3. Algebra

Exercise 3.1

1. Question

Solve the following system of equation by elimination method.

$x + 2y = 7$, $x - 2y = 1$

Answer

The given equations are

$x + 2y = 7$ … (1)

$x - 2y = 1$ … (2)

Adding (1) and (2),

$\Rightarrow x + 2y + x - 2y = 7 + 1$

$\Rightarrow 2x = 8$

$\Rightarrow x = 4$

Substituting $x = 4$ in (1),

$\Rightarrow 4 + 2y = 7$

$\Rightarrow 2y = 7 - 4 = 3$

$\Rightarrow y = \frac{3}{2}$

$\therefore (4, \frac{3}{2})$ is the solution to the given system.

2. Question

Solve the following system of equation by elimination method.

$3x + y = 8$, $5x + y = 10$

Answer

The given equations are

$3x + y = 8$ … (1)

$5x + y = 10$ … (2)

Here, the coefficients of y in both equations are numerically equal.

Subtracting (2) from (1),

$\Rightarrow (3x + y) - (5x + y) = 8 - 10$

$\Rightarrow 3x + y - 5x - y = -2$

$\Rightarrow -2x = -2$

$\Rightarrow x = 1$

Substituting $x = 1$ in (1),

$\Rightarrow 3(1) + y = 8$

$\Rightarrow y = 8 - 3$

$\Rightarrow y = 5$

$\therefore (1, 5)$ is the solution to the given system.

3. Question

Solve the following system of equation by elimination method.

$$x + \frac{y}{2} = 4, \frac{x}{3} + 2y = 5$$

Answer

The given equations are

$x + \frac{y}{2} = 4$ … (1)

$\frac{x}{3} + 2y = 5$ … (2)

(1) becomes 2x + y = 8

(2) becomes x + 6y = 15

Now, (2) × 2 – (1)

⇒ 2x + 12y – (2x + y) = 30 – 8

⇒ 2x + 12y – 2x – y = 22

⇒ 11y = 22

⇒ y = 2

Substituting y = 2 in (1),

⇒ 2x + 2 = 8

⇒ 2x = 6

⇒ x = 3

∴ (3, 2) is the solution to the given system.

4. Question

Solve the following system of equation by elimination method.

11x – 7y = xy, 9x – 4y = 6xy

Answer

The given equations are

11x – 7y = xy … (1)

9x – 4y = 6xy … (2)

Dividing both sides of the equation by xy,

⇒ $\frac{11}{y} - \frac{7}{x} = 1$ i.e. $\frac{-7}{x} + \frac{11}{y} = 1$ … (3)

⇒ $\frac{9}{y} - \frac{4}{x} = 6$ i.e. $\frac{-4}{x} + \frac{9}{y} = 6$ … (4)

Let $a = \frac{1}{x}$ and $b = \frac{1}{y}$.

Equations (3) and (4) become

⇒ – 7a + 11b = 1 … (5)

⇒ – 4a + 9y = 6 … (6)

Now, (6) × 7 – (5) × 4

⇒ – 28a + 63b – (– 28a + 44b) = 42 – 4

⇒ – 28a + 63b + 28a – 44b = 38

⇒ 19b = 38

⇒ b = 2

Substituting b = 2 in (5),

⇒ – 7a + 11(2) = 1

⇒ – 7a = 1 – 22 = – 21

⇒ a = 3

When a = 3, we have $\frac{1}{x}$ = 3. Thus, x = $\frac{1}{3}$

When b = 2, we have $\frac{1}{y}$ = 2. Thus, y = $\frac{1}{2}$

∴ ($\frac{1}{3}$, $\frac{1}{2}$) is the solution for the given system.

5. Question

Solve the following system of equation by elimination method.

$$\frac{3}{x}+\frac{5}{y}=\frac{20}{xy}, \frac{2}{x}+\frac{5}{y}=\frac{15}{xy}, x \neq 0, y \neq 0$$

Answer

The given equations are

$\frac{3}{x}+\frac{5}{y}=\frac{20}{xy}$ … (1)

$\frac{2}{x}+\frac{5}{y}=\frac{15}{xy}$ … (2)

Multiplying both sides of the equation with xy,

3y + 5x = 20 i.e. 5x + 3y = 20 … (3)

2y + 5x = 15 i.e. 5x + 2y = 15 … (4)

Subtracting (4) from (3),

⇒ 5x + 3y – (5x + 2y) = 20 – 15

⇒ 5x + 3y – 5x – 2y = 5

⇒ y = 5

Substituting y = 5 in (3),

⇒ 5x + 3 (5) = 20

⇒ 5x = 20 – 15 = 5

⇒ x = 1

∴ (1, 5) is the solution for the given system.

6. Question

Solve the following system of equation by elimination method.

8x – 3y = 5xy, 6x – 5y = – 2xy

Answer

The given equations are

8x – 3y = 5xy … (1)

6x – 5y = – 2xy … (2)

Dividing both sides of the equation by xy,

⇒ $\frac{8}{y}$ – $\frac{3}{x}$ = 5 i.e. $\frac{-3}{x}$ + $\frac{8}{y}$ = 5 … (3)

⇒ $\frac{6}{y}$ – $\frac{5}{x}$ = – 2 i.e. $\frac{-5}{x}$ + $\frac{6}{y}$ = – 2 … (4)

Let a = $\frac{1}{x}$ and b =$\frac{1}{y}$.

Equations (3) and (4) become

⇒ – 3a + 8b = 5 … (5)

⇒ – 5a + 6y = – 2 … (6)

Now, (5) × 5 – (6) × 3

⇒ – 15a + 40b – (– 15a + 18b) = 25 – (– 6)

⇒ – 15a + 40b + 15a – 18b = 31

⇒ 22b = 31

⇒ b = $\frac{31}{22}$

Substituting b = $\frac{31}{22}$ in (5),

⇒ – 3a + 8($\frac{31}{22}$) = 5

⇒ – 3a = 5 – $\frac{124}{11}$ = $\frac{-69}{11}$

⇒ a = $\frac{23}{11}$

When a =$\frac{23}{11}$, we have $\frac{1}{x}$ =$\frac{23}{11}$. Thus, x = $\frac{11}{23}$

When b =$\frac{31}{22}$, we have $\frac{1}{y}$ =$\frac{31}{22}$. Thus, y = $\frac{31}{22}$

∴ ($\frac{11}{23}$, $\frac{31}{22}$) is the solution for the given system.

7. Question

Solve the following system of equation by elimination method.

13x + 11y = 70, 11x + 13y = 74

Answer

The given equations are

13x + 11y = 70 … (1)

11x + 13y = 74 … (2)

Adding (1) and (2),

⇒ 24x + 24y = 144

Dividing by 24,

⇒ x + y = 6 … (3)

Subtracting (2) from (1),

⇒ $2x + (-2y) = -4$

Dividing by 2,

⇒ $x - y = -4$ … (4)

Solving (3) and (4),

⇒ $x + y + (x - y) = 6 - 4$

⇒ $2x = 2$

⇒ $x = 1$

Substituting $x = 1$ in (3),

⇒ $1 + y = 6$

⇒ $y = 5$

∴ (1, 5) is the solution to the given system.

8. Question

Solve the following system of equation by elimination method.

$65x - 33y = 97$, $33x - 65y = 1$

Answer

The given equations are

$65x - 33y = 97$ … (1)

$33x - 65y = 1$ … (2)

Adding (1) and (2),

⇒ $98x - 98y = 98$

Dividing by 98,

⇒ $x - y = 1$ … (3)

Subtracting (1) and (2),

⇒ $32x + 32y = 96$

Dividing by 32,

⇒ $x + y = 3$ … (4)

Solving (3) and (4),

⇒ $x - y + (x + y) = 1 + 3$

⇒ $2x = 4$

⇒ $x = 2$

Substituting $x = 2$ in (4),

⇒ $2 + y = 3$

⇒ $y = 1$

∴ (2, 1) is the solution to the given system.

9. Question

Solve the following system of equation by elimination method.

$$\frac{15}{x}+\frac{2}{y}=17, \frac{1}{x}+\frac{1}{y}=\frac{36}{5}, x \neq 0, y \neq 0$$

Answer

The given equations are

$\frac{15}{x}+\frac{2}{y}=17$ … (1)

$\frac{1}{x}+\frac{1}{y}=\frac{36}{5}$ … (2)

Let a = $\frac{1}{x}$ and b = $\frac{1}{y}$.

⇒ 15a + 2b = 17 … (3)

⇒ a + b = $\frac{36}{5}$ … (4)

Now, (3) – (4) × 2

⇒ 15a + 2b – (2a + 2b) = 17 – $\frac{72}{5}$

⇒ 15a + 2b – 2a – 2b = $\frac{85-72}{5}$

⇒ 13a = $\frac{13}{5}$

⇒ a = $\frac{1}{5}$

Substituting a = $\frac{1}{5}$ in (4),

⇒ $\frac{1}{5}$ + b = $\frac{36}{5}$

⇒ b = $\frac{35}{5}$ = 7

When a = $\frac{1}{5}$, $\frac{1}{x}=\frac{1}{5}$. Thus, x = 5.

When b = 7, $\frac{1}{y}$ = 7. Thus, y = $\frac{1}{7}$

∴ (5, $\frac{1}{7}$) is the solution to the given system.

10. Question

Solve the following system of equation by elimination method.

$$\frac{2}{x}+\frac{2}{3y}=\frac{1}{6}, \frac{3}{x}+\frac{2}{y}=0, x \neq 0, y \neq 0$$

Answer

The given equations are

$\frac{2}{x}+\frac{2}{3y}=\frac{1}{6}$ … (1)

$\frac{3}{x}+\frac{2}{y}$ = 0 … (2)

Let a = $\frac{1}{x}$ and b = $\frac{1}{y}$.

⇒ 2a + $\frac{2}{3}$b = $\frac{1}{6}$ … (3)

⇒ 3a + 2b = 0 … (4)

Now, (3) × 3 – (4) × 2

⇒ 6a + 2b – (6a + 4b) = 1/2 – 0

⇒ 6a + 2b – 6a – 4b = 1/2

⇒ – 2b = 1/2

⇒ b = $\frac{-1}{4}$

Substituting b = $\frac{-1}{4}$ in (4),

⇒ 3a + 2($\frac{-1}{4}$) = 0

⇒ 3a = 1/2

⇒ a = $\frac{1}{6}$

When a =$\frac{1}{6}$, $\frac{1}{x}$ =$\frac{1}{6}$. Thus, x = 6.

When b =$\frac{-1}{4}$, $\frac{1}{y}$ =$\frac{-1}{4}$. Thus, y = – 4

∴ (6, – 4) is the solution to the given system.

Exercise 3.2

1 A. Question

Solve the following systems of equation using cross multiplication method.

3x + 4y = 24, 20x – 11y = 47

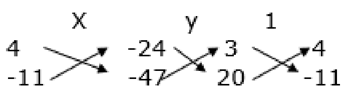
Answer

The given system of equations is

3x + 4y – 24 = 0 and

20x – 11y – 47 = 0

For cross multiplication method, we write the coefficients as



Hence, we get $\frac{x}{4(-47)-(-11)(-24)} = \frac{y}{(-24)(20)-3(-47)} = \frac{1}{3(-11)-4(20)}$

⇒ $\frac{x}{-188-264} = \frac{y}{-480+141} = \frac{1}{-33-80}$

⇒ $\frac{x}{-452} = \frac{y}{-339} = \frac{1}{-113}$

⇒ x = $\frac{-452}{-113}$ = 4

⇒ y = $\frac{-339}{-113}$ = 3

∴ (4, 3) is the solution to the given system.

1 B. Question

Solve the following systems of equation using cross multiplication method.

0.5x + 0.8y = 0.44, 0.8x + 0.6y = 0.5

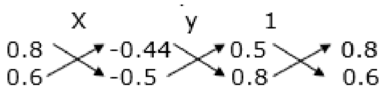
Answer

The given equations are

0.5x + 0.8y – 0.44 = 0

0.8x +0.6y – 0.5 = 0

For cross multiplication method, we write the coefficients as



Hence, we get $\frac{x}{0.8(-0.5)-(-0.44)(0.6)} = \frac{y}{(-0.44)(0.8)-0.5(-0.5)} = \frac{1}{0.5(0.6)-0.8(0.8)}$

$\Rightarrow \frac{x}{-0.4+0.264} = \frac{y}{-0.352+0.25} = \frac{1}{0.3-0.64}$

$\Rightarrow \frac{x}{-0.136} = \frac{y}{-0.102} = \frac{1}{-0.34}$

$\Rightarrow x = \frac{-0.136}{-0.34} = 0.4$

$\Rightarrow y = \frac{-0.102}{-0.34} = 0.3$

∴ (0.4, 0.3) is the solution to the given system.

1 C. Question

Solve the following systems of equation using cross multiplication method.

$$\frac{3x}{2} - \frac{5y}{3} = -2, \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

Answer

The given equations are

$\frac{3x}{2} - \frac{5y}{3} = -2$ … (1)

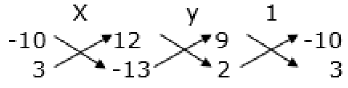
$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ … (2)

By taking LCM,

(1) becomes 9x – 10y + 12 = 0

(2) becomes 2x + 3y – 13 = 0

For cross multiplication method, we write the coefficients as



Hence, we get $\frac{x}{(-10)(-13)-(3)(12)} = \frac{y}{(12)(2)-9(-13)} = \frac{1}{9(3)-2(-10)}$

$\Rightarrow \frac{x}{130-36} = \frac{y}{24+117} = \frac{1}{27+20}$

$\Rightarrow \frac{x}{94} = \frac{y}{141} = \frac{1}{47}$

$\Rightarrow x = \frac{94}{47} = 2$

$\Rightarrow y = \frac{141}{47} = 3$

∴ (2, 3) is the solution to the given system.

1 D. Question

Solve the following systems of equation using cross multiplication method.

$$\frac{5}{x} - \frac{4}{y} = -2, \frac{2}{x} + \frac{3}{y} = 13$$

Answer

The given equations are

$\frac{5}{x} - \frac{4}{y} = -2$ … (1)

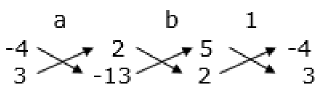
$\frac{2}{x} + \frac{3}{y} = 13$ … (2)

Let $a = \frac{1}{x}$ and $b = \frac{1}{y}$.

⇒ 5a – 4b + 2 = 0 … (3)

⇒ 2a + 3b – 13 = 0 … (4)

For cross multiplication method, we write the coefficients as



Hence, we get $\frac{a}{(-4)(-13)-(3)(2)} = \frac{b}{(2)(2)-5(-13)} = \frac{1}{5(3)-2(-4)}$

$\Rightarrow \frac{a}{52-6} = \frac{b}{4+65} = \frac{1}{15+8}$

$\Rightarrow \frac{a}{46} = \frac{b}{69} = \frac{1}{23}$

$\Rightarrow a = \frac{46}{23} = 2$

$\Rightarrow b = \frac{69}{23} = 3$

When a = 2, $\frac{1}{x} = 2$. Thus, x = 1/2.

When b = 3, $\frac{1}{y} = 3$. Thus, $y = \frac{1}{3}$.

∴ $(\frac{1}{2}, \frac{1}{3})$ is the solution to the given system.

2 A. Question

Formulate the following problems as a pair of equations, and hence find their solutions:

One number is greater than thrice the other number by 2. If 4 times the smaller number exceeds the greater by 5, find the numbers.

Answer

Let x be the greater number and y be the smaller number.

First condition is x = 3y + 2

Equation is x – 3y – 2 = 0 … (1)

Second condition is x = 4y – 5

Equation is x – 4y + 5 = 0 … (2)

For cross multiplication method, we write the coefficients as

	x		y		1	
-3		-2		1		-3
-4		5		1		-4

Hence, we get $\frac{x}{(-3)(5)-(-2)(-4)} = \frac{y}{(-2)(1)-1(5)} = \frac{1}{1(-4)-1(-3)}$

$\Rightarrow \frac{x}{-15-8} = \frac{y}{-2-5} = \frac{1}{-4+3}$

$\Rightarrow \frac{x}{-23} = \frac{y}{-7} = \frac{1}{-1}$

$\Rightarrow x = \frac{-23}{-1} = 23$

$\Rightarrow y = \frac{-7}{-1} = 7$

∴ The numbers are 23 and 7.

2 B. Question

Formulate the following problems as a pair of equations, and hence find their solutions:

The ratio of income of two persons is 9: 7 and the ratio of their expenditure is 4: 3. If each of them manages to save ₹ 2000 per month, find their monthly income.

Answer

Let the incomes be x and expenditure be y.

We know that savings = income – expenditure

First condition is

9x – 4y = 2000

Second condition is

7x – 3y = 2000

For cross multiplication method, we write the coefficients as

	x		y		1	
-4		-2000		9		-4
-3		-2000		7		-3

Hence, we get $\frac{x}{(-4)(-2000)-(-2000)(-3)} = \frac{y}{(-2000)(7)-9(-2000)} = \frac{1}{9(-3)-7(-4)}$

$\Rightarrow \frac{x}{8000-6000} = \frac{y}{-14000+18000} = \frac{1}{-27+28}$

$\Rightarrow \frac{x}{2000} = \frac{y}{4000} = \frac{1}{1}$

⇒ x = 2000

⇒ y = 4000

∴ Income of first person = 9x = 9 × 2000 = Rs. 18, 000

Income of second person = 7x = 7 × 2000 = Rs. 14, 000

2 C. Question

Formulate the following problems as a pair of equations, and hence find their solutions:

A two digit number is seven times the sum of its digits. The number formed by reversing the digits is 18 less than the given number. Find the given number.

Answer

Let x denote the digit in the tenth place and y denote the digit in the unit place. So the number may be written as 10x + y.

When digits are reversed, the number becomes 10y + x.

First condition is

10x + y = 7 (x + y)

⇒ 10x + y – 7x – 7y = 0

⇒ 3x – 6y = 0

⇒ x – 2y = 0

Second condition is

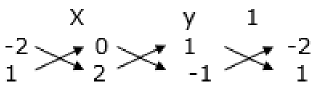
10y + x = (10x + y) – 18

⇒ 10y + x – 10x – y + 18 = 0

⇒ – 9x + 9y + 18 = 0

⇒ – x + y + 2 = 0

For cross multiplication method, we write the coefficients as



Hence, we get $\frac{x}{(-2)(2)-(0)(1)} = \frac{y}{(0)(-1)-1(2)} = \frac{1}{1(1)-(-1)(-2)}$

$\Rightarrow \frac{x}{-4-0} = \frac{y}{0-2} = \frac{1}{1-2}$

$\Rightarrow \frac{x}{-4} = \frac{y}{-2} = \frac{1}{-1}$

$\Rightarrow x = \frac{-4}{-1} = 4$

$\Rightarrow y = \frac{-2}{-1} = 2$

∴ The number is 10x + y = 10 (4) + 2 = 40 + 2 = 42.

2 D. Question

Formulate the following problems as a pair of equations, and hence find their solutions:

Three chairs and two tables cost ₹ 700 and five chairs and three tables cost ₹1100. What is the total cost of 2 chairs and 3 tables?

Answer

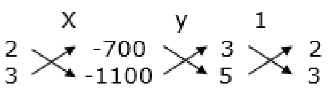
Let chairs be x and tables be y.

Then the equations are

3x + 2y = 700 i.e. 3x + 2y – 700 = 0

5x + 3y = 1100 i.e. 5x + 3y – 1100 = 0

For cross multiplication method, we write the coefficients as



Hence, we get $\frac{x}{(2)(-1100)-(3)(-700)} = \frac{y}{(-700)(5)-3(-1100)} = \frac{1}{3(3)-5(2)}$

$\Rightarrow \frac{x}{-2200+2100} = \frac{y}{-3500+3300} = \frac{1}{9-10}$

$\Rightarrow \frac{x}{-100} = \frac{y}{-200} = \frac{1}{-1}$

$\Rightarrow x = \frac{-100}{-1}$ = 100 (Cost of one chair)

$\Rightarrow y = \frac{-200}{-1}$ = 200 (Cost of one table)

Now, total cost of two chairs and three tables,

2x + 3y = 2 (100) + 3 (200) = 200 + 600 = Rs. 800

2 E. Question

Formulate the following problems as a pair of equations, and hence find their solutions:

In a rectangle, if the length is increased and the breadth is reduced each by 2 cm then the area is reduced by 28 cm^2. If the length is reduced by 1 cm and the breadth increased by 2 cm , then the area increases by 33 cm^2. Find the area of the rectangle.

Answer

Let length be l and breadth be b.

Then the first condition is

(l + 2) (b – 2) = lb – 28

⇒ lb – 2l + 2b – 4 – lb + 28 = 0

⇒ – 2l + 2b + 24 = 0

⇒ – l + b + 12 = 0

The second condition is

(l – 1) (b + 2) = lb + 33

⇒ lb + 2l – b – 2 – lb – 33 = 0

⇒ 2l – b – 35 = 0

For cross multiplication method, we write the coefficients as

l b 1

1 ⨯ 12 ⨯ -1 ⨯ 1

-1 ⨯ -35 ⨯ 2 ⨯ -1

Hence, we get $\frac{l}{(1)(-35)-(-1)(12)} = \frac{b}{(12)(2)-(-1)(-35)} = \frac{1}{-1(-1)-1(2)}$

$\Rightarrow \frac{l}{-35+12} = \frac{b}{24-35} = \frac{1}{1-2}$

$\Rightarrow \frac{l}{-23} = \frac{b}{-11} = \frac{1}{-1}$

$\Rightarrow l = \frac{-23}{-1} = 23$

$\Rightarrow b = \frac{-11}{-1} = 11$

∴ Area of rectangle= l × b = 23 × 11 = 253 cm^2

2 F. Question

Formulate the following problems as a pair of equations, and hence find their solutions:

A train travelled a certain distance at a uniform speed. If the train had been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. If the train were slower by 6 km/hr, then it would have taken 6 hours more than the scheduled time. Find the distance covered by the train.

Answer

Let the speed be x km/hr and distance travelled be y km.

<u>We know that distance = speed × time</u>

Scheduled time to cover distance = y/x hr

Then the first condition is

$\Rightarrow \frac{y}{x+6} = \frac{y}{x} - 4$

$\Rightarrow \frac{y}{x+6} = \frac{y-4x}{x}$

$\Rightarrow xy = (x + 6)(y - 4x)$

$\Rightarrow xy = xy - 4x^2 + 6y - 24x$

$\Rightarrow 4x^2 - 6y + 24x = 0$

$\Rightarrow 2x^2 - 3y + 12x = 0$ i.e. $12x - 3y + 2x^2 = 0$

Second condition is

$\Rightarrow \frac{y}{x-6} = \frac{y}{x} + 6$

$\Rightarrow \frac{y}{x-6} = \frac{y+6x}{x}$

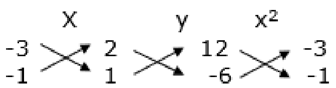
$\Rightarrow xy = (x - 6)(y + 6x)$

$\Rightarrow xy = xy + 6x^2 - 6y - 36x$

$\Rightarrow 6x^2 - 6y - 36x = 0$

$\Rightarrow x^2 - y - 6x = 0$ i.e. $-6x - y + x^2 = 0$

For cross multiplication method, we write the coefficients as



Hence, we get $\frac{x}{(-3)(1)-(-1)(2)} = \frac{y}{(-6)(2)-1(12)} = \frac{x^2}{12(-1)-(-3)(-6)}$

$\Rightarrow \frac{x}{-3+2} = \frac{y}{-12-12} = \frac{x^2}{-12-18}$

$\Rightarrow \frac{x}{-1} = \frac{y}{-24} = \frac{x^2}{-30}$

$\Rightarrow \frac{x}{-1} = \frac{x^2}{-30}$

$\Rightarrow -30x = -x^2$

$\Rightarrow$ 30km/hr = x

Now, $\frac{y}{-24} = \frac{x^2}{-30}$

$\Rightarrow \frac{y}{-24} = \frac{30^2}{-30}$

$\Rightarrow y = 24 \times 30 = 720$ km

∴ The distance covered by train = 720 km

Exercise 3.3

1 A. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$x^2 – 2x – 8$

Answer

Let $f(x) = x^2 – 2x – 8$

To find out zeros of the given polynomial.

We put $f(x) = 0$

$\Rightarrow x^2 – 2x – 8 = 0$

To find out roots of this polynomial we use splitting of middle term method.

According to this method we need to find two numbers whose sum is – 2 and product is 8.

$\therefore x^2 – (4 – 2)x – 8 = 0$

$\Rightarrow x^2 – 4x + 2x – 8 = 0$

$\Rightarrow x(x – 4) + 2(x – 4) = 0$

$\Rightarrow (x + 2)(x – 4) = 0$

$\therefore x = – 2$ and $x = 4$.

$\Rightarrow$ Our zeros are $\alpha = – 2$ and $\beta = 4$.

$\Rightarrow$ sum of zeros $= \alpha + \beta = – 2 + 4 = 2$.

$\Rightarrow$ Product of zeros $= \alpha\beta = (– 2) \times 4 = – 8$.

$\Rightarrow$ Comparing $f(x) = x^2 – 2x – 8$ with standard equation $ax^2 + bx + c = 0$.

We get, $a = 1$, $b = – 2$ and $c = – 8$

We can verify,

$\Rightarrow$ Sum of zeros $= -\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{-2}{1}$

$\therefore \alpha + \beta = 2$

$\Rightarrow$ Product of zeros $= \frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha\beta = -\frac{8}{1}$

$\alpha\beta = – 8$.

Hence, relationship between zeros and coefficient is verified.

1 B. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$4x^2 – 4x + 1$

Answer

Let $f(x) = 4x^2 – 4x + 1$

To find out zeros of the given polynomial.

We put f(x) = 0

⇒ $4x^2 - 4x + 1 = 0$

To find out roots of this polynomial we use splitting of middle term method.

According to this method we need to find two numbers whose sum is – 4 and product is 4.

∴ $4x^2 - (2 + 2)x + 1 = 0$

⇒ $4x^2 - 2x - 2x + 1 = 0$

⇒ $2x(2x - 1) - 1(2x - 1) = 0$

⇒ $(2x - 1)(2x - 1) = 0$

∴ $2x - 1 = 0$

∴ $x = \frac{1}{2}$.

Again, $2x - 1 = 0$

∴ $x = \frac{1}{2}$

⇒ Our zeros are $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$.

⇒ sum of zeros = $\alpha + \beta = \frac{1}{2} + \frac{1}{2} = 1$.

⇒ Product of zeros = $\alpha\beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Now, Comparing $f(x) = 4x^2 - 4x + 1$ with standard equation $ax^2 + bx + c = 0$.

We get, a = 4, b = – 4 and c = 1

We can verify,

⇒ Sum of zeros = $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{(-4)}{4}$

∴ $\alpha + \beta = 1$

⇒ Product of zeros = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha\beta = \frac{1}{4}$

Hence, relationship between zeros and coefficient is verified.

1 C. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$6x^2 - 3 - 7x$

Answer

Let $f(x) = 6x^2 - 3 - 7x$

Arranging equation in proper form.

Now, $f(x) = 6x^2 - 7x - 3$

To find out zeros of the given polynomial.

We put f(x) = 0

⇒ $6x^2 - 7x - 3 = 0$

To find out roots of this polynomial we use splitting of middle term method.

According to this method we need to find two numbers whose sum is – 7 and product is – 18.

∴ $6x^2 - (9 - 2)x - 3 = 0$

⇒$6x^2 - 9x + 2x - 3 = 0$

⇒ $3x(2x - 3) + 1(2x - 3) = 0$

⇒ $(3x + 1)(2x - 3) = 0$

∴ $3x + 1 = 0$

∴ $x = \frac{-1}{3}$.

Again, $2x - 3 = 0$

∴ $x = \frac{3}{2}$

⇒ Our zeros are $\alpha = \frac{-1}{3}$ and $\beta = \frac{3}{2}$.

⇒ sum of zeros = $\alpha + \beta = \frac{-1}{3} + \frac{3}{2}$

⇒ sum of zeros = $\alpha + \beta = \frac{-2+9}{6} = \frac{7}{6}$

⇒ Product of zeros = $\alpha\beta = \frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6} = -\frac{1}{2}$.

Now, Comparing $f(x) = 6x^2 - 7x - 3$ with standard equation $ax^2 + bx + c$.

We get, a = 6, b = – 7 and c = – 3.

We can verify,

⇒ Sum of zeros = $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{(-7)}{6}$

∴ $\alpha + \beta = \frac{7}{6}$

⇒ Product of zeros = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha\beta = \frac{-3}{6} = -\frac{1}{2}$

Hence, relationship between zeros and coefficient is verified.

1 D. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$4x^2 + 8x$

Answer

Let $f(x) = 4x^2 + 8x$

Arranging equation in proper form.

Now, $f(x) = 4x^2 + 8x + 0$

To find out zeros of the given polynomial.

We put f(x) = 0

⇒ f(x) = $4x^2 + 8x + 0 = 0$

∴$4x^2 + 8x = 0$

⇒4x(x + 2) = 0

Now, 4x = 0

∴ x = 0

When, (x + 2) = 0

Then, x = – 2

⇒ Our zeros are α = 0 and β = – 2.

⇒ sum of zeros = α + β = 0 + (– 2)

⇒ sum of zeros = α + β = – 2.

⇒ Product of zeros = αβ = 0 × (– 2) = 0.

Now, Comparing f(x) = $4x^2 + 8x + 0$ with standard equation $ax^2 + bx + c$.

We get, a = 4, b = 8 and c = 0.

We can verify,

⇒ Sum of zeros = $-\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$

i.e. α + β = $-\frac{8}{4}$

∴ α + β = – 2

⇒ Product of zeros = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

αβ = $\frac{0}{4} = 0$

Hence, relationship between zeros and coefficient is verified.

1 E. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$x^2 – 15$

Answer

Let f(x) = $x^2 – 15$

Arranging equation in proper form.

Now, f(x) = $x^2 + 0x – 15$

To find out zeros of the given polynomial.

We put f(x) = 0

⇒ $x^2 – 15 = 0$

∴ $x^2 - (\sqrt{15})^2 = 0$

So, $(x + \sqrt{15})(x - \sqrt{15}) = 0$

When, $(x + \sqrt{15}) = 0$

Then, $x = -\sqrt{15}$.

When. $(x - \sqrt{15}) = 0$

Then, $x = \sqrt{15}$

⇒ Our zeros are $\alpha = -\sqrt{15}$ and $\beta = \sqrt{15}$.

⇒ sum of zeros = $\alpha + \beta = -\sqrt{15} + \sqrt{15}$

⇒ sum of zeros = $\alpha + \beta = 0$

⇒ Product of zeros = $\alpha\beta = -\sqrt{15} \times \sqrt{15} = -15$

Now, Comparing $f(x) = x^2 + 0x - 15$ with standard equation $ax^2 + bx + c$.

We get, a = 1, b = 0 and c = – 15.

We can verify,

⇒ Sum of zeros $= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{0}{1}$

∴ $\alpha + \beta = 0$

⇒ Product of zeros $= \frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha\beta = \frac{-15}{1} = -15$

Hence, relationship between zeros and coefficient is verified.

1 F. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$3x^2 - 5x + 2$

Answer

Let $f(x) = 3x^2 - 5x + 2$.

To find out zeros of the given polynomial.

We put f(x) = 0

⇒ $3x^2 - 5x + 2 = 0$

To find out roots of this polynomial we use splitting of middle term method.

According to this method we need to find two numbers whose sum is – 5 and product is 6.

∴ $3x^2 - (3 + 2)x + 2 = 0$

⇒$3x^2 - 3x - 2x + 2 = 0$

⇒ $3x(x - 1) - 2(x - 1) = 0$

⇒ $(3x - 2)(x - 1) = 0$

When, $3x - 2 = 0$

Then, $x = \frac{2}{3}$.

Again when, x – 1 = 0

∴ then, x = 1

⇒ Our zeros are $\alpha = \frac{2}{3}$ and $\beta = 1$.

⇒ sum of zeros = $\alpha + \beta = \frac{2}{3} + 1$

⇒ sum of zeros = $\alpha + \beta = \frac{2+3}{3} = \frac{5}{3}$

⇒ Product of zeros = $\alpha\beta = \frac{2}{3} \times 1 = \frac{2}{3}$.

Now, Comparing $f(x) = 3x^2 - 5x + 2$ with standard equation $ax^2 + bx + c$.

We get, a = 3, b = – 5 and c = 2.

We can verify,

⇒ Sum of zeros = $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{(-5)}{3}$

∴ $\alpha + \beta = \frac{5}{3}$

⇒ Product of zeros = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha\beta = \frac{2}{3}$.

Hence, relationship between zeros and coefficient is verified.

1 G. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$2x^2 - 2\sqrt{2}\,x + 1$

Answer

Let $f(x) = 2x^2 - 2\sqrt{2}\,x + 1$

To find out zeros of the given polynomial.

We put f(x) = 0

⇒ $2x^2 - 2\sqrt{2}\,x + 1 = 0$

To find out roots of this polynomial we use splitting of middle term method.

According to this method we need to find two numbers whose sum is $-2\sqrt{2}$ and product is 2.

∴ $2x^2 - (\sqrt{2} + \sqrt{2})x + 1 = 0$

⇒ $2x^2 - \sqrt{2}x - \sqrt{2}x + 1 = 0$

⇒ $\sqrt{2}x(\sqrt{2}x - 1) - 1(\sqrt{2}x - 1) = 0$

⇒ $(\sqrt{2}x - 1)(\sqrt{2}x - 1) = 0$

⇒ $(\sqrt{2}x - 1)^2 = 0$

∴ $x = \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

⇒ Our zeros are $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = \frac{1}{\sqrt{2}}$.

⇒ sum of zeros = $\alpha + \beta = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

⇒ sum of zeros = $\alpha + \beta = \frac{2}{\sqrt{2}} = \sqrt{2}$

⇒ Product of zeros = $\alpha\beta = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$.

Now, Comparing $f(x) = 2x^2 - 2\sqrt{2}x + 1$ with standard equation $ax^2 + bx + c$.

We get, $a = 2$, $b = -2\sqrt{2}$ and $c = 1$.

We can verify,

⇒ Sum of zeros = $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{(-2\sqrt{2})}{2}$

∴ $\alpha + \beta = \sqrt{2}$

⇒ Product of zeros = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha\beta = \frac{1}{2}$.

Hence, relationship between zeros and coefficient is verified.

1 H. Question

Find the zeros of the following quadratic polynomials and verify the basic relationships between the zeros and the coefficients.

$x^2 + 2x - 143$

Answer

Let $f(x) = x^2 + 2x - 143$

To find out zeros of the given polynomial.

We put $f(x) = 0$

⇒ $x^2 + 2x - 143 = 0$

To find out roots of this polynomial we use splitting of middle term method.

According to this method we need to find two numbers whose sum is 2 and product is – 143.

∴ $x^2 + (13 - 11)x - 143 = 0$

⇒$x^2 + 13x - 11x - 143 = 0$

⇒ $x(x + 13) - 11(x + 13) = 0$

⇒ $(x - 11)(x + 13) = 0$

∴ When, $(x - 11) = 0$

∴ Then, $x = 11$.

Again, When, $(x + 13) = 0$

∴ Then, $x = -13$

⇒ Our zeros are $\alpha = 11$ and $\beta = -13$.

⇒ sum of zeros = $\alpha + \beta = 11 + (-13)$

⇒ sum of zeros = $\alpha + \beta = -2$

⇒ Product of zeros = $\alpha\beta = 11 \times (-13) = -143$

Now, Comparing $f(x) = x^2 + 2x - 143$ with standard equation $ax^2 + bx + c$.

We get, a = 1, b = 2 and c = – 143.

We can verify,

⇒ Sum of zeros = $-\frac{\text{coefficient of x}}{\text{coefficient of } x^2}$

i.e. $\alpha + \beta = -\frac{2}{1}$

∴ $\alpha + \beta = -2$

⇒ Product of zeros = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$\alpha\beta = \frac{-143}{1} = -143$

Hence, relationship between zeros and coefficient is verified.

2 A. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

3, 1

Answer

Formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = 3 and product of roots = 1

∴ Quadratic equation is,

⇒ $x^2 - 3x + 1 = 0$

Hence, Quadratic equation is $x^2 - 3x + 1 = 0$.

2 B. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

2, 4

Answer

Formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = 2 and product of roots = 4

∴ Quadratic equation is,

⇒ $x^2 - 2x + 4 = 0$

Hence, Quadratic equation is $x^2 - 2x + 4 = 0$.

2 C. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

0, 4

Answer

Formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = 0 and product of roots = 4

∴ Quadratic equation is,

⇒ $x^2 - 0x + 4 = 0$

⇒ $x^2 + 4 = 0$

Hence, Quadratic equation is $x^2 + 4 = 0$.

2 D. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

$\sqrt{2}, \frac{1}{5}$

Answer

Formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = $\sqrt{2}$ and product of roots = $\frac{1}{5}$

∴ Quadratic equation is,

⇒ $x^2 - \sqrt{2}\,x + \frac{1}{5} = 0$

Hence, Quadratic equation is $x^2 - \sqrt{2}\,x + \frac{1}{5} = 0$.

2 E. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

$\frac{1}{3}, 1$

Answer

Formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = $\frac{1}{3}$ and product of roots = 1

∴ Quadratic equation is,

⇒ $x^2 - \frac{1}{3}x + 1 = 0$

⇒ $x^2 - \frac{x}{3} + 1 = 0$

Hence, Quadratic equation is $x^2 - \frac{x}{3} + 1 = 0$

2 F. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

$\frac{1}{2}, -4$

Answer

formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = $\frac{1}{2}$ and product of roots = – 4

∴ Quadratic equation is,

⇒ $x^2 - \frac{1}{2}x - 4 = 0$

⇒ $x^2 - \frac{x}{2} - 4 = 0$

Hence, Quadratic equation is $x^2 - \frac{x}{2} - 4 = 0$.

2 G. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

$\frac{1}{2}, -\frac{1}{3}$

Answer

Formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = $\frac{1}{2}$ and product of roots = $-\frac{1}{3}$

∴ Quadratic equation is,

⇒ $x^2 - \frac{1}{2}x - \frac{1}{3} = 0$

⇒ $x^2 - \frac{x}{2} - \frac{1}{3} = 0$

Hence, Quadratic equation is $x^2 - \frac{x}{2} - \frac{1}{3} = 0$.

2 H. Question

Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

$\sqrt{3}, 2$

Answer

Formula for quadratic equation is,

x^2 – (sum of roots) x + Product of roots = 0

Given, sum of roots = $\sqrt{3}$ and product of roots = 2

∴ Quadratic equation is,

⇒ $x^2 - \sqrt{3}x + 2 = 0$

Hence, Quadratic equation is $x^2 - \sqrt{3}x + 2 = 0$.

Exercise 3.4

1 A. Question

Find the quotient and remainder using synthetic division.

$x^3 + x^2 – 3x + 5) \div (x – 1)$

Answer

Let p(x) = $x^3 + x^2 – 3x + 5$ be the dividend. Arranging p(x) according to the descending powers of x.

$p(x) = x^3 + x^2 – 3x + 5$

Divisor, q(x) = x – 1

⇒ To find out Zero of the divisor –

q(x) = 0

x – 1 = 0

x = 1

So, zero of divisor is 1.

⇒ $p(x) = x^3 + x^2 – 3x + 5$

Put zero for the first entry in the second row.

1	1	1	- 3	5
	0	1× 1 =1	2× 1 = 2	-1× 1 =-1
	1+0 = 1	1+1 =2	-3+2 =-1	5 +(-1) =4 ← Remainder

∴ Quotient = $x^2 + 2x – 1$

Hence, when p(x) is divided by (x – 1) the quotient is $x^2 + 2x – 1$ and remainder is 4.

1 B. Question

Find the quotient and remainder using synthetic division.

$(3x^3 – 2x^2 + 7x – 5) \div (x + 3)$

Answer

Let p(x) = $3x^3 – 2x^2 + 7x – 5$ be the dividend and arranging p(x) according to the descending powers of x.

Divisor, q(x) = x + 3

⇒ To find out Zero of the divisor –

q(x) = 0

x + 3 = 0

x = – 3

So, zero of divisor is – 3.

And, $p(x) = 3x^3 – 2x^2 + 7x – 5$

Put zero for the first entry in the second row.

-3	3	-2	7	-5
	0	3× (-3) =-9	(-11)× (-3) =33	40× (-3) = -120
	3+0 = 3	-2+(-9) =-11	7+33 =40	-5 + (-120) = -125 ← Remainder

∴ Quotient = $3x^2 - 11x + 40$

Hence, when p(x) is divided by (x – 1) the quotient is $3x^2 - 11x + 40$ and remainder is – 125.

1 C. Question

Find the quotient and remainder using synthetic division.

$(3x^3 + 4x^2 - 10x + 6) \div (3x - 2)$

Answer

Let p(x) = $3x^3 + 4x^2 - 10x + 6$ be the dividend and arranging p(x) according to the descending powers of x.

Divisor, q(x) = 3x – 2

⇒ To find out Zero of the divisor –

q(x) = 0

3x – 2= 0

$x = \frac{2}{3}$

So, zero of divisor is $\frac{2}{3}$.

And, p(x) = $3x^3 + 4x^2 - 10x + 6$

Put zero for the first entry in the second row.

$\frac{2}{3}$	3	4	-10	6
	0	$3\times\frac{2}{3}$ = 2	$6\times\frac{2}{3}$ = 4	$-6\times\frac{2}{3}$ = -4
	3+0 = 3	4+2 =6	-10+4 =-6	6+(-4) =2 ← Remainder

∵ p(x) = (Quotient)×q(x) + remainder.

So, $3x^3 + 4x^2 - 10x + 6 = (x - \frac{2}{3})(3x^2 + 6x - 6) + 2$

$= (3x - 2)\frac{1}{3}(3x^2 + 6x - 6) + 2$

Thus, the Quotient = $\frac{1}{3}(3x^2 + 6x - 6) = x^2 + 2x - 2$ and remainder is 2.

Hence, when p(x) is divided by (3x – 2) the quotient is $x^2 + 2x - 2$ and remainder is 2.

1 D. Question

Find the quotient and remainder using synthetic division.

$(3x^3 - 4x^2 - 5) \div (3x + 1)$

Answer

Let p(x) = $3x^3 - 4x^2 - 5$ be the dividend. Arranging p(x) according to the descending powers of x and insert zero for missing term.

p(x) = $3x^3 - 4x^2 + 0x - 5$

Divisor, q(x) = $3x + 1$

⇒ To find out Zero of the divisor –

q(x) = 0

3x + 1 = 0

$x = -\frac{1}{3}$

zero of divisor is $\frac{-1}{3}$.

And, p(x) = $3x^3 - 4x^2 + 0x - 5$

Put zero for the first entry in the 2nd row.

$\frac{-1}{3}$	3	-4	0	-5	
	0	$3\times\frac{-1}{3}$ = -1	$-5\times\frac{-1}{3} = \frac{5}{3}$	$\frac{5}{3}\times-\frac{1}{3} = -\frac{5}{9}$	
	3+0 = 3	-4+(-1) =-5	$0+\frac{5}{3} = \frac{5}{3}$	$-5+(-\frac{5}{9}) = -\frac{50}{9}$	← Remainder

∵ p(x) = (Quotient)×q(x) + remainder.

So, $3x^3 - 4x^2 - 5 = (x + \frac{1}{3})(3x^2 - 5x + \frac{5}{3}) + (-\frac{50}{9})$

$= (3x + 1)\frac{1}{3}(3x^2 - 5x + \frac{5}{3}) - \frac{50}{9}$

Thus, the Quotient = $\frac{1}{3}(3x^2 - 5x + \frac{5}{3}) = (x^2 - \frac{5}{3}x + \frac{5}{9})$ and remainder is $-\frac{50}{9}$.

Hence, when p(x) is divided by (3x + 1) the quotient is $(x^2 - \frac{5}{3}x + \frac{5}{9})$ and remainder is $-\frac{50}{9}$.

1 E. Question

Find the quotient and remainder using synthetic division.

$(8x^4 - 2x^2 + 6x + 5) \div (4x + 1)$

Answer

Let p(x) = $8x^4 - 2x^2 + 6x - 5$ be the dividend. Arranging p(x) according to the descending powers of x and insert zero for missing term.

p(x) = $8x^4 + 0x^3 - 2x^2 + 6x - 5$

Divisor, q(x) = $4x + 1$

⇒ To find out Zero of the divisor –

q(x) = 0

4x + 1 = 0

$x = -\frac{1}{4}$

zero of divisor is $\frac{-1}{4}$.

And, p(x) = $8x^4 + 0x^3 - 2x^2 + 6x - 5$

Put zero for the first entry in the 2nd row.

$\frac{-1}{4}$	8	0	-2	6	-5
	0	$8 \times \frac{-1}{4}$ $= -2$	$-2 \times \frac{-1}{4}$ $= \frac{1}{2}$	$\frac{-3}{2} \times -\frac{1}{4}$ $= \frac{3}{8}$	$\frac{51}{8} \times -\frac{1}{4}$ $= -\frac{51}{32}$
	8+0 =8	0+(-2) =-2	$-2+\frac{1}{2}$ $=-\frac{3}{2}$	$6+\frac{3}{8}$ $= \frac{51}{8}$	$-5+(-\frac{51}{32})$ $=-\frac{211}{32}$ ← Remainder

∵ p(x) = (Quotient)×q(x) + remainder.

So, $8x^4 - 2x^2 + 6x - 5 = (x + \frac{1}{4})(8x^3 - 2x^2 - \frac{3}{2}x + \frac{51}{8}) + (-\frac{211}{32})$

$= (4x + 1)\frac{1}{4}(8x^3 - 2x^2 - \frac{3}{2}x + \frac{51}{8}) - \frac{211}{32}$

Thus, the Quotient = $\frac{1}{4}(8x^3 - 2x^2 - \frac{3}{2}x + \frac{51}{8}) = (2x^3 - \frac{1}{2}x^2 - \frac{3}{8}x + \frac{51}{32})$ and remainder is $-\frac{211}{32}$.

Hence, when p(x) is divided by (4x + 1) the quotient is $(2x^3 - \frac{1}{2}x^2 - \frac{3}{8}x + \frac{51}{32})$ and remainder is $-\frac{211}{32}$.

1 F. Question

Find the quotient and remainder using synthetic division.

$(2x^4 - 7x^3 - 13x^2 + 63x - 48) \div (2x - 1)$

Answer

Let p(x) = $2x^4 - 7x^3 - 13x^2 + 63x - 48$ be the dividend. Arranging p(x) according to the according descending powers of x.

p(x) = $2x^4 - 7x^3 - 13x^2 + 63x - 48$

Divisor, q(x) = 2x – 1

⇒ To find out Zero of the divisor –

q(x) = 0

2x – 1 = 0

$x = \frac{1}{2}$

zero of divisor is $\frac{1}{2}$.

And, p(x) = $2x^4 - 7x^3 - 13x^2 + 63x - 48$

Put zero for the first entry in the 2nd row.

$\frac{1}{2}$	2	-7	-13	63	-48
	0	$2 \times \frac{1}{2}$ =1	$-6 \times \frac{1}{2}$ = -3	$-16 \times \frac{1}{2}$ =-8	$55 \times \frac{1}{2}$ $= \frac{55}{2}$
	2+0 =2	-7+1 =-6	-13+(-3) =-16	63+(-8) = 55	$-48+(\frac{55}{2})$ $=-\frac{41}{2}$ ← Remainder

∴ Quotient = $2x^3 - 6x^2 - 16x + 55$

∵ p(x) = (Quotient)×q(x) + remainder.

So, $2x^4 - 7x^3 - 13x^2 + 63x - 48$

$= (x - \frac{1}{2})(8x^3 - 2x^2 - \frac{3}{2}x + \frac{51}{8}) + (-\frac{41}{2})$

$= (2x - 1)\frac{1}{2}(2x^3 - 6x^2 - 16x + 55) - \frac{41}{2}$

Thus, the Quotient $= \frac{1}{2}(2x^3 - 6x^2 - 16x + 55) = (x^3 - 3x^2 - 8x + \frac{55}{2})$ and remainder is $-\frac{41}{2}$.

Hence, when p(x) is divided by (2x – 1) the quotient is $(x^3 - 3x^2 - 8x + \frac{55}{2})$ and remainder is $-\frac{41}{2}$.

2. Question

If the quotient on dividing $x^4 + 10x^3 + 35x^2 + 50x + 29$ by x + 4 is $x^3 - ax^2 + bx + 6$, then find a, b and also the remainder.

Answer

Let $p(x) = x^4 + 10x^3 + 35x^2 + 50x + 29$ be the dividend. Arranging p(x) according to the descending powers of x.

$p(x) = x^4 + 10x^3 + 35x^2 + 50x + 29$

Divisor, q(x) = x + 4

⇒ To find out Zero of the divisor –

q(x) = 0

x + 4 = 0

x = – 4

zero of divisor is – 4.

And, $p(x) = x^4 + 10x^3 + 35x^2 + 50x + 29$

Put zero for the first entry in the 2nd row.

-4	1	10	35	50	29
	0	1× (-4) =-4	6× (-4) = -24	11× (-4) =-44	6× (-4) = -24
	1+0 =1	10+(-4) =6	35+(-24) =11	50+(-44) = 6	29-24 =5 ← Remainder

∴ Quotient = $x^3 + 6x^2 + 11x + 6$

Hence, when p(x) is divided by (x + 4) the quotient is $x^3 + 6x^2 + 11x + 6$ and remainder is 5.

Comparing $x^3 + 6x^2 + 11x + 6$ with $x^3 - ax^2 + bx + 6$ we get,

a = – 6 and b = 11.

3. Question

If the quotient on dividing, $8x^4 - 2x^2 + 6x - 7$ by 2x + 1 is $4x^3 + px^2 - qx + 3$,then find p, q and also the remainder.

Answer

Let $p(x) = 8x^4 - 2x^2 + 6x - 7$ be the dividend. Arranging p(x) according to the descending powers of x and write zero in place of missing term.

$p(x) = 8x^4 + 0x^3 – 2x^2 + 6x – 7$

Divisor, $q(x) = 2x + 1$

⇒ To find out Zero of the divisor –

$q(x) = 0$

$2x + 1 = 0$

$x = -\frac{1}{2}$.

zero of divisor is $-\frac{1}{2}$.

And, $p(x) = 8x^4 + 0x^3 – 2x^2 + 6x – 7$

Put zero for the first entry in the 2nd row.

$-\frac{1}{2}$	8	0	-2	6	-7
	0	$8\times(-\frac{1}{2})$ =-4	$-4\times-\frac{1}{2}$ = 2	$0\times-\frac{1}{2}$ =0	$6\times-\frac{1}{2}$ =-3
	8+0 =8	0+(-4) =-4	-2+2 =0	6+0 = 6	-7+(-3) =-10 ← Remainder

∴ Quotient $= 8x^3 – 4x^2 + 0x + 6$

Hence, when p(x) is divided by (2x + 1) the quotient is $8x^3 – 4x^2 + 0x + 6$

6 and remainder is – 10.

Comparing $8x^3 – 4x^2 + 0x + 6$ with $4x^3 + px^2 – qx + 3$ we get,

p = – 4 and q = 0.

Exercise 3.5

1 A. Question

Factorize each of the following polynomials.

$x^3 – 2x^2 – 5x + 6$

Answer

Given,

$x^3 – 2x^2 – 5x + 6$, put x = 1

then, 1 – 2 – 5 + 6 = 0, since,

this equation is divisible by (x – 1)

∴ according to the question,

$x^3 – 2x^2 – 5x + 6 = (x – 1)(x^2 – x – 6)$

$= (x – 1)[x^2 – 3x + 2x – 6]$

$= (x – 1)[x(x – 3) + 2(x – 3)]$

$= (x – 1)(x + 2)(x – 3)$

1 B. Question

Factorize each of the following polynomials.

$4x^3 – 7x + 3$

Answer

Given,

$4x^3 – 7x + 3$,put x = 1

Then, 4 × 1 – 7 + 3 = 0.

since, this equation is divisible by (x – 1).

∴ according to the question,

$4x^3 – 7x + 3 = (x – 1)(4x^2 + 4x – 3)$

$= (x – 1)[4x^2 + 6x – 2x – 3]$

= (x – 1)[2x(2x + 3) – 1(2x + 3)]

= (x – 1)(2x + 3)(2x – 1)

1 C. Question

Factorize each of the following polynomials.

$x^3 – 23x^2 + 142x – 120$

Answer

Given,

$x^3 – 23x^2 + 142x – 120$, put x = 1

then, 1 – 23 × 1 + 142 – 120 = – 143 + 143 = 0

since, this equation is divisible by (x – 1).

∴ according to the question,

$x^3 – 23x^2 + 142x – 120 = (x – 1)[x^2 – 22x + 120]$

$= (x – 1)[x^2 – 12x – 10x + 120]$

= (x – 1)[x(x – 12) – 10(x – 12)]

= (x – 1)(x – 12)(x – 10).

1 D. Question

Factorize each of the following polynomials.

$4x^3 – 5x^2 + 7x – 6$

Answer

Given,

$4x^3 – 5x^2 + 7x – 6$,put x = 1

Then, 4 × 1 – 5 + 7 – 6 = 0

since, this equation is divisible by (x – 1).

∴ according to the question,

$4x^3 – 5x^2 + 7x – 6 = (x – 1)[4x^2 – x + 6]$

1 E. Question

Factorize each of the following polynomials.

$x^3 - 7x + 6$

Answer

Given,

$x^3 - 7x + 6$,

put x = 1

then, 1 – 7 + 6 = 0

since, this equation is divisible by (x – 1).

∴ according to the question,

$x^3 - 7x + 6 = (x - 1)(x^2 + x - 6)$

$= (x - 1)[x^2 + 3x - 2x - 6]$

$= (x - 1)[x(x + 3) - 2(x + 3)]$

$= (x - 1)(x - 2)(x + 3)$

1 F. Question

Factorize each of the following polynomials.

$x^3 + 13x^2 + 32x + 20$

Answer

Given,

$x^3 + 13x^2 + 32x + 20$,

put x = – 1

then, – 1 + 13 – 32 + 20 = 0

since, this equation is divisible by (x + 1).

∴ according to the question,

$x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$

$= (x + 1)[x^2 + 10x + 2x + 20]$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 2)(x + 10)$

1 G. Question

Factorize each of the following polynomials.

$2x^3 - 9x^2 + 7x + 6$

Answer

Given,

$2x^3 - 9x^2 + 7x + 6$,

put x = 2

Then, 16 – 36 + 14 + 6 = 0

since, this equation is divisible by (x – 2).

∴ according to the question,

$2x^3 – 9x^2 + 7x + 6 = (x – 2)(2x^2 – 5x – 3)$

$= (x – 2)[2x^2 – 6x + x – 3]$

$= (x – 2)[2x(x – 3) + 1(x – 3)]$

$= (x – 2)(x – 3)(2x + 1)$

1 H. Question

Factorize each of the following polynomials.

$x^3 – 5x + 4$

Answer

Given,

$x^3 – 5x + 4$,put x = 1

then,1 – 5 + 4 = 0

since, this equation is divisible by (x – 1).

∴ according to the question,

$x^3 – 5x + 4 = (x – 1)(x^2 + x – 4)$.

1 I. Question

Factorize each of the following polynomials.

$x^3 – 10x^2 – x + 10$

Answer

Given,

$x^3 – 10x^2 – x + 10$,put x = 1

then, 1 – 10 – 1 + 10 = 0

since, this equation is divisible by (x – 1).

∴ according to the question,

$x^3 – 10x^2 – x + 10 = (x – 1)(x^2 – 9x – 10)$

$= (x – 1)[x^2 – 10x + x – 10]$

$= (x – 1)[x(x – 10) + 1(x – 10)]$

$= (x – 1)(x + 1)(x – 10)$

1 J. Question

Factorize each of the following polynomials.

$2x^3 + 11x^2 – 7x – 6$

Answer

Given,

$2x^3 + 11x^2 – 7x – 6$,

put x = 1

Then, 2 + 11 – 7 – 6 = 0

since, this equation is divisible by (x – 1).

∴ according to the question,

$2x^3 + 11x^2 – 7x – 6 = (x – 1)(2x^2 + 13x + 6)$

$= (x – 1)[2x^2 + 12x + x + 6]$

$= (x – 1)[2x(x + 6) + 1(x + 6)]$

$= (x – 1)(2x + 1)(x + 6)$

1 K. Question

Factorize each of the following polynomials.

$x^3 + x^2 + x – 14$

Answer

Given,

$x^3 + x^2 + x – 14$,

put x = 2

then, 8 + 4 + 2 – 14 = 0

since, this equation is divisible by (x – 2).

∴ according to the question,

$x^3 + x^2 + x – 14 = (x – 2)(x^2 + 3x + 7)$.

1 L. Question

Factorize each of the following polynomials.

$x^3 – 5x^2 – 2x + 24$

Answer

Given,

$x^3 – 5x^2 – 2x + 24$,put x = – 2

then, – 8 – 20 + 4 + 24 = 0

since, this equation is divisible by (x + 2).

∴ according to the question,

$x^3 – 5x^2 – 2x + 24 = (x + 2)(x^2 – 7x + 12)$

$= (x + 2)[x^2 – 4x – 3x + 12]$

$= (x + 2)[x(x – 4) – 3(x – 4)]$

$= (x + 2)(x – 4)(x – 3)$

Exercise 3.6

1 A. Question

Find the greatest common divisor of

$7x^2 yz^4, 21x^2 y^5 z^3$

Answer

Given,

$7x^2 yz^4 = 7x^2 yz^3 \times z$

$21x^2 y^5 z^3 = 3 \times 7x^2 y \times y^4 z^3$

Greatest common divisor = $7x^2yz^3$

1 B. Question

Find the greatest common divisor of

x^2y, x^3y, x^2y^2

Answer

Given,

$x^2 y = x \times x \times y$

$x^3 y = x \times x \times x \times y$

$x^2 y^2 = x \times x \times y \times y$

Greatest common divisor = x^2y

1 C. Question

Find the greatest common divisor of

$25bc^4 d^3, 35b2c^5, 45c^3 d$

Answer

Given,

$25bc^4 d^3 = 5 \times 5 \times b \times c^3 \times c \times d^3$

$35b2c^5 = 5 \times 7 \times b \times 2 \times c^3 \times c^2$

$45c^3 d = 5 \times 3 \times 3 \times c^3 \times d$

Greatest common divisor = $5c^3$

1 D. Question

Find the greatest common divisor of

$35x^5 y^3 z^4, 49x^2 yz^3, 14xy^2 z^2$

Answer

Given,

$35x^5 y^3 z^4 = 5 \times 7 \times x \times x^4 \times y^2 \times y \times z^2 \times z^2$

$49x^2 yz^3 = 7 \times 7 \times x^2 \times y \times z \times z^2$

$14xy^2 z^2 = 2 \times 7 \times x \times y \times y \times z^2$

Greatest common divisor = $7xyz^2$

2 A. Question

Find the GCD of the following

$c^2 – d^2, – c(c – d)$

Answer

Given,

$c^2 – d^2 = (c + d) \times (c – d)$

$– c(c – d) = – c \times (c – d)$

Greatest common divisor = (c – d)

2 B. Question

Find the GCD of the following

$x^4 – 27a^3 x,(x – 3a)^2$

Answer

Given,

$x^4 – 27a^3 x = x[x^3 – (3a)^3]$

$= x[(x – 3a)(x^2 + 9a^2 + 3ax)]$

$= x(x – 3a)(x^2 + 9a^2 + 3ax)$

$(x – 3a)^2 = (x – 3a)(x – 3a)$

Greatest common divisor = (x – 3a)

2 C. Question

Find the GCD of the following

$m^2 – 3m – 18, m^2 + 5m + 6$

Answer

Given,

$m^2 – 3m – 18 = m^2 – 6m + 3m – 18 = m(m – 6) + 3(m – 6)$

$= (m + 3)(m – 6)$

$m^2 + 5m + 6 = m^2 + 3m + 2m + 6 = m(m + 3) + 2(m + 3)$

$= (m + 3)(m + 2)$

Greatest common divisor = (m + 3)

2 D. Question

Find the GCD of the following

$x^2 + 14x + 33, x^3 + 10x^2 – 11x$

Answer

Given,

$X^2 + 14x + 33 = x^2 + 3x + 11x + 33 = x(x + 3) + 11(x + 3)$

$= (x + 11)(x + 3)$

$X^3 + 10x^2 – 11x = x(x^2 + 10x – 11) = x(x^2 + 11x – x – 11)$

$= x[x(x + 11) – 1(x + 11)]$

$= x(x + 11)(x – 1)$

Greatest common divisor = (x + 11)

2 E. Question

Find the GCD of the following

$x^2 + 3xy + 2y^2, x^2 + 5xy + 6y^2$

Answer

Given,

$X^2 + 3xy + 2y^2 = x^2 + xy + 2xy + 2y^2$

$= x(x + y) + 2y(x + y)$

$= (x + 2y)(x + y)$

$x^2 + 5xy + 6y^2 = x^2 + 3xy + 2xy + 6y^2$

$= x(x + 3y) + 2y(x + 3y)$

$= (x + 2y)(x + 3y)$

Greatest common divisor = $(x + 2y)$

2 F. Question

Find the GCD of the following

$2x^2 – x – 1, 4x^2 + 8x + 3$

Answer

Given,

$2x^2 – x – 1 = 2x^2 – 2x + x – 1 = 2x(x – 1) + 1(x – 1)$

$= (2x + 1)(x – 1)$

$4x^2 + 8x + 3 = 4x^2 + 2x + 6x + 3 = 2x(x + 1) + 3(2x + 1)$

$= (2x + 1)(2x + 3)$

Greatest common divisor = $(2x + 1)$

2 G. Question

Find the GCD of the following

$x^2 – x – 2, x^2 + x – 6, 3x^2 – 13x + 14$

Answer

Given,

$x^2 – x – 2 = x^2 – 2x + x – 2 = x(x – 2) + 1(x – 2) = (x + 1)(x – 2)$

$x^2 + x – 6 = x^2 + 3x – 2x – 6$

$= x(x + 3) – 2(x + 3) = (x – 2)(x + 3)$

$3x^2 – 13x + 14 = 3x^2 – 6x – 7x + 14 = 3x(x – 2) – 7(x – 2)$

$= (x – 2)(3x – 7)$

Greatest common divisor = $(x – 2)$

1 H. Question

Find the GCD of the following

$x^3 – x^2 + x – 1, x^4 – 1$

Answer

$x^3 – x^2 + x – 1$, put $x = 1$

Then, $1 – 1 + 1 – 1 = 0$

Since, this equation is divisible by $x – 1$

$(x - 1)(x^2 + 1) = (x + 1)(x^2 + 1)$

In second equation,

$x^4 - 1 = (x^2)^2 - 1 = (x^2 - 1)(x^2 + 1) = (x + 1)(x - 1)(x^2 + 1)$

[using $a^2 - b^2 = (a - b)(a + b)$]

Greatest common divisor = $(x + 1)(x^2 + 1)$

2 I. Question

Find the GCD of the following

$24(6x^4 - x^3 - 2x^2), 20(2x^6 + 3x^5 + x^4)$

Answer

Given,

$24(6x^4 - x^3 - 2x^2) = 2 \times 2 \times 2 \times 3[x^2(6x^2 - x - 2)]$

$= 2 \times 2 \times 2 \times 3[x^2(6x^2 - 4x + 3x - 2)]$

$= 2 \times 2 \times 2 \times 3 \times x^2 [2x(3x - 2) + 1(3x - 2)]$

$= 2 \times 2 \times 2 \times 3 \times x^2 (2x + 1)(3x - 2)$

$20(2x^6 + 3x^5 + x^4) = 2 \times 2 \times 5[x^4(2x^2 + 3x + 1)]$

$= 2 \times 2 \times 5 \times x^2 \times x^2[2x(x + 1) + 1(x + 1)]$

$= 2 \times 2 \times 5 \times x^2 \times x^2(2x + 1)(x + 1)$

Greatest common divisor = $2 \times 2 \times x^2 \times (2x + 1) = 4x^2(2x + 1)$

2 J. Question

Find the GCD of the following

$(a - 1)^5 (a + 3)^2, (a - 2)^2 (a - 1)^3(a + 3)^4$

Answer

Given,

$(a - 1)^5 (a + 3)^2 = (a - 1)^3(a - 1)^2(a + 3)^2$

$(a - 2)^2(a - 1)^3(a + 3)^4 = (a - 2)^2(a - 1)^3(a + 3)^2(a + 3)^2$

Greatest common divisor = $(a - 1)^3(a + 3)^2$

3 A. Question

Find the GCD of the following pairs of polynomials using division algorithm.

$x^3 - 9x^2 + 23x - 15,\ 4x^2 - 16x + 12$

Answer

$x^3 - 9x^2 + 23x - 15$

put x = 1 in polynomials equations is $1 - 9(1) + 23 - 15 = 0$

∴ This equation is divisible by (x – 1),

then using division algorithm method.

Since,

$(x - 1)(x^2 - 8x + 15) = (x - 1)[x^2 - 5x - 3x + 15]$

$= (x - 1)[x(x - 5) - 3(x - 5)] = (x - 1)(x - 3)(x - 5)$

$4x^2 - 16x + 12 = 4x^2 - 12x - 4x + 12 = 4x(x - 3) - 4(x - 3)$

$= 4(x - 1)(x - 3)$

Greatest common divisor $= (x - 1)(x - 3) = x^2 - 4x + 3$

3 B. Question

Find the GCD of the following pairs of polynomials using division algorithm.

$3x^3 + 18x^2 + 33x + 18, 3x^2 + 13x + 10$

Answer

According to the question,

$3x^3 + 18x^2 + 33x + 18$ then put $x = - 1$ in that equation

$\therefore$ put $x = - 1$

then, $- 3 + 18 - 33 + 18 = 0$

Then the equation is divisible by $x + 1$, using division algorithm method.

$(x + 1)(3x^2 + 15x + 18) = (x + 1)[3x^2 + 9x + 6x + 18]$

$= (x + 1)[3x(x + 3) + 6(x + 3)]$

$= 3(x + 1)(x + 2)(x + 3)$

$3x^2 + 13x + 10 = 3x^2 + 3x + 10x + 10 = 3x(x + 1) + 10(x + 1)$

$= (x + 1)(3x + 10)$

Greatest common divisor $= (x + 1)$

3 C. Question

Find the GCD of the following pairs of polynomials using division algorithm.

$2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12$

Answer

$2x^3 + 2x^2 + 2x + 2$ then put $x = - 1$

Since, $- 2 + 2 - 2 + 2 = 0$

Then the equation is divisible by $x + 1$, using division algorithm method.

$(x + 1)(2x^2 + 2) = 2(x + 1)(x^2 + 1)$

In equation second,

$6x^3 + 12x^2 + 6x + 12$,put $x = - 2$

Then, $- 48 + 48 - 12 + 12 = 0$

Then the equation is divisible by $x + 2$, using division algorithm method.

$(x + 2)(6x^2 + 6) = 2 \times 3(x + 2)(x^2 + 1)$

Greatest common divisor $= 2(x^2 + 1)$

3 D. Question

Find the GCD of the following pairs of polynomials using division algorithm.

$x^3 - 3x^2 + 4x - 12, x^4 + x^3 + 4x^2 + 4x$

Answer

According to the question,

$x^3 – 3x^2 + 4x – 12$,then put x = 3

then,27 – 27 + 12 – 12 = 0

Then the equation is divisible by x – 3, using division algorithm method.

$(x – 3)(x^2 + 4) = (x – 3)(x^2 + 4)$

In equation second,

$x^4 + x^3 + 4x^2 + 4x$,put x = – 1

then, 1 – 1 + 4 – 4 = 0,

Then the equation is divisible by x + 1, using division algorithm method.

$(x + 1)(x^3 + 4x) = x(x + 1)(x^2 + 4)$

Greatest common divisor = $(x^2 + 4)$

Exercise 3.7

1. Question

Find the LCM of the following

$x^3 y^2$, xyz

Answer

Given terms: –

x^3, y^2 , xyz

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be divided completely;

$x^3 = x \times x \times x$

$y^2 = y \times y$

$xyz = x \times y \times z$

⇒ first find the common factors in all terms

Common factor = x × y

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (x \times y) \times [(x^2)(y)(z)]$

$= x^3y^2z$

Conclusion: –

The LCM of given terms [x^3 , y^2 , xyz] is x^3y^2z

2. Question

Find the LCM of the following

$3x^2yz$, $4x^3y^3$

Answer

Given terms: –

$3x^2yz, 4x^3y^3$

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be

divided completely;

$3x^2yz, = 3 \times x \times x \times y \times z$

$4x^3y^3 = 4 \times x \times x \times x \times y \times y \times y$

⇒ first find the common factors in all terms

Common factor = $x \times x \times y$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (x \times y \times x) \times [(3yz)(4xy^2)]$

$= 12x^3y^3z$

Conclusion: –

The LCM of given terms $[3x^2yz, 4x^3y^3]$ is $12x^3y^3z$

3. Question

Find the LCM of the following

a^2bc, b^2ca, c^2ab

Answer

Given terms: –

a^2bc, b^2ca, c^2ab

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be

divided completely;

$a^2bc = a \times a \times b \times c$

$b^2ca = a \times b \times b \times c$

$c^2ab = a \times b \times c \times c$

⇒ first find the common factors in all terms

Common factor = $a \times b \times c$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (a \times b \times c) \times [(a)(b)(c)]$

$= a^2b^2c^2$

Conclusion: –

The LCM of given terms [a^2bc, b^2ca, c^2ab] is $a^2b^2c^2$

4. Question

Find the LCM of the following

$66a^4b^2c^3$, $44a^3b^4c^2$, $24a^2b^3c^4$

Answer

Given terms: –

$66a^4b^2c^3$, $44a^3b^4c^2$, $24a^2b^3c^4$

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be

divided completely;

$66a^4b^2c^3 = 3 \times 2 \times 11 \times a \times a \times a \times a \times b \times b \times c \times c \times c$

$44a^3b^4c^2 = 2 \times 2 \times 11 \times a \times a \times a \times b \times b \times b \times b \times c \times c$

$24a^2b^3c^4 = 2 \times 2 \times 2 \times 3 \times a \times a \times b \times b \times b \times c \times c \times c \times c$

⇒ first find the common factors in all terms

Common factor in all terms = $2 \times a \times a \times b \times b \times c \times c$

Common factors from any 2 terms

$2a^2b^2c^2 \times [(3 \times 11 \times a \times a \times c)(2 \times 11 \times a \times b \times b)(2 \times 2 \times 3 \times b \times c \times c)]$

$2a^2b^2c^2 \times (3 \times 11 \times 2 \times a \times b \times c)[(a)(b)(2c)]$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= 2a^2b^2c^2 \times (66abc) \times (2abc)$

$(66 \times 2 \times 2)(a^2b^2c^2 \times abc \times abc)$

$264\ a^4b^4c^4$

Conclusion: –

The LCM of given terms [$66a^4b^2c^3$, $44a^3b^4c^2$, $24a^2b^3c^4$] is $264\ a^4b^4c^4$

5. Question

Find the LCM of the following

a^{m+1}, a^{m+2}, a^{m+3}

Answer

Given terms: –

a^{m+1}, a^{m+2}, a^{m+3}

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be divided completely;

$a^{m+1} = a^m \times a$

$a^{m+2} = a^m \times a \times a$

$a^{m+3} = a^m \times a \times a \times a$

⇒ first find the common factors in all terms

Common factor = $a^m \times a$

Common factor from any 2 terms

$= (a^m \times a) \times [(1)(a)(a^2)]$

$= (a^m \times a) \times a[(1)(1)(a)]$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (a^m \times a^2)(a)$

$= a^m a^3$

Conclusion: –

The LCM of given terms [a^{m+1}, a^{m+2}, a^{m+3}] is $a^m a^3$

6. Question

Find the LCM of the following

$x^2y + xy^2$, $x^2 + xy$

Answer

Given terms: –

$x^2y + xy^2$, $x^2 + xy$

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be

divided completely;

$x^2y + xy^2 = x \times y \times x + x \times y \times y$

$= x \times y \times (x + y)$

$x^2 + xy = x \times x + x \times y$

$= x \times (x + y)$

⇒ first find the common factors in all terms

Common factor = $(x + y) \times x$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (x + y) \times x \times (x)$

$= x^3 + yx^2$

Conclusion: –

The LCM of given terms [$x^2y + xy^2$, $x^2 + xy$] is $\underline{x^3 + yx^2}$

7. Question

Find the LCM of the following

$3(a – 1), 2(a – 1)^2, (a^2 – 1)$

Answer

Given terms: –

$3(a – 1), 2(a – 1)^2, (a^2 – 1)$

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be

divided completely;

$3(a – 1) = 3 \times (a – 1)$

$2(a – 1)^2 = 2 \times (a – 1) \times (a – 1)$

$(a^2 – 1) = (a^2 – 1^2) = (a – 1) \times (a + 1)$

⇒ first find the common factors in all terms

Common factor = $(a – 1)$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (a – 1) \times [(3) \times (2(a – 1)) \times (a + 1)]$

$= 6(a + 1)(a – 1)^2$

Conclusion: –

The LCM of given terms [$3(a – 1), 2(a – 1)^2, (a^2 – 1)$] is

$\underline{6(a + 1)(a – 1)^2}$

8. Question

Find the LCM of the following

$2x^2 – 18, 5x^2y + 15xy^2, x^3 + 27y^3$

Answer

Given terms: –

$2x^2 – 18, 5x^2y + 15xy^2, x^3 + 27y^3$

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be

divided completely;

$2x^2 – 18y^2 = 2 \times (x^2 – 9y^2) = 2(x^2 – (3y)^2) = 2 \times (x – 3y) \times (x + 3y)$

$5x^2y + 15xy^2 = 5 \times x \times y \times (x + 3y)$

$x^3 + 27y^3 = (x^3 + (3y)^3) = (x + 3y)(x^2 – 3xy + 9y^2)$

⇒ first find the common factors in all terms

Common factor = (x + 3y)

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (x + 3y) \times [2 \times (x - 3y) \times 5xy \times (x^2 - 3xy + 9y^2)]$

$= 10xy(x + 3y)(x - 3y)(x^2 - 3xy + 9y^2)]$

Conclusion: –

The LCM of given terms $[2x^2 - 18, 5x^2y + 15xy^2, x^3 + 27y^3]$ is

$10xy(x + 3y)(x - 3y)(x^2 - 3xy + 9y^2)]$

9. Question

Find the LCM of the following

$(x + 4)^2 (x - 3)^3, (x - 1)(x + 4)(x - 3)^2$

Answer

Given terms: –

$(x + 4)^2 (x - 3)^3, (x - 1)(x + 4)(x - 3)^2$

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be divided completely;

$(x + 4)^2 (x - 3)^3 = (x + 4) \times (x + 4) \times (x - 3) \times (x - 3) \times (x - 3)$

$(x - 1)(x + 4)(x - 3)^2 = (x - 1) \times (x + 4) \times (x - 3) \times (x - 3)$

⇒ first find the common factors in all terms

Common factor = $(x + 4) \times (x - 3) \times (x - 3)$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= (x + 4)(x - 3)(x - 3) \times [(x + 4) \times (x - 3) \times (x - 1)]$

$= (x + 4)^2 (x - 3)^3(x - 1)$

Conclusion: –

The LCM of given terms $[(x + 4)^2 (x - 3)^3, (x - 1)(x + 4)(x - 3)^2]$ is

$(x + 4)^2 (x - 3)^3(x - 1)$

10. Question

Find the LCM of the following

$10(9x^2 + 6xy + y^2), 12(3x^2 - 5xy - 2y^2), 14(6x^4 + 2x^3)$

Answer

Given terms: –

$10(9x^2 + 6xy + y^2), 12(3x^2 - 5xy - 2y^2), 14(6x^4 + 2x^3)$

Formula used: –

LCM = Least Common Multiple

Means it is the lowest term by which every element must be

divided completely;

$10(9x^2 + 6xy + y^2) = 2 \times 5 \times ((3x)^2 + 2 \times 3x \times y + y^2)$

$= 2 \times 5 \times (3x + y)^2$

$= 2 \times 5 \times (3x + y) \times (3x + y)$

$12(3x^2 – 5xy – 2y^2) = 2 \times 2 \times 3 \times (3x^2 – 6xy + xy – 2y^2)$

$= 2 \times 2 \times 3 \times (3x(x – 2y) + y(x – 2y))$

$= 2 \times 2 \times 3 \times (x – 2y) \times (3x + y)$

$14(6x^4 + 2x^3) = 2 \times 7 \times 2 \times x \times x \times x \times (3x + 1)$

⇒ first find the common factors in all terms

Common factor = 2

Common factors in any 2 terms

$2 \times [(5(3x + 4)^2)(2 \times 3 \times (x – 2y)(3x + y))(7 \times 2 \times x^3 \times (3x + 1))]$

$2 \times 2 \times (3x + y)[(5(3x + 4))(3 \times (x – 2y))(7 \times x^3 \times (3x + 1))]$

⇒ then multiply the remaining factors of terms in common

factor to get the LCM

$= 2 \times 2 \times 5 \times 3 \times 7 \times x^3 \times (3x + y)(3x + y)(x – 2y)(3x + 1)$

$= 420x^3(3x + y)^2(x – 2y)(3x + 1)$

Conclusion: –

The LCM of given terms $[10(9x^2 + 6xy + y^2), 12(3x^2 – 5xy – 2y^2), 14(6x^4 + 2x^3)]$ is $\underline{420x^3(3x + y)^2(x – 2y)(3x + 1)}$

Exercise 3.8

1 A. Question

Find the LCM of each pair of the following polynomials.

$x^2 – 5x + 6, x^2 + 4x – 12$ whose GCD is $x – 2$.

Answer

Given: –

Polynomials $x^2 – 5x + 6, x^2 + 4x – 12$

And GCD[Greatest Common Divisor] = $(x – 2)$

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

Product of 2 polynomial = $(x^2 – 5x + 6) \times (x^2 + 4x – 12)$

$= (x^2 – 2x – 3x + 6)(x^2 + 6x – 2x – 12)$

$= (x(x - 2) - 3(x - 2))(x(x + 6) - 2(x + 6))$

$= (x - 3)(x - 2)(x - 2)(x + 6)$

Product of 2 polynomial = LCM × GCD

LCM = $\frac{\text{Productof 2 polynomial}}{\text{GCD}}$

LCM = $\frac{(x-3)(x-2)(x-2)(x+6)}{x-2}$

LCM = $(x - 3)(x - 2)(x + 6)$

Conclusion: –

The LCM of polynomial $[x^2 - 5x + 6, x^2 + 4x - 12]$ is

$(x - 3)(x - 2)(x + 6)$

1 B. Question

Find the LCM of each pair of the following polynomials.

$x^4 + 3x^3 + 6x^2 + 5x + 3$, $x^4 + 2x^2 + x + 2$ whose GCD is $x^2 + x + 1$

Answer

Given: –

Polynomials $x^4 + 3x^3 + 6x^2 + 5x + 3$, $x^4 + 2x^2 + x + 2$

And GCD[Greatest Common Divisor] = $(x^2 + x + 1)$

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

Product of 2 polynomial = $(x^4 + 3x^3 + 6x^2 + 5x + 3) \times (x^4 + 2x^2 + x + 2)$

Product of 2 polynomial = LCM × GCD

LCM = $\frac{\text{Productof 2 polynomial}}{\text{GCD}}$

LCM = $\frac{(x^4 + 3x^3 + 6x^2 + 5x + 3) \times (x^4 + 2x^2 + x + 2)}{(x^2 + x + 1)}$

LCM = $\frac{(x^4 + 3x^3 + 6x^2 + 5x + 3)}{(x^2 + x + 1)} \times (x^4 + 2x^2 + x + 2)$

LCM = $(x^2 + 2x + 3)(x^4 + 2x^2 + x + 2)$

Conclusion: –

The LCM of polynomial $[x^4 + 3x^3 + 6x^2 + 5x + 3, x^4 + 2x^2 + x + 2]$ is

$(x^2 + 2x + 3)(x^4 + 2x^2 + x + 2)$

1 C. Question

Find the LCM of each pair of the following polynomials.

$2x^3 + 15x^2 + 2x - 35$, $x^3 + 8x^2 + 4x - 21$ whose GCD is $x + 7$.

Answer

Given: –

Polynomials $2x^3 + 15x^2 + 2x - 35$, $x^3 + 8x^2 + 4x - 21$

And GCD[Greatest Common Divisor] = (x + 7)

<u>Formula used: –</u>

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

Product of 2 polynomial = $(2x^3 + 15x^2 + 2x - 35) \times (x^3 + 8x^2 + 4x - 21)$

Product of 2 polynomial = LCM × GCD

LCM = $\frac{\text{Productof 2 polynomial}}{\text{GCD}}$

LCM = $\frac{(2x^3 + 15x^2 + 2x - 35) \times (x^3 + 8x^2 + 4x - 21)}{(x + 7)}$

LCM = $\frac{(2x^3 + 15x^2 + 2x - 35)}{(x + 7)} \times (x^3 + 8x^2 + 4x - 21)$

LCM = $(2x^2 + x - 5)(x^3 + 8x^2 + 4x - 21)$

<u>Conclusion: –</u>

The LCM of given polynomials [$2x^3 + 15x^2 + 2x - 35$, $x^3 + 8x^2 + 4x - 21$] is <u>$(2x^2 + x - 5)(x^3 + 8x^2 + 4x - 21)$</u>

1 D. Question

Find the LCM of each pair of the following polynomials.

$2x^3 - 3x^2 - 9x + 5$, $2x^4 - x^3 - 10x^2 - 11x + 8$ whose GCD is 2x – 1

Answer

<u>Given: –</u>

Polynomials $2x^3 - 3x^2 - 9x + 5$, $2x^4 - x^3 - 10x^2 - 11x + 8$

And GCD[Greatest Common Divisor] = (x + 7)

<u>Formula used: –</u>

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

Product of 2 polynomial = $(2x^3 - 3x^2 - 9x + 5) \times (2x^4 - x^3 - 10x^2 - 11x + 8)$

Product of 2 polynomial = LCM × GCD

LCM = $\frac{\text{Productof 2 polynomial}}{\text{GCD}}$

LCM = $\frac{(2x^4 - x^3 - 10x^2 - 11x + 8) \times (2x^3 - 3x^2 - 9x + 5)}{(2x - 1)}$

LCM = $\frac{(2x^4 - x^3 - 10x^2 - 11x + 8)}{(2x - 1)} \times (2x^3 - 3x^2 - 9x + 5)$

LCM = $(x^3 - 5x - 8)(2x^3 - 3x^2 - 9x + 5)$

<u>Conclusion: –</u>

The LCM of given polynomials [$2x^3 - 3x^2 - 9x + 5$, $2x^4 - x^3 - 10x^2 - 11x + 8$] is <u>$(x^3 - 5x - 8)(2x^3 - 3x^2 - 9x + 5)$</u>

2 A. Question

Find the other polynomial q(x) of each of the following, given that LCM and GCD and one polynomial p(x) respectively.

$(x + 1)^2 (x + 2)^2$, $(x + 1) (x + 2)$, $(x + 1)^2 (x + 2)$

Answer

Given: –

Polynomials $p(x) = (x + 1)^2 (x + 2)$

And GCD[Greatest Common Divisor] = $(x + 1) (x + 2)$

And LCM[Lowest Common Multiple] = $(x + 1)^2 (x + 2)^2$

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

p(x) × q(x) = LCM × GCD

LCM × GCD = $(x + 1)^2 \times (x + 2)^2 \times (x + 1) \times (x + 2)$

p(x) × q(x) = LCM × GCD

$q(x) = \frac{LCM \times GCD}{p(x)}$

$q(x) = \frac{(x+1)^2 \times (x+2)^2 \times (x+1) \times (x+2)}{(x+1)^2 (x+2)}$

$q(x) = \frac{(x+1)^2 \times (x+2)[\,(x+1) \times (x+2)^2]}{(x+1)^2 (x+2)}$

$q(x) = (x + 1)(x + 2)^2$

Conclusion: –

The other polynomial term q(x) is $(x + 1)(x + 2)^2$

2 B. Question

Find the other polynomial q(x) of each of the following, given that LCM and GCD and one polynomial p(x) respectively.

$(4x + 5)^3 (3x - 7)^3$, $(4x + 5) (3x - 7)^2$, $(4x + 5)^3 (3x - 7)^2$

Answer

Given: –

Polynomials $p(x) = (4x + 5)^3 (3x - 7)^2$

And GCD[Greatest Common Divisor] = $(4x + 5) (3x - 7)^2$

And LCM[Lowest Common Multiple] = $(4x + 5)^3 (3x - 7)^3$

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

p(x) × q(x) = LCM × GCD

LCM × GCD = $(4x + 5)^3 \times (3x - 7)^3 \times (4x + 5) \times (3x - 7)^2$

= $(4x + 5)^4(3x - 7)^5$

p(x) × q(x) = LCM × GCD

q(x) = $\frac{\text{LCM} \times \text{GCD}}{p(x)}$

q(x) = $\frac{(4x+5)^4 \times (3x-7)^5}{(4x+5)^3 \times (3x-7)^2}$

q(x) = $\frac{(4x+5)^3 \times (3x-7)^2[(4x+5)(3x-7)^3]}{(4x+5)^3 \times (3x-7)^2}$

q(x) = $(4x + 5)(3x - 7)^3$

Conclusion: –

The other polynomial term q(x) is $(4x + 5)(3x - 7)^3$

2 C. Question

Find the other polynomial q(x) of each of the following, given that LCM and GCD and one polynomial p(x) respectively.

$(x^4 - y^4)(x^4 + x^2y^2 + y^4)$, $x^2 - y^2$, $x^4 - y^4$.

Answer

Given: –

Polynomials p(x) = $x^4 - y^4$

And GCD[Greatest Common Divisor] = $x^2 - y^2$

And LCM[Lowest Common Multiple] = $(x^4 - y^4)(x^4 + x^2y^2 + y^4)$

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

p(x) × q(x) = LCM × GCD

LCM × GCD = $(x^4 - y^4)(x^4 + x^2y^2 + y^4) \times (x^2 - y^2)$

p(x) × q(x) = LCM × GCD

q(x) = $\frac{\text{LCM} \times \text{GCD}}{p(x)}$

q(x) = $\frac{(x^4-y^4)(x^4+x^2y^2+y^4)(x^2-y^2)}{(x^4-y^4)}$

q(x) = $\frac{(x^4-y^4)[(x^4+x^2y^2+y^4)(x^2-y^2)]}{(x^4-y^4)}$

q(x) = $(x^4 + x^2y^2 + y^4)(x^2 - y^2)$

Conclusion: –

The other polynomial term q(x) is $(x^4 + x^2y^2 + y^4)(x^2 - y^2)$

2 D. Question

Find the other polynomial q(x) of each of the following, given that LCM and GCD and one polynomial p(x) respectively.

$(x^3 - 4x)$ $(5x + 1)$, $(5x^2 + x)$, $(5x^3 - 9x^2 - 2x)$.

Answer

Given: –

Polynomials $p(x) = (5x^3 - 9x^2 - 2x)$

And GCD[Greatest Common Divisor] = $(5x^2 + x)$

And LCM[Lowest Common Multiple] = $(x^3 - 4x)(5x + 1)$

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

p(x) × q(x) = LCM × GCD

$LCM \times GCD = (x^3 - 4x)(5x + 1) \times (5x^2 + x)$

$= x(x^2 - 4)(5x + 1) \times x(5x + 1)$

$= x^2(x + 2)(x - 2)(5x + 1)(5x + 1)$

$p(x) = (5x^3 - 9x^2 - 2x)$

$= x(5x^2 - 9x - 2)$

$= x(5x^2 - 10x + x - 2)$

$= x[5x(x - 2) + 1(x - 2)]$

$= x(5x + 1)(x - 2)$

p(x) × q(x) = LCM × GCD

$q(x) = \frac{LCM \times GCD}{p(x)}$

$q(x) = \frac{(x^3 - 4x)(5x + 1).(5x^2 + x)}{(5x^3 - 9x^2 - 2x)}$

$q(x) = \frac{x^2(x + 2)(x - 2)(5x + 1)(5x + 1)}{x(5x + 1)(x - 2)}$

$q(x) = x(x + 2)(5x + 1)$

Conclusion: –

The other polynomial term q(x) is $\underline{x(x + 2)(5x + 1)}$

2 E. Question

Find the other polynomial q(x) of each of the following, given that LCM and GCD and one polynomial p(x) respectively.

$(x - 1)(x - 2)(x^2 - 3x + 3)$, $(x - 1)$, $(x^3 - 4x^2 + 6x - 3)$.

Answer

Given: –

Polynomials $p(x) = (x^3 - 4x^2 + 6x - 3)$

And GCD[Greatest Common Divisor] = $(x - 1)$

And LCM[Lowest Common Multiple] = $(x - 1)(x - 2)(x^2 - 3x + 3)$

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

p(x) × q(x) = LCM × GCD

LCM × GCD = (x – 1) (x – 2) (x^2 – 3x + 3) × (x – 1)

= $(x-1)^2$ (x – 2) (x^2 – 3x + 3)

p(x) × q(x) = LCM × GCD

$$q(x) = \frac{LCM \times GCD}{p(x)}$$

$$q(x) = \frac{(x-1)(x-2)(x^2-3x+3) \times (x-1)}{(x^3-4x^2+6x-3)}$$

$$q(x) = \frac{(x-1)(x-2) \times (x-1)}{\frac{x^3-4x^2+6x-3}{x^2-3x+3}}$$

$$q(x) = \frac{(x-1)(x-2) \times (x-1)}{(x-1)}$$

q(x) = (x – 1)(x – 2)

Conclusion: –

The other polynomial term q(x) is (x – 1)(x – 2)

2 F. Question

Find the other polynomial q(x) of each of the following, given that LCM and GCD and one polynomial p(x) respectively.

2(x + 1) (x^2 – 4), (x + 1), (x + 1) (x – 2).

Answer

Given: –

Polynomials p(x) = (x + 1)(x – 2)

And GCD[Greatest Common Divisor] = (x + 1)

And LCM[Lowest Common Multiple] = 2(x + 1) (x^2 – 4)

Formula used: –

The product of 2 polynomial is equal to product of their LCM

and GCD.

Product of 2 polynomial = LCM × GCD

p(x) × q(x) = LCM × GCD

LCM × GCD = 2(x + 1) (x^2 – 4) × (x + 1)

= $2(x+1)^2$ (x^2 – 2^2)

= $2(x+1)^2$ (x – 2)(x + 2)

p(x) × q(x) = LCM × GCD

$q(x) = \frac{LCM \times GCD}{p(x)}$

$q(x) = \frac{2(x+1)(x^2-4) \times (x+1)}{(x+1)(x-2)}$

$q(x) = \frac{2(x+1)(x-2)(x+2)(x+1)}{(x+1)(x-2)}$

q(x) = 2(x + 2)(x + 1)

<u>Conclusion: –</u>

The other polynomial term q(x) is <u>2(x + 2)(x + 1)</u>

Exercise 3.9

1 A. Question

Simplify the following into their lowest forms.

$$\frac{6x^2+9x}{3x^2-12x}$$

Answer

$$\frac{6x^2 + 9x}{3x^2 - 12x}$$

[dividing numerator and denominator by 3x]

$$= \frac{3x(2x + 3)}{3x(x-4)}$$

$$= \frac{2x + 3}{x - 4}$$

1 B. Question

Simplify the following into their lowest forms.

$$\frac{x^2+1}{x^4-1}$$

Answer

$$\frac{(x^2 + 1)}{x^4 - 1}$$

$$= \frac{x^2 + 1}{(x^2 + 1)(x^2 - 1)}$$

The like terms are cancelled.

$= \frac{1}{x^2-1}$ which is the required answer.

1 C. Question

Simplify the following into their lowest forms.

$$\frac{x^3-1}{x^2+x+1}$$

Answer

$$\frac{x^3 - 1}{x^2 + x + 1}$$

$$= \frac{(x-1)(x^2 + x + 1)}{x^2 + x + 1}$$

The like terms are cancelled.

=x – 1 which is the required answer.

1 D. Question

Simplify the following into their lowest forms.

$$\frac{x^3 - 27}{x^2 - 9}$$

Answer

$$\frac{x^3 - 27}{x^2 - 9}$$

$$= \frac{(x-3)(x^2 + 3x + 9)}{(x-3)(x + 3)}$$

The like terms are cancelled.

$= \frac{x^2 + 3x + 9}{x + 3}$ which is the required answer.

1 E. Question

Simplify the following into their lowest forms.

$\frac{x^4 + x^2 + 1}{x^2 + x + 1}$ (Hint : $x^4 + x^2 + 1 = (x^2 + 1)^2 - x^2$)

Answer

$$\frac{x^4 + x^2 + 1}{x^2 + x + 1}$$

$$= [(x^2 + 1)^2 - x^2)/(x^2 + x + 1)$$

$$= \frac{(x^2 + x + 1)(x^2 - x + 1)}{x^2 + x + 1}$$

The like terms are cancelled.

$= x^2 + x + 1$ which is the required answer.

1 F. Question

Simplify the following into their lowest forms.

$$\frac{x^3 + 8}{x^4 + 4x^2 + 16}$$

Answer

$$\frac{x^3 + 8}{x^4 + 4x^2 + 16}$$

$$= \frac{(x + 2)(x^2 - 2x + 4)}{(x^2 + 4)^2 - (2x)\text{^}2}$$

$$= \frac{(x + 2)(x^2 - 2x + 4)}{(x^2 - 2x + 4)(x^2 + 2x + 4)}$$

The like terms are cancelled.

$$= \frac{x + 2}{x^2 + 2x + 4}$$

Required solution.

1 G. Question

Simplify the following into their lowest forms.

$$\frac{2x^2 + x - 3}{2x^2 + 5x + 3}$$

Answer

$$\frac{2x^2 + x - 3}{2x^2 + 5x + 3}$$

$$= \frac{2x^2 - 2x + 3x - 3}{2x^2 + 2x + 3x + 3}$$

$$= \frac{(2x + 3)(x - 1)}{(2x + 3)(x + 1)}$$

The like terms are cancelled.

$$= \frac{x - 1}{x + 1}$$

Required solution.

1 H. Question

Simplify the following into their lowest forms.

$$\frac{2x^4 - 162}{(x^2 + 9)(2x - 6)}$$

Answer

$$\frac{2x^4 - 162}{(x^2 + 9)(2x - 6)}$$

$$= \frac{2(x^4 - 81)}{(x^2 + 9)2(x - 3)}$$

$$= \frac{(x^2 - 9)(x^2 + 9)}{(x^2 + 9)(x - 3)}$$

$$= \frac{(x - 3)(x + 3)}{x - 3}$$

The like terms are cancelled.

$= x + 3$

Required solution.

1 I. Question

Simplify the following into their lowest forms.

$$\frac{(x-3)(x^2-5x+4)}{(x-4)(x^2-2x-3)}$$

Answer

$$\frac{(x-3)(x^2-5x+4)}{(x-4)(x^\wedge 2-2x-3)}$$

$$=\frac{(x-3)(x^2-4x-x-4)}{(x-4)(x^2+x-3x-3)}$$

$$=\frac{(x-3)(x-4)(x-1)}{(x-4)(x-3)(x+1)}$$

The like terms are cancelled.

$$=\frac{x-1}{x+1}$$

Required answer.

1 J. Question

Simplify the following into their lowest forms.

$$\frac{(x-8)(x^2-5x-50)}{(x+10)(x^2-13x+40)}$$

Answer

$$\frac{(x-8)(x^2-5x-50)}{(x-10)(x^2-13x+40)}$$

$$=\frac{(x-8)(x^2-10x+5x-50)}{(x-10)(x^2-8x-5x+40)}$$

$$=\frac{(x-8)(x-5)(x-10)}{(x-10)(x-8)(x-5)}$$

The like terms are cancelled.

=1 required solution.

1 K. Question

Simplify the following into their lowest forms.

$$\frac{4x^2+9x+5}{8x^2+6x-5}$$

Answer

$$\frac{4x^2+9x+5}{8x^2+6x-5}$$

$$=\frac{4x^2+4x+5x+5}{8x^2+10x-4x-5}$$

$$=\frac{(4x+5)(x+1)}{(2x-1)(4x+5)}$$

The like terms are cancelled.

$$= \frac{x + 1}{2x - 1}$$

Required solution

1 L. Question

Simplify the following into their lowest forms.

$$\frac{(x-1)(x-2)(x^2-9x+14)}{(x-7)(x^2-3x+2)}$$

Answer

$$\frac{(x-1)(x-2)(x^2-9x+14)}{(x-7)(x^2-3x+2)}$$

$$= \frac{(x-1)(x-2)(x^2-7x-2x+14)}{(x-7)(x^2-2x-x+2)}$$

$$= \frac{(x-1)(x-2)(x-7)(x-2)}{(x-7)(x-1)(x-2)}$$

The like terms are cancelled.

=x – 2 required solution.

Exercise 3.10

1. Question

Multiply the following and write your answer in lowest terms.

(i) $\frac{x^2-2x}{x+2} \times \frac{3x+6}{x-2}$ (ii) $\frac{x^2-81}{x^2-4} \times \frac{x^2+6x+8}{x^2-5x-36}$

(iii) $\frac{x^2-3x-10}{x^2-x-20} . \frac{x^2-2x+4}{x^3+8}$ (iv) $\frac{x^2-16}{x^2-3x+2} \times \frac{x^2-4}{x^3+64} \times \frac{x^2-4x+16}{x^2-2x-8}$

(v) $\frac{3x^2-2x-1}{x^2-x-2} \times \frac{2x^2-3x-2}{3x^2+5x-2}$ (vi) $\frac{2x-1}{x^2-2x+4} \times \frac{x^4-8x}{2x^2+5x-3} \times \frac{x+3}{x^2-2x}$

Answer

(i) $\frac{x^2-2x}{x+2} . \frac{3x+6}{x-2}$

$$= \frac{x(x-2)}{x+2} . \frac{3(x+2)}{x-2}$$

The like terms are cancelled.

=3x required solution

(ii) $\frac{x^2-81}{x^2-4} . \frac{x^2-6x+8}{x^2-5x-36}$

We know $a^2 - b^2 = (a-b)(a+b)$

So,

$$\frac{x^2-81}{x^2-4} = \frac{(x-9)(x+9)}{(x-2)(x+2)}$$

Also,

$x^2 + 6x + 8 = x^2 + 4x + 2x + 8$

$= x(x+4) + 2(x+4)$

$= (x+2)(x+4)$

And

$x^2 - 5x - 36 = x^2 - 9x + 4x - 36$

$= x(x-9)+4(x-9)$

$= (x+4)(x-9)$

So,

$$= \frac{(x-9)(x+9)}{(x-2)(x+2)} \cdot \frac{(x-4)(x+2)}{(x-9)(x+4)}$$

The like terms are cancelled.

$$= \frac{x+9}{x-2}$$

Required solution

(iii) $\frac{x^2-3x-10}{x^2-x-20} \cdot \frac{x^2-2x+4}{x^3+8}$

$x^2 - 3x - 10 = x^2 - 5x + 2x - 10$

$= x(x-5)+2(x-5)$

$= (x+2)(x-5)$

And,

$x^2 - x - 20 = x^2 - 5x + 4x - 20$

$= x(x-5)+4(x-5)$

$= (x+4)(x-5)$

We know the formula $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

So $x^3 + 8 = x^3 + 2^3$

$= (x+2)(x^2 + 4 - 2x)$

$$\frac{(x+2)(x-5)}{(x+4)(x-5)} \cdot \frac{x^2-2x+4}{(x+2)(x2 + 4 - 2x)}$$

The like terms are cancelled.

$$= \frac{1}{x+4}$$

Required solution

(iv) $\frac{x^2-16}{x^2-3x+2} \cdot \frac{(x^2-4)}{(x^3+64)} \cdot \frac{x^2-4x+16}{x^2-2x-8}$

We know $a^2 - b^2 = (a-b)(a+b)$

$x^2 - 16 = (x-4)(x+4)$

$x^2 - 4 = (x-2)(x+2)$

$x^2 - 3x + 2 = x^2 - 2x - x + 2$

$= x(x-2)-1(x-2)$

$= (x-1)(x-2)$

$x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

$= x(x-4)+2(x-4)$

$= (x-4)(x+2)$

We know the formula $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

So $x^3 + 64 = x^3 + 4^3$

$= (x+4)(x^2 + 16 - 4x)$

$$= \frac{(x-4)(x+4)}{(x-2)(x-1)} \cdot \frac{(x-2)(x+2)}{(x+2)(x^2-4x+16)} \cdot \frac{x^2-4x+16}{(x-4)(x+2)}$$

The like terms are cancelled.

$$= \frac{1}{x-4}$$

Required solution

(v) $\frac{3x^2-2x-1}{x^2-x-2} \cdot \frac{2x^2-3x-2}{3x^2+5x-2}$

$$= \frac{3x^2-3x+x-1}{x^2-2x+x-2} \cdot \frac{2x^2-4x+x-2}{3x^2+6x-x-2}$$

The like terms are cancelled.

$$= \frac{(3x+1)(x-1)}{(x-2)(x+1)} \cdot \frac{(2x+1)(x-2)}{(3x-1)(x+2)}$$

$$= \frac{(3x+1)(x-1)(2x+1)}{(x+1)(3x-1)(x+2)}$$

Required solution

(vi) $\frac{2x-1}{x^2-2x+4} \cdot \frac{x^4-8x}{2x^2+5x-3} \cdot \frac{x+3}{x^2-2x}$

$= \frac{2x-1}{x^2-2x+4} \cdot \frac{x(x^3-8)}{2x^2+6x-x-3} \cdot \frac{x+3}{x(x-2)}$ We know $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$= \frac{2x-1}{x^2-2x+4} \cdot \frac{x(x-2)(x^2+2x+4)}{(2x-1)(x+3)} \cdot \frac{x+3}{x(x-2)}$$

The like terms are cancelled.

$= (x^2 + 2x + 4)/(x^2 - 2x + 4)$

Required solution.

2 A. Question

Divide the following and write your answer in lowest terms.

$$\frac{x}{x+1} \div \frac{x^2}{x^2-1}$$

Answer

$$\frac{x}{x + 1} \cdot \frac{x^2 - 1}{x^2}$$

$$= \frac{\frac{x}{x + 1}(x + 1)(x - 1)}{x^2}$$

The like terms are cancelled.

$$= \frac{x - 1}{x}$$

Required solution

2 B. Question

Divide the following and write your answer in lowest terms.

$$\frac{x^2 - 36}{x^2 - 49} \div \frac{x + 6}{x + 7}$$

Answer

$$\frac{\frac{(x^2 - 36)}{x^2 - 49}(x + 7)}{x + 6}$$

$$= \frac{\frac{(x - 6)(x + 6)}{(x - 7)(x + 7)}(x + 7)}{(x + 6)}$$

The like terms are cancelled.

$$= \frac{x - 6}{x - 7}$$

Required solution.

2 C. Question

Divide the following and write your answer in lowest terms.

$$\frac{x^2 - 4x - 5}{x^2 - 25} \div \frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

Answer

$$\frac{x^2 - 4x - 5}{x^2 - 5} \cdot \frac{x^2 + 7x + 10}{x^2 - 3x - 10}$$

$$= \frac{(x - 5)(x + 1)}{(x - 5)(x + 5)} \cdot \frac{(x + 5)(x + 2)}{(x - 5)(x + 2)}$$

The like terms are cancelled.

$$= \frac{x + 1}{x - 5}$$

Required solution.

2 D. Question

Divide the following and write your answer in lowest terms.

$$\frac{x^2+11x+28}{x^2-4x-77} \div \frac{x^2+7x+12}{x^2-2x-15}$$

Answer

$$\frac{x^2+11x+28}{x^2-4x-77} \cdot \frac{x^2-2x-15}{x^2+7x+12}$$

$$= \frac{(x+7)(x+4)}{(x-11)(x+7)} \cdot \frac{(x+3)(x-5)}{(x+4)(x+3)}$$

The like terms are cancelled.

$$= \frac{x-5}{x-11}$$

Required solution

2 E. Question

Divide the following and write your answer in lowest terms.

$$\frac{2x^2+13x+15}{x^2+3x-10} \div \frac{2x^2-x-6}{x^2-4x+4}$$

Answer

$$\frac{2x^2+13x+15}{x^2+3x-10} \cdot \frac{x^2-4x+4}{2x^2-x-6}$$

$$= \frac{2x^2+10x+3x+15}{x^2+5x-2x+10} \cdot \frac{x^2-2.x.2+2^2}{2x^2-4x+3x-6}$$

$$= \frac{(2x+3)(x+5)}{(x+5)(x-2)} \cdot \frac{(x-2)(x-2)}{(x-2)(2x+3)}$$

The like terms are cancelled.

=1 required solution.

2 F. Question

Divide the following and write your answer in lowest terms.

$$\frac{3x^2-x-4}{9x^2-16} \div \frac{4x^2-4}{3x^2-2x-1}$$

Answer

$$\frac{3x^2-x-4}{9x^2-16} \cdot \frac{3x^2-2x-1}{4x^2-4}$$

$$= \frac{3x^2+3x-4x-4}{(3x+4)(3x-4)} \cdot \frac{3x^2-3x+x-1}{4(x-1)(x+1)}$$

$$= \frac{(3x-4)(x+1)}{(3x+4)(3x-4)} \cdot \frac{(3x+1)(x-1)}{4(x-1)(x+1)}$$

The like terms are cancelled.

$$= \frac{3x+1}{4(3x+4)}$$

Required solution,

2 G. Question

Divide the following and write your answer in lowest terms.

$$\frac{2x^2+5x-3}{2x^2+9x+9} \div \frac{2x^2+x-1}{2x^2+x-3}$$

Answer

$$\frac{2x^2 + 5x - 3}{2x^2 + 9x + 9} \cdot \frac{2x^2 + x - 3}{2x^2 + x - 1}$$

$$= \frac{2x^2 + 6x - x - 3}{2x^2 + 6x + 3x + 9} \cdot \frac{2x^2 - 2x + 3x - 3}{2x^2 + 2x - x - 1}$$

$$= \frac{(2x - 1)(x + 3)}{(2x + 3)(x + 3)} \cdot \frac{(2x + 3)(x - 1)}{(2x - 1)(x + 1)}$$

The like terms are cancelled.

$$= \frac{x - 1}{x + 1}$$

Required solution.

Exercise 3.11

1. Question

Simplify the following as a quotient of two polynomials in the simplest form.

(i) $\frac{x^3}{x-2} + \frac{8}{2-x}$ (ii) $\frac{x+2}{x^2+3x+2} + \frac{x-3}{x^2-2x-3}$

(iii) $\frac{x^2-x-6}{x^2-9} + \frac{x^2+2x-24}{x^2-x-12}$ (iv) $\frac{x-2}{x^2-7x+10} + \frac{x+3}{x^2-2x-15}$

(v) $\frac{2x^2-5x+3}{x^2-3x+2} - \frac{2x^2-7x-4}{2x^2-3x-2}$ (vi) $\frac{x^2-4}{x^2+6x+8} - \frac{x^2-11x+30}{x^2-x-20}$

(vii) $\left[\frac{2x+5}{x+1} + \frac{x^2+1}{x^2-1}\right] - \left(\frac{3x-2}{x-1}\right)$ (viii) $\frac{1}{x^2+3x+2} + \frac{1}{x^2+5x+6} - \frac{2}{x^2+4x+3}$

Answer

(i) $\frac{x^3}{x-2} + \frac{8}{2-x}$

$$= \left(\frac{x^3}{x - 2} - \frac{8}{x - 2}\right)$$

$$= \frac{x^3 - 8}{x - 2}$$

$$= \frac{(x - 2)(x^2 + 2x + 4)}{x - 2}$$

The like terms are cancelled.

$=x^2 + 2x + 4$ required solution.

(ii) $\frac{x+2}{x^2+3x+2} + \frac{x-3}{x^2-2x-3}$

$$= \frac{x+2}{(x+2)(x+1)} + \frac{x-3}{(x-3)(x+1)}$$

The like terms are cancelled.

$$= \frac{1}{x+1} + \frac{1}{x+1}$$

$$= \frac{2}{x+1}$$

(iii) $\frac{x^2-x-6}{x^2-9} + \frac{x^2+2x-24}{x^2-x-12}$

$$= \frac{x^2-3x+2x-6}{(x-3)(x+3)} + \frac{x^2-4x+6x-24}{x^2-4x+3x-12}$$

$$= \frac{(x+2)(x-3)}{(x-3)(x+3)} + \frac{(x-4)(x+6)}{(x-4)(x+3)}$$

The like terms are cancelled.

$$= \frac{x+2}{x+3} + \frac{x+6}{x+3}$$

$$= \frac{2x+8}{x+3}$$

$$= \frac{2(x+4)}{x+3}$$

Required solution

(iv) $\frac{x-2}{x^2-7x+10} + \frac{x+3}{x^2-2x-15}$

$$= \frac{x-2}{(x-5)(x-2)} + \frac{x+3}{(x-5)(x+3)}$$

The like terms are cancelled.

$$= \frac{1}{x-5} + \frac{1}{x-5}$$

$$= \frac{2}{x-5}$$

Required solution.

(v) $\frac{2x^2-5x+3}{x^2-3x+2} - \frac{2x^2-7x-4}{2x^2-3x-2}$

$$= \frac{(2x-3)(x-1)}{(x-2)(x-1)} - \frac{(2x+1)(x-4)}{(2x+1)(x-2)}$$

The like terms are cancelled.

$$= \frac{2x-3}{x-2} - \frac{x-4}{x-2}$$

Required Solution $= \frac{x+1}{x-2}$

(vi) $\frac{x^2-4}{x^2+6x+8} - \frac{x^2-11x+30}{x^2-x-20}$

$$= \frac{(x+2)(x-2)}{(x+4)(x+2)} - \frac{(x-6)(x-5)}{(x-5)(x+4)}$$

Like terms are cancelled

$$= \frac{x-2}{x+4} - \frac{x-6}{x+4}$$

Required solution

$$= \frac{4}{x+4}$$

(vii) $\left[\frac{2x+5}{x+1} + \frac{x^2+1}{x^2-1}\right] - \frac{3x-2}{x-1}$

$$= \left[\frac{2x+5}{x+1} + \frac{x^2+1}{(x+1)(x-1)}\right] - \frac{3x-2}{x-1}$$

$$= \frac{1}{x+1}\frac{[(2x+5)(x-1)+(x^2+1)]}{x-1} - \frac{3x-2}{x-1}$$

$$= \frac{1}{(x+1)(x-1)}[(2x^2+3x-5+x^2+1)-(3x-2)(x+1)]$$

$$= \frac{[3x^2+3x-4-(3x^2+x-2)]}{(x+1)(x-1)}$$

$$= \frac{2(x-1)}{(x+1)(x-1)}$$

Like terms are cancelled

$$= \frac{2}{x+1}$$

(viii) $\frac{1}{x^2+3x+2} + \frac{1}{x^2+5x+6} - \frac{2}{x^2+4x+3}$

$$= \frac{1}{(x+2)(x+1)} + \frac{1}{(x+3)(x+2)} - \frac{2}{(x+3)(x+1)}$$

$$= \frac{[(x+3)+(x+1)-2(x+2)]}{(x+1)(x+2)(x+3)}$$

$$= \frac{2x+4-(2x+4)}{(x+1)(x+2)(x+3)}$$

= 0

2. Question

Which rational expression should be added to $\frac{x^3-1}{x^2+2}$ to get $\frac{3x^3+2x^2+4}{x^2+2}$?

Answer

Let p(x)= $\frac{3x^3+2x^2+4}{x^2+2}$, q(x)=$\frac{x^3-1}{x^2+2}$

And the rational expression be r(x),

So according to question,

p(x) = q(x) + r(x)

or, r(x) = p(x) – q(x)

$$=\frac{3x^3 + 2x^2 + 4}{x^2 + 2} - \frac{x^3 - 1}{x^2 + 2}$$

$$=\frac{3x^3 + 2x^2 + 4 - x^3 + 1}{x^2 + 2}$$

So required rational expression r(x) $= \frac{2x^3 + 2x^2 + 5}{x^2 + 2}$

3. Question

Which rational expression should be subtracted from

$\frac{4x^3 - 7x^2 + 5}{2x - 1}$ to get $2x^2 - 5x + 1$?

Answer

Let p(x) = $\frac{4x^3 - 7x^2 + 5}{2x-1}$, q(x) $= 2x^2 - 5x + 1$

And rational expression be r(x)

According to question,

p(x) – r(x)=q(x)

or, r(x) = p(x) – q(x)

r(x) $= \frac{4x^3 - 7x^2 + 5}{2x-1} + 2x^2 - 5x + 1$

$$= \frac{4x^3 - 7x^2 + 5 - (2x - 1)(2x^2 - 5x + 1)}{2x - 1}$$

$$= \frac{4x^3 - 7x^2 + 5 - 4x^3 + 8x^2 + 3x + 1}{2x - 1}$$

So required rational expression $= \frac{x^2 + 3x + 6}{2x-1}$

4. Question

If $P = \frac{x}{x+y}$, $Q = \frac{y}{x+y}$, then find $\frac{1}{P-Q} - \frac{2Q}{P^2 - Q^2}$.

Answer

P – Q $= \frac{x}{x+y} - \frac{y}{x+y} = \frac{x-y}{x+y}$

$P^2 - Q^2$ =(P + Q)(P – Q) $= \left(\frac{x}{x+y} + \frac{y}{x+y}\right)\left(\frac{x-y}{x+y}\right)$

$$= \frac{x - y}{x + y}$$

=P – Q

So $= \frac{1}{P-Q} - \frac{2Q}{P^2 - Q^2} = \frac{1-2Q}{P-Q}$ ($\because P^2 - Q^2 = P - Q$)

$$= \frac{x + y}{x - y}\left(1 - \frac{2y}{x + y}\right)$$

$$= \frac{x + y}{x - y} \times \frac{x - y}{x + y}$$

=1

Exercise 3.12

1 A. Question

Find the square root of the following

$196a^6b^8c^{10}$

Answer

In Square root the power of each term is divided by 2

$\sqrt{(196a^6b^8c^{10})} = \sqrt{(14^2\ a^6b^8c^{10})}$

Square Root = $|14a^3b^4c^5|$

1 B. Question

Find the square root of the following

$289\ (a–b)^4\ (b–c)^6$

Answer

$289 = 17^2$

$\Rightarrow$Square Root = $\sqrt{[17^2(a–b)^4\ (b–c)^6]}$

Square Root = $|17(a–b)^2(b–c)^3|$

1 C. Question

Find the square root of the following

$(x + 11)^2–44x$

Answer

$(x + 11)^2–44x$

$\Rightarrow x^2 + 22x + 121–44x$

$\Rightarrow x^2–22x + 121 = (x–11)^2$

$\sqrt{[(x + 11)^2–44x}$

Square root = $|x–11|$

1 D. Question

Find the square root of the following

$(x–y)^2 + 4xy$

Answer

$(x–y)^2 + 4xy$

$\Rightarrow x^2 + y^2–2xy + 4xy$

$\Rightarrow x^2 + y^2 + 2xy = (x + y)^2$

$\sqrt{[(x–y)^2 + 4xy]}$

Square Root = $|x + y|$

1 E. Question

Find the square root of the following

$121x^8y^6 \div 81x^4y^8$

Answer

$$\frac{121x^8y^6}{81x^4y^8} = \frac{121x^4}{81y^2}$$

Square Root $= \sqrt{\frac{121x^4}{81y^2}}$

⇒Square Root $= \sqrt{\frac{11^2x^4}{9^2y^2}}$

Square Root $= \left|\frac{11x^2}{9y}\right|$

1 F. Question

Find the square root of the following

$$\frac{64(a+b)^4(x-y)^8(b-c)^6}{25(x+y)^4(a-b)^6(b+c)^{10}}$$

Answer

$$\frac{64(a+b)^4(x-y)^8(b-c)^6}{25(x+y)^4(a-b)^6(b+c)^{10}} = \frac{8^2(a+b)^4(x-y)^8(b-c)^6}{5^2(x+y)^4(a-b)^6(b+c)^{10}}$$

Square Root $= \sqrt{\frac{8^2(a+b)^4(x-y)^8(b-c)^6}{5^2(x+y)^4(a-b)^6(b+c)^{10}}}$

$$= \left|\frac{8(a+b)^2(x-y)^4(b-c)^3}{5(x+y)^2(a-b)^3(b+c)^5}\right|$$

2 A. Question

Find the square root of the following:

$16x^2 - 24x + 9$

Answer

$16x^2 - 24x + 9$

The above expression can be rewritten as

$(4x)^2 - 2\times3\times4x + 3^2$

It is in the form of $(a-b)^2$

$= (4x-3)^2$

Square root $= \sqrt{(4x-3)^2}$

$|4x-3|$

2 B. Question

Find the square root of the following:

$(x^2 - 25)(x^2 + 8x + 15)(x^2 - 2x - 15)$

Answer

We factorize each of the above polynomials

$x^2 - 25 = x^2 - 5^2$

Since it is in the form of $a^2 - b^2 = (a-b)(a+b)$

$\Rightarrow x^2 - 25 = (x-5)(x+5)$ ...(i)

$x^2 + 8x + 15 = x^2 + 5x + 3x + 15$

$\Rightarrow x^2 + 8x + 15 = x(x+5) + 3(x+5)$

$\Rightarrow x^2 + 8x + 15 = (x+3)(x+5)$ ...(ii)

$x^2 - 2x - 15 = x^2 - 5x + 3x - 15$

$\Rightarrow x^2 - 2x - 15 = x(x-5) + 3(x-5)$

$\Rightarrow x^2 - 2x - 15 = (x+3)(x-5)$ ... (iii)

Combining (i), (ii) & (iii) we get

$(x^2 - 25)(x^2 + 8x + 15)(x^2 - 2x - 15) = = (x-5)^2(x+5)^2(x+3)^2$

Square Root = $\sqrt{[(x-5)^2(x+5)^2(x+3)^2]}$

$|(x-5)(x+5)(x+3)|$

2 C. Question

Find the square root of the following:

$4x^2 + 9y^2 + 25z^2 - 12xy + 30yz - 20zx$

Answer

The above expression can be rewritten as

$(2x)^2 + (-3y)^2 + (-5z)^2 + 2((-3y)\times(2x) + (-5z)\times(-3y) + (2x)\times(-5z))$

The above expression is in the form of

$(a-b-c)^2 = a^2 + b^2 + c^2 + 2(-ab + bc - ca)$

So the expression becomes $(2x-3y-5z)^2$

Square Root = $\sqrt{(2x-3y-5z)^2}$

$|2x-3y-5z|$

2 D. Question

Find the square root of the following:

$$x^4 + \frac{1}{x^4} + 2$$

Answer

The equation can be written as

$$(x^2)^2 + \frac{1}{(x^2)^2} + 2 \times x^2 \times \frac{1}{x^2}$$

The above equation is in the form of $(a+b)^2 = a^2 + b^2 + 2ab$

So it becomes

$$x^4 + \frac{1}{x^4} + 2 = \left(x^2 + \frac{1}{x^2}\right)^2$$

Square root $= \sqrt{\left(x^2 + \frac{1}{x^2}\right)^2}$

$\left|x^2 + \frac{1}{x^2}\right|$

2 E. Question

Find the square root of the following:

$(6x^2 + 5x - 6)(6x^2 - x - 2)(4x^2 + 8x + 3)$

Answer

We factorize each of the above polynomials

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$\Rightarrow 6x^2 + 5x - 6 = 3x(2x + 3) - 2(2x + 3)$

$\Rightarrow 6x^2 + 5x - 6 = (3x - 2)(2x + 3)$ ...(i)

$6x^2 - x - 2 = 6x^2 - 4x + 3x - 2$

$\Rightarrow 6x^2 - x - 2 = 2x(3x - 2) + 1(3x - 2)$

$\Rightarrow 6x^2 - x - 2 = (2x + 1)(3x - 2)$ ...(ii)

$4x^2 + 8x + 3 = 4x^2 + 6x + 2x + 3$

$\Rightarrow 4x^2 + 8x + 3 = 2x(2x + 3) + 1(2x + 3)$

$\Rightarrow 4x^2 + 8x + 3 = (2x + 1)(2x + 3)$ ...(iii)

Combining (i), (ii) & (iii) we get

$(6x^2 + 5x - 6)(6x^2 - x - 2)(4x^2 + 8x + 3) = (3x - 2)^2(2x + 3)^2(2x + 1)^2$

Square Root = $\sqrt{(3x - 2)^2(2x + 3)^2(2x + 1)^2}$

$|(3x - 2)(2x + 3)(2x + 1)|$

2 F. Question

Find the square root of the following:

$(2x^2 - 5x + 2)(3x^2 - 5x - 2)(6x^2 - x - 1)$

Answer

We factorize each of the above polynomials

$2x^2 - 5x + 2 = 2x^2 - 4x - x + 2$

$\Rightarrow 2x^2 - 5x + 2 = 2x(x - 2) - 1(x - 2)$

$\Rightarrow 2x^2 - 5x + 2 = (2x - 1)(x - 2)$ ...(i)

$3x^2 - 5x - 2 = 3x^2 - 6x + x - 2$

$\Rightarrow 3x^2 - 5x - 2 = 3x(x - 2) + 1(x - 2)$

$\Rightarrow 3x^2 - 5x - 2 = (3x + 1)(x - 2)$ ...(ii)

$6x^2 - x - 1 = 6x^2 - 3x + 2x - 1$

$\Rightarrow 6x^2 - x - 1 = 3x(2x - 1) + 1(2x - 1)$

$\Rightarrow 6x^2 - x - 1 = (3x + 1)(2x - 1)$ ...(iii)

Combining (i), (ii) & (iii) we get

$(2x^2 - 5x + 2)(3x^2 - 5x - 2)(6x^2 - x - 1) = (2x - 1)^2(x - 2)^2(3x + 1)^2$

Square Root = $\sqrt{(2x-1)^2(x-2)^2(3x+1)^2}$

$|(2x-1)(x-2)(3x+1)|$

Exercise 3.13

1 A. Question

Find the square root of the following polynomials by division method.

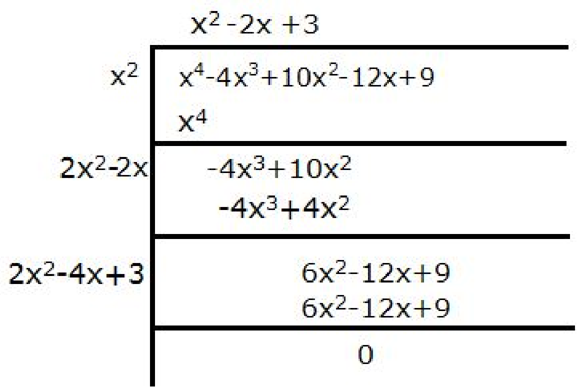
$x^4 - 4x^3 + 10x^2 - 12x + 9$

Answer

Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient .To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.



$|x^2 - 2x + 3|$

1 B. Question

Find the square root of the following polynomials by division method.

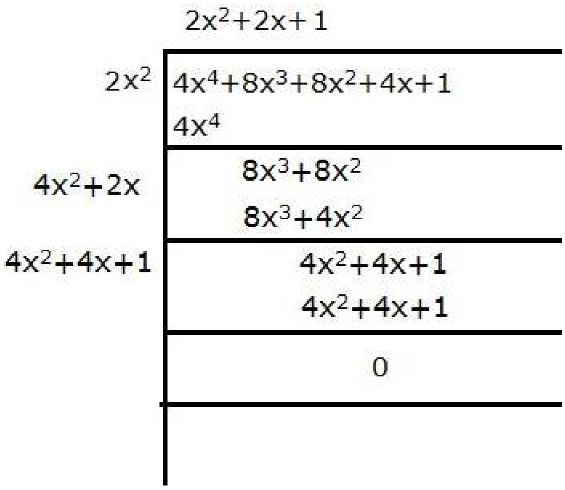
$4x^4 + 8x^3 + 8x^2 + 4x + 1$

Answer

Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient .To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.



$|2x^2 + 2x + 1|$

1 C. Question

Find the square root of the following polynomials by division method.

$9\ x^4 – 6\ x^3 + 7\ x^2 – 2x + 1$

Answer

Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient. To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.

	$3x^2-x+1$
$3x^2$	$9x^4-6x^3+7x^2-2x+1$
	$9x^4$
$6x^2-x$	$-6x^3+7x^2$
	$-6x^3+x^2$
$6x^2-2x+1$	$6x^2-2x+1$
	$6x^2-2x+1$
	0

$|3x^2 – x + 1|$

1 D. Question

Find the square root of the following polynomials by division method.

$4 + 25x^2 12x – 24x^3 + 16x^4$

Answer

Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient .To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.

	$4x^2-3x+2$
$4x^2$	$16x^4-24x^3+25x^2-12x+4$
	$16x^4$
$8x^2-3x$	$-24x^3+25x^2$
	$-24x^3+9x^2$
$8x^2-6x+2$	$16x^2-12x+4$
	$16x^2-12x+4$
	0

$|4x^2 – 3x + 2|$

2 A. Question

Find the values of a and b if the following polynomials are perfect squares.

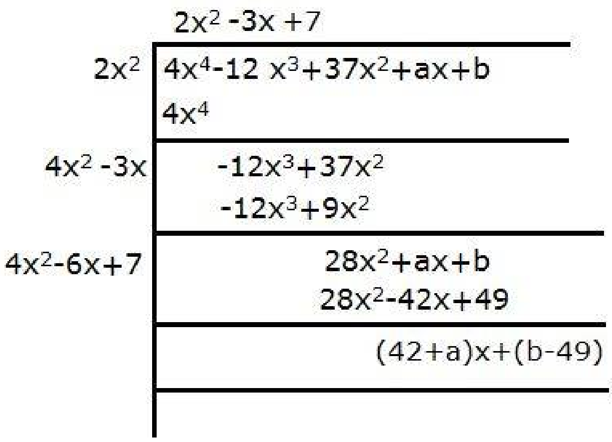
$4x^4 – 12\ x^3 + 37x^2 + ax + b$

Answer

Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient .To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.



Since it is a perfect square the remainder is 0

a = –42, b = 49

2 B. Question

Find the values of a and b if the following polynomials are perfect squares.

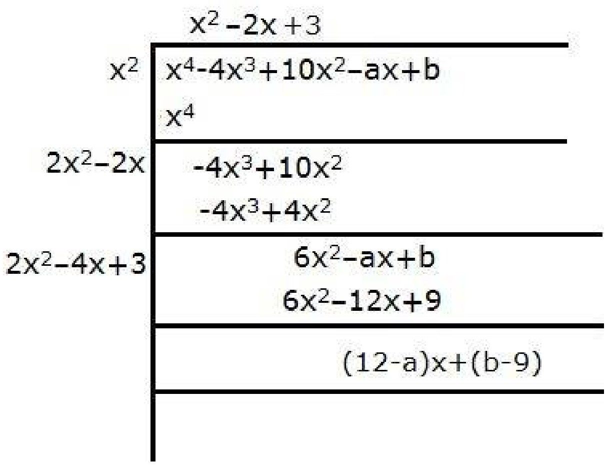
$x^4 – 4x^3 + 6x^2 – ax + b$

Answer

Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient .To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.



Since it is a perfect square the remainder is 0

a = 12, b = 9

2 C. Question

Find the values of a and b if the following polynomials are perfect squares.

$ax^4 + bx^3 + 109x^2 – 60x + 36$

Answer

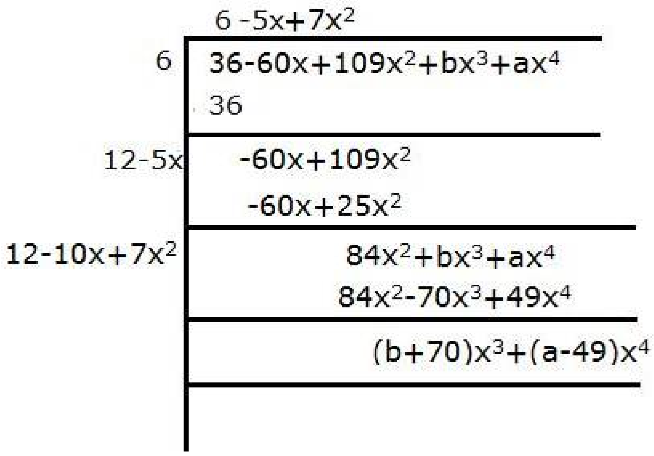
Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient .To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.

We rearrange the equation in the increasing order of power of x.

The polynomial becomes $36–60x + 109x^2 + bx^3 + ax^4$



Since it is a perfect square the remainder is 0

a = 49, b = –70

2 D. Question

Find the values of a and b if the following polynomials are perfect squares.

$a x^4–bx^3 + 40 x^2 + 24x + 36$

Answer

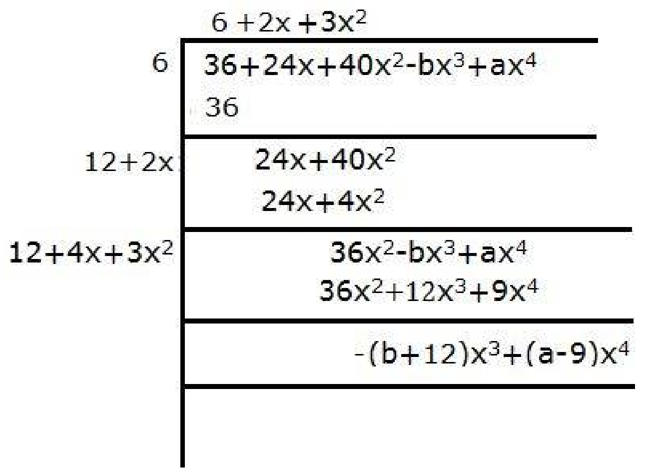
Step 1: Find the algebraic expression whose square gives the first term.

Step 2: Add the divisor and quotient .To this sum add a suitable expression and add the same expression in the quotient such that the product gives the next term.

Step 3: Continue the process until all the terms are divided.

We rearrange the equation in the increasing order of power of x.

The polynomial becomes $36 + 24x + 40x^2–bx^3 + ax^4$



Since it is a perfect square the remainder is 0

a = 9, b = –12

Exercise 3.14

1 A. Question

Solve the following quadratic equations by factorization method.

$(2x + 3)^2 – 81 = 0$

Answer

$(2x + 3)^2 – 81 = 0$

$= (2x)^2 + 2(2x)(3) + 3^2 – 81 = 0$

$= 4x^2 + 12x + 9 –81 = 0$

$= 4x^2 + 12x – 72 = 0$

Divide by 4 both sides

$\Rightarrow \frac{4x^2}{4} + \frac{12x}{4} - \frac{72}{4} = \frac{0}{4}$

$= x^2 + 3x – 18 = 0$

$= x^2 + 6x – 3x – 18 = 0$

$= x (x + 6) – 3 (x + 6) = 0$

$= (x + 6) (x – 3) = 0$

$x + 6 = 0$ or $x – 3 = 0$

$x = –6$ or $x = 3$

1 B. Question

Solve the following quadratic equations by factorization method.

$3x^2 –5x –12 = 0$

Answer

$3x^2 – 5x –12 = 0$

$= 3x^2 – 9x + 4x – 12 = 0$

$= 3x (x – 3) + 4(x – 3) = 0$

$= (3x + 4) (x – 3) = 0$

$3x + 4 = 0$ or $x –3 = 0$

$3x = –4$ or $x = 3$

$x = -\frac{4}{3}$ or $x = 3$

1 C. Question

Solve the following quadratic equations by factorization method.

$\sqrt{5}x^2 + 2x - 3\sqrt{5} = 0$

Answer

$\sqrt{5}x^2 + 2x – 3\sqrt{5} = 0$

$= \sqrt{5}x^2 + 5x – 3x – 3\sqrt{5} = 0$

$= \sqrt{5}x (x + \sqrt{5}) – 3(x + \sqrt{5}) = 0$

$= (\sqrt{5}x - 3)(x + \sqrt{5}) = 0$

$\sqrt{5}x - 3 = 0$ or $x + \sqrt{5} = 0$

$\sqrt{5}x = 3$ or $x = -\sqrt{5}$

$x = \frac{3}{\sqrt{5}}$ or $x = -\sqrt{5}$

1 D. Question

Solve the following quadratic equations by factorization method.

$3(x^2 - 6) = x(x + 7) - 3$

Answer

$3(x^2 - 6) = x(x + 7) - 3$

$= 3x^2 - 18 = x^2 + 7x - 3$

$= 3x^2 - 18 - x^2 - 7x + 3 = 0$

$= 2x^2 - 7x - 15 = 0$

$= 2x^2 - 10x + 3x - 15 = 0$

$= 2x(x - 5) + 3(x - 5) = 0$

$= (2x + 3)(x - 5) = 0$

$2x + 3 = 0$ or $x - 5 = 0$

$2x = -3$ or $x = 5$

$x = -\frac{3}{2}$ or $x = 5$

1 E. Question

Solve the following quadratic equations by factorization method.

$$3x - \frac{8}{x} = 2$$

Answer

$$3x - \frac{8}{x} = 2$$

$$= \frac{3x \times x - 8}{x} = 2$$

$= 3x^2 - 8 = 2x$

$= 3x^2 - 2x - 8 = 0$

$= 3x^2 - 6x + 4x - 8 = 0$

$= 3x(x - 2) + 2(x - 2) = 0$

$= (3x + 2)(x - 2) = 0$

$3x + 2 = 0$ or $x - 2 = 0$

$3x = -2$ or $x = 2$

$x = -\frac{2}{3}$ or $x = 2$

1 F. Question

Solve the following quadratic equations by factorization method.

$$x + \frac{1}{x} = \frac{26}{5}$$

Answer

$$x + \frac{1}{x} = \frac{26}{5}$$

$= \frac{x \times x + 1}{x} = \frac{26}{5}$

= $5(x^2 + 1) = 26x$

= $5x^2 + 5 = 26x$

= $5x^2 - 26x + 5 = 0$

= $5x^2 - 25x - x + 5 = 0$

= $5x(x - 5) - (x - 5) = 0$

= $(5x - 1)(x - 5) = 0$

$5x - 1 = 0$ or $x - 5 = 0$

$5x = 1$ or $x = 5$

$x = \frac{1}{5}$ or $x = 5$

1 G. Question

Solve the following quadratic equations by factorization method.

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$$

Answer

$$\frac{x}{x + 1} + \frac{x + 1}{x} = \frac{34}{15}$$

$= \frac{\{x \times x + (x+1)(x+1)\}}{[x(x+1)]} = \frac{34}{15}$

$= \frac{[x^2 + (x+1)^2]}{x^2 + x} = \frac{34}{15}$

= $15(x^2 + x^2 + 2x + 1) = 34(x^2 + x)$

= $15(2x^2 + 2x + 1) = 34(x^2 + x)$

= $30x^2 + 30x + 15 = 34x^2 + 34x$

= $34x^2 + 34x - 30x^2 - 30x - 15 = 0$

= $4x^2 - 4x - 15 = 0$

= $4x^2 - 10x + 6x - 15 = 0$

= $2x(2x - 5) + 3(2x - 5) = 0$

= $(2x + 3)(2x - 5) = 0$

$2x + 3 = 0$ or $2x - 5 = 0$

$2x = -3$ or $2x = 5$

$x = -\frac{3}{2}$ or $x = \frac{5}{2}$

1 H. Question

Solve the following quadratic equations by factorization method.

$a^2b^2x^2 – (a^2 + b^2) x + 1 = 0$

Answer

$a^2b^2x^2 – (a^2 + b^2) x + 1 = 0$

$= a^2b^2x^2 – a^2x – b^2x + 1 = 0$

$= a^2x (b^2x – 1) – (b^2x – 1) = 0$

$= (a^2x – 1) (b^2x – 1) = 0$

$a^2x – 1 = 0$ or $b^2x – 1 = 0$

$a^2x = 1$ or $b^2x = 1$

$x = \frac{1}{a^2}$ or $x = \frac{1}{b^2}$

1 I. Question

Solve the following quadratic equations by factorization method.

$2(x + 1)^2 –5 (x + 1) = 12$

Answer

$2(x + 1)^2 – 5(x + 1) = 12$

$= 2(x^2 + 2x + 1) – 5x – 5 = 12$

$= 2x^2 + 4x + 2 – 5x – 5 –12 = 0$

$= 2x^2 – x – 15 = 0$

$= 2x^2 – 6x + 5x – 15 = 0$

$= 2x (x – 3) + 5(x –3) = 0$

$= (2x + 5) (x – 3) = 0$

$2x + 5 = 0$ or $x – 3 = 0$

$2x = –5$ or $x = 3$

$x = -\frac{5}{2}$ or $x = 3$

1 J. Question

Solve the following quadratic equations by factorization method.

$3(x–4)^2 – 5 (x – 4) = 12$

Answer

$3(x – 4)^2 – 5(x – 4) = 12$

$= 3(x^2 – 8x + 16) – 5x + 20 = 12$

$= 3x^2 – 24x + 48 –5x + 20 – 12 = 0$

$= 3x^2 – 29x + 56 = 0$

$= 3x^2 - 21x - 8x + 56 = 0$

$= 3x(x - 7) - 8(x - 7) = 0$

$= (x - 7)(3x - 8) = 0$

$x - 7 = 0$ or $3x - 8 = 0$

$x = 7$ or $3x = 8$

$x = 7$ or $x = \frac{8}{3}$

Exercise 3.15

1 A. Question

Solve the following quadratic equations by completing the square.

$x^2 + 6x - 7 = 0$

Answer

$x^2 + 6x - 7 = 0$

$= x^2 + 6x = 7$

Add 9 on both sides

$= x^2 + 6x + 9 = 7 + 9$

$= x^2 + 2(3)(x) + 3^2 = 16$

$= (x + 3)^2 = 16$

$= x + 3 = \sqrt{16}$

$= x + 3 = \pm 4$

$x + 3 = 4$ or $x + 3 = -4$

$x = 4 - 3$ or $x = -4 - 3$

$x = 1$ or $x = -7$

1 B. Question

Solve the following quadratic equations by completing the square.

$x^2 + 3x + 1 = 0$

Answer

$x^2 + 3x + 1 = 0$

$= x^2 + 3x = -1$

Add $\frac{9}{4}$ on both sides

$= x^2 + 3x + \frac{9}{4} = -1 + \frac{9}{4}$

$= x^2 + 2(\frac{3}{2})(x) + \left(\frac{3}{2}\right)^2 = \frac{-4 + 9}{4}$

$= \left(x + \frac{3}{2}\right)^2 = \frac{5}{4}$

$= x + \frac{3}{2} = \sqrt{\frac{5}{4}}$

$= x + \frac{3}{2} = \pm\frac{\sqrt{5}}{2}$

$x + \frac{3}{2} = \frac{\sqrt{5}}{2}$ or $x + \frac{3}{2} = -\frac{\sqrt{5}}{2}$

$x = \frac{\sqrt{5}}{2} - \frac{3}{2}$ or $x = -\frac{\sqrt{5}}{2} - \frac{3}{2}$

$x = \frac{\sqrt{5} - 3}{2}$ or $x = -\frac{\sqrt{5} + 3}{2}$

1 C. Question

Solve the following quadratic equations by completing the square.

$2x^2 + 5x - 3 = 0$

Answer

$2x^2 + 5x - 3 = 0$

$= 2x^2 + 5x = 3$

Add $\frac{25}{4}$ on both sides

$= 2x^2 + 5x + \frac{25}{4} = 3 + \frac{25}{4}$

$= 2x^2 + 2\left(\frac{5}{2}\right)(x) + \frac{25}{4} = \frac{3 + 25}{4}$

$= (\sqrt{2}x + \frac{5}{2})2 = \frac{28}{4}$

$= \sqrt{2}x + \frac{5}{2} = \sqrt{\frac{28}{4}}$

$= \sqrt{2}x + \frac{5}{2} = \pm\frac{\sqrt{28}}{2}$

$\sqrt{2}x + \frac{5}{2} = \frac{\sqrt{28}}{2}$ or $\sqrt{2}x + \frac{5}{2} = -\frac{\sqrt{28}}{2}$

$\sqrt{2}x = \frac{\sqrt{28}}{2} - \frac{5}{2}$ or $\sqrt{2}x = -\frac{\sqrt{28}}{2} - \frac{5}{2}$

$\sqrt{2}x = \frac{\sqrt{28} - 5}{2}$ or $\sqrt{2}x = \frac{-\sqrt{28} - 5}{2}$

$x = \frac{\left[\frac{\sqrt{28} - 5}{2}\right]}{\sqrt{2}}$ or $x = \frac{\left[\frac{-\sqrt{28} - 5}{2}\right]}{\sqrt{2}}$

$x = \frac{2\sqrt{7} - 5}{2\sqrt{2}}$ or $x = \frac{-2\sqrt{7} - 5}{2\sqrt{2}}$

1 D. Question

Solve the following quadratic equations by completing the square.

$4x^2 + 4bx - (a^2 - b^2) = 0$

Answer

$4x^2 + 4bx - (a^2 - b^2) = 0$

Divide the whole equation by 4

$$= x^2 + bx - \frac{a^2 - b^2}{4} = 0$$

$$= x^2 + \left(\frac{2}{2}\right)(b)x - \frac{a^2 - b^2}{4} = 0$$

$$= x^2 + \left(\frac{2}{2}\right)(b)x = \frac{a^2 - b^2}{4}$$

Add $\frac{b^2}{4}$ on both sides

$$= x^2 + \left(\frac{2}{2}\right)(b)x + \frac{b^2}{4} = \frac{a^2 - b^2}{4} + \frac{b^2}{4}$$

$$= x^2 + \left(\frac{2}{2}\right)(b)x + \left(\frac{b}{2}\right)^2 = \frac{a^2 - b^2}{4} + \frac{b^2}{4}$$

$$= \left(x + \frac{b}{2}\right)^2 = \frac{a^2 - b^2 + b^2}{4}$$

$$= \left(x + \frac{b}{2}\right)^2 = \frac{a^2}{4}$$

$$= x + \frac{b}{2} = \sqrt{\frac{a^2}{4}}$$

$$= x + \frac{b}{2} = \pm\frac{a}{2}$$

$$x + \frac{b}{2} = \frac{a}{2} \text{ or } x + \frac{b}{2} = -\frac{a}{2}$$

$$x = \frac{a}{2} - \frac{b}{2} \text{ or } x = -\frac{a}{2} - \frac{b}{2}$$

$$x = \frac{a - b}{2} \text{ or } x = \frac{-a - b}{2}$$

1 E. Question

Solve the following quadratic equations by completing the square.

$x^2 - (\sqrt{3} + 1)\, x + \sqrt{3} = 0$

Answer

$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

$$= x^2 - \frac{2}{2}(\sqrt{3} + 1)\, x + \sqrt{3} = 0$$

$= x^2 - \frac{2}{2}(\sqrt{3} + 1)x = -\sqrt{3}$

Add $\left(\frac{\sqrt{3} + 1}{2}\right)^2$ on both sides

$= x^2 - \frac{2}{2}(\sqrt{3} + 1)x + \left(\frac{\sqrt{3} + 1}{2}\right)^2 = \left(\frac{\sqrt{3} + 1}{2}\right)^2 - \sqrt{3}$

$= \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \frac{(\sqrt{3} + 1)^2}{4} - \sqrt{3}$

$= \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \frac{(\sqrt{3})^2 + 2\sqrt{3} + 1 - 4\sqrt{3}}{4}$

$= \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \frac{(\sqrt{3})^2 - 2\sqrt{3} + 1}{4}$

$= \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \frac{(\sqrt{3} - 1)^2}{4}$

$= \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \frac{(\sqrt{3} - 1)^2}{4}$

$= \left(x - \frac{\sqrt{3} + 1}{2}\right)^2 = \frac{(\sqrt{3} - 1)^2}{4}$

$= x - \frac{\sqrt{3} + 1}{2} = \pm \frac{\sqrt{3} - 1}{2}$

$x - \frac{\sqrt{3} + 1}{2} = \frac{\sqrt{3} - 1}{2}$ or $x - \frac{\sqrt{3} + 1}{2} = -\frac{\sqrt{3} - 1}{2}$

$x = \frac{\sqrt{3} - 1}{2} + \frac{\sqrt{3} + 1}{2}$ or $x = -\frac{\sqrt{3} - 1}{2} + \frac{\sqrt{3} + 1}{2}$

$x = \frac{\sqrt{3} - 1 + \sqrt{3} + 1}{2}$ or $x = \frac{-\sqrt{3} + 1 + \sqrt{3} + 1}{2}$

$x = \frac{2\sqrt{3}}{2}$ or $x = \frac{2}{2}$

$x = \sqrt{3}$ or $x = 1$

1 F. Question

Solve the following quadratic equations by completing the square.

$\frac{5x + 7}{x - 1} = 3x + 2$

Answer

$\frac{5x + 7}{x - 1} = 3x + 2$

$= 5x + 7 = (3x + 2)(x - 1)$

$= 5x + 7 = 3x(x - 1) + 2(x - 1)$

$= 5x + 7 = 3x^2 – 3x + 2x –2$

$= 5x + 7 = 3x^2 – x – 2$

$= 3x^2 – x – 2 – 5x – 7 =0$

$= 3x^2 – 6x – 9 = 0$

Divide whole equation by 3

$= x^2 – 2x – 3 = 0$

$= x^2 – 3x + x – 3 = 0$

$= x (x – 3) + (x – 3) = 0$

$= (x – 3) (x + 1) = 0$

$x – 3 = 0$ or $x + 1 = 0$

$x = 3$ or $x = –1$

2 A. Question

Solve the following quadratic equations using quadratic formula.

$x^2 7x + 12= 0$

Answer

$x^2 – 7x + 12 = 0$

$\Rightarrow\ x^2 - 7x + 12 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow\ a = 1, b = -7$ and $c = 12$

$\therefore\ b^2 - 4ac = (–7)^2 – 4 (1) (12)$

$=49 – 48$

$=1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{1}}{2 \times 1} = \frac{7 \pm 1}{2}$$

$$\Rightarrow\ x = \frac{7 + 1}{2} \text{ or } x = \frac{7 - 1}{2}$$

$$\Rightarrow\ x = \frac{8}{2} \text{ or } x = \frac{6}{2}$$

$\Rightarrow\ x = 4$ or $x = 3$

2 B. Question

Solve the following quadratic equations using quadratic formula.

$15x^2 – 11x + 2 = 0$

Answer

$15x^2 – 11x + 2 = 0$

$\Rightarrow\ x^2 - 7x + 12 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 15 , b = –11$ and $c = 2$

$\therefore\ b^2 - 4ac = (-11)^2 - 4\ (15)\ (2)$

$= 121 - 120$

$= 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-11) \pm \sqrt{1}}{2 \times 15} = \frac{11 \pm 1}{30}$$

$$\Rightarrow\ x = \frac{11\ +\ 1}{30}\ \text{or}\ x = \frac{11 - 1}{30}$$

$$\Rightarrow\ x = \frac{12}{30}\ \text{or}\ x = \frac{10}{30}$$

$$\Rightarrow\ x = \frac{2}{5}\ \text{or}\ x = \frac{1}{3}$$

2 C. Question

Solve the following quadratic equations using quadratic formula.

$$x + \frac{1}{x} = 2\frac{1}{2}$$

Answer

$$x\ +\ \frac{1}{x} = 2\frac{1}{2}$$

$$= \frac{x^2 + 1}{x} = \frac{5}{2}$$

$\Rightarrow 2(x^2 + 1) = 5x$

$\Rightarrow 2x^2 + 2 = 5x$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow\ 2x^2 - 5x\ +\ 2 = 0$ compare with $ax^2\ +\ bx\ +\ c = 0$

$\Rightarrow\ a = 2, b = -5$ and $c = 2$

$\therefore\ b^2 - 4ac = (-5)^2 - 4\ (2)\ (2)$

$= 25 - 16$

$= 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{9}}{2 \times 2} = \frac{5 \pm 3}{4}$$

$$\Rightarrow\ x = \frac{5\ +\ 3}{4}\ \text{or}\ x = \frac{5 - 3}{4}$$

$$\Rightarrow\ x = \frac{8}{4}\ \text{or}\ x = \frac{2}{4}$$

$$\Rightarrow\ x = 2\ \text{or}\ x = \frac{1}{2}$$

2 D. Question

Solve the following quadratic equations using quadratic formula.

$3a^2x^2 – ax –2b^2 = 0$

Answer

$3a^2x^2 – abx – 2b^2 = 0$

$\Rightarrow 3a^2x^2 - abx - 2b^2 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 3a^2$, $b = –ab$ and $c = –2b^2$

$\therefore b^2 - 4ac = (–ab)^2 – 4 (3a^2) (–2b^2)$

$= a^2b^2 + 24a^2b^2$

$=25a^2b^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-ab) \pm \sqrt{25a^2b^2}}{2 \times 3a^2} = \frac{ab \pm 5ab}{6a^2}$$

$$\Rightarrow x = \frac{ab + 5ab}{6a^2} \text{ or } x = \frac{ab - 5ab}{6a^2}$$

$$\Rightarrow x = \frac{6ab}{6a^2} \text{ or } x = \frac{-4ab}{6a^2}$$

$$\Rightarrow x = \frac{b}{a} \text{ or } x = -\frac{2b}{3a}$$

2 E. Question

Solve the following quadratic equations using quadratic formula.

$a (x^2 + 1) = x (a^2 + 1)$

Answer

$a(x^2 + 1) = x(a^2 + 1)$

$\Rightarrow ax^2 + a = a^2x + x$

$\Rightarrow ax^2 + a – a^2x – x = 0$

$\Rightarrow ax^2 – x(a^2 + 1) + a = 0$

$\Rightarrow ax^2 - x(a^2 + 1) + a = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = a$, $b = – a^2 – 1$ and $c = a$

$\therefore b^2 - 4ac = (–a^2 – 1)^2 – 4 (a)(a)$

$= a^4 + 2a^2 + 1 – 4a^2$

$= (a^2)^2 – 2a^2 + 1$

$= (a^2 – 1)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-a^2 - 1) \pm \sqrt{(a^2 - 1)^2}}{2 \times a} = \frac{a^2 + 1 \pm a^2 - 1}{2a}$$

$$\Rightarrow x = \frac{a^2 + 1 + a^2 - 1}{2a} \text{ or } x = \frac{a^2 + 1 - a^2 + 1}{2a}$$

$$\Rightarrow x = \frac{2a^2}{2a} \text{ or } x = \frac{2}{2a}$$

$$\Rightarrow x = a \text{ or } x = \frac{1}{a}$$

2 F. Question

Solve the following quadratic equations using quadratic formula.

$36x^2 - 12ax + (a^2 - b^2) = 0$

Answer

$36x^2 - 12ax + (a^2 - b^2) = 0$

$\Rightarrow 36x^2 - 12ax + (a^2 - b^2) = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 36, b = -12a$ and $c = a^2 - b^2$

$\therefore b^2 - 4ac = (-12a)^2 - 4(36)(a^2 - b^2)$

$= 144a^2 - 144(a^2 - b^2)$

$= 144a^2 - 144a^2 + 144b^2$

$= 144b^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12a) \pm \sqrt{144b^2}}{2 \times 36} = \frac{12a \pm 12b}{72}$$

$$\Rightarrow x = \frac{12a + 12b}{72} \text{ or } x = \frac{12a - 12b}{72}$$

$$\Rightarrow x = \frac{12(a + b)}{72} \text{ or } x = \frac{12(a - b)}{72}$$

$$\Rightarrow x = \frac{a + b}{6} \text{ or } x = \frac{a - b}{6}$$

2 G. Question

Solve the following quadratic equations using quadratic formula.

$$\frac{x-1}{x+1} + \frac{x-3}{x-4} = \frac{10}{3}$$

Answer

$$\frac{x-1}{x+1} + \frac{x-3}{x-4} = \frac{10}{3}$$

$$\Rightarrow \frac{(x-1)(x-4) + (x-3)(x+1)}{(x+1)(x-4)} = \frac{10}{3}$$

$$\Rightarrow \frac{(x^2 - 5x + 4 + x^2 - 2x - 3)}{x^2 - 3x - 4} = \frac{10}{3}$$

$\Rightarrow 3(2x^2 - 7x + 1) = 10(x^2 - 3x - 4)$

$\Rightarrow 6x^2 - 21x + 3 = 10x^2 - 30x - 40$

$\Rightarrow 10x^2 - 6x^2 - 30x + 21x - 40 - 3 = 0$

$\Rightarrow 4x^2 - 9x - 43 = 0$

$\Rightarrow\ 4x^2 - 9x - 43 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow\ a = 4, b = -9$ and $c = -43$

$\therefore\ b^2 - 4ac = (-9)^2 - 4\ (4)\ (-43)$

$= 91 + 688$

$= 769$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{769}}{2 \times 4} = \frac{9 \pm \sqrt{769}}{8}$$

$$\Rightarrow\ x = \frac{9 + \sqrt{769}}{8} \text{ or } x = \frac{9 - \sqrt{769}}{8}$$

2 H. Question

Solve the following quadratic equations using quadratic formula.

$a^2x^2 + (a^2 - b^2)\ x - b^2 = 0$

Answer

$a^2x^2 + (a^2 - b^2)x - b^2 = 0$

$\Rightarrow\ a^2x^2 + x(a^2 - b^2) - b^2 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = a^2$, $b = a^2 - b^2$ and $c = -b^2$

$\therefore\ b^2 - 4ac = (a^2 - b^2)^2 - 4\ (a^2)(-b^2)$

$= (a^2)^2 - 2a^2b^2 + (b^2)^2 + 4a^2b^2$

$= (a^2)^2 + 2a^2b^2 + (b^2)^2$

$= (a^2 + b^2)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(a^2 - b^2) \pm \sqrt{(a^2 + b^2)^2}}{2 \times a^2} = \frac{-a^2 + b^2 \pm a^2 + b^2}{2a^2}$$

$$\Rightarrow\ x = \frac{-a^2 + b^2 + a^2 + b^2}{2a^2} \text{ or } x = \frac{-a^2 + b^2 - a^2 - b^2}{2a^2}$$

$$\Rightarrow\ x = \frac{2b^2}{2a^2} \text{ or } x = \frac{-2a^2}{2a^2}$$

$$\Rightarrow\ x = \frac{b^2}{a^2} \text{ or } x = -1$$

Exercise 3.16

1. Question

The sum of a number and its reciprocal is $\frac{65}{8}$. Find the number.

Answer

Let x be the required number. Then, the reciprocal is $\frac{1}{x}$.

⇒ sum of a number and its reciprocal is $\frac{65}{8}$

⇒ $x + \frac{1}{x} = \frac{65}{8}$

⇒ $\frac{x^2 + 1}{x} = \frac{65}{8}$

⇒ $8(x^2 + 1) = 65x$

⇒ $8x^2 + 8 = 65x$

⇒ $8x^2 - 65x + 8 = 0$

⇒ $8x^2 - x - 64x + 8 = 0$

⇒ $x(8x - 1) - 8(8x - 1) = 0$

⇒ $(x - 8)(8x - 1) = 0$

$x - 8 = 0$ or $8x - 1 = 0$

$x = 8$ or $8x = 1$

$x = 8$ or $x = \frac{1}{8}$

Therefore, the two required numbers are 8 and $\frac{1}{8}$.

2. Question

The difference of the squares of two positive numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

Answer

Let ‘x’ be the larger number and ‘y’ be the smaller number.

$x^2 - y^2 = 45$ …(1)

$y^2 = 4x$ …(2)

Now, put the value of y^2 in equation (1).

$x^2 - 4x = 45$

⇒ $x^2 - 4x - 45 = 0$

⇒ $x^2 - 9x + 5x - 45 = 0$

⇒ $x(x - 9) + 5(x - 9) = 0$

⇒ $(x + 5)(x - 9) = 0$

$x + 5 = 0$ or $x - 9 = 0$

$x = -5$ or $x = 9$

Here positive 9 only admissible. From this we need to find the value of y for that we are going to aplly this value in the second equation.

$y^2 = 4x$

$\Rightarrow y^2 = 4 \times 9$

$\Rightarrow y^2 = 36$

$\Rightarrow y = \sqrt{36}$

$\Rightarrow y = \pm 6$

Here, positive 6 only admissible.

Therefore, the required numbers are 6 and 9.

3. Question

A farmer wishes to start a 100 sq. rectangular vegetable garden. Since he has only 30 m barbed wire, he fences the sides of the rectangular garden letting his house compound wall act as the fourth side fence. Find the dimension of the garden.

Answer

Let 'x' and 'y' are the dimension of the vegetable garden.

Area of rectangle = Length × Width

$x \times y = 100$

$x = \frac{100}{y}$

we are going to cover the barbed wire for fencing only. So, it must be the perimeter of vegetable garden. Usually perimeter always covers all the four side. Bute here we are going to cover only three sides, because one side of the vegetable garden will act as the compound wall.

$x + x + y = 30$

$\Rightarrow 2x + y = 30$

$\Rightarrow 2\left(\frac{100}{y}\right) + y = 30$

$\Rightarrow \frac{200 + y^2}{y} = 30$

$\Rightarrow 200 + y^2 = 30y$

$\Rightarrow y^2 - 30y + 200 = 0$

$\Rightarrow y^2 - 10y - 20y + 200 = 0$

$\Rightarrow y(y - 10) - 20(y - 10) = 0$

$\Rightarrow (y - 10)(y - 20) = 0$

$y - 10 = 0$ or $y - 20 = 0$

$y = 10$ or $y = 20$

Now we are going to apply these values in $x = \frac{100}{y}$ to get the values of x.

If $y = 10$ if $y = 20$

$x = \frac{100}{10}$ $x = \frac{100}{20}$

$x = 10$ $x = 5$

Therefore, the required dimensions are 10m and 10m or 20m and 5m.

4. Question

A rectangular field is 20 m long and 14 m wide. There is a path of equal width all around it having an area of 111 sq. meters. Find the width of the path on the outside.

Answer

Length of the rectangular field = 20m

Breadth of the rectangular field = 14m

Let x be the uniform width all around the path.

Length of the rectangular field including the path

= 20 + x + x

= 20 + 2x

Width of the rectangular field including path

= 14 + x + x

= 14 + 2x

Area of path = area of rectangular field including path – area of rectangular field

⇒ 111 = (20 + 2x)(14 + 2x) – (20 × 14)

⇒ 111 = 280 + 40x + 28x + $4x^2$ – 280

⇒ 111 = 68x + $4x^2$

⇒ $4x^2$ + 68x – 111 = 0

⇒ $4x^2$ + 74x – 6x – 111 = 0

⇒ 2x(2x + 37) – 3(2x + 37) = 0

⇒ (2x + 37)(2x – 3) = 0

2x + 37 = 0 or 2x – 3 = 0

2x = –37 or 2x = 3

$x = -\frac{37}{2} \text{ or } x = \frac{3}{2}$

x = –18.5 or x = 1.5

Therefore, width of the path = 1.5m

5. Question

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Answer

Let x be the usual speed of the train.

Let T1 be the time taken to cover the distance 90 km in the speed x km/hr.

Let T2 be the time taken to cover the distance 90 km in the speed x + 15 km/hr.

$\text{time} = \frac{\text{distance}}{\text{speed}}$

$T1 = \frac{90}{x}$

$T2 = \frac{90}{x + 15}$

By using the given condition

$$T1 - T2 = \frac{30}{60}$$

$$\frac{90}{x} - \frac{90}{x + 15} = \frac{1}{2}$$

Taking 90commonly from two fractions

$$\Rightarrow 90\left[\frac{1}{x} - \frac{1}{x+15}\right] = \frac{1}{2}$$

$$\Rightarrow \frac{x+5-x}{x(x+15)} = \frac{1}{2\times 90}$$

$$\Rightarrow \frac{15}{x^2+15x} = \frac{1}{180}$$

⇒ $15 \times 180 = x^2 + 15x$

⇒ $x^2 + 15x = 2700$

⇒ $x^2 + 15x – 2700 = 0$

⇒ $x^2 + 60x –45x – 2700 = 0$

⇒ $x(x + 60) –45(x + 60) = 0$

⇒ $(x – 45)(x + 60) = 0$

$x – 45 = 0$ or $x + 60 = 0$

$x = 45$ or $x= –60$

therefore speed of the train is 45 km/hr.

6. Question

The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and return downstream to the original point in 4 hrs 30 minutes. Find the speed of the stream.

Answer

Let x km/hr be the speed of water

Speed of boat is 15km/hr.

So, speed in upstream = (15 + x) km/hr.

speed in downstream = (15 – x) km/hr.

Let T1 be the time taken to cover the distance 30 km in upstream.

Let T2 be the time taken to cover the distance 30 km in downstream.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$T1 = \frac{30}{15 + x}$$

$$T2 = \frac{30}{15 - x}$$

T1 + T2 = 4hours 30minutes

$$T1 + T2 = 4\frac{1}{2}$$

$$\Rightarrow \frac{30}{15+x} + \frac{30}{15-x} = \frac{9}{2}$$

$\Rightarrow 30\left[\frac{1}{15+x} + \frac{1}{15-x}\right] = \frac{9}{2}$

$\Rightarrow 30\left[\frac{1}{15+x} + \frac{1}{15-x}\right] = \frac{9}{2}$

$\Rightarrow 30\left[\frac{15+x+15-x}{(15+x)(15-x)}\right] = \frac{9}{2}$

$\Rightarrow \frac{30\times 30}{15^2-x^2} = \frac{9}{2}$

$\Rightarrow \frac{900}{225-x^2} = \frac{9}{2}$

$\Rightarrow 900 \times 2 = 9(225 - x^2)$

Now, let us divide the entire equation by 9.

So, that we will get,

$200 = 225 - x^2$

$200 + x^2 = 225$

$x^2 = 225 - 200$

$x^2 = 25$

$x = \sqrt{25}$

$x = \pm 5$

Speed must be positive so x =5 is the requied speed.

Speed of water = 5km/hr.

7. Question

One year ago, a man was 8 times as old as his son. Now his age is equal to the square of his son's age. Find their present ages.

Answer

Let x be the present age of son

Let 'y' be the present age of father

So, x – 1 be the age of son one year age

y – 1 be the age of father one year ago.

By using the given information

$y = x^2$

$y - 1 = 8(x - 1)$

$\Rightarrow y = 8x - 8 + 1$

$\Rightarrow y = 8x - 7$

$\Rightarrow x^2 = 8x - 7$

$\Rightarrow x^2 - 8x + 7 = 0$

$\Rightarrow x^2 - x - 7x + 7 = 0$

$\Rightarrow x(x - 1) - 7(x - 1) = 0$

$\Rightarrow (x - 1)(x - 7) = 0$

$X - 1 = 0$ or $x - 7 = 0$

$x = 1$ or $x = 7$

Therefore, age of father is 49.

8. Question

A chess board contains 64 equal squares and the area of each square is 6.25 cm^2. An order around the board is 2 cm wide. Find the length of the side of the chess board

Answer

Let x be the side length of the square board

Area of one square in the chess board = $6.25cm^2$

Area of 64square = 64×6.25

$(x - 4)^2 = 400$

$\Rightarrow x - 4 = \sqrt{400}$

$\Rightarrow x - 4 = \pm 20$

$x - 4 = 20$ or $x - 4 = -20$

$x = 20 + 4$ or $x = -20 + 4$

$x = 24$ or $x = -16$

Therefore, side length of square shaped chess board is 24cm.

9. Question

A takes 6 day less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time that B would take to finish this work by himself.

Answer

Let x be the tune taken by A to finish the work.

So, $x - 6$ be the time taken by B to finish the work.

Work done by A in one day = $\frac{1}{x}$

Work done by B in one day = $\frac{1}{x-6}$

Number of days taken by both to finish the work = ¼

$$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$

$\Rightarrow \frac{x-6+x}{x(x-6)} = \frac{1}{4}$

$\Rightarrow \frac{2x-6}{x^2-6x} = \frac{1}{4}$

$\Rightarrow 4(2x - 6) = x^2 - 6x$

$\Rightarrow 8x - 24 = x^2 - 6x$

$\Rightarrow x^2 - 6x - 8x + 24 = 0$

$\Rightarrow x^2 - 14x + 24 = 0$

$\Rightarrow x^2 - 12x - 2x + 24 = 0$

$\Rightarrow x(x - 12) -2(x - 12) = 0$

$\Rightarrow (x - 12)(x - 2) = 0$

x – 12 = 0 or x – 2 = 0

x = 12 or x = 2

Here, 2 is not admissible.

So, B is taking 12 days to finish the work.

10. Question

Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than the second train. If after two hours, they are 50 km apart, find the average speed of each train.

Answer

Let x km/hr. be the speed of second train.

So, speed of first train will be (x + 15) km/hr.

Distance covered by first train in 2 hours = 2(x + 5)

Distance covered by the second train in 2 hours = 2x

By using Pythagoras theorem

$[2(x + 5)]^2 + (2x)^2 = 50^2$

$\Rightarrow (2x + 10)^2 + (2x)^2 = 50^2$

$\Rightarrow (4x^2 + 100 + 40x) + 4x^2 = 2500$

$\Rightarrow 8x^2 + 40x + 100 = 2500$

$\Rightarrow 8x^2 + 40x + 100 - 2500 = 0$

$\Rightarrow 8x^2 + 40x - 2400 = 0$

Divide by 8 both sides

$\Rightarrow x^2 + 5x - 300 = 0$

$\Rightarrow x^2 + 20x - 15x - 300 = 0$

$\Rightarrow x(x + 20) - 15 (x + 20) = 0$

$\Rightarrow (x - 15)(x + 20) = 0$

x – 15 = 0 or x + 20 = 0

x = 15 or x = –20

Therefore, speed of the second train is 15 km/hr.

Exercise 3.17

1 A. Question

Determine the nature of the roots of the equation.

$x^2 - 8x + 12 = 0$

Answer

$x^2 - 8x + 12 = 0$

$\Rightarrow x^2 - 8x + 12 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow$ a = 1 , b = –8 and c = 12

$\therefore b^2 - 4ac = (-8)^2 - 4(1)(12)$

$= 64 – 48$

$= 16$

$\therefore b^2 – 4ac > 0$. hence, roots are real.

1 B. Question

Determine the nature of the roots of the equation.

$2x^2 – 3x + 4 = 0$

Answer

$2x^2 – 3x + 4 = 0$

$\Rightarrow 2x^2 - 3x + 4 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 2, b = –3$ and $c = 4$

$\therefore b^2 - 4ac = (–3)^2 – 4(2)(4)$

$= 9 – 32$

$= –23$

$\therefore b^2 – 4ac < 0$. hence, roots are not real.

1 C. Question

Determine the nature of the roots of the equation.

$9x^2 + 12x + 4 = 0$

Answer

$9x^2 – 12x + 4 = 0$

$\Rightarrow 9x^2 - 12x + 4 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 9, b = –12$ and $c = 4$

$\therefore b^2 - 4ac = (–12)^2 – 4(9)(4)$

$= 144 – 144$

$= 0$

$\therefore b^2 – 4ac > 0$. hence, roots are real and equal.

1 D. Question

Determine the nature of the roots of the equation.

$3x^2 –2\sqrt{6}x + 2 = 0$

Answer

$3x^2 – 2\sqrt{6}\, x + 2 =$

$\Rightarrow 3x^2 - 2\sqrt{6}x + 2 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 3, b = –2\sqrt{6}$ and $c = 2$

$\therefore b^2 - 4ac = (–2\sqrt{6}\,)^2 – 4(3)(2)$

$= 24 – 24$

$= 0$

∴ b^2 – 4ac = 0. hence, roots are real and equal.

1 E. Question

Determine the nature of the roots of the equation.

$$\frac{3}{5}x^2 - \frac{12}{3}x + 1 = 0$$

Answer

$$\frac{3}{5}x^2 - \frac{12}{3}x + 1 = 0$$

$\Rightarrow \frac{(3\times3)x^2-(12\times5)x + 15}{15} = 0$

⇒ $9x^2$ – 60x + 15 = 0

⇒ $9x^2 - 60x + 15 = 0$ compare with $ax^2 + bx + c = 0$

⇒ a = 9, b = –60 and c = 15

∴ $b^2 - 4ac$ = $(-60)^2$– 4(9)(15)

= 3600 – 540

= 3060

∴ b^2 – 4ac > 0. hence, roots are real.

1 F. Question

Determine the nature of the roots of the equation.

(x – 2a) (x – 2b) = 4ab

Answer

(x – 2a)(x – 2b) = 4ab

⇒ x(x –2b) – 2a(x – 2b) = 4ab

⇒ x^2 – 2bx – 2ax + 4ab – 4ab = 0

⇒ x^2 – 2x(b + a) = 0

⇒ $x^2 - 2x(b + a) = 0$ compare with $ax^2 + bx + c = 0$

⇒ a = 1, b = –b – a and c = 0

∴ $b^2 - 4ac$ = $(-b - a)^2$– 4(1)(0)

= $b^2 + a^2$ + 2ab

∴ b^2 – 4ac > 0. hence, roots are real.

2 A. Question

Find the values of k for which the roots are real and equal in each of the following equations

$2x^2$ – 10x + k = 0

Answer

$2x^2$ – 10x + k = 0

⇒ $2x^2 - 10x + k = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow\ a = 2 - 10\,, b = \text{and } c = k$

$\therefore\ b^2 - 4ac = (-10)^2 - 4(2)(k)$

$= 100 - 8k$

If roots are equal and real then, $\therefore b^2 - 4ac = 0$

⇒ 100 – 8k = 0

⇒ 100 = 8k

$\Rightarrow\ k = \frac{100}{8} = \frac{25}{2}$

$\therefore k = \frac{25}{2}$

2 B. Question

Find the values of k for which the roots are real and equal in each of the following equations

$12x^2 + 4kx + 3 = 0$

Answer

$12x^2 + 4kx + 3 = 0$

$\Rightarrow\ 12x^2 - 4kx\ +\ 3\ = 0$ compare with $ax^2\ +\ bx\ +\ c = 0$

$\Rightarrow\ a = 12, b\ =\ 4k$ and $c = k3$

$\therefore\ b^2 - 4ac = (4k)^2 - 4(12)(3)$

$= 16k^2 - 144$

If roots are equal and real then, $\therefore b^2 - 4ac = 0$

$\Rightarrow 16k^2 - 144 = 0$

$\Rightarrow 16k^2 = 144$

$$\Rightarrow\ k^2 = \frac{144}{16}$$

$\Rightarrow k\ =\ \sqrt{\frac{144}{16}}$

$\Rightarrow k\ =\ \pm\frac{12}{4} = \pm 3$

∴ k = 3 and k = -3

2 C. Question

Find the values of k for which the roots are real and equal in each of the following equations

$x^2 + 2k(x - 2) + 5 = 0$

Answer

$x^2 + 2k(x - 2) + 5 = 0$

$\Rightarrow x^2 + 2kx - 4k + 5 = 0$

$\Rightarrow\ x^2\ +\ 2kx - (4k - 5)\ \ = 0$ compare with $ax^2\ +\ bx\ +\ c = 0$

$\Rightarrow\ a = 1, b\ =\ 2k$ and $c = (-4k\ +\ 50$

$\therefore\ b^2 - 4ac = (2k)^2 - 4(1)(-4k\ +\ 5)$

$= 4k^2 + 16k - 20$

If roots are equal and real then, $\therefore b^2 - 4ac = 0$

$\Rightarrow 4k^2 + 16k - 20 = 0$

Divide by 4

$\Rightarrow k^2 + 4k - 5 = 0$

$\Rightarrow k^2 + 5k - k - 5 = 0$

$\Rightarrow k(k + 5) - (k + 5) = 0$

$\Rightarrow (k + 5)(k - 1) = 0$

$k + 5 = 0$ or $k - 1 = 0$

$k = -5$ or $k = 1$

$\therefore k = -5$ and $k = 1$

2 D. Question

Find the values of k for which the roots are real and equal in each of the following equations

$(k + 1) x^2 - 2 (k - 1) x + 1 = 0$

Answer

$(k + 1) x^2 - 2 (k - 1) x + 1 = 0$

$\Rightarrow (k + 1)x^2 - 2(k - 1)x + 1 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = (k + 1), b = -2k + 2$ and $c = 1$

$\therefore b^2 - 4ac = (2 - 2k)^2 - 4(k + 1)(1)$

$= 4k^2 - 8k + 4 - 4k - 4$

If roots are equal and real then, $\therefore b^2 - 4ac = 0$

$\Rightarrow 4k^2 - 8k + 4 - 4k - 4 = 0$

$\Rightarrow 4k^2 - 12k = 0$

Divide by 4

$\Rightarrow k^2 - 3k = 0$

$\Rightarrow k(k - 3) = 0$

$\Rightarrow k = 0$ or $k - 3 = 0$

$k = 0$ or $k = 3$

$\therefore k = 0$ and $k = 3$

3. Question

Show that the roots of the equation $x^2 + 2(a + b) x + 2 (a^2 + b^2) = 0$ are unreal.

Answer

$x^2 + 2(a + b) x + 2 (a^2 + b^2) = 0$

Compare this equation with $ax^2 + bx + c = 0$

$\therefore a = 1, b = 2(a + b)$ and $c = 2(a^2 + b^2)$

$b^2 - 4ac = (2a + 2b)^2 - 4(1)[2(a^2 + b^2)]$

$= 4a^2 + 8ab + 4b^2 - 8a^2 - 8b^2$

$= 8ab - 4a^2 - 4b^2$

$= -4a^2 + 8ab - 4b^2$

$= -4(a^2 - 2ab + b^2)$

$= -4(a - b)^2$

Since squared quantity is always positive.

Hence, $(a - b)^2 \geq 0$

Now, it is given $a \neq b$, so $(a - b)^2 > 0$

So, $D = -4(a - b)^2$ will be negative.

Hence the equation has no real roots.

4. Question

Show that the roots of the equation $3p^2x^2 - 2pqx + q^2 = 0$ are not real.

Answer

$3p^2x^2 - 2pqx + q^2 = 0$

Compare this equation with $ax^2 + bx + c = 0$

$\therefore a = 3p^2$, $b = 2pq$ and $c = q^2$

$b^2 - 4ac = (2pq)^2 - 4(3p^2)[q^2]$

$= 4p^2q^2 - 12p^2q^2$

$= -8p^2q^2$

Since squared quantity is always positive.

Hence, $p^2q^2 \geq 0$

Now, it is given $p \neq q$, so $p^2q^2 > 0$

So, $D = -8p^2q^2$ will be negative.

Hence the equation has no real roots.

5. Question

If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$, where a, b, c and $d \neq 0$, are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

Answer

Given: $(a^2 + b^2)x^2 - 2(ac + bd)x + c^2 + d^2 = 0$

To prove: $\frac{a}{b} = \frac{c}{d}$

Proof:

We know that,

$D = b^2 - 4ac$

If roots are equal, then $b^2 = 4ac$

$\Rightarrow \{-2(ac + bd)\}^2 = 4\{(a^2 + b^2)(c^2 + d^2)\}$

$\Rightarrow 4(a^2c^2 + b^2d^2 + 2acbd) = 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$\Rightarrow 2acbd = a^2d^2 + b^2c^2$

$\Rightarrow a^2d^2 + b^2c^2 - 2acbd = 0$

$\Rightarrow (ad - bc)^2 = 0$

$\Rightarrow ad - bc = 0$

$\Rightarrow ad = bc$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

Hence proved.

6. Question

Show that the roots of the equation

$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are always real and they cannot be unless a = b = c.

Answer

$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$\Rightarrow x(x - b) - a(x - b) + x(x - c) - b(x - c) + x(x - a) - c(x - a) = 0$

$\Rightarrow x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$

$\Rightarrow 3x^2 - 2x(a + b + c) + ab + bc + ac = 0$

$D = b^2 - 4ac$

$D = (a + b + c)^2 - 4(3)(ab + bc + ac) = 0$

$D = 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 3ab - 3bc - 3ca)$

$D = 4(a^2 + b^2 + c^2 - ab - bc - ca)$

$D = 2[(a - b)^2 + (b - c)^2 + (c - a)^2]$

Which is always greater than zero so the roots are real.

Roots are equal if D = 0

i.e. $(a - b)^2 + (b + c)^2 + (c - a)^2 = 0$

since sum of three perfect square is equal to zero so each of them separately equal to zero.

So, a – b = 0, b – c = 0, c – a = 0

a = b , b = c, c = a

so, a = b = c.

7. Question

If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then prove that $c^2 = a^2(1 + m^2)$

Answer

Given: $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

To prove: $c^2 = a(1 + m^2)$

Proof: it is being that equation has equal roots, therefore

$D = b^2 - 4ac = 0$ ...(1)

From the equation, we have

$a = (1 + m^2)$, $b = 2mc$, $c = c^2 – a^2$

putting values of a, b and c in (1), we get

$D = (2mc)^2 – 4(1 + m^2)(c^2 – a^2)$

$\Rightarrow 4m^2c^2 – 4(c^2 + c^2m^2 – a^2 – a^2m^2) = 0$

$\Rightarrow 4m^2c^2 – 4c^2 – 4c^2m^2 + 4a^2 + 4a^2m^2 = 0$

$\Rightarrow –4c^2 + 4a^2 + 4a^2m^2 = 0$

$\Rightarrow 4c^2 = 4a^2 + 4a^2m^2$

$\Rightarrow c^2 = a^2 + a^2m^2$

$\Rightarrow c^2 = a^2 (1 + m^2)$

Hence proved.

Exercise 3.18

1 A. Question

Find the sum and the product of the roots of the following equation.

$x^2 – 6x + 5 = 0$

Answer

$x^2 – 6x + 5 = 0$

Compare this equation with $ax^2 + bx + c = 0$

$a = 1$, $b = –6$ and $c = 5$

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{b}{a}$

$= -\frac{-6}{1} = 6$

$\alpha\beta = \frac{c}{a}$

$= \frac{5}{1} = 5$

1 B. Question

Find the sum and the product of the roots of the following equation.

$kx^2 + ax + pk = 0$

Answer

$kx^2 + ax + pk = 0$

Compare this equation with $ax^2 + bx + c = 0$

$a = k$, $b = –a$ and $c = pk$

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{b}{a}$

$= -\frac{-a}{k} = \frac{a}{k}$

$\alpha\beta = \frac{c}{a}$

$= \frac{pk}{k} = p$

1 C. Question

Find the sum and the product of the roots of the following equation.

$3x^2 - 5x = 0$

Answer

$3x^2 - 5x = 0$

Compare this equation with $ax^2 + bx + c = 0$

a = 3, b = –5 and c = 0

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{b}{a}$

$= -\frac{-5}{3} = \frac{5}{3}$

$\alpha\beta = \frac{c}{a}$

$= \frac{0}{3} = 0$

1 D. Question

Find the sum and the product of the roots of the following equation.

$8x^2 - 25 = 0$

Answer

$8x^2 - 25 = 0$

Compare this equation with $ax^2 + bx + c = 0$

a = 8, b = 0 and c = –25

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{b}{a}$

$= -\frac{0}{8} = 0$

$\alpha\beta = \frac{c}{a}$

$= \frac{-25}{8}$

2. Question

Form a quadratic equation whose roots are

(i) 3, 4 (ii) 3 + √7, 3 – √7 (iii) $\frac{4+\sqrt{7}}{2}, \frac{4-\sqrt{7}}{2}$

Answer

i: 3 and 4

Let $\alpha = 3$ and $\beta = 4$

$\therefore \alpha + \beta = 3 + 4 = 7$ and $\alpha\beta = (3)(4) = 12$

$\therefore$ and quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (7)x + (12) = 0$

$\therefore x^2 - 7x + 12 = 0$

ii: 3 + √7 and 3 – √7

Let α=3 + √7 and β=3–√7

∴ α + β=3 + √7 + 3–√7=6 and αβ=(3 + √7)(3–√7)

=9–7=2

$\therefore$ and quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (6)x + (2) = 0$

$\therefore x^2 - 6x + 2 = 0$

iii: $\frac{4 + \sqrt{7}}{2}$ and $\frac{4-\sqrt{7}}{2}$

Let $\alpha = \frac{4 + \sqrt{7}}{2}$ and $\beta = \frac{4 - \sqrt{7}}{2}$

$\therefore \alpha + \beta = \frac{4 + \sqrt{7}}{2} + \frac{4 - \sqrt{7}}{2} = \frac{4 + \sqrt{7} + 4 - \sqrt{7}}{2} = \frac{8}{2} = 4$

and $\alpha\beta = \frac{4 + \sqrt{7}}{2} \times \frac{4 - \sqrt{7}}{2} = \frac{16 - 7}{4} = \frac{9}{4}$

$\therefore$ and quadratic equation is,x^2 $- (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (4)x + (\frac{9}{4}) = 0$

$\Rightarrow \frac{(4x^2 - 16x + 9)}{4} = 0$

$\Rightarrow 4x^2 - 16x + 9 = 0$

3. Question

If α and β are the roots of the equation $3x^2 - 5x + 2 = 0$, then find the values of

(i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (ii) α – β (iii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Answer

$3x^2 - 5x + 2 = 0$ compare this with $ax^2 - bx + c = 0$

∴ a = 3 , b = –5 and c = 2

$$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$$

$$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$$

$$\alpha + \beta = -\frac{-5}{3}$$

$$= -\frac{5}{3}$$

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{2}{3}$$

i). $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$\Rightarrow \frac{(\alpha^2 + \beta^2)}{\alpha\beta}$$

$$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$\Rightarrow \frac{\left(-\frac{5}{3}\right)^2 - 2\times\frac{2}{3}}{\frac{2}{3}} = \frac{\frac{25}{9} - \frac{4}{3}}{\frac{2}{3}}$$

$$\Rightarrow \frac{\frac{25+12}{9}}{\frac{2}{3}} = \frac{\frac{13}{9}}{\frac{2}{3}}$$

$$\Rightarrow \frac{13}{9} \times \frac{3}{2} = \frac{13}{6}$$

ii). α – β

$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$$= \sqrt{\left(\frac{5}{3}\right)^2 - 4 \times \frac{2}{3}}$$

$$= \sqrt{\frac{25}{9} - \frac{8}{3}}$$

$$= \frac{5}{3} - \frac{8}{3}$$

$= \frac{3}{3} = 1$

iii). $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

$= \frac{\alpha^3}{\beta}\frac{\beta^3}{\alpha}$

$= \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(\frac{5}{3}\right)^3 - 3 \times \frac{2}{3} \times \left(\frac{5}{3}\right)$

$= \frac{125}{27} - \frac{10}{3}$

$= \frac{125 - 90}{27} = \frac{35}{27}$

$= \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{\frac{35}{27}}{\frac{2}{3}}$

$= \frac{35}{27} \times \frac{3}{2} = \frac{35}{9 \times 2} = \frac{35}{18}$

4. Question

If α and β are the roots of the equation $3x^2 - 6x + 4 = 0$, find the value of $\alpha^2 - \beta^2$.

Answer

$3x^2 - 6x + 4 = 0$ compare this with $ax^2 - bx + c = 0$

∴ a = 3 , b = -6 and c = 4

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{-6}{3}$

$= \frac{6}{3} = 2$

$\alpha\beta = \frac{c}{a}$

$= \frac{4}{3}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 2^2 - 2 \times \frac{4}{3}$

$= 4 - \frac{8}{3}$

$= \frac{12 - 8}{3} = \frac{4}{3}$

5. Question

If α, β are roots of $2x^2 - 3x - 5 = 0$, from an equation whose roots are α^2 and β^2.

Answer

$2x^2 - 3x - 5 = 0$ compare this with $ax^2 - bx + c = 0$

∴ a = 2 , b = –3 and c = –5

$$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$$

$$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$$

$$\alpha + \beta = -\frac{-3}{2}$$

$$= \frac{3}{2}$$

$$\alpha\beta = \frac{c}{a}$$

$$= -\frac{5}{2}$$

Here $\alpha = \alpha^2$ and $\beta = \beta^2$

General form of quadratic equation whose roots are α^2 and β^2

$\Rightarrow x^2 - (\alpha^2 + \beta^2)\,x + \alpha^2\beta^2 = 0$

$\Rightarrow x^2 - (\alpha^2 + \beta^2)\,x + (\alpha\beta)^2 = 0$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2(\alpha\beta)$

$$= \left(\frac{3}{2}\right)^2 - 2 \times \left(-\frac{5}{2}\right)$$

$$= \frac{9}{4} + 5$$

$$= \frac{9 + 20}{4} = \frac{29}{4}$$

$x^2 - (\alpha^2 + \beta^2)\,x + (\alpha\beta)^2 = 0$

$$x^2 - \frac{29}{4}x + \left(-\frac{5}{2}\right)^2 = 0$$

$$x^2 - \frac{29}{5}x + \frac{25}{4} = 0$$

$$\frac{4x^2 - 29x + 25}{4} = 0$$

$4x^2 - 29x + 25 = 0$

Therefore the required equation is $4x^2 - 29x + 25 = 0$

6. Question

If α, β are roots of $x^2 - 3x + 2 = 0$, form a quadratic equation whose roots are –α and –β

Answer

$x^2 - 3x + 2 = 0$ compare this with $ax^2 - bx + c = 0$

∴ a = 1 , b = –3 and c = 2

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{-3}{1}$

$= 3$

$\alpha\beta = \frac{c}{a}$

$= \frac{2}{1} = 2$

Here α = –αand β = – β

General form of quadratic equation whose roots are α^2 and β^2

⇒ x^2 – (–α – β) x + (–α) (–β) = 0

⇒ x^2 + (α + β) x + (αβ) = 0

⇒ x^2 + (3)x + (2) = 0

Therefore, the required quadratic equation is x^2 – 3x + 2 = 0

7. Question

If α and β are roots of x^2 – 3x–1 = 0, then form a quadratic equation whose roots are

$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$

Answer

x^2 – 3x – 1 = 0 compare this with ax^2 – bx + c = 0

∴ a = 1 , b = –3 and c = –1

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{-3}{1}$

$= 3$

$\alpha\beta = \frac{c}{a}$

$= \frac{-1}{1} = -1$

Here $\alpha = \frac{1}{\alpha^2}$ and $\beta = \frac{1}{\beta^2}$

General form of quadratic equation whose roots are α^2 and β^2

$\Rightarrow x^2 - \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)x + \left(\frac{1}{\alpha^2}\right)\left(\frac{1}{\beta^2}\right) = 0$

$\Rightarrow x^2 - \left(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}\right)x + \left(\frac{1}{\alpha\beta}\right)^2 = 0$

$\Rightarrow x^2 - \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}x + \left(\frac{1}{\alpha\beta}\right)^2 = 0$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 3^2 - 2 \times (-1)$

$= 9 + 2$

$= 11$

$\Rightarrow x^2 - \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}x + \left(\frac{1}{\alpha\beta}\right)^2 = 0$

$\Rightarrow x^2 - \frac{11}{(-1)^2}x + \left(\frac{1}{-1}\right)^2 = 0$

$\Rightarrow x^2 - 11x + 1 = 0$

Therefore, the required equation is $x^2 + 11x + 1 = 0$

8. Question

If α and β are roots of $3x^2 - 6x + 1 = 0$, then form a quadratic equation whose roots are

(i) $\frac{1}{\alpha}, \frac{1}{\beta}$ (ii) $\alpha^2\beta, \beta^2\alpha$ (iii) $2\alpha + \beta, 2\beta + \alpha$

Answer

$3x^2 - 6x + 1 = 0$ compare this with $ax^2 - bx + c = 0$

$\therefore a = 3, b = -6$ and $c = 1$

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$

$\alpha + \beta = -\frac{-6}{3}$

$= 2$

$\alpha\beta = \frac{c}{a}$

$= \frac{1}{3}$

i). Here $\alpha = \frac{1}{\alpha}$ and $\beta = \frac{1}{\beta}$

General form of quadratic equation whose roots are α^2 and β^2

$\Rightarrow x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = 0$

$\Rightarrow x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$

$\Rightarrow x^2 - \frac{2}{\frac{1}{3}}x + \frac{1}{\frac{1}{3}} = 0$

$\Rightarrow x^2 - 2 \times \frac{3}{1}x + 1 \times \frac{3}{1} = 0$

$\Rightarrow x^2 - 6x + 3 = 0$

Therefore, required equation is $x^2 - 6x + 3 = 0$

ii). Here, $\alpha = \alpha^2\beta$ and $\beta = \beta^2\alpha$

General form of quadratic equation whose roots are $\alpha^2\beta$ and $\beta^2\alpha$

$x^2 - (\alpha^2\beta + \beta^2\alpha)x + (\alpha^2\beta)(\beta^2\alpha) = 0$

$\Rightarrow x^2 - \alpha\beta(\alpha + \beta)x + (\alpha^3\beta^3) = 0$

$\Rightarrow x^2 - \alpha\beta(\alpha + \beta)x + (\alpha\beta)^3 = 0$

$\Rightarrow x^2 - \left(\frac{1}{3} \times 2\right)x + \left(\frac{1}{3}\right)^3 = 0$

$\Rightarrow x^2 - \frac{2}{3}x + \frac{1}{27} = 0$

$\Rightarrow \frac{27x^2 - 18x + 1}{27} = 0$

$\Rightarrow 27x^2 - 18x + 1 = 0$

Therefore, required equation is $27x^2 - 18x + 1 = 0$

iii). Here, $\alpha = 2\alpha + \beta$ and $\beta = 2\beta + \alpha$

General form of equation whose roots are $2\alpha + \beta$ and $2\beta + \alpha$

$x^2 - (2\alpha + \beta + 2\beta + \alpha)x + (2\alpha + \beta)(2\beta + \alpha) = 0$

$x^2 - (3\alpha + 3\beta)x + (4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta) = 0$

$x^2 - (3\alpha + 3\beta)x + (2(\alpha^2 + \beta^2) + 5\alpha\beta) = 0$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 2^2 - 2 \times \frac{1}{3}$

$= 4 - \frac{2}{3}$

$= \frac{12 - 2}{3} = \frac{10}{3}$

$x^2 - (3\alpha + 3\beta)x + (2(\alpha^2 + \beta^2) + 5\alpha\beta) = 0$

$\Rightarrow x^2 - 3(2)x + \left(\left(2 \times \frac{10}{3}\right) + 5 \times \frac{10}{3}\right) = 0$

$\Rightarrow x^2 - 6x + \left(\frac{20}{3} + \frac{50}{3}\right) = 0$

$\Rightarrow x^2 - 6x + \frac{70}{3} = 0$

$\Rightarrow \frac{3x^2 - 18x + 70}{3} = 0$

$\Rightarrow 3x^2 - 18x + 70 = 0$

Therefore, the required equation is $3x^2 - 18x + 70 = 0$

9. Question

Find a quadratic equation whose roots are the reciprocal of the roots of the equation

$4x^2 - 3x - 1 = 0$

Answer

$4x^2 - 3x - 1 = 0$ compare this with $ax^2 - bx + c = 0$

$\therefore$ a = 4 , b = –3 and c = –1

$$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$$

$$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$$

$$\alpha + \beta = -\frac{-3}{4}$$

$$= \frac{3}{4}$$

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{-1}{4}$$

Here $\alpha = \frac{1}{\alpha}$ and $\beta = \frac{1}{\beta}$

General form of quadratic equation whose roots are α^2 and β^2

$$\Rightarrow x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = 0$$

$$\Rightarrow x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - \frac{\frac{3}{4}}{\frac{1}{4}}x + \frac{1}{\frac{-1}{4}} = 0$$

$$\Rightarrow x^2 + \frac{3}{4} \times \frac{4}{1}x - 1 \times \frac{4}{1} = 0$$

$\Rightarrow x^2 - 3x - 4 = 0$

Therefore, required equation is $x^2 - 3x - 4 = 0$

10. Question

If one root of the equation $3x^2 + kx - 81 = 0$ is the square of the other, find k.

Answer

Two roots of any quadratic equation are α and β.

Here, one root is square of the other i.e $\alpha = \beta^2$

$3x^2 - kx - 81 = 0$ compare this with $ax^2 - bx + c = 0$

$\therefore$ a = 3, b = –k and c = –81

$$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$$

$$\text{product of the roots } (\alpha\beta) = \frac{c}{a}$$

$$\alpha + \beta = -\frac{-k}{3}$$

$$= \frac{k}{3}$$

$\alpha\beta = \frac{c}{a}$

$= \frac{-81}{3} = -27$

$\alpha + \beta = \frac{k}{3}$

$\beta^2 + \beta = \frac{k}{3} \ldots (1)$

$\alpha\beta = -27$

$\beta^2(\beta) = -27$

$\beta^3 = -27$

$\beta^3 = (-3)^3$

$\beta = -3$

now, we are going to apply in first equation

$\beta^2 + \beta = \frac{k}{3}$

$\Rightarrow (-3)^2 - 3 = \frac{k}{3}$

$\Rightarrow 9 - 3 = \frac{k}{3}$

$\Rightarrow 6 = \frac{k}{3}$

$\Rightarrow 6 \times 3 = k$

$\Rightarrow k = 18$

11. Question

If one root of the equation $2x^2 - ax + 64 = 0$ is twice the other, then find the value of a

Answer

Roots of any quadratic equation are α and β

Here one root is twice the other.

$\alpha = 2\beta$

by comparing the given equation with general form of quadratic equation we get,

a = 2, b = –a and c = 64

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$= -\frac{-a}{2} = \frac{a}{2}$

$\text{Products of roots } (\alpha\beta) = \frac{c}{a}$

$= \frac{64}{2} = 32$

$\alpha + \beta = \frac{a}{2}$

$2\beta + \beta = \frac{a}{2}$

$3\beta = \frac{a}{2}$ ...(1)

$\alpha\beta = 32$

$2\beta(\beta) = 32$

$2\beta^2 = 32$

$\Rightarrow \beta^2 = \frac{32}{2}$

$\Rightarrow \beta^2 = 16$

$\Rightarrow \beta = \sqrt{16}$

$\Rightarrow \beta = \pm 4$

$3\beta = \frac{a}{2}$

$\Rightarrow 3 \times 4 = \frac{a}{2}$

$\Rightarrow 12 = \frac{a}{2}$

$\Rightarrow 12 \times 2 = a$

$\Rightarrow a = 24$

12. Question

If α and β are roots of $5x^2 - px + 1 = 0$ and $\alpha - \beta = 1$, then find P.

Answer

Roots of any quadratic equation are α and β

By comparing the given equation with $ax^2 + bx + c = 0$

a = 5, b = –p and c = 1

$\text{sum of the roots } (\alpha + \beta) = -\frac{b}{a}$

$= -\frac{-p}{5} = \frac{p}{5}$

$\text{Products of roots } (\alpha\beta) = \frac{c}{a}$

$= \frac{1}{5}$

$\alpha - \beta = 1$

$\alpha - \beta = (\alpha + \beta)^2 - 4\alpha\beta$

$\left(\frac{p}{5}\right)^2 - 4 \times \frac{1}{5} = 1$

$\Rightarrow \frac{p^2}{25} - \frac{4}{5} = 1$

$\Rightarrow \frac{p^2 - 20}{25} = 1$

$\Rightarrow p^2 - 20 = 25$

⇒ $p^2 = 25 + 20$

⇒ $p^2 = 45$

⇒ p = √45

⇒ p = 3√5

Exercise 3.19

1. Question

If the system 6x – 2y = 3, kx – y = 2 has a unique solution, then

A. k = 3

B. k ≠ 3

C. k = 4

D. k ≠ 4

Answer

Given: Two equations: 6x – 2y = 3 and kx – y = 2

Required: To find the value of k such that system of equations have unique solutions

To have unique solution to the System of equations, the required condition is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ $\frac{6}{k} \neq \frac{-2}{-1}$

⇒ $\frac{6}{k} \neq 2$

⇒ $k \neq \frac{6}{2}$

⇒ $k \neq 3$

∴ For every value of k except 3 the system equations have unique solutions.

∴ Correct option is – Option (B)

2. Question

A system of two linear equations in two variables is consistent, if their graphs

A. coincide

B. intersect only at a point

C. do not intersect at any point

D. cut the x-axis

Answer

Given: A system of two linear equations in two variables is consistent

We know that if a system of two linear equations in two variables is consistent then their graph do not intersect at any point

∴ Correct option is – Option (C)

3. Question

The system of equations x –4y = 8 , 3x –12y = 24

A. has infinitely many solutions

B. has no solution

C. has a unique solution

D. may or may not have a solution

Answer

Given: system of equations x –4y = 8 , 3x –12y = 24

Here,

$a_1 = 1$, $b_1 = -4$, $c_1 = 8$ and $a_2 = 3$, $b_2 = -12$, $c_2 = 24$

Now,

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-4}{-12} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{8}{24} = \frac{1}{3}$

Here, we can clearly see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ which is the condition for infinitely many solutions.

∴ The system of equations have infinitely many solutions.

∴ Correct option is – Option (A)

4. Question

If one zero of the polynomial $p(x) = (k + 4)x^2 + 13x + 3k$ is reciprocal of the other, then k is equal to

A. 2

B. 3

C. 4

D. 5

Answer

Given: A Quadratic equation $p(x) = (k + 4)x^2 + 13x + 3k$

Required: To find the value of k

Let the roots of the given Quadratic equation be: α and $\frac{1}{\alpha}$

∴ Product of roots of the given Quadratic equation is $\alpha \times \frac{1}{\alpha} = 1$

We know that, Product of roots of a given Quadratic equation is $\frac{c}{a}$

∴ $\frac{c}{a} = \frac{3k}{k+4} = 1$

⇒ $\frac{3k}{k+4} = 1$

⇒ 3k = k + 4

⇒ 2k = 4

⇒ k = 2

∴ The value of k is 2

∴ Correct option is – Option (A)

5. Question

The sum of two zeros of the polynomial $f(x) = 2x^2 + (p + 3)x + 5$ is zero, then the value of p is

A. 3

B. 4

C. –3

D. –4

Answer

Given: A Quadratic equation f(x) = $2x^2 + (p + 3)x + 5$ and sum of roots is zero.

Required: to find the value of p

We know that, sum of roots = $\frac{-b}{a}$

∴ $\frac{-(p+3)}{2} = 0$

⇒ -(p + 3) = 0

⇒ P + 3 = 0

∴ p = –3

∴ The value of p is –3

∴ Correct option is – Option (C)

6. Question

The remainder when $x^2 – 2x + 7$ is divided by x + 4 is

A. 28

B. 29

C. 30

D. 31

Answer

Given: $x^2 – 2x + 7$

Required: Reminder of $x^2 – 2x + 7$ at x + 4

By synthetic division we can find reminder of

$x^2 – 2x + 7$ at x = -4

-4	1	-2	7
	0	-4	24
	1	-6	31

∴ Reminder of $x^2 – 2x + 7$ at x + 4 = 31

∴ Correct option is –Option (D)

7. Question

The quotient when $x^3 – 5x^2 + 7x – 4$ is divided by x–1 is

A. $x^2 + 4x + 3$

B. $x^2 – 4x + 3$

C. $x^2 – 4x -3$

D. $x^2 + 4x – 3$

Answer

Given: $x^2 – 2x + 7$

Required: Reminder of $x^3 – 5x^2 + 7x – 4$ at $x – 1$

By synthetic division we can find Quotient of $x^3 – 5x^2 + 7x – 4$ at $x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 7 & -4 \\ & 0 & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & -1 \end{array}$$

∴ Quotient of $x^2 – 2x + 7$ when divided by $x–1 = x^2–4x + 3$

∴ Correct option is –Option(B)

8. Question

The GCD of $(x^3 + 1)$ and $x^4 – 1$ is

A. $x^3 – 1$

B. $x^3 + 1$

C. $-(x + 1)$

D. $x–1$

Answer

Given two polynomials: $(x^3 + 1)$ and $x^4 – 1$

Required: To find GCD of the given two polynomials

Let $f(x) = x^3 + 1$ and $g(x) = X^4 – 1$

Here, degree of $g(x) > f(x)$ ∴ Devisor $= x^3 + 1$

$$\begin{array}{r|l} & x \\ x^3+1 & x^4 + 0x - 1 \\ & x^4 + x \\ \hline & -x - 1 \end{array}$$

Here reminder $= –(x + 1)$ (Reminder ≠ zero)

$$\begin{array}{r|l} & -x^2 + x - 1 \\ -x-1 & x^3 + 0x^2 + 0x + 1 \\ & x^3 + x^2 \\ \hline & -x^2 + 0x + 1 \\ & -x^2 - x \\ \hline & x + 1 \\ & x + 1 \\ \hline & 0 \end{array}$$

∴ The GCD of given two polynomials is $-(x + 1)$

∴ Correct Option is - Option(C)

9. Question

The GCD of $x^2 – 2xy + y^2$ and $x^4 – y^4$ is

A. 1

B. $x + y$

C. $x – y$

D. $x^2 – y^2$

Answer

Given two polynomials: $x^2 – 2xy + y^2$ and $x^4 – y^4$

Required: To find GCD of the given two polynomials

Let $f(x) = x^2 – 2xy + y^2$ and $g(x) = x^4 – y^4$

$f(x) = x^2 – 2xy + y^2$

$\Rightarrow f(x) = (x–y)^2$

$g(x) = x^4 – y^4$

$\Rightarrow g(x) = (x^2)^2 – (y^2)^2$

$\Rightarrow g(x) = (x^2 – y^2)(x^2 + y^2)$ ($\because a^2–b^2 = (a–b)(a + b)$)

$\Rightarrow g(x) = (x–y)(x + y)(x^2 + y^2)$ ($\because a^2–b^2 = (a–b)(a + b)$)

$\therefore$ The GCD of given two polynomials is (x–y)

$\therefore$ Correct Option is - Option (C)

10. Question

The LCM of $x^3 – a^3$ and $(x - a)^2$ is

A. $(x^3 – a^3) (x + a)$

B. $(x^3 – a^3) (x - a)^2$

C. $(x - a)^2 (x^2 + ax + a^2)$

D. $(x + a)^2(x^2 + ax + a^2)$

Answer

Given two polynomials: $x^3 – a^3$ and $(x - a)^2$

Required: To find LCM of the given two polynomials

Let $f(x) = x^3 – a^3$ and $g(x) = (x - a)^2$

here,

$f(x) = x^3 – a^3$

$\Rightarrow f(x) = (x-a)(x^2 + a^2 + xa)$

$g(x) = (x - a)^2$

$\Rightarrow g(x) = (x-a)(x-a)$

$\therefore$ LCM = $(x-a) (x^2 + a^2 + xa) (x-a) = (x-a)^2(x^2 + a^2 + xa)$

$\therefore$ Correct Option is - Option (C)

11. Question

The LCM of $a^k, a^{k + 3}, a^{k + 5}$ where $k \in \mathbb{N}$ is

A. $a^{k + 9}$

B. a^k

C. $a^{k + 6}$

D. $a^{k + 5}$

Answer

Given three polynomials: a^k , $a^{k + 3}$ and $a^{k + 5}$

Required: To find LCM of the given two polynomials

Let f(x) = a^k , g(x) = a^{k+3} and h(x) = a^{k+5}

here,

f(x) = a^k

⇒f(x) = a^k

g(x) = a^{k+3}

⇒g(x) = $a^k \times a^3$

h(x) = a^{k+5}

⇒h(x) = $a^k \times a^{3+2} = a^k \times a^3 \times a^2$

∴ LCM = $a^k \times a^3 \times a^2 = a^{k+5}$

∴ Correct Option is - Option (D)

12. Question

The lowest form of the rational expression $\frac{x^2+5x+6}{x^2-x-6}$ is

A. $\frac{x-3}{x+3}$

B. $\frac{x+3}{x-3}$

C. $\frac{x+2}{x-3}$

D. $\frac{x-3}{x+2}$

Answer

Given: Rational Expression: $\frac{x^2+5x+6}{x^2-x-6}$

Required: The lowest form of the given Rational Expression.

Let f(x) = $\frac{x^2+5x+6}{x^2-x-6}$

⇒ f(x) = $\frac{x^2+2x+3x+6}{x^2-3x+2x-6}$ (∵ factorization)

⇒ f(x) = $\frac{x(x+2)+3(x+2)}{x(x-3)+2(x-3)}$

⇒ f(x) = $\frac{(x+3)(x+2)}{(x+2)(x-3)}$

⇒ f(x) = $\frac{(x+3)}{(x-3)}$

∴ The lowest form of the given rational expression is: $\frac{(x+3)}{(x-3)}$

∴ Correct Option is - Option (B)

13. Question

If $\frac{a+b}{a-b}$ and $\frac{a^3-b^3}{a^3+b^3}$ are the two rational expressions, then their product is

A. $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

B. $\frac{a^2-ab+b^2}{a^2+ab+b^2}$

C. $\frac{a^2-ab-b^2}{a^2+ab+b^2}$

D. $\frac{a^2+ab+b^2}{a^2-ab-b^2}$

Answer

Given: Two rational expressions: $\frac{a+b}{a-b}$ and $\frac{a^3-b^3}{a^3+b^3}$

Required: Product of the given two rational numbers

Let f(x) = $\frac{a+b}{a-b}$ and g(x) = $\frac{a^3-b^3}{a^3+b^3}$

Now, f(x)×g(x) = $\frac{a+b}{a-b} \times \frac{a^3-b^3}{a^3+b^3}$

⇒ f(x)×g(x) = $\frac{a+b}{a-b} \times \frac{(a-b)(a^2+b^2+ab)}{(a-b)(a^2+b^2-ab)}$

⇒ f(x)×g(x) = $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

∴ Product of the given rational expressions is: $\frac{a^2+ab+b^2}{a^2-ab+b^2}$

∴ Correct Option is - Option (A)

14. Question

On dividing $\frac{x^2-25}{x+3}$ by $\frac{x+5}{x^2-9}$ is equal to

A. (x – 5) (x – 3)

B. (x – 5) (x + 3)

C. (x + 5) (x – 3)

D. (x + 5) (x + 3)

Answer

Given: Two polynomials $\frac{x^2-25}{x+3}$ and $\frac{x+5}{x^2-9}$

Required: Divide $\frac{x^2-25}{x+3}$ by $\frac{x+5}{x^2-9}$

Let f(x) = $\frac{x^2-25}{x+3}$ and g(x) = $\frac{x+5}{x^2-9}$

Now, $\frac{f(x)}{g(x)} = \frac{\frac{x^2-25}{x+3}}{\frac{x+5}{x^2-9}}$

$\Rightarrow \frac{f(x)}{g(x)} = \frac{x^2-25}{x+3} \times \frac{x^2-9}{x+5} \left(\because \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}\right)$

$\Rightarrow \frac{f(x)}{g(x)} = \frac{x^2-5^2}{x+3} \times \frac{x^2-3^2}{x+5}$

$\Rightarrow \frac{f(x)}{g(x)} = \frac{(x-5)(x+5)}{x+3} \times \frac{(x-3)(x+3)}{x+5}$ ($\because a^2-b^2 = (a-b)(a + b)$)

$\Rightarrow \frac{f(x)}{g(x)} = (x-5) \times (x-3)$

∴ Quotient when f(x) is divided g(x) we get is: (x-5)(x-3)

∴ Correct Option is - Option (A)

15. Question

If $\frac{a^3}{a-b}$ is added with $\frac{b^3}{b-a}$, then the new expression is

A. $a^2 + ab + b^2$

B. $a^2 - ab + b^2$

C. $a^3 + b^3$

D. $a^3 - b^3$

Answer

Given: Two polynomials: $\frac{a^3}{a-b}$ and $\frac{b^3}{b-a}$

Required: Two add the given two polynomials

Let f(x) = $\frac{a^3}{a-b}$ and g(x) = $\frac{b^3}{b-a}$

Now, f(x) + g(x) = $\frac{a^3}{a-b} + \frac{b^3}{b-a}$

$\Rightarrow$ f(x) + g(x) = $\frac{a^3}{a-b} + \frac{b^3}{b-a}$

$\Rightarrow$ f(x) + g(x) = $\frac{a^3}{a-b} + \frac{b^3}{-(a-b)}$

$\Rightarrow$ f(x) + g(x) = $\frac{a^3}{a-b} - \frac{b^3}{a-b}$

$\Rightarrow$ f(x) + g(x) = $\frac{a^3-b^3}{a-b}$

$\Rightarrow$ f(x) + g(x) = $\frac{(a-b)(a^2+ab+b^2)}{a-b}$ ($\because a^3-b^3 = (a-b)(a^2 + ab + b^2)$)

∴ f(x) + g(x) = $a^2 + ab + b^2$

That is, the sum of given two polynomials is $a^2 + ab + b^2$

∴ Correct Option is - Option (A)

16. Question

The square root of 49 $(x^2 - 2x + y^2)^2$ is

A. 7 |x – y|

B. 7(x + y) (x – y)

C. $7(x + y)^2$

D. $7(x - y)^2$

Answer

Given: A polynomial: 49 $\left(x^2 - 2xy + y^2\right)^2$

Required: to find the square root of the given polynomial

Let f(x) = 49 $\left(x^2 - 2xy + y^2\right)^2$

Now, $\sqrt{f(x)} = \sqrt{49 \times (x^2 - 2xy + y^2)^2}$

⇒ $\sqrt{f(x)} = \sqrt{49 \times ((x-y)^2)^2}$ (∵ $(x-y)^2 = x^2 - 2xy + y^2$)

⇒ $\sqrt{f(x)} = 7 \times \sqrt{((x-y)^2)^2}$

⇒ $\sqrt{f(x)} = 7 \times (x-y)^2$

∴ Square root of the given polynomial is: $7 \times (x-y)^2$

∴ Correct Option is - Option (D)

17. Question

The square root of $x^2 + y^2 + z^2 - 2xy + 2yz - 2zx$

A. |x + y – z|

B. |x – y + z|

C. |x + y + z|

D. |x – y – z|

Answer

Given: A polynomial: $x^2 + y^2 + z^2 - 2xy + 2yz - 2zx$

Required: to find the square root of the given polynomial

Let f(x) = $x^2 + y^2 + z^2 - 2xy + 2yz - 2zx$

Now, $\sqrt{f(x)} = \sqrt{x^2 + y^2 + z^2 - 2xy + 2yz - 2zx}$

⇒ $\sqrt{f(x)} = \sqrt{(x-y-z)^2}$ (∵ $(x-y-z)^2 = x^2 + y^2 + z^2 - 2xy + 2yz - 2zx$)

⇒ $\sqrt{f(x)} = |x - y - z|$

∴ Square root of the given polynomial is: $|x - y - z|$

∴ Correct Option is - Option (D)

18. Question

The square root of 121 $x^4y^8z^6 (l - m)^2$ is

A. $11x^2y^4z^4|l - m|$

B. $11x^4y^4|z^3(l - m)$

C. $11x^2y^4z^6|(l - m)|$

D. $11x^2y^4|z^3(l - m)|$

Answer

Given: A polynomial: 121 $x^4y^8z^6(l-m)^2$

Required: to find the square root of the given polynomial

Let f(x) = 121 $x^4y^8z^6(l-m)^2$

Now, $\sqrt{f(x)} = \sqrt{121 \times x^4y^8z^6(l-m)^2}$

⇒ $\sqrt{f(x)} = 11\sqrt{(x^2)^2(y^4)^2(z^3)^2(l-m)^2}$

⇒ $\sqrt{f(x)} = 11x^2y^4|z^3(l-m)|$

∴ Square root of the given polynomial is: $11x^2y^4|z^3(l-m)|$

∴ Correct Option is - Option (D)

19. Question

If $ax^2 + bx + c = 0$ has equal roots, then c is equal

A. $\frac{b^2}{2a}$

B. $\frac{b^2}{4a}$

C. $-\frac{b^2}{2a}$

D. $-\frac{b^2}{4a}$

Answer

Given: The quadratic equation has equal roots

Here,

$b^2 - 4ac = 0$ (∵ The quadratic equation has equal roots)

⇒ $b^2 - 4ac = 0$

⇒ $b^2 = 4ac$

⇒ $4ac = b^2$

⇒ $c = \frac{b^2}{4a}$

∴ The value of $c = \frac{b^2}{4a}$

∴ Correct Option is - Option (B)

20. Question

If $x^2 + 5kx + 16 = 0$ has no real roots, then

A. $k > \frac{8}{5}$

B. $k > -\frac{8}{5}$

C. $-\frac{8}{5} < k < \frac{8}{5}$

D. $0 < k < \frac{8}{5}$

Answer

Given: The quadratic equation $x^2 + 5kx + 16 = 0$ and has no real roots.

Required: To find the value of k

Here,

$b^2 - 4ac < 0$ (∵ Quadratic equation has no real roots)

$\Rightarrow (5k)^2 - 4(1)(16) < 0$

$\Rightarrow 25k^2 < 64$

$\Rightarrow k^2 < \frac{64}{25}$

Applying Square root on both sides

$\Rightarrow \sqrt{k^2} < \sqrt{\frac{64}{25}}$

$\Rightarrow -\frac{8}{5} < k < \frac{8}{5}$

∴ The value of k is $-\frac{8}{5} < k < \frac{8}{5}$

∴ Correct Option is - Option (C)

21. Question

A quadratic equation whose one root is 3 is

A. $x^2 - 6x - 5 = 0$

B. $x^2 + 6x - 5 = 0$

C. $x^2 - 5x - 6 = 0$

D. $x^2 - 5x + 6 = 0$

Answer

Given: One of the root of the Quadratic Equation that is 3

Required: To find the Quadratic equation, which satisfies the given root.

Now,

We substitute the zero in every equation given in the options

∴ Case (i): $x^2 - 6x - 5 = 0$

⇒ $(3)^2 - 6(3) - 5 = 0$

⇒ $9 - 18 - 5 = -14 \neq 0$

Case (ii): $x^2 + 6x - 5 = 0$

⇒ $(3)^2 + 6(3) - 5 = 0$

⇒ $9 + 18 - 5 = 22 \neq 0$

Case(iii): $x^2 - 5x - 6 = 0$

⇒ $(3)^2 - 5(3) - 6 = 0$

⇒ $9 - 15 - 6 = -12 \neq 0$

Case(iv): $x^2 - 5x + 6 = 0$

⇒ $(3)^2 - 5(3) + 6 = 0$

⇒ $9 - 15 + 6 = 0$

∴ The Quadratic equation with one root as 3 is $x^2 - 5x - 6 = 0$

∴ Correct Option is - Option (D)

22. Question

The common root of the equation $x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ is

A. $\frac{c+a}{2b}$

B. $\frac{c-a}{2b}$

C. $\frac{c+a}{2a}$

D. $\frac{a+b}{2c}$

Answer

Given: Two Quadratic Equations $x^2 - bx + c = 0$ and $x^2 + bx - a = 0$

Required: to find the common root of the given Quadratic Equations

Let the common root be α

A is the root of $x^2 - bx + c = 0$

∴ $x^2 - bx + c = 0$

⇒ $\alpha^2 - b\alpha + c = 0$ -eq(1)

Also, α is the root of $x^2 + bx - a = 0$

∴ $\alpha^2 + b\alpha - a = 0$ -eq(2)

Here, eq(1) = eq(2)

∴ $\alpha^2 + b\alpha - a = \alpha^2 - b\alpha + c$

⇒ $2b\alpha = c + a$

$\Rightarrow \alpha = \frac{c+a}{2b}$

∴ The common root is $\alpha = \frac{c+a}{2b}$

∴ Correct Option is - Option(A)

23. Question

If α, β are the roots of $ax^2 + bx + c = 0$ a ≠ 0, then the wrong statement is

A. $\alpha^2 + \beta^2 = \frac{b^2 - 2ac}{a^2}$

B. $\alpha\beta = \frac{c}{a}$

C. $\alpha + \beta = \frac{b}{a}$

D. $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{b}{c}$

Answer

Given: α, β are the roots of $ax^2 + bx + c = 0$ $a \neq 0$

Required: To find the wrong statement in the given options

We know that sum of roots is given by :$\frac{-b}{a}$

$\therefore \alpha + \beta = \frac{-b}{a}$

We, also know that product of roots is given by: $\frac{c}{a}$

$\therefore \alpha\beta = \frac{c}{a}$

Now, $(\alpha + \beta)^2 = \left(\frac{-b}{a}\right)^2$

$\Rightarrow \alpha^2 + 2\alpha\beta + \beta^2 = \frac{b^2}{a^2}$

$\Rightarrow \alpha^2 + \frac{2c}{a} + \beta^2 = \frac{b^2}{a^2}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{b^2 - 2ac}{a^2}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$

∴ We can clearly see in the option that $\alpha + \beta = \frac{b}{a}$ is wrong.

∴ Correct Option is - Option (C)

24. Question

If α and β are the roots of $ax^2 + bx + c = 0$, then one of the quadratic equations whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, is

A. $ax^2 + bx + c = 0$

B. $bx^2 + ax + c = 0$

C. $cx^2 + bx + a = 0$

D. $cx^2 + ax + b = 0$

Answer

Given: α and β are roots of $ax^2 + bx + c = 0$

Required:- Quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Sum of roots of given quadratic equation = $\frac{-b}{a}$

∴ $\frac{-b}{a} = \alpha + \beta$ -eq(1)

Product of roots of given quadratic equation = $\frac{c}{a}$

∴ $\frac{c}{a} = \alpha\beta$ -eq(2)

Sum of roots of required quadratic equation = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

Product of roots of required quadratic equation = $\frac{1}{\alpha\beta}$

Here,

Dividing eq(1) by eq(2) we get,

$$\frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

∴ Sum of roots of the required quadratic equation = $\frac{-b}{c}$

Again by making the reciprocal of eq(2), we get

$$\frac{1}{\alpha\beta} = \frac{a}{c}$$

∴ Product of roots of the required quadratic equation = $\frac{a}{c}$

We know that, when roots of the quadratic equation are known, we can calculate the quadratic equation as:

x^2-(sum of roots)x + (product of roots) = 0

∴ Required quadratic equation: $x^2 -(\frac{-b}{c}) + (\frac{a}{c}) = 0$

⇒ $\frac{cx^2 + bx + a}{c} = 0$

⇒ $cx^2 + bx + a = 0$

∴ Required quadratic equation is: $cx^2 + bx + a = 0$

∴ Correct option is -Option(C)

25. Question

Let b = a + c. Then the equation $ax^2 + bx + c = 0$ has equal roots, if

A. a = c

B. a = - c

C. a = 2 c

D. a = -2c

Answer

Given: b = a + c and the quadratic equation has equal roots

Here,

$b^2 – 4ac = 0$ (∵ The quadratic equation has equal roots)

⇒ $b^2 – 4ac = 0$

⇒ $(a + c)^2 – 4ac = 0$ (∵ b = a + c)

⇒ $a^2 + 2ac + c^2 – 4ac = 0$

⇒ $a^2 – 2ac + c^2 = 0$

⇒ $(a – c)^2 = 0$

Applying sq.rt on both sides

⇒ $\sqrt{(a – c)^2} = \sqrt{0}$

⇒ a – c = 0

⇒ a = c

∴ a = c

∴ Correct option is -Option (A)