



# ANSWERS

## Chapter 7 EXERCISE 7.1

- |                                    |   |
|------------------------------------|---|
| (1) (i) 21 m/s                     | (ii) 15 m/s and 27 m/s                                      |
| (2) (2)(i) 5 sec                   | (ii) 128 ft/s (iii) 160 ft/s                                |
| (3) (i) 1.2 sec                    | (ii) 34 m (iii) $-6 \text{ m/s}^2, 6 \text{ m/s}^2$         |
| (4) 75 units                       | (5) $\frac{1}{2} \text{ kg/m}, \frac{1}{6} \text{ kg/m}$    |
| (6) $20\pi \text{ sq.cm/s}$        | (7) $2\pi \text{ km/s}$ (8) $\frac{9}{10\pi} \text{ m/min}$ |
| (9) (i) $\frac{-8}{3} \text{ m/s}$ | (ii) 26.83 sq.m/sec (10) 70 km/hr.                          |

## EXERCISE 7.2

- |  |                                  |            |
|--|----------------------------------|------------|
| (1) (i) 7  | (ii) $\infty$                    | (2) (1, 0) |
| (3) (0, 3) and (4, -25)  | (4) (2, -1) and (-2, 1)          |            |
| (5) (i) $2x + y = 2; x - 2y = 1$   | (ii) $2x - y = -2; x + 2y = 4$   |            |
| (iii) $x - y = 0; x + y = \pi$   | (iv) $4x + 2y = 5; 2x - 4y = -5$ |            |
| (6) $12x - y = 15; 12x - y = -17$  | (7) $x + 2y = 7; x + 2y = -1$    |            |
| (8) $(2 \cos t)x + (7 \sin t)y = 14; (7 \sin t)x - (2 \cos t)y = 45 \sin t \cos t$ | (9) $\tan^{-1}(3)$               |            |

## EXERCISE 7.3

- |                                   |   |                        |
|-----------------------------------|---|------------------------|
| (1) (i) not continuous at $x = 0$ | (ii) not continuous at $x = \frac{\pi}{2}$                  | (iii) $f(2) \neq f(7)$ |
| (2) (i) $\frac{1}{2}$             | (ii) $-2 + 2\sqrt{2}$                                       | (iii) $\frac{9}{4}$    |
| (3) (i) not continuous at $x = 0$ | (ii) not differentiable at $x = \frac{-1}{3}$               |                        |
| (4) (i) $\pm 1$                   | (ii) 7  |                        |
| (6) 320 km                        | (8) No. Since $f'(x)$ cannot be 2.5 at any point in (0, 2). |                        |

## EXERCISE 7.4

- |  |   |
|--|---|
| (1) (i) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$                                  | (ii) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$            |
| (iii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$              |   |
| (iv) $\log(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$ |   |
| (v) $\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$             | (vi) $\cos^2 x = 1 - \frac{2x^2}{2!} + \frac{2^3 x^4}{4!} - \frac{2^5 x^6}{6!} + \dots$ |

(2)  $\log x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$

(3)  $\frac{\sqrt{2}}{2} \left( 1 + \frac{1}{1!} \left( x - \frac{\pi}{4} \right) - \frac{1}{2!} \left( x - \frac{\pi}{4} \right)^2 - \frac{1}{3!} \left( x - \frac{\pi}{4} \right)^3 + \dots \right)$

(4)  $f(x) = -(x-1) + (x-1)^2$



## EXERCISE 7.5

- (1)  $\frac{1}{2}$       (2) 2      (3)  $\infty$       (4) 1      (5) 0      (6) 0  
(7) -1      (8) 1      (9)  $e$       (10)  $\frac{1}{e}$       (11)  $\frac{1}{\sqrt{e}}$

## EXERCISE 7.6

- (1) (i) absolute maximum = -1, absolute minimum = -26  
(ii) absolute maximum = 16, absolute minimum = -1  
(iii) absolute maximum = 9, absolute minimum =  $-\frac{9}{8}$   
(iv) absolute maximum =  $\frac{3\sqrt{3}}{2}$ , absolute minimum = 0  
(2) (i) strictly increasing on  $(-\infty, -2)$  and  $(1, \infty)$ , strictly decreasing on  $(-2, 1)$   
local maximum = 20      local minimum = -7

- (ii) strictly decreasing on  $(-\infty, 5)$  and  $(5, \infty)$ . No local extremum.  
(iii) strictly increasing on  $(-\infty, \infty)$ . No local extremum.

(iv) strictly decreasing on  $(0, 1)$ , strictly increasing on  $(1, \infty)$ . local minimum =  $\frac{1}{3}$   
(v) strictly increasing on  $\left(0, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ , and  $\left(\frac{7\pi}{4}, 2\pi\right)$ .

strictly decreasing on  $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$  and  $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$ . local maximum =  $\frac{11}{2}$  at  $x = \frac{\pi}{4}, \frac{5\pi}{4}$ .  
local minimum =  $\frac{9}{2}$  at  $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ .

## EXERCISE 7.7

- (1) (i) concave upwards on  $(-\infty, 2)$  and  $(4, \infty)$ . Concave downwards on  $(2, 4)$

Points of inflection  $(2, -16)$  and  $(4, 0)$

(ii) concave upwards on  $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$ . Concave downwards on  $\left(0, \frac{3\pi}{4}\right)$  and  $\left(\frac{7\pi}{4}, 2\pi\right)$

Points of inflection  $\left(\frac{3\pi}{4}, 0\right)$  and  $\left(\frac{7\pi}{4}, 0\right)$

- (iii) concave upwards on  $(0, \infty)$ . Concave downward on  $(-\infty, 0)$

Points of inflection  $(0, 0)$

- (2) (i) local minimum = -2 ; local maximum = 2      (ii) local minimum =  $-\frac{1}{e}$

(iii) local minimum = 0 ; local maximum =  $\frac{1}{e^2}$

- (3) strictly increasing on  $(-\infty, -1)$  and  $\left(\frac{1}{2}, \infty\right)$ . strictly increasing on  $\left(-1, \frac{1}{2}\right)$

local maximum = 6 , local minimum =  $-\frac{3}{4}$

concave upwards on  $\left(-\infty, -\frac{1}{4}\right)$ ; concave downwards on  $\left(-\frac{1}{4}, \infty\right)$ .

point of inflection  $\left(-\frac{1}{4}, \frac{21}{8}\right)$



## EXERCISE 7.8

- (1) 36      (2)  $4\sqrt{5}$       (3) 50      (4)  $100\text{m}^2$       (5) 9cm, 6cm      (6) 1200m  
(7)  $20\sqrt{2}$ ,  $20\sqrt{2}$       (9)  $\sqrt{2}r, \frac{r}{\sqrt{2}}$       (10) 6cm, 6cm, 3cm      (11)  $32\pi, 0$

## EXERCISE 7.9

- (1) (i)  $x = -1, x = 1, y = 1$       (ii)  $x = -1, y = x - 1$ ,      (iii)  $y = -3, y = 3$   
(iv)  $y = -9x - 1, x = -3$       (v)  $y = \frac{1}{3}x + \frac{8}{3}, x = 2$

## EXERCISE 7.10

1	2	3	4	5	6	7	8	9	10
(1)	(2)	(2)	(2)	(1)	(2)	(3)	(4)	(3)	(4)
11	12	13	14	15	16	17	18	19	20
(3)	(4)	(3)	(4)	(2)	(3)	(3)	(1)	(4)	(3)

## Chapter 8

### Exercise 8.1

1. (i) 3.0074      2. (i) 24.73      (ii) 1.9688      (iii) 2.963  
3. (i)  $7x - 4$       (ii)  $\frac{9-4x}{5}$       (iii)  $\frac{x+1}{4}$   
4. (i)  $0.0225\pi \text{ cm}^2$ ,      (ii)  $0.006 \text{ cm}^2$       (iii) 0.6%  
5. (i) Volume decreases by  $80\pi \text{ cm}^3$       (ii) Surface area decreases by  $16\pi \text{ cm}^2$       6. 1%

### Exercise 8.2

1. (i)  $\frac{2(1-2x)^2(8x-7)}{(3-4x)^2} dx$  (ii)  $\frac{4}{3} \frac{\cos 2x}{(3+\sin 2x)^{\frac{1}{3}}} dx$  (iii)  $e^{x^2-5x+7} [(2x-5)\cos(x^2-1)-2x\sin(x^2-1)] dx$   
2. (i) 0.7      (ii) 0.18      3. (i)  $\Delta f = 2.125$ ,  $df = 2.0$  (ii)  $\Delta f = 0.83$ ,  $df = 0.1$   
4. 3.0013029      5. (i)  $\frac{6}{\pi} \text{ cm}$       (ii)  $\frac{40}{\pi}\%$       6.  $30\pi \text{ mm}^3$  7.  $0.4\pi \text{ mm}^2$       8. 8000  
9. (i)  $\approx 3$  words      (ii)  $\approx 1$  word      10.  $5.25\pi, 4.76\%$       11.  $60 \text{ cm}^3, 61.2 \text{ cm}^3$

### Exercise 8.3

1.  $\frac{1}{8}$       2. 1      4.  $\cos(1)$

### Exercise 8.4

1. (i) 27, -14      (ii) 11, -4      (iii) 2, 0, 4      (iv)  $e^2((\log 2)-1), e^2(1+\log 8)$   
3.  $\frac{x^2-y^2}{x^2y}, \frac{y^2-x^2}{y^2x} + 3z^2, 6yz$       4.  $\frac{3(x^2+y^2+z^2)}{(x^3+y^3+z^3)}$   
5. (i)  $e^y + 6x, 6y, xe^y, e^y + 6x$   
(ii)  $\frac{-15}{(5x+3y)^2}, \frac{-25}{(5x+3y)^2}, \frac{-9}{(5x+3y)^2}, \frac{-15}{(5x+3y)^2}$       (iii) 3,  $2 - 25\cos 5x, 0, 3$   
10. (i)  $72x + 84y + 0.04xy - 0.05x^2 - 0.05y^2 - 2000$       (ii) 24, -48, Keeping y constant and increasing x increases profit.



### Exercise 8.5

1.  $6x - 7y - 7$

2.  $-(x + 20y + 16)$

3.  $(2x - y)dx + \left(x + \frac{1}{2}y\right)dy$

4.  $5x - 2y + 3z - 6$

5.  $(y + z)dx + (x + z)dy + (y + x)dz$

### Exercise 8.6

1.  $e^t(2e^t \sin t + 3 \sin^4 t + e^t \cos t + 12 \sin^3 t \cos t), 1$

2.  $(1+e^{2t})^2 [\cos^3 t (1+e^{2t}) - \sin t \sin 2t (1+e^{2t}) + 6e^{2t} \sin t \cos^2 t]$

3.  $4e^{2t}$       4.  $-e^{-2t} [\sin 2t - \cos 2t]$       5.  $18e^{3s} - 3e^s \cos s + 3e^s \sin s - 4 \sin s \cos s, 15$

6.  $\frac{3e}{1+e^2} + 2 \tan^{-1} e, \frac{e}{1+e^2}$

7.  $te^{st^2} [t \sin(s^2 t) + 2s \cos(s^2 t)], \frac{du}{dt} = se^{st^2} [2t + \sin(s^2 t) + s \cos(s^2 t)], e[2 \sin(1) + \cos(1)]$

8.  $3s^3(e^{3t} + e^{-t}), 3s^2e^t(e^{2t} - 2s^2e^{-2t} - 3s^2)$       9.  $2u(1+2v), 2(u^2 - v), 3, \frac{-3}{2}$

### Exercise 8.7

1. (i) not homogeneous

(ii) Homogeneous, deg.3

(iii) homogeneous, deg.0

(iv) not homogeneous

6. 5

### Exercise 8.8

1	2	3	4	5	6	7	8
(2)	(2)	(2)	(4)	(3)	(2)	(4)	(2)
9	10	11	12	13	14	15	
(3)	(1)	(2)	(3)	(2)	(4)	(1)	

## Chapter 9

### Exercise 9.1

1. 0.6

2. 0.855

3. 0.375

### Exercise 9.2

1. (i)  $\frac{13}{2}$       (ii)  $\frac{25}{3}$

### Exercise 9.3

1. (i)  $\frac{1}{4} \log \frac{5}{3}$       (ii)  $\frac{\pi}{8}$       (iii)  $\frac{\pi}{2} - 1$       (iv)  $e^{\frac{\pi}{2}}$       (v)  $\frac{8}{21}$       (vi)  $\frac{1}{2}$   
2. (i) 0      (ii)  $\pi$       (iii)  $\frac{\pi-2}{4}$       (iv) 0      (v) 0      (vi)  $\frac{13}{10}$       (vii)  $\frac{\pi}{4}$   
(viii)  $\frac{\pi}{8} \log 2$       (ix)  $\frac{\pi}{2}(\pi-2)$       (x)  $\frac{\pi}{8}$       (xi)  $\frac{\pi^2}{2}$

### Exercise 9.4

1.  $\frac{3}{8} - \frac{19}{8}e^{-2}$

2.  $\frac{1}{\sqrt{2}} \left( \frac{\pi}{12} + \frac{1}{9} \right)$

3.  $1 + e^{\frac{\pi}{4}} \left[ \frac{\pi}{4} - 1 \right]$

4.  $-\frac{\pi}{4}$

### Exercise 9.5

1. (i)  $\frac{\pi}{2\sqrt{6}}$

(ii).  $\frac{\pi}{2\sqrt{30}}$





## Exercise 9.6

1. (i)  $\frac{63}{512}$  (ii)  $\frac{16}{105}$  (iii)  $\frac{5\pi}{64}$  (iv)  $\frac{8}{45}$  (v)  $\frac{\pi}{24}$  (vi)  $\frac{64}{35}$  (vii)  $\frac{1}{24}$  (viii)  $\frac{1}{60}$

## Exercise 9.7

1. (i)  $\frac{5!}{3^6}$  (ii) 29 (2)  $\frac{1}{8}$

## Exercise 9.8

1. 7.5 2. 2 3. 16 4. 36 5.  $2\sqrt{2}$  6.  $\log 2$  7.  $\frac{9}{2}$  8. yes,  $\frac{16}{3}$  9.  $\frac{4}{3}$  10.  $\frac{4}{3}(4\pi + \sqrt{3})$

## Exercise 9.9

1.  $\frac{4\pi}{5}$  2.  $\frac{\pi}{4}[1 - e^{-4}]$  3.  $8\pi$  4.  $\frac{\pi}{12}$  5.  $\frac{14}{3}\pi m^3$  6.  $\frac{1000}{3}\pi cm^3$

1	2	3	4	5	6	7	8	9	10
(1)	(3)	(3)	(1)	(4)	(3)	(3)	(1)	(2)	(1)
11	12	13	14	15	16	17	18	19	20
(4)	(2)	(2)	(4)	(2)	(4)	(3)	(4)	(2)	(1)

## Chapter 10

### Exercise 10.1

1. (i) 1,1 (ii) 3,2 (iii) 2, does not exist (iv) 1, 2 (v) 1,4  
(vi) 2,2 (vii) 2,6 (viii) 2, does not exist (ix) 3,1 (x) 1, doest not exist

### Exercise 10.2

1. (i)  $\frac{dQ}{dt} = kQ$  (ii)  $\frac{dP}{dt} = kP(500000 - P)$  (iii)  $\frac{dP}{dt} = \frac{kP}{T^2}$  (iv)  $\frac{dx}{dt} = \frac{2x}{25} + 400$  2.  $\frac{dr}{dt} = -k$

### Exercise 10.3

1. (i)  $\frac{d^2y}{dx^2} = 0$  (ii)  $\frac{d^2x}{dy^2} = 0$  2.  $r^2 \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right] = \left( x \frac{dy}{dx} - y \right)^2$   
3.  $x^2 + 2xy \frac{dy}{dx} - y^2 = 0$  4.  $2ay'' + y'^3 = 0$  5.  $xy' - 2y - 2 = 0$  6.  $xy'^2 + xyy'' - yy' = 0$   
7.  $\frac{d^2y}{dx^2} = 64y$  8.  $xy'' + 2y' + x^2 - xy - 2 = 0$

### Exercise 10.4

2. (i)  $m = -2$  (ii)  $m = 2, 3$  3.  $2y^2 = x + 48$

### Exercise 10.5

1.  $F = (F - kV)e^{\frac{kt}{M}}$  2.  $k^2 \left( 1 - e^{-\frac{2gx}{k^2}} \right) = v^2$  3.  $y = \frac{1-x}{1+x}$   
4. (i)  $\sin^{-1} y = \sin^{-1} x + C$  (ii)  $y \tan^{-1} x = C$  (iii)  $\sin \left( \frac{y-1}{x} \right) = a$  (iv)  $e^x + e^{-y} + \frac{x^4}{4} = C$   
(v)  $(e^y + 1) \sin x = C$  (vi)  $\sin \left( \frac{x}{y} \right) = e^{nx+c}$  (vii)  $3y = -(25 - x^2)^{\frac{3}{2}} + 3C$  (viii)  $\sin y = e^x \log x + C$



$$(ix) \sec y = 2 \sin x + C \quad (x) \frac{1}{2} [(x+y) + \sin(x+y)\cos(x+y)] = x + C$$

### Exercise 10.6

$$\begin{array}{lll} 1. \sin\left(\frac{y}{x}\right) = \log|Cx| & 2. y = Ce^{\frac{x^3}{3y^3}} & 3. e^{\frac{x}{y}} = \log|Cy| \\ 5. xy^2 - x^2y = C & 6. C = xe^{\tan\left(\frac{y}{x}\right)} & 7. y + 3xe^{\frac{y}{x}} = 3 \\ & & 8. x_0 = \pm\sqrt{3}e \end{array}$$

### Exercise 10.7

$$\begin{array}{lll} 1. y = \sin x + C \cos x & 2. y = \frac{\sin^{-1} x}{\sqrt{1-x^2}} + C(1-x^2)^{-\frac{1}{2}} & 3. (y + \cos x)x = \sin x + C \\ 4. y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + \frac{1}{2}\log|x+\sqrt{x^2+4}| + C & 5. xy^2 = 2y^5 + C & \\ 6. xy \sin x + \cos x = C & 7. ye^{\sin^{-1} x} = \frac{e^{2\sin^{-1} x}}{2} + C & 8. y\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = x + \frac{2}{3}x\sqrt{x} + C \\ 9. xy + \tan^{-1} y = C & 10. y \log x + \frac{\cos 2x}{2} = C & 11. 2y = (x+a)^4 + 2C(x+a)^2 \\ 12. y(1+x^3) = \frac{x}{2} - \frac{\sin 2x}{4} + C & 13. 4yx = 2x^2 \log x - x^2 + 4C & 14. x^2y = \frac{x^4}{4} \log x - \frac{x^4}{16} + C \\ 15. 2x^3y = x^2 + 3 & & \end{array}$$

### Exercise 10.8

- After 10 hours the number of bacteria as 9 times the original number of bacteria.
- $P = 300000\left(\frac{4}{3}\right)^{\frac{t}{40}}$
- $i = Ce^{-\frac{Rt}{L}}$
- $v = \frac{10}{e^2}$
- $P = 10000e^{0.075}$
- $\frac{9^{10}}{10^8}$  % of the radioactive element will remain after 1000 years.
- (i)  $65.33^\circ C$  (ii)  $53.46 \text{ mts}$
- (i)  $T \approx 151^\circ F$  (ii)  $t = 22.523$ . She drunk the coffee between 10.22 and 10.30 approximately.
- $11^\circ$
- $x = 100\left(1 - e^{-\frac{3t}{50}}\right)$

### Exercise 10.9

Q	1	2	3	4	5	6	7	8	9	10
A	(1)	(2)	(3)	(1)	(2)	(3)	(3)	(2)	(2)	(3)
Q	11	12	13	14	15	16	17	18	19	20
A	(3)	(3)	(1)	(1)	(2)	(3)	(2)	(4)	(2)	(4)
Q	21	22	23	24	25					
A	(1)	(1)	(2)	(2)	(1)					

## Chapter 11

### EXERCISE 11.1

(1)	Values of Random Variable	0	1	2	3	Total
	Number of points in inverse image	1	3	3	1	8



(2)	Values of Random Variable	0	1	2	Total
	Number of points in inverse image	650	26	650	1326
(3)	Values of Random Variable	0	1	2	3
	Number of points in inverse image	1	3	3	1
(4)	Values of Random Variable	0	15	30	Total
	Number of points in inverse image	1	2	1	4
(5)	Values of Random Variable	4	5	6	7
	Number of points in inverse image	1	2	3	2
					Total
					9.

## EXERCISE 11.2

$$(1) \quad f(x) = \begin{cases} \frac{1}{8} & \text{for } x = 0, 3 \\ \frac{3}{8} & \text{for } x = 1, 2 \end{cases}$$

(2) (i)

x	2	4	6	8	10	Total
f(x)	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{8}{25}$	$\frac{8}{25}$	$\frac{4}{25}$	1

$$(2) \text{ (ii)} \quad F(x) = \begin{cases} 0 & \text{for } x < 2 \\ \frac{1}{25} & \text{for } x \leq 2 \\ \frac{5}{25} & \text{for } x \leq 4 \\ \frac{13}{25} & \text{for } x \leq 6 \\ \frac{21}{25} & \text{for } x \leq 8 \\ 1 & \text{for } x \leq 10 \end{cases}$$

(iii)  $\frac{4}{5}$       (iv)  $\frac{24}{25}$

$$(3) \quad f(x) = \begin{cases} \frac{1}{4} & \text{for } x = 1, 3 \\ \frac{1}{16} & \text{for } x = 0, 4, \\ \frac{3}{8} & \text{for } x = 2 \end{cases}, \quad F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{16} & \text{for } x \leq 0 \\ \frac{5}{16} & \text{for } x \leq 1 \\ \frac{11}{16} & \text{for } x \leq 2 \\ \frac{15}{16} & \text{for } x \leq 3 \\ 1 & \text{for } x \leq 4 \end{cases}$$

$$(4) \text{ (i)} 8 \quad \text{(ii)} \quad F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{8} & \text{for } x \leq 0 \\ \frac{3}{8} & \text{for } x \leq 1 \\ \frac{1}{8} & \text{for } x \leq 2 \end{cases}$$

(iii)  $\frac{7}{8}$

(5) (i)

x	-1	0	1	2	3
f(x)	0.15	0.20	0.25	0.25	0.15

(ii)  $P(X < 1) = 0.35$     (iii)  $P(X \geq 2) = 0.40$



(6) (i)  $\frac{1}{6}$       (ii)  $\frac{17}{36}$       (iii)  $\frac{5}{36}$

(7) (a)

$x$	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

(b)  $\frac{4}{5}$       (c)  $\frac{2}{5}$

### EXERCISE 11.3

(1) 4

(2) (i) 0.16      (ii) 0.3      (iii) 0.75

(3) (i)  $\frac{1}{400}$       (ii)  $F(x) = \begin{cases} 0 & \text{for } x < 200 \\ \frac{x}{400} - \frac{1}{2} & \text{for } 200 \leq x \leq 600 \\ 0 & \text{for } x > 600 \end{cases}$

(iii)  $\frac{1}{2}$

(4) (i)  $\frac{1}{3}$       (ii)  $1 - e^{-\frac{x}{3}}$       (iii)  $1 - e^{-1}$       (iv)  $e^{-\frac{5}{3}} - e^{-\frac{x}{3}}$       (v)  $1 - e^{-\frac{4}{3}}$

(5) (i)  $F(x) = \begin{cases} 0 & x \leq -1 \\ \frac{x^2}{2} + x + \frac{1}{2} & -1 \leq x < 0 \\ -\frac{x^2}{2} + x & 0 \leq x < 1 \\ x - 1 & 1 \leq x \end{cases}$

(ii) 0.75

(6) (i)  $f(x) = F'(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(2x+1) & 0 \leq x < 1 \\ 0 & 1 \leq x \end{cases}$

(ii) 0.099

### EXERCISE 11.4

(1) (i) 2.3, 2.81      (ii) 1.67, 0.56      (iii)  $\frac{5}{3}, \frac{1}{18}$       (iv) 2, 8

(2)  $\frac{8}{7}$

$x$	0	1	2
$f(x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

(3) 7, 16

(4) 2, 1

$x$	0	1	2	3	4
$f(x)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

(5) 15 minutes      (6)  $\frac{1}{3}, \frac{2}{9}$       (7)  $\frac{1}{2}, \frac{1}{8}$       (8) Loss ₹. 0.50



## EXERCISE 11.5

- (1) (i)  $\frac{160}{729}$    (ii)  $210\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6$    (iii)  $\binom{9}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^2$    (2)   (i)  $\binom{10}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^6$    (ii)  $1 - \frac{3^{10}}{4^{10}}$   
 (3) (i) 25   (ii)  $\frac{100}{3}$    (4)  $\frac{270}{1024}$    (5) (i)  $1 - 0.95^{10}$    (ii)  $\binom{10}{2}(0.05)^2(0.95)^8$   
 (6) (i)  $\binom{12}{10}(0.9)^{10}(0.1)^2$    (ii)  $2.1(0.9)^{11}$    (iii)  $1 - [2.1(0.9)^{11}]$   
 (7) (i)  $\binom{18}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{18-x}$    (ii)  $\binom{18}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{15}$    (iii)  $1 - \frac{20}{3}\left(\frac{2}{3}\right)^{17}$    (8) 2,  $\frac{4}{3}$    (9)  $\frac{5}{2}, 54$

## EXERCISE 11.6

1	2	3	4	5	6	7	8	9	10
(2)	(4)	(2)	(4)	(4)	(2)	(4)	(3)	(2)	(1)
11	12	13	14	15	16	17	18	19	20
(4)	(4)	(1)	(2)	(1)	(1)	(4)	(4)	(2)	(1)

## Chapter 12

### Exercise 12.1

1. (i) Yes, \* is binary on  $\mathbb{R}$       (ii) Yes, \* is binary on A  
 (iii) No, \* is not binary on  $\mathbb{R}$
2. No,  $\otimes$  is not binary on  $\mathbb{Z}$       3.  $\frac{-88}{15}$
4. Yes, usual multiplication is binary on A
5. (i) The given operation \* is closure and commutative but not associative on  $\mathbb{Q}$ .  
 (ii) Identity does not exist and so inverse does not exist.

*	a	b	c
a	b	c	a
b	c	b	a
c	a	a	c

7. No. The given operation is not commutative and associative

8. (i)  $A \vee B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$       (ii)  $A \wedge B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

(iii)  $(A \vee B) \wedge C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$       (iv)  $(A \wedge B) \vee C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$



10. (i) It is commutative and associative.  
(ii) Identity and Inverse exist.

### Exercise 12.2

1. (i)  $\neg p$ : Jupiter is not a planet    (ii)  $p \wedge \neg q$ : Jupiter is a planet and India is not an Island.  
(iii)  $\neg p \vee q$ : Jupiter is not a planet or India is an Island.  
(iv)  $p \rightarrow \neg q$ : If Jupiter is a planet then India is not an Island.  
(v)  $p \leftrightarrow q$  Jupiter is a planet if and only if India is an Island.
2. (i)  $\neg p \wedge q$     (ii)  $p \vee \neg q$     (iii)  $p \wedge q$  (iv)  $\neg p$
3. (i)  $p \rightarrow q$  is T    (ii)  $p \vee q$  is F    (iii)  $\neg p \vee q$  is T    (iv)  $p \wedge q$  is F
4. (i), (iii) and (iv) are propositions
5. (i) **Converse:** If  $x$  and  $y$  are numbers such that  $x^2 = y^2$  then  $x = y$ .  
**Inverse:** If  $x$  and  $y$  are numbers such that  $x \neq y$  then  $x^2 \neq y^2$ .  
**Contra positive:** If  $x$  and  $y$  are numbers such that  $x^2 \neq y^2$  then  $x \neq y$ .  
(ii) **Converse:** If a quadrilateral is a rectangle then it is a square.  
**Inverse:** If a quadrilateral is not a square then it is not a rectangle.  
**Contrapositive:** If a quadrilateral is not a rectangle then it is not a square.
6. (i) Truth table for  $\neg p \wedge \neg q$

$p$	$q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(ii) Truth table for  $\neg(\neg p \wedge \neg q)$

$p$	$q$	$\neg q$	$p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

(iii) Truth table for  $(p \vee q) \vee \neg q$

$p$	$q$	$\neg q$	$p \vee q$	$(p \vee q) \vee \neg q$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	T



(iv) Truth table for  $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$

<b>p</b>	<b>q</b>	<b>r</b>	$\neg p$	$(\neg p \rightarrow r)$	$p \leftrightarrow q$	$(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

7. (i) Contradiction      (ii) Tautology      (iii) Contingency      (iv) Tautology

12.  $p \rightarrow (q \rightarrow p)$  is a Tautology.

13. Yes. The statements are logically equivalent.

### Exercise 12.3

Choose the appropriate answer from the given distractors.

Q	1	2	3	4	5	6	7	8	9	10
A	(2)	(3)	(2)	(4)	(2)	(2)	(3)	(4)	(3)	(2)
Q	11	12	13	14	15	16	17	18	19	20
A	(4)	(1)	(3)	(3)	(3)	(2)	(4)	(3)	(1)	(4)



## GLOSSARY

### CHAPTER 7

#### Application of Differential Calculus

related rates	சார்ந்த வீதங்கள்
mean value theorem	இடை மதிப்புத் தேற்றம்
indeterminate forms	தேறப்பெறாத வடிவங்கள்
stationary points	நிலைப் புள்ளிகள்
critical points	மாறுநிலைப் புள்ளிகள்
monotonicity of functions	ஒழியல்புச் சார்புகள்
absolute extremum	மீப்பெரு அறுதி
relative extremum	இடஞ்சார்ந்த அறுதி
Concave	குழிவு
Convex	குவிவு
point of inflection	வளைவு மாற்றப் புள்ளி
Symmetry	சமச்சீர்த் தன்மை

### CHAPTER 8

#### Differential and Partial Derivatives

Differential	வகையீடு
Partial Derivatives	பகுதி வகைக்கெழு
Harmonic	சீரான
Homogeneous	சமச்சீரான
Absolute error	தனிப்பிழை
Relative error	சார் பிழை
Percentage error	சதவிகித பிழை

### CHAPTER 9

#### Applications of Integration

Definite integral	வரையறுத்தத் தொகை
Reduction formula	குறைப்பு சூத்திரம்
Gamma integral	காமா தொகையிடல்
Bounded region	இடைப்பட்ட பகுதி

### CHAPTER 10

#### Ordinary Differential Equations

order	வரிசை
Linear	நேரியல்
Degree	படி
arbitrary constant	ஏதேனுமொரு மாறிலி
dependent variable	சார்ந்த மாறி
independent variable	சாரா மாறி
integrating factor	தொகையீட்டுக் காரணி
homogeneous function	சமபடித்தான சார்பு



## CHAPTER 11

### Probability Distributions

bernoulli random variable	பெர்நோவி சமவாய்ப்பு மாறி
binomial distribution	ஈருறுப்பு பரவல்
binomial random variable	ஈருறுப்பு சமவாய்ப்பு மாறி
continuous random variable	தொடர்நிலை சமவாய்ப்பு மாறி
cumulative distribution function	சேர்ப்புப் பரவல் சார்பு
discrete random variable	தனிநிலை சமவாய்ப்பு மாறி
mathematical expectation	கணித எதிர்பார்ப்பு
probability density function	நிகழ்தகவு அடர்த்திச் சார்பு
probability mass function	நிகழ்தகவு நிறைச்(செறிவு) சார்பு
random variable	சமவாய்ப்பு மாறி

## CHAPTER 12

### Discrete Mathematics

Absorption law	ஈர்ப்பு விதி
Algebraic structure	இயற்கணித அமைப்பு
Biconditional statement	இரு நிபந்தனைக் கூற்று
Binary Operation	ஈருறுப்பு செயலி
Boolean Algebra	பூலியன் இயற்கணிதம்
Boolean Matrix	பூலியன் அணி
Coding theory	குறியீட்டுக் கோட்பாடு
Compound statement	கூட்டுக் கூற்று
Conditional statement	நிபந்தனைக் கூற்று
Conjunction	இணையல்
Contradiction	முரண்பாடு
Contra positive	நேர்மாறு
Disjunction	பிரிப்பிணையல்
Duality	இருமை இயல்பு (அ) இரட்டைத் தன்மை
Hypothesis	கருதுகோள்
Involution law	உட்சமற்சி விதி
Logical connectives	தர்க்க இணைப்புகள்
Logical equivalent	தர்க்க சமானமானவை
Negation	மறுப்பு
Paradox	முரண்பாடு மெய்மை
Simple statement	தனிக்கூற்று
Tautology	மெய்மை
Truth table	மெய் அட்டவணை



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