CBSE Class 11 Mathematics

Important Questions

Chapter 14

Mathematical Reasoning

4 Marks Questions

1. Give three examples of sentences which are not statements. Give reasons for the answers.

Ans. (i) The sentence "Rani is a beautiful girl" is not a statement. To some Rani may look beautiful and to other she may not look beautiful. We cannot say on logic whether or not this sentence is true.

(ii) The sentence 'shut the door' is not a statement. It is only an imperative sentence giving a direction to someone. There is no question of it being true or false.

(iii) The sentence 'yesterday was Friday' is not a statement. It is an ambiguous sentence which is true if spoken on Saturday and false if spoken on other days. Truth or false hood of the sentence depends on the time at which it is spoken and not on mathematical reasoning.

2. Write the negation of the following statements

(i) Chennai is the capital of Tamil Nadu.

(ii) Every natural number is an integer.

Ans. (i) Chennai is not the capital of Tamil Nadu.

(ii) Every natural number is not an integer.

3. Find the component statements of the following compound statements and check whether they are true or false.

(i) The number 3 is prime or it is odd.

Ans. The component statements of the given statement are

p: “The number 3 is prime”

q: “number 3 is odd”

There two have been connected by using the connective “or”

The given statement is true as both the statements are true.

4. Check whether the following pair of statements are negations of each other Give reasons for your answer.

(i) $x+y=y+x$ **is true for every real numbers** x **and** y**.**

(ii) There exists real numbers x **and** y **for which** $x+y=y+x$**.**

Ans. The given statements are

$p:$" $x+y=y+x$ is true for every real number x and y"

$q:$ " There exists real numbers x and y for which $x+y=y+x$."

These statements are not negations of each other as they can be true at the same time. Infact, negation of p is

$\sim p:$" There are real numbers x and y for which $x+y\neq y+x$."

Note that p is always true whatever x and y may be and $-p$ is always false.

5. Write the contra-positive and converse of the following statements.

(i) If x **is a prime number, then** x **is odd.**

(ii) if the two lines are parallel, then they do not intersect in the same plane.

Ans. If statement is $p\Rightarrow q$, then its contra-positive is $\sim q\Rightarrow\sim p$ and its converse is $q\Rightarrow p$.

(i) Contra-positive : “If x is not odd, then x is not a prime number.”

Converse : “If x is odd, then x is a prime number.”

(ii) Contra-positive : “If two lines intersect in the same plane, then they are not parallel.”

Converse: “If two lines do not intersect in the same plane, then they are parallel.”

6. Show that the statement

P : “If x is a real number such that $x^3 + 4x = 0$, then x is 0” is true by

(i) direct method, (ii) method of contradiction, (iii) method of contra-positive

Ans. Given statement is p: “If x is a real number such that $x^3 + 4x = 0$, then $x = 0$"

(i) Direct method: Let $x^3 + 4x = 0, x \in R$

$\Rightarrow x\left(x^2 + 4\right) = 0, x \in R \Rightarrow x = 0 \left(\because \text{if } x \in R \text{ then } x^2 + 4 \geq 4\right)$

Note that if the product of two numbers is zero then atleast one of them is surely zero.

Thus, we find that p is a true statement.

(ii) Method of contradiction.

Let x be a nonzero real number

$\Rightarrow x^2 > 0$ ($\because$ Square of a non- zero real number is always positive)

$\Rightarrow x^2 + 4 > 4 \Rightarrow x^2 + 4 \neq 0$

$\Rightarrow x\left(x^2 + 4\right) \neq 0 \left(\because x \neq 0 \text{ and } x^2 + 4 \neq 0\right)$

$\Rightarrow x^3 + 4x \neq 0$, which is a contradiction.

Hence, $x = 0$

(iii) Method of contra-positive:

Let q : " $x \in R$ and $x^3 + 4x = 0$"

r : " $x = 0$"

$\therefore$ Given statement p is $q \Rightarrow r$

Its contra-positive is $\sim r \Rightarrow \sim q$

i.e. "if x is a non- zero real number then $x^3 + 4x$ is also nonzero"

Now $x \neq 0, x \in R \Rightarrow x^2 > 0 \Rightarrow x^2 + 4 > 4 \Rightarrow x^2 + 4 \neq 0$

$\Rightarrow x(x^2 + 4) \neq 0 \Rightarrow x^3 + 4x \neq 0$ i.e. $\sim r \Rightarrow \sim q$.

Thus the statement $\sim r \Rightarrow \sim q$ is always true

Hence, $q \Rightarrow r$ is always true

Note: Infact, 'Method of contradiction' is another form of 'contra-positive method' while proving an implication.

7. Given below are two statements

P : 25 is a multiple of 5.

q: 25 is a multiple of 8

Write the compound statements connecting these two statements with "and" and "OR". In both cases check the validity of the compound statement.

Ans. Case I. Using the connective 'and', we obtain the compound statement " p and q ".

i.e., "25 is a multiple of 5 and 8".

It is false statement as q is always false. $(\because 25$ is not a multiple of $8)$

Case II. Using the connective 'or', we obtain the compound statement " p or q ".

i.e., "25 is a multiple of 5 or 8".

It is a true statement as p always true. $(\because 25 \text{ is a multiple of } 5)$

8. Write the following statement in five different ways, conveying the same meaning.

P : If a triangle is equiangular, then it is an obtuse angled triangle.

Ans. Given statement is

"If a triangle is equiangular, then it is an obtuse angled triangle". Its five equivalents are as follows:

(i) "A triangle is equiangular only if it is an obtuse angled triangle".

(ii) "If a triangle is not obtuse angled triangle then it is not an equiangular triangle."

(iii) "*equiangularity* is a sufficient condition for triangle to be obtuse angled."

(iv) "A triangle being obtuse angled, is necessary condition for it to be equiangular".

(v) A triangle is obtuse is obtuse angled if it is equiangular.