Trigonometric Functions

Q1 :

Find the radian measures corresponding to the following degree measures:

(i) 25° (ii) - 47° 30' (iii) 240° (iv) 520°

Answer :

(i) 25°

We know that 180° = π radian

$$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) â€"47° 30'

$$â€"47° 30' = \frac{-47\frac{1}{2}}{} \text{ degree } [1° = 60']$$

$$= \frac{-95}{2} \text{ degree}$$

Since 180° = π radian

$$\frac{-95}{2} \text{ deg ree} = \frac{\pi}{180} \times \left(\frac{-95}{2} \right) \text{ radian} = \left(\frac{-19}{36 \times 2} \right) \pi \text{ radian} = \frac{-19}{72} \pi \text{ radian}$$

$$\therefore -47° 30' = \frac{-19}{72} \pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3} \pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Q2 :

Find the degree measures corresponding to the following radian measures

$$\left(\text{Use } \pi = \frac{22}{7}\right).$$

(i) $\dfrac{11}{16}$ (ii) – 4 (iii) $\dfrac{5\pi}{3}$ (iv) $\dfrac{7\pi}{6}$

Answer :

(i) $\dfrac{11}{16}$

We know that π radian = 180°

$$\therefore \frac{11}{16} \text{ radain} = \frac{180}{\pi} \times \frac{11}{16} \text{ deg ree} = \frac{45 \times 11}{\pi \times 4} \text{deg ree}$$

$$= \frac{45 \times 11 \times 7}{22 \times 4} \text{ deg ree} = \frac{315}{8} \text{ deg ree}$$

$$= 39\frac{3}{8} \text{ deg ree}$$

$$= 39° + \frac{3 \times 60}{8} \text{ min utes} \qquad \left[1° = 60'\right]$$

$$= 39° + 22' + \frac{1}{2} \text{ min utes}$$

$$= 39°22'30" \qquad \left[1' = 60"\right]$$

(ii) – 4

We know that π radian = 180°

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ deg ree} = \frac{180 \times 7(-4)}{22} \text{ deg ree}$$

$$= \frac{-2520}{11} \text{ deg ree} = -229\frac{1}{11} \text{ deg ree}$$

$$= -229° + \frac{1 \times 60}{11} \text{ min utes} \qquad \left[1° = 60'\right]$$

$$= -229° + 5' + \frac{5}{11} \text{ min utes}$$

$$= -229°5'27" \qquad \left[1' = 60"\right]$$

(iii) $\dfrac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3} \text{radian} = \frac{180}{\pi} \times \frac{5\pi}{3} \text{ deg ree} = 300°$$

(iv) $\dfrac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6} \text{radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

Q3 :

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer :

Number of revolutions made by the wheel in 1 minute = 360

∴Number of revolutions made by the wheel in 1 second = $\dfrac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e.,

12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Q4 :

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm $\left(\text{Use } \pi = \dfrac{22}{7} \right)$.

Answer :

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, for r = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100} \text{ radian} = \frac{180}{\pi} \times \frac{22}{100} \text{ deg ree} = \frac{180 \times 7 \times 22}{22 \times 100} \text{ deg ree}$$

$$= \frac{126}{10} \text{ deg ree} = 12\frac{3}{5} \text{ deg ree} = 12°36' \qquad \left[1° = 60' \right]$$

Thus, the required angle is 12°36"².
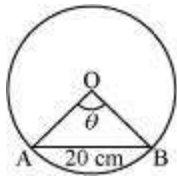

Q5 :

In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Answer :

Diameter of the circle = 40 cm

∴Radius (r) of the circle = $\dfrac{40}{2}$ cm $= 20$ cm

Let AB be a chord (length = 20 cm) of the circle.

In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

∴θ = 60° = $\dfrac{\pi}{3}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \dfrac{l}{r}$.

$$\dfrac{\pi}{3} = \dfrac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \dfrac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\dfrac{20\pi}{3}$ cm.

Q6 :

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer :

Let the radii of the two circles be r_1 and r_2. Let an arc of length l subtend an angle of 60° at the centre of the circle of radiusr_1, while let an arc of length l subtend an angle of 75° at the centre of the circle of radius r_2.

Now, $60° = \dfrac{\pi}{3}$ radian and $75° = \dfrac{5\pi}{12}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then

$\theta = \dfrac{l}{r}$ or $l = r\theta$.

$\therefore l = \dfrac{r_1\pi}{3}$ and $l = \dfrac{r_2 5\pi}{12}$

$\Rightarrow \dfrac{r_1\pi}{3} = \dfrac{r_2 5\pi}{12}$

$\Rightarrow r_1 = \dfrac{r_2 5}{4}$

$\Rightarrow \dfrac{r_1}{r_2} = \dfrac{5}{4}$

Thus, the ratio of the radii is 5:4.

Q7 :

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Answer :

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \dfrac{l}{r}$.

It is given that $r = 75$ cm

(i) Here, $l = 10$ cm

$\theta = \dfrac{10}{75}$ radian $= \dfrac{2}{15}$ radian

(ii) Here, $l = 15$ cm

$\theta = \dfrac{15}{75}$ radian $= \dfrac{1}{5}$ radian

(iii) Here, $l = 21$ cm

$\theta = \dfrac{21}{75}$ radian $= \dfrac{7}{25}$ radian

Q1 :

Find the values of other five trigonometric functions if $\cos x = -\dfrac{1}{2}$, x lies in third quadrant.

Answer :

$$\cos x = -\frac{1}{2}$$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\dfrac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$$

Since x lies in the 3rd quadrant, the value of sin x will be negative.

$$\therefore \sin x = -\frac{\sqrt{3}}{2}$$

$$\cos ecx = \frac{1}{\sin x} = \frac{1}{\left(-\dfrac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\dfrac{\sqrt{3}}{2}\right)}{\left(-\dfrac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

Q2 :

Find the values of other five trigonometric functions if $\sin x = \dfrac{3}{5}$**, x lies in second quadrant.**

Answer :

$$\sin x = \frac{3}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\dfrac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \cos^2 x = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm\frac{4}{5}$$

Since x lies in the 2nd quadrant, the value of cos x will be negative

$$\therefore \cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\dfrac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\dfrac{3}{5}\right)}{\left(-\dfrac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

Q3 :

Find the values of other five trigonometric functions if $\cot x = \dfrac{3}{4}$**, x lies in third quadrant.**

Answer :

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\dfrac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm\frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of sec x will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\dfrac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\dfrac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\csc x = \frac{1}{\sin x} = -\frac{5}{4}$$

Q4 :

Find the values of other five trigonometric functions if $\sec x = \dfrac{13}{5}$, x lies in fourth quadrant.

Answer :

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since x lies in the 4th quadrant, the value of sin x will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\csc x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

Q5 :

$$\tan x = -\frac{5}{12}$$

Find the values of other five trigonometric functions if , *x* **lies in second quadrant.**

Answer :

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\dfrac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since x lies in the 2nd quadrant, the value of sec x will be negative.

$$\therefore \sec x = \frac{-\dfrac{13}{12}}{}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\dfrac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\dfrac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\dfrac{5}{13}\right)} = \frac{13}{5}$$

Q6 :

Find the value of the trigonometric function sin 765°

Answer :

It is known that the values of sin x repeat after an interval of 2π or 360°.

$$\therefore \sin 765° = \sin\left(2 \times 360° + 45°\right) = \sin 45° = \frac{1}{\sqrt{2}}$$

Q7 :

Find the value of the trigonometric function cosec (-1410°)

Answer :

It is known that the values of cosec x repeat after an interval of 2π or 360°.

$$\therefore \cosec(-1410°) = \cosec(-1410° + 4 \times 360°)$$
$$= \cosec(-1410° + 1440°)$$
$$= \cosec 30° = 2$$

Q8 :

Find the value of the trigonometric function $\tan \dfrac{19\pi}{3}$

Answer :

It isknown that the values of tan x repeat after an interval of π or 180°.

$$\therefore \tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \tan 60° = \sqrt{3}$$

Q9 :

Find the value of the trigonometric function $\sin\left(-\dfrac{11\pi}{3}\right)$

Answer :

It is known that the values of sin x repeat after an interval of 2π or 360°.

$$\therefore \sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Q10 :

Find the value of the trigonometric function $\cot\left(-\dfrac{15\pi}{4}\right)$

Answer :

It is known that the values of cot x repeat after an interval of π or 180°.

$$\therefore \cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right) = \cot\frac{\pi}{4} = 1$$

Exercise 3.3 : Solutions of Questions on Page Number : 73

Q1 :

$$\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$$

Answer :

L.H.S. = $\sin^2\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{3} - \tan^2\dfrac{\pi}{4}$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$$

$$= \text{R.H.S.}$$

Q2 :

Prove that $2\sin^2\dfrac{\pi}{6} + \cos ec^2\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3} = \dfrac{3}{2}$

Answer :

L.H.S. = $2\sin^2\dfrac{\pi}{6} + \cos ec^2\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3}$

$$= 2\left(\frac{1}{2}\right)^2 + \cos ec^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$= 2 \times \frac{1}{4} + \left(-\cos ec\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2}$$

$$= \text{R.H.S.}$$

Q3 :

Prove that $\cot^2\frac{\pi}{6} + \cos ec\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Answer :

L.H.S. = $\cot^2\frac{\pi}{6} + \cos ec\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$$= \left(\sqrt{3}\right)^2 + \cos ec\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \cos ec\frac{\pi}{6} + 3 \times \frac{1}{3}$$

$$= 3 + 2 + 1 = 6$$

$$= \text{R.H.S}$$

Q4 :

Prove that $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

Answer :

L.H.S = $2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$$= 2\left\{\sin\left(\pi - \frac{\pi}{4}\right)\right\}^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\left\{\sin\frac{\pi}{4}\right\}^2 + 2 \times \frac{1}{2} + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8$$

$$= 1 + 1 + 8$$

$$= 10$$

$$= \text{R.H.S}$$

Q5 :

Find the value of:

(i) sin 75°

(ii) tan 15°

Answer :

(i) sin 75° = sin (45° + 30°)

= sin 45° cos 30° + cos 45° sin 30°

[sin (x + y) = sin x cos y + cos x sin y]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) tan 15° = tan (45° â€" 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \right]$$

$$= \frac{1 - \dfrac{1}{\sqrt{3}}}{1 + 1\left(\dfrac{1}{\sqrt{3}}\right)} = \frac{\dfrac{\sqrt{3}-1}{\sqrt{3}}}{\dfrac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$$

$$= \frac{4 - 2\sqrt{3}}{3 - 1} = 2 - \sqrt{3}$$

Q6 :

Prove that: $\cos\left(\dfrac{\pi}{4} - x\right)\cos\left(\dfrac{\pi}{4} - y\right) - \sin\left(\dfrac{\pi}{4} - x\right)\sin\left(\dfrac{\pi}{4} - y\right) = \sin(x+y)$

Answer :

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$\begin{bmatrix} \because\ 2\cos A \cos B = \cos(A+B) + \cos(A-B) \\ -2\sin A \sin B = \cos(A+B) - \cos(A-B) \end{bmatrix}$$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left[\frac{\pi}{2} - (x+y)\right]$$

$$= \sin(x+y)$$

$$= R.H.S$$

Q7 :

Prove that:
$$\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$$

Answer :

It is known that
$$\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}\ \text{ and }\ \tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$$

∴L.H.S. =
$$\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\dfrac{\left(\dfrac{\tan\dfrac{\pi}{4}+\tan x}{1-\tan\dfrac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\dfrac{\pi}{4}-\tan x}{1+\tan\dfrac{\pi}{4}\tan x}\right)}=\dfrac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$$

Q8 :

Prove that
$$\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}=\cot^2 x$$

Answer :

$$\text{L.H.S.}=\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}$$
$$=\dfrac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$
$$=\dfrac{-\cos^2 x}{-\sin^2 x}$$
$$=\cot^2 x$$
$$=\text{R.H.S.}$$

Q9 :

$$\cos\left(\frac{3\pi}{2}+x\right)\cos\left(2\pi+x\right)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot\left(2\pi+x\right)\right]=1$$

Answer :

L.H.S. = $\cos\left(\dfrac{3\pi}{2}+x\right)\cos\left(2\pi+x\right)\left[\cot\left(\dfrac{3\pi}{2}-x\right)+\cot\left(2\pi+x\right)\right]$

$= \sin x \cos x \left[\tan x + \cot x\right]$

$= \sin x \cos x \left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)$

$= (\sin x \cos x)\left[\dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right]$

$= 1 =$ R.H.S.

Q10 :

Prove that sin (*n* + 1)x sin (*n* + 2)x + cos (*n* + 1)x cos (*n* + 2)x = cos x

Answer :

L.H.S. = sin (*n* + 1)x sin(*n* + 2)x + cos (*n* + 1)x cos(*n* + 2)x

$= \dfrac{1}{2}\left[2\sin\left(n+1\right)x\sin\left(n+2\right)x + 2\cos\left(n+1\right)x\cos\left(n+2\right)x\right]$

$= \dfrac{1}{2}\left[\begin{array}{l}\cos\{(n+1)x-(n+2)x\}-\cos\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{array}\right]$

$\left[\begin{array}{l}\because -2\sin A\sin B = \cos\left(A+B\right)-\cos\left(A-B\right)\\2\cos A\cos B = \cos\left(A+B\right)+\cos\left(A-B\right)\end{array}\right]$

$= \dfrac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$

$= \cos(-x) = \cos x =$ R.H.S.

Q11 :

Prove that $$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$$

Answer :

It is known that $$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right).$$

$\therefore$ L.H.S. = $$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

$$=-2\sin\left\{\frac{\left[\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)\right]}{2}\right\}.\sin\left\{\frac{\left[\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)\right]}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$=-2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$=-2\sin\frac{\pi}{4}\sin x$$

$$=-2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$=-\sqrt{2}\sin x$$

= R.H.S.

Q12 :

Prove that sin² 6x - sin² 4x = sin 2x sin 10x

Answer :

It is known that $$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$ L.H.S. = sin²6x – sin²4x

= (sin 6x + sin 4x) (sin 6x – sin 4x)

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

= (2 sin 5x cos x) (2 cos 5x sin x)

= (2 sin 5x cos 5x) (2 sin x cos x)

= sin 10x sin 2x

= R.H.S.

Q13 :

Prove that cos² 2x - cos² 6x = sin 4x sin 8x

Answer :

It is known
that
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S. = cos² 2x – cos² 6x

= (cos 2x + cos 6x) (cos 2x – 6x)

$$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\frac{(2x-6x)}{2}\right]$$

$$= \left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$$

= [2 cos 4x cos 2x] [–2 sin 4x (–sin 2x)]

= (2 sin 4x cos 4x) (2 sin 2x cos 2x)

= sin 8x sin 4x

= R.H.S.

Q14 :

Prove that sin 2x + 2sin 4x + sin 6x = 4cos² x sin 4x

Answer :

L.H.S. = sin 2x + 2 sin 4x + sin 6x

= [sin 2x + sin 6x] + 2 sin 4x

$$= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

= 2 sin 4x cos (– 2x) + 2 sin 4x

= 2 sin 4x cos 2x + 2 sin 4x

= 2 sin 4x (cos 2x + 1)

= 2 sin 4x (2 cos² x – 1 + 1)

= 2 sin 4x (2 cos² x)

= 4cos² x sin 4x

= R.H.S.

Q15 :

Prove that cot 4x (sin 5x + sin 3x) = cot x (sin 5x - sin 3x)

Answer :

L.H.S = cot 4x (sin 5x + sin 3x)

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

= 2 cos 4x cos x

R.H.S. = cot x (sin 5x – sin 3x)

$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{\cos x}{\sin x}\left[2\cos 4x\sin x\right]$$

= 2 cos 4x. cos x

L.H.S. = R.H.S.

Q16 :

Prove that $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

Answer :

It is known that

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

∴L.H.S = $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\left(\dfrac{9x+5x}{2}\right).\sin\left(\dfrac{9x-5x}{2}\right)}{2\cos\left(\dfrac{17x+3x}{2}\right).\sin\left(\dfrac{17x-3x}{2}\right)}$$

$$= \frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$= -\frac{\sin 2x}{\cos 10x}$$

= R.H.S.

Q17 :

Prove that $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Answer :

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \frac{2\sin\left(\dfrac{5x+3x}{2}\right).\cos\left(\dfrac{5x-3x}{2}\right)}{2\cos\left(\dfrac{5x+3x}{2}\right).\cos\left(\dfrac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$= \frac{\sin 4x}{\cos 4x}$$

= tan 4x = R.H.S.

Q18 :

Prove that $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan \dfrac{x-y}{2}$

Answer :

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\dfrac{x+y}{2}\right).\sin\left(\dfrac{x-y}{2}\right)}{2\cos\left(\dfrac{x+y}{2}\right).\cos\left(\dfrac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\dfrac{x-y}{2}\right)}{\cos\left(\dfrac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Q19 :

Prove that $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Answer :

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

∴L.H.S. = $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$$= \frac{2\sin\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}{2\cos\left(\dfrac{x+3x}{2}\right)\cos\left(\dfrac{x-3x}{2}\right)}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= \text{R.H.S}$$

Q20 :

Prove that $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

Answer :

It is known that

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \quad \cos^2 A - \sin^2 A = \cos 2A$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\cos\left(\dfrac{x+3x}{2}\right)\sin\left(\dfrac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2\times(-\sin x)$$

$$= 2\sin x = \text{R.H.S.}$$

Q21 :

Prove that $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Answer :

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\dfrac{4x + 2x}{2}\right)\cos\left(\dfrac{4x - 2x}{2}\right) + \cos 3x}{2\sin\left(\dfrac{4x + 2x}{2}\right)\cos\left(\dfrac{4x - 2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right), \quad \sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x (2\cos x + 1)}{\sin 3x (2\cos x + 1)}$$

$$= \cot 3x = \text{R.H.S.}$$

Q22 :

Prove that cot *x* cot 2*x* - cot 2*x* cot 3*x* - cot 3*x* cot *x* = 1

Answer :

L.H.S. = cot *x* cot 2*x* – cot 2*x* cot 3*x* – cot 3*x* cot *x*

= cot *x* cot 2*x* – cot 3*x* (cot 2*x* + cot *x*)

= cot *x* cot 2*x* – cot (2*x* + *x*) (cot 2*x* + cot *x*)

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

$$\left[\because \cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

= cot *x* cot 2*x* – (cot 2*x* cot *x* – 1)

= 1 = R.H.S.

Q23 :

Prove that $\tan 4x = \dfrac{4\tan x \left(1 - \tan^2 x\right)}{1 - 6\tan^2 x + \tan^4 x}$

Answer :

It is known that $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$.

$\therefore$ L.H.S. $= \tan 4x = \tan 2(2x)$

$= \dfrac{2 \tan 2x}{1 - \tan^2(2x)}$

$= \dfrac{2\left(\dfrac{2 \tan x}{1 - \tan^2 x}\right)}{1 - \left(\dfrac{2 \tan x}{1 - \tan^2 x}\right)^2}$

$= \dfrac{\left(\dfrac{4 \tan x}{1 - \tan^2 x}\right)}{\left[1 - \dfrac{4 \tan^2 x}{\left(1 - \tan^2 x\right)^2}\right]}$

$= \dfrac{\left(\dfrac{4 \tan x}{1 - \tan^2 x}\right)}{\left[\dfrac{\left(1 - \tan^2 x\right)^2 - 4 \tan^2 x}{\left(1 - \tan^2 x\right)^2}\right]}$

$= \dfrac{4 \tan x \left(1 - \tan^2 x\right)}{\left(1 - \tan^2 x\right)^2 - 4 \tan^2 x}$

$= \dfrac{4 \tan x \left(1 - \tan^2 x\right)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x}$

$= \dfrac{4 \tan x \left(1 - \tan^2 x\right)}{1 - 6 \tan^2 x + \tan^4 x} = $ R.H.S.

Q24 :

Prove that cos 4x = 1 - 8sin² x cos² x

Answer :

L.H.S. = cos 4x

= cos 2(2x)

= 1 - 2 sin² 2x [cos 2A = 1 - 2 sin² A]

= 1 - 2(2 sin x cos x)² [sin2A = 2sin A cosA]

= 1 - 8 sin²x cos²x

= R.H.S.

Q25 :

Prove that: cos 6x = 32 cos⁶ x - 48 cos⁴ x + 18 cos² x - 1

Answer :

L.H.S. = cos 6x

= cos 3(2x)

= 4 cos³ 2x - 3 cos 2x [cos 3A = 4 cos³ A - 3 cos A]

= 4 [(2 cos² x - 1)³ - 3 (2 cos² x - 1) [cos 2x = 2 cos² x - 1]

= 4 [(2 cos² x)³ - (1)³ - 3 (2 cos² x)² + 3 (2 cos² x)] - 6cos² x + 3

= 4 [8cos⁶x - 1 - 12 cos⁴x + 6 cos²x] - 6 cos²x + 3

= 32 cos⁶x - 4 - 48 cos⁴x + 24 cos² x - 6 cos²x + 3

= 32 cos⁶x - 48 cos⁴x + 18 cos²x - 1

= R.H.S.

Exercise 3.4 : Solutions of Questions on Page Number : **78**

Q1 :

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

Answer :

$\tan x = \sqrt{3}$

It is known that $\tan\dfrac{\pi}{3} = \sqrt{3}$ and $\tan\left(\dfrac{4\pi}{3}\right) = \tan\left(\pi + \dfrac{\pi}{3}\right) = \tan\dfrac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \dfrac{\pi}{3}$ and $\dfrac{4\pi}{3}$.

Now, $\tan x = \tan\dfrac{\pi}{3}$

$\Rightarrow x = n\pi + \dfrac{\pi}{3}$, where n ∈ Z

Therefore, the general solution is $x = n\pi + \dfrac{\pi}{3}$, where n ∈ Z

Q2 :

Find the principal and general solutions of the equation $\sec x = 2$

Answer :

$\sec x = 2$

It is known that $\sec \dfrac{\pi}{3} = 2$ and $\sec \dfrac{5\pi}{3} = \sec\left(2\pi - \dfrac{\pi}{3}\right) = \sec \dfrac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \dfrac{\pi}{3}$ and $\dfrac{5\pi}{3}$.

Now, $\sec x = \sec \dfrac{\pi}{3}$

$\Rightarrow \cos x = \cos \dfrac{\pi}{3}$ $\left[\sec x = \dfrac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \dfrac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \dfrac{\pi}{3}$, where $n \in Z$

Q3 :

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Answer :

$\cot x = -\sqrt{3}$

It is known that $\cot \dfrac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \dfrac{\pi}{6}\right) = -\cot \dfrac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \dfrac{\pi}{6}\right) = -\cot \dfrac{\pi}{6} = -\sqrt{3}$

i.e., $\cot \dfrac{5\pi}{6} = -\sqrt{3}$ and $\cot \dfrac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \dfrac{5\pi}{6}$ and $\dfrac{11\pi}{6}$..

Now, $\cot x = \cot \dfrac{5\pi}{6}$

$\Rightarrow \tan x = \tan \dfrac{5\pi}{6}$ $\left[\cot x = \dfrac{1}{\tan x} \right]$

$\Rightarrow x = n\pi + \dfrac{5\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \dfrac{5\pi}{6}$, where $n \in Z$

Q4 :

Find the general solution of cosec $x = -2$

Answer :

cosec x= â€"2

It is known that

$\operatorname{cosec} \dfrac{\pi}{6} = 2$

$\therefore \operatorname{cosec}\left(\pi + \dfrac{\pi}{6} \right) = -\operatorname{cosec}\dfrac{\pi}{6} = -2$ and $\operatorname{cosec}\left(2\pi - \dfrac{\pi}{6} \right) = -\operatorname{cosec}\dfrac{\pi}{6} = -2$

i.e., $\operatorname{cosec}\dfrac{7\pi}{6} = -2$ and $\operatorname{cosec}\dfrac{11\pi}{6} = -2$

Therefore, the principal solutions are $x = \dfrac{7\pi}{6}$ and $\dfrac{11\pi}{6}$.

Now, $\operatorname{cosec} x = \operatorname{cosec}\dfrac{7\pi}{6}$

$\Rightarrow \sin x = \sin \dfrac{7\pi}{6}$ $\left[\operatorname{cosec} x = \dfrac{1}{\sin x} \right]$

$\Rightarrow x = n\pi + (-1)^n \dfrac{7\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + (-1)^n \dfrac{7\pi}{6}$, where $n \in Z$

Q5 :

Find the general solution of the equation $\cos 4x = \cos 2x$

Answer :

$\cos 4x = \cos 2x$

$\Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow -2\sin\left(\dfrac{4x+2x}{2}\right)\sin\left(\dfrac{4x-2x}{2}\right) = 0$

$\left[\because \cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)\right]$

$\Rightarrow \sin 3x \sin x = 0$

$\Rightarrow \sin 3x = 0 \quad$ or $\quad \sin x = 0$

$\therefore 3x = n\pi \quad$ or $\quad x = n\pi,\ \text{where } n \in Z$

$\Rightarrow x = \dfrac{n\pi}{3} \quad$ or $\quad x = n\pi,\ \text{where } n \in Z$

Q6 :

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

Answer :

$\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\dfrac{3x+x}{2}\right)\cos\left(\dfrac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\right]$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0 \quad$ or $\quad 2\cos x - 1 = 0$

$\Rightarrow \cos 2x = 0 \quad$ or $\quad \cos x = \dfrac{1}{2}$

$\therefore 2x = (2n+1)\dfrac{\pi}{2} \quad$ or $\quad \cos x = \cos\dfrac{\pi}{3},\ \text{where } n \in Z$

$\Rightarrow x = (2n+1)\dfrac{\pi}{4} \quad$ or $\quad x = 2n\pi \pm \dfrac{\pi}{3},\ \text{where } n \in Z$

Q7 :

Find the general solution of the equation $\sin 2x + \cos x = 0$

Answer :

$$\sin 2x + \cos x = 0$$

$$\Rightarrow 2\sin x \cos x + \cos x = 0$$

$$\Rightarrow \cos x (2\sin x + 1) = 0$$

$$\Rightarrow \cos x = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$

Now, $\cos x = 0 \Rightarrow \cos x = (2n+1)\dfrac{\pi}{2}$, where $n \in Z$

$$2\sin x + 1 = 0$$

$$\Rightarrow \sin x = \dfrac{-1}{2} = -\sin\dfrac{\pi}{6} = \sin\left(\pi+\dfrac{\pi}{6}\right) = \sin\left(\pi+\dfrac{\pi}{6}\right) = \sin\dfrac{7\pi}{6}$$

$$\Rightarrow x = n\pi + (-1)^n\dfrac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $(2n+1)\dfrac{\pi}{2}$ or $n\pi + (-1)^n\dfrac{7\pi}{6}, n \in Z$.

Q8 :

Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

Answer :

$$\sec^2 2x = 1 - \tan 2x$$

$$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$$

$$\Rightarrow \tan^2 2x + \tan 2x = 0$$

$$\Rightarrow \tan 2x (\tan 2x + 1) = 0$$

$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$$

Now, $\tan 2x = 0$

$$\Rightarrow \tan 2x = \tan 0$$

$$\Rightarrow 2x = n\pi + 0, \text{ where } n \in Z$$

$$\Rightarrow x = \dfrac{n\pi}{2}, \text{ where } n \in Z$$

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1 = -\tan\dfrac{\pi}{4} = \tan\left(\pi - \dfrac{\pi}{4}\right) = \tan\dfrac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \dfrac{3\pi}{4}$, where $n \in Z$

$\Rightarrow x = \dfrac{n\pi}{2} + \dfrac{3\pi}{8}$, where $n \in Z$

Therefore, the general solution is $\dfrac{n\pi}{2}$ or $\dfrac{n\pi}{2} + \dfrac{3\pi}{8}$, $n \in Z$.

Q9 :

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

Answer :

$\sin x + \sin 3x + \sin 5x = 0$

$(\sin x + \sin 5x) + \sin 3x = 0$

$\Rightarrow \left[2\sin\left(\dfrac{x+5x}{2}\right)\cos\left(\dfrac{x-5x}{2}\right)\right] + \sin 3x = 0$ $\qquad \left[\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\right]$

$\Rightarrow 2\sin 3x\cos(-2x) + \sin 3x = 0$

$\Rightarrow 2\sin 3x\cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0 \quad$ or $\quad 2\cos 2x + 1 = 0$

Now, $\sin 3x = 0 \Rightarrow 3x = n\pi$, where $n \in Z$

i.e., $x = \dfrac{n\pi}{3}$, where $n \in Z$

$2\cos 2x + 1 = 0$

$\Rightarrow \cos 2x = \dfrac{-1}{2} = -\cos\dfrac{\pi}{3} = \cos\left(\pi - \dfrac{\pi}{3}\right)$

$\Rightarrow \cos 2x = \cos\dfrac{2\pi}{3}$

$\Rightarrow 2x = 2n\pi \pm \dfrac{2\pi}{3}$, where $n \in Z$

$\Rightarrow x = n\pi \pm \dfrac{\pi}{3}$, where $n \in Z$

$$\frac{n\pi}{3} \text{ or } n\pi \pm \frac{\pi}{3}, \ n \in Z$$

Therefore, the general solution is

Q1 :

Prove that: $2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13} = 0$

Answer :

L.H.S.

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13} - \frac{5\pi}{13}}{2}\right) \quad \left[\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13} + \frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13} - \frac{4\pi}{13}}{2}\right)\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$= 2\cos\frac{\pi}{13} \times 2 \times 0 \times \cos\frac{5\pi}{26}$$

$$= 0 = \text{R.H.S}$$

Q2 :

Prove that: (sin 3x + sin x) sin x + (cos 3x - cos x) cos x = 0

Answer :

L.H.S.

= (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$

$= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x)$

$= \cos(3x - x) - \cos 2x \qquad \left[\cos(A - B) = \cos A \cos B + \sin A \sin B\right]$

$= \cos 2x - \cos 2x$

$= 0$

= RH.S.

Q3 :

Prove that: $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \dfrac{x+y}{2}$

Answer :

L.H.S. = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$

$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)$

$= 1 + 1 + 2\cos(x + y) \qquad \left[\cos(A + B) = (\cos A \cos B - \sin A \sin B)\right]$

$= 2 + 2\cos(x + y)$

$= 2\left[1 + \cos(x + y)\right]$

$= 2\left[1 + 2\cos^2\left(\dfrac{x+y}{2}\right) - 1\right] \qquad \left[\cos 2A = 2\cos^2 A - 1\right]$

$= 4\cos^2\left(\dfrac{x+y}{2}\right) = $ R.H.S.

Q4 :

Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \dfrac{x-y}{2}$

Answer :

L.H.S. = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$= \left(\cos^2 x + \sin^2 x \right) + \left(\cos^2 y + \sin^2 y \right) - 2 \left[\cos x \cos y + \sin x \sin y \right]$$

$$= 1 + 1 - 2 \left[\cos(x-y) \right] \qquad \left[\cos(A-B) = \cos A \cos B + \sin A \sin B \right]$$

$$= 2 \left[1 - \cos(x-y) \right]$$

$$= 2 \left[1 - \left\{ 1 - 2\sin^2 \left(\frac{x-y}{2} \right) \right\} \right] \qquad \left[\cos 2A = 1 - 2\sin^2 A \right]$$

$$= 4\sin^2 \left(\frac{x-y}{2} \right) = \text{R.H.S.}$$

Q5 :

Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Answer :

It is known that $\sin A + \sin B = 2\sin \left(\dfrac{A+B}{2} \right) \cdot \cos \left(\dfrac{A-B}{2} \right)$.

$\therefore$L.H.S. $= \sin x + \sin 3x + \sin 5x + \sin 7x$

$$= \left(\sin x + \sin 5x \right) + \left(\sin 3x + \sin 7x \right)$$

$$= 2\sin \left(\frac{x+5x}{2} \right) \cdot \cos \left(\frac{x-5x}{2} \right) + 2\sin \left(\frac{3x+7x}{2} \right) \cos \left(\frac{3x-7x}{2} \right)$$

$$= 2\sin 3x \cos(-2x) + 2\sin 5x \ \cos(-2x)$$

$$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$$

$$= 2\cos 2x \left[\sin 3x + \sin 5x \right]$$

$$= 2\cos 2x \left[2\sin \left(\frac{3x+5x}{2} \right) \cdot \cos \left(\frac{3x-5x}{2} \right) \right]$$

$$= 2\cos 2x \left[2\sin 4x \cdot \cos(-x) \right]$$

$$= 4\cos 2x \sin 4x \cos x = \text{R.H.S.}$$

Q6 :

Prove that: $\dfrac{\left(\sin 7x + \sin 5x \right) + \left(\sin 9x + \sin 3x \right)}{\left(\cos 7x + \cos 5x \right) + \left(\cos 9x + \cos 3x \right)} = \tan 6x$

Answer :

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right).$$

L.H.S. = $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$

$$= \frac{\left[2\sin\left(\dfrac{7x+5x}{2}\right)\cdot\cos\left(\dfrac{7x-5x}{2}\right)\right] + \left[2\sin\left(\dfrac{9x+3x}{2}\right)\cdot\cos\left(\dfrac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\dfrac{7x+5x}{2}\right)\cdot\cos\left(\dfrac{7x-5x}{2}\right)\right] + \left[2\cos\left(\dfrac{9x+3x}{2}\right)\cdot\cos\left(\dfrac{9x-3x}{2}\right)\right]}$$

$$= \frac{[2\sin 6x \cdot \cos x] + [2\sin 6x \cdot \cos 3x]}{[2\cos 6x \cdot \cos x] + [2\cos 6x \cdot \cos 3x]}$$

$$= \frac{2\sin 6x[\cos x + \cos 3x]}{2\cos 6x[\cos x + \cos 3x]}$$

= tan 6*x*

= R.H.S.

Q7 :

Prove that: $\sin 3x + \sin 2x - \sin x = 4\sin x \cos\dfrac{x}{2}\cos\dfrac{3x}{2}$

Answer :

L.H.S. = $\sin 3x + \sin 2x - \sin x$

$$= \sin 3x + (\sin 2x - \sin x)$$

$$= \sin 3x + \left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right) \right] \qquad \left[\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \right]$$

$$= \sin 3x + \left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right) \right]$$

$$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2} \qquad \left[\sin 2A = 2\sin A \cdot \cos B \right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right) \right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{ \frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2} \right\}\cos\left\{ \frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2} \right\} \right] \qquad \left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \right]$$

$$= 2\cos\left(\frac{3x}{2}\right).2\sin x \cos\left(\frac{x}{2}\right)$$

$$= 4\sin x \cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right) = \text{R.H.S.}$$

Q8 :

$$\tan x = -\frac{4}{3} \text{, } x \text{ in quadrant II}$$

Answer :

Here, x is in quadrant II.

i.e., $\dfrac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\dfrac{x}{2}$, $\cos\dfrac{x}{2}$ and $\tan\dfrac{x}{2}$ are all positive.

It is given that $\tan x = -\dfrac{4}{3}$.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\dfrac{-4}{3}\right)^2 = 1 + \dfrac{16}{9} = \dfrac{25}{9}$$

$$\therefore \cos^2 x = \dfrac{9}{25}$$

$$\Rightarrow \cos x = \pm\dfrac{3}{5}$$

As x is in quadrant II, $\cos x$ is negative.

$$\therefore \cos x = \dfrac{-3}{5}$$

Now, $\cos x = 2\cos^2\dfrac{x}{2} - 1$

$$\Rightarrow \dfrac{-3}{5} = 2\cos^2\dfrac{x}{2} - 1$$

$$\Rightarrow 2\cos^2\dfrac{x}{2} = 1 - \dfrac{3}{5}$$

$$\Rightarrow 2\cos^2\dfrac{x}{2} = \dfrac{2}{5}$$

$$\Rightarrow \cos^2\dfrac{x}{2} = \dfrac{1}{5}$$

$$\Rightarrow \cos\dfrac{x}{2} = \dfrac{1}{\sqrt{5}} \qquad \left[\because \cos\dfrac{x}{2} \text{ is positive}\right]$$

$$\therefore \cos\dfrac{x}{2} = \dfrac{\sqrt{5}}{5}$$

$$\sin^2\dfrac{x}{2} + \cos^2\dfrac{x}{2} = 1$$

$$\Rightarrow \sin^2\dfrac{x}{2} + \left(\dfrac{1}{\sqrt{5}}\right)^2 = 1$$

$$\Rightarrow \sin^2\dfrac{x}{2} = 1 - \dfrac{1}{5} = \dfrac{4}{5}$$

$$\Rightarrow \sin\dfrac{x}{2} = \dfrac{2}{\sqrt{5}} \qquad \left[\because \sin\dfrac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\dfrac{x}{2} = \dfrac{2\sqrt{5}}{5}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{5}$, and 2.

Q9 :

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\cos x = -\frac{1}{3}$, x in quadrant III

Answer :

Here, x is in quadrant III.

i.e., $\pi < x < \frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative, whereas $\sin\frac{x}{2}$ is positive.

It is given that $\cos x = -\frac{1}{3}$.

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2} = \frac{1-\left(-\frac{1}{3}\right)}{2} = \frac{\left(1+\frac{1}{3}\right)}{2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

$$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \qquad \left[\because \cos\frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{6}}{3}$, $\frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$.

Q10 :

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\sin x = \frac{1}{4}$, *x* in quadrant II

Answer :

Here, *x* is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$, and $\tan\frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$.

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos x = -\frac{\sqrt{15}}{4} \qquad [\cos x \text{ is negative in quadrant II}]$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} = \frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4+\sqrt{15}}{8}$$

$$\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}} \qquad\qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4+\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4-\sqrt{15}}{8}$$

$$\Rightarrow \cos\frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}} \qquad\qquad \left[\because \cos\frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4-\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$= \sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

$$= \sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$$

Thus, the respective values of $\sin\dfrac{x}{2}$, $\cos\dfrac{x}{2}$ and $\tan\dfrac{x}{2}$ are $\dfrac{\sqrt{8+2\sqrt{15}}}{4}$, $\dfrac{\sqrt{8-2\sqrt{15}}}{4}$, and $4+\sqrt{15}$