CBSE Board
Class XII Mathematics
Board Paper 2013
Delhi Set – 1

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three Section A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

2. Write the value of tan $\left(2\tan^{-1}\frac{1}{5}\right)$

3. Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

4. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

5. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

6. Write the degree of the differential equation $x^3\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 = 0$

7. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of x + y + z.

8. If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and acute angle θ with $\hat{k}$, then find the value of θ.

9. Find the Cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

10. The amount of pollution content added in air in a city due to x-diesel vehicles is given by P(x) = 0.005 x^3 + 0.02 x^2 + 30x. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above questions.

SECTION – B

11. Show that the function f in A = **R** - $\left\{\frac{2}{3}\right\}$ defined as f(x) = $\frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.

12. Find the value of the following:

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0 \text{ and } xy<1.$$

OR

Prove that $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

13. Using properties of determinants prove the following:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = {1-x^3}^{\,2}$$

14. Differentiate the following function with respect to x:

$(\log x)^x + x^{\log x}$

15. If y = log $\left[x+\sqrt{x^2+a^2}\right]$, show that $(x^2 + a^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.

16. Show that the function $f(x) = |x-3|, x \in \mathbf{R}$, is continuous but not differentiable at x = 3.

OR

If x = a sin t and y = a $\left(\cos t + \log \tan \frac{t}{2}\right)$ find $\frac{d^2y}{dx^2}$

17. Evaluate: $\int \frac{\sin(x-a)}{\sin(x+a)} dx$

OR

Evaluate: $\int \frac{5x-2}{1+2x+3x^2} dx$

18. Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

19. Evaluate: $\int_0^4 (|x|+|x-2|+|x-4|) dx$

20. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a}+\vec{b}$ is perpendicular to vector $\vec{b}$.

21. Find the coordinates of the point, where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ intersects the plane x – y + z – 5 = 0. Also find the angle between the line and the plane.

OR

Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k}) - 4 = 0$ and $\bar{r}.(2\hat{i}+\hat{j}-\hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k}) + 8 = 0$.

22. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

SECTION – C

23. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs 6,000. Three times the award money for Hard work added to that given for honesty amounts to Rs 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

24. Show that the height of the cylinder of maximum volume, which can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Find the equation of the normal at a point on the curve $x^2 = 4y$ which passes through the point (1, 2). Also, find the equation of the corresponding tangent.

25. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

OR

Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

26. Show that the differential equation $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that x = 0 when y = 1.

27. Find the vector equation of the plane passing through three points with position vectors $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$.

28. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment?

29. Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation

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SECTION – A

1. Let $\tan^{-1}(1) = y$

$\Rightarrow \tan y = 1 = \tan\left(\frac{\pi}{4}\right) \Rightarrow y = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

$\cos^{-1}\left(-\frac{1}{2}\right) = z$

$\Rightarrow \cos z = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow z = \frac{2\pi}{3} \Rightarrow \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{11\pi}{12}$

2. We know: $2\tan^{-1} x = \tan^{-1} \frac{2x}{1 - x^2}$

$\Rightarrow 2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{2\left(\frac{1}{5}\right)}{1 - \left(\frac{1}{5}\right)^2} = \tan^{-1}\frac{\frac{2}{5}}{\frac{24}{25}} = \tan^{-1}\frac{5}{12}$

$\therefore \tan\left(2\tan^{-1}\frac{1}{5}\right) = \tan\left(\tan^{-1}\frac{5}{12}\right) = \frac{5}{12}$

3. $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Equating the corresponding elements, we get,

$\Rightarrow$ a - b = -1, 2a + c = 5, 2a - b = 0, 3c + d = 13

Now, consider the equations:

a - b = -1 and 2a - b = 0

Subtracting first equation from second, we get: a = 1

4. $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$\Rightarrow$ (x + 1) (x + 2) - (x - 1) (x - 3) = 12 + 1

$\Rightarrow$ $x^2 + 3x + 2 - x^2 + 4x - 3 = 13$

$\Rightarrow$ 7x - 1 = 13

$\Rightarrow$ 7x = 14

$\Rightarrow$ x = 2

5. $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 9-1 & -1-2 & 4+1 \\ -2-0 & 1-4 & 3-9 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

6. $x^3\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 = 0$

We know that the degree of a differential equation is the highest power (exponent) of the highest order derivative in it.

The highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$. Its power is 2. So, the degree of the given differential equation is 2.

7. Given, $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are equal vectors.

$\therefore x\hat{i} + 2\hat{j} - zk = 3\hat{i} - y\hat{j} + k$

$\Rightarrow x = 3,\ y = -2,\ z = -1$

$\therefore x + y + z = 3 + (-2) + (-1) = 0$

8. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ be the unit vector.

$\therefore x = \cos\frac{\pi}{3} = \frac{1}{2}$

$y = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Now, $x^2 + y^2 + z^2 = 1$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + z^2 = 1$

$\Rightarrow z^2 = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$

$\Rightarrow z = \frac{1}{2}$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

9. The equation of the given line is:

$\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$

i.e., $\frac{x+3}{3} = \frac{y-4}{-5} = \frac{z+8}{6}$

The required line is parallel to the given line. Therefore, direction ratios of the required line are same as the direction ratio of the given line. So, the direction ratios of the required line are 3, -5, and 6.

Thus, the equation of the straight line passing through (-2, 4, -5) and having direction ratios 3, -5, 6 is

$\frac{x - \; -2}{3} = \frac{y-4}{-5} = \frac{z - \; -5}{6}$

i.e., $\frac{x+2}{3} = \frac{4-y}{5} = \frac{z+5}{6}$

10. P(x) = 0.005x3 + 0.02x2 + 30x

Differentiating w.r.t. x,

Marginal increase in pollution content = $\frac{dP \; x}{dx} = 0.015x^2 + 0.04x + 30$

Putting x = 3 in (1), $\left(\frac{dp \; x}{dx}\right)_{x=3} = 0.015 \times 9 + 0.04 \times 3 + 30 = 30.255$

Therefore, the value of marginal increase pollution content is 30.255.

Increase in number of diesel vehicles increases the pollution. We should aim at saving the environment by reducing the pollution by decreasing the vehicle density on road.

SECTION – B

11. $f(x) = \frac{4x+3}{6x-4}$

Let $f(x_1) = f(x_2)$

$\Rightarrow \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$

$\Rightarrow$ $24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 + 18x_1 - 16x_2 - 12$

$\Rightarrow$ $18x_2 + 16x_2 = 18x_1 + 16x_1$

$\Rightarrow$ $34x_2 = 34x_1$

$\Rightarrow$ $x_1 = x_2$

Since, $\frac{4x+3}{6x-4}$ is a real number, therefore, for every y in the co-domain of f, there exists a number x in **R** - $\left\{\frac{2}{3}\right\}$ such that f(x) = $y = \frac{4x+3}{6x-4}$

Therefore, f(x) is onto.

Hence, f^{-1} exists.

Now, let $y = \frac{4x+3}{6x-4}$

$\Rightarrow$ $6xy - 4y = 4x + 3$

$\Rightarrow 6xy - 4x = 4y + 3$

$\Rightarrow x\ 6y - 4\ = 4y + 3$

$\Rightarrow x = \frac{4y+3}{6y-4}$

$\Rightarrow y = \frac{4x+3}{6x-4}$ int erchanging the variables x and y

$\Rightarrow f^{-1}\ x\ = \frac{4x+3}{6x-4}$ $\left[\text{put } y=f^{-1}\ x\ \right]$

12.

We know that:

$$\sin^{-1}\frac{2x}{1+x^2}=2\tan^{-1}x \text{ for } |x|\le 1 \qquad \text{... } 1$$

$$\cos^{-1}\frac{1-y^2}{1+y^2}=2\tan^{-1}y \text{ for } y>0 \qquad \text{... } 2$$

$$\therefore \sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}=2\tan^{-1}x+2\tan^{-1}y.$$

$$\Rightarrow \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$=\tan\frac{1}{2}(2\tan^{-1}x+2\tan^{-1}y)$$

$$=\tan(\tan^{-1}x+\tan^{-1}y)$$

$$=\tan\left(\tan^{-1}\frac{x+y}{1-xy}\right) \qquad \left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}, \text{for } xy<1\right]$$

$$=\frac{x+y}{1-xy}$$

OR

We know that:

$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \dfrac{x+y}{1-xy}, xy < 1$

We have:

$$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\left[\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)\right]+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}\right)+\tan^{-1}\left(\frac{1}{8}\right) \quad \left(\because \frac{1}{2}\times\frac{1}{5}<1\right)$$

$$=\tan^{-1}\left(\frac{7}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)$$

$$=\tan^{-1}\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}$$

$$=\tan^{-1}\frac{56+9}{72-7} \quad \left(\because \frac{7}{9}\times\frac{1}{8}<1\right)$$

$$=\tan^{-1}\frac{65}{65}=\tan^{-1}1=\frac{\pi}{4}$$

Hence, $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

13. $\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= 1+x+x^2 \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = 1+x+x^2 \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$$

$$= 1+x+x^2 \quad 1-x \quad 1-x \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

$$= 1-x^3 \quad 1-x \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = 1-x^3 \quad 1-x \quad 1 \begin{vmatrix} 1+x & x \\ -x & 1 \end{vmatrix}$$

$$= 1-x^3 \quad 1-x \quad 1+x+x^2$$

$$= 1-x^3 \quad 1-x^3$$

$$= 1-x^3 \;{}^{2}$$

Hence proved.

14. Let $y = (\log x)^x + x^{\log x}$ ...(1)

Now let $y_1 = (\log x)^x$ and $y_2 = x^{\log x}$

$\Rightarrow y = y_1 + y_2$ ...(2)

Differentiating (2) w.r.t. x,

$$\frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx} \quad ...(3)$$

Now consider $y_1 = (\log x)^x$

Taking log on both sides,

$\log y_1 = x \log (\log x)$

Differentiating w.r.t. x, we get

$$\frac{1}{y_1}\frac{dy_1}{dx} = x \times \frac{1}{\log x} \times \frac{1}{x} + 1 \times \log \ \log x$$

$$\Rightarrow \frac{dy_1}{dx} = y_1\left(\frac{1}{\log x} + \log \ \log x\right)$$

$$\Rightarrow \frac{dy_1}{dx} = \ \log x \ ^{x}\left(\frac{1}{\log x} + \log \ \log x\right) \quad ...(4)$$

Now, consider $y_2 = x^{\log x}$

$\log y_2 = (\log x)(\log x) = (\log x)^2$

Differentiating w.r.t. x, we get

$$\frac{1}{y_2}\frac{dy_2}{dx} = 2 \ \log x \ \times \frac{1}{x}$$

$$\Rightarrow \frac{dy_2}{dx} = y_2\left(\frac{2\log x}{x}\right) = x^{\log x}\left(\frac{2\log x}{x}\right) \quad ....(5)$$

Using equations (3), (4) and (5), we get:

$$\frac{dy}{dx} = \ \log x \ ^{x}\left(\frac{1}{\log x} + \log \ \log x\right) + x^{\log x}\left(\frac{2\log x}{x}\right)$$

15. y = log $\left[x+\sqrt{x^2+a^2}\right]$ ...(1)

y = log $\left[x+\sqrt{x^2+a^2}\right]$ ...(1)

Differentiating (1) w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1+\frac{x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{x^2+a^2}} \quad ...(2)$$

$$\Rightarrow x\frac{dy}{dx}=\frac{x}{\sqrt{x^2+a^2}} \quad ...(3)$$

Again, differentiating (2) w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\frac{-\frac{2x}{2\left(x^2+a^2\right)^{\frac{1}{2}}}}{x^2+a^2}$$

$$\Rightarrow \frac{d^2y}{dx^2}=-\frac{x}{\left(x^2+a^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \left(x^2+a^2\right)\frac{d^2y}{dx^2}=-\frac{x}{\sqrt{x^2+a^2}} \quad ...(4)$$

Adding equation (3) and (4), we get

$$\left(x^2+a^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-\frac{x}{\sqrt{x^2+a^2}}+\frac{x}{\sqrt{x^2+a^2}}=0$$

$$\Rightarrow \left(x^2+a^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$$

16. f(x) = $|x-3| = \begin{cases} 3-x, & x<3 \\ x-3, & x\geq 3 \end{cases}$

Let c be a real number.

Case I: c < 3. Then f(c) = 3 – c.

$\lim_{x\to c} f(x) = \lim_{x\to c} (3-x) = 3-c$

Since, $\lim_{x\to c} f(x) = f(c)$, f is continuous at all negative real numbers.

Case II: c = 3. Then f(c) = 3 – 3 = 0

$\lim_{x\to c} f(x) = \lim_{x\to c} (x-3) = 3-3 = 0$

Since, $\lim_{x\to c} f(x) = f(3)$, f is continuous at x = 3.

Case III: c > 3. Then f(c) = c – 3.

$\lim_{x\to c} f(x) = \lim_{x\to c} (x-3) = c-3.$

Since, $\lim_{x\to c} f(x) = f(c)$, f is continuous at all positive real numbers.

Therefore, f is continuous function.

Now, we need to show that f(x) = $|x-3|$, $x \in \mathbf{R}$ is not differentiable at x = 3.

Consider the left hand limit of f at x = 3

$$\lim_{h\to 0^-}\frac{f\ 3+h\ -f\ 3}{h} = \lim_{h\to 0^-}\frac{|3+h-3|-|3-3|}{h} = \lim_{h\to 0^-}\frac{|h|-0}{h} = \lim_{h\to 0^-}\frac{-h}{h} = -1$$

$h<0 \Rightarrow |h| = -h$

Consider the right hand limit of f at x = 3

$$\lim_{h\to 0^+}\frac{f\ 3+h\ -f\ 3}{h}\ \lim_{h\to 0^+}\frac{|3+h-3|-|3-3|}{h} = \lim_{h\to 0^+}\frac{|h|-0}{h} = \lim_{h\to 0^+}\frac{h}{h} = 1$$

$h>0 \Rightarrow |h| = h$

Since the left and right hand limits are not equal, f is not differentiable at x = 3.

OR

$$y = a\left(\cos t + \log\tan\frac{t}{2}\right)$$

$$\Rightarrow \frac{dy}{dt} = a\left[\frac{d}{dt}\cos t + \frac{d}{dt}\left(\log\tan\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \cot\frac{t}{2}\times\sec^2\frac{t}{2}\times\frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right]$$

$$= a\left(-\sin t + \frac{1}{\sin t}\right) = a\left(\frac{-\sin^2 t + 1}{\sin t}\right) = a\frac{\cos^2 t}{\sin t}$$

x = a sin t

$$\frac{dx}{dt} = a\frac{d}{dt}\sin t = a\cos t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(a\frac{\cos^2 t}{\sin t}\right)}{a\cos t} = \frac{\cos t}{\sin t} = \cot t$$

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 t\frac{dt}{dx} = -\operatorname{cosec}^2 t\times\frac{1}{a\cos t} = -\frac{1}{a\sin^2 t\cos t}$$

17. $I=\int \frac{\sin\ \ x-a}{\sin\ \ x+a}dx$

Let (x + a) = t $\Rightarrow$ dx = dt

$\therefore I=\int \frac{\sin\ \ t-2a}{\sin t}dt$

$=\int \frac{\sin t\cos 2a-\cos t\sin 2a}{\sin t}dt$

$=\int\ \ \cos 2a-\cot t\sin 2a\ \ dt$

$=\ \ \cos 2a\ \ t-\sin 2a\ \log\left|\sin t\right|+C$

$=\cos 2a\ \ x+a\ \ -\sin 2a\ \log\left|\sin\ \ x+a\ \ \right|+C$

OR

$\int \frac{5x-2}{1+2x+3x^2}dx$

$=5\int \frac{x-\frac{2}{5}}{1+2x+3x^2}dx$

$=\frac{5}{6}\int \frac{6x-\frac{12}{5}}{1+2x+3x^2}dx$

$=\frac{5}{6}\int \frac{6x+2-\frac{12}{5}-2}{1+2x+3x^2}dx$

$=\frac{5}{6}\int \frac{6x+2-\frac{22}{5}}{1+2x+3x^2}dx$

$=\frac{5}{6}\int \frac{6x+2}{1+2x+3x^2}dx-\frac{5}{6}\times\frac{22}{5}\int \frac{1}{3\left\{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right\}}dx$

$=\frac{5}{6}\log\left|1+2x+3x^2\right|-\frac{11}{9}\int \frac{1}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}dx$

$=\frac{5}{6}\log\left|1+2x+3x^2\right|-\frac{11}{9}\times\frac{3}{\sqrt{2}}\tan^{-1}\frac{\left(x+\frac{1}{3}\right)}{\frac{\sqrt{2}}{3}}+C$

$=\frac{5}{6}\log\left|1+2x+3x^2\right|-\frac{11}{3\sqrt{2}}\times\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$

18. Let $x^2 = y$

$$\frac{x^2}{x^2+4 \quad x^2+9} = \frac{y}{y+4 \quad y+9} = \frac{A}{y+4} + \frac{B}{y+9}$$

$y = A(y+9) + B(y+4)$

Comparing both sides,

$A + B = 1$ and $9A + 4B = 0$

Solving, we get $A = \frac{-4}{5}$ and $B = \frac{9}{5}$

$$\therefore I = \int \left[\frac{-4}{5 \ x^2+4} + \frac{9}{5 \ x^2+9} \right] dx$$

$$= -\frac{4}{5} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$= -\frac{2}{5} \tan^{-1}\frac{x}{2} + \frac{3}{5} \tan^{-1}\frac{x}{3} + C$$

19. $\int_0^4 |x| + |x-2| + |x-4| dx$

$$= \int_0^2 f \ x \ dx + \int_2^4 f \ x \, dx$$

$$= \int_0^2 \ x - x + 2 - x + 4 \ dx + \int_2^4 \ x + x - 2 - x + 4 \ dx$$

$$= \int_0^2 \ 6 - x \ dx + \int_2^4 \ x + 2 \ dx$$

$$= \left[6x - \frac{x^2}{2} \right]_0^2 + \left[\frac{x^2}{2} + 2x \right]_2^4$$

$$= \ 12 - 2 \ + \ 8 + 8 - 2 - 4 \ = 20$$

20.

$$\left|\vec{a}+\vec{b}\right|=\left|\vec{a}\right|$$

$$\Rightarrow \left|\vec{a}+\vec{b}\right|^2=\left|\bar{a}\right|^2$$

$$\Rightarrow \left|\vec{a}\right|^2+2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2=\left|\vec{a}\right|^2$$

$$\Rightarrow 2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2=0 \qquad \text{... 1}$$

Now, $2\bar{a}+\vec{b}\cdot\vec{b}=2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}=2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2=0$

[Using (1)]

We know that if the dot product of two vectors is zero, then either of the two vectors is zero or the vectors are perpendicular to each other.

Thus, $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$.

21. The equation of the given line is $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ ....(1)

Any point on the given line is $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$.

If this point lies on the given plane $x - y + z - 5 = 0$, then

$3\lambda + 2 - (4\lambda - 1) + 2\lambda + 2 - 5 = 0$

$\Rightarrow \lambda = 0$

Putting $\lambda = 0$ in $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$, we get the point of intersection of the given line and the plane is (2, -1, 2).

Let θ be the angle between the given line and the plane.

$$\therefore \sin\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\frac{(3\vec{i}+4\vec{j}+2\vec{k}).(\vec{i}-\vec{j}+\vec{k})}{\sqrt{3^2+4^2+2^2}\sqrt{1^2+1^2+1^2}}=\frac{3-4+2}{\sqrt{29}\sqrt{3}}=\frac{1}{\sqrt{87}}$$

$$\Rightarrow \theta=\sin^{-1}\left(\frac{1}{\sqrt{87}}\right)$$

Thus, the angle between the given line and the given plane is $\sin^{-1}\left(\frac{1}{\sqrt{87}}\right)$.

OR

The equation of the given planes are

$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$ ... (1)

$\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ ... (2)

The equation of the plane passing through the intersection of the planes (1) and (2) is

$$\left[\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4\right]+\lambda\left[\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5\right]=0$$

$$\Rightarrow \vec{r}\cdot\left[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}\right]=4-5\lambda \quad ...(3)$$

Given that plane (3) is perpendicular to the plane $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

$$(1+2\lambda)\times5+(2+\lambda)\times3+(3-\lambda)\times(-6)=0$$

$$\Rightarrow 19\lambda-7=0$$

$$\Rightarrow \lambda=\frac{7}{19}$$

Putting $\lambda=\frac{7}{19}$ in (3), we get

$$\vec{r}\left[\left(1+\frac{14}{19}\right)\hat{i}+\left(2+\frac{7}{19}\right)\hat{j}+\left(3-\frac{7}{19}\right)\hat{k}\right]=4-\frac{35}{19}$$

$$\Rightarrow \vec{r}\cdot\left(\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right)=\frac{41}{19}$$

$\Rightarrow \vec{r}.(33\hat{i}+45\hat{j}+50\hat{k})=41$ .....This is the equation of the required plane.

22. Let the probability that A and B speak truth be P(A) and P(B) respectively.

Therefore, $P(A)=\frac{60}{100}=\frac{3}{5}$ and $P(B)=\frac{90}{100}=\frac{9}{10}$.

A and B can contradict in stating a fact when one is speaking the truth and other is not speaking the truth.

Case 1: A is speaking the truth and B is not speaking the truth.

Required probability = $P(A)\times(1-P(B))=\frac{3}{5}\times\left(1-\frac{9}{10}\right)=\frac{3}{50}$.

Case 2: A is not speaking the truth and B is speaking the truth.

Required probability = $(1-P(A))\times P(B)=\left(1-\frac{3}{5}\right)\times\frac{9}{10}=\frac{9}{25}$.

$\therefore$ Percentage of cases in which they are likely to contradict in stating the same fact =

$$\left(\frac{3}{50}+\frac{9}{25}\right)\times100\%=\left(\frac{3+18}{50}\right)\times100\%=42\%$$

From case 1, it is clear that it is not necessary that the statement of B will carry more weight as he speaks truth in more number of cases than A.

SECTION – C

23. Let the award money given for honesty, regularity and hard work be Rs. x, Rs. y and Rs. z respectively.

Since total cash award is Rs. 6,000.

$\therefore$ x + y + z = 6,000 ...(1)

Three times the award money for hard work and honesty amounts to Rs. 11,000.

$\therefore$ x + 3z = 11,000

$\Rightarrow$ x + 0×y + 3 z = 11,000 ...(2)

Award money for honesty and hard work is double that given for regularity.

$\therefore$ x + z = 2y

$\Rightarrow$ x – 2y + z = 0 ...(3)

The above system of equations can be written in matrix form AX = B as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$|A| = 1(0+6) - 1(1-3) + 1(-2-0) = 6 \neq 0$$

Thus, A is non-singular. Hence, it is invertible.

$$\text{Adj } A = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{ adj} A = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 36000 - 33000 + 0 \\ 12000 + 0 - 0 \\ -12000 + 33000 - 0 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

Hence, x = 500, y = 2000, and z = 3500.

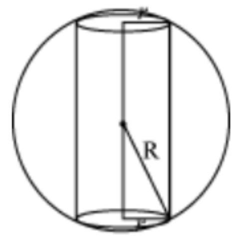
Thus, award money given for honesty, regularity and hard work is Rs. 500, Rs. 2000 and Rs. 3500 respectively.

The school can include awards for obedience.

24. Given, radius of the sphere is R.

Let r and h be the radius and the height of the inscribed cylinder respectively.

We have:



$h = 2\sqrt{R^2 - r^2}$

Let Volume of cylinder = V

$$V = \pi r^2 h$$

$$= \pi r^2 \times 2\sqrt{R^2 - r^2}$$

$$= 2\pi r^2 \sqrt{R^2 - r^2}$$

Differentiating the above function w.r.t. r, we have,

$$V = 2\pi r^2 \sqrt{R^2 - r^2}$$

$$\frac{dV}{dr} = 4\pi r \ \sqrt{R^2 - r^2} \ - \frac{4\pi r^3}{2\sqrt{R^2 - r^2}}$$

$$= \frac{4\pi r \ R^2 - r^2 \ - 4\pi r^3}{2\sqrt{R^2 - r^2}}$$

$$\frac{dV}{dr} = \frac{4\pi r R^2 - 4\pi r^3 - 2\pi r^3}{2\sqrt{R^2 - r^2}}$$

$$= \frac{4\pi r R^2 - 6\pi r^3}{2\sqrt{R^2 - r^2}}$$

For maxima or minima, $\frac{dV}{dr} = 0 \Rightarrow 4\pi r R^2 - 6\pi r^3 = 0$

$$\Rightarrow 6\pi r^3 = 4\pi r R^2$$

$$\Rightarrow r^2 = \frac{2R^2}{3}$$

$$\frac{dV}{dr} = \frac{4\pi r R^2 - 6\pi r^3}{2\sqrt{R^2 - r^2}}$$

Now, $\frac{d^2V}{dr^2} = \frac{1}{2}\left[\frac{\sqrt{R^2 - r^2} \ 4\pi R^2 - 18\pi r^2 \ - \ 4\pi r R^2 - 6\pi r^3 \ \frac{-2r}{2\sqrt{R^2 - r^2}}}{R^2 - r^2}\right]$

$$=\frac{1}{2}\left[\frac{R^2-r^2 \quad 4\pi R^2-18\pi r^2 \quad +r \quad 4\pi rR^2-6\pi r^3}{R^2-r^2 \quad ^{\frac{3}{2}}}\right]$$

$$=\frac{1}{2}\left[\frac{4\pi R^4-22\pi r^2R^2+12\pi r^4+4\pi r^2R^2}{R^2-r^2 \quad ^{\frac{3}{2}}}\right]$$

Now, when $r^2 = \frac{2R^2}{3}, \frac{d^2V}{dr^2}<0$.

$\therefore$ Volume is the maximum when $r^2 = \frac{2R^2}{3}$.

When $r^2=\frac{2R^2}{3}$, $h = 2\sqrt{R^2-\frac{2R^2}{3}}=2\sqrt{\frac{R^2}{3}}=\frac{2R}{\sqrt{3}}$.

Hence, the volume of the cylinder is the maximum when the height of the cylinder is $\frac{2R}{\sqrt{3}}$.

OR

The equation of the given curve is $x^2 = 4y$.

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{x}{2}$$

Let (h, k) be the co-ordinates of the point of contact of the normal to the curve $x^2 = 4y$.

Now, slope of the tangent at (h, k) is given by

$$\left.\frac{dy}{dx}\right]_{h,k} = \frac{h}{2}$$

Hence, slope of the normal at $(h, k) = \frac{-2}{h}$

Therefore, the equation of normal at (h, k) is

$$y - k = \frac{-2}{h}(x - h) \qquad \text{...(1)}$$

Since, it passes through the point (1, 2) we have

$$2 - k = \frac{-2}{h}(1 - h) \text{ or } k = 2 + \frac{2}{h}(1 - h) \qquad \text{...}(2)$$

Now, (h, k) lies on the curve $x^2 = 4y$, so, we have:

$h^2 = 4k$ ...(3)

Solving (2) and (3), we get,

h = 2 and k = 1.

From (1), the required equation of the normal is:

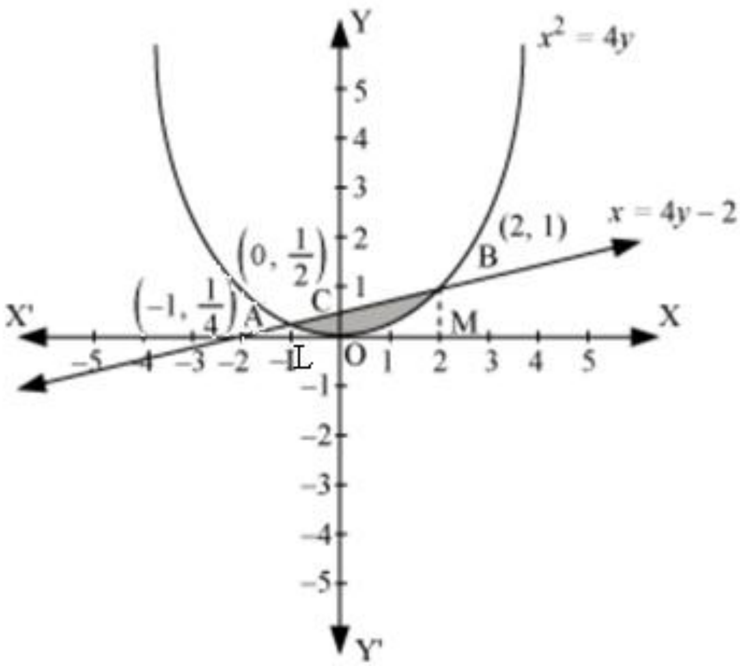
$$y - 1 = \frac{-2}{2}(x - 2) \text{ or } x + y = 3$$

Also, slope of the tangent = 1

$\therefore$ Equation of tangent at (1, 2) is:

y – 2 = 1(x – 1) or y = x + 1

25. The shaded area OBAO represents the area bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.



Let A and B be the points of intersection of the line and parabola.

Co-ordinates of point A are $\left(-1, \frac{1}{4}\right)$. Co-ordinates of point B are (2, 1).

Area OBAO = Area OBCO + Area OACO ... (1)

Area OBCO =

$$= \int_0^2 \frac{x+2}{4}dx - \int_0^2 \frac{x^2}{4}dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{4}\ 2 + 4\ - \frac{1}{4}\left[\frac{8}{3}\right]$$

$$= \frac{3}{2} - \frac{2}{3} = \frac{5}{6}$$

Area OACO =

$$=\int_{-1}^{0}\frac{x+2}{4}dx-\int_{-1}^{0}\frac{x^2}{4}dx$$

$$=\frac{1}{4}\left[\frac{x^2}{2}+2x\right]_{-1}^{0}-\frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{0}$$

$$=\frac{1}{4}\left[-\frac{-1^{\ 2}}{2}-2\ -1\ \right]-\frac{1}{4}\left[-\left(\frac{-1^{\ 3}}{3}\right)\right]$$

$$=\frac{1}{4}\left[-\frac{1}{2}+2\right]-\frac{1}{4}\left[\frac{1}{3}\right]$$

$$=\frac{3}{8}-\frac{1}{12}=\frac{7}{24}$$

Therefore, required area = $\left(\frac{5}{6}+\frac{7}{24}\right)=\frac{9}{8}$ sq. units

OR

Given equations of the circles are

$x^2+y^2=4$ ... 1

$x-2^{\ 2}+y^2=4$ ... 2

Equation (1) is a circle with centre O at the origin and radius 2. Equation (2) is a circle with centre C (2, 0) and radius 2.

Solving (1) and (2), we have:

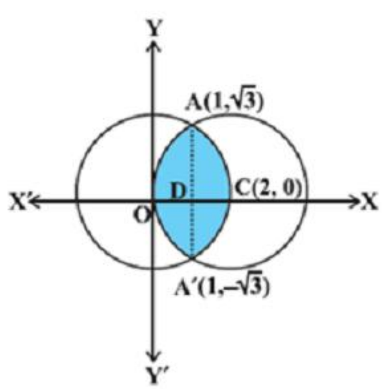
$(x - 2)^2 + y^2 = x^2 + y^2$

$x^2 - 4x + 4 + y^2 = x^2 + y^2$

x = 1

This gives y = $\pm\sqrt{3}$

Thus, the points of intersection of the given circles are $A\ 1,\sqrt{3}$ and $A'\ 1,-\sqrt{3}$ as shown in the figure.



Required area

= Area of the region OACA'O

= 2 [area of the region ODCAO]

= 2 [area of the region ODAO + area of the region DCAD]

$$=2\left[\int_0^1 y\,dx+\int_1^2 y\,dx\right]$$

$$=2\left[\int_0^1\sqrt{4-(x-2)^2}\,dx+\int_1^2\sqrt{4-x^2}\,dx\right]$$

$$=2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2}+\frac{1}{2}\times 4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1+2\left[\frac{1}{2}x\sqrt{4-x^2}+\frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2$$

$$=\left[(x-2)\sqrt{4-(x-2)^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^1+\left[x\sqrt{4-x^2}+4\sin^{-1}\frac{x}{2}\right]_1^2$$

$$=\left[\left(-\sqrt{3}+4\sin^{-1}\left(\frac{-1}{2}\right)\right)-4\sin^{-1}(-1)\right]+\left[4\sin^{-1}1-\sqrt{3}-4\sin^{-1}\frac{1}{2}\right]$$

$$=\left[\left(-\sqrt{3}-4\times\frac{\pi}{6}\right)+4\times\frac{\pi}{2}\right]+\left[4\times\frac{\pi}{2}-\sqrt{3}-4\times\frac{\pi}{6}\right]$$

$$=\frac{8\pi}{3}-2\sqrt{3}$$

26. $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$

$$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}} \qquad \ldots 1$$

$$\text{Let } F(x,y) = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}$$

$$\text{Then, } F(\lambda x, \lambda y) = \frac{\lambda\left(2xe^{\frac{x}{y}} - y\right)}{\lambda\left(2ye^{\frac{x}{y}}\right)} = \lambda^{o}\left[F(x,y)\right]$$

Thus, F(x, y) is a homogeneous function of degree zero. Therefore, the given differential equation is a homogeneous differential equation.

Let x = vy

Differentiating w.r.t. y, we get

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

Substituting the value of x and $\frac{dx}{dy}$ in equation (1), we get

$$v + y\frac{dv}{dy} = \frac{2vye^{v} - y}{2ye^{v}} = \frac{2ve^{v} - 1}{2e^{v}}$$

$$\text{or } y\frac{dv}{dy} = \frac{2ve^{v} - 1}{2e^{v}} - v$$

$$\text{or } y\frac{dv}{dy} = -\frac{1}{2e^{v}}$$

$$\text{or } 2e^{v}dv = \frac{-dy}{y}$$

$$\text{or } \int 2e^{v}.dv = -\int \frac{dy}{y}$$

or $2e^{v} = -\log|y| + C$

Substituting the value of v, we get

$$2e^{\frac{x}{y}} + \log|y| = C \qquad \ldots 2$$

Substituting x = 0 and y = 1 in equation (2), we get

$$2e^{o} + \log|1| = C \Rightarrow C = 2$$

Substituting the value of C in equation (2), we get

$2e^{\frac{x}{y}} + \log|y| = 2$, which is the particular solution of the given differential equation.

27. Let the position vectors of the three points be,

$\vec{a}=\hat{i}+\hat{j}-2\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}+\hat{k}$.

So, the equation of the plane passing through the points $\vec{a}, \vec{b}$ and $\vec{c}$ is

$$(\vec{r}-\vec{a}).\left[(\vec{b}-\vec{c}) \times (\vec{c}-\vec{a})\right]=0$$

$$\Rightarrow\left[\vec{r}-(\hat{i}+\hat{j}-2\hat{k})\right].\left[(\hat{i}-3\hat{j}) \times (\hat{j}+3\hat{k})\right]=0$$

$$\Rightarrow\left[\vec{r}-(\hat{i}+\hat{j}-2\hat{k})\right].(\hat{k}-3\hat{j}-9\hat{i})=0$$

$$\Rightarrow \vec{r}.(-9\hat{i}-3\hat{j}+\hat{k})+14=0$$

$$\Rightarrow \vec{r}.(9\hat{i}+3\hat{j}-\hat{k})=14 \qquad \dots(1)$$

So, the vector equation of the required plane is $\vec{r}.(9\hat{i}+3\hat{j}-\hat{k})=14$.

The equation of the given line is $\vec{r}=(3\hat{i}-\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$.

Position vector of any point on the given line is

$$\vec{r}=(3+2\lambda)\hat{i}+(-1-2\lambda)\hat{j}+(-1+\lambda)\hat{k} \qquad \dots(2)$$

The point (2) lies on plane (1) if,

$$\left[(3+2\lambda)\hat{i}+(-1-2\lambda)\hat{j}+(-1+\lambda)\hat{k}\right].(9\hat{i}+3\hat{j}-\hat{k})=14$$

$$\Rightarrow 9(3+2\lambda)+3(-1-2\lambda)-(-1+\lambda)=14$$

$$\Rightarrow 11\lambda+25=14$$

$$\Rightarrow \lambda=-1$$

Putting $\lambda=-1$ in (2), we have

$$\vec{r}=(3+2\lambda)\hat{i}+(-1-2\lambda)\hat{j}+(-1+\lambda)\hat{k}$$

$$=(3+2(-1))\hat{i}+(-1-2(-1))\hat{j}+(-1+(-1))\hat{k}$$

$$=\hat{i}+\hat{j}-2\hat{k}$$

Thus, the position vector of the point of intersection of the given line and plane (1) is $\hat{i}+\hat{j}-2\hat{k}$ and its co-ordinates are $(1, 1, -2)$.

28. Let the land allocated for crop A be x hectares and crop B be y hectares.
Maximum area of the land available for two crops is 50 hectares.
$\therefore x + y \leq 50$
Liquid herbicide to be used for crops A and B are at the rate of 20 litres and 10 litres per hectare respectively. Maximum amount of herbicide to be used is 800 litres.
$\therefore 20x + 10y \leq 800$
$\Rightarrow 2x + y \leq 80$
The profits from crops A and B per hectare are Rs 10,500 and Rs 9,000 respectively.
Thus, total profit = Rs (10,500x + 9,000y) = Rs 1500 (7x + 6y)
Thus, the linear programming problem is:
Maximize Z = 1500 (7x + 6y) subject to the constraints

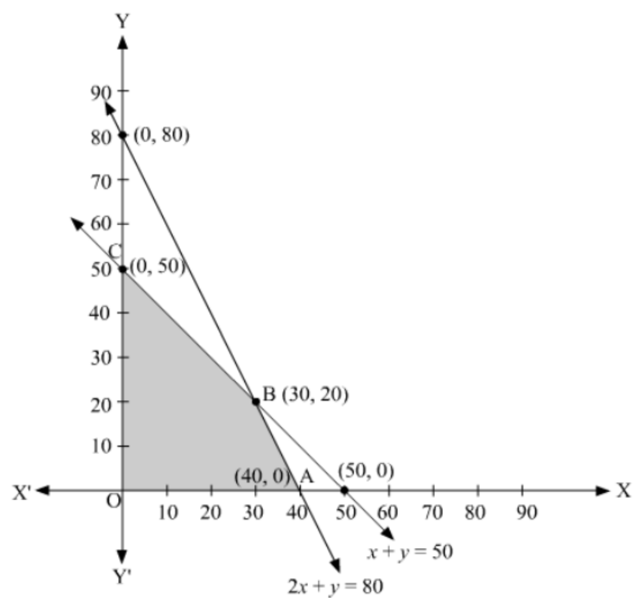
$x + y \leq 50$ ... 1

$2x + y \leq 80$ ... 2

$x \geq 0$ ... 3

$y \geq 0$ ... 4

The feasible region determined by constraints is represented by the shaded region in the following graph:



The corner points of the feasible region are O (0, 0), A (40, 0), B (30, 20) and C (0, 50).
The value of Z at these corner points are

Corner point	**Z = 1500 (7x + 6y)**	
O (0, 0)	0	
A (40, 0)	420000	
B (30, 20)	495000	Maximum
C (0, 50)	450000	

The maximum profit is at point B (30, 20).
Thus, 30 hectares of land should be allocated for crop A and 20 hectares of land should be allocated for crop B.
The maximum profit is Rs 495000.
Yes, the protection of wildlife is utmost necessary to preserve the balance in environment.

29. Let A, E_1, and E_2, respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$\therefore P(A) = 0.40$

$P(E_1) = P(E_2) = \frac{1}{2}$

$P(A|E_1) = 0.40 \times 0.70 = 0.28$

$P(A|E_2) = 0.40 \times 0.75 = 0.30$

Probability that the patient suffering a heart attack followed a course of meditation and yoga =

$P(E_1|A)$

$= \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$

$= \frac{\frac{1}{2} \times 0.28}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30}$

$= \frac{28}{28+30}$

$= \frac{28}{58}$

$= \frac{14}{29}$

Now, calculate $P(E_2|A)$.

$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$

$= \frac{\frac{1}{2} \times 0.30}{\frac{1}{2} \times 0.28 + \frac{1}{2} \times 0.30}$

$= \frac{30}{28+30}$

$= \frac{30}{58}$

$= \frac{15}{29}$

Since $P(E_1|A) < P(E_2|A)$, the course of yoga and meditation is more beneficial for a patient.