Lecture - 9

$$\nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E \quad \text{— (III)}$$

$$\nabla^2 H = j\omega\mu(\sigma + j\omega\epsilon)H \quad \text{— (IV)}$$

Consider ∇ in one ~~direction~~ dimension of z only

E is in x direction, H is in y-direction

Consider $\sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \gamma$

$$\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \quad \text{— (III)}$$

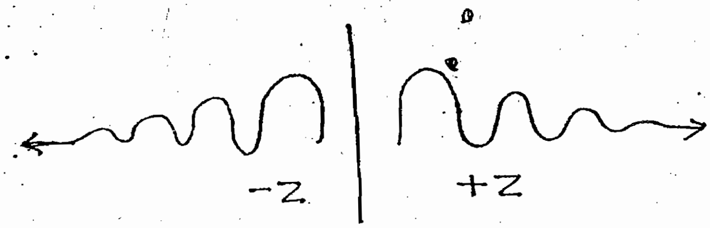
$$\frac{\partial^2 H_y}{\partial z^2} = \gamma^2 H_y \quad \text{— (IV)}$$

The E(z) and H(z) solution is.

$$E(z)_x = (C_1 e^{-\gamma z} + C_2 e^{\gamma z})\, a_z \quad \text{— (V)}$$

$$H(z)_y = (\underbrace{C_3 e^{-\gamma z}}_{+z} + \underbrace{C_4 e^{\gamma z}}_{-z})\, a_y \quad \text{— (VI)}$$

The solution physically represent two waves travelling on either sides from a planar source and both decaying. This is called as uniform plane wave



Considering only the +z solution and C_1 & C_3 being the initial source values. The final E/H wave solution is

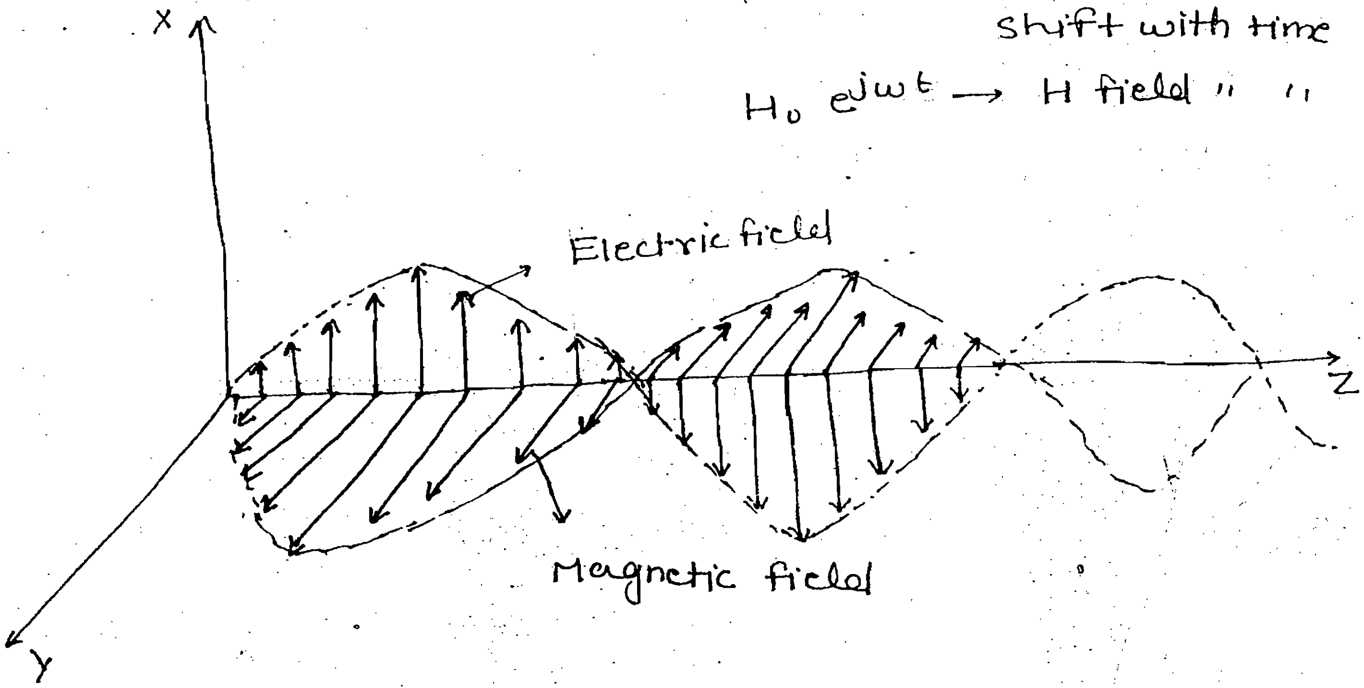
$$E(z,t)_x = (E_0 e^{j\omega t} \cdot e^{-\gamma z})\, a_x$$

$$H(z,t)_y = (H_0 e^{j\omega t} \cdot e^{-\gamma z})\, a_y$$

The solution is a product solution of time and space harmonics.

$E_0 e^{j\omega t} \rightarrow$ electric field shift with time

$H_0 e^{j\omega t} \rightarrow$ H field " "



→ Electric and H field perpendicular and shifts simultaneously with time

→ Cross product result in transverse and dot product result in longitudinal.

<u>Propogation constant (γ)</u> :-

γ is the constant of the exponential with z that decides the course of propagation i.e. γ is called as propagation constant.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

$$E(z,t) = \left(E_0 e^{-\alpha z} \cdot e^{j\omega t} e^{-j\beta z}\right) a_x$$

$$\boxed{\begin{aligned} E(z,t)_x &= (E_0 e^{-\alpha z})\, e^{j(\omega t - \beta z)}\, a_x \\ H(z,t)_y &= (H_0 e^{-\alpha z})\, e^{j(\omega t - \beta z)}\, a_y \end{aligned}}$$

field direction

Amplitude

Phase

Note-1:-

Every EM wave is harmonic whose amplitude exponentially decays at α rate

α = attenuation constant

Note-2:-

Every EM wave is a harmonic whose phase independtly but linearly changes with time and space.

Note-3:-

The harmonic E & H have direction and obeying the basic transverse nature with propogation

$$\boxed{\underset{(\text{Dir})}{E} \times \underset{(\text{Dir})}{H} = \underset{(\text{Dir.})}{\text{Propogation constant}}}$$

Intrinsic Wave Impedance (η):-

$$E \rightleftharpoons H$$

$$\eta = \frac{E}{H} = \frac{\text{Volts/m}}{\text{Amp/m}} = \text{ohm}$$

When E is transformed to H & vice-versa the rate of transformation is called as intrinsic impedance of the medium

$$\nabla \times E = -j\omega\mu H$$

$$\frac{\partial}{\partial z} a_z \times E a_x = -j\omega\mu H a_y$$

$$\frac{\partial E}{\partial z} = -j\omega\mu H$$

As $E = E_0 e^{-\gamma z}$

$$\Rightarrow \quad -\gamma E = -j\omega\mu H$$

$$\Rightarrow \quad \frac{E}{H} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\Rightarrow \boxed{\frac{E}{H} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \text{complex} = R + jX$$

The transformation has a resistance which is a loss (α) in transformation and a reactance which leads to phase shift (β)

$$\gamma = \alpha + j\beta$$

$$\eta = R + jX$$

Workbook-2

1) $V = -\frac{d\Psi_m}{dt} = -\frac{d}{dt}\left(\frac{\lambda}{3}t^3\right) = -\frac{\lambda}{3}\cdot 3t^2 \Big|_{t=3s} = 9V$

$$\Rightarrow \boxed{\lambda = -1}$$

(2) $V = -\frac{d\Psi_m}{dt} = -\frac{d}{dt}(BA) = -\pi r^2 \frac{dB}{dt}$

$$= -\pi \times (0.1)^2\, 10 \left(-\sin(120\pi t)\cdot 120\pi\right)$$

$$= 118 \sin(120\pi t) \quad \text{Ans}$$

3) B → Linearly decrease with time $B(t) = -kt$

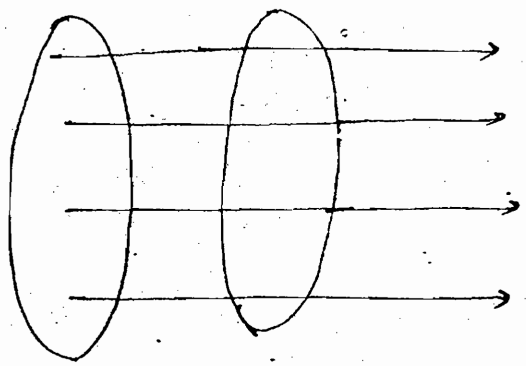
→ Uniform decrease with space

$\therefore V = -\frac{d\Psi_m}{dt} = -\frac{d}{dt}(BA) = -A(-k) = \text{Constant}$

$= \text{DC voltage}$

→ Upto open circuit E flows and emf is induced

→ Induced emf converts into current in coil 1

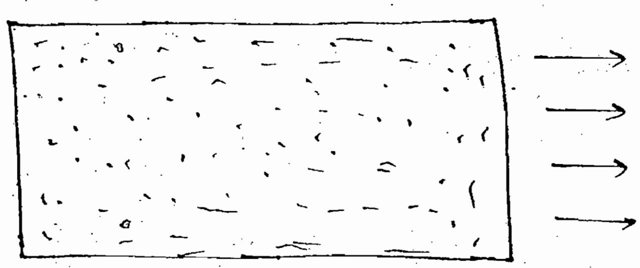


Ans- D

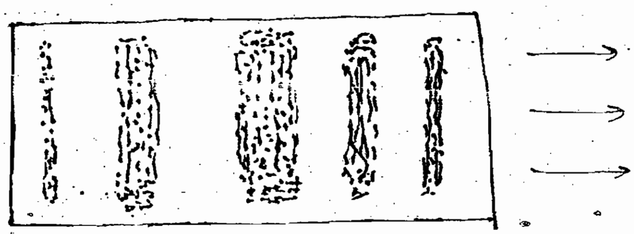
V = 0   V ≠ 0

I ≠ 0   I = 0

Note:-



DC current

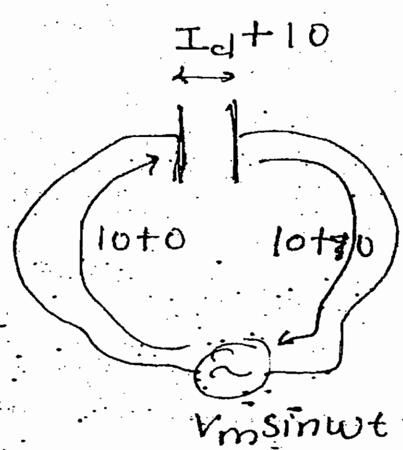


AC current

Q.4.

$I_c = I_d$

$\oint H \cdot dl = I_c + I_d \neq 2I_d$ or $2I_c$

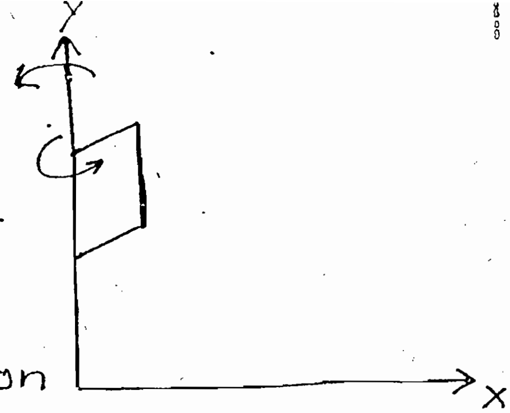


$J_d = \epsilon \dfrac{\partial E}{\partial t}$

$I_d = \epsilon A \cdot j\omega E = j\omega \dfrac{\epsilon A}{d} V$

Ans-(a)

5. $B(t)x$



Uniform and Harmonic in time

$\downarrow$

Transformer Action

$\downarrow$

AC Voltages and harmonic $B(t)$ being involved

$$V = -\frac{d\psi_m}{dt} = -\frac{d}{dt}(BA) = \underbrace{-B\frac{dA}{dt}}_{\text{Rotation}} - A\frac{dB}{dt}$$

$B(t) = \cos(\omega t) \longrightarrow$ AC voltage

Ans – (c)

6. Radiation $\rightarrow$ EM waves $\rightarrow$ generate due to cross product

7. $\gamma = \alpha + j\beta$

Amplitude $= |E|$

$|E| = E_0 e^{-\alpha z} = \frac{1}{e} E_0$ when $z = 20m$

$\Rightarrow \alpha z = 1 \Rightarrow \alpha = \frac{1}{20}$

$\theta \propto z \Rightarrow \theta = \beta z$

$\Rightarrow \beta = \frac{\pi/6}{20m} = \frac{\pi}{120}$

$$\boxed{\gamma = \frac{1}{20} + j\frac{\pi}{120}}$$

$|E| \longrightarrow 20\%$ of initial value

$|E| \longrightarrow 20 \longrightarrow \overset{E_0}{100} \longrightarrow z = 5m$

$|E| \longrightarrow 40 \longrightarrow 100 \longrightarrow z = ?$

It is not a linear function

$$|E| = E_0 e^{-\alpha z}$$

$$\Rightarrow 20 = 100 e^{-\alpha 5} \qquad \Rightarrow e^{5\alpha} = 5$$

$$\Rightarrow 5\alpha = \ln 5$$

$$\Rightarrow \alpha = \frac{\ln 5}{5}$$

$$40 = 100 e^{-\alpha z}$$

$$\Rightarrow e^{\alpha z} = \frac{100}{40} = 2.5$$

$$\Rightarrow \alpha z = \ln 2.5$$

$$\Rightarrow z = \frac{5 \ln 2.5}{\ln 5} = 2.8 \text{ , Ans.}$$

Note:-

Change is depending on function value

Case-(1):-

EM Wave Propogation in free space

$$|E| = E_0 e^{-\alpha z}$$

$E_0 = 100$

$z = \frac{1}{\alpha} \downarrow$

$\frac{100}{e} = 37$

$z = \frac{1}{\alpha} \downarrow$

$\frac{100}{e^2} = 13$

$z = \frac{1}{\alpha} \downarrow$

$\frac{100}{e^3} = 4$

$\frac{2}{\alpha}$

$\frac{3}{\alpha}$

Case-(i) :-

EM Wave Propogation in free space $\begin{bmatrix} \sigma = 0, \epsilon = \epsilon_0 \\ \mu = \mu_0 \end{bmatrix}$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0} = \text{purely imaginary}$$

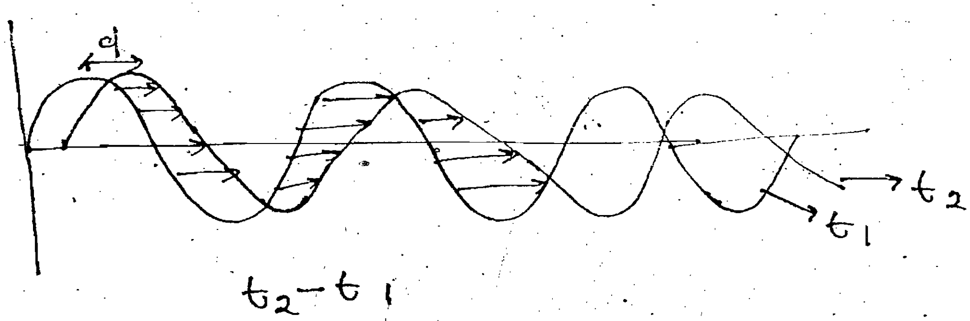
$\Rightarrow$ $\alpha = 0$ No attenuation of EM wave in free space

$\beta = \omega\sqrt{\mu_0\epsilon_0}$ = only phase shift and propogation constant exists

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \text{real}$$

$$= \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^{9}} = 120\pi = 377$$

Phase Velocity (V_p) :-



The distance travelled by any in-phase point per unit time is called as phase velocity.

$$V_p = \frac{d}{t}$$

$$= \frac{\lambda}{T} = \lambda f = \frac{2\pi\lambda}{2\pi T}$$

$$\Rightarrow V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\Rightarrow V_p = 3 \times 10^{8}\ m/s$$

case-(II) :-

EM Wave propogation in Ideal dielectrics

→ Ideal dielectrics = Perfect / lossless dielectrics

→ $\sigma = 0$, $\epsilon = \epsilon_0 \epsilon_R$, $\mu = \mu_0$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu_0\epsilon_0\epsilon_R}$$

$$= \text{purely imaginary}$$

$\Rightarrow$ $\alpha = 0$ i.e. No attenuation of EM wave in ideal dielectrics

$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}$ = only phase shift and property exist

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_R}} = \text{real}$$

$$= \sqrt{\frac{4\pi\times10^{-7}\times36\pi\times10^{9}}{\epsilon_R}} = \frac{120\pi}{\sqrt{\epsilon_R}} = \frac{377}{\sqrt{\epsilon_R}}$$

Phase Velocity (V_p) :-

$$V_p = \frac{\lambda}{T} = \lambda f = \frac{2\pi\lambda}{2\pi T}$$

$$\Rightarrow V_p = \frac{\omega}{\beta_0} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}}$$

$$\Rightarrow \boxed{V_P = \frac{3\times10^8}{\sqrt{\epsilon_r}}} \text{ m/s}$$

Q. $E(x,t) = 25\sin(\omega t + 4x)\,a_y$

By comparison, propogation direction → $-a_x$

$$\beta = 4 = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{\pi}{2}$$

$$f = \frac{3\times10^8}{\frac{\pi}{2}} = \frac{600}{\pi} \text{ MHz}$$

Method-1 :-

$$\nabla \times E = -j\omega\mu H = -\mu\frac{\partial H}{\partial t}$$

$$H = -\frac{1}{\mu}\int(\nabla \times E)\,dt$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

Method 2 :-

$$E(x,t) = 25\sin(\omega t + 4x)\,a_y$$

Step-1 :- $\frac{|E|}{|H|} = |\eta|$

Step-2 :- when η is real E & H have the same harmonic and phase

Step-3 :- E dir × H dir = Prop dir. (Basic Transverse Nature)

$$H(x,t) = \left(\frac{25}{120\pi}\right)\sin(\omega t + 4x)(-a_x \times a_y)$$

$$= \left(\frac{-25}{120\pi}\right)\sin(\omega t + 4x)\,a_z$$

10. $E(y,t) = 25\sin(10^8 t - y)\,a_z$

25 → const → free space, lossless dielectrics

$\omega = 10^8$

$\beta = 1$

$V_p = \frac{\omega}{\beta} = 10^8 \neq 3\times10^8$

or

$$V_p = \frac{3\times10^8}{\sqrt{\epsilon_R}} = 10^8 \Rightarrow \epsilon_R = 9$$

Propogation direction $\longrightarrow a_y$

$$f = \frac{10^8}{2\pi}$$

$$\lambda = \frac{\cancel{c}}{\cancel{2\pi}} 2\pi$$

$$H(y,t) = \left(\frac{\frac{25}{120\pi}}{\sqrt{9}}\right) \sin(10^8 t - y)(a_y \times a_z)$$

$$= \left(\frac{5}{8\pi}\right) \sin(10^8 t - y)\, a_x$$

1. $H = (0.5\, e^{-0.1x}) \cos(10^6 t - 2x)\, a_z$

$\omega = 10^6$ rad/second

$\beta = 2 = \frac{2\pi}{\lambda} \Rightarrow \lambda = 3.14$

Propogation $\longrightarrow +a_x$

$\rightarrow$ Polarization ~~Propogation~~ means E field direction.

Note :-
Polarization direction is always E field orientation and direction

2. Free Space

$$E = 50 \sin(10^7 t + \beta z)\, a_y$$

$\rightarrow$ $\beta = \frac{10^7}{3 \times 10^8} = \frac{1}{30} = \frac{2\pi}{\lambda}$

$\Rightarrow \lambda = 60\pi = 188m$

$\rightarrow$ $k = \beta = \frac{1}{30} = 0.033$

$\rightarrow$ Free space, no attenuation

13. $H = 0.5\, e^{-0.1x} \sin(10^6 t - 2x)\, a_z$

$$V_p = \frac{10^6}{2} = 5 \times 10^5$$

$$\gamma = 0.1 + j2$$

direction $\longrightarrow$ $+a_x$

14. $\sigma = 0 \quad \epsilon_R = 81$

$$\omega = 6\pi \times 10^8, \quad \beta = ?$$

$$V = \frac{3 \times 10^8}{\sqrt{81}} = \frac{6\pi \times 10^8}{\beta}$$

$$\Rightarrow \beta = 18\pi$$

15. $\sigma = 0 \quad \mu_R = 2 \quad \epsilon_R = 8$

$$\eta = \sqrt{\frac{\mu_0 \mu_R}{\epsilon_0 \epsilon_R}} = 120\pi \sqrt{\frac{2}{8}} \doteq 188\,\Omega$$

16. $H = 0.1 \cos(\omega t - \beta z)\, a_x$

$$E = (0.1 \times 377) \cos(\omega t - \beta z)\,(a_x \times a_z)$$

$$\doteq -37.7 \cos(\omega t - \beta z)\, a_y$$

Ans – D

17. $\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_R} = \frac{2\pi}{\lambda}$

$$\lambda \propto \frac{1}{\sqrt{\epsilon}}$$

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{\epsilon_0 \epsilon_R}{\epsilon_0}} = \frac{2}{1} \Rightarrow \epsilon_R = 4$$

18. $E = 20\, e^{-j5z} a_x - 25\, e^{-j5z} a_y$

$$H = \left(\frac{20}{120\pi}\right) e^{-j5z} a_y - 25\, e^{-j5z} (-a_x)$$

$$= \left(\frac{25}{120\pi}\right) e^{-j5z} a_x + \left(\frac{20}{120\pi}\right) e^{-j5z} a_y$$

19.

$$\beta = \frac{\theta}{z} = \frac{2\pi}{\lambda}$$

3mm

90

$$= \frac{\pi/2}{3\,mm} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = 12\,mm$$

$$\lambda f = 12\times 10^{-3} \times 10 \times 10^{9} = \frac{3\times 10^{8}}{\sqrt{\epsilon_R}}$$

$$\Rightarrow \epsilon_R = 6.25$$

20.

$$E = A\cos\left(\omega t - \frac{\omega}{c}z\right)a_y$$

$$\frac{\omega}{c} = \beta$$

$$H = \left[\frac{A}{\sqrt{\mu/\epsilon}}\right]\cos\left(\omega t - \frac{\omega}{c}z\right)(-a_x)$$

$$= -A\sqrt{\frac{\epsilon}{\mu}}\cos\left(\omega t - \frac{\omega}{c}z\right)a_x$$

Ans-D

21.

$$\nabla^2 E + K^2 E = 0$$

$$\nabla^2 E = \gamma^2 E \qquad D^2 m^2 = 0$$

$$e^{-\gamma z} + e^{\gamma z} \longleftarrow \nabla^2 E - \gamma^2 E = 0$$

if $\gamma = j\beta$

$$e^{-j\beta z} + e^{j\beta z} \longleftarrow \nabla^2 E + \beta^2 E = 0$$

$(+z)$ $\qquad (D^2 + m^2 = 0)$

Ans-C

22.

$$E(x, z, t)_y = 25\sin(\omega t - 3x + 4z)\,a_y$$

Unit Prop. direction $= \dfrac{3a_x - 4a_z}{5}$

$$\beta = \sqrt{\beta_x^2 + \beta_z^2} = 5 = \frac{2\pi}{\lambda}$$

Note:-

$$\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial z^2} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(z,t) = E_0 e^{j\omega t} \cdot e^{-j\beta z}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} - \mu \epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$E(x,z,t) = E_0 e^{j\omega t} \cdot e^{-j\beta_x \cdot x} \cdot e^{-j\beta_z \cdot z}$$

It is a product solution of time, x & z harmonics

$$E(x,z,t) = 25 \sin(\omega t - 3x + 4z)\, a_y$$

$$H(x,z,t) = \left(\frac{25}{120\pi}\right) \sin(\omega t - 3x + 4z) \cdot \frac{(3a_x - 4a_z)}{5} \times a_y$$

$$= \left(\frac{25}{120\pi}\right) \sin(\omega t - 3x + 4z) \left(\frac{3a_z + 4a_x}{5}\right)$$

$E(x,z,t)_y$

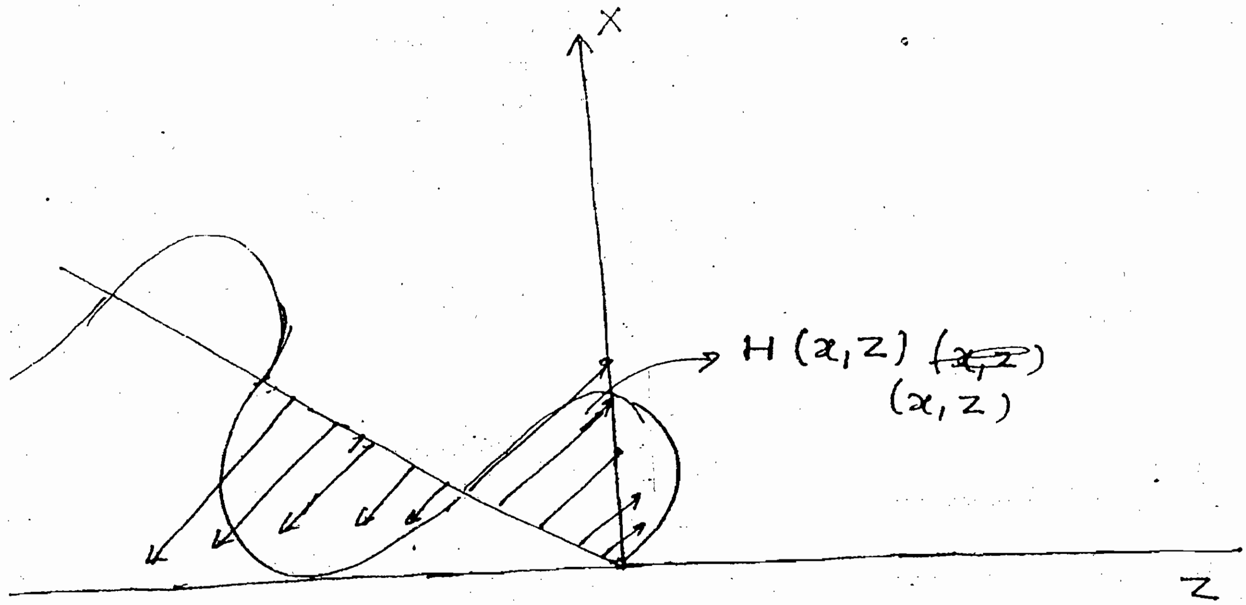
$H(x,z,t)\,(x,z)$

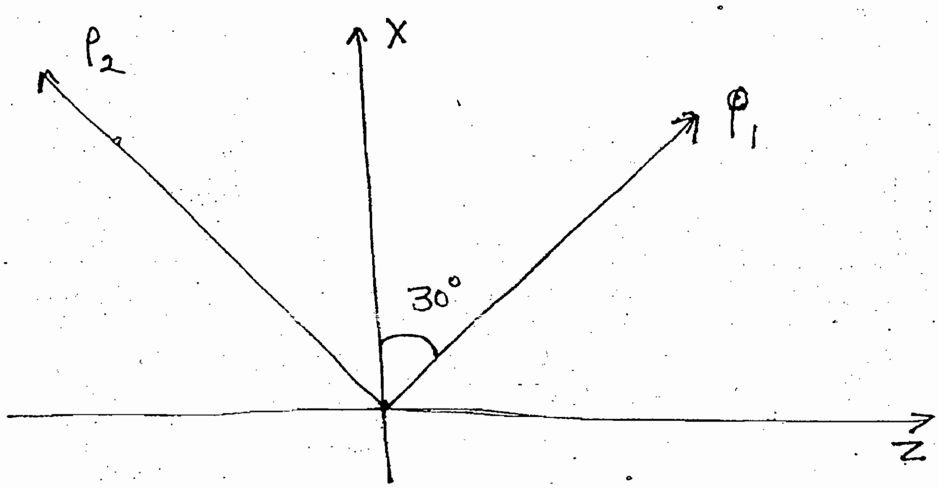
Prop. direction $\rightarrow 3a_x - 4a_z$

H direction $\rightarrow 4a_x + 3a_z$

E direction $\rightarrow a_y$ ($\because$ dot product $= 0$)

$\boxed{E \perp H \perp P}$ $\rightarrow$ Basic Transverse Nature





90° to Y-axis ⇒ ZX Plane

30° to the X-axis

$$E(x, z, t) = E_0 e^{j\omega t} \cdot e^{-j\beta_x \cdot x} \cdot e^{\pm j \beta_z \cdot z}$$

$$\tan 30 = \frac{\beta_z}{\beta_x} = \frac{1}{\sqrt{3}}$$

$$\beta_x = \sqrt{3}\,\beta_z$$

$$\beta = \frac{2\pi}{\lambda} = \sqrt{\beta_x^2 + \beta_z^2} = \sqrt{(\sqrt{3}\,\beta_z)^2 + \beta_z^2}$$

$$\Rightarrow \beta_z = \frac{\pi}{\lambda} \quad \beta_x = \frac{\sqrt{3}\,\pi}{\lambda}$$

Ans - (a)

24.

$$\beta = 280\pi = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{1}{140}$$

$$f = 14 \times 10^9$$

$$\lambda f = 10^8 = \frac{3 \times 10^8}{\sqrt{\epsilon_R}} \Rightarrow \epsilon_R = 9$$

$$\frac{E_p}{3} = \frac{120\pi}{\sqrt{9}} \Rightarrow E_p = 120\pi \text{, Ans}$$