Chapter 5 Quadratic Functions

Ex 5.1

Answer 1e.

a. If the bases of the exponential expressions being multiplied are the same, then you can combine them into a single expression by adding the exponents.

One of the properties of exponents states that for any real numbers a and b, and integers m and n,
$a^m \cdot a^n = a^{m+n}$.

This property is called the product-of-powers property and it is illustrated in the given equation.

b. Another property of exponents, called the negative exponent property, enables us to rewrite an expression with negative exponent, say, a^{-m} such that it contains only positive exponent.

$$a^{-m} = \frac{1}{a^m}$$

Thus, the property illustrated is the negative-exponent property.

c. In the given expression, each factor of the product ab is raised to the power m.

The power-of-a-product property states that $(ab)^m = a^m b^m$.

Therefore, the property illustrated is the power-of-a-product property.

Answer 1gp.

In the given expression, a power is raised to a power.

Apply the power-of-a-power property by which $(a^m)^n = a^{mn}$.

$$\begin{aligned}(4^2)^3 &= 4^{2\cdot 3}\\ &= 4^6\\ &= 4,096\end{aligned}$$

Therefore, the given expression evaluates to 4, 096, and the property used is the power-of-a-power property.

Answer 2e.

Consider the number : 25.2×10^{-3}.

We need to determine the number 25.2×10^{-3} is in the form of scientific notation or not.

A number is expressed in scientific notation if it is in the form $c\times10^{n}$ where $1\le c<10$ and n is an integer.

we compare this number with scientific notation $c\times10^{n}$.

We get $c=25.2$ and $n=-3$.

But $c>10$ which is a contradiction.

Therefore, the number 25.2×10^{-3} is not in the form of scientific notation.

Answer 2gp.

Consider the expression : $\left(4^{2}\right)^{3}$.

We need to evaluate the following expression using properties of exponents.

$$\begin{aligned}\left(4^{2}\right)^{3} &= 4^{2\cdot3} && \text{(Using power of a power property)}\\ &= 4^{6}\\ &= 4096\end{aligned}$$

Hence the result is $\boxed{4096}$.

Answer 2q.

Now

$$\begin{aligned}\left(2^{4}\right)^{2} &= 2^{4\cdot2} && \text{[Power of a power property]}\\ &= 2^{8} && \text{[Multiply the powers]}\\ &= \boxed{256} && \text{[Simplify and evalute power]}\end{aligned}$$

Answer 3e.

We are required to find the product of two powers in which the bases are the same.

The property of the product-of-powers states that for any real numbers a and b, and integers m and n,

$a^{m}\cdot a^{n} = a^{m+n}$.

Apply the property to find the product. For this, add the exponents keeping the base.

$$\begin{aligned}3^{3}\cdot3^{2} &= 3^{3+2}\\ &= 3^{5}\\ &= 243\end{aligned}$$

Therefore, the given expression evaluates to 243, and we used the product-of-powers property.

Answer 3gp.

In the given expression, a quotient is raised to a power.

Raise both the numerator and denominator to the power using the power-of-a-quotient property.

$$\left(\frac{2}{9}\right)^3 = \frac{2^3}{9^3}$$

Simplify.

$$\frac{2^3}{9^3} = \frac{8}{729}$$

Therefore, the given expression evaluates to $\frac{8}{729}$, and the property that we used is the power-of-a-quotient property.

Answer 4e.

Consider the expression : $\left(4^{-2}\right)^3$.

We need to evaluate the following expression using properties of exponents.

$$\left(4^{-2}\right)^3 = 4^{-2\times3} \quad \text{(Using power of a power property)}$$

$$=4^{-6}$$

$$=\frac{1}{4^6} \quad \text{(Using negative exponent property)}$$

$$=\frac{1}{4096}$$

Hence the result is $\boxed{\frac{1}{4096}}$.

Answer 4gp.

Consider the number : $\frac{6.10^{-4}}{9.10^{7}}$.

We need to evaluate the following expression using properties of exponents.

$$\frac{6.10^{-4}}{9.10^{7}} = \frac{6}{9}\times10^{-4-7} \quad \text{(Using quotient of a power property)}$$

$$=\frac{2}{3}\times10^{-11}$$

Hence the result is $\boxed{\frac{2}{3}\times10^{-11}}$.

Answer 5e.

We are required to find the product of two powers which have like bases.

The property of the product-of-powers states that for any real numbers a and b, and integers m and n,
$a^m \cdot a^n = a^{m+n}$.

Apply the property to find the product. For this, add the exponents keeping the base.

$$\begin{aligned}(-5)(-5)^4 &= (-5)^{1+4}\\ &= (-5)^5\\ &= -3125\end{aligned}$$

Therefore, the given expression evaluates to -3125, and we used the product-of-powers property.

Answer 5gp.

We are required to find the product of three powers, all of which have like bases.

Apply the product-of-powers property by which $a^m \cdot a^n = a^{m+n}$.
For this, add the exponents keeping the base.

$$\begin{aligned}x^{-6}x^5x^3 &= x^{-6+5+3}\\ &= x^2\end{aligned}$$

Therefore, the given expression simplifies to x^2, and the property that we used is the product-of-powers property.

Answer 6e.

Consider the expression : $(2^4)^2$.

We need to evaluate the following expression using properties of exponents.

$$\begin{aligned}(2^4)^2 &= 2^{4\cdot 2} \quad \text{(Using power of a power property)}\\ &= 2^8\end{aligned}$$

Hence the result is $\boxed{2^8}$.

Answer 6gp.

Consider the expression : $(7y^2z^5)(y^{-4}z^{-1})$.

We need to simplify the following expression using properties of exponents.

$$(7y^2z^5)(y^{-4}z^{-1}) = 7(y^2y^{-4})(z^5z^{-1}) \quad \text{(Using product of powers property)}$$
$$= 7y^{-2}z^4$$
$$= \frac{7z^4}{y^2} \quad \text{(Using negative exponent property)}$$

Hence the result is $\boxed{\frac{7z^4}{y^2}}$.

Answer 6q.

Now

$$(a^2b^{-5})^{-3} = (a^2)^{-3}(b^{-5})^{-3} \quad \left[\text{Power of a product property}, (a^m)^n = a^{m \cdot n}\right]$$
$$= a^{2(-3)}b^{(-5)(-3)} \quad [\text{Power of a power property}]$$
$$= a^{-6}b^{15} \quad [\text{Multiply the powers}]$$
$$= \boxed{\frac{b^{15}}{a^6}} \quad \left[\text{Negative exponent property}, a^{-m} = \frac{1}{a^m}, a \neq 0\right]$$

Answer 7e.

We are required to find the quotient of two powers which have like bases.

The property of the quotient-of-powers states that for any real numbers a and b, and integers m and n,

$$\frac{a^m}{a^n} = a^{m-n}, a \neq 0.$$

Apply the property to find the quotient. For this, subtract the exponents keeping the base.

$$\frac{5^2}{5^5} = 5^{2-5}$$
$$= 5^{-3}$$

The expression has a negative exponent. Rewrite it using the rule $a^{-m} = \frac{1}{a^m}, a \neq 0$.

$$5^{-3} = \frac{1}{5^3}$$
$$= \frac{1}{125}$$

Therefore, the given expression evaluates to $\frac{1}{125}$, and we used the quotient-of-powers property and negative-exponent property.

Answer 7gp.

Raise the numerator and the denominator to the second power using the power of a quotient property.

$$\left(\frac{s^3}{t^{-4}}\right)^2 = \frac{\left(s^3\right)^2}{\left(t^{-4}\right)^2}$$

Now, apply the power-of-a-power property by which $(a^m)^n = a^{mn}$.

$$\frac{\left(s^3\right)^2}{\left(t^{-4}\right)^2} = \frac{s^{3\cdot 2}}{t^{-4\cdot 2}}$$

Simplify the exponents.

$$\frac{s^{3\cdot 2}}{t^{-4\cdot 2}} = \frac{s^6}{t^{-2}}$$

Rewrite the expression with positive exponents using the negative-exponent rule.

$$\frac{s^6}{t^{-2}} = s^6t^2$$

Therefore, the given expression simplifies to s^6t^2, and we used the power-of-a-quotient property, the power-of-a-power property, and the negative-exponent property.

Answer 8e.

Consider the expression : $\left(\frac{3}{5}\right)^4$.

We need to evaluate the following expression using properties of exponents.

$$\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4} \quad \text{(Using power of a quotient property)}$$
$$= \frac{81}{625}$$

Hence the result is $\boxed{\frac{81}{625}}$.

Answer 8gp.

Consider the expression : $\left(\frac{x^4y^{-2}}{x^3y^6}\right)^3$.

We need to simplify the following expression using properties of exponents.

$$\left(\frac{x^4y^{-2}}{x^3y^6}\right)^3 = \left(x^{4-3}y^{-2-6}\right)^3 \quad \text{(Using quotient of a power property)}$$

$$= \left(x^1y^{-8}\right)^3$$

$$= \left(xy^{-8}\right)^3$$

$$= \left(\frac{x}{y^8}\right)^3 \quad \text{(Using negative exponent property)}$$

$$= \frac{x^3}{y^{8\cdot 3}} \quad \text{(Using power of a quotient property)}$$

$$= \frac{x^3}{y^{24}}$$

Hence the result is $\boxed{\frac{x^3}{y^{24}}}$.

Answer 8q.

Now

$$\frac{c^3d^{-2}}{c^5d^{-1}} = \frac{c^3}{c^5}\cdot\frac{d^{-2}}{d^{-1}}$$

$$= c^{3-5}\cdot d^{-2+1} \quad \left[\text{Quotient of powers property, } \frac{a^m}{a^n} = a^{m-n}, a \neq 0\right]$$

$$= c^{-2}\cdot d^{-1}$$

$$= \frac{1}{c^2}\cdot\frac{1}{d} \quad \left[\text{Negative exponent property,} a^{-m} = \frac{1}{a^m}, a \neq 0\right]$$

$$= \boxed{\frac{1}{c^2d}} \quad \left[\text{Multiply the factors}\right]$$

Answer 9e.

In the given expression, a quotient is raised to a power.

The property of the power-of-a-quotient states that for any real numbers a and b, and integers m and n,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0.$$

Apply the property by raising both the numerator and the denominator to the power.

$$\left(\frac{2}{7}\right)^{-3} = \frac{(2)^{-3}}{(7)^{-3}}$$

Now, rewrite the expression with positive exponents using the rule $a^{-m} = \frac{1}{a^m},\ a \neq 0.$

$$\frac{(2)^{-3}}{(7)^{-3}} = \frac{7^3}{2^3}$$

Simplify.

$$\frac{7^3}{2^3} = \frac{343}{8}$$

Therefore, the given expression evaluates to $\frac{343}{8}$, and we used the power-of-a-quotient property and the negative-exponent property.

Answer 10e.

Consider the expression: $9^3 \cdot 9^{-1}$.

We need to evaluate the following expression using properties of exponents.

$$\begin{aligned} 9^3 \cdot 9^{-1} &= 9^{3-1} \\ &= 9^2 \\ &= 81 \qquad \text{(Using product of powers property)} \end{aligned}$$

Hence the result is $\boxed{81}$.

Answer 10q.

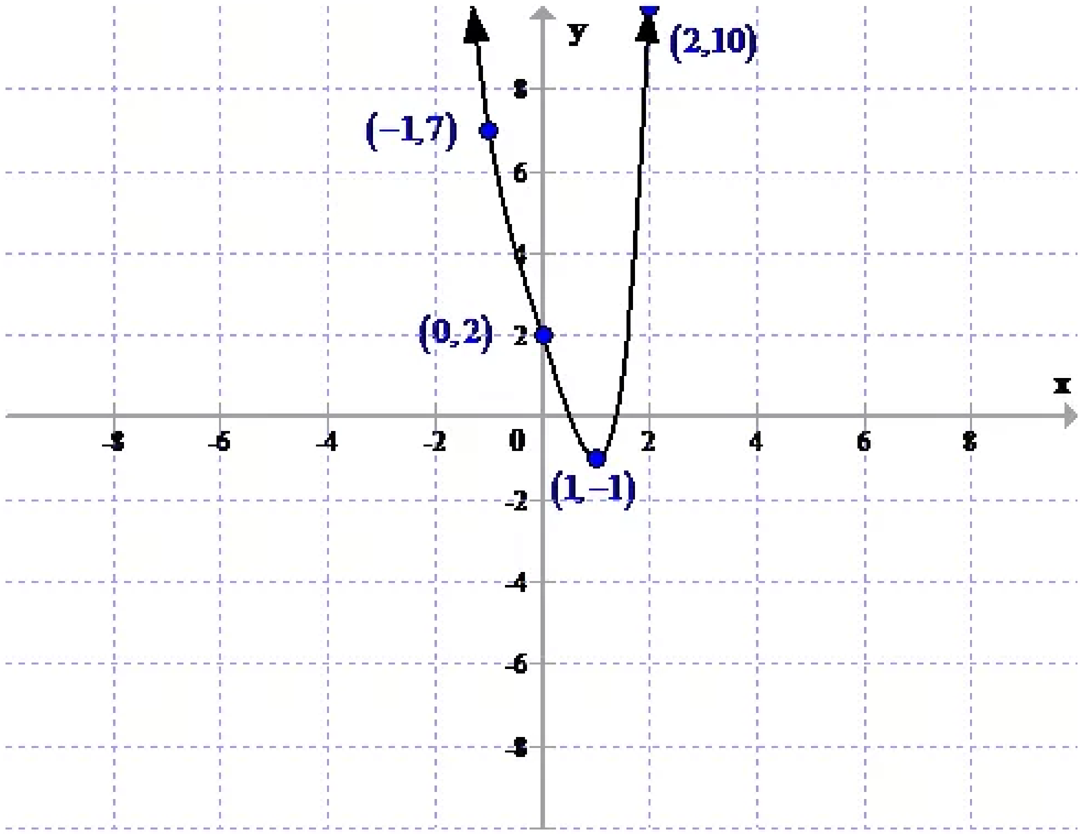
Now we will make a table of values

x	-2	-1	0	1	2
$h(x)$	26	7	2	-1	10

Plot the points and connect the points with a smooth curve and check the end behavior.

The degree is even and the leading coefficient is positive, so $h(x) \to +\infty$ as $x \to -\infty$ and $h(x) \to +\infty$ as $x \to +\infty$

The graph of the polynomial function is



Answer 11e.

We are required to find the quotient of two powers which have like bases.

The property of the quotient-of-powers states that for any real numbers a and b, and integers m and n,

$$\frac{a^m}{a^n} = a^{m-n},\ a \neq 0.$$

Apply the property to find the quotient. For this, subtract the exponents keeping the base.

$$\frac{3^4}{3^{-2}} = 3^{4-(-2)}$$
$$= 3^6$$
$$= 729$$

Therefore, the given expression evaluates to 729, and we used the quotient-of-powers property.

Answer 12e.

Consider the expression : $\left(\frac{2}{3}\right)^{-5} \cdot \left(\frac{2}{3}\right)^{4}$.

We need to evaluate the following expression using properties of exponents.

$$\left(\frac{2}{3}\right)^{-5} \cdot \left(\frac{2}{3}\right)^{4} = \left(\frac{2}{3}\right)^{-5+4} \quad \text{(by using product of powers property)}$$
$$= \left(\frac{2}{3}\right)^{-1} \quad \text{(Simplify)}$$
$$= \frac{3}{2} \quad \text{(Using negative exponent property)}$$

Hence the result is $\boxed{\frac{3}{2}}$.

Answer 12q.

$$(x^3 + x^2 - 6) - (2x^2 + 4x - 8)$$

$$= x^3 + x^2 - 6 - 2x^2 - 4x + 8 \quad \left[\begin{array}{l}\text{Change the sign of} \\ \text{each terms of } 2x^2 + 4x - 8 \\ \text{and drop the parentheses}\end{array}\right]$$

$$= x^3 + (x^2 - 2x^2) - 4x + (-6 + 8) \quad [\text{Combine like terms}]$$

$$= x^3 - x^2 - 4x + 2 \quad [\text{Add the coefficients of like terms}]$$

The polynomials can be subtracted by writing them so that like terms are in column.

$$\begin{array}{r} x^3 + x^2 - 6 \\ \underline{-(2x^2 + 4x - 8)} \end{array} \rightarrow \begin{array}{rr} & x^3 + \ \ x^2 - 6 \\ \underline{+} & \underline{-2x^2 + 8 - 4x} \\ & x^3 - \ \ x^2 + 2 - 4x \end{array} \quad [\text{Change sign and add column by column}]$$

Answer 13e.

We are required to find the product of three powers which have like bases.

The property of the product-of-powers states that for any real numbers a and b, and integers m and n,
$a^m \cdot a^n = a^{m+n}$.

Apply the property to find the product. For this, add the exponents keeping the base.

$$\begin{aligned} 6^3 \cdot 6^0 \cdot 6^{-5} &= 6^{3+0+(-5)} \\ &= 6^{-2} \end{aligned}$$

Now, rewrite the expression with positive exponents using the rule $a^{-m} = \dfrac{1}{a^m}$, $a \neq 0$.

$$\begin{aligned} 6^{-2} &= \frac{1}{6^2} \\ &= \frac{1}{36} \end{aligned}$$

Therefore, the given expression evaluates to $\dfrac{1}{36}$, and we used the product-of-powers property and the negative-exponent property.

Answer 14e.

Consider the expression : $\left(\left(\frac{1}{2}\right)^{-5}\right)^2$.

We need to evaluate the following expression using properties of exponents.

$$\begin{aligned} \left(\left(\frac{1}{2}\right)^{-5}\right)^2 &= \left(2^5\right)^2 && \text{(Using negative exponent property)} \\ &= 2^{5\times 2} && \text{(Using power of a power property)} \\ &= 2^{10} \\ &= 1024 \end{aligned}$$

Hence the result is $\boxed{1024}$.

Answer 15e.

Begin by applying the properties of multiplication to rearrange the terms.

$$(4.2\times10^3)(1.5\times10^6) = (4.2\times1.5)(10^3\times10^6)$$

Multiply the terms within the first set of parentheses.

$$(4.2\times1.5)(10^3\times10^6) = 6.3(10^3\times10^6)$$

The powers 10^3 and 10^6 have like bases, and can be simplified using the product-of-powers property of exponents, $a^m \cdot a^n = a^{m+n}$.

$$\begin{aligned} 6.3(10^3\times10^6) &= 6.3\times10^{3+6} \\ &= 6.3\times10^9 \end{aligned}$$

Therefore, the result is 6.3×10^9.

Answer 16e.

Consider the expression : $(1.2\times10^{-3})(6.7\times10^{-7})$.

We need to write the following expression in scientific notation.

$$\begin{aligned} (1.2\times10^{-3})(6.7\times10^{-7}) &= 1.2\times6.7\times10^{-3}\times10^{-7} \\ &= 8.04\times10^{-3-7} \quad \text{(Using the product of powers property)} \\ &= 8.04\times10^{-10} \end{aligned}$$

A number is expressed in scientific notation if it is in the form $c\times10^n$ where $1\le c<10$ and n is an integer.

Compare the number 8.04×10^{-10} with $c\times10^n$, we get $c=8.04$ and $c=-10$.

Here the value of $c=-10$ is in the interval $[1,10)$.

Therefore, 8.04×10^{-10} is in the form of scientific notation.

Answer 17e.

Rearrange the terms by applying the properties of multiplication.

$$(6.3\times10^5)(8.9\times10^{-12}) = (6.3\times8.9)(10^5\times10^{-12})$$

Multiply the terms within the first set of parentheses.

$$(6.3\times8.9)(10^5\times10^{-12}) = 56.07(10^5\times10^{-12})$$

The powers 10^5 and 10^{-12} have like bases, and can be simplified using the product-of-powers property of exponents, $a^m \cdot a^n = a^{m+n}$.

$$56.07\left(10^5 \times 10^{-12}\right) = 56.07\left(10^{5+(-12)}\right)$$
$$= 56.07 \times 10^{-7}$$

Rewrite 56.07 in scientific notation.

$$56.07 \times 10^{-7} = 5.607 \times 10^1 \times 10^{-7}$$

Apply the product-of-powers property once again to simplify.

$$5.607 \times 10^1 \times 10^{-7} = 5.607 \times 10^{1+(-7)}$$
$$= 5.607 \times 10^{-6}$$

Therefore, the result is 5.607×10^{-6}.

Answer 18e.

Consider the expression : $\left(7.2\times10^9\right)\left(9.4\times10^8\right)$.

We need to write the following expression in scientific notation.

$$\left(7.2\times10^9\right)\left(9.4\times10^8\right) = 7.2\times9.4\times10^9\times10^8$$
$$= 67.68\times10^{9+8} \quad \text{(Using product of powers property)}$$
$$= 67.68\times10^{17}$$
$$= \frac{67.68}{10}\times10^{17}\times10^1 \quad \text{(Using product of powers property)}$$
$$= 6.768\times10^{17+1}$$
$$= 6.768\times10^{18}$$

Therefore, $\left(7.2\times10^9\right)\left(9.4\times10^8\right) = 6.768\times10^{18}$.

Here we need to determine the number 6.768×10^{18} is in the notation of $c\times10^n$ or not.

Compare the number 6.768×10^{18} with $c\times10^n$, we get

$c = 6.768$ and $n = 18$. And the value $c = 6.768$ is in the interval $[1,10)$.

Therefore, the number 6.768×10^{18} is in the form of scientific notation.

Hence our expression $\left(7.2\times10^9\right)\left(9.4\times10^8\right)$ is in the form of scientific notation.

Answer 19e.

In the given expression, a product is raised to a power.

Raise each term in the product to the power using the power-of-a-product property of exponents, $(ab)^m = a^m b^m$.

$$\left(2.1\times 10^{-4}\right)^3 = (2.1)^3\times\left(10^{-4}\right)^3$$

Simplify.

$$(2.1)^3\times\left(10^{-4}\right)^3 = 9.261\times\left(10^{-4}\right)^3$$

Now, apply the power-of-a-power property, $(a^m)^n = a^{mn}$.

$$\begin{aligned} 9.261\times\left(10^{-4}\right)^3 &= 9.261\times 10^{-4(3)} \\ &= 9.261\times 10^{-12} \end{aligned}$$

Therefore, the result is 9.261×10^{-12}.

Answer 20e.

Consider the expression : $\left(4.0\times 10^3\right)^4$.

We need to write the following expression in scientific notation.

$$\begin{aligned} \left(4.0\times 10^3\right)^4 &= 4^4\times\left(10^3\right)^4 && \text{(Using power of a product property)} \\ &= 256\times 10^{3\cdot 4} && \text{(Using power of a power property)} \\ &= 256\times 10^{12} \\ &= \frac{256}{10^2}\times 10^{12}\times 10^2 \\ &= 2.56\times 10^{12+2} && \text{(Using product of powers property)} \\ &= 2.56\times 10^{14} \end{aligned}$$

Therefore, $\left(4.0\times 10^3\right)^4 = 2.56\times 10^{14}$.

Compare the number 2.56×10^{14} with $c\times 10^n$, we get $c = 2.56$ and $n = 14$.

The number $c = 2.56$ is in the interval $[1,10)$.

Therefore, the number 2.56×10^{14} is in the form of scientific notation.

Hence our expression $\left(4.0\times 10^3\right)^4$ is in the form of scientific notation.

Answer 21e.

Begin by rewriting the given quotient as a product of two fractions.

$$\frac{8.1\times10^{12}}{5.4\times10^{9}} = \frac{8.1}{5.4}\times\frac{10^{12}}{10^{9}}$$

Simplify.

$$\frac{8.1}{5.4}\times\frac{10^{12}}{10^{9}} = 1.5\times\frac{10^{12}}{10^{9}}$$

Now, apply the quotient-of-powers property to $\frac{10^{12}}{10^{9}}$.

Subtract the exponents keeping the base.

$$1.5\times\frac{10^{12}}{10^{9}} = 1.5\times10^{12-9}$$

$$= 1.5\times10^{3}$$

Therefore, the answer is 1.5×10^{3}.

Answer 22e.

Consider the number : $\frac{1.1\times10^{-3}}{5.5\times10^{-8}}$.

We need to write the following expression in scientific notation.

$$\frac{1.1\times10^{-3}}{5.5\times10^{-8}} = \frac{1.1}{5.5}\times10^{-3-(-8)}$$ (Using quotient of a power property)

$$= \frac{1}{5}\times10^{5}$$

$$= \frac{2}{10}\times10^{5}$$

$$= 2\times\frac{10^{5}}{10^{1}}$$ (Using quotient of a power property)

$$= 2\times10^{4}$$

$$\frac{1.1\times10^{-3}}{5.5\times10^{-8}} = 2\times10^{4}.$$

Here we need to determine the number 2×10^{4} is in the form of scientific notation or not.

A number is expressed in scientific notation if it is in the form $c \times 10^n$ where $1 \le c < 10$ and n is an integer.

Compare the number 2×10^4 with $c \times 10^n$, we get $c = 2$ and $n = 4$.
The value of $c = 2$ is in the interval $[1,10)$ and $n = 4$ is an integer.

Therefore the number 2×10^4 is in the form of scientific notation.

Hence our expression $\frac{1.1 \times 10^{-3}}{5.5 \times 10^{-8}}$ is in the form of scientific notation.

Answer 23e.

Apply the properties of multiplication on the numerator.

$$\frac{(7.5 \times 10^8)(4.5 \times 10^{-4})}{1.5 \times 10^7} = \frac{(7.5 \times 4.5)(10^8 \times 10^{-4})}{1.5 \times 10^7}$$

$$= \frac{33.75(10^8 \times 10^{-4})}{1.5 \times 10^7}$$

The powers 10^8 and 10^{-4} have like bases, and can be simplified using the product-of-powers property of exponents.

$$\frac{33.75(10^8 \times 10^{-4})}{1.5 \times 10^7} = \frac{33.75 \times 10^{8+(-4)}}{1.5 \times 10^7}$$

$$= \frac{33.75 \times 10^4}{1.5 \times 10^7}$$

Now, rewrite the quotient as a product of two fractions.

$$\frac{33.75 \times 10^4}{1.5 \times 10^7} = \frac{33.75}{1.5} \times \frac{10^4}{10^7}$$

Simplify.

$$\frac{33.75}{1.5} \times \frac{10^4}{10^7} = 22.5 \times \frac{10^4}{10^7}$$

Apply the quotient-of-powers property to $\frac{10^4}{10^7}$.

Subtract the exponents keeping the base.

$$22.5 \times \frac{10^4}{10^7} = 22.5 \times 10^{4-7}$$

$$= 22.5 \times 10^{-3}$$

Answer 24e.

Express 22.5 in scientific notation.

$22.5 \times 10^{-3} = 2.25 \times 10^{1} \times 10^{-3}$

Apply the product-of-powers property again to simplify.

$2.25 \times 10^{1} \times 10^{-3} = 2.25 \times 10^{-2}$

Therefore, the answer is 2.25×10^{-2}.

Answer 25e.

In the given expression, a product is raised to a power. We can simplify such expressions using the power-of-a-product property which states that $(ab)^m = a^m b^m$.

$(2^2 y^3)^5 = (2^2)^5 (y^3)^5$

By the power-of-a-power property, $(a^m)^n = a^{mn}$.

Raise each power to the power using this property and simplify.

$$\left(2^2\right)^5 \left(y^3\right)^5 = 2^{2\cdot 5} y^{3\cdot 5}$$
$$= 2^{10} y^{15}$$

Therefore, the given expression simplifies to $2^{10} y^{15}$, and we used the power-of-a-product property and the power-of-a-power property.

Answer 26e.

Consider the expression : $\left(p^3 q^2\right)^{-1}$.

We need to simplify the following expression using properties of exponents.

$$\left(p^3 q^2\right)^{-1} = \frac{1}{\left(p^3 q^2\right)^1} \qquad \text{(Using negative exponent property)}$$
$$= \frac{1}{p^3 q^2}$$

Hence the result is $\boxed{\dfrac{1}{p^3 q^2}}$.

Answer 27e.

Use the properties of multiplication to rearrange the terms.

$$\left(w^3 x^{-2}\right)\left(w^6 x^{-1}\right) = \left(w^3 \cdot w^6\right)\left(x^{-2} \cdot x^{-1}\right)$$

Now, simplify using the product-of-powers property, $a^m \cdot a^n = a^{m+n}$.

$$\left(w^3 \cdot w^6\right)\left(x^{-2} \cdot x^{-1}\right) = w^{3+6} \cdot x^{-2+(-1)}$$
$$= w^9 \cdot x^{-3}$$

Rewrite the expression using positive exponents by applying the negative exponent property, $a^{-m} = \frac{1}{a^m}, a \neq 0$.

$$w^9 \cdot x^{-3} = \frac{w^9}{x^3}$$

Therefore, the given expression simplifies to $\frac{w^9}{x^3}$, and we used the product-of-powers property and the negative-exponent property.

Answer 28e.

Consider the expression : $\left(5s^{-2}t^4\right)^{-3}$.

We need to simplify the following expression using properties of exponents.

$$\left(5s^{-2}t^4\right)^{-3} = 5^{-3}s^{(-2)(-3)}t^{4(-3)} \quad \text{(Using power of a product property)}$$
$$= 5^{-3}s^6t^{-12}$$
$$= \left(\frac{1}{5^3}\right)s^6\left(\frac{1}{t^{12}}\right) \quad \text{(Using negative exponent property)}$$
$$= \frac{s^6}{125t^{12}}$$

Hence the result is $\boxed{\frac{s^6}{125t^{12}}}$.

Answer 29e.

In the given expression, a product is raised to a power. We can simplify such expressions using the power-of-a-product property which states that $(ab)^m = a^m b^m$.

$$\left(3a^3b^5\right)^{-3} = \left(3a^3\right)^{-3}\left(b^5\right)^{-3}$$
$$= (3)^{-3}\left(a^3\right)^{-3}\left(b^5\right)^{-3}$$

By the power-of-a-power property, $(a^m)^n = a^{mn}$. Raise each power to the respective exponent using this property and simplify.

$$(3)^{-3}\left(a^3\right)^{-3}\left(b^5\right)^{-3} = 3^{-3} \cdot a^{3(-3)} \cdot b^{5(-3)}$$
$$= 3^{-3} \cdot a^{-9} \cdot b^{-15}$$

Now, rewrite the expression using positive exponents by applying the negative exponent property, $a^{-m} = \frac{1}{a^m}, a \neq 0$.

$$3^{-3} \cdot a^{-9} \cdot b^{-15} = \frac{1}{3^3} \cdot \frac{1}{a^9} \cdot \frac{1}{b^{15}}$$
$$= \frac{1}{27a^9b^{15}}$$

Therefore, the given expression simplifies to $\frac{1}{27a^9b^{15}}$, and we used the power-of-a-product property, the power-of-a-power property, and the negative-exponent property.

Answer 30e.

Consider the expression : $\frac{x^{-1}y^2}{x^2y^{-1}}$.

We need to simplify the following expression using properties of exponents.

$$\frac{x^{-1}y^2}{x^2y^{-1}} = x^{-1-2}y^{2-(-1)} \quad \text{(Using quotient of a power property)}$$

$$= x^{-3}y^3$$

$$= \frac{y^3}{x^3} \quad \text{(Using negative exponent property)}$$

Hence the result is $\boxed{\frac{y^3}{x^3}}$.

Answer 31e.

Begin by rewriting the given quotient as a product of fractions.

$$\frac{3c^3d}{9cd^{-1}} = \frac{3}{9} \cdot \frac{c^3}{c} \cdot \frac{d}{d^{-1}}$$

Simplify.

$$\frac{3}{9} \cdot \frac{c^3}{c} \cdot \frac{d}{d^{-1}} = \frac{1}{3} \cdot \frac{c^3}{c} \cdot \frac{d}{d^{-1}}$$

Now, apply the quotient-of-powers property which states that $\frac{a^m}{a^n} = a^{m-n}$.

$$\frac{1}{3} \cdot \frac{c^3}{c} \cdot \frac{d}{d^{-1}} = \frac{1}{3} \cdot c^{3-1} \cdot d^{1-(-1)}$$

$$= \frac{1}{3} \cdot c^2 \cdot d^2$$

$$= \frac{c^2d^2}{3}$$

Therefore, the given expression simplifies to $\frac{c^2d^2}{3}$, and we used the quotient-of-powers property.

Answer 32e.

Consider the expression : $\frac{4r^4s^5}{24r^4s^{-5}}$.

We need to simplify the following expression using properties of exponents.

$$\frac{4r^4s^5}{24r^4s^{-5}} = \frac{r^{4-4}s^{5-(-5)}}{6} \quad \text{(Using quotient of a power property)}$$

$$= \frac{r^0s^{10}}{6}$$

$$= \frac{(1)s^{10}}{6}$$

$$= \frac{s^{10}}{6}$$

Hence the result is $\boxed{\frac{s^{10}}{6}}$.

Answer 33e.

Begin by rewriting the given quotient as a product of fractions.

$$\frac{2a^3b^{-4}}{3a^5b^{-2}} = \frac{2}{3} \cdot \frac{a^3}{a^5} \cdot \frac{b^{-4}}{b^{-2}}$$

Now, apply the quotient-of-powers property which states that $\frac{a^m}{a^n} = a^{m-n}$.

$$\frac{2}{3} \cdot \frac{a^3}{a^5} \cdot \frac{b^{-4}}{b^{-2}} = \frac{2}{3} \cdot a^{3-5} \cdot b^{-4-(-2)}$$

$$= \frac{2}{3} \cdot a^{-2} \cdot b^{-2}$$

Rewrite the expression with positive exponents by applying the negative-exponent property, $a^{-m} = \frac{1}{a^m}$, $a \neq 0$.

$$\frac{2}{3} \cdot a^{-2} \cdot b^{-2} = \frac{2}{3} \cdot \frac{1}{a^2} \cdot \frac{1}{b^2}$$

$$= \frac{2}{3a^2b^2}$$

Therefore, the given expression simplifies to $\frac{2}{3a^2b^2}$, and we used the quotient-of-powers and the negative-exponent properties.

Answer 34e.

Consider the expression : $\frac{y^{11}}{4z^3} \cdot \frac{8z^7}{y^7}$

We need to simplify the following expression using properties of exponents.

$$\frac{y^{11}}{4z^3} \cdot \frac{8z^7}{y^7} = 2\left(\frac{y^{11}}{y^7}\right)\left(\frac{z^7}{z^3}\right)$$

$$= 2y^{11-7}z^{7-3} \quad \text{(Using quotient of a power property)}$$

$$= 2y^4z^4$$

Hence the result is $\boxed{2y^4z^4}$.

Answer 35e.

Begin by rewriting the given quotient as a product of fractions.

$$\frac{x^2y^{-3}}{3y^2} \cdot \frac{y^2}{x^{-4}} = \frac{1}{3} \cdot \frac{x^2}{x^{-4}} \cdot \frac{y^{-3}y^2}{y^2}$$

Now, apply the product-of-powers property which states that $a^m \cdot a^n = a^{m+n}$, on the numerator.

$$\frac{1}{3} \cdot \frac{x^2}{x^{-4}} \cdot \frac{y^{-3}y^2}{y^2} = \frac{1}{3} \cdot \frac{x^2}{x^{-4}} \cdot \frac{y^{-3+2}}{y^2}$$

$$= \frac{1}{3} \cdot \frac{x^2}{x^{-4}} \cdot \frac{y^{-1}}{y^2}$$

Simplify using the quotient-of-powers property $\frac{a^m}{a^n} = a^{m-n}$.

$$\frac{1}{3} \cdot \frac{x^2}{x^{-4}} \cdot \frac{y^{-1}}{y^2} = \frac{1}{3} \cdot x^{2-(-4)} \cdot y^{-1-2}$$

$$= \frac{1}{3} \cdot x^6 \cdot y^{-3}$$

Rewrite the expression with positive exponents by applying the negative-exponent property, $a^{-m} = \frac{1}{a^m}$, $a \neq 0$.

$$\frac{1}{3} \cdot x^6 \cdot y^{-3} = \frac{1}{3} \cdot x^6 \cdot \frac{1}{y^3}$$

$$= \frac{x^6}{3y^3}$$

Therefore, the given expression simplifies to $\frac{x^6}{3y^3}$, and we used the product-of-powers, quotient-of-powers, and the negative-exponent properties.

Answer 36e.

Consider the table :

A	B	C	D
$\frac{y^2}{3}$	$\frac{xy^2}{3}$	$\frac{x}{3}$	$\frac{1}{3}$

We need to identify the simplified form the expression $\frac{2x^2y}{6xy^{-1}}$ using properties of exponents.

$$\frac{2x^2y}{6xy^{-1}} = \frac{1}{3}x^{2-1}y^{1-(-1)} \qquad \text{(Using quotient of a power property)}$$

$$= \frac{1}{3}x^1y^2$$

$$= \frac{xy^2}{3}$$

Therefore, the answer is option (B).

Answer 37e.

The expression on the left is the quotient of two powers having like bases.
We can simplify expressions of such form using the quotient-of-powers property which states that $\frac{a^m}{a^n} = a^{m-n}$.

$$\frac{x^{10}}{x^2} = x^{10-2}$$

$$= x^8$$

We have to subtract the exponents keeping the base. The result on the right in the given simplification is x^5, which means that the exponents have been divided instead of being subtracted. As a result, the simplification is not correct.

Therefore, the correct simplification is $\frac{x^{10}}{x^2} = x^8$.

Answer 38e.

Consider the expression : $x^5 \cdot x^3 = x^{15}$ ✗.
The error in the given equation is $x^5 \cdot x^3 \neq x^{5\cdot 3}$ .x

Therefore,

$$x^5 \cdot x^3 = x^{5+3} \qquad \text{(Using product of powers property)}$$

$$= x^8$$

Hence the correction of error is $\boxed{x^5 \cdot x^3 = x^8}$.

Answer 39e.

The expression on the left is the product of two powers having like bases.
We can simplify expressions of such form using the quotient-of-powers property which states that $a^m \cdot a^n = a^{m+n}$.

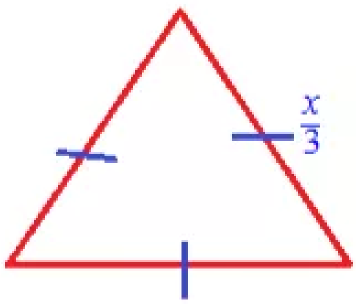
$$(-3)^2(-3)^4 = (-3)^{2+4}$$
$$= (-3)^6$$

We have to add the exponents keeping the base. The result on the right in the given simplification is 9^6, in which the base has changed. As a result, the simplification is not correct.

Therefore, the correct simplification is $(-3)^2(-3)^4 = (-3)^6$.

Answer 40e.

Consider the diagram :



Above triangle is an equilateral triangle.

We know that the area of an equilateral triangle is $\frac{\sqrt{3}}{4}s^2$, where s is side of the triangle.

An expression for the area of the figure in terms of x is

$$\frac{\sqrt{3}}{4}s^2 = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 \quad \text{Substitute } s = \frac{x}{3}$$
$$= \frac{\sqrt{3}}{4}\left(\frac{x^2}{3^2}\right) \quad \text{(Using power of a quotient property)}$$
$$= \frac{\sqrt{3}}{4}\left(\frac{x^2}{9}\right)$$
$$= \frac{\sqrt{3}}{36}x^2$$

Hence the result is $\boxed{\frac{\sqrt{3}}{36}x^2}$.

Answer 41e.

From the given figure, we get the radius of the figure as x, and the height as $\frac{x}{2}$.

Substitute r for x, and $\frac{x}{2}$ for h in $V = \pi r^2 h$.

$$V = \pi(x)^2\left(\frac{x}{2}\right)$$

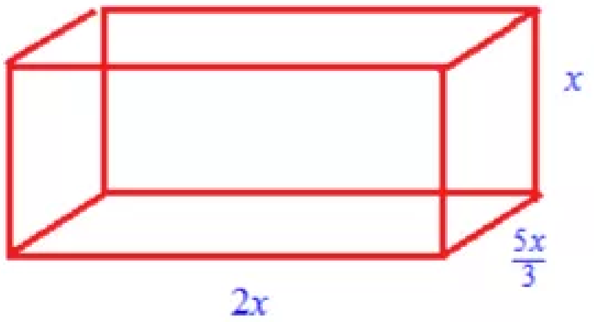
Simplify using the product-of-powers property.

$$V = \frac{1}{2}\pi x^{2+1}$$
$$= \frac{1}{2}\pi x^3$$

Therefore, the volume of the region is $\frac{1}{2}\pi x^3$.

Answer 42e.

Consider the diagram :



An expression for the volume of the figure in terms of x is

$$V = (2x)\left(\frac{5x}{3}\right)(x)$$
$$= 2\left(\frac{5}{3}\right)(x \cdot x \cdot x) \quad \text{(Separate the nubers and the variables)}$$
$$= \frac{10}{3}x^{1+1+1} \quad \text{(Using product of powers property)}$$
$$= \frac{10}{3}x^3$$

Hence the result is $\boxed{\frac{10}{3}x^3}$.

Answer 43e.

Divide $x^{15}y^{12}z^{6}$ by $x^{4}y^{7}z^{11}$ to find the expression that will make the statement true.

$$\frac{x^{15}y^{12}z^{8}}{x^{4}y^{7}z^{11}}$$

Now, use the quotient-of-powers property by which $\frac{a^m}{a^n} = a^{m-n}$.

$$\frac{x^{15}y^{12}z^{8}}{x^{4}y^{7}z^{11}} = x^{15-4}y^{12-7}z^{8-11}$$

Simplify the exponents.

$$x^{15-4}y^{12-7}z^{8-11} = x^{11}y^{5}z^{-3}$$

Therefore, the expression that makes the given statement true is $x^{11}y^{5}z^{-3}$.

Answer 44e.

Consider the expression : $3x^3y^2 = \frac{12x^2y^5}{?}$.

After simplification of the expression $3x^3y^2 = \frac{12x^2y^5}{?}$, we need to find the expression $\frac{12x^2y^5}{3x^3y^2}$, using properties of exponents.

$$\frac{12x^2y^5}{3x^3y^2} == 4x^{2-3}y^{5-2} \quad \text{(by using quotient of a power property)}$$

$$= 4x^{-1}y^3$$

$$= \frac{4y^3}{x} \quad \text{(Using negative exponent property)}$$

Hence the result is $\boxed{\frac{4y^3}{x}}$.

Answer 45e.

Begin by simplifying the expression on the left by applying the power-of-a-power property which states that $(a^m)^n = a^{mn}$.

$$a^{5\cdot 2}b^{4\cdot 2} = a^{14}b^{-1}\cdot ?$$

$$a^{10}b^{8} = a^{14}b^{-1}\cdot ?$$

Divide $a^{10}b^{8}$ by $a^{14}b^{-14}$ to find the expression that will make the statement true.

$$\frac{a^{10}b^{8}}{a^{14}b^{-1}}$$

Now, use the quotient-of-powers property by which $\frac{a^m}{a^n} = a^{m-n}$.

$$\frac{a^{10}b^8}{a^{14}b^{-1}} = a^{10-14}b^{8-(-1)}$$

Simplify the exponents.

$$a^{10-14}b^{8-(-1)} = a^{-4}b^9$$

Therefore, the expression that makes the given statement true is $a^{-4}b^9$.

Answer 46e.

Consider the expression : $x^{12}y^{16} = (x^?y^?)(x^?y^?)$.

The possible ways of making the given statement are

$x^{12}y^{16} = (x^{10}y^{10})(x^2y^6)$ (by using product of powers property),

$x^{12}y^{16} = (x^{11}y^{11})(xy^5)$ (by using product of powers property), and

$x^{12}y^{16} = (x^6y^8)(x^6y^8)$ (by using product of powers property)

Answer 47e.

The negative exponent property states that $\frac{1}{a^m} = a^{-m}$.

Let us start with the left side of the equation and try deriving the expression on the right.

By the zero exponent rule, $a^0 = 1$. Replace 1 with a^0 with in $\frac{1}{a^m}$.

$$\frac{1}{a^m} = \frac{a^0}{a^m}$$

Now, the quotient of the two powers can be found by applying the quotient of powers property,

$$\frac{a^m}{a^n} = a^{m-n}.$$

$$\frac{a^0}{a^m} = a^{0-m}$$
$$= a^{-m}$$

The left side simplifies to the right side.

Thus, we have derived the negative exponent property the quotient of powers property and the zero exponent property.

Answer 48e.

We need to show that the quotient of powers property can be derived from the product of powers property and the negative exponent property.

$$\frac{x^m}{x^n} = x^m\left(\frac{1}{x^n}\right), \text{where } x \neq 0$$

$$= x^m \cdot x^{-n} \qquad \text{(Using negative exponent property)}$$

$$= x^{m-n} \qquad \text{(Using product of powers property)}$$

Hence the quotient of powers property can be derived from the product of powers property and the negative exponent property.

Answer 49e.

From the given table, it can be seen that the surface area of the first ocean is 1.56×10^{14} square meters, and its average depth is 4.03×10^3 meters.

The volume of the ocean is given by the product of its surface area and average depth.

$$\text{Volume} = \text{Surface area} \times \text{Average depth}$$

$$= \left(1.56 \times 10^{14}\right)\left(4.03 \times 10^3\right)$$

Apply the properties of multiplication to rearrange the terms.

$$\left(1.56 \times 10^{14}\right)\left(4.03 \times 10^3\right) = \left(1.56 \times 4.03\right)\left(10^{14} \times 10^3\right)_8$$

$$= 6.2868\left(10^{14} \times 10^3\right)$$

Now, simplify using a property of exponents $a^m \cdot a^n = a^{m+n}$.

$$6.2868\left(10^{14} \times 10^3\right) = 6.2858 \times 10^{14+3}$$

$$= 6.2858 \times 10^{17}$$

Therefore, the volume of the first ocean is 6.2858×10^{17} cubic meters.

In a similar way, calculate the volumes of the other three oceans.

$$\begin{aligned}\text{Volume of the second ocean} &= \left(7.68 \times 10^{13}\right)\left(3.93 \times 10^{3}\right) \\ &= 30.1824 \times 10^{16} \\ &= 3.01824 \times 10^{17}\end{aligned}$$

$$\begin{aligned}\text{Volume of the third ocean} &= \left(6.86 \times 10^{13}\right)\left(3.96 \times 10^{3}\right) \\ &= 27.1656 \times 10^{16} \\ &= 2.71656 \times 10^{17}\end{aligned}$$

$$\begin{aligned}\text{Volume of the fourth ocean} &= \left(1.41 \times 10^{13}\right)\left(1.21 \times 10^{3}\right) \\ &= 1.7061 \times 10^{16}\end{aligned}$$

Therefore, the volumes of the four oceans are 6.2858×10^{17} cubic meters, 3.01824×10^{17} cubic meters, 2.71656×10^{17} cubic meters, and 1.7061×10^{16} cubic meters .

Answer 50e.

Consider that a continent has been moving about 0.000022 mile per year for the past 125,000,000 years.

We need to find how far has the continent moved in that time.

$$= 0.000022 \times 125,000,000$$
$$= 2.2 \times 10^{-5} \times 1.25 \times 10^{8}$$
$$= 2.2 \times 1.25 \times 10^{-5} \times 10^{8}$$
$$= 2.75 \times 10^{-5+8} \quad \text{(Using product of powers property)}$$
$$= 2.75 \times 10^{3}$$

The above number is in scientific notation because it is in the form $c \times 10^{n}$, where $1 \le c < 10$ and n is an integer.

Answer 51e.

It is given that the diameter of the bead is 6 millimeters. From this, we get the radius as 3 millimeters.

Similarly, since the diameter of the pearl is 9 millimeters, its radius is 4.5 millimeters.

We know that the volume of a spherical region is $\frac{4}{3}\pi r^3$, where r is the radius. Divide the volume of the pearl by the volume of the bead so that we can compare and determine the greatest among the two.

$$\frac{\text{Volume of the pearl}}{\text{Volume of the bead}} = \frac{\frac{4}{3}\pi\left(\frac{9}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{6}{2}\right)^3}$$

Simplify.

$$\frac{\frac{4}{3}\pi\left(\frac{9}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{6}{2}\right)^3} = \frac{\cancel{\frac{4}{3}\pi}\left(\frac{9}{2}\right)^3}{\cancel{\frac{4}{3}\pi}\left(\frac{6}{2}\right)^3}$$
$$= \frac{729}{216}$$
$$= \frac{27}{8}$$

Therefore, it can be seen that the volume of the pearl is $\frac{27}{8}$ times as large as the volume of the bead.

Answer 52e.

Consider that a can of tennis balls consists of three spheres of radius r stacked vertically inside a cylinder of radius r and height h.

(a) Volume of a single tennis ball is $\frac{4}{3}\pi r^3$.

Therefore, the total volume of three tennis balls is

$$3\left(\frac{4}{3}\pi r^3\right) = 4\pi r^3$$

(b) Volume of the cylinder is $\pi r^2 h$.

(c) Since, the height h of the cylinder is the sum of the diameters of the three tennis balls.

$\Rightarrow \quad h = 3(2r)$

$\Rightarrow \quad h = 6r$

(d) The fraction of can's volume that is taken up by the tennis balls is

$$\frac{4\pi r^3}{\pi r^2 h} = \frac{4r^{3-2}}{h} \qquad \text{(Using quotient of a power property)}$$

$$= 4\left(\frac{r}{h}\right)$$

$$= 4\left(\frac{r}{6r}\right) \qquad \text{(Using } h = 6r\text{)}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

Hence the result is $\boxed{\frac{2}{3}}$.

Answer 53e.

A penny is a cylinder with radius 9.53 millimeters and height is 1.55 millimeters.

(a) We need to find an approximate volume of a penny.
Let V denote the volume of the penny.
Then,

$$V = \pi\left(9.53\times10^{-3}\right)\left(1.55\times10^{-3}\right)$$

$$\approx \left(3.14\times9.53\times1.55\right)\times\left(10^{-3}\times10^{-3}\right)$$

$$\approx 46.4\times10^{-3-3} \qquad \text{(Using product of powers property)}$$

$$= 46.4\times10^{-6}$$

$$= 4.64\times10^{-7} \quad \text{cubic meters.}$$

(b) The dimension of my classroom is $5m \times 3m \times 4m$
So, the volume of my classroom is

$$\begin{aligned} V_c &= (5)(3)(4) \\ &= 60 \text{ cubic meters} \end{aligned}$$

(c) we need to find the number of pennies it would take to fill my classroom.

Number of pennies required is $\dfrac{60}{4.64 \times 10^{-7}} \approx 129310344$.

The above approximation is an under estimated of the actual number.

Answer 54e.

(a) Given that Earth's radius is about 5 times as great as the radius of Earth's inner core.
We need to find the ratio of Earth's total volume to the volume of Earth's inner core.
The required ratio is

$$\begin{aligned} \frac{\frac{4}{3}\pi(5r)^3}{\frac{4}{3}\pi r^3} &= \frac{(5r)^3}{r^3} \\ &= \left(\frac{5r}{r}\right)^3 \qquad \text{(Using power of a quotient property)} \\ &= 5^3 \\ &= 125 \end{aligned}$$

Hence the result is $\boxed{125}$.

(a) Given that Earth's radius is about 5 times as great as the radius of Earth's inner core.
We need to find the ratio of Earth's total volume to the volume of Earth's inner core.
The required ratio is

$$\begin{aligned} \frac{\frac{4}{3}\pi(5r)^3}{\frac{4}{3}\pi r^3} &= \frac{(5r)^3}{r^3} \\ &= \left(\frac{5r}{r}\right)^3 \qquad \text{(Using power of a quotient property)} \\ &= 5^3 \\ &= 125 \end{aligned}$$

Hence the result is $\boxed{125}$.

(b) We need to find the ratio of volume of Earth's outer core to the volume of Earth's inner core.
The required ratio is

$$\frac{\frac{4}{3}\pi\left\{\left(1+\frac{11}{6}\right)^3 r^3 - r^3\right\}}{\frac{4}{3}\pi r^3} = \frac{\left(\frac{17}{6}\right)^3 r^3 - r^3}{r^3}$$

$$= \left(\frac{17}{6}\right)^3 - 1$$

$$= \frac{4697}{216}$$

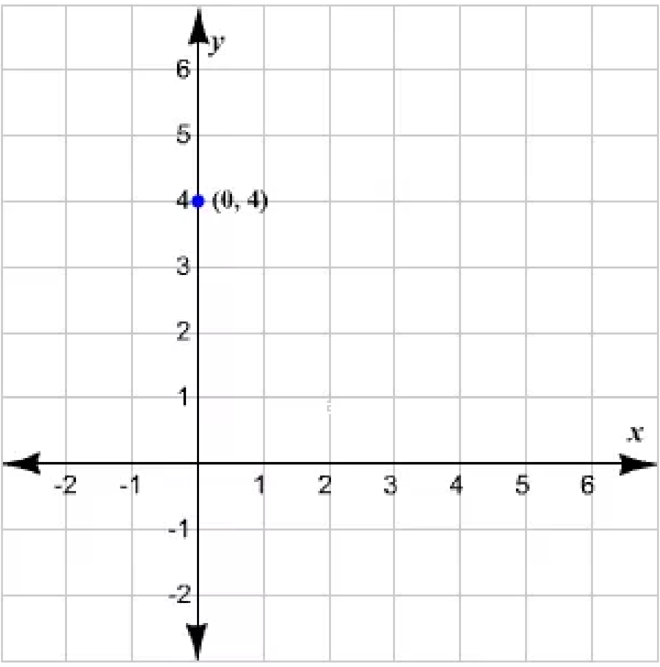
Hence the result is $\boxed{\frac{4697}{216}}$.

Answer 55e.

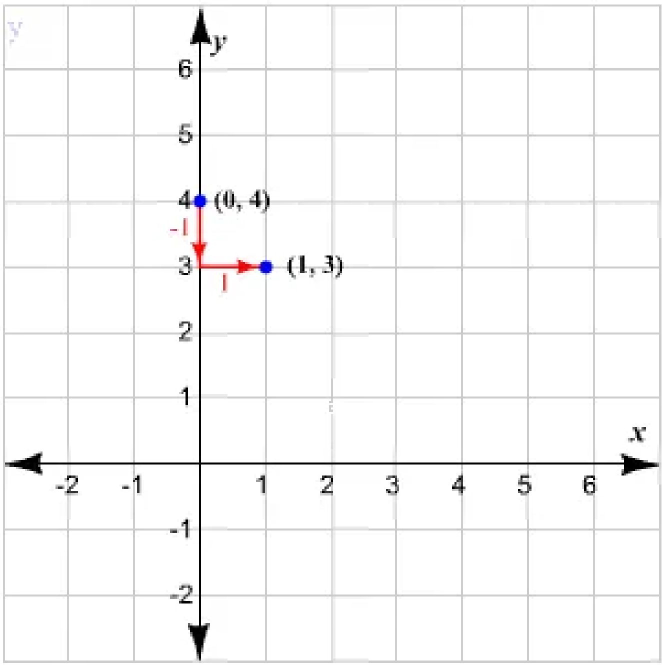
STEP 1 The slope-intercept form of a linear equation is $y = mx + b$. The given equation is already in the slope-intercept form.

On comparing the given equation with $y = mx + b$, we find that m is -1, and b is 4.

STEP 2 The y-intercept is 4. Plot the point (0, 4) on a coordinate plane where the line crosses the y-axis.

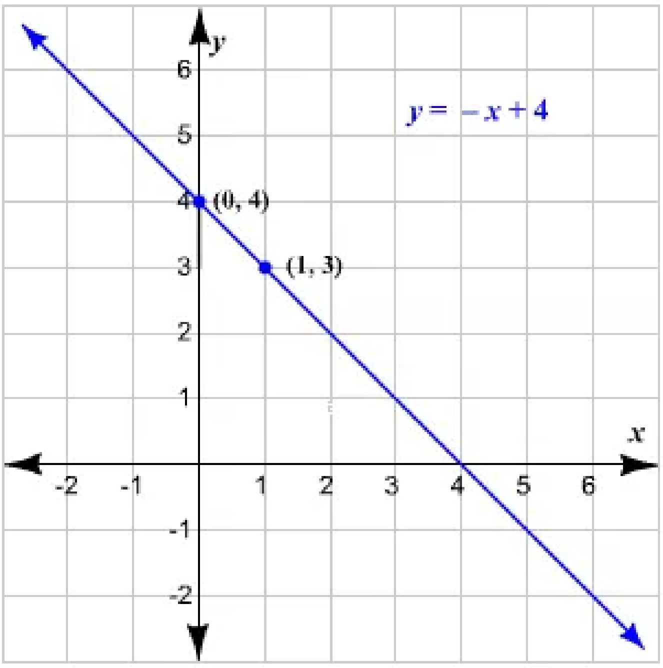


STEP 3 Use the slope to plot a second point on the line. Since the slope is −1, or $\frac{-1}{1}$, start at (0, 4) and move 1 unit down. Now, move 1 unit to the right.



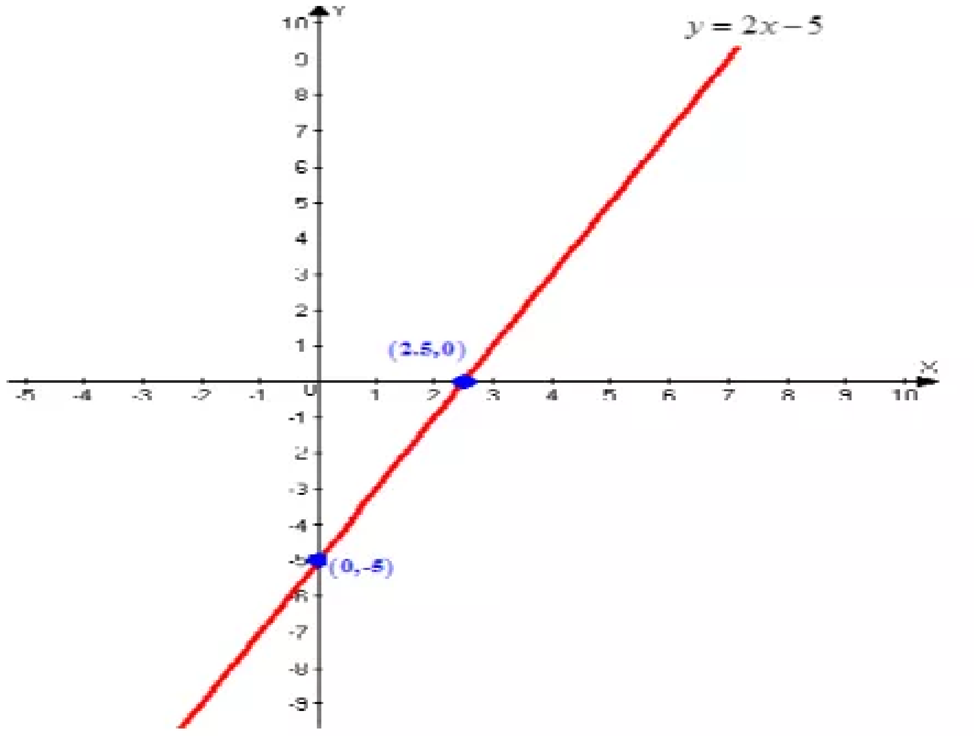
The second point is (1, 3).

STEP 4 Finally, draw a line through the two points.



Answer 56e.

Consider the function: $y = 2x - 5$.



The graph of $y = 2x - 5$ represents a straight line.

It intersects x-axis at $(2.5, 0)$ and intersects y-axis at $(0, -5)$.

Answer 57e.

STEP 1 We need to find some points to graph the function. For this, make a table of values for the given function.

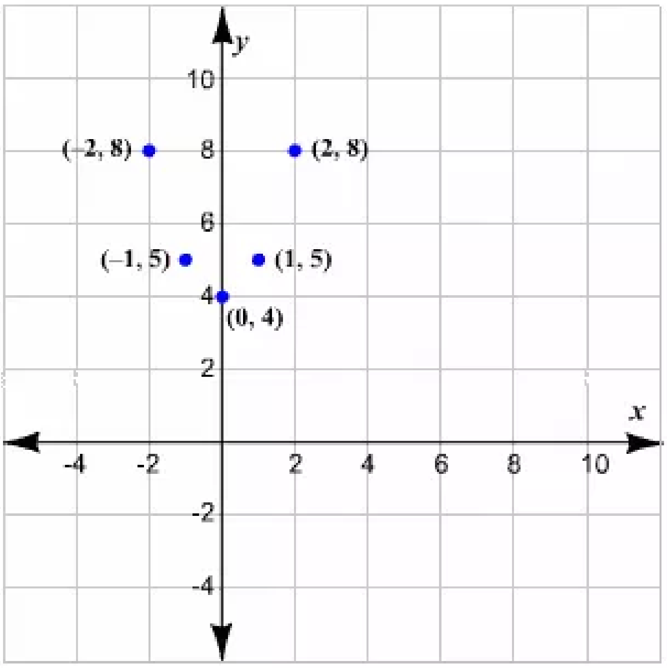
Substitute any value for x, say, -2 and evaluate y.

$$f(-1) = (-1)^2 + 4$$
$$= 1 + 4$$
$$= 5$$

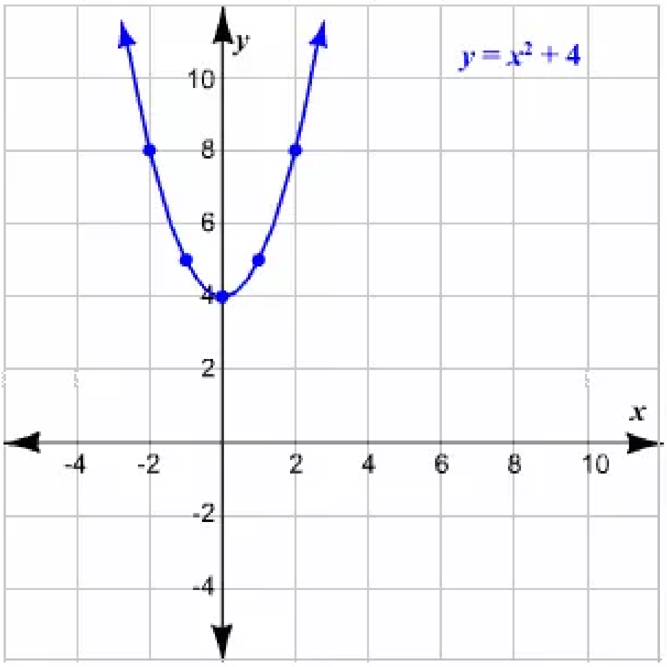
Choose some more x-values and find the corresponding y-values. Organize the results in a table.

x	-2	-1	0	1	2
y	8	5	4	5	8

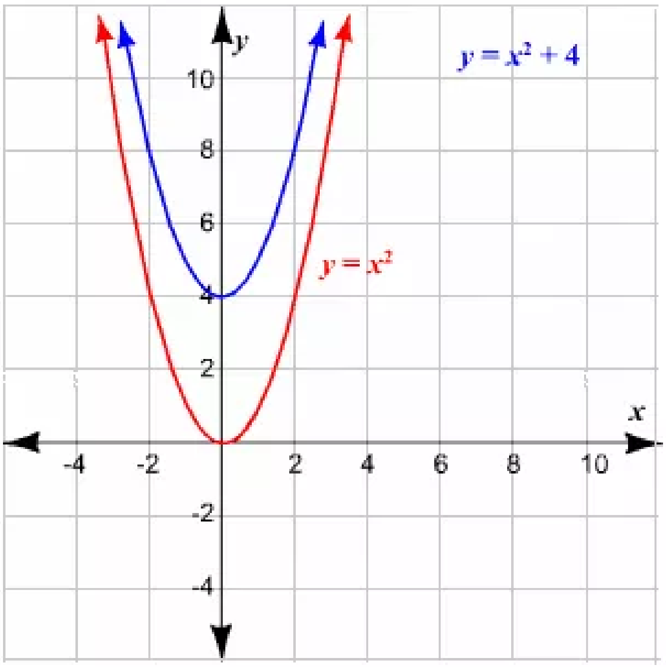
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



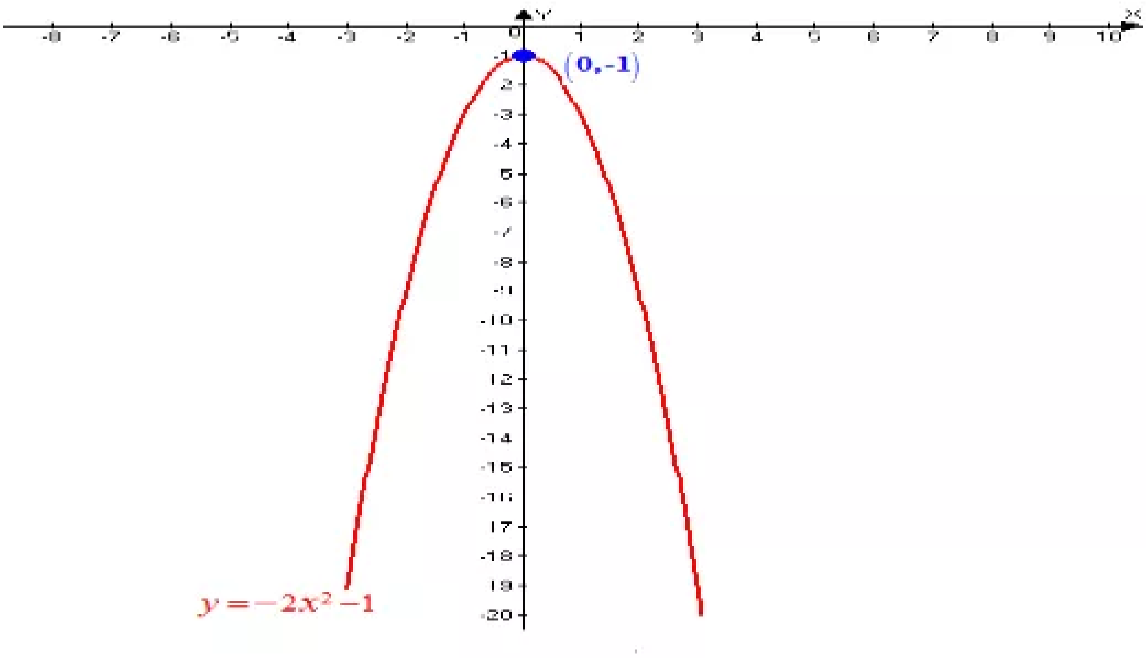
STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



On comparing, it can be seen that both the graphs have the same axis of symmetry. The graph of $y = x^2 + 4$ opens up, but its vertex is 4 units lower than the graph of $y = x^2$.

Answer 58e.

Consider the equation : $y = -2x^2 - 1$.



The equation $y = -2x^2 - 1$ represents parabola.

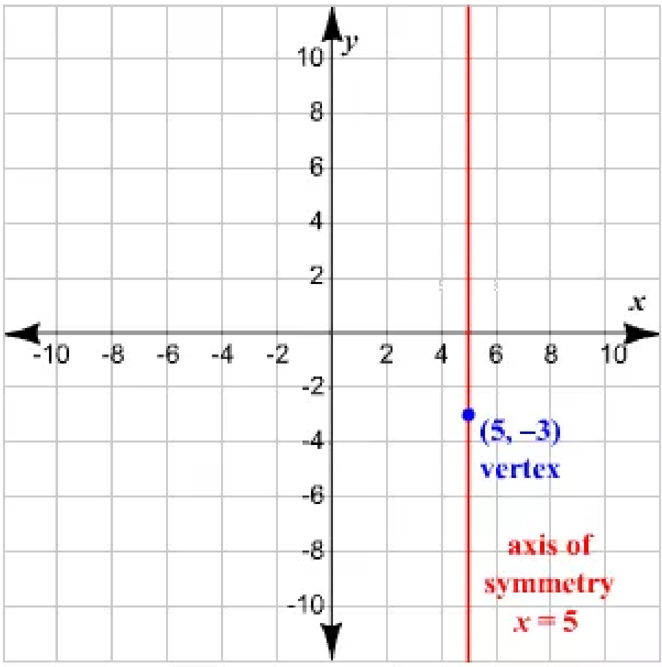
It intersects y-axis at $(0, -1)$.

Answer 59e.

STEP 1 The graph of a quadratic function in the vertex form $y = a(x - h)^2 + k$ has its vertex at (h, k) and has $x = h$ as the axis of symmetry.

In order to graph the given function, first we have to identify the constants.
On comparing the given equation with the vertex form, we find that $a = 1$, $h = 5$, and $k = -3$. Thus, the vertex is $(h, k) = (5, -3)$, and the axis of symmetry is $x = 5$. Since $a > 0$, the parabola opens up.

STEP 2 Plot the vertex $(5, -3)$ on a coordinate plane and draw the axis of symmetry $x = 5$.



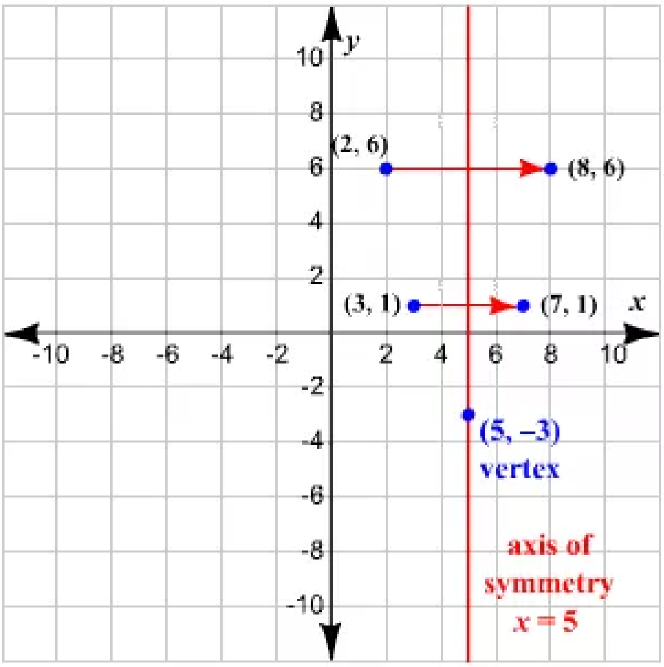
STEP 3 Evaluate the function for any two values of x.

$x = 2: \quad y = (2 - 5)^2 - 3 = 6$

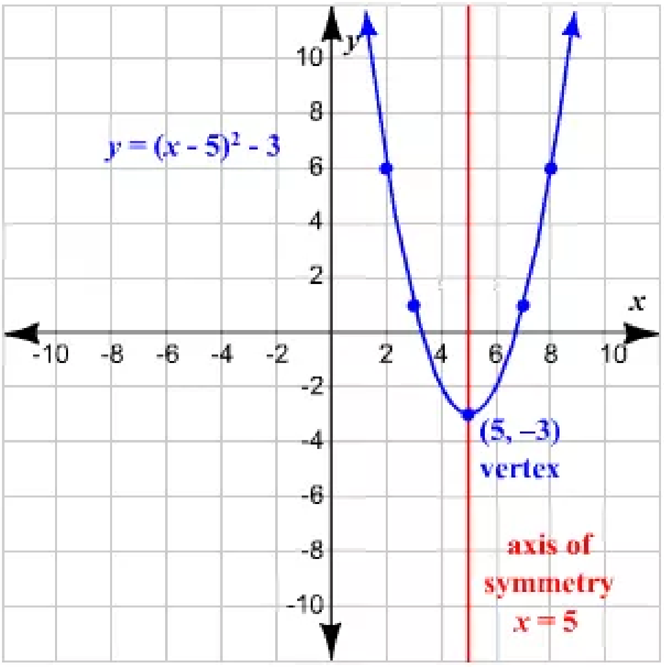
$x = 3: \quad y = (3 - 5)^2 - 3 = 1$

Thus, $(2, 6)$ and $(3, 1)$ are two points on the graph.

Now, plot the points (2, 6) and (3, 1) and their reflections in the axis of symmetry.

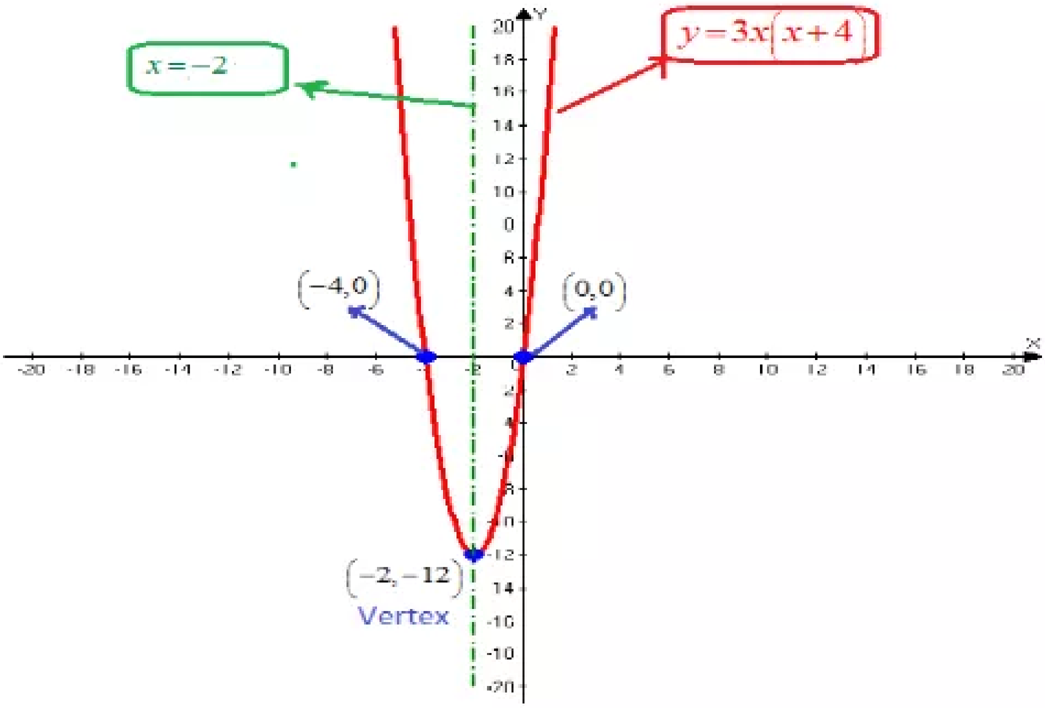


STEP 4 Draw a parabola through the points plotted.



Answer 60e.

Consider the function : $y = 3x(x+4)$.



The graph represents parabola.

The vertex of parabola is $(-2,-12)$, and x-intercepts are -4 and 0.

And the axis of parabola is $x=-2$.

Answer 61e.

First, name the equations.

$x + y = 2$ Equation 1

$7x + 8y = 21$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)	matrix of variables (X)	matrix of constants (B)

$$\begin{bmatrix} 1 & 1 \\ 7 & 8 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 21 \end{bmatrix}$$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 1 \\ 7 & 8 \end{vmatrix} \\ &= (1)(8) - (7)(1) \\ &= 8 - 7 \\ &= 1 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix}.$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B = \begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 21 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{aligned} \begin{bmatrix} 8 & -1 \\ -7 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 21 \end{bmatrix} &= \begin{bmatrix} 8(2) - (-1)(21) \\ -7(2) - 1(21) \end{bmatrix} \\ &= \begin{bmatrix} 16 - 21 \\ -14 + 21 \end{bmatrix} \\ &= \begin{bmatrix} -5 \\ 7 \end{bmatrix} \end{aligned}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$-5 + 7 \stackrel{?}{=} 2$$
$$7 - 5 \stackrel{?}{=} 2$$
$$2 = 2 \checkmark$$

Equation 2

$$7(-5) + 8(7) \stackrel{?}{=} 21$$
$$-35 + 56 \stackrel{?}{=} 21$$
$$21 = 21 \checkmark$$

Therefore, the solution of the system is (–5, 7).

Answer 62e.

Consider the linear system $\begin{cases} -x-2y=3 \\ 2x+8y=1 \end{cases}$

The system $\begin{cases} -x-2y=3 \\ 2x+8y=1 \end{cases}$ can be expressed as $AX=B$.

Where $A=\begin{bmatrix} -1 & -2 \\ 2 & 8 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$ and $B=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Then

$$A^{-1}(AX)=A^{-1}B$$
$$(A^{-1}A)X=A^{-1}B$$
$$IX=A^{-1}B$$
$$X=A^{-1}B$$

Where, $A^{-1}=\frac{1}{|A|}(adjA)$.

Now find A^{-1}, which is follows.

$$A^{-1} = \frac{1}{|A|}(adjA)$$
$$= \frac{1}{\begin{vmatrix} -1 & -2 \\ 2 & 8 \end{vmatrix}} \begin{bmatrix} 8 & 2 \\ -2 & -1 \end{bmatrix}$$
$$= -\frac{1}{4}\begin{bmatrix} 8 & 2 \\ -2 & -1 \end{bmatrix}$$

Therefore,

$$X = A^{-1}B$$
$$X = -\frac{1}{4}\begin{bmatrix} 8 & 2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 26 \\ -7 \end{bmatrix}$$
$$= \begin{bmatrix} -26/4 \\ 7/4 \end{bmatrix}$$
$$= \begin{bmatrix} -13/2 \\ 7/4 \end{bmatrix}$$

Hence the solution of the linear system is $\boxed{\left(-\frac{13}{2}, \frac{7}{4}\right)}$.

Answer 63e.

First, name the equations.

$4x + 3y = 6$ Equation 1

$6x - 2y = 10$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 4 & 3 \\ 6 & -2 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} 6 \\ 10 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{bmatrix} 4 & 3 \\ 6 & -2 \end{bmatrix} \\ &= (4)(-2) - (6)(3) \\ &= -8 - 18 \\ &= -26 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\begin{aligned} A^{-1} &= \frac{1}{-26}\begin{bmatrix} -2 & -3 \\ -6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{13} & \frac{3}{26} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix} \end{aligned}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B = \begin{bmatrix} \frac{1}{13} & \frac{3}{26} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix}\begin{bmatrix} 6 \\ 10 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} \frac{1}{13} & \frac{3}{26} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix}\begin{bmatrix} 6 \\ 10 \end{bmatrix} = \begin{bmatrix} \frac{1}{13}(6) + \frac{3}{26}(10) \\ \frac{3}{13}(6) - \frac{2}{13}(10) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{13} + \frac{15}{13} \\ \frac{18}{13} - \frac{20}{13} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{21}{13} \\ -\frac{2}{13} \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{21}{13} \\ -\frac{2}{13} \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1	**Equation 2**
$4\left(\frac{21}{13}\right) + 3\left(-\frac{2}{13}\right) \stackrel{?}{=} 6$	$6\left(\frac{21}{13}\right) - 2\left(-\frac{2}{13}\right) \stackrel{?}{=} 10$
$\frac{84}{13} - \frac{6}{13} \stackrel{?}{=} 6$	$\frac{126}{13} + \frac{4}{13} \stackrel{?}{=} 10$
$\frac{78}{13} = 6$	$\frac{130}{13} = 10$
$6 = 6$ ✓	$10 = 10$ ✓

Therefore, the solution of the system is $\left(\frac{21}{13}, -\frac{2}{13}\right)$.

Answer 64e.

Consider the expression : $(8+3i)-(7+4i)$.

We need to write the following expression as a complex number in standard form.

$$\begin{aligned}(8+3i)-(7+4i) &= 8+3i-7-4i \\ &= (8-7)+(3-4)i \\ &= 1+(-1)i \\ &= 1-i\end{aligned}$$

Hence the result is $\boxed{1-i}$.

Answer 65e.

For subtracting two complex numbers, subtract their real parts and their imaginary parts separately.

$(5-2i)-(-9+6i) = [5-(-9)]+(-2-6)i$

Simplify.

$[5-(-9)]+(-2-6)i = 14+(-8)i$

Write in standard form.

$14+(-8)i = 14-8i$

Thus, the given expression simplifies to $14-8i$.

Answer 66e.

Consider the number $i(3+i)$.

We need to write the following expression as a complex number in standard form.

$$\begin{aligned}i(3+i) &= 3i+i^2 \\ &= 3i+(-1) \qquad (\text{Since } i^2=-1) \\ &= -1+3i\end{aligned}$$

The number $-1+3i$,which is in the form of $a+bi$.

Therefore the standard from of the expression $i(3+i)$ is $\boxed{-1+3i}$.

Answer 67e.

For subtracting two complex numbers, subtract their real parts and their imaginary parts separately.

$(12+5i)-(7-8i) = (12-7)+[5-(-8)i]$

Simplify.
$(12 - 7) + [5 - (-8)i] = 5 + 13i$

Thus, the given expression simplifies to $5 + 13i$.

Answer 68e.

Consider the expression : $(8-4i)(1+6i)$.

We need to write the following expression as a complex number in standard form.

$$\begin{aligned}(8-4i)(1+6i) &= 8+48i-4i-24i^2 && \text{(Using method of FOIL)}\\ &= 8+(48-4)i-24(-1) && (\text{Since } i^2=-1)\\ &= 32+44i\end{aligned}$$

Therefore, the standard form of the expression $(8-4i)(1+6i)$ is $\boxed{32+44i}$.

Which is in the standard form as $a+bi$.

Answer 69e.

Apply the FOIL method to multiply $8 - 4i$ and $1 + 6i$.
$(8 - 4i)(1 + 6i) = (8 + 48i - 4i - 24i^2)$

We know that $i^2 = -1$.
$(8 + 48i - 4i - 24i^2) = [8 + 48i - 4i - 24(-1)]$

Simplify within the brackets.

$$\begin{aligned}\left[8 + 48i - 4i - 24(-1)\right] &= 8 + 48i - 4i + 24\\ &= 32 + 4i\end{aligned}$$

Thus, the given expression simplifies to $32 + 4i$.