3 Rational Indices

◆ **Let's remember :**

Let's remember the laws of rational indices which we have learnt in Std. 7.

● **Fill in the blanks :**

(1) $5^4 \times 5^3 = 5^{....}$ (2) $7^5 \div 7^2 = 7^{....}$

(3) $\frac{11^3}{11^5} = \frac{1}{11^{...}}$ (4) $(9^3)^2 = 9^{....}$

(5) $(...)^3 = 4^3 \times m^3$ (6) $\left(\frac{2}{....}\right)^5 = \frac{2^5}{3^5}$

◆ **Let's learn new :**

● **Laws of natural indices :**

(1) Law of multiplication : In the earlier class we have learnt that 'during multiplication of two rational indices with same base then their indices are added with same base.'

Multiplication of indices	Repeated multiplication	Result
$2^2 \times 2^3$	$\underline{2 \times 2} \times \underline{2 \times 2 \times 2}$	$2^5 = 2^{2+3}$
$(-8)^3 \times (-8)^4$	$\underline{(-8)\times(-8)\times(-8)} \times \underline{(-8)\times(-8)\times(-8)\times(-8)}$	$(-8)^7 = (-8)^{3+4}$
$x^2 \times x^4$	$\underline{x \times x} \times \underline{x \times x \times x \times x}$	$x^6 = x^{2+4}$
$\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^5$	$\underline{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}\times\underline{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}$	$\left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^{2+5}$
$(\sqrt{3})^2\times(\sqrt{3})^3\times(\sqrt{3})^4$	$\underline{(\sqrt{3})\times(\sqrt{3})}\times\underline{(\sqrt{3})\times(\sqrt{3})\times(\sqrt{3})}\times$ $\underline{(\sqrt{3})\times(\sqrt{3})\times(\sqrt{3})\times(\sqrt{3})}$	$(\sqrt{3})^9 = (\sqrt{3})^{2+3+4}$

Therefore, $a^m \times a^n = a^{m+n}$; where $a \in$ R and $m, n \in$ N

Example 1 : Simplify :

(1) $(0.3)^2 \times (0.3)^4 \times (0.3)$
$= (0.3)^{2+4+1}$
$= (0.3)^7$

(2) $(\sqrt{7})^2 \times (\sqrt{7})^7 \times (\sqrt{7})^3$
$= (\sqrt{7})^{2+7+3}$
$= (\sqrt{7})^{12}$

(3) $\left(-\frac{4}{5}\right)^6 \times \left(-\frac{4}{5}\right)^{18} \times \left(-\frac{4}{5}\right)$
$= \left(-\frac{4}{5}\right)^{6+18+1}$
$= \left(-\frac{4}{5}\right)^{25}$

(2) Law of division : During division of two rational indices with same base their indices are subtracted at the same base.

Division of indices	Repeated multiplication form	Result
$(-3)^6 \div (-3)^4$	$\frac{(-3)^6}{(-3)^4} = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{(-3)\times(-3)\times(-3)\times(-3)}$	$(-3)^2 = (-3)^{6-4}$
$\left(\frac{1}{2}\right)^3 \div \left(\frac{1}{2}\right)^4$	$\frac{\left(\frac{1}{2}\right)^3}{\left(\frac{1}{2}\right)^4} = \frac{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}$	$\frac{1}{\left(\frac{1}{2}\right)^1} = \frac{1}{\left(\frac{1}{2}\right)^{4-3}}$
$(\sqrt{m})^4 \div (\sqrt{m})^4$ where $m \neq 0$	$\frac{(\sqrt{m})^4}{(\sqrt{m})^4} = \frac{(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})}{(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})} = 1$	$1 = (\sqrt{m})^{4-4}$ $= (\sqrt{m})^0$

Therefore, (i) If $m > n$ and $a \neq 0$, then $a^m \div a^n = a^{m-n}$

(ii) If $m < n$ and $a \neq 0$, then $a^m \div a^n = \frac{1}{a^{n-m}}$

(iii) If $a \neq 0$, then $a^m \div a^m = a^{m-m} = a^0 = 1$

For any $a \neq 0$, $a^0 = 1$.

Example 2 : Simplify :

(1) $\left(\frac{1}{3}\right)^5 \div \left(\frac{1}{3}\right)^3$
$= \left(\frac{1}{3}\right)^{5-3}$
$= \left(\frac{1}{3}\right)^2$

(2) $\frac{(-7)^4}{(-7)^6}$
$= \frac{1}{(-7)^{6-4}}$
$= \frac{1}{(-7)^2}$

(3) $(-5)^7 \div (-5)^3$
$= (-5)^{7-3}$
$= (-5)^4$

(4) $\frac{a^7}{a^{14}} \times a^{12}$

First method : $\frac{a^7}{a^{14}} \times a^{12}, (a \neq 0)$

$= \frac{1}{a^{14-7}} \times a^{12}$

$= \frac{a^{12}}{a^7}$

$= a^{12-7}$

$= a^5$

Second method : $\frac{a^7}{a^{14}} \times a^{12}, (a \neq 0)$

$= \frac{a^7 \times a^{12}}{a^{14}}$

$= \frac{a^{7+12}}{a^{14}}$

$= \frac{a^{19}}{a^{14}}$

$= a^{19-14}$

$= a^5$

(5) $y^{19} \div y^{19}; (y \neq 0)$

$= y^{19-19}$

$= y^0$

$= 1$

(3) Law of power of a power :

For taking power of a power, the rational indices are multiplied on the same base.

Example : (1) $(2^2)^3 = 2^{2\times3} = 2^6$

(2) $[(-3)^4]^5 = (-3)^{4\times5} = (-3)^{20}$

(3) $\left[\left(\frac{1}{2}\right)^3\right]^4 = \left(\frac{1}{2}\right)^{3\times4} = \left(\frac{1}{2}\right)^{12}$

- **Try yourself :**

(1) $(7^3)^4 = 7^{....} = ...^{....}$ (2) $(a^5)^2 = a^{....} = ...^{....}$

Therefore, $(a^m)^n = a^{mn}$; where $a \in$ R and $m, n \in$ N

Example 3 : Simplify :

(1) $(a^{10})^3 \div (a^6)^5,\ (a \neq 0)$

$= a^{10\times 3} \div a^{6\times 5}$

$= a^{30} \div a^{30}$

$= a^{30-30}$

$= a^0 = 1$

(2) $\dfrac{\left[(-2)^3 \times (-2)^5\right]^2}{(-2)^7}$

$= \dfrac{\left[(-2)^{3+5}\right]^2}{(-2)^7}$

$= \dfrac{\left[(-2)^8\right]^2}{(-2)^7}$

$= \dfrac{(-2)^{16}}{(-2)^7}$

$= (-2)^{16-7} = (-2)^9$

(4) Law of power of product :

The power of multiplication of two numbers is multiplication of powers of two numbers.

Example : (1) $(3 \times 4)^5 = 3^5 \times 4^5$

(2) $[(-3) \times 2]^4 = (-3)^4 \times 2^4$

Try yourself : (1) $(8 \times 10)^3 = 8^{\cdots\cdots} \times 10^{\cdots\cdots}$ (2) $(a \times b)^5 = a^{\cdots\cdots} \times b^{\cdots\cdots}$

Therefore, $(ab)^n = a^n b^n$; where $a,\ b \in$ R and $n \in$ N

Note : If there is multiplication of more than two numbers, then this law can be applied.

Example 4 : Simplify :

(1) $(-2ab)^5$

$= (-2)^5 a^5 b^5$

(2) $(ab^2c)^3$

$= a^3(b^2)^3c^3$

$= a^3b^6c^3$

(3) $(6a^3)^2$

$= (6 \times a^3)^2$

$= 6^2 \times (a^3)^2$

$= 36 \times a^6$

$= 36a^6$

(5) Law of power of quotient :

The power of division of two numbers is the division of two numbers with same power, where divisor is not zero.

Example : (1) $\left(\frac{2}{5}\right)^{10} = \frac{2^{10}}{5^{10}}$, (2) $\left(\frac{-3}{4}\right)^{5} = \frac{(-3)^5}{4^5}$

- **Try yourself :** (1) $\left(\frac{7}{9}\right)^{8} = \frac{7^{\cdots\cdots}}{9^{\cdots\cdots}}$ (2) $\left(\frac{-2}{4}\right)^{6} = \frac{(-2)^{\cdots\cdots}}{4^{\cdots\cdots}}$

Therefore, $b \neq 0$ and $a, b \in$ R, then $\left(\frac{a}{b}\right)^{n} = \frac{a^n}{b^n}$.

Example 5 : Simplify :

(1) $[(a^6)^3 (b^4)^7] \div [(a^2)^9 (b^{14})^2]$; $(a \neq 0, b \neq 0)$

$= [a^{18}b^{28}] \div [a^{18}b^{28}]$

$= \frac{a^{18}b^{28}}{a^{18}b^{28}}$

$= a^{18-18}\ b^{28-28}$

$= a^0 b^0$

$= 1 \times 1$

$= 1$

(2) $\frac{(3x^2)^2 \times (2x^3)^4}{(6x^2)^2}$

$= \frac{3^2 \times (x^2)^2 \times 2^4 \times (x^3)^4}{(2 \times 3 \times x^2)^2}$

$= \frac{3^2 \times x^4 \times 2^4 \times x^{12}}{2^2 \times 3^2 \times x^4}$

$= 2^{4-2} \times x^{12}$

$= 2^2 \times x^{12}$

$= 4x^{12}$

Practice 1

1. Fill in the blanks :

(1) $(-10)^6 \times (-10)^6 = (-10)^{\cdots\cdots}$

(2) $4^5 \div 4^3 = 4^{\cdots\cdots}$

(3) $\dfrac{x^3}{x^6} = \dfrac{1}{x^{\cdots\cdots}} = \cdots\cdots$

(4) $(21^3)^3 = 21^{\cdots\cdots}$

(5) $(\sqrt{3})^0 = \cdots\cdots$

(6) $[(-3) \times 4]^2 = (-3)^{\cdots\cdots} \times \cdots\cdots^2$

(7) $\left(\dfrac{5}{x}\right)^4 = \dfrac{5^{\cdots\cdots}}{x^{\cdots\cdots}}$

(8) $\left(\dfrac{1}{2}\right)^3 \div \left(\dfrac{1}{2}\right)^3 = \cdots\cdots$

(9) $12x^4 \div 3x^6 = \cdots\cdots$

(10) $a^{10} \div \cdots\cdots = a^8$

2. Simplify :

(1) $(2^7)^3 \times (2^4)^6$

(2) $(-3)^4 \times (-3)$

(3) $\dfrac{(a^{14})^4 \times (a^2)^3}{(a^7)^3}$; $(a \neq 0)$

(4) $\dfrac{(y^7)^3}{(y^6)^5}$; $(y \neq 0)$

(5) $\left(\dfrac{1}{3} \times x\right)^3$

(6) $\dfrac{(2a^2b^3)^5 \times (2a^2b^2)^3}{(5a^4b)^6}$; $(a,\ b \neq 0)$

- **Negative Rational indices :**

For number x except zero according to the law of division of power the indices are subtracted in the numerator only.

$$\frac{x^3}{x} = x^{3-1} = x^2$$

$$\frac{x^2}{x} = x^{2-1} = x^1$$

$$\frac{x}{x} = x^{1-1} = x^0 = 1$$

$$\frac{1}{x} = \frac{x^0}{x} = x^{0-1} = x^{-1} \quad (\because\ 1 = x^0)$$

$$\frac{1}{x} \div x = \frac{1}{x} \times \frac{1}{x} = \frac{1}{x \times x} = \frac{x^0}{x^2} = x^{0-2} = x^{-2} \quad (\because\ 1 = x^0)$$

Therefore, for any positive real number a, $\dfrac{1}{a} = a^{-1}$, $\dfrac{1}{a^2} = a^{-2}$, $\dfrac{1}{a^3} = a^{-3}$

$\because$ **is read as 'because'**

Similarly (i) $a^{-1} = \frac{1}{a}$, $a^{-5} = \frac{1}{a^5}$, $a^{-7} = \frac{1}{a^7}$ (ii) $5^{-1} = \frac{1}{5}$, $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

(iii) $\left(\frac{1}{4}\right)^{-3} = \frac{1^{-3}}{4^{-3}} = \frac{4^3}{1^3} = 4^3 = 64$ (iv) $\left(\frac{2}{7}\right)^{-5} = \frac{2^{-5}}{7^{-5}} = \frac{7^5}{2^5}$

Therefore, if $a \neq 0$ and $a \in R$, then for $n \in N$, $\boldsymbol{a^{-n} = \frac{1}{a^n}}$

Besides this, all the laws of rational indices for positive integers are also true for negative integers.

$a, b \neq 0$, $m, n \in Z$, then

(1) $a^m \times a^n = a^{m+n}$ (2) $(a^m)^n = a^{mn}$ (3) $\frac{a^m}{a^n} = a^{m-n}$

(4) $(ab)^n = a^n b^n$ (5) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 6 : Simplify :

(1) 4^{-3}

$= \frac{1}{4^3}$

$= \frac{1}{64}$

(2) $\left(\frac{1}{3}\right)^{-4}$

$= \frac{1}{\left(\frac{1}{3}\right)^4} = \frac{1}{\left(\frac{1^4}{3^4}\right)}$

$= \frac{1}{\frac{1}{81}} = \frac{81}{1} = 81$

(3) $\left(x^{-1}\right)^3$

$= x^{-3}$

$= \frac{1}{x^3}$

(4) $2^3 \times \left(\frac{1}{2}\right)^5 \times 2^{-6}$

First method : $2^3 \times \left(\frac{1}{2}\right)^5 \times 2^{-6}$

$= 2^3 \times (2^{-1})^5 \times 2^{-6}$

$= 2^3 \times 2^{-5} \times 2^{-6}$

$= 2^{3-5-6}$

$= 2^{-8}$

$= \frac{1}{2^8}$

$= \frac{1}{256}$

Second method : $2^3 \times \left(\frac{1}{2}\right)^5 \times 2^{-6}$

$= 2^3 \times \frac{1^5}{2^5} \times \frac{1}{2^6}$

$= \frac{2^3 \times 1 \times 1}{2^5 \times 2^6}$

$= \frac{2^3}{2^{5+6}}$

$= \frac{2^3}{2^{11}}$

$= \frac{1}{2^{11-3}}$

$= \frac{1}{2^8} = \frac{1}{256}$

Example 7 : Simplify :

(1) $(5x^{-2})^3 \times (3x^3)^4 \div (15x^2)^2$

$= \dfrac{5^3 \times x^{(-2)\times 3} \times 3^4 \times x^{3\times 4}}{(3\times 5\times x^2)^2}$

$= \dfrac{5^3 \times x^{-6} \times 3^4 \times x^{12}}{3^2 \times 5^2 \times x^{2\times 2}}$

$= \dfrac{3^4 \times 5^3 \times x^{-6+12}}{3^2 \times 5^2 \times x^4}$

$= \dfrac{3^4 \times 5^3 \times x^6}{3^2 \times 5^2 \times x^4}$

$= 3^{4-2} \times 5^{3-2} \times x^{6-4}$

$= 3^2 \times 5^1 \times x^2$

$= 9 \times 5 \times x^2$

$= 45x^2$

(2) $\left(\dfrac{x}{y}\right)^a (xy)^b \div \left(\dfrac{x}{y}\right)^b$

$= \dfrac{x^a}{y^a} \times x^b \times y^b \div \dfrac{x^b}{y^b}$

$= \dfrac{x^a \times x^b \times y^b \times y^b}{y^a \times x^b}$

$= \dfrac{x^a \times x^b \times y^{b+b}}{x^b \times y^a}$

$= x^a \times \dfrac{x^b}{x^b} \times y^{2b-a}$

$= x^a \,.\, y^{2b-a}$

Practice 2

1. Fill in the blanks by choosing proper options :

(1) $4^{-3} =$ .... [(a) 64, (b) −12, (c) $\frac{1}{64}$]

(2) $\frac{1}{5} =$ .... [(a) 5^{-1}, (b) 5^{-2}, (c) 5^{-3}]

(3) $(8^{-1})^{-2} =$ .... [(a) $\frac{1}{8}$, (b) $\frac{1}{64}$, (c) 64]

(4) $a^{-2} \times \dfrac{1}{a^{-2}} =$ .... [(a) a^0, (b) a^2, (c) a^{-4}]

(5) $\dfrac{1}{(2^{-1})^2} =$ .... [(a) $\frac{1}{4}$, (b) 2^2, (c) $\frac{1}{2}$]

(6) $(\sqrt{5})^3 \div (\sqrt{5})^3 =$ .... [(a) $\sqrt{5}$, (b) 1, (c) $(\sqrt{5})^6$]

2. Simplify :

(1) $2^{-3} \times \left(\dfrac{1}{4}\right)^5 \times 8^{-3}$ (2) $\left(\dfrac{a}{b}\right)^{m+n} \left(\dfrac{a}{b}\right)^{m-n} (ab)^m$ (3) $4a^{-2} \times \left(\dfrac{a}{2}\right)^5 \div (2a)^2$

3. Evaluate :

(1) $2^2 \times 2^{-3} \times 2^{-1}$ (2) $\left(\frac{1}{3}\right)^3 \times 3^{-2} \times 3^5$ (3) $(8^{-2} \times 12^4) \div 27^2$

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Fractional (rational) indices :

Number	Square	Square root of square
1	1	$\sqrt{1} = \sqrt{1^2}$
2	4	$\sqrt{4} = \sqrt{2^2}$
3	9	$\sqrt{9} = \sqrt{3^2}$
4	16	$\sqrt{16} = \sqrt{4^2}$
5	25	$\sqrt{25} = \sqrt{5^2}$

Now, from the table comparing number and square root of square of the number we get,

$1 = \sqrt{1^2}$

$2 = \sqrt{2^2}$

$3 = \sqrt{3^2}$

$4 = \sqrt{4^2}$

$5 = \sqrt{5^2}$

Now, $\sqrt{3^2} = 3$ **Result (1)**

Here, 3 is square root of 3^2.

Now, let's find such indices of 3^2 such that value is 3.

If, we take such rational indices of 3^2 which when multiplied with 3, then rational indices is 1. (Law of power of power)

The reciprocal of two inverse numbers is 1 and so reciprocal of $2 = \frac{1}{2}$.

Thus, $(3^2)^{\frac{1}{2}} = 3^{2\times\frac{1}{2}}$

$= 3^1 = 3$ **Result (2)**

Comparing result (1) and result (2) we get,

$$3 = \sqrt{3^2} = (3^2)^{\frac{1}{2}}$$

Here, the symbol of square root ($\sqrt{\ }$) is denoted by index $\frac{1}{2}$.

Therefore, $\sqrt{2} = 2^{\frac{1}{2}}$, $\sqrt{3} = 3^{\frac{1}{2}}$, $\sqrt{5} = 5^{\frac{1}{2}}$ and $\sqrt{x} = x^{\frac{1}{2}}$; where x is a real positive number.

Similarly, for symbol of cube root $\sqrt[3]{\ }$, $\sqrt[3]{x} = x^{\frac{1}{3}}$, for symbol of fourth root $\sqrt[4]{\ }$, $\sqrt[4]{x} = x^{\frac{1}{4}}$, for symbol of fifth root $\sqrt[5]{\ }$, $\sqrt[5]{x} = x^{\frac{1}{5}}$, for symbol of nth root $\sqrt[n]{\ }$, $\sqrt[n]{x} = x^{\frac{1}{n}}$, where $n \in$ N.

Therefore, if x is a positive real number and $n \in$ N, then $\sqrt[n]{x} = x^{\frac{1}{n}}$ can be written.

Example 8 : For $a = 64$, $m = \frac{3}{2}$, $n = \frac{1}{3}$ verify $a^m \times a^n = a^{m+n}$.

L.H.S. $= a^m \times a^n$

$= (64)^{\frac{3}{2}} \times (64)^{\frac{1}{3}}$

$= (2^6)^{\frac{3}{2}} \times (2^6)^{\frac{1}{3}}$

$= 2^{6 \times \frac{3}{2}} \times 2^{6 \times \frac{1}{3}}$

$= 2^9 \times 2^2$

$= 2^{9+2}$

$= 2^{11}$

R.H.S. $= a^{m+n}$

$= (64)^{\frac{3}{2}+\frac{1}{3}}$

$= (64)^{\frac{11}{6}}$

$= (2^6)^{\frac{11}{6}}$

$= 2^{11}$

Therefore, $a^m \times a^n = a^{m+n}$

Note : To show exponent (indices) form of any number, write that number as exponent form of prime number.

Example 9 : For $a = 64$, $m = \frac{2}{3}$, $n = \frac{1}{2}$, verify $(a^m)^n = a^{mn}$.

L.H.S. $= (a^m)^n$

$= (64^{\frac{2}{3}})^{\frac{1}{2}}$

$= ((2^6)^{\frac{2}{3}})^{\frac{1}{2}}$

$= (2^4)^{\frac{1}{2}}$

$= 2^2$

$= 4$

R.H.S. $= a^{mn}$

$= (64)^{\frac{2}{3} \times \frac{1}{2}}$

$= (2^6)^{\frac{1}{3}}$

$= 2^{6 \times \frac{1}{3}}$

$= 2^2$

$= 4$

Therefore, $(a^m)^n = a^{mn}$

Example 10 : For $a = 27$, $b = 8$, $m = \frac{1}{3}$ verify $(ab)^m = a^m b^m$

L.H.S. $= (ab)^m$

$= (27\times8)^{\frac{1}{3}}$

$= (3^3\times2^3)^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}}(2^3)^{\frac{1}{3}}$

$= 3^{3\times\frac{1}{3}}2^{3\times\frac{1}{3}}$

$= 3\times2 = 6$

R.H.S. $= a^m b^m$

$= 27^{\frac{1}{3}}\times8^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}}(2^3)^{\frac{1}{3}}$

$= 3\times2$

$= 6$

$\therefore$ L.H.S. = R.H.S. Therefore, $(ab)^m = a^m b^m$

Example 11 : Simplify :

(1) $(343)^{\frac{1}{3}}$

$= (7^3)^{\frac{1}{3}}$

$= 7^{3\times\frac{1}{3}}$

$= 7$

(2) $\left(\frac{81}{625}\right)^{\frac{3}{4}}$

$= \left(\frac{3^4}{5^4}\right)^{\frac{3}{4}}$

$= \frac{3^{4\times\frac{3}{4}}}{5^{4\times\frac{3}{4}}}$

$= \frac{3^3}{5^3}$

$= \frac{27}{125}$

(3) $\left(\frac{32}{243}\right)^{\frac{-2}{5}}$

$= \left(\frac{2^5}{3^5}\right)^{\frac{-2}{5}}$

$= \frac{2^{5\times\frac{-2}{5}}}{3^{5\times\frac{-2}{5}}}$

$= \frac{2^{-2}}{3^{-2}}$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

Second method :

$\left(\frac{32}{243}\right)^{\frac{-2}{5}}$

$= \left(\frac{243}{32}\right)^{\frac{2}{5}}$

$= \left(\frac{3^5}{2^5}\right)^{\frac{2}{5}}$

$= \frac{3^{5\times\frac{2}{5}}}{2^{5\times\frac{2}{5}}}$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

Example 12 : Simplify : $\left(\dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}\right)^{\frac{1}{5}}\left(\dfrac{x^{\frac{1}{5}}}{x^{\frac{1}{3}}}\right)^{\frac{1}{2}}\left(\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{5}}}\right)^{\frac{1}{3}}$, where $(x > 0)$.

$$= \frac{(x^{\frac{1}{3}})^{\frac{1}{5}}}{(x^{\frac{1}{2}})^{\frac{1}{5}}} \times \frac{(x^{\frac{1}{5}})^{\frac{1}{2}}}{(x^{\frac{1}{3}})^{\frac{1}{2}}} \times \frac{(x^{\frac{1}{2}})^{\frac{1}{3}}}{(x^{\frac{1}{5}})^{\frac{1}{3}}}$$

$$= \frac{x^{\frac{1}{3}\times\frac{1}{5}}}{x^{\frac{1}{2}\times\frac{1}{5}}} \times \frac{x^{\frac{1}{5}\times\frac{1}{2}}}{x^{\frac{1}{3}\times\frac{1}{2}}} \times \frac{x^{\frac{1}{2}\times\frac{1}{3}}}{x^{\frac{1}{5}\times\frac{1}{3}}}$$

$$= \frac{x^{\frac{1}{15}}}{x^{\frac{1}{10}}} \times \frac{x^{\frac{1}{10}}}{x^{\frac{1}{6}}} \times \frac{x^{\frac{1}{6}}}{x^{\frac{1}{15}}}$$

$$= \frac{x^{\frac{1}{15}}}{x^{\frac{1}{15}}} \times \frac{x^{\frac{1}{10}}}{x^{\frac{1}{10}}} \times \frac{x^{\frac{1}{6}}}{x^{\frac{1}{6}}}$$

$$= 1$$

Example 13 : If $x = 64$, then find the value of $x^{\frac{1}{6}} + x^{-\frac{1}{6}}$.

$$x^{\frac{1}{6}} + x^{-\frac{1}{6}} = 64^{\frac{1}{6}} + 64^{-\frac{1}{6}}$$

$$= (2^6)^{\frac{1}{6}} + (2^6)^{-\frac{1}{6}}$$

$$= 2^{6\times\frac{1}{6}} + 2^{6\times\frac{-1}{6}}$$

$$= 2^1 + 2^{-1}$$

$$= 2 + \frac{1}{2}$$

$$= 2\frac{1}{2}$$

Example 14 : If $x = \frac{9}{4}$, then prove $\left(x\sqrt{x}\right)^x = x^{x\sqrt{x}}$.

$$\left(x\sqrt{x}\right)^x = \left(\frac{9}{4}\sqrt{\frac{9}{4}}\right)^{\frac{9}{4}} = \left(\frac{3^2}{2^2}\times\frac{3}{2}\right)^{\frac{9}{4}} = \left[\left(\frac{3}{2}\right)^3\right]^{\frac{9}{4}} = \left(\frac{3}{2}\right)^{\frac{27}{4}}$$

$$x^{x\sqrt{x}} = \left(\frac{9}{4}\right)^{\frac{9}{4}\sqrt{\frac{9}{4}}} = \left(\frac{9}{4}\right)^{\frac{9}{4}\times\frac{3}{2}} = \left(\frac{9}{4}\right)^{\frac{27}{8}} = \left[\left(\frac{3}{2}\right)^2\right]^{\frac{27}{8}} = \left(\frac{3}{2}\right)^{\frac{27}{4}}$$

Therefore, $\left(x\sqrt{x}\right)^x = x^{x\sqrt{x}}$.

Practice 3

1. Evaluate :

(1) $\sqrt[5]{243}$ (2) $729^{\frac{1}{3}}$ (3) $\left(\frac{125}{343}\right)^{\frac{1}{3}}$ (4) $64^{\frac{1}{6}}$

(5) $625^{\frac{1}{4}}$ (6) $\left(\frac{81}{144}\right)^{\frac{1}{2}}$ (7) $\left(\frac{81}{625}\right)^{\frac{-3}{4}}$ (8) $\frac{32^{\frac{1}{5}}}{81^{\frac{1}{4}}}$

2. Prove that, $5^{\frac{1}{3}}\times\left(\frac{2}{5}\right)^{\frac{1}{3}}\times\frac{64^{\frac{1}{3}}}{3^{\frac{1}{3}}}\times\frac{9^{\frac{1}{6}}}{2^{\frac{1}{3}}} = 4$.

3. If $x = 243$, then find the value of $x^{\frac{1}{5}}\times x^{-\frac{1}{5}}$.

4. Verify : (1) $\sqrt{625} - 5\sqrt[3]{27} - (100)^{\frac{1}{2}} = 0$

(2) $\left[(81)^{\frac{1}{2}}+1\right]\left[(81)^{\frac{1}{4}}-1\right] = 20$

- **What did you learn ?**
  - $a, b \neq 0$ and $m, n \in$ Q

    (1) $a^m \times a^n = a^{m+n}$ (2) $(a^m)^n = a^{mn}$ (3) $\dfrac{a^m}{a^n} = a^{m-n}$

    (4) $(ab)^n = a^n b^n$ (5) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
  - If $a \neq 0$, then $a^0 = 1$
  - If $a \neq 0$ and $a \in$ R, then for $n \in$ N, $a^{-n} = \dfrac{1}{a^n}$.
  - If x is a positive real number and $n \in$ N, then $\sqrt[n]{x} = x^{\frac{1}{n}}$.

Exercise

1. Fill in the blanks :

(1) $(-51)^0 =$ ....

(2) $x^5 \times x^{-4} \div x^2 =$ ....

(3) $(a^3)^{-4} =$ ....

(4) $(\sqrt{y}\,)^5 =$ ....

(5) $4^{-2} \times \dfrac{1}{4^{-2}} =$ ....

(6) $\dfrac{1}{(3 \times 4)^{-1}} =$ ....

(7) $\left[\left(\dfrac{2}{3}\right)^2\right]^{-2} =$ ....

(8) $\left[\dfrac{16}{81}\right]^{\frac{1}{4}} =$ ....

2. Simplify : (1) $\left(\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^2 \left(\dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{4}}}\right)^3 \left(\dfrac{x^{\frac{1}{4}}}{x^{\frac{1}{2}}}\right)^4$, $(x > 0)$ (2) $\dfrac{(x^5)^{\frac{1}{6}} \times x^{\frac{1}{7}} \times (x^{\frac{2}{3}})^2}{(x^2)^{\frac{2}{3}} \times (x^{\frac{1}{6}})^5 \times x^{\frac{1}{7}}}$

3. Evaluate :

(1) $\left[\dfrac{49}{16}\right]^{\frac{1}{2}} \times \left(\dfrac{4^2}{7^2}\right) \times 4^{\frac{1}{2}}$

(2) $\left(\dfrac{25}{16}\right)^{\frac{1}{4}} \times \left(\dfrac{27}{8}\right)^{\frac{1}{6}} \times \left(\dfrac{2}{15}\right)^{\frac{1}{2}}$

(3) $\dfrac{\sqrt[3]{108} \times \sqrt[6]{4}}{\sqrt[4]{81}}$

(4) $\left(\dfrac{8}{27}\right)^{\frac{1}{3}} \times \left(\dfrac{9}{25}\right)^{\frac{1}{2}} \times \left(\dfrac{2}{5}\right)^{-1}$

4. Prove that : $\left(\dfrac{2^{\frac{1}{3}}}{2^{\frac{-1}{3}}}\right)^3 + \dfrac{3^{\frac{1}{2}}}{3^{\frac{-1}{2}}} = 7$

5. Prove that $\dfrac{(16)^{\frac{1}{4}}}{(27)^{\frac{1}{3}}} + \dfrac{(625)^{\frac{1}{4}}}{(81)^{\frac{1}{4}}} - \dfrac{1}{(243)^{\frac{1}{5}}} = 2$

6. Prove : $[(a^x)^y(a^y)^x]^z = a^{2xyz}$ $(x, y, z \in Q)$

7. If $x > 0$ and $x, y, z \in Q$ and a, b, c are non-zero integers, then prove that

$$\left[\left(\frac{x^a}{x^b}\right)^{\frac{1}{a}}\right]^{\frac{1}{b}}\left[\left(\frac{x^b}{x^c}\right)^{\frac{1}{b}}\right]^{\frac{1}{c}}\left[\left(\frac{x^c}{x^a}\right)^{\frac{1}{c}}\right]^{\frac{1}{a}} = 1.$$

Answers

Practice 1

1. (1) 12 (2) 2 (3) 3 (4) 9 (5) 1 (6) 2, 4 (7) 4, 4 (8) $\left(\frac{1}{2}\right)^0$ or 1 (9) $\frac{4}{x^2}$ (10) a^2

2. (1) 2^{45} (2) $(-3)^5$ (3) a^{41} (4) $\frac{1}{y^9}$ (5) $\frac{x^3}{3^3}$ (6) $\frac{2^8 b^{15}}{5^6 a^8}$

Practice 2

1. (1) c (2) a (3) c (4) a (5) b (6) b

2. (1) $\frac{1}{2^{22}}$ (2) $a^{3m} \div b^m$ (3) $\frac{a}{32}$

3. (1) $\frac{1}{4}$ (2) 1 (3) $\frac{4}{9}$

Practice 3

1. (1) 3 (2) 9 (3) $\frac{5}{7}$ (4) 2 (5) 5 (6) $\frac{3}{4}$ (7) $\frac{125}{27}$ (8) $\frac{2}{3}$ **3.** 1

Exercise

1. (1) 1 (2) $\frac{1}{x}$ (3) $\frac{1}{a^{12}}$ (4) $y^{\frac{5}{2}}$ (5) 1 (6) 12 (7) $5\frac{1}{16}$ (8) $\frac{2}{3}$

2. (1) $\frac{1}{x^{\frac{5}{12}}}$ (2) 1 **3.** (1) $\frac{8}{7}$ or $1\frac{1}{7}$ (2) $\frac{1}{2}$ (3) 2 (4) 1