting y = 0 and z = 3 in equation (i) , we get

x + 0 + 3 = 1

∴ x = 1 – 3 = – 2
∴ x = – 2, y = 0 and z = 3 is the required solution.

Exercise 2.6 | Q 3 | Page 80

The total cost of 3 T.V. and 2 V.C.R. is ₹ 35,000. The shopkeeper wants profit of ₹1000 per television and ₹ 500 per V.C.R. He can sell 2 T.V. and 1 V.C.R. and get the total revenue as ₹ 21,500. Find the cost price and the selling price of a T.V. and a V.C.R.

Solution: Let the cost of a T.V. be ₹ x and the cost of a V.C.R. be ₹ y.
According to the first condition,
3x + 2y = 35000 ...(i)
The required profit per T.V. is ₹ 1000 and per V.C.R. is ₹ 500.
∴ Selling price of a T.V. is ₹ (x + 1000) and selling price of a V.C.R. is ₹ (y + 500).
According to the second condition,
2(x + 1000) + 1(y + 500) = 21500
∴ 2x + 2000 + y + 500 = 21500
∴ 2x + y = 21500 – 2500
∴ 2x + y = 19000 ...(ii)
Matrix form of equations (i) and (ii) is

$$\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 19000 \end{bmatrix}$$

Applying $R_2 \to 2R_2 - R_1$, we get

$$\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 35000 \\ 3000 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2 - R_1$, we get

$$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 300 \\ 35000 \end{bmatrix}$$

Hence, the original matrix is reduced to a lower triangular matrix.

$\therefore \begin{bmatrix} x+0 \\ 3x+2y \end{bmatrix} = \begin{bmatrix} 3000 \\ 35000 \end{bmatrix}$

∴ By equality of matrices, we get

x = 3000 ...(iii)

3x + 2y = 35000 ...(iv)

Substituting x = 3000 in equation (iv), we get

3(3000) + 2y = 35000

∴ 2y = 35000 – 9000

∴ y = $\frac{35000 - 9000}{2}$

= $\frac{26000}{2}$

= 13000.

∴ The cost price of a T.V. is ₹ 3,000 and the cost price of a V.C.R. is ₹ 13,000.

Hence, the selling price of a T.V.

= ₹ (3,000 + 1,000)

= ₹ 4,000

and the selling price of a V.C.R.

= ₹ (13,000 + 500)

= ₹ 13, 500.

Exercise 2.6 | Q 4 | Page 80

The sum of the cost of one Economic book, one Co-operation book and one account book is ₹ 420. The total cost of an Economic book, 2 Co-operation books and an Account book is ₹ 480. Also the total cost of an Economic book, 3 Co-operation books and 2 Account books is ₹ 600. Find the cost of each book.

Solution: Let the cost of one economic book, one co-operation book and one account book be ₹ x, ₹ y and ₹ z respectively. According to the first condition, x + y + z = 420

According to the second condition,

x + 2y + z = 480

According to the third condition,

x + 3y + 2z = 600

Matrix form of the above system of equations is

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 1\\1 & 3 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}420\\480\\600\end{bmatrix}$$

Applying $R_3 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 0\\0 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}420\\60\\180\end{bmatrix}$$

Applying $R_3 \to R_3 - 2R_2$, we get

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}420\\60\\60\end{bmatrix}$$

Hence, the original matrix is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix}x+y+z\\0+y+0\\0+0+z\end{bmatrix} = \begin{bmatrix}420\\60\\60\end{bmatrix}$$

∴ By equality o matrices, we get

x + y + z = 420 ...(i)

y = 60

z = 60

Substituting y = 60 and z = 60 in equation (i), we get x + 60 + 60 = 420

∴ x = 420 – 120 = 300

∴ The cost of one economic book is ₹ 300, one co-operation book is ` 60 and one account book is ₹ 60.

MISCELLANEOUS EXERCISE 2 [PAGES 81 - 86]

Miscellaneous Exercise 2 | Q 1.01 | Page 81

Choose the correct alternative.

If AX = B, where A = $\begin{bmatrix}-1 & 2\\ 2 & -1\end{bmatrix}, \mathrm{B} = \begin{bmatrix}1\\ 1\end{bmatrix}$, then X = ______

Options

$\begin{bmatrix}\frac{3}{5}\\ \frac{3}{7}\end{bmatrix}$

$\begin{bmatrix}\frac{7}{3}\\ \frac{5}{3}\end{bmatrix}$

$\begin{bmatrix}1\\ 1\end{bmatrix}$

$\begin{bmatrix}1\\ 2\end{bmatrix}$

Solution:

AX = $\begin{bmatrix}-1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}$

= $\begin{bmatrix}-1+2\\ 2-1\end{bmatrix}$

$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= B$

$\therefore$ **X =** $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 1.02 | Page 81

Choose the correct alternative.

The matrix $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ is ______

1. identity matrix
2. **scalar matrix**
3. null matrix
4. diagonal matrix

Solution:

The matrix $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ is **scalar matrix**.

Miscellaneous Exercise 2 | Q 1.03 | Page 81

Choose the correct alternative.

The matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is ______

1. identity matrix
2. diagonal matrix
3. scalar matrix
4. **null matrix**

Solution:

The matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is **null matrix**.

Miscellaneous Exercise 2 | Q 1.04 | Page 81

Choose the correct alternative.

If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then |adj.A| = ______

1. a^{12}
2. a^{9}
3. **a^{6}**
4. a^{-3}

Solution:

adj A = $\begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$

$\therefore$ |adj A| = $\begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$

= $\mathbf{a^6}$.

Miscellaneous Exercise 2 | Q 1.05 | Page 81

Choose the correct alternative.

Adjoint of $\begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}$ is ______

Options

$\begin{bmatrix} -6 & 3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} 6 & 3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$\begin{bmatrix} -6 & 3 \\ 4 & -2 \end{bmatrix}$

Solution:

Adjoint of $\begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}$ is $\begin{bmatrix} -6 & 3 \\ -4 & 2 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 1.06 | Page 82

Choose the correct alternative.

If A = diag $[d_1, d_2, d_3, \dots, d_n]$, where $d_i \neq 0$, for i = 1, 2, 3,...,n, then A^{-1} = _______

Options

$$\text{diag.}\left[\frac{1}{d_1},\frac{1}{d_2},\frac{1}{d_3},\ldots\ldots,\frac{1}{d_n}\right],$$

D

I

O

Solution:

If A = diag $[d_1, d_2, d_{3,\ldots,}d_n]$, where di ≠ 0, for i = 1, 2, 3,...,n, then A^{-1} = diag. $\left[\frac{1}{d_1},\frac{1}{d_2},\frac{1}{d_3},\ldots\ldots,\frac{1}{d_n}\right]$,

Miscellaneous Exercise 2 | Q 1.07 | Page 82

Choose the correct alternative.

If $A^2 + mA + nI = O$ and $n \neq 0$, $|A| \neq 0$, then A^{-1} = ________

Options

$$\frac{-1}{m}(A + nI)$$

$$\frac{-1}{n}(A + mI)$$

$$\frac{-1}{n}(I + mA)$$

(A + mnI)

Solution:

$A^2 + mA + nI = O$

$\therefore A^{-1}A2 + mA^{-1}A + n\,A^{-1}\,I = 0$

$\therefore (A^{-1}\,A)A + mI + n\,A^{-1} = 0$

$\therefore IA + mI + nA^{-1} = 0$

$\therefore \mathbf{A^{-1}} = \dfrac{-1}{n}(A + mI).$

Miscellaneous Exercise 2 | Q 1.08 | Page 82

Choose the correct alternative.

If a 3 x 3 matrix B has it inverse equal to B, thenB^2 = ________

Options

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution:

$B^{-1} = B$

$\therefore B^{-1}B = B.B$

$$\therefore B^2 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Miscellaneous Exercise 2 | Q 1.09 | Page 82

Choose the correct alternative.

If $A = \begin{vmatrix} \alpha & 4 \\ 4 & \alpha \end{vmatrix}$ and $|A^3| = 729$, then $\alpha =$

1. ±3
2. ±4
3. **±5**
4. ±6

Solution:

$$|A| = \begin{vmatrix} \alpha & 4 \\ 4 & \alpha \end{vmatrix} = \alpha^2 - 16$$

$|A|^3 = 729 = 9^3$

$\therefore |A| = 9$

$\therefore \alpha^2 - 16 = 9$

$\therefore \alpha^2 = 25$

$\therefore \boldsymbol{\alpha = \underline{\pm 5}}.$

Miscellaneous Exercise 2 | Q 1.1 | Page 82

Choose the correct alternative.

If A and B are square matrices of order n × n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?

1. **AB = BA**

2. either of A or B is a zero matrix
3. either of A and B is an identity matrix
4. A = B

Solution: $A^2 - B^2 = (A - B)(A + B)$

$\therefore A^2 - B^2 = A^2 + AB - BA - B^2$

$\therefore 0 = AB - BA$

$\therefore$ **AB = BA**.

Miscellaneous Exercise 2 | Q 1.11 | Page 82

Choose the correct alternative.

If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then A^{-1} = ______

Options

$\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

$\begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}$

$\begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}$

Solution:

If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 1.12 | Page 82

Choose the correct alternative.

If A is a 2x2 matrix such that A(adj.A) = $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$, then |A| = ______

1. 0
2. **5**
3. 10
4. 25

Solution: a(adj A) = |A| I

a(adj A) = $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore$ **|A| = 5**.

Miscellaneous Exercise 2 | Q 1.13 | Page 82

If A is a no singular matrix, then det (A^{-1}) = _______

1. 1
2. 0
3. det(A)
4. **1/det(A)**

Solution:

If A is a no singular matrix, then det (A^{-1}) = $\frac{1}{\det(A)}$.

Miscellaneous Exercise 2 | Q 1.14 | Page 82

If A = $\begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 1 & 5 \end{bmatrix}$, then AB =

Options

$\begin{bmatrix} 1 & -10 \\ 1 & 20 \end{bmatrix}$

$\begin{bmatrix} 1 & 10 \\ -1 & 20 \end{bmatrix}$

$\begin{bmatrix} 1 & 10 \\ 2 & -5 \end{bmatrix}$

$\begin{bmatrix} 1 & 10 \\ -1 & -20 \end{bmatrix}$

Solution:

If A = $\begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 1 & 5 \end{bmatrix}$, then AB = $\begin{bmatrix} 1 & 10 \\ 2 & -5 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 1.15 | Page 82

Choose the correct alternative.

If x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6, then (y, z) = ________

1. (–1, 0)
2. **(1, 0)**
3. (1, –1)
4. (–1, 1)

Solution: If x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6, then (y, z) = **(1, 0)**.

Miscellaneous Exercise 2 | Q 2.01 | Page 83

Fill in the blank:

A = $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is .... matrix.

Solution:

A = $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is **Column** matrix.

Miscellaneous Exercise 2 | Q 2.02 | Page 83

Fill in the blank :

Order of matrix $\begin{bmatrix} 2 & 1 & 1 \\ 5 & 1 & 8 \end{bmatrix}$ is ______

Solution:

Order of matrix $\begin{bmatrix} 2 & 1 & 1 \\ 5 & 1 & 8 \end{bmatrix}$ is **2 x 3**.

Miscellaneous Exercise 2 | Q 2.03 | Page 83

Fill in the blank :

If A = $\begin{bmatrix} 4 & x \\ 6 & 3 \end{bmatrix}$ is a singular matrix, then x is ______

Solution: |A| = 0

$\therefore \begin{vmatrix} 4 & x \\ 6 & 3 \end{vmatrix} = 0$

$\therefore$ 12 – 6x = 0

$\therefore$ **x = 2**.

Miscellaneous Exercise 2 | Q 2.04 | Page 83

Fill in the blank :

Matrix B = $\begin{bmatrix} 0 & 3 & 1 \\ -3 & 0 & -4 \\ p & 4 & 0 \end{bmatrix}$ is skew symmetric, then the value of p is ______

Solution: Matrix B is skew symmetric.

$\therefore$ B = – B^T

$$\therefore \begin{bmatrix} 0 & 3 & 1 \\ -3 & 0 & -4 \\ p & 4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 & -p \\ -3 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

$\therefore$ **P = – 1**.

Miscellaneous Exercise 2 | Q 2.05 | Page 83

Fill in the blank :

If A = $[a_{ij}]_{2x3}$ and B = $[b_{ij}]_{mx1}$ and AB is defined, then m = ________

Solution: If A = $[a_{ij}]_{2x3}$ and B = $[b_{ij}]_{mx1}$ and AB is defined, then m = **3**.

Miscellaneous Exercise 2 | Q 2.06 | Page 83

Fill in the blank :

If A = $\begin{bmatrix} 3 & -5 \\ 2 & 5 \end{bmatrix}$, then co-factor of a_{12} is ______

Solution:

If A = $\begin{bmatrix} 3 & -5 \\ 2 & 5 \end{bmatrix}$, then co-factor of a_{12} is **– 2**.

Miscellaneous Exercise 2 | Q 2.07 | Page 83

Fill in the blank :

If A = $[a_{ij}]_{mxm}$ is a non-singular matrix, then $A^{-1} = \dfrac{1}{\text{......}}$ adj(A).

Solution:

If A = $[a_{ij}]_{m \times m}$ is a non-singular matrix, then $A^{-1} = \frac{1}{|A|}$ adj(A).

Miscellaneous Exercise 2 | Q 2.08 | Page 83

Fill in the blank :

$(A^T)^T$ = ________

Solution: $(A^T)^T$ = **A**.

Miscellaneous Exercise 2 | Q 2.09 | Page 83

Fill in the blank :

If A = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{and} A^{-1} = \begin{bmatrix} 1 & 1 \\ x & 2 \end{bmatrix}$, then x = ______

Solution:

If A = $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{and} A^{-1} = \begin{bmatrix} 1 & 1 \\ x & 2 \end{bmatrix}$, then x = **– 1**.

Miscellaneous Exercise 2 | Q 2.1 | Page 83

Fill in the blank :

If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, then matrix form is

$$\begin{bmatrix} \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ldots\ldots \\ \ldots\ldots \end{bmatrix}$$

Solution:

If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, then matrix form is

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Miscellaneous Exercise 2 | Q 3.01 | Page 83

State whether the following is True or False :

Single element matrix is row as well as column matrix.

1. **True**
2. False

Solution: Single element matrix is row as well as column matrix **True**.

Miscellaneous Exercise 2 | Q 3.02 | Page 83

State whether the following is True or False :

Every scalar matrix is unit matrix.

1. True
2. **False**

Solution: Every unit matrix is a scalar matrix **False**.

Miscellaneous Exercise 2 | Q 3.03 | Page 83

State whether the following is True or False :

A = $\begin{bmatrix} 4 & 5 \\ 6 & 1 \end{bmatrix}$ is no singular matrix.

1. **True**
2. False

Solution:

$|A| = \begin{vmatrix} 4 & 5 \\ 6 & 1 \end{vmatrix}$

$= 4 - 30$

$= -26 \neq 0$

True.

Miscellaneous Exercise 2 | Q 3.04 | Page 83

State whether the following is True or False :

If A is symmetric, then $A = -A^T$.

1. True
2. **False**

Solution: If A is symmetric, then $A = A^T$ **False**.

Miscellaneous Exercise 2 | Q 3.05 | Page 83

State whether the following is True or False :

If AB and BA both exist, then AB = BA.

1. True
2. **False**

Solution: If AB and BA both exist, then AB = BA **False**.

Miscellaneous Exercise 2 | Q 3.06 | Page 83

State whether the following is True or False :

If A and B are square matrices of same order, then $(A + B)^2 = A^2 + 2AB + B^2$.

1. True
2. **False**

Solution: $(A + B)^2 = A^2 + AB + BA + B^2$ **False**.

Miscellaneous Exercise 2 | Q 3.07 | Page 83

State whether the following is True or False :

If A and B are conformable for the product AB, then $(AB)^T = A^T B^T$.

1. True
2. **False**

Solution: $(AB)^T = B^T A^T$ **False**.

Miscellaneous Exercise 2 | Q 3.08 | Page 83

State whether the following is True or False :

Singleton matrix is only row matrix.

1. True
2. **False**

Solution: Singleton matrix is also column matrix **False**.

Miscellaneous Exercise 2 | Q 3.09 | Page 83

State whether the following is True or False :

A = $\begin{bmatrix} 2 & 1 \\ 10 & 5 \end{bmatrix}$ is invertible matrix.

1. True
2. **False**

Solution:

A = $\begin{vmatrix} 2 & 1 \\ 10 & 5 \end{vmatrix}$

= 10 – 10

= 0

False.

Miscellaneous Exercise 2 | Q 3.1 | Page 83

State whether the following is True or False :

A(adj. A) = |A| I, where I is the unit matrix.

1. **True**
2. False

Solution: A(adj. A) = |A| I, where I is the unit matrix **True**.

Miscellaneous Exercise 2 | Q 4.01 | Page 84

Solve the following :

Find k, if $\begin{bmatrix} 7 & 3 \\ 5 & k \end{bmatrix}$ is a singular matrix.

Solution:

Let A = $\begin{bmatrix} 7 & 3 \\ 5 & \mathrm{k} \end{bmatrix}$

Since A is singular matrix,

|A| = 0

$\therefore \begin{vmatrix} 7 & 3 \\ 5 & \mathrm{k} \end{vmatrix} = 0$

∴ 7k – 15 = 0

∴ 7k = 15

$\therefore \mathrm{k} = \dfrac{15}{7}$.

Miscellaneous Exercise 2 | Q 4.02 | Page 84

Solve the following :

Find x, y, z if $\begin{bmatrix} 2 & x & 5 \\ 3 & 1 & z \\ y & 5 & 8 \end{bmatrix}$ is a symmetric matrix.

Solution:

Let A = $\begin{bmatrix} 2 & x & 5 \\ 3 & 1 & z \\ y & 5 & 8 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 2 & 3 & y \\ x & 1 & 5 \\ 5 & z & 8 \end{bmatrix}$

Since A is a symmetric matrix,

A = A^T

$$\therefore \begin{bmatrix} 2 & x & 5 \\ 3 & 1 & z \\ y & 5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & 3 & y \\ x & 1 & 5 \\ 5 & z & 8 \end{bmatrix}$$

∴ By equality of matrices, we get

x = 3, y = 5, z = 5.

Miscellaneous Exercise 2 | Q 4.03 | Page 84

Solve the following :

If A = $\begin{bmatrix} 1 & 5 \\ 7 & 8 \\ 9 & 5 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ -8 & 6 \end{bmatrix} \mathrm{C} = \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 7 & 8 \end{bmatrix}$ then show that (A + B) + C = A + (B + C).

Solution:

(A + B) + C = $\left\{ \begin{bmatrix} 1 & 5 \\ 7 & 8 \\ 9 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ -8 & 6 \end{bmatrix} \right\} + \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 7 & 8 \end{bmatrix}$

= $\begin{bmatrix} 1+2 & 5+4 \\ 7+1 & 8+5 \\ 9-8 & 5+6 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 7 & 8 \end{bmatrix}$

= $\begin{bmatrix} 3 & 9 \\ 8 & 13 \\ 1 & 11 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 7 & 8 \end{bmatrix}$

$$= \begin{bmatrix} 3-2 & 9+3 \\ 8+1 & 13-5 \\ 1+7 & 11+8 \end{bmatrix}$$

$$\therefore (A + B) + C = \begin{bmatrix} 1 & 12 \\ 9 & 8 \\ 8 & 19 \end{bmatrix} \quad \text{...(i)}$$

$$A + (B + C) = \begin{bmatrix} 1 & 5 \\ 7 & 8 \\ 9 & 5 \end{bmatrix} + \left\{ \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ -8 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 1 & -5 \\ 7 & 8 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 1 & 5 \\ 7 & 8 \\ 9 & 5 \end{bmatrix} + \begin{bmatrix} 2-2 & 4+3 \\ 1+1 & 5-5 \\ -8+7 & 6+8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 7 & 8 \\ 9 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 7 \\ 2 & 0 \\ -1 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 5+7 \\ 7+2 & 8+0 \\ 9-1 & 5+14 \end{bmatrix}$$

$$\therefore A + (B + C) = \begin{bmatrix} 1 & 12 \\ 9 & 8 \\ 8 & 19 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

$(A + B) + C = A + (B + C)$.

Miscellaneous Exercise 2 | Q 4.04 | Page 84

Solve the following :

If A = $\begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}, \mathrm{B} = 4\begin{bmatrix} 1 & 7 \\ -3 & 0 \end{bmatrix}$, find matrix A – 4B + 7I, where I is the unit matrix of order 2.

Solution:

A – 4B + 7I = $\begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} - 4\begin{bmatrix} 1 & 7 \\ -3 & 0 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

= $\begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} - \begin{bmatrix} 4 & 28 \\ -12 & 0 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

= $\begin{bmatrix} 2-4+7 & 5-28+0 \\ 3+12+0 & 7-0+7 \end{bmatrix}$

= $\begin{bmatrix} 5 & -23 \\ 15 & 14 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 4.05 | Page 84

Solve the following :

If A = $\begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$, verify $(A + 2B^T)^T = A^T + 2B$.

Solution:

A = $\begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$

$\therefore$ $A^T = \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} \text{and } B^T = \begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$

$\therefore$ $A + 2B^T = \begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} + 2\begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 4 \\ 8 & -2 \\ 2 & -6 \end{bmatrix}$

$= \begin{bmatrix} 2-6 & -3+4 \\ 3+8 & -2-2 \\ -1-2 & 4-6 \end{bmatrix}$

$\therefore$ $A + 2B^T = \begin{bmatrix} -4 & 1 \\ 11 & -4 \\ 1 & -2 \end{bmatrix}$

$\therefore$ $*A + 2B^T)^T = \begin{bmatrix} -4 & 11 & 1 \\ 1 & -4 & -2 \end{bmatrix}$ ...(i)

$A + 2B = \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} + 2\begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 8 & 2 \\ 4 & -2 & -6 \end{bmatrix}$

$= \begin{bmatrix} -4 & 11 & 1 \\ 1 & -4 & -2 \end{bmatrix}$ ...(iii)

From (i) and (ii), we get

$(A + 2B^T)^T = A^T + 2B.$

Miscellaneous Exercise 2 | Q 4.05 | Page 84

Solve the following :

If A = $\begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$, verify $(3A - 5B^T)^T = 3A^T - 5B$.

Solution:

$3A - 5B^T = 3\begin{bmatrix} 2 & -3 \\ 3 & -2 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 4 & -1 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 6 & - \\ 9 & -6 \\ -3 & 12 \end{bmatrix} - \begin{bmatrix} -15 & 10 \\ 20 & -5 \\ 5 & -15 \end{bmatrix}$

$= \begin{bmatrix} 6 + 15 & -9 - 10 \\ 9 - 20 & -6 + 5 \\ -3 - 5 & 12 + 15 \end{bmatrix}$

$\therefore 3A - 5B^T = \begin{bmatrix} 21 & -19 \\ -11 & -1 \\ -8 & 27 \end{bmatrix}$

$\therefore (3A - 5B^T)^T = \begin{bmatrix} 21 & -11 & -8 \\ -19 & -1 & 27 \end{bmatrix}$ ...(i)

$3A^T - 5B = 3\begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 4 \end{bmatrix} - 5\begin{bmatrix} -3 & 4 & 1 \\ 2 & -1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 6 & 9 & -3 \\ -9 & 16 & 12 \end{bmatrix} - \begin{bmatrix} -15 & 20 & 5 \\ 10 & -5 & -15 \end{bmatrix}$

$$= \begin{bmatrix} 6+15 & 9-20 & -3-5 \\ -9-10 & -6+5 & 12+15 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -11 & -8 \\ -19 & -1 & 27 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

$(3A - 5B^T)^T = 3A^T - 5B.$

Miscellaneous Exercise 2 | Q 4.06 | Page 84

Solve the following :

If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$, then show that AB and BA are bothh singular martices.

Solution:

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6-6 & -1+4+3 & 1-2+0 \\ 2-12-12 & -2+8+6 & 2-4+0 \\ 1-6-6 & -1+4+3 & 1-2+0 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{bmatrix}$$

$$\therefore |AB| = \begin{vmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{vmatrix}$$

= 0 ...[$\because$ R_1 an R_3 are identical]

$\therefore$ AB is a singular matrix.

BA = $\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

= $\begin{bmatrix} 1-2+1 & 2-4+2 & 3-6+3 \\ -3+4-1 & -6+8-2 & -9+12-3 \\ -2+2+0 & -4+4+0 & -6+6+0 \end{bmatrix}$

= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ |BA| = 0

$\therefore$ BA is a singular matrix.

Miscellaneous Exercise 2 | Q 4.07 | Page 84

Solve the following :

If A = $\begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$, verify |AB| = |A| |B|.

Solution:

AB = $\begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$

= $\begin{bmatrix} 3+5 & 6-2 \\ 1+25 & 2-10 \end{bmatrix}$

= $\begin{bmatrix} 8 & 4 \\ 26 & -8 \end{bmatrix}$

$\therefore$ |AB| = $\begin{vmatrix} 8 & 4 \\ 26 & -8 \end{vmatrix}$

= – 64 – 104

= – 168

|A| = $\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}$

= 15 – 1

= 14

|B| = $\begin{vmatrix} 1 & 2 \\ 5 & -2 \end{vmatrix}$

= 2 – 10

= – 12

$\therefore$ |A| · |B| = 14(– 12) = – 168

$\therefore$ |AB| = |A| · |B|.

Miscellaneous Exercise 2 | Q 4.08 | Page 84

Solve the following :

If A = $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 3I = 0$.

Solution: $A^2 - 4A + 3I$

= A.A – 4A + 3I

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-8+3 & -4+4+0 \\ -4+4+0 & 5-8+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

= 0.

Miscellaneous Exercise 2 | Q 4.09 | Page 84

Solve the following :

If A = $\begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix}$ and (A + B) (A – B) = $A^2 - B^2$, find a and b.

Solution: (A + B) (A – B) = $A^2 - B^2$

∴ $A^2 - AB + BA - B^2 = A^2 - B^2$

∴ –AB + BA = 0

∴ AB = BA

$$\therefore \begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix} = \begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -3+2b & -3a+0 \\ 2+4b & 2a+0 \end{bmatrix} = \begin{bmatrix} -3+2a & 2+4a \\ -3b+0 & 2b+0 \end{bmatrix}$$

∴ By equality of matrices, we get

– 3a = 2 + 4a

∴ 7a = – 2

∴ a = $\dfrac{-2}{7}$

and 2 + 4b = – 3b

∴ 7b = – 2

$\therefore$ b = $\dfrac{-2}{7}$.

Miscellaneous Exercise 2 | Q 4.1 | Page 84

Solve the following :

if A = $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, then find A^3.

Solution:

$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & 2+6 \\ -1-3 & -2+9 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} -1 & 8 \\ -4 & 7 \end{bmatrix}$

$\therefore A^3 = A^2 \cdot A = \begin{bmatrix} -1 & 8 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1-8 & -2+24 \\ -4-7 & -8+21 \end{bmatrix}$

$\therefore A^3 = \begin{bmatrix} -9 & 22 \\ -11 & 13 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 4.11 | Page 84

Find x, y, z, if $\left\{5\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -2 \\ 1 & 3 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x-1 \\ y+1 \\ 2z \end{bmatrix}$

Solution:

$$\left\{5\begin{bmatrix}0 & 1\\1 & 0\\1 & 1\end{bmatrix}-\begin{bmatrix}2 & 1\\3 & -2\\1 & 3\end{bmatrix}\right\}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\left\{\begin{bmatrix}0 & 5\\5 & 0\\5 & 5\end{bmatrix}-\begin{bmatrix}2 & 1\\3 & -2\\1 & 3\end{bmatrix}\right\}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}0-2 & -1\\5-3 & 0+2\\5-1 & -3\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}-2 & 4\\2 & 2\\4 & 2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}-4+4\\4+2\\8+2\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$$\therefore\begin{bmatrix}0\\6\\10\end{bmatrix}=\begin{bmatrix}x-1\\y+1\\2z\end{bmatrix}$$

$\therefore$ By equality of matrices, we get

$x - 1 = 0$ $\quad\therefore$ $x = 1$

$y + 1 = 6$ $\quad\therefore$ $y = 5$

$2z = 10$ $\quad\therefore$ $z = 5$.

Miscellaneous Exercise 2 | Q 4.12 | Page 84

Solve the following :

If A = $\begin{bmatrix} 2 & -4 \\ 3 & -2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 0 \end{bmatrix}$, then show that $(AB)^T = B^TA^T$.

Solution:

AB = $\begin{bmatrix} 2 & -4 \\ 3 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 0 \end{bmatrix}$

= $\begin{bmatrix} 2+8 & -2-4 & 4+0 \\ 3+4 & -3-2 & 6-0 \\ 0-2 & 0+1 & 0+0 \end{bmatrix}$

= $\begin{bmatrix} 10 & -6 & 4 \\ 7 & -5 & 6 \\ -2 & 1 & 0 \end{bmatrix}$

∴ $(AB)^T = \begin{bmatrix} 10 & 7 & -2 \\ -6 & -5 & 1 \\ 4 & 6 & 0 \end{bmatrix}$ ...(i)

Now,AT = $\begin{bmatrix} 2 & 3 & 0 \\ -4 & -2 & 1 \end{bmatrix}$ and $B^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}$

∴ $B^TA^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ -4 & -2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2+8 & 3+4 & 0-2 \\ -2-4 & -3-2 & 0+1 \\ 4-0 & 6-0 & 0+0 \end{bmatrix}$$

$$\therefore B^T A^T = \begin{bmatrix} 10 & 7 & -2 \\ -6 & -5 & 1 \\ 4 & 6 & 0 \end{bmatrix} \quad \text{...(ii)}$$

From (i) and (ii), we get

$(AB)^T = B^T A^T$.

Miscellaneous Exercise 2 | Q 4.13 | Page 85

Solve the following :

If A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$, the reduce it to unit matrix by using row transformations.

Solution:

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\therefore |A| = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

= 1(1 – 0) –0(2 –0) + 0(6 – 3)

= 1 – 0 + 0

= 1 ≠ 0

∴ A is non-singular matrix.

Hence, row transformations are possible.

Now, A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we get

A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

Applying $R_3 \to R_3 - 3R_2$, we get

A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 4.14 | Page 85

Solve the following :

Two farmers Shantaram and Kantaram cultivate three crops rice, wheat and groundnut. The sale (in Rupees) of these crops by both the farmers for the month of April and May 2016 is given below,

April 2016 (in ₹.)			
	Rice	**Wheat**	**Groundnut**
Shantaram	15000	13000	12000
Kantaram	18000	15000	8000

May 2016 (in ₹.)			
	Rice	**Wheat**	**Groundnut**
Shantaram	18000	15000	12000

Kantaram	21000	16500	16000

Find : The total sale in rupees for two months of each farmer for each crop.

Solution: Total sale for Shantaram:
For rice = 15000 + 18000 = ₹ 33000.
For wheat = 13000 + 15000 = ₹ 28000.
For groundnut = 12000 + 12000 = ₹ 24000.
Total sale for Kantaram:
For rice = 18000 + 21000 = ₹ 39000
For wheat = 15000 + 16500 = ₹ 31500
For groundnut = 8000 + 16000 = ₹ 24000

Alternate method:
Matrix form

$$= \begin{bmatrix} 15000 & 13000 & 12000 \\ 18000 & 15000 & 8000 \end{bmatrix} + \begin{bmatrix} 18000 & 15000 & 12000 \\ 21000 & 16500 & 16000 \end{bmatrix}$$

$$= \begin{bmatrix} 33000 & 28000 & 24000 \\ 39000 & 31500 & 24000 \end{bmatrix}$$

∴ The total sale of April and May of Shantaram in ₹ is ₹ 33000 (rice), ₹ 28000 (wheat), ₹ 24000 (groundnut) and that of Kantaram in ₹ is ₹ 39000(rice), ₹ 31500(wheat), and ₹ 24000 (groundnut).

Miscellaneous Exercise 2 | Q 4.14 | Page 85

Solve the following :

Two farmers Shantaram and Kantaram cultivate three crops rice, wheat and groundnut. The sale (in Rupees) of these crops by both the farmers for the month of April and May 2016 is given below,

April 2016 (in ₹.)			
	Rice	**Wheat**	**Groundnut**
Shantaram	15000	13000	12000
Kantaram	18000	15000	8000

May 2016 (in ₹.)			
	Rice	**Wheat**	**Groundnut**

Shantaram	18000	15000	12000
Kantaram	21000	16500	16000

Find : the increase in sale from April to May for every crop of each farmer.

Solution: Increase in sale from April to May for Shantaram:
For rice = 18000 – 15000 = 3000
For wheat = 15000 – 13000 = 2000
For groundnut = 12000 – 12000 = 0
Increase in sale from April to May for Kantaram:
For rice = 21000 – 18000 = 3000
For wheat = 16500 – 15000 = 1500
For groundnut = 16000 – 8000 = 8000

Alternate method:
Matrix form

$$= \begin{bmatrix} 18000 & 15000 & 12000 \\ 21000 & 16500 & 16000 \end{bmatrix} - \begin{bmatrix} 15000 & 13000 & 12000 \\ 18000 & 15000 & 8000 \end{bmatrix}$$

$$= \begin{bmatrix} 3000 & 2000 & 0 \\ 3000 & 1500 & 8000 \end{bmatrix}$$

∴ The increase in sales for Shantaram from April to May in each crop is ₹ 3000 (rice), ₹ 2000(wheat), 0 (groundnut) and that for Kantaram is ₹ 3000 (rice), ₹ 1500 (wheat) and ₹ 8000 (groundnut).

Miscellaneous Exercise 2 | Q 4.15 | Page 85

Check whether the following matrices are invertible or not:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solution:

let A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then, |A| = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

= 1 – 0

= 1 ≠ 0

∴ A is a non-singular matrix.

∴ A is invertible.

Miscellaneous Exercise 2 | Q 4.15 | Page 85

Check whether the following matrices are invertible or not:

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

let A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then, |A| = $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

= 1 – 1

= 0

∴ A is a singular matrix.

∴ A is not invertible.

Miscellaneous Exercise 2 | Q 4.15 | Page 85

Check whether the following matrices are invertible or not:

$$\begin{bmatrix} 3 & 4 & 5 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 3 & 4 & 5 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{bmatrix}$

Then, |A| = $\begin{vmatrix} 3 & 4 & 5 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{vmatrix}$

= 3(5 – 0) – 4(5 – 0) + 3(4 – 1)

= 15 – 20 + 9

= 4 ≠ 0

∴ A is a non-singular matrix.

∴ A is invertible.

Miscellaneous Exercise 2 | Q 4.15 | Page 85

Check whether the following matrices are invertible or not:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 2 & 4 & 6 \end{vmatrix}$

$= 1(24 - 20) - 2(12 - 10) + 3(8 - 8)$

$= 4 - 4 + 0$

$= 0$

$\therefore$ A is a singular matrix.

$\therefore$ A is not invertible.

Miscellaneous Exercise 2 | Q 4.16 | Page 85

Find inverse of the following matrices (if they exist) by elementary transformations :

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$

$= 3 + 2$

$= 5 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - 2R_1$, we get

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Applying $R_2 \to \left(\frac{1}{5}\right) R_2$, we get

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}.$$

Miscellaneous Exercise 2 | Q 4.16 | Page 85

Find inverse of the following matrices (if they exist) by elementary transformations :

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 1 \\ 7 & 4 \end{vmatrix}$

$= 8 - 7$

$= 1 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $R_1 \to 4R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 7 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 \\ 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - 7R_1$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

Applying $R_2 \to \left(\frac{1}{4}\right) R_2$, we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}.$$

Miscellaneous Exercise 2 | Q 4.16 | Page 85

Find inverse of the following matrices (if they exist) by elementary transformations :

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Solution:

$$\text{Let } A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

$\therefore |A| = \begin{vmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{vmatrix}$

= 2(4 + 6) + 3(4 – 9) + 3(–4 – 6)

= 20 – 15 – 30

= – 25 ≠ 0

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to 2R_1 - R_3$, we get

$$\begin{bmatrix} 1 & -4 & 4 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} A^{1} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_1 - 2R_1$ and $R_3 \to R_1 - 2R_1$, we get

$$\begin{bmatrix} 1 & -4 & 4 \\ 0 & 10 & -5 \\ 0 & 10 & -10 \end{bmatrix} A^{-1} \begin{bmatrix} 2 & 0 & -1 \\ -4 & 1 & 2 \\ -6 & 0 & 4 \end{bmatrix}$$

Applying $R_2 \to \left(\frac{1}{10}\right) R_2$ and $R_3 \to \left(-\frac{1}{10}\right) R3$, we get

$$\begin{bmatrix} 1 & -4 & 4 \\ 0 & 1 & -\frac{1}{2} \\ 0 & -1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ -\frac{4}{10} & \frac{1}{10} & \frac{2}{10} \\ \frac{6}{10} & 0 & -\frac{4}{10} \end{bmatrix}$$

Applying $R_1 \to R_1 - 4R_2$ and $R_3 \to R_3 + R_2$, we get

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{4}{10} & \frac{4}{10} & -\frac{2}{10} \\ -\frac{4}{10} & \frac{1}{10} & \frac{2}{10} \\ \frac{2}{10} & \frac{1}{10} & -\frac{2}{10} \end{bmatrix}$$

Applying $R_3 \to 2R_3$, we get

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{4}{10} & \frac{4}{10} & -\frac{2}{10} \\ -\frac{4}{10} & \frac{1}{10} & \frac{2}{10} \\ \frac{4}{10} & \frac{2}{10} & -\frac{4}{10} \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_3$ and $R_2 \to R_2 + \left(\frac{1}{2}\right)R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} \begin{bmatrix} -\frac{4}{10} & 0 & \frac{6}{10} \\ -\frac{2}{10} & \frac{2}{10} & 0 \\ \frac{4}{10} & \frac{2}{10} & -\frac{4}{10} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

Miscellaneous Exercise 2 | Q 4.16 | Page 85

Find inverse of the following matrices (if they exist) by elementary transformations :

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$

= 2(3 – 0) –0 –1(5 – 0)

= 6 – 0 – 5

= 1 $\neq$ 0

$\therefore$ A^{-1} existts.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 0 & 1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \leftrightarrow R_2$, we get

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 - 2R_2$, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - 3R_3$, we get

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$, we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

Applying $R_3 \to (-1) R_3$, we get

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ 5 & -2 & 2 \end{bmatrix}$$

Applying $R_1 \to R_1 + 2R_3$ and $R_2 \to R_2 - 4R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.$$

Miscellaneous Exercise 2 | Q 4.17 | Page 85

Find the inverse of $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{bmatrix}$ by adjoint method.

Solution:

Let A = $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{vmatrix}$

= 3(35 – 16) – 1(10 –8) + 5(4 – 7)

= 3(19) –1(2) + 5(– 3)

= 57 – 2 – 15

= 40 ≠ 0

∴ A^{-1} exists.

$$A_{11} = (-1)^{1+1} M_{11} = 1\begin{vmatrix} 7 & 8 \\ 2 & 5 \end{vmatrix}$$

= 1(35 – 16) = 19

$$A_{12} = (-1)^{1+2} M_{12} = -1\begin{vmatrix} 2 & 8 \\ 1 & 5 \end{vmatrix}$$

= 1(10 – 8) = – 2

$$A_{13} = (-1)^{1+3} M_{13} = 1\begin{vmatrix} 2 & 7 \\ 1 & 2 \end{vmatrix}$$

= 1(4 – 7) = – 3

$$A_{21} = (-1)^{2+1} M_{21} = -1\begin{vmatrix} 1 & 5 \\ 2 & 5 \end{vmatrix}$$

= –1(5 – 10) = 5

$$A_{22} = (-1)^{2+2} M_{22} = 1\begin{vmatrix} 3 & 5 \\ 1 & 5 \end{vmatrix}$$

= 1(15 – 5) = 10

$$A_{23} = (-1)^{2+3} M_{23} = -1\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$

= –1(6 – 1) = – 5

$$A_{31} = (-1)^{3+1} M_{31} = 1\begin{vmatrix} 1 & 5 \\ 7 & 8 \end{vmatrix}$$

= 1(8 – 35) = – 27

A_{32} = (– 1)$^{3+2}$ M_{32} = $-1\begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix}$

= – 1(24 – 10) = – 14

A_{33} = (– 1)$^{3+3}$ M_{33} = $1\begin{vmatrix} 3 & 1 \\ 2 & 7 \end{vmatrix}$

= 1(21 – 2_ = 19

∴ The matrix of the co-factors is

$[A_{ij}]_{3x3}$ = $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 19 & -2 & -3 \\ 5 & 10 & -5 \\ -27 & -14 & 19 \end{bmatrix}$

Now, adj A = $[A_{ij}]^T_{3\times3} = \begin{bmatrix} 19 & 5 & -27 \\ -2 & 10 & -14 \\ -3 & -5 & 19 \end{bmatrix}$

∴ A^{-1} = $\dfrac{1}{|A|}(\text{adj A}) = \dfrac{1}{40}\begin{bmatrix} 19 & 5 & -27 \\ -2 & 10 & -14 \\ -3 & -5 & 19 \end{bmatrix}$.

Miscellaneous Exercise 2 | Q 4.18 | Page 85

Solve the following equations by method of inversion : 4x – 3y – 2 = 0, 3x – 4 + 6 = 0

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

This of the form AX = B, where

A = $\begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{and } B = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

To determine X, we have to find A^{-1}.

$|A| = \begin{vmatrix} 4 & -3 \\ 3 & -4 \end{vmatrix}$

$= -16 + 9$

$= -7 \neq 0$

$\therefore A^{-1}$ exists

Consider $A^{A-1} = I$

$$\begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - 3R_1$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & -7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

Applying $R_2 \to \left(\frac{1}{7}\right) R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -1 \\ \frac{3}{7} & \frac{-4}{7} \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{4}{7} & \frac{-3}{7} \\ \frac{3}{7} & \frac{-4}{7} \end{bmatrix}$$

$\therefore A^{-1} = \frac{1}{7}\begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix}$

Pre-multiplyig AX = by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)\, X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = A^{-1}B$

$\therefore X = \frac{1}{7}\begin{bmatrix} 4 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 2 \\ -6 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 8+18 \\ 6+24 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 26 \\ 30 \end{bmatrix}$

$= \begin{bmatrix} \frac{26}{7} \\ \frac{30}{7} \end{bmatrix}$

$\therefore$ By equality of martices, we get

$x = \frac{26}{7} \text{ and } y = \frac{30}{7}$.

Miscellaneous Exercise 2 | Q 4.18 | Page 85

Solve the following equations by method of inversion : x + y – z = 2, x – 2y + z = 3 and 2x – y – 3z = – 1

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \end{bmatrix} \text{and } \mathrm{B} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

To determine X, we have to find A^{-1}.

|A|= $\begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{vmatrix}$

= 1(6 + 1) – 1(–3 – 2) –1(–1 + 4)

= 1(7) –1(–5)–1(3)

= 7 + 5 – 3

= 9 ≠ 0

∴ A^{-1} exists.

Consider AA^{-1} =

$$\therefore \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 2 & -1 & -3 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 2R_1$, we get

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 2 \\ 0 & -3 & -1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to \left(\frac{-1}{3}\right) R_2$, we get

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -\frac{2}{3} \\ 0 & -3 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{-1}{3} & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + 3R_2$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{-2}{3} \\ 0 & 0 & -3 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{3} & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

Applying $R_3 \to \left(-\frac{1}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

Applying $R_1 \to R_1 + \left(-\frac{1}{3}\right) R_3$ and $R_2 \to R_2 + \left(\frac{2}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{7}{9} & \frac{4}{9} & -\frac{1}{9} \\ \frac{5}{9} & -\frac{1}{9} & -\frac{2}{9} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{9} \begin{bmatrix} 7 & 4 & -1 \\ 5 & -1 & -2 \\ 3 & 3 & -3 \end{bmatrix}$$

Pre-multiplying AX = B by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = A^{-1}B$

$$\therefore X = \frac{1}{9}\begin{bmatrix} 7 & 4 & -1 \\ 5 & -1 & -2 \\ 3 & 3 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 14+12+1 \\ 10-3+2 \\ 6+9+3 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 27 \\ 9 \\ 18 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ By equality of martices, we get

x = 3, y = 1 and z = 2.

Miscellaneous Exercise 2 | Q 4.18 | Page 85

Solve the following equations by method of inversion : x – y + z = 4, 2x + y –z = 0 , x + y + z = 2

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

To determine X, we have to find A^{-1}.

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 1(1 + 3) + 1(2 + 3) + 1(2 - 1)$

$= 1(4) + 1(5) + 1(1)$

$= 4 + 5 + 1$

$= 10 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{A} - 1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - R_1$, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow \left(\frac{1}{3}\right) R_2$, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & \frac{-5}{3} \\ 0 & 2 & 0 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-2}{3} & \frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 2R_2$, we get

$$\begin{bmatrix} 1 & 0 & \frac{-2}{3} \\ 0 & 1 & \frac{-5}{3} \\ 0 & 0 & \frac{10}{3} \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{-2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{-2}{3} & 1 \end{bmatrix}$$

Applying $R_3 \to \left(\frac{3}{10}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & \frac{-2}{3} \\ 0 & 1 & \frac{-5}{3} \\ 0 & 0 & 1 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{-2}{3} & \frac{1}{3} & 0 \\ \frac{1}{10} & \frac{-1}{3} & \frac{3}{10} \end{bmatrix}$$

Applying $R_1 \to R_1 + \left(\frac{2}{3}\right) R_3$ and $R_2 \to R_2 + \left(\frac{5}{3}\right) R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{-1}{2} & 0 & \frac{1}{2} \\ \frac{1}{10} & \frac{-1}{5} & \frac{3}{10} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Pre-multiplying $AX = B$ by A^{-1}, we get

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)\, X = A^{-1}B$

$\therefore$ IX = A^{-1}B

$\therefore$ X = A^{-1}B

$$\therefore X = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore$ By equality of martices, we get

x = 2, y = – 1 and z = 1.

Miscellaneous Exercise 2 | Q 4.19 | Page 85

Solve the following equations by method of reduction : 2x + y = 5, 3x + 5y = – 3

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

This is of the form AX = B,

where A = $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \end{bmatrix} \text{and } \mathrm{B} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

Applying $R_2 \to 2R_2$, we get

$$\begin{bmatrix} 2 & 1 \\ 6 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

Applying $R_2 \to R_2 - 3R_1$, we get

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

Hence, the original matrix A is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix} 2x + y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

∴ By equality of matrices, we get

$2x + y = 5$ ...(i)

$7y = -21$ ...(ii)

From equation (ii), $y = -3$

Sunstituting $y = -3$ in equation (i), we get

$2x - 3 = 5$

$\therefore 2x = 5 + 3$

$\therefore 2x = 8$

$\therefore x = 4$

∴ $x = 4$ and $y = -3$ is the required solution.

Miscellaneous Exercise 2 | Q 4.19 | Page 85

Solve the following equations by method of reduction :

$x + 2y - z = 3$, $3x - y + 2z = 1$ and $2x - 3y + 3z = 2$

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix}1 & 2 & 1\\ 3 & -1 & 2\\ 2 & -3 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ 1\\ 2\end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix}1 & 2 & 1\\ 3 & -1 & 2\\ 2 & -3 & 3\end{bmatrix}, \mathrm{X} = \begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } \mathrm{B} = \begin{bmatrix}3\\ 1\\ 2\end{bmatrix}$

Applying $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 - 2R_1$, we get

$$\begin{bmatrix}1 & 2 & 1\\ 0 & -7 & -1\\ 0 & -7 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ -8\\ -4\end{bmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get

$$\begin{bmatrix}1 & 2 & 1\\ 0 & -7 & -1\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ -8\\ 4\end{bmatrix}$$

Hence, the original matrix A is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix}x + 2y + z\\ 0 - 7y - zx\\ 0 + 0 + 2z\end{bmatrix} = \begin{bmatrix}3\\ -8\\ 4\end{bmatrix}$$

By equality of martices, we get

$x + 2y + z = 3$ ...(i)

$-7y - z = -8$

i.e., $7y + z = 8$ ...(ii)

$2z = 4$

∴ z = 2

Substituting z = 2 in equation (ii), we get

7y + 2 = 8

∴ 7y = 6

∴ y = $\frac{6}{7}$

Substituting y = $6\frac{)}{7}$ and z = 2 in equation (i), we get

$$x + 2\left(\frac{6}{7}\right) + 2 = 3$$

$$\therefore x + \frac{12}{7} + 2 = 3$$

$$\therefore x = 3 - 2 - \frac{12}{7} = \frac{-5}{7}$$

∴ x = $\frac{-5}{7}, y = \frac{6}{7}$ and z = 2 is the required solution.

Miscellaneous Exercise 2 | Q 4.19 | Page 85

Solve the following equations by method of reduction : x – 3y + z = 2 , 3x + y + z = 1 and 5x + y + 3z = 3

Solution: Matrix form of the given system of equations is

$$\begin{bmatrix} 1 & -3 & 1 \\ 3 & 1 & 1 \\ 5 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix} 1 & -3 & 1 \\ 3 & 1 & 1 \\ 5 & 1 & 3 \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{and } \mathrm{B} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

Applying $R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 - 5R_1$, we get

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 10 & -2 \\ 0 & 16 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -7 \end{bmatrix}$$

Applying $R_3 \to R_3 - \left(\frac{8}{5}\right) R_2$, we get

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 10 & -2 \\ 0 & 0 & \frac{6}{5} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$$

Hence, the original matrix A is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix} x - 3y + z \\ 0 + 10y - 2z \\ 0 + 0 + \frac{6}{5}z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$$

∴ By equality of martices, we get

$x - 3y + z = 2$ ...(i)

$10y - 2z = -5$ ...(ii)

$\frac{6}{5}z = 1$

$\therefore z = \frac{5}{6}$

Substituting $z = \frac{5}{6}$ in equation (ii), we get

$$10y - 2\left(\frac{5}{6}\right) = -5$$

$$\therefore 10y - \frac{10}{6} = -5$$

$\therefore$ 10y = $-5 + \frac{10}{6} = \frac{-20}{6}$

$\therefore$ 10y = $\frac{-10}{3}$

$\therefore$ y = $\frac{-1}{3}$

Sunstituting y = $\frac{-1}{3}$ and z = $\frac{5}{6}$ in equation (i), we get

$x - 3\left(\frac{-1}{3}\right) + \frac{5}{6}$ = 2

$\therefore x + 1 + \frac{5}{6}$ = 2

$\therefore$ x = $2 - 1 - \frac{5}{6} = \frac{1}{6}$

$\therefore$ x = $\frac{1}{6}, y = \frac{-1}{3} \text{and} z = \frac{5}{6}$ is the required solution.

Miscellaneous Exercise 2 | Q 4.2 | Page 86

The sum of three numbers is 6. If we multiply third number by 3 and add it to the second number we get 11. By adding the first and third number we get a number which is double the second number. Use this information and find a system of linear equations. Find the three numbers using matrices.

Solution: Let the three numbers be x, y and z respectively. According to the first condition,

x + y + z = 6

According to the second condition,

3z + y = 11 i.e., y + 3z = 11

According to the third condition,

x + z = 2y i.e., x – 2y + z = 0

Matrix form of the above system of equations is

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

Applying $R_2 \leftrightarrow R_3$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 11 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 11 \end{bmatrix}$$

Applying $R_3 \to 3R_3 + R_2$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 27 \end{bmatrix}$$

Hence, the original matrix is reduced to an upper triangular matrix.

$$\therefore \begin{bmatrix} x + y + z \\ 0 - 3y + 0 \\ 0 + 0 + 9z \end{bmatrix} \begin{bmatrix} 6 \\ -6 \\ 27 \end{bmatrix}$$

∴ By equality of martices, we get

$x + y + z = 6$ ...(i)

$-3y = -6$ ...(ii)

i.e., $y = 2$

$9z = 27$ ...(iii)

i.e., $z = 3$

Substituting $y = 2$ and $z = 3$ in equation (i), we get $x + 2 + 3 = 6$

$\therefore x = 1$

$\therefore$ 1, 2 and 3 are the required numbers.