# 13. Variation

## Let us Work Out 13

### 1. Question

Corresponding values of two variables A and B are :

| A | 25 | 30 | 45 | 250 |
|---|---|---|---|---|
| B | 10 | 12 | 18 | 100 |

If there is any relation of variation between A and B, let us determine it and write the value of variation constant.

**Answer**

Given:

| A | 25 | 30 | 45 | 250 |
|---|---|---|---|---|
| B | 10 | 12 | 18 | 100 |

We see in the given table, as the value of A increases or decreases the corresponding value of B is also increasing or decreasing.

$$\text{again,} \frac{A}{B} = \frac{25}{10} = \frac{30}{12} = \frac{45}{18} = \frac{250}{100} = \frac{5}{2}$$

$$\text{i.e.} A = \frac{5}{2}B$$

Hence, A ∝ B and here value of variation constant is $\frac{5}{2}$.

Here, the value of variation constant is positive. i.e. for value of A increases or decreases the corresponding value of B will also increases or decreases.

### 2. Question

Corresponding values of two variables x and y

| x | 18 | 8 | 12 | 6 |
|---|---|---|---|---|
| y | 3 | $\frac{27}{4}$ | $\frac{9}{2}$ | 9 |

If there is any relation of variation between x and y, let us write it by understanding.

**Answer**

Given:

| x | 18 | 8 | 12 | 6 |
|---|---|---|---|---|
| y | 3 | $\frac{27}{4}$ | $\frac{9}{2}$ | 9 |

We see in the given table, as the value of x increases or decreases the corresponding value of y is also decreasing or increasing respectively.

$$\text{again}, x \times y = 18 \times 3 = 8 \times \frac{27}{4} = 12 \times \frac{9}{2} = 6 \times 9 = 54$$

i.e. xy = 54 = constant

Hence, $A \propto \frac{1}{B}$ and here value of variation constant is 54.

Here, the value of variation constant is positive. i.e. for value of A increases or decreases the corresponding value of B will also decreases or increases respectively.

**3 A. Question**

A taxi of Bipin uncle travels 14 km path in 25 minute. Let us calculate by applying theory of variation how much path he will go in 5 hours by driving taxi with same speed.

**Answer**

Given:$x_1$ = 14km and $t_1$ = 25min.

Let the distance cover = x

Time taken = t

$2^{nd}$ time $t_2$ = 5hrs = 5× 60 = 300min.

$2^{nd}$ distance = $x_2$

$$\text{as, speed} = \frac{\text{distance}}{\text{time}}$$

As speed is constant,

So, x ∝ t

$$\text{i.e.} \frac{x}{t} = \frac{14}{25} = \frac{x_2}{300}$$

⇒ 25 × $x_2$ = 14 × 300

$$\Rightarrow x_2 = \frac{14 \times 300}{25}$$

$\Rightarrow x_2 = 14\times 12$

$\Rightarrow x_2 = 168$ km

Hence it will cover 168km in 300min or 5hrs.

**3 B. Question**

A box of sweets is divided among 24 children of class one of our school, they will get 5 sweets each. Let us calculate by applying theory of variation how many sweets would each get, if the number of the children is reduced by 4.

**Answer**

Given: number of children (x) = 24

Number of sweets each get (y) = 5

New number of children ($x_1$) = 24 – 4 = 20

Let new Number of sweets each get = $y_1$

As per given condition if the number of students reduced then each will get more sweets than previously given. Hence, it is the question of indirect proportion.

i.e. $xy = x_1y_1$

or $24\times5 = 20 \times y_1$

$$\Rightarrow y_1 = \frac{24\times 5}{20}$$

$\Rightarrow y_1 = 6$

Hence, each children will get 6 sweets.

**3 C. Question**

50 villagers had taken 18 days to dig a pond. Let us calculate by using theory of variation how many extra persons will be required to dig the pond is 15 days.

**Answer**

Given: number of villagers (x) = 50

Number of days to dig a pond (y) = 18

Let New number of villagers ($x_1$) = ($x_1$)

New Number of days to dig = 15

As per given condition if the number of days reduced then number of villagers will increases than the previously given. Hence, it is the question of indirect proportion.

i.e. $xy = x_1y_1$

or $50 \times 18 = 15 \times x_1$

$$\Rightarrow x_1 = \frac{50 \times 18}{15}$$

$\Rightarrow x_1 = 60$ days

Hence, 15 villagers will need 60 days.

**4 A. Question**

y varies directly with square root of x and y = 9 when x = 9. Let us find the value of x when y = 6.

**Answer**

Given: $y \propto \sqrt{x}$

$y = k\sqrt{x}$

Acc. To given values y = 9 and x = 9,

$9 = k\sqrt{9}$

$\Rightarrow 9 = k\,(3)$

$\Rightarrow k = 3$

So, $y = 3\sqrt{x}$ ...(1)

Put y = 6 in (1)

$\Rightarrow 6 = 3\sqrt{x}$

$\Rightarrow \sqrt{x} = 2$

Square on both sides,

x = 4.

**4 B. Question**

x varies directly with y and inversely with z. When y=4, z=5, then x=3. Again, if y=16, z = 30, let us write by calculating the value of x.

**Answer**

Given: $x \propto \frac{y}{z}$

or $x = k\frac{y}{z}$

Acc. To given values y = 4, z = 5 and x = 3,

$$3 = k\frac{4}{5}$$

$$\Rightarrow k = 3 \times \frac{5}{4}$$

$$\Rightarrow k = \frac{15}{4}$$

So, $x = \frac{15}{4} \times \frac{y}{z}$ ...(1)

Put y = 16 and z = 30 in (1)

$$\Rightarrow x = \frac{15}{4} \times \frac{16}{30}$$

⇒ x = 2

**4 C. Question**

x varies directly with y and inversely with z. When y=5, z=9 then $x = \frac{1}{6}$. Let us find the relation among three variables x, y and z and if y = 6 and $z = \frac{1}{5}$, let us write by calculating the value of x.

**Answer**

Given: $x \propto \frac{y}{z}$

or $x = k\frac{y}{z}$

Acc. To given values y = 5, z = 9 and x = 1/6,

$$\frac{1}{6} = k\frac{5}{9}$$

$$\Rightarrow k = \frac{1}{6} \times \frac{9}{5}$$

$$\Rightarrow k = \frac{9}{30} = \frac{3}{10}$$

So, $x = \frac{3}{10} \times \frac{y}{z}$ ...(1)

Put y = 6 and z = 1/5 in (1)

$$\Rightarrow x = \frac{3}{10} \times \frac{6}{\frac{1}{5}}$$

$$\Rightarrow x = \frac{3}{10} \times 30$$

$\Rightarrow x = 9$

**5 A. Question**

If $x \propto y$, let us show that $x + y \propto x - y$.

**Answer**

Given:

$x \propto y$

$x = ky$

$$\frac{x}{y} = k$$

Do componendo and dividend,

$$\frac{x}{y} = \frac{k}{1}$$

$$\frac{x + y}{x - y} = \frac{k + 1}{k - 1} = \text{non zero constant}$$

$$\Rightarrow \frac{x + y}{x - y} = k_1$$

i.e. $x + y \propto x - y$

hence proved.

**5 B. Question**

$A \propto \frac{1}{C}, C \propto \frac{1}{B}$ let us show that $A \propto B$.

**Answer**

Given:

$A \propto \frac{1}{C}$ and $C \propto \frac{1}{B}$

It can be written as,

$$A = k\frac{1}{C} \text{ and } C = m\frac{1}{B}$$

Or

$$C = k\frac{1}{A} \text{ and } C = m\frac{1}{B}$$

$$\Rightarrow C = k\frac{1}{A} = m\frac{1}{B}$$

$$\Rightarrow \frac{k}{A} = \frac{m}{B}$$

$$\Rightarrow \frac{A}{k} = \frac{B}{m}$$

$$\Rightarrow A = \frac{B}{m}.k$$

$$\Rightarrow A = \frac{k}{m}.B$$

i.e. A = constant B

Or $A \propto B$

Hence proved.

**5 C. Question**

If $a \propto b, b \propto \frac{1}{c}$ and $c \propto d$, let us write the relation of variation between a and d.

**Answer**

Given:

$$a \propto b, b \propto \frac{1}{c} \text{ and } c \propto d$$

It can be written as,

$$a = kb, b = m\frac{1}{c} \text{ and } c = nd$$

Put the value of b in a,

$$a = \frac{km}{c} \text{ and } c = nd$$

Now, put the value of c in a,

We get,

$$a = \frac{km}{nd}$$

$$\Rightarrow a = \frac{\text{constant}}{d}$$

$$\Rightarrow a \propto \frac{1}{d}$$

This is the required relation between a and d.

**5 D. Question**

If $x \propto y$, $y \propto z$ and $z \propto x$. Let us find the relation among three constants of variation.

**Answer**

Given: $x \propto y$ , $y \propto z$ and $z \propto x$

It can be written as,

$x = k\,y$, $y = m\,z$ and $z = n\,x$

where k, m and n are constant of variations.

Put the value of y in x,

We get,

$x = k\,m\,z$

Now put the value of z in x,

$\Rightarrow x = k\,m\,n\,x$

$\Rightarrow k\,m\,n = 1$

This is the required relation among three constants of variation i.e. k, m and n.

**6 A. Question**

If $x + y \propto x - y$, let us show that

$x^2 + y^2 \propto xy$

**Answer**

Given: $x + y \propto x - y$

It can be written as,

$x + y = k(x - y)$

$$\frac{x + y}{x - y} = k$$

Square on both sides,

$$\frac{(x + y)^2}{(x - y)^2} = k^2$$

$$\frac{x^2 + y^2 + 2xy}{x^2 + y^2 - 2xy} = k = \text{constant}$$

$\Rightarrow x^2 + y^2 + 2xy = k(x^2 + y^2 + 2xy)$

$\Rightarrow x^2 + y^2 + 2xy = kx^2 + ky^2 + k2xy$

$\Rightarrow x^2 - kx^2 + y^2 - ky^2 = k2xy - 2xy$

$\Rightarrow x^2 (1-k) + y^2 (1-k) = 2xy (k-1)$

$\Rightarrow (1-k) \{x^2 + y^2\} = -2xy(1-k)$

$\Rightarrow x^2 + y^2 = -2xy$

As -2 is also a constant term.

Hence,

$\Rightarrow x^2 + y^2 \propto xy$

Hence proved.

**6 B. Question**

If $x + y \propto x - y$, let us show that

$x^3 + y^3 \propto x^3 - y^3$

**Answer**

Given: $x + y \propto x - y$

It can be written as,

$x + y = k(x - y)$

$$\frac{x + y}{x - y} = k$$

Apply componendo dividend,

$$\frac{x + y + x - y}{x + y - x + y} = \frac{k + 1}{k - 1} = \text{constant}$$

$$\frac{2x}{2y} = k_1$$

$$\frac{x}{y} = k_1$$

Cube on both sides,

$$\frac{x^3}{y^3} = (k_1)^3 = \text{constant}$$

Again apply componendo dividend,

$$\frac{x^3 + y^3}{x^3 - y^3} = k_2$$

i.e. $x^3 + y^3 \propto x^3 - y^3$

Hence,

$x^3 + y^3 \propto x^3 - y^3$

Hence proved.

**6 C. Question**

If $x + y \propto x - y$, let us show that

$ax + by \propto px + qy$

[where a, b, p, q are non zero constant]

**Answer**

Given: $x + y \propto x - y$

$ax + by \propto px + qy$

**7 A. Question**

If $a^2 + b^2 \propto ab$, let us prove that $a + b \propto a - b$.

**Answer**

Given: $a^2 + b^2 \propto ab$ to prove $a + b \propto a-b$

$a^2 + b^2 = 2kab$

as 2k is a constant.

It can be written as:

$$\frac{a^2 + b^2}{2ab} = k$$

Apply componendo dividend,

$$\frac{a^2 + b^2 + 2ab}{a^2 + b^2 - 2ab} = \frac{k + 1}{k - 1} = \text{constant}$$

$$\frac{(a + b)^2}{(a - b)^2} = k_1$$

Square root on both sides,

$$\frac{a + b}{a - b} = \sqrt{k_1}$$

i.e a + b ∝ a-b

hence proved.

**7 B. Question**

If $x^3 + y^3 \propto x^3 - y^3$, let us prove that $x + y \propto x - y$.

**Answer**

Given:

$x^3 + y^3 \propto x^3 - y^3$

$$x^3 + y^3 = k(x^3 - y^3)$$

$$\frac{x^3 + y^3}{x^3 - y^3} = k$$

Apply componendo dividend,

$$\frac{x^3 + y^3 + x^3 - y^3}{x^3 + y^3 - x^3 + y^3} = \frac{k + 1}{k - 1} = \text{constant}$$

$$\frac{2x^3}{2y^3} = k_1$$

$$\frac{x^3}{y^3} = k_1$$

Cube root on both sides,

$$\frac{x}{y} = \sqrt[3]{k_1} = \text{constant}$$

$$\frac{x}{y} = \frac{k_2}{1}$$

Again apply componendo dividend,

$$\frac{x + y}{x - y} = \frac{k_2 + 1}{k_2 - 1} = \text{constant}$$

$$\frac{x + y}{x - y} = k_3$$

i.e. $x + y \propto x-y$

**8. Question**

If 15 farmers can cultivate 18 bighas of land in 5 days, let us determine by using theory of variation the number of days required by 10 farmers to cultivate 12 bighas of land.

**Answer**

Given:

More will be the number of farmers, more bighas of land will be cultivated i.e. direct proportion.

More will be farmers, less number of days will be needed i.e. indirect proportion.

$$\text{Farmers (F)} \propto \frac{\text{bighas of land(B)}}{\text{days (D)}}$$

$$\text{i.e.}\, F \propto \frac{B}{D}$$

$$F = k\frac{B}{D}$$

Acc. To given values,

$$15 = k\frac{18}{5}$$

$$\Rightarrow k = \frac{75}{18} = \frac{25}{6}$$

For, F = 10, Bighas of land = 12

$$10 = \frac{25}{6} \times \frac{12}{D}$$

$$\Rightarrow D = \frac{25}{6} \times \frac{12}{10}$$

$\Rightarrow$ D = 5days

**9. Question**

Volume of a sphere varies directly with the cube of its radius. Three solid spheres having length of $1\frac{1}{2}, 2$ and $2\frac{1}{2}$ metre diameter are melted and a new solid sphere is formed. Let us find the length of diameter of the new sphere. [let us consider that the volume of sphere remains same before and after melting]

**Answer**

Given:

Radius of first sphere = $\frac{\text{Diameter}}{2} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$

Radius of second sphere = $\frac{\text{Diameter}}{2} = \frac{2}{2} = 1$

Radius of third sphere = $\frac{\text{Diameter}}{2} = \frac{\frac{5}{2}}{2} = \frac{5}{4}$

Let volume is v and radius is r

Acc . to given condition:

$V \propto r^3$

$V = kr^3$

Volume of sphere of radius 3/4 m is = $k\left(\frac{3}{4}\right)^3 = \frac{27k}{64}$

Volume of sphere of radius 1 m is = $k\,(1)^3 = 1k$

Volume of sphere of radius 5/4m is = $k\left(\frac{5}{4}\right)^3 = \frac{125k}{64}$

Total volume will be : $\frac{27k}{64} + k + \frac{125k}{64}$

$$= \frac{27k + 64k + 125k}{64} = \frac{216k}{64}$$

If the radius of new sphere is R

$$k\,(R)^3 = \frac{216k}{64}$$

$$(R)^3 = \frac{216}{64}$$

$$R = \frac{6}{4} = \frac{3}{2}$$

Diameter of new sphere will be = 2R = 3 m

**10. Question**

y is a sum of two variables, one of which varies directly with x and another varies inversely with x. With x = - 1, then y = 1 and when x = 3, then y = 5. Let us find the relation between x and y.

**Answer**

Given:

y = a + b

Acc. To given condition:

$a \propto x$

a = kx

and $b \propto \frac{1}{x}$

$$b = m\frac{1}{x}$$

Put the values of a and b in y.

$$y = kx + m\frac{1}{x}$$

x = -1 then y = 1

$$1 = k(-1) + m\frac{1}{(-1)}$$

⇒ 1 = -k – m ....(1)

x = 3 then y = 5

$$5 = k(3) + m\frac{1}{(3)}$$

$$5 = \frac{9k + 3m}{3}$$

⇒ 15 = 9k + 3m ...(2)

Multiply (1) by 3

We get,

3 = -3k – 3m ...(3)

Add (2) and (3)

18 = 6k

⇒ k = 3

Put it in (1)

1 = -3 –m

⇒ m = -4

$$y = (3)x + (-4)\frac{1}{x}$$

$$y = 3x - \frac{4}{x}$$

This is the required relation between x and y.

**11. Question**

If a ∝ b, b ∝ c let us show that $a^3b^3 + b^3c^3 + c^3a^3 \propto abc\,(a^3 + b^3 + c^3)$.

**Answer**

Given: a ∝ b, b ∝ c

a = kb and b = mc

it can be written as:

a = kb = kmc and b = mc

To prove: $a^3b^3 + b^3c^3 + c^3a^3 \propto abc(a^3 + b^3 + c^3)$

$$\frac{a^3b^3 + b^3c^3 + c^3a^3}{abc(a^3 + b^3 + c^3)} = \frac{(kmc)^3(mc)^3 + (mc)^3c^3 + c^3(kmc)^3}{(kmc)(mc)c((kmc)^3 + (mc)^3 + c^3)}$$

$$= \frac{(km)^3(m)^3c^6 + (m)^3c^6 + c^6(km)^3}{(km)(m)c^3.c^3((km)^3 + (m)^3 + 1)}$$

$$= \frac{c^6\{(km)^3(m)^3 + (m)^3 + (km)^3\}}{c^6\{(km)(m)((km)^3 + (m)^3 + 1)\}}$$

$$= \frac{\{(km)^3(m)^3 + (m)^3 + (km)^3\}}{\{(km)(m)((km)^3 + (m)^3 + 1)\}} = \text{non zero constant}$$

Hence, $a^3b^3 + b^3c^3 + c^3a^3 \propto abc(a^3 + b^3 + c^3)$

**12. Question**

To dig a well of x dcm deep. One part of the total expenses varies directly with x and other part varies directly with $x^2$. If the expenses of digging wells of 100 dcm and 200 dcm depths are ₹ 5000 and ₹ 12000 respectively, let us write by calculating the expenses of digging a well of 250 dcm depth.

**Answer**

Given:

Acc. To given condition:

First part of expense $(t_1) \propto x$

Where x is the depth of well.

$t_1 = kx$

second part of expense $(t_2) \propto x^2$

$t_2 = mx^2$

total expense $(t) = t_1 + t_2$

It can be written as:

$t = k\,x + m\,x^2$

First depth = 100 dcm and first expense = Rs.5000

$5000 = k\,100 + m(100)^2)$

$5000 = 100k + 10000m$ ...(1)

First depth = 200 dcm and first expense = Rs.12000

$12000 = k\,200 + m(200)^2)$

$12000 = 200k + 40000m$ ...(2)

Multiply (1) by 2 :

$10000 = 200k + 20000m$ .....(3)

Subtract (3) from (2)

$2000 = 20000m$

$$\Rightarrow m = \frac{1}{10}$$

Put m in (1)

$$5000 = 100k + 10000\left(\frac{1}{10}\right)$$

$5000 = 100k + 1000$

$100k = 5000 - 1000$

$100k = 4000$

$\Rightarrow k = 40$

$$\text{so}, t = 40x + \frac{1}{10}x^2$$

For x = 250 dcm

$$t = 40(250) + \frac{1}{10}(250)^2$$

$$t = 10000 + \frac{1}{10} \times 62500$$

$\Rightarrow t = 10000 + 6250$

t = Rs16250

So total expene will be Rs. 16250

**13. Question**

Volume of a cylinder is in joint variation with square of the length of radius of base and its height, Ratio of radii of bases of two cylinders is 2 : 3 and ratio of their heights is 5 : 4, let us find the ratio of their volumes.

**Answer**

Given:

Acc. To given condition:

Volume of cylinder $\propto$ (radius)$^2$(height)

$V \propto (r)^2(h)$

Radius of two cylinders = $r_1 : r_2 = 2 : 3$

Heights of two cylinders = $h_1 : h_2 = 5 : 4$

Volume of two cylinders = $v_1 : v_2$

$$\frac{v_1}{v_2} = \frac{(r_1)^2}{(r_2)^2} \times \frac{h_1}{h_2}$$

$$\frac{v_1}{v_2} = \frac{(2)^2}{(3)^2} \times \frac{5}{4}$$

$$\frac{v_1}{v_2} = \frac{4}{9} \times \frac{5}{4}$$

$$\frac{v_1}{v_2} = \frac{5}{9}$$

Then, the ratio of their volumes = 5 : 9.

**14. Question**

An agricultural Co-operative Society of village of Pachla has purchased a tractor. Previously 2400 bighas of land were cultivated by 25 ploughs in 36 days. Now half of the land can be cultivated only by that-tractor in 30 days. Let us calculate by using the theory of variation, the number of ploughs work equally with one tractor.

**Answer**

Given:

More will be the number of ploughs, more bighas of land will be cultivated i.e. direct proportion.

More will be ploughs, less number of days will be needed i.e. indirect proportion.

$$\text{ploughs (P)} \propto \frac{\text{bighas of land(B)}}{\text{days (D)}}$$

$$\text{i. e. } P \propto \frac{B}{D}$$

$$P = k\frac{B}{D}$$

Acc. To given values,

$$25 = k\frac{2400}{36}$$

$$\Rightarrow k = \frac{25 \times 36}{2400}$$

$$\Rightarrow k = \frac{900}{2400}$$

$$\Rightarrow k = \frac{3}{8}$$

$$P = \frac{3}{8} \times \frac{B}{D}$$

For, D = 30, Bighas of land = 1200

$$P = \frac{3}{8} \times \frac{1200}{30}$$

$$\Rightarrow P = 15$$

So, 15 ploughs are needed for 1200 bighas of land in 30 days.

**15. Question**

Volume of a sphere varies directly with cube of length of its radius and surface area of sphere varies directly with the square of the length of radius. Let us prove that the square of volume of sphere varies directly with cube of its surface area.

**Answer**

Given:

Acc. To given conditions:

Volume of sphere ∝ $(\text{radius})^3$

$V \propto (r)^3$

$V = k\,(r)^3$ ....(1)

Surface area of sphere ∝ $(\text{radius})^2$

$S \propto (r)^2$

$S = m\,(r)^2$

$$\Rightarrow r^2 = \frac{s}{m}$$

$$\Rightarrow r = \sqrt{\frac{s}{m}}$$

Put the value of r in (1)

$$\Rightarrow v = k\left(\sqrt{\frac{s}{m}}\right)^3$$

$$\Rightarrow v = k\left(\frac{s}{m}\right)^{\frac{3}{2}}$$

Square on both sides,

$$\Rightarrow v^2 = k^2\left(\frac{s}{m}\right)^{\frac{3}{2}\times 2}$$

$$\Rightarrow v^2 = k^2\left(\frac{s}{m}\right)^3$$

Where k and m are non zero constants.

So,

$v^2 \propto s^3$

hence proved.

**16 A1. Question**

$x \propto \frac{1}{y}$, then

A. $x = \frac{1}{y}$

B. $y = \frac{1}{x}$

C. xy = 1

D. xy = non zero constant.

**Answer**

Given:

$$x \propto \frac{1}{y}$$

$$x = k\frac{1}{y}$$

$\Rightarrow$ xy = k = non zero constant

Option (d) is correct.

**16 A2. Question**

If $x \propto y$ then

A. $x^2 \propto y^3$

B. $x^3 \propto y^2$

C. $x \propto y^3$

D. $x^2 \propto y^2$

**Answer**

Given:

$x \propto y$

it can written as,

$x = ky$

square on both sides,

$x^2 = k^2 y^2$

as $k^2$ is also a constant.

$x^2 = k y^2$

or

$x^2 \propto y^2$

option (D) is correct.

**16 A3. Question**

If $x \propto y$ and $y = 8$ when $x = 2$; if $y = 16$, then the value of x if

A. 2

B. 4

C. 6

D. 8

**Answer**

Given:

$x \propto y$

it can written as,

$x = ky$

$y = 8$ and $x = 2$

$2 = k (8)$

$\Rightarrow k = \frac{1}{4}$

$\Rightarrow x = \frac{1}{4} y$

When y = 16

$$\Rightarrow x = \frac{1}{4} \times 16$$

⇒ x = 4

Option (b) is correct.

**16 A4. Question**

If x ∝ $y^2$ and y = 4 when x = 8; if x = 32, then the value of y is

A. 4

B. 8

C. 16

D. 32

**Answer**

Given:

x ∝ $y^2$

it can written as,

x = $ky^2$

y = 4 and x = 8 (given)

8 = k $(4)^2$

$$\Rightarrow k = \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{2}y^2$$

When x = 32,

$$\Rightarrow 32 = \frac{1}{2}y^2$$

⇒ $y^2$ = 64

⇒ y = 8

Option (b) is correct.

**16 A5. Question**

If $y - z \propto \frac{1}{x}, z - x \propto \frac{1}{y}$ and $x - y \propto \frac{1}{z}$, sum of three variation constants is

A. 0

B. 1

C. -1

D. 2

**Answer**

Given:

$$y - z \propto \frac{1}{x}, z - x \propto \frac{1}{y} \text{ and } x - y \propto \frac{1}{z}$$

$$y - z = k\frac{1}{x}..eq(1)$$

$$z - x = m\frac{1}{y}...eq(2)$$

$$x - y = n\frac{1}{z}..eq(3)$$

Where k, m and n are constants,

Adding all the three equations we get,

$$y\text{-}z + z - x + x - y = \left(\frac{k}{x} + \frac{m}{y} + \frac{n}{z}\right)$$

$$\Rightarrow 0 = \left(\frac{k}{x} + \frac{m}{y} + \frac{n}{z}\right)$$

Now, k, m, and n are also constants.

Then, the sum of these variation constants will also be 0

**16 B. Question**

Let us write whether the following statements are true or false :

i. If $y \propto \frac{1}{x}, \frac{y}{x} =$non-zero constant.

ii. If $x \propto z$ and $y \propto z$ then $y \propto z$

**Answer**

(i) Given:

$y \propto \frac{1}{x}$

$y = k\frac{1}{x}$

$\Rightarrow xy = k =$ non zero constant

Hence, given statement is not true.

(ii) Given: $x \propto z$ , $y \propto z$

It can be written as,

$x = k\,y$, $y = m\,z$

where k, m are constant of variations.

Put the value of y in x,

We get,

$x = k\,m\,z$

$\Rightarrow x \propto z$

Multiply both sides by y.

We get,

$xy \propto yz$

Hence, given statement is not true.

**16 C. Question**

Let us fill in the blanks :

i. If $x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$, then $x\propto$ ______

ii. If $x \propto y$, $x^n\propto$ _______

iii. If $x \propto y$ and $x \propto z$, then $(y + z)\propto$ _______

**Answer**

(i) Given:

$x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$

$x = k\frac{1}{y}$ and $y = m\frac{1}{z}$

Put the value of y in x.

$$x = k\frac{1}{\left(\frac{m}{z}\right)}$$

$$x = k\frac{z}{(m)}$$

As k and m are constant.

$\Rightarrow x \propto z$

Hence, If $x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$, then $x \propto$ _z.

(ii) Given: $x \propto y$

$x = ky$

$x^n = k^n.y^n$

where $k^n$ is a constant.

Hence $x^n \propto y^n$

Hence, If $x \propto y, x^n \propto$ _$y^n$_

(iii) Given: $x \propto y$ and $x \propto z$

$x = ky$ and $x = mz$

$$\Rightarrow y = \frac{x}{k} \text{ and } z = \frac{x}{m}$$

Hence, y + z = ?

$$\Rightarrow y + z = \frac{x}{k} + \frac{x}{m}$$

$$\Rightarrow y + z = x\left(\frac{1}{k} + \frac{1}{m}\right)$$

$\left(\frac{1}{k} + \frac{1}{m}\right)$ is a constant

So, $y + z \propto x$

If $x \propto y$ and $x \propto z$, then $(y + z) \propto$ _x_

**17 A. Question**

If $x \propto y^2$ and y = 2a when x = a; x and y let us find the relation between x and y.

**Answer**

Given:

$x \propto y^2$

$x = ky^2$ ……(1)

when x = a then y = 2a

put the value of x and y in equ.(1)

we get,

$a = k\,(2a)^2$

$a = 4a^2k$

$\Rightarrow k = \frac{1}{4a}$

Put the value of k in (1)

$x = \frac{1}{4a}y^2$

This is the required relation between x and y.

**17 B. Question**

If $x \propto y$, $y \propto z$ and $z \propto x$, let us find the product of three non zero constants.

**Answer**

Given: $x \propto y$ , $y \propto z$ and $z \propto x$

It can be written as,

x = k y, y = m z and z = n x

where k, m and n are constant of variations.

Put the value of y in x,

We get,

x = k m z

Now put the value of z in x,

$\Rightarrow$ x = k m n x

$\Rightarrow$ k m n = 1

Thus the product of three non zero constants is 1.

**17 C. Question**

If $x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$, let us find if there be any relation of direct or inverse variation between x and z.

**Answer**

Given:

$x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$

$x = k\frac{1}{y}$ and $y = m\frac{1}{z}$

Put the value of y in x.

$x = k\frac{1}{\left(\frac{m}{z}\right)}$

$x = k\frac{z}{(m)}$

As k and m are constant.

$\Rightarrow x \propto z$

So, x and z are in direct variation.

**17 D. Question**

If $x \propto yz$ and $y \propto zx$ let us show that z is a non zero constant.

**Answer**

Given: $x \propto yz$ and $y \propto zx$

$x = kyz$

$y = mzx$

Put the value of y in x.

$x = k(mzx)z$

$x = kmxz^2$

$$\Rightarrow z^2 = \frac{1}{km}$$

$$z = \sqrt{\frac{1}{km}}$$

As k and m are constant.

$\Rightarrow z \propto$ non zero constant

Hence proved.

**17 E. Question**

If $b \propto a^3$ and a increases in the ratio of 2 : 3, let us find in what ratio b will be increased.

**Answer**

Given: $b \propto a^3$

$b = ka^3$

$$\frac{b_1}{b_2} = \frac{k}{k}\frac{a_1{}^3}{a_2{}^3}$$

$$\frac{b_1}{b_2} = \frac{(2)^3}{(3)^3}$$

$$\frac{b_1}{b_2} = \frac{8}{27}$$

Hence, b must increase in ratio 8 : 27.