Signals and Systems

GATE: (9 to 12 Marks)

Reference Book: S & S by Ranjan and HSU

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Signals and Systems

Analysis	Approximation	Transformation $t \to f$
1. Introduction	3. Fourier series	4. C.T.F.T. 't', 'T'
2. LTI system		5. L.T.
		6. D.T.F.T. 'n', 'N'
		7. Z.T.
		8. DFT/FFT

1. Introduction to Signals and Systems

→ What is signal?

→ What is system?

→ Characteristics of signals.

→ Types of signals

→ Some standard signals

→ Transformation on signals ($x(t)$)

① Time shifting $x(t \pm t_0)$

② Time scaling $x(\alpha t)$

③ Time reversal $x(-t)$

④ Amplitude scaling $k \cdot x(\alpha)$

→ Problems

→ Classification of signal

→ Classification of system

→ Summary

→ Problems

What is signal?

- It is the indication from which some amount of information is to be conveyed from one place to another.

What is system?

i/p $\longrightarrow$ $\boxed{\text{system}}$ $\longrightarrow$ proper o/p

- System is nothing but group of elements/physical components arranged in such a way that it gives proper output to given input.

Eg:- A fan without Blades: No air flow : not proper o/p : Not a system

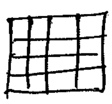
A fan with blades : Air flow : proper o/p : It is system

If we add controller from controlling purpose then it is called <u>CONTROL SYSTEM</u>.

* Characteristics of signals

(1) Dimension

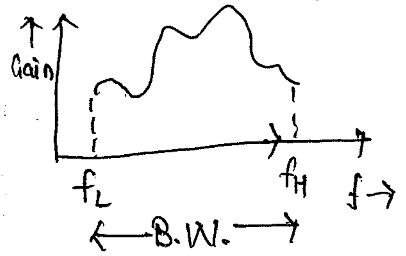
1D $\rightarrow$ point (·) x-coordinate only

2D $\rightarrow$ image  (x, y)

3D $\rightarrow$ TV (x, y, t)

(2) Bandwidth

Range of frequencies occupy by signals.



(3) Randomness

- more the randomness more the information

$$I = \log_2 \frac{1}{P_i} = -\log_2 P_i$$

$P_i = 1/8 \Rightarrow I = 3$ bits

more random $\rightarrow P_i = \frac{1}{32} \Rightarrow I = 5$ bits

* Types of Signals *

Based on

Time & Amplitude

- Time → C (continuous), D (discrete)
- Amplitude → C, D

$t \rightarrow$ Continuous $\rightarrow t = 0.1, t = 0.01, t = 0.02, t = 2$

$n \rightarrow$ discrete $\rightarrow n = 0, 1, 2, 3, 4, -1, -2, -3, -4$

$n = 1.5$ ✗

$A \rightarrow$ continuous $\rightarrow -\infty$ to $+\infty$

$A \rightarrow$ discrete $\rightarrow [-2, -1, 0, 0.5, 2]$

1) Continuous signal

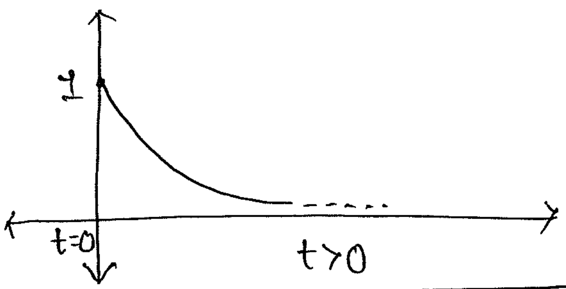
time → continuous, amplitude → continuous

For eg:- $x(t) = e^{-3t} \cdot u(t)$

$= e^{-3t} \cdot 1, \quad t \geq 0$

$= 0, \quad t < 0$

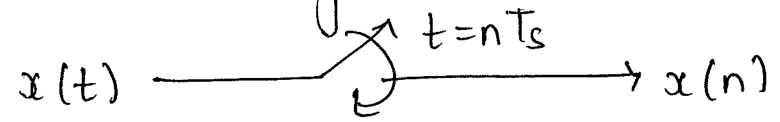
$$\boxed{\begin{aligned} u(t) &= 1, \quad t > 0 \\ &= 0, \quad t < 0 \\ &= \text{not defined at } t = 0 \end{aligned}}$$



$$x(t)\Big|_{t \to \infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

A signal which is continuous in both amplitude and time at any instant amplitude and time is known as continuous signal.

(2) Discrete signal



$t = T_s$, $n = 1$ → sampling period

$f_s = \frac{1}{T_s}$ = sampling frequency

$x(t) = e^{-3t} \cdot u(t)$

$\downarrow t = nT_s$

$x(nT_s) = e^{-3nT_s} \cdot u[nT_s]$

Let $T_s = 1$

$x[n] = e^{-3n} u[n]$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$x[n] = e^{-3n}$, $n \geq 0$, $n = 0, \pm 1, +2, +3, \ldots$

$= 0$, $n < 0$

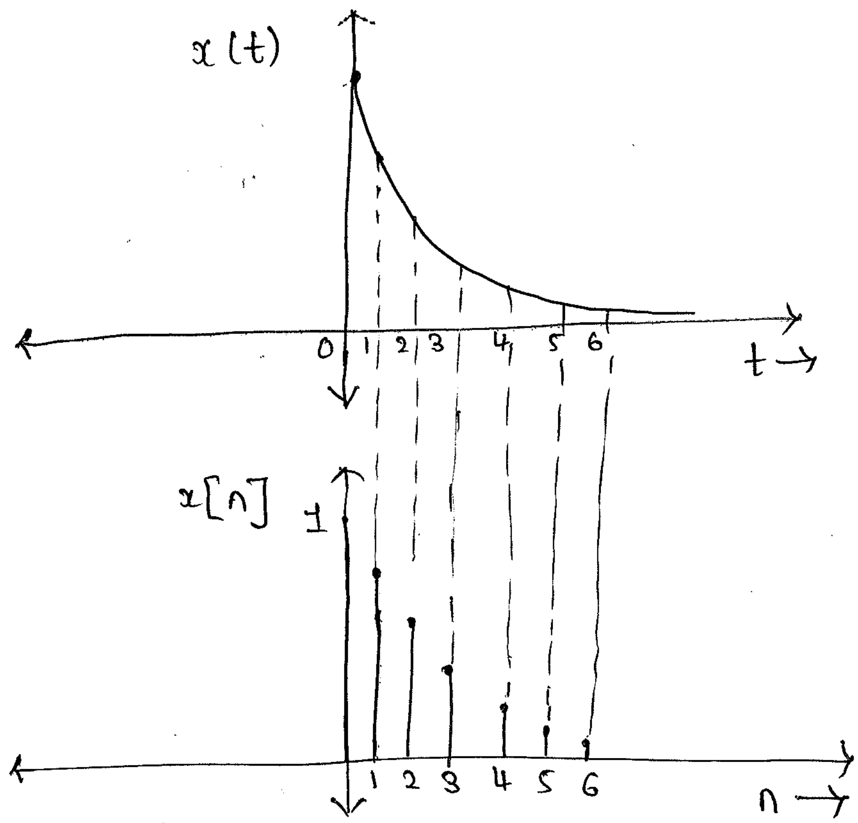
If $t = 10$ sec

$t = 0, 2, 4, 6, 8$

$n = 5$

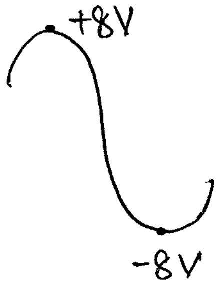
$T_s = 2$

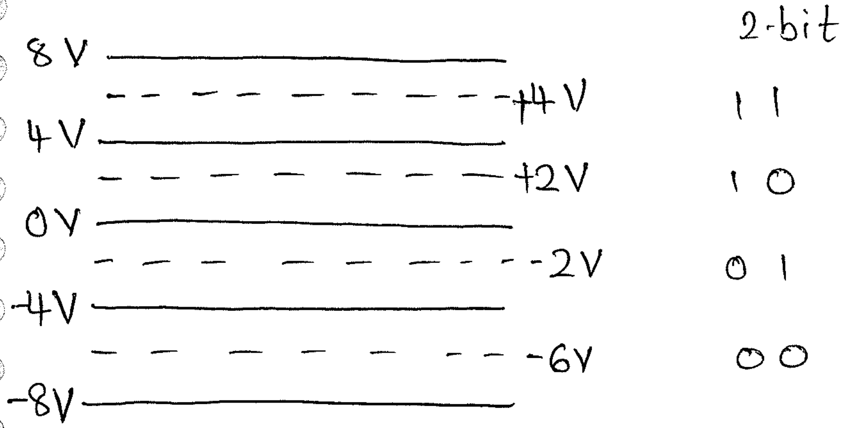
$\boxed{t = nT_s}$



(3) Quantized Signal

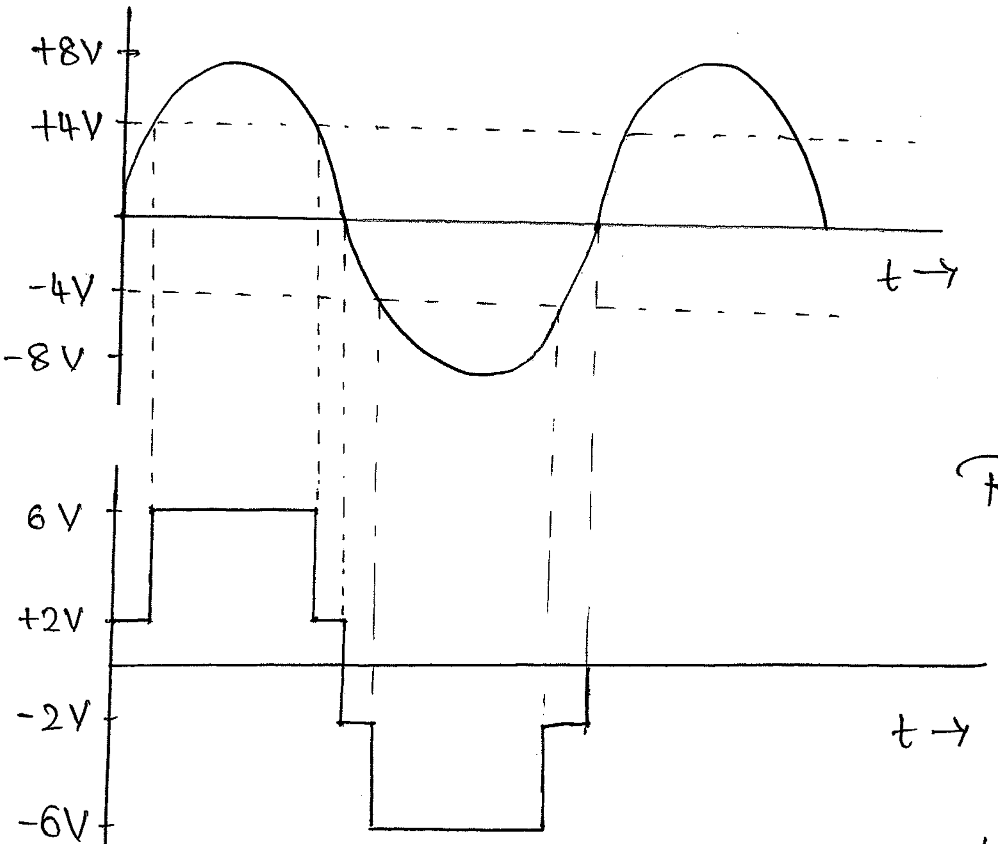
- A signal which is continuous in time and discrete in amplitude is called quantized signal.





$n = 2$

$2^2 = 4$

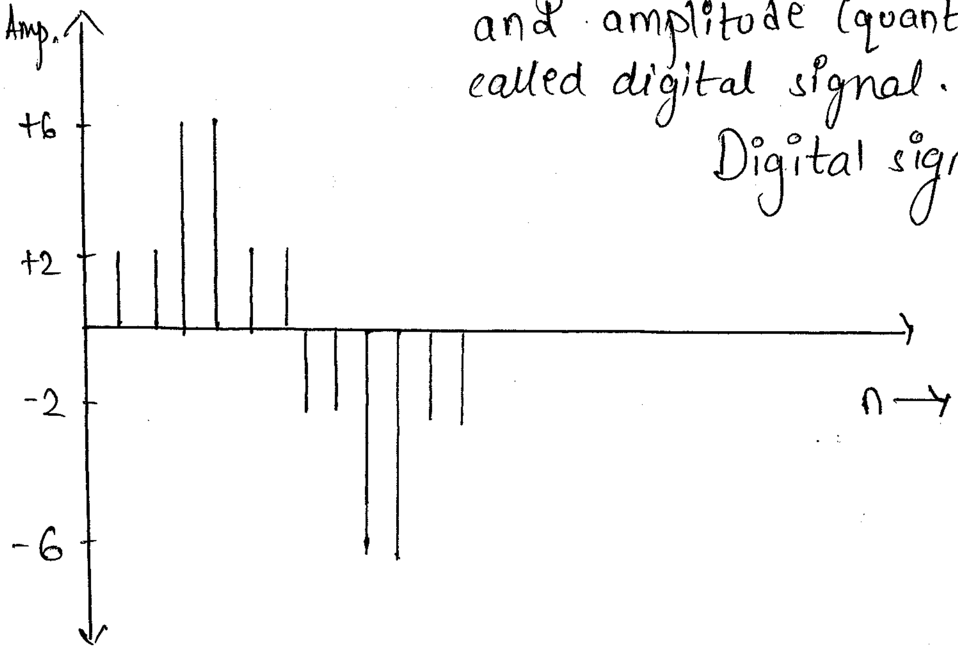


Range : [-6, -2, 2, 6]

t → continuous

A → discrete

A signal which is discrete in time and amplitude (quantized amp.) is called digital signal.



Digital signal.

Types of signals	Amplitude	Time
Continuous time signal	C	C Sampling
Discrete time signal	C Quantization	D
Quatised signal	D	C
Digital signal	D	D

NOTE: ~~Time~~ Amplitude conversion from $C \rightarrow D \Rightarrow$ Quantizer

~~Amplitude~~ Time conversion from $C \rightarrow D \Rightarrow$ Sampling

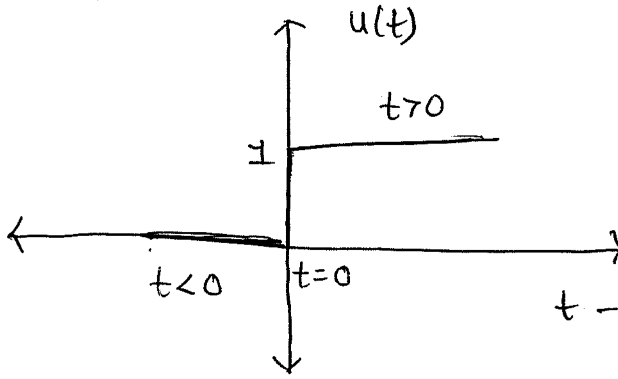
* Some standard signals *

[1] Continous step signal

$$u(t) = A \;,\; t > 0$$
$$= 0 \;,\; t < 0$$

if $A = 1 \Rightarrow$ unit step signal

$$u(t) = 1 \;,\; t > 0$$
$$= 0 \;,\; t < 0$$
$$= \text{not defined} \;,\; t = 0$$

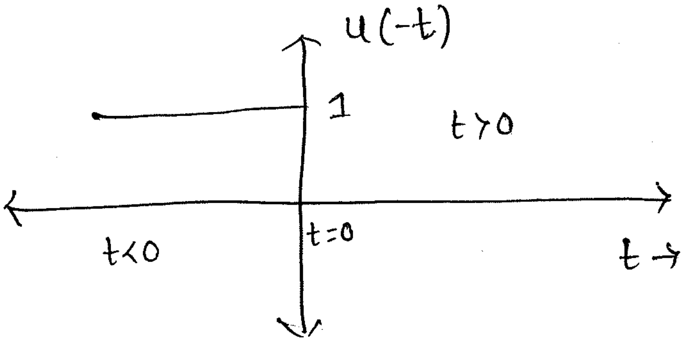


Practically $u(t)\big|_{t=0} = 1/2$

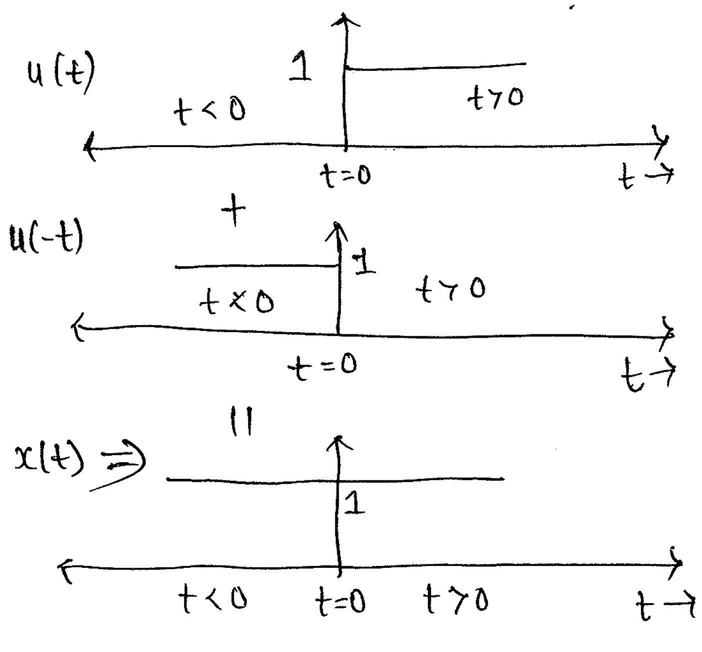
① $u(-t) = 1$

$$u(-t) = 0 \;,\; -t < 0$$
$$= 1 \;,\; -t > 0$$

$$u(-t) = 1 \;,\; t < 0$$
$$= 0 \;,\; t > 0$$



② $u(t) + u(-t) = x(t) = P = 1 \;, \forall t$



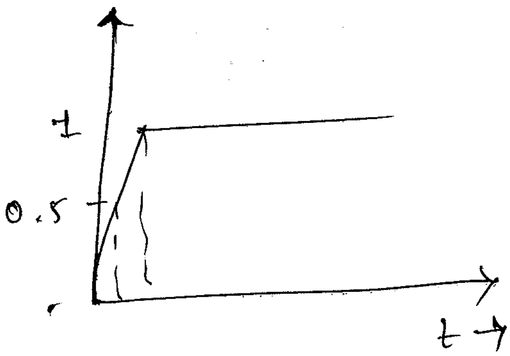
$u(t) + u(-t) = 1$

$u(0) + u(0) = 1$

$2u(0) = 1$

$u(0) = 1/2$

$u(t)\Big|_{t=0} = 1/2$



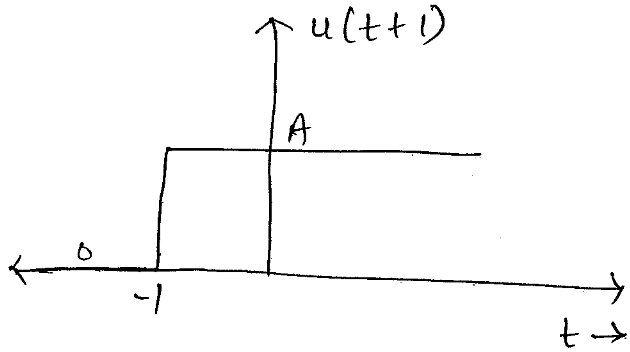
④ $u(t+1)$

$u(t+1) = 1, \quad t+1 > 0$

$\quad = 0, \quad t+1 < 0$

$u(t+1) = 1, \quad t > -1$

$\quad = 0, \quad t < -1$



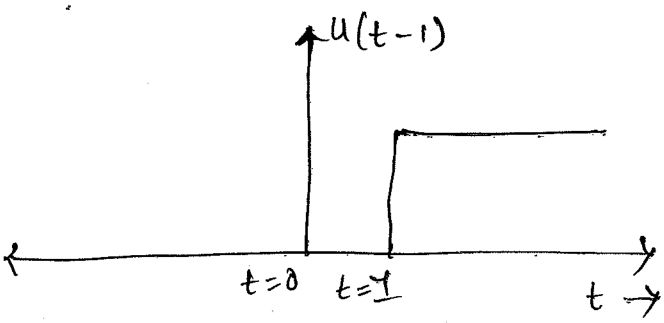
⑤ $u(t-1)$

$u(t-1) = 1, \quad t-1 > 0$

$\quad = 0, \quad t-1 < 0$

$u(t-1) = 1, \quad t > 1$

$\quad = 0, \quad t < 1$



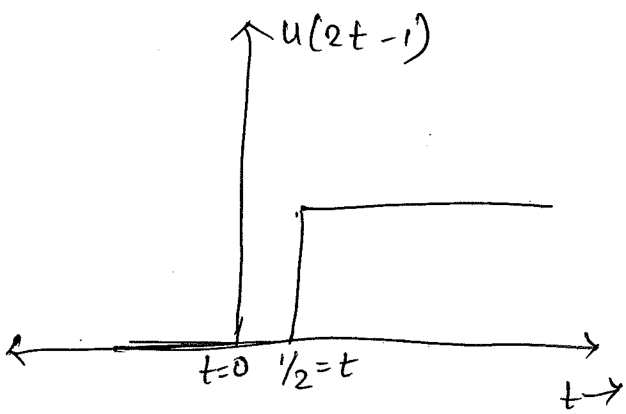
⑥ $u(2t-1)$

$u(2t-1) = 1, \quad 2t-1 > 0$

$\quad = 0, \quad 2t-1 < 0$

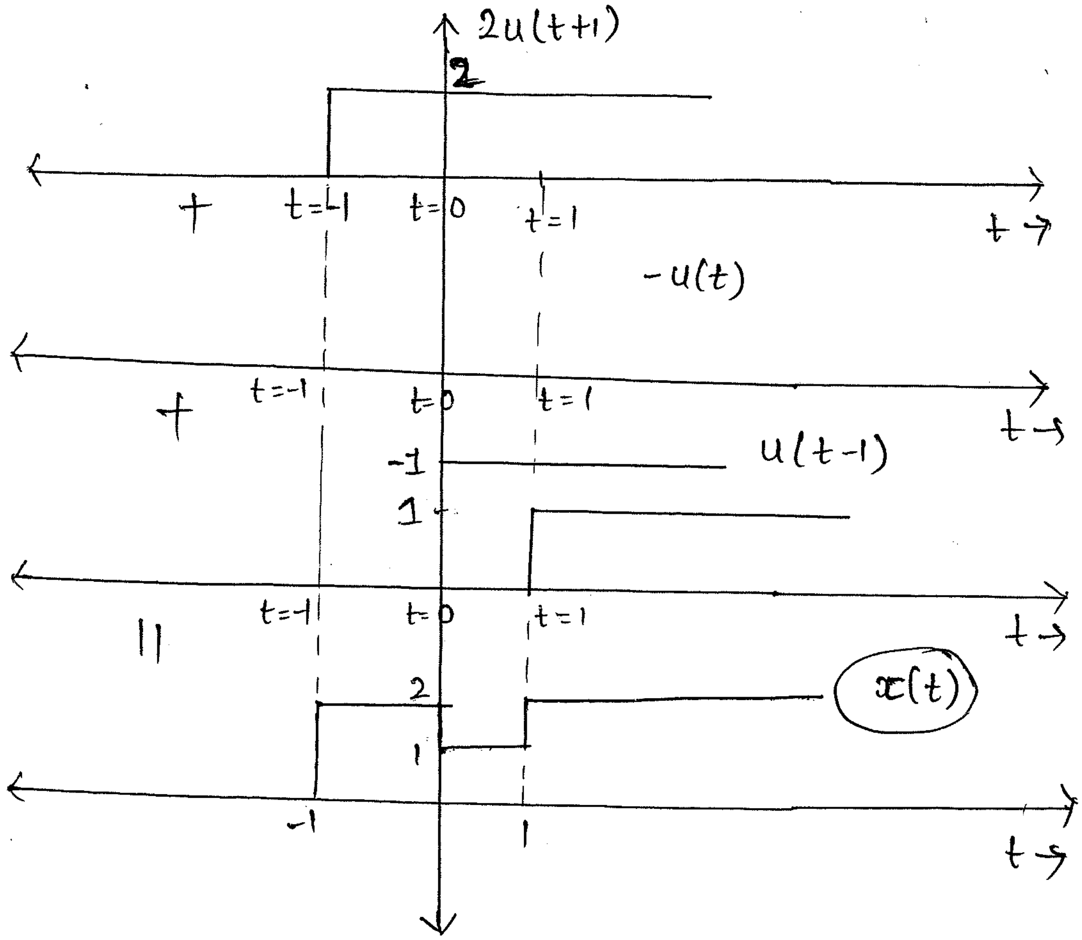
$u(2t-1) = 1, \quad t > 1/2$

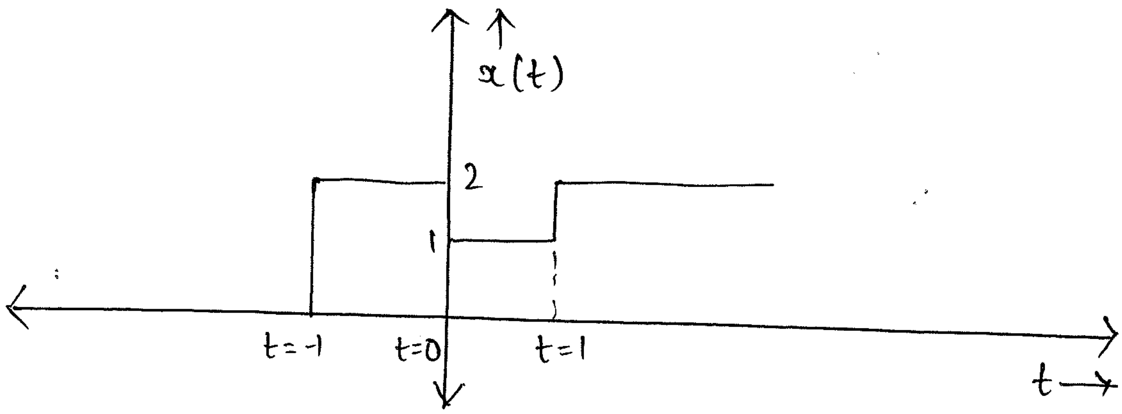
$\quad = 0, \quad t < 1/2$



Examples

(1) $x(t) = 2u(t+1) - u(t) + u(t-1)$



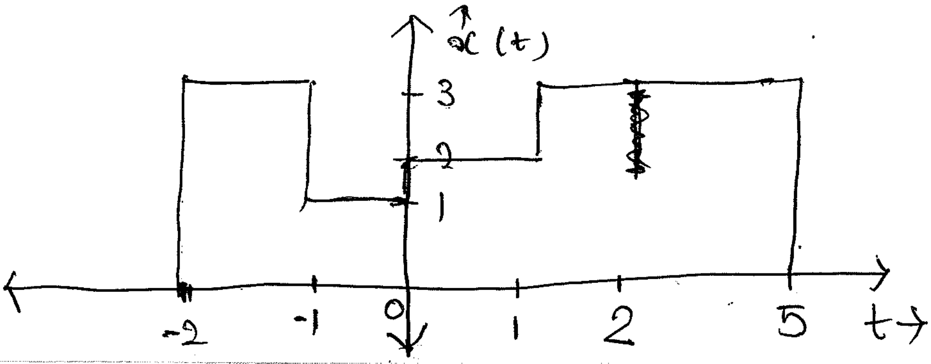


(2) $x(t) = u(t-2) + u(t) + 3u(t+2) - 2u(t+1) - 3u(t-5)$

$t=2 \qquad t=0 \qquad t=-2 \qquad t=-1 \qquad t=5$

Arrange in ascending order

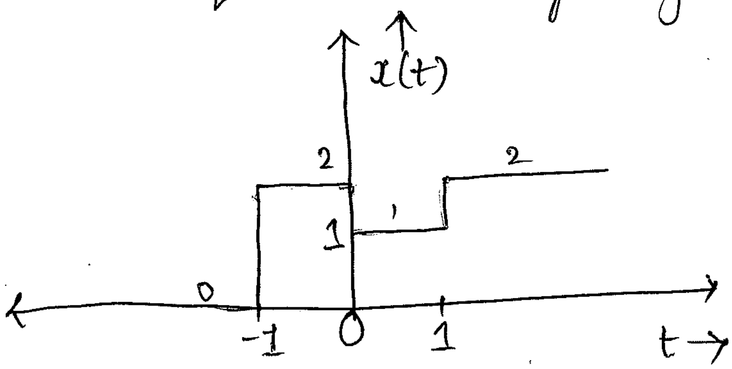
$= 3u(t+2) - 2u(t+1) + u(t) + u(t-2) - 3u(t-5)$



From graph to equation :-

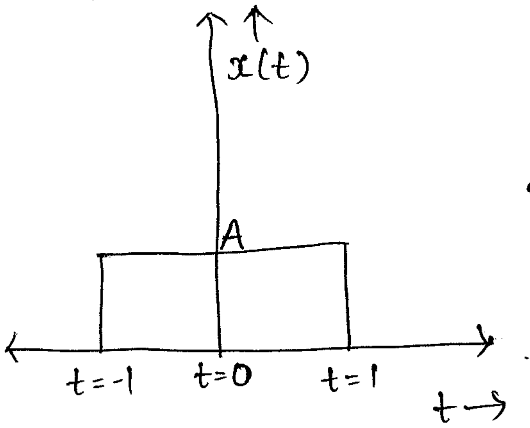
NOTE: $\boxed{\text{Change in amplitude = Next amp. - Previous amp.}}$

Q:- Write eq^n of following signal?



① First mark sudden change points
② Amplitude check
③ Write individual signal eq^n.

Sol^n: $2u(t+1) - u(t) + 1 \cdot u(t-1)$

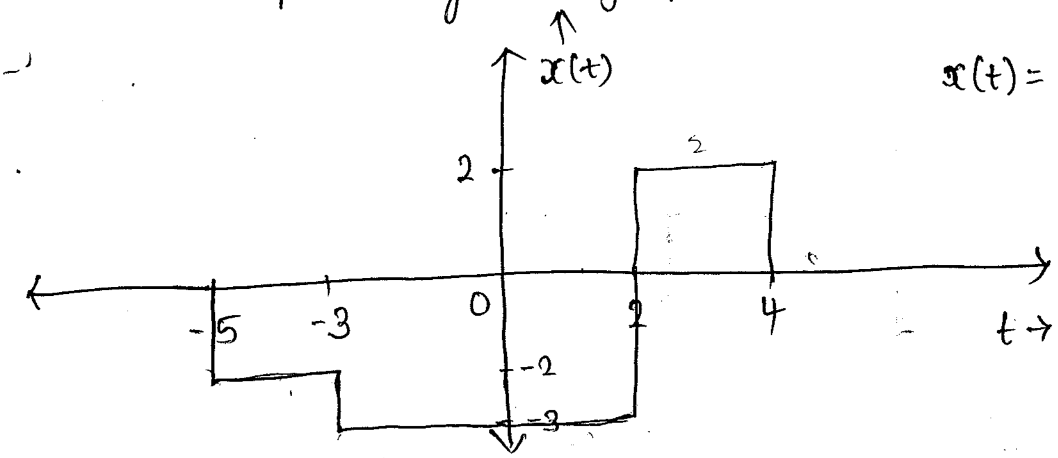


$x(t) = A \cdot u(t+1) - A\,u(t-1)$

$$\boxed{x(t) = A\left(u(t+1) - u(t-1)\right)}$$

NOTE: $\boxed{\text{Whenever there is sudden change in signal step. signal exists.}}$

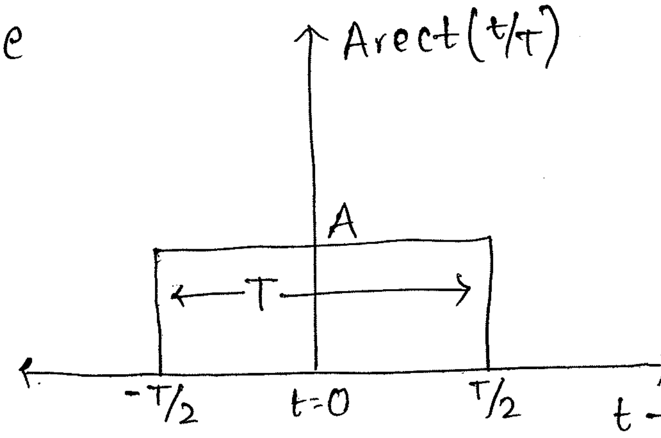
Q:- Write eq^n for given graph:- (in terms of step signal)



$x(t) = -2u(t+5) - 1 \cdot u(t+3)$
$+5u(t-2)$
$-2u(t-4)$.

(2) Rectangular function OR Gate function OR Π-function

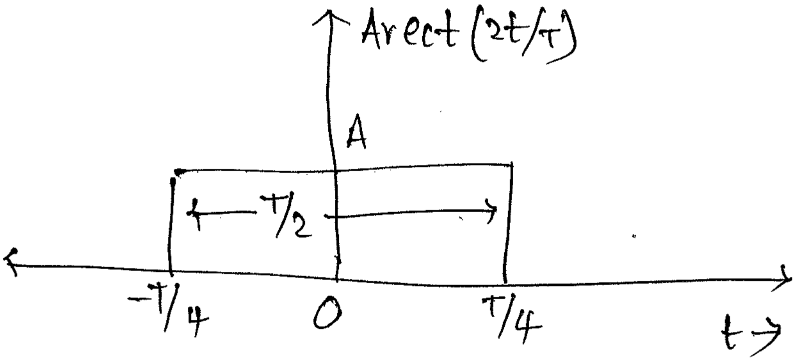
$A\,\text{rect}(t/T)$; T = pulse width
A = amplitude
t = time



$$A\,\text{rect}(t/T) = A \ , \ -T/2 < t < T/2$$
$$= 0 \ , \ \text{otherwise}$$

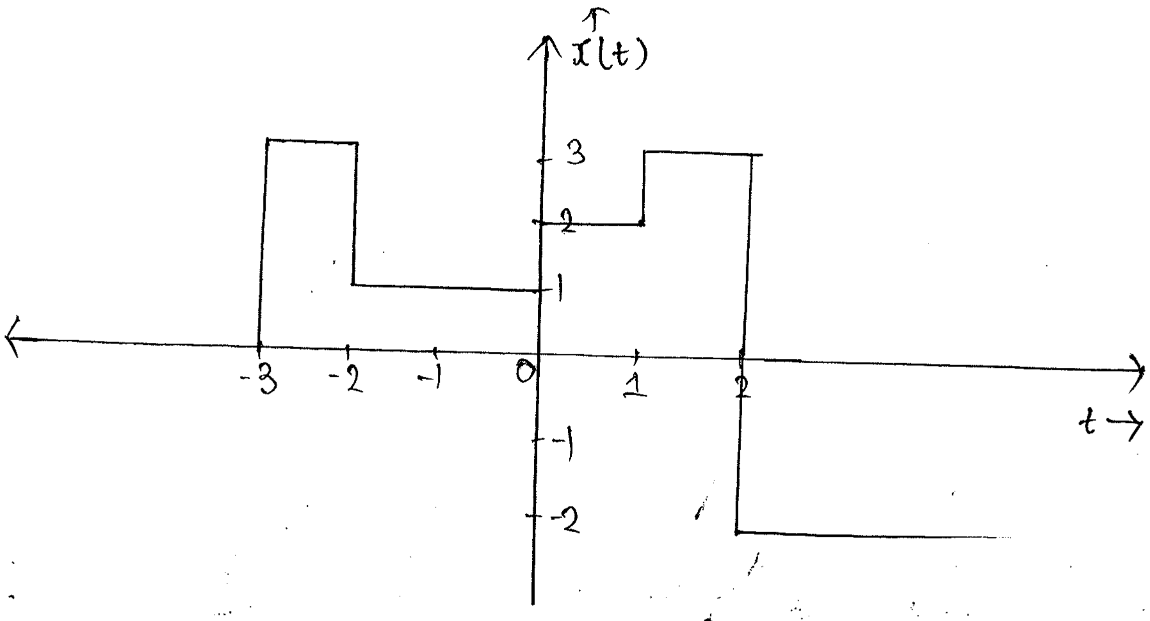
EX:- $x(t) = A\,\text{rect}(2t/T)$ OR $A\,\text{rect}\left(t/\tfrac{T}{2}\right)$

$$= A \ , \ -T/2 < 2t < T/2$$
$$= A \ , \ -T/4 < t < T/4$$
$$= 0 \ , \ \text{otherwise}$$



Q:- $x(t) = 3u(t+3) - 2u(t+2) + u(t) + u(t-1) - 5u(t-2)$

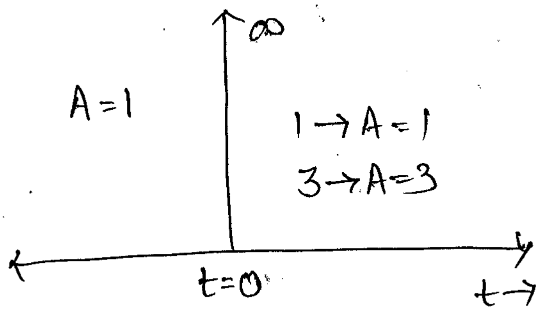
t = -3 t = -2 t = 0 t = 1 t = 2



(3) Continuous impulse function OR Direct Delta function

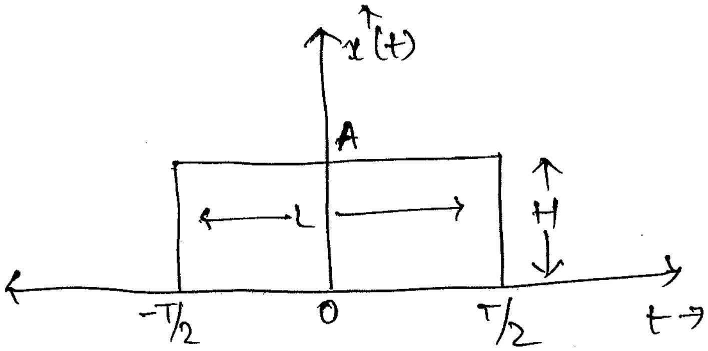
$\delta(t)$ ← denoted by.

$$\delta(t) = \infty \ , \ t=0 \quad (t \to 0)$$
$$= 0 \ , \ t \neq 0$$



Using unit area rectangular function

$$x(t) = A\,\mathrm{rect}(t/\tau) = A \ , \ -T/2 < t < T/2$$
$$= 0 \ , \ \text{otherwise}$$



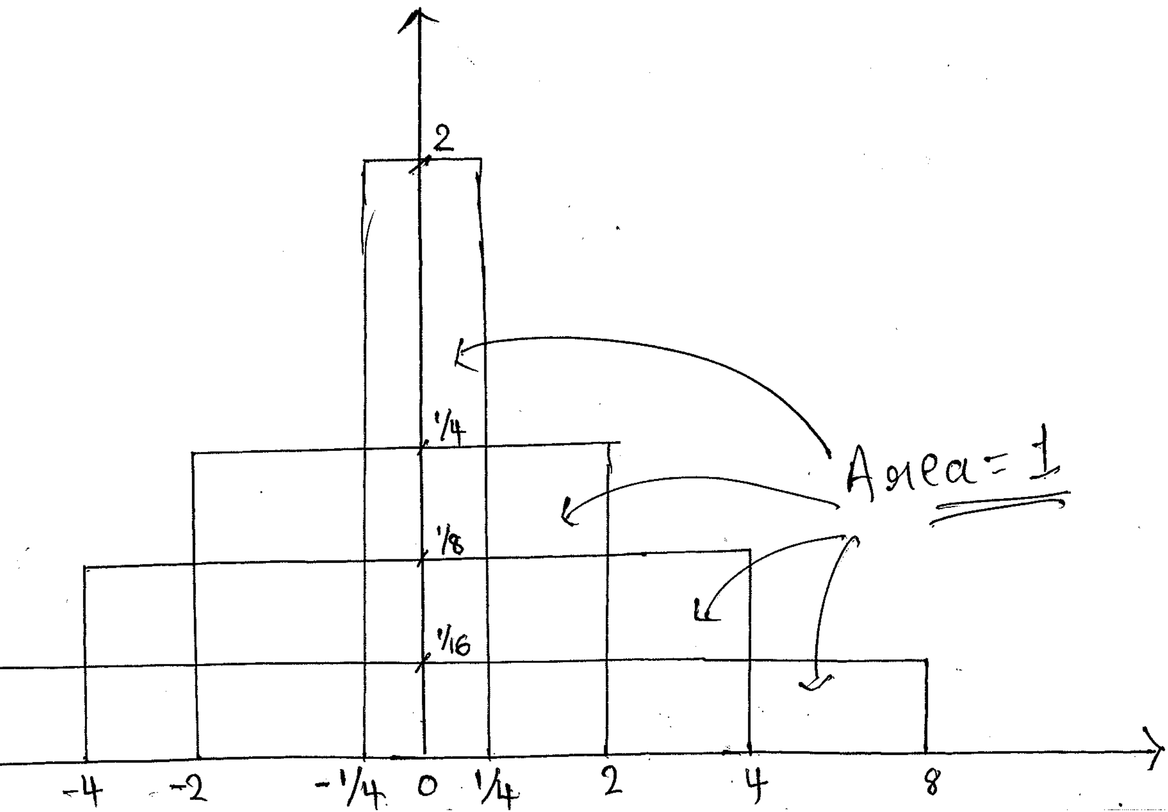
L = length
H = height

$A = L \times H = 1$

$\uparrow L \Rightarrow H \downarrow$

Suppose $L = 16$

$\therefore H = 1/16$.



$A = L \times H = 1$

$\downarrow$

$L \to 0 \Rightarrow H \to \infty$

$\downarrow$

$t \to 0 \Rightarrow x(t) \longrightarrow \infty$

$t \to 0 \Rightarrow \delta(t) \Rightarrow \infty$

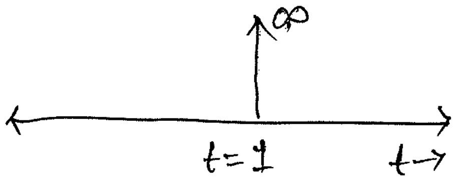
$x(t) = 3\,\delta(t) = 3 \quad , t = 0$

$\qquad\qquad\qquad\quad 0 \quad , t \neq 0$

$\downarrow$

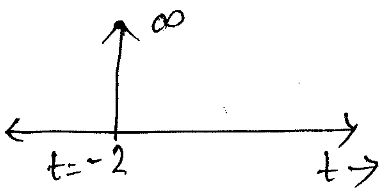
amplitude = 3.

Ex:- (1) $\delta(t-1)$



(2) $x(t) = \delta(t+2) = \infty \quad , t+2 = 0$

$\qquad\qquad\qquad\qquad = 0 \quad , t+2 \neq 0$



NPTEL

$\delta(t) = \lim_{\varepsilon \to 0} \delta_\varepsilon(t)$

Impulse function

As $\varepsilon \to 0$

width $\varepsilon \to 0$

Height $1/\varepsilon \to \infty$

But Area = 1
= constant

*PROPERTIES OF $\delta(t)$ *

(1) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$ $\boxed{\text{AREA}}$

PRODUCT PROPERTY

(2) $x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$

↳ $x(t)$ is continuous at $t = t_0$

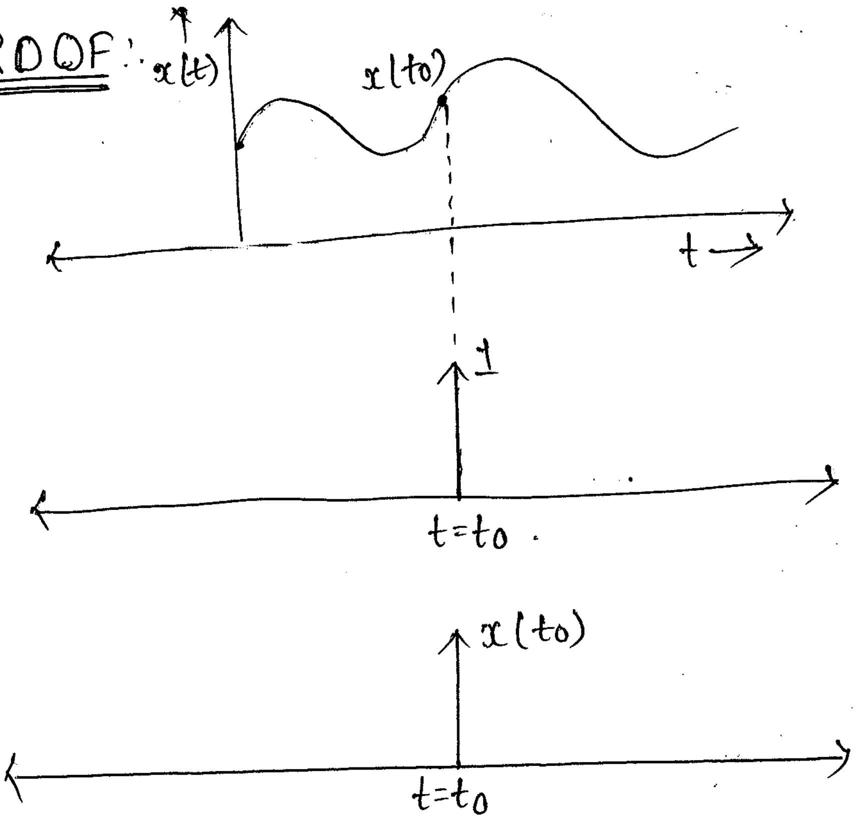
Eg:- (1) $y(t) = \cos t \cdot \delta(t-\pi) = ?$

$\underbrace{\cos t}_{x(t)} \cdot \underbrace{\delta(t-\pi)}_{\delta(t-t_0)}$

$= \cos \pi \cdot \delta(t-\pi)$

$= (-1) \cdot \delta(t-\pi)$ //

PROOF:



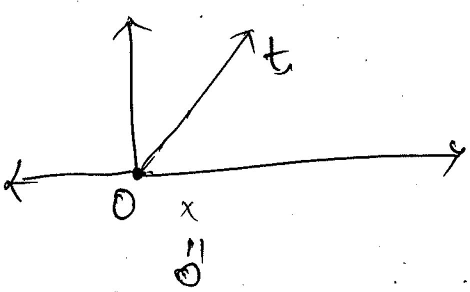
$\Rightarrow x(t_0) \cdot \delta(t-t_0)$

(2) $x(t) \cdot \delta(t) = ?$

$= x(0) \cdot \delta(t)$ //

(3) $t \cdot \delta(t)$

$= 0$



(3) $x(t)\cdot\delta(t-t_0) = x(t_0)\cdot\delta(t-t_0)$

$$\int_{-\infty}^{\infty} x(t)\cdot\delta(t-t_0) = \int_{-\infty}^{\infty} x(t_0)\cdot\delta(t-t_0)\cdot dt$$

$$= x(t_0)\int_{-\infty}^{\infty}\delta(t-t_0)\cdot dt \quad \leftarrow \text{Area} = 1$$

$$= x(t_0)$$

$$\boxed{\int_{t_1}^{t_2} x(t)\cdot\delta(t-t_0)\,dt = \begin{cases} x(t_0), & t_1 \le t_0 \le t_2 \\ 0, & \text{otherwise}\end{cases}}$$ Sampling property.

Q:- $\int_{-\infty}^{\infty}(t+\cos\pi t)\cdot\delta(t-1)\,dt$

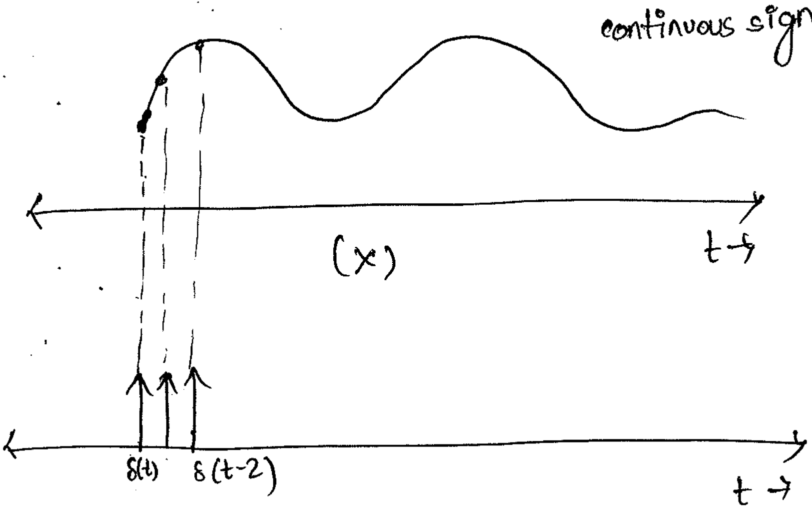
$t_0 = 1$

$= x(t_0)$
$= x(1)$
$= 1+\cos\pi = 1-1 = 0$

Q:- $\int_{-\infty}^{\infty} x(2-t)\cdot\delta(t-4)\,dt$

$t_0 = 4$

$= x(t_0)$
$= x(4)$
$= x(-2)$



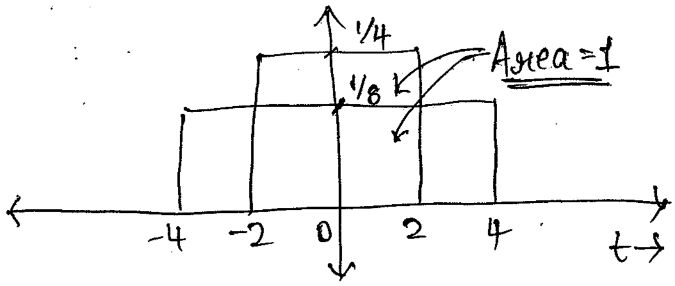
$$\int_{-\infty}^{\infty} \begin{matrix} x(0)\cdot\delta(t-0)\,dt + \\ x(1)\cdot\delta(t-1)\,dt + \\ x(2)\cdot\delta(t-2)\,dt \end{matrix} = x(t)$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\cdot\delta(t-\tau)\,d\tau$$

||

Discrete signal.

③ $\boxed{\delta(\alpha t) = \frac{1}{|\alpha|} \cdot \delta(t)}$ ⇒ Time scaling.



$A = L \times H = 1$

$= \uparrow t \cdot x(t) \downarrow = 1$

Eg:- (1) $\delta(2t)$

$= \frac{1}{2}\,\delta(t)$

★ (2) $\delta(-t) = \delta(t)$ ⇒ EVEN SIGNAL

(3) $\delta(at \pm b)$

$= \delta(a(t \pm b/a))$

$\delta(at \pm b) = \frac{1}{|a|}\,\delta(t \pm b/a).$

Eg:- $\delta(-t+2) = \delta(-(t-2))$

$= \delta(t-2)$

Q:- $\int_{-\infty}^{\infty} e^{3(t-3)} \cdot \delta(3t-6)\,dt$

$= \int_{-\infty}^{\infty} \frac{1}{3} \cdot e^{3(t-3)} \cdot \delta(t-2)\,dt$

$= \frac{1}{3} \cdot e^{3(2-3)}$

$= \frac{1}{3} \cdot e^{-3} = \underline{0.01659}.$

SUMMARY OF $\delta(t)$

NPTEL

$$\boxed{\delta(-t) = \delta(t)}$$
$\Downarrow$
even function

(1) $\displaystyle\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

(2) $\delta(\alpha t) = \dfrac{1}{|\alpha|} \cdot \delta(t)$

$\rightarrow \delta(-t) = \delta(t)$

$\rightarrow \delta(at \pm b) = \dfrac{1}{|a|} \cdot \delta(t \pm b/a)$

(3) $x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)$

(4) $\displaystyle\int_{-\infty}^{\infty} x(t) \cdot \delta(t - t_0)\,dt = x(t_0)$, $t_1 \leq t_0 \leq t_2$

$= 0$, otherwise

(5) $x(t) = \displaystyle\int_{-\infty}^{\infty} x(t) \cdot \delta(t - \tau)\,d\tau$ ⇒ Shifting property of Impulse Function

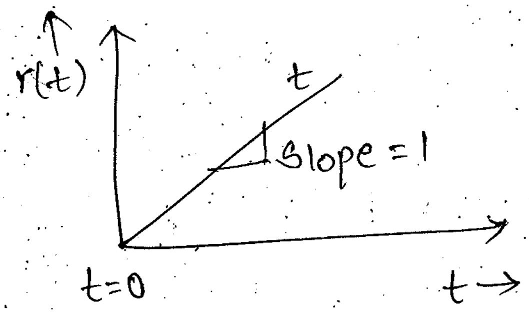
(6) $u(t) = \displaystyle\int_{0}^{\infty} \delta(t - \tau)\,d\tau$

$$\boxed{\delta(t) = \frac{du(t)}{dt}}$$

⇒ The function which do not possess higher derivative are singularity function.

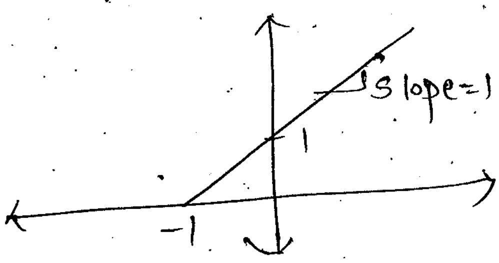
④ RAMP signal.

$r(t) = t \quad , t \geq 0$
$\quad\;\; = 0 \quad , t < 0$

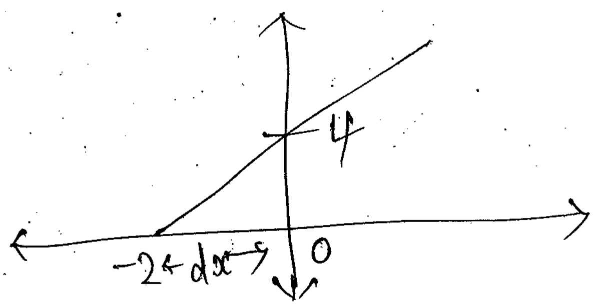


$\frac{d}{dt} \cdot r(t) = 1 = u(t) = 1 \quad , t = 0$
$\qquad\qquad\qquad\qquad\quad = 0 \quad , t \neq 0$

Eg:- $r(t+1) = t+1 \quad , t \geq -1$
$\qquad\qquad\quad = 0 \quad ; t < -1$

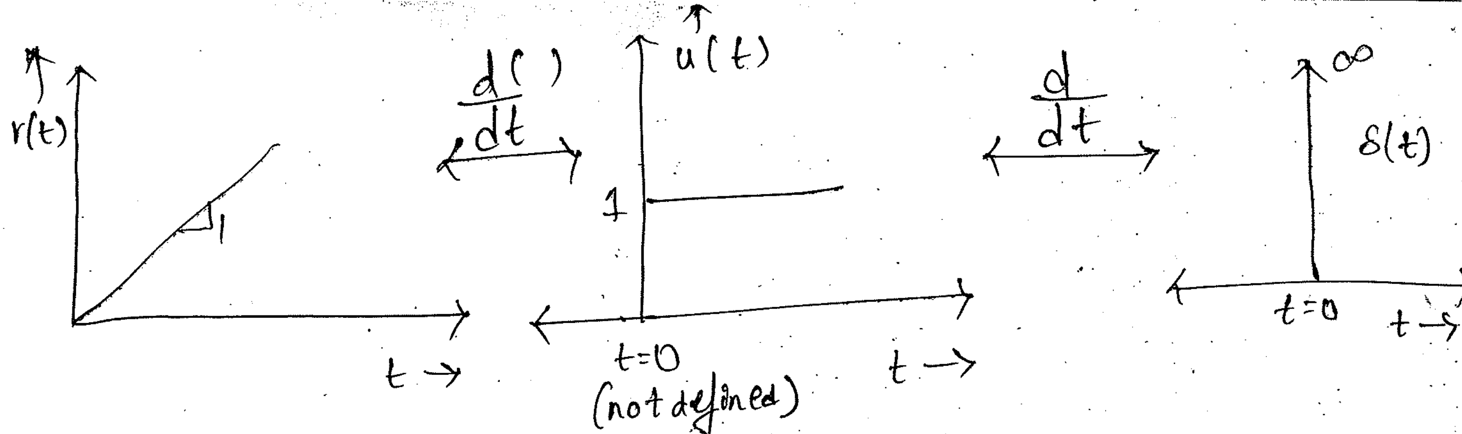


Q:- Draw $2r(t+2)$
Slope = 2
Ramp starting from -2

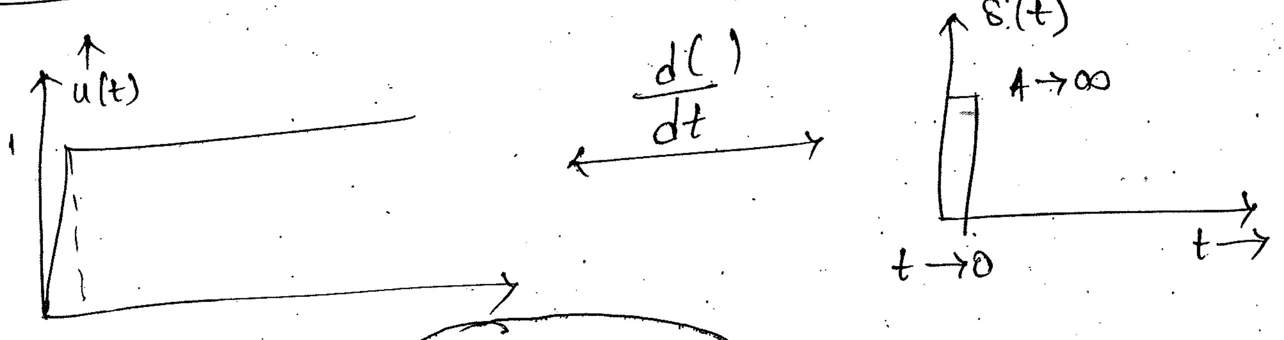


$\left|\frac{dy}{dx}\right| = \text{slope} = 2$

$dx = |-2| = 2$
so, dy should be 4 only.



IDEALLY

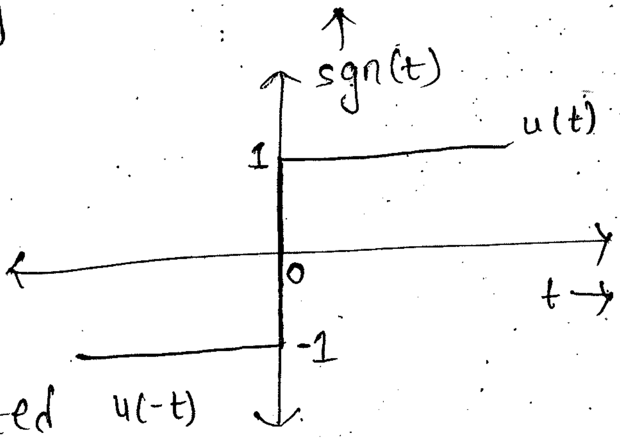


PRACTICALLY

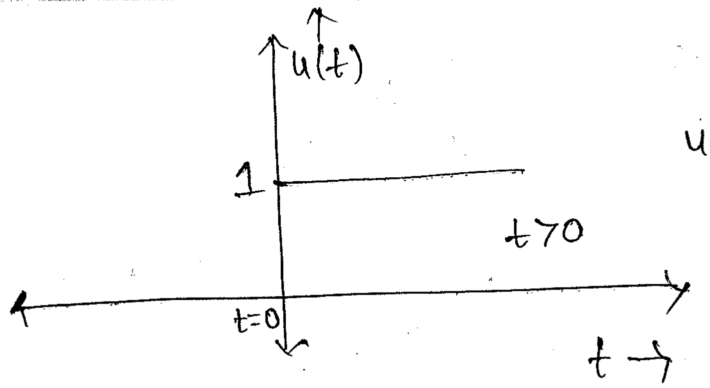
$$\delta(t) = \frac{d\,u(t)}{dt}$$

[5] SIGNUM FUNCTION

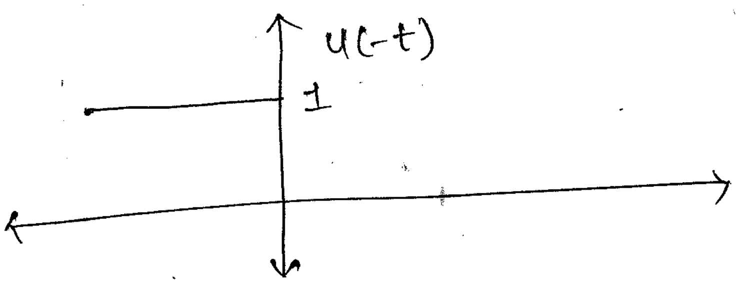
$$\text{sgn}(t) = 1 \quad , t > 0$$
$$\quad\quad = -1 \quad , t < 0$$



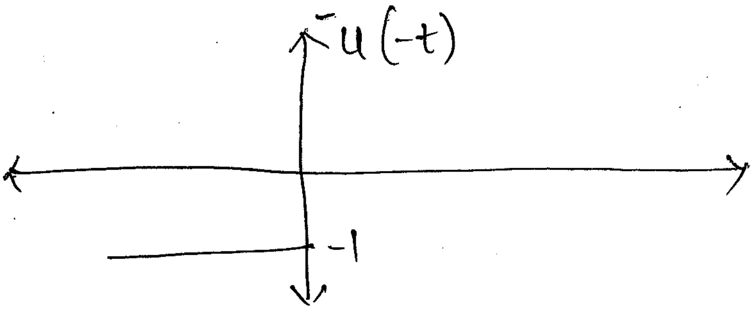
Signum function can be represented in form of step signal function.



u(t) ①



②



③

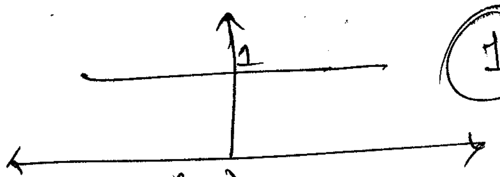
$$① + ② = 1 = u(t) + u(-t)$$

$$\text{sgn}(t) = u(t) - u(-t)$$

$$= 1 - u(-t) - u(-t)$$

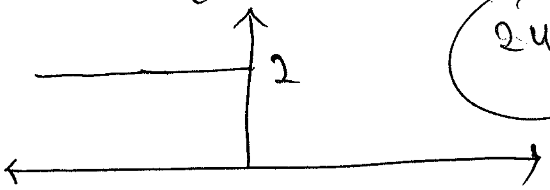
$$= 1 - 2u(-t)$$

PROOF:



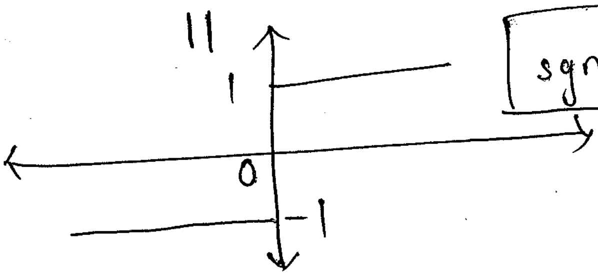
1

(−)



2u(-t)

||



$\boxed{\text{sgn}(t)}$

Q:- Do following operations on given signal

$$\int_{-\infty}^{\infty} x(\tau)\cdot\delta(t-\tau)\,d\tau = x(t)$$

① $\delta(\omega)$

$= \delta(2\pi f)$

$= \frac{1}{2\pi}\delta(f)$

② $e^{-3t}\cdot\delta(3-t)$

$= e^{-3t}\cdot\delta(t-3)$

$= e^{-9}\cdot\delta(t-3)$

③ $(1+3t+t^2)\cdot\delta(2t-3)$

$= (1+3t+t^2)\cdot\delta\left(2\left(t-3/2\right)\right)$

$= \frac{1}{2}(1+3t+t^2)\cdot\delta(t-3/2)$

$= \frac{1}{2}\left[1+3\cdot\frac{3}{2}+\frac{9}{4}\right]\cdot\delta(t-3/2)$

$= \frac{1}{2}\left[\frac{4+18+9}{4}\right]\cdot\delta(t-3/2)$

$= \frac{1}{2}\left(\frac{31}{4}\right)\cdot\delta(t-3/2)$

$= \frac{31}{8}\cdot\delta(t-3/2)$

④ $t\cdot\delta(t)$

$= 0 \quad [t_0=0]$

⑤ $\delta(-3t)$

$= \frac{1}{3}\cdot\delta(t)$

⑥ $\cos 3\tau\cdot\delta(\tau-\pi/3)$

$= \cos 3\cdot\frac{\pi}{3}\cdot\delta(\tau-\pi/3)$

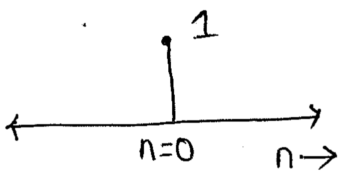
$= \cos\pi\cdot\delta(\tau-\pi/3)$

$= -\delta(\tau-\pi/3)$

* Discrete Impulse function *

$\delta[n] = 1, \; n=0$

$\quad\;\;\; = 0, \; n\neq 0$

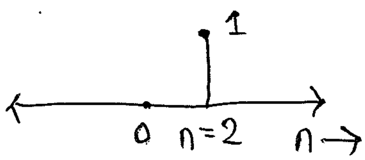


→ $\delta[n-2] = 1, \; n-2=0$

$\quad = 0, \; n-2\neq 0$

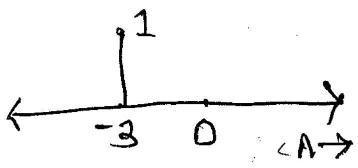
$\quad = 1, \; n=2$

$\quad = 0, \; n\neq 2$



$\delta[n+3] = 0, \; n\neq -3$

$\quad = 1, \; n=-3$



* Property of discrete impulse function *

1] $\delta[kn] = 1, \; kn = 0$

$= 1, \; n = 0$

$= 0, \; n \neq 0$

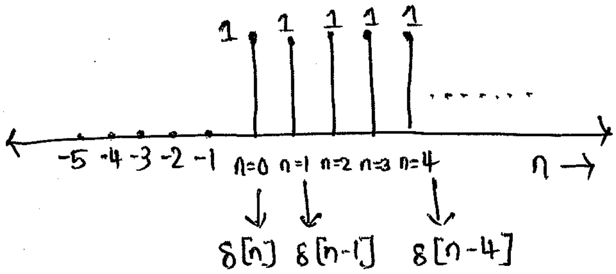
Eg:- $\delta[2n] = \delta[n]$

2] $x[n] \cdot \delta[n-n_0] = x[n_0] \cdot \delta[n-n_0]$

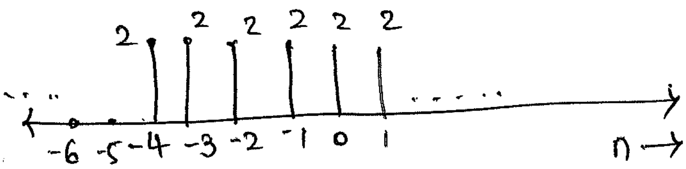
* UNIT STEP SEQUENCE *

$u[n] = 1, \; n \geq 0$

$= 0, \; n < 0$



Eg: $2u[n+4]$



NOTE

$\Rightarrow \sum_{k=-\infty}^{\infty} \delta[n-k] = 1, \; \forall n$

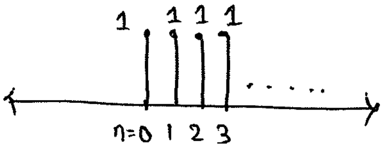
$\Rightarrow \sum_{k=0}^{\infty} \delta[n-k] = u[n]$

$\Rightarrow \sum_{k=-\infty}^{-2} \delta[n-k] = u[-n-2]$

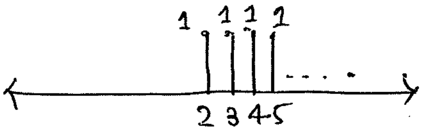
$\Rightarrow \sum_{k=1}^{\infty} \delta[n-k] = u[n-1]$

$\Rightarrow \sum_{k=-\infty}^{0} \delta[n-k] = u[-n]$

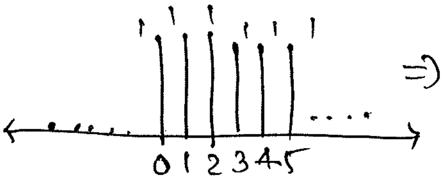
$u[n] = 1 \,, n = 0$
$\quad = 0 \,, n \neq 0$



$u[n-2] = 1 \,, n = 2$
$\quad = 0 \,, n \neq 2$



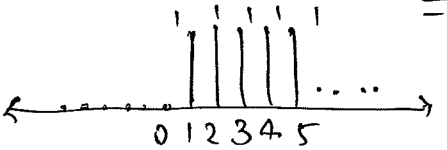
$x[n] = u[n] - u[n-1]$:- PROVE



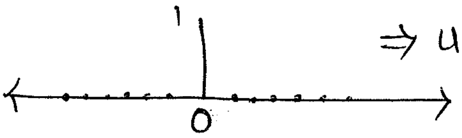
$\Rightarrow u[n]$

$(-)$

$\Rightarrow u[n-1]$



$(=)$

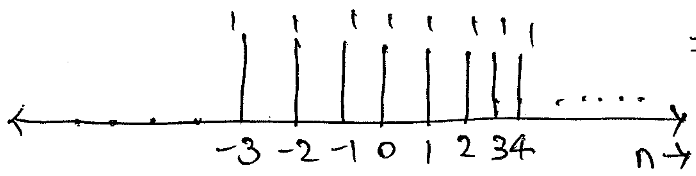


$\Rightarrow u[n] - u[n-1]$

NOTE:-

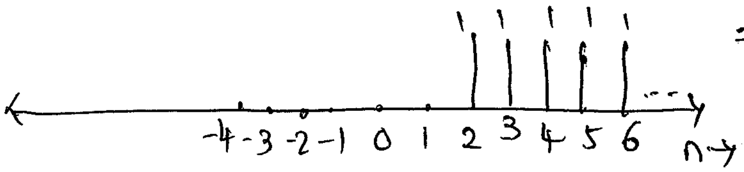
For discrete	For continuous
$\delta[n] = u[n] - u[n-1]$	$\delta(t) = \dfrac{d\,u(t)}{dt}$
$u[n] = \sum_{K=0}^{\infty} \delta[n-k]$	$u(t) = \int_{\tau=0}^{\tau=\infty} \delta(t-\tau) \cdot d\tau$

Q:- Draw $x[n] = u[n+3] - u[n-2]$

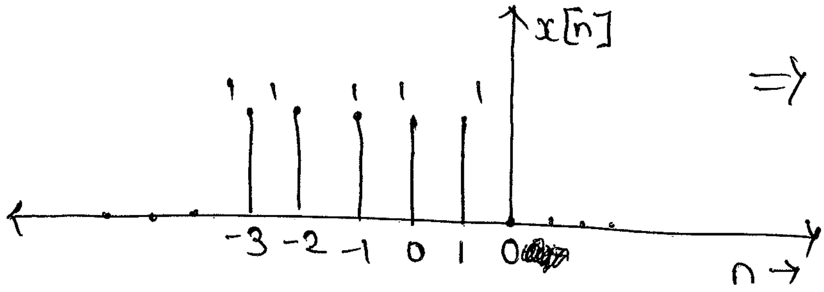


$\Rightarrow u[n+3]$

$(-)$



$\Rightarrow u[n-2]$

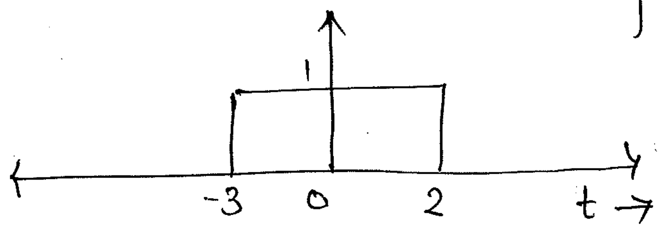


$\Rightarrow u[n+3] - u[n-2]$

For continuous, $x(t) = u(t+3) - u(t-2)$

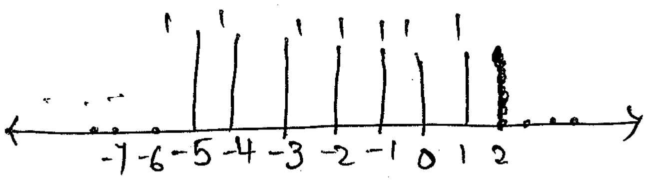
Sudden change at $t=-3$ & $t=2$

$\therefore$ There will be two step signal.



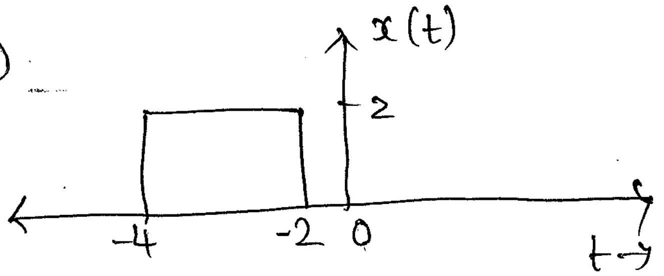
Q:- Write equation from graph:-

(1)



$x[n] = u[n+5] - u[n-2]$

(2)



$x(t) = 2u(t+4) - 2u(t+2)$.

* SUMMARY OF PROPERTY *

CONTINUOUS	DISCRETE
1] $x(t)\cdot\delta(t-t_0) = x(t_0)\cdot\delta(t-t_0)$	1] $x[n]\cdot\delta[n-n_0] = x[n_0]\cdot\delta[n-n_0]$
2] $\int_{-\infty}^{\infty} x(t)\cdot\delta(t-t_0)\,dt_0 = x(t_0)$	2] $\sum_{k=-\infty}^{\infty} x[n]\cdot\delta[n-k] = x[k]$
3] $x(t) = \int_{-\infty}^{\infty} x(\tau)\cdot\delta(t-\tau)\cdot d\tau$	3] $x[n] = \sum_{k=-\infty}^{\infty} x[k]\cdot\delta[n-k]$
4] $\delta(t) = \frac{d}{dt}u(t)$	4] $\delta[u] = u[n] - u[n-1]$
5] $u(t) = \int_{0}^{\tau} \delta(t-\tau)\,d\tau$	5] $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$

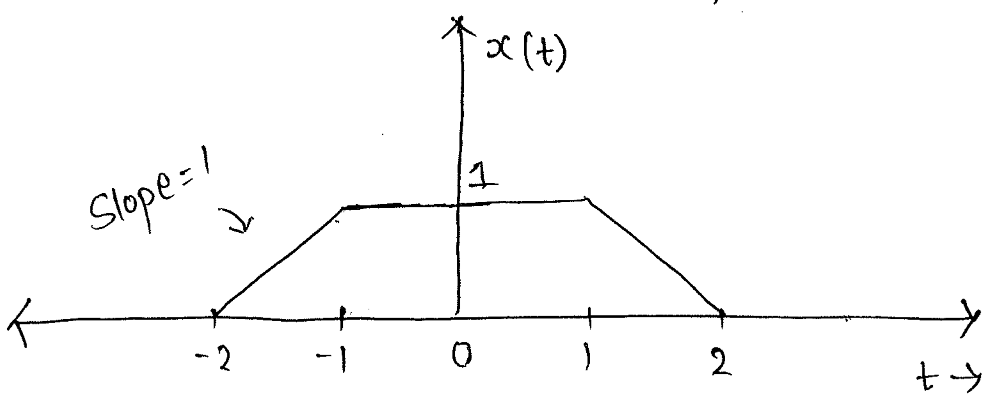
Q :- Draw $x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$

$r \rightarrow$ ramp signal

Sol^n :- Key

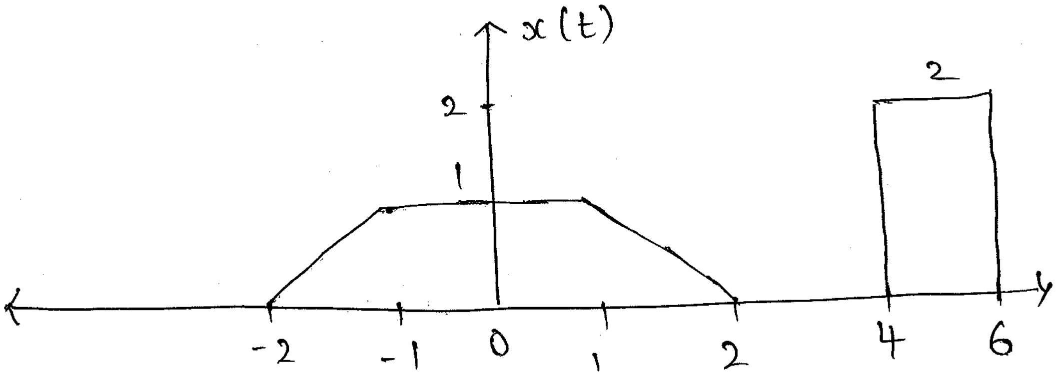
1. Arrange signals in ascending order
2. Allocate slope $\therefore$ $\underline{\text{Change in slope} = \text{Next slope} - \text{Previous slope}}$
3. Whenever $\underline{\text{sudden change}}$ = step signal

   " $\underline{\text{slope}}$ = ramp.

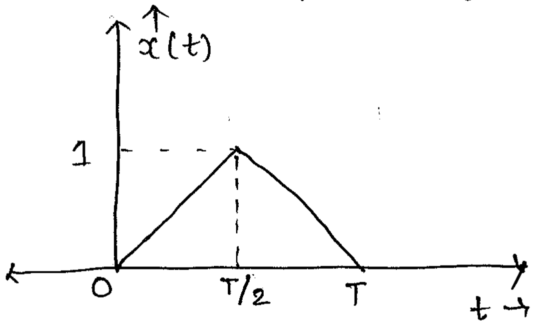


Q:- Draw $x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2) + 2u(t-4) - 2u(t-6)$

Soln:-



Q:- Write eqn for given graph:-

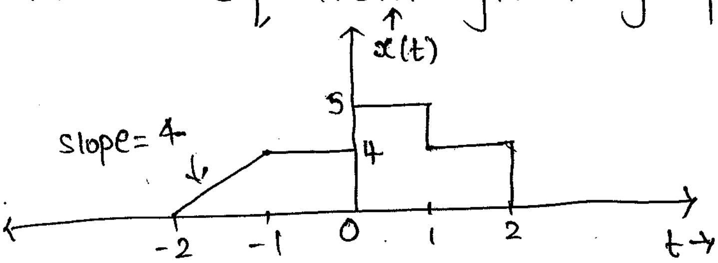


Soln:-

$$\text{Slope} = \frac{dy}{dx} = \frac{1}{T/2} = \frac{2}{T}$$

$$\frac{2}{T} r(t) - \frac{4}{T} r(t - T/2) + \frac{2}{T} r(t-T)$$

Q:- Write eqn from given graph:-



$$x(t) = 4r(t+2) - 4r(t+1) + u(t) - 1\,u(t-1) - 4\,u(t-2)$$

* Operations on signals *

I] Operations on amplitude of signal

(i) Amplitude shifting

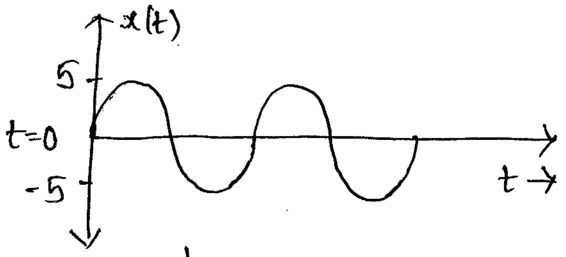
$\Rightarrow k + x(t)$

$\Rightarrow x(t) + k = y(t)$

$k > 0 \Rightarrow$ up-shift

$k < 0 \Rightarrow$ down-shift

Eg:- (1) $y(t) = x(t) + 5$



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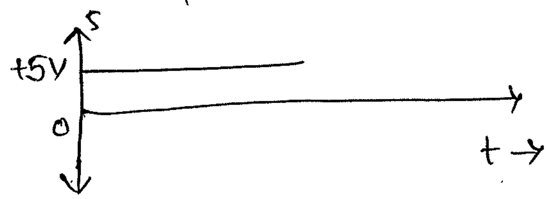
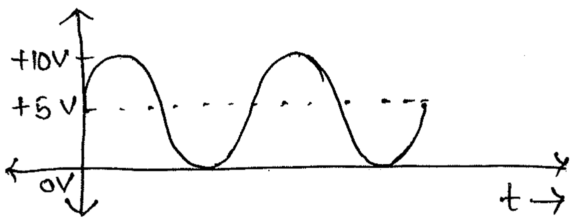
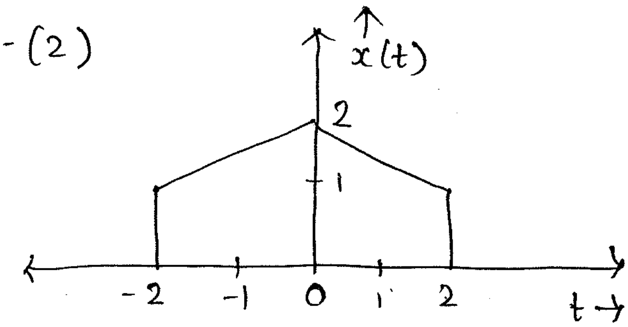
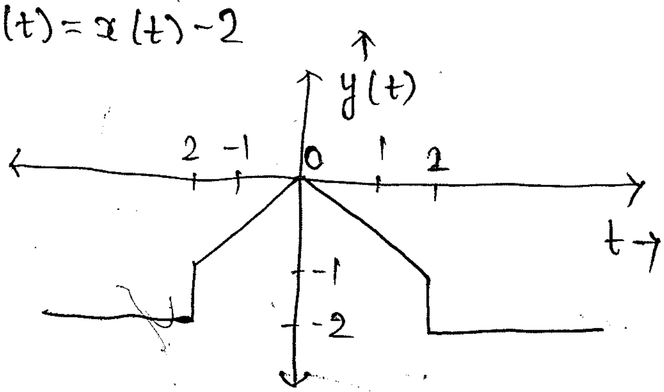




Eg:- (2)



$y(t) = x(t) - 2$



(ii) Amplitude scaling

$x(t) \longrightarrow k\cdot x(t)$

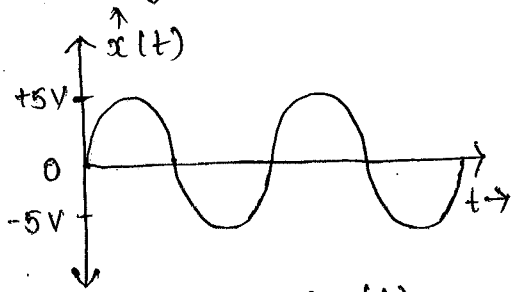
$y(t) = k\cdot x(t)$

- $K > 1$ → Amplification
- $K < 1$ → Attenuation

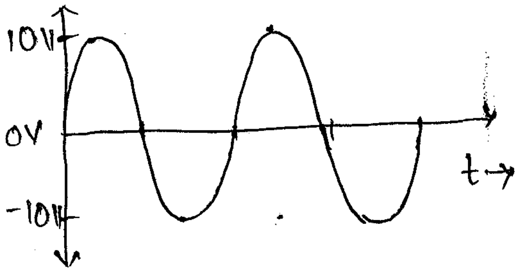
Let $x(t) = 5\sin\omega t$

$K = 2$

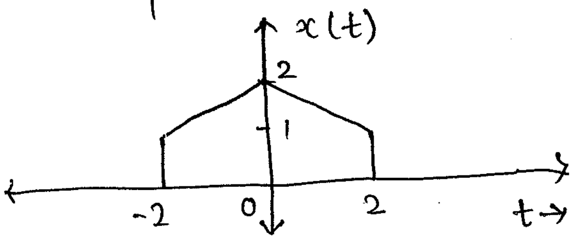
$y(t) = k\cdot x(t) = 10\sin\omega t$

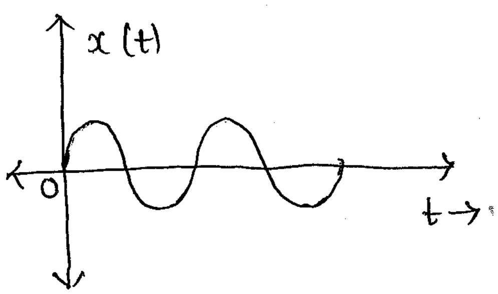


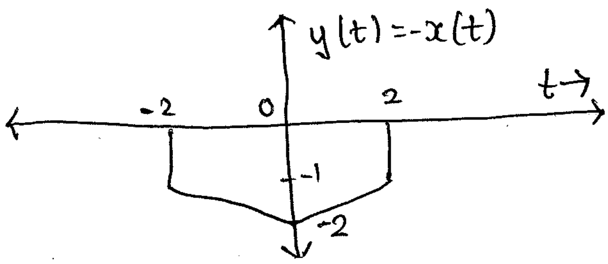
$y(t) = 2x(t)$

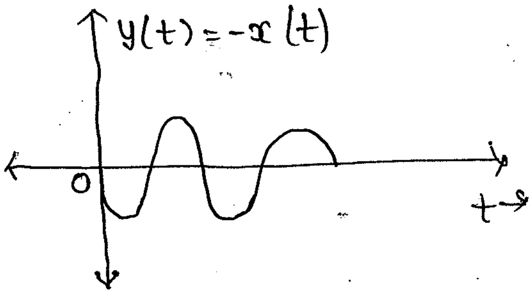


(iii) Amplitude reversal [folding w.r.t. X-axis]









* Operations on time of signal *

(i) Time shifting

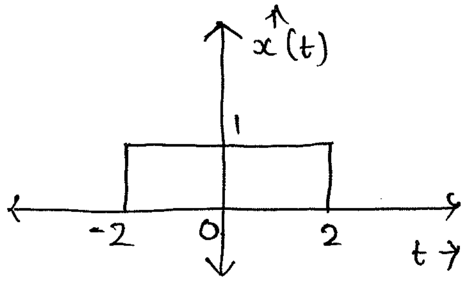
$$y(t) = x(t)$$

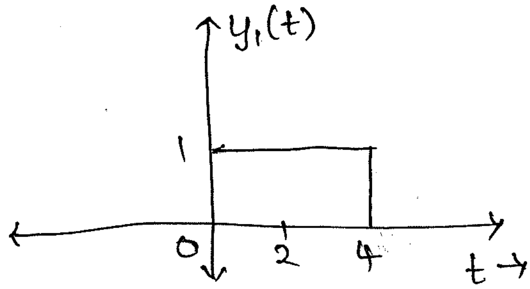
$$y(t) = x(t - t_0)$$

$t_0 > 0$ → Right shift [Delay]

$t_0 < 0$ → Left shift [Advance]

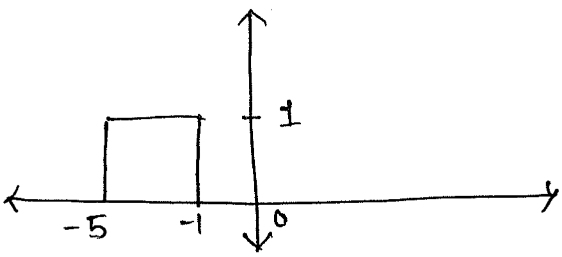
Eg:- (1) $y_1(t) = x(t-2)$



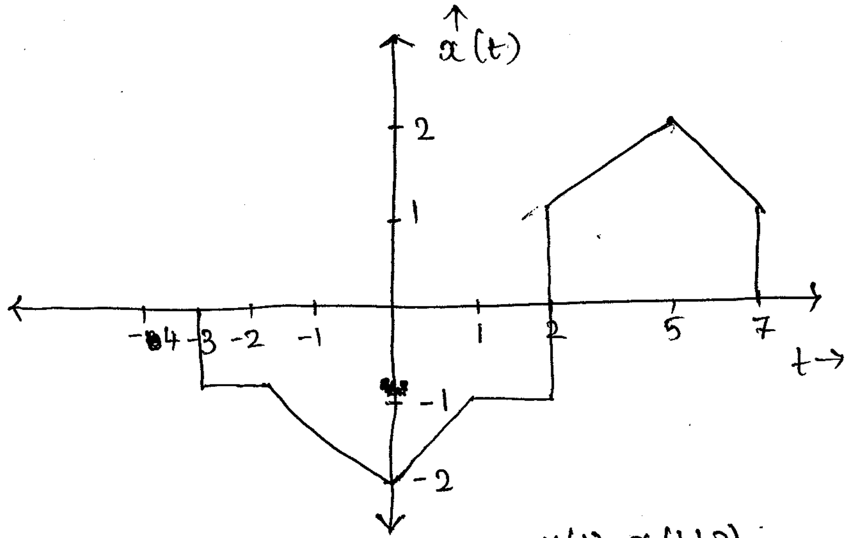


$$x(t) = 1 \quad , \; -2 < t < 2$$
$$= 0 \quad , \; \text{otherwise}$$

(2) $y_2(t) = x(t+3)$

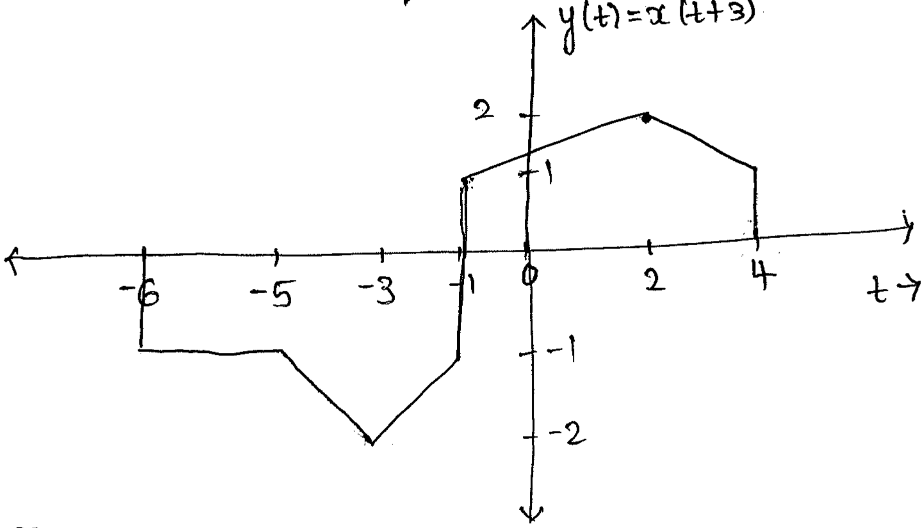


Q:-



Find $x(t+3)$

Sol^n:-

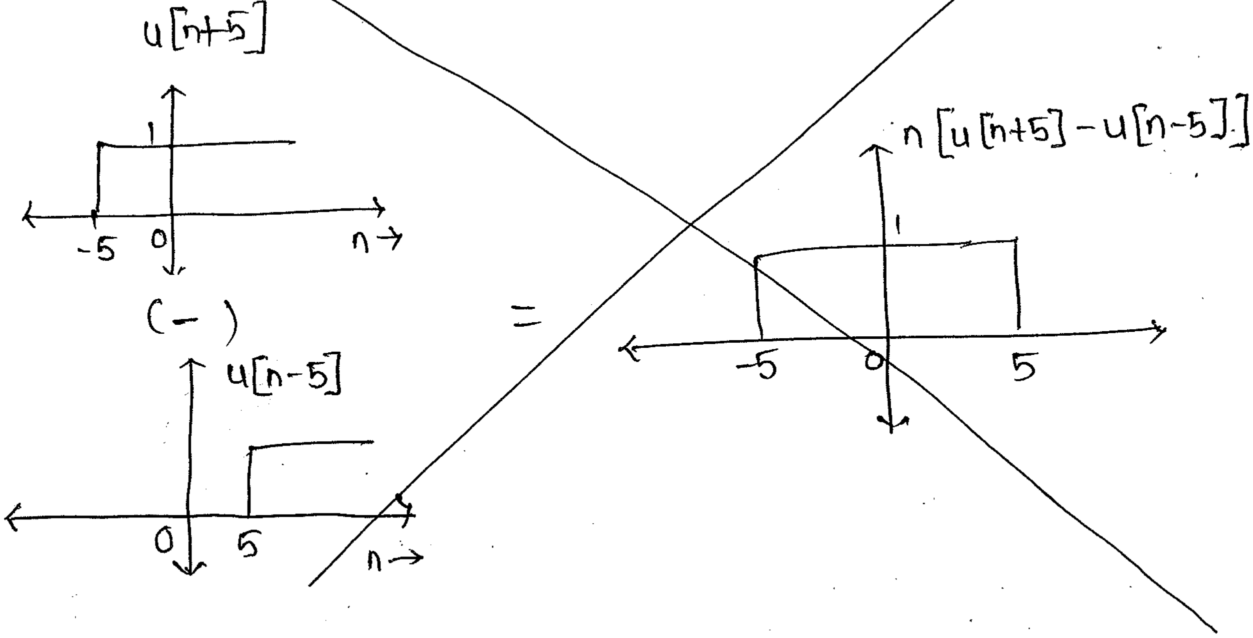


Q:- $\int_{-\infty}^{\infty} \delta(1-t)\cdot(t^3+4)\,dt$

$= \int_{-\infty}^{\infty} \delta(t-1)\,(t^3+4)\,dt$

$= 1+4 = 5$

Q:- Sketch $x[n] = n[u(n+5) - u(n-5)]$



Q:- $\int_{-\infty}^{\infty} t^2 \cdot \delta\left(-\frac{t}{2}+\frac{1}{2}\right) dt$

$= \int_{-\infty}^{\infty} t^2 \cdot \delta\left(\frac{1}{2}(t-1)\right) dt$

$= \frac{1}{1/2} \int_{-\infty}^{\infty} t^2 \cdot \delta(t-1)\, dt$

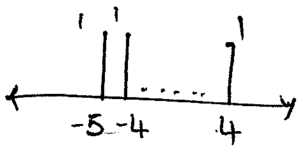
$= 2 \cdot 1 = 2$

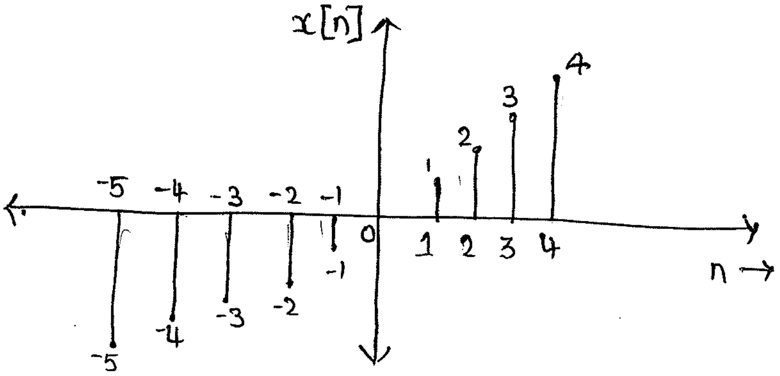
Q:- $\int_{0}^{\infty} \cos(t-\tau)\, \delta(\tau+1)\, d\tau$

$=$

Q:- $x[n] = n\left[u[n+5] - u[n-5]\right]$

$= n - 1 \quad , \; -5 \leq n \leq 4$





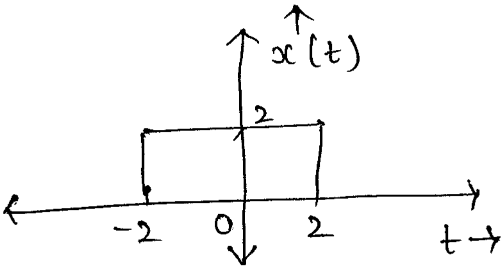
(ii) Time Scaling.

$$y(t) = x(\alpha t)$$

$\alpha < 1$ $\rightarrow$ expansion

$\alpha > 1$ $\rightarrow$ compression

Q:- $x(t) = 2 \quad , -2 < t < 2$
$\quad\quad = 0 \quad ,$ otherwise

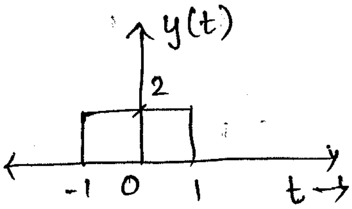


(i) $x(2t) = ?$

$\downarrow$

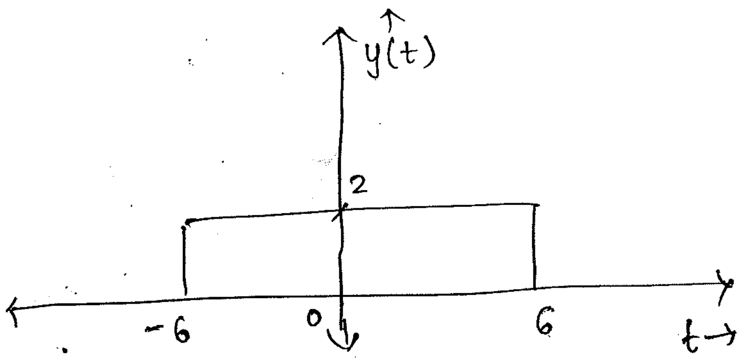
$\alpha = 2 > 1$

$$y(t) = x(2t) = 2 \quad , -2 < 2t < 2$$
$$= 2 \quad , -1 < t < 1$$
$$= 0 \quad , \text{otherwise}$$



(ii) $y(t) = x(t/3)$

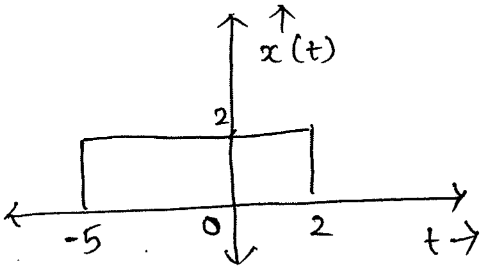
$$= 2 \quad , -2 < t/3 < 2$$
$$= 2 \quad , -6 < t < 6$$
$$= 0 \quad , \text{otherwise}$$



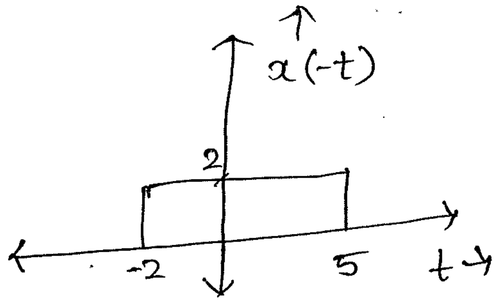
* Time Reversal *

$y(t) = x(-t)$ [folding w.r.t. Y-axis]

Q:- Let $x(t) = 2 \ , \ -5 < t < 2$
$= 0$, otherwise



$x(-t) = 2 \ , \ -5 < -t < 2$
$= 2 \ , \ -2 < t < 5$
$= 0$, otherwise



Q:- $y(t) = x(\alpha t)$

✓(i) $y(t-t_0) = x(\alpha(t-t_0))$

(ii) $y(t-t_0) = x(\alpha t - t_0)$

Sol^n:- Replace t by $t-t_0$.

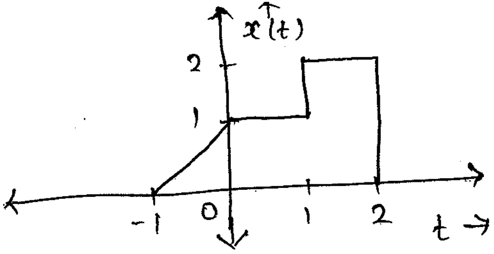
Q:- $y(t) = x(t-t_0)$

(i) $y(\alpha t) = x(\alpha(t-t_0))$

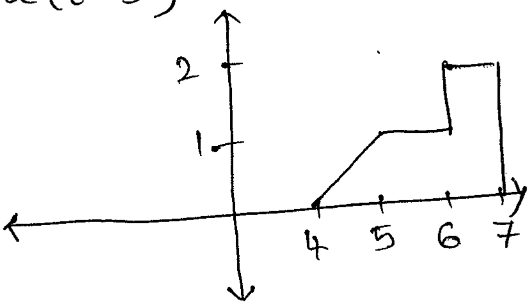
✓(ii) $y(\alpha t) = x(\alpha t - t_0)$

sol^n:- Replace t by αt

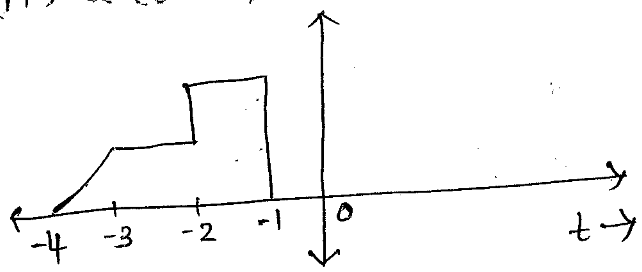
Q:-



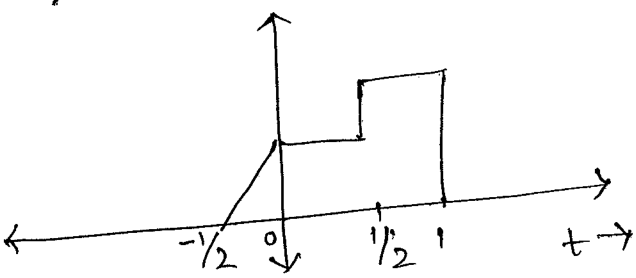
(i) $x(t-5)$



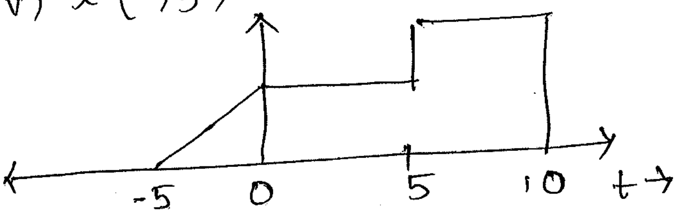
(ii) $x(t+3)$



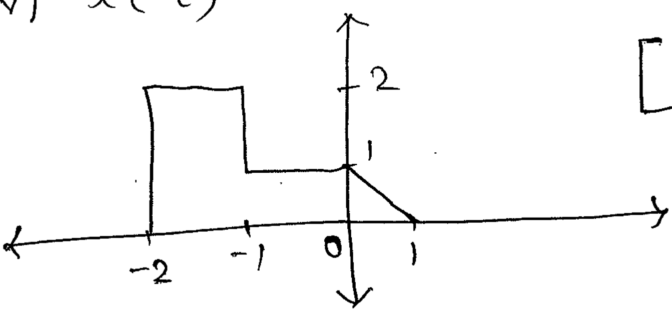
(iii) $x(2t)$



(iv) $x(t/5)$

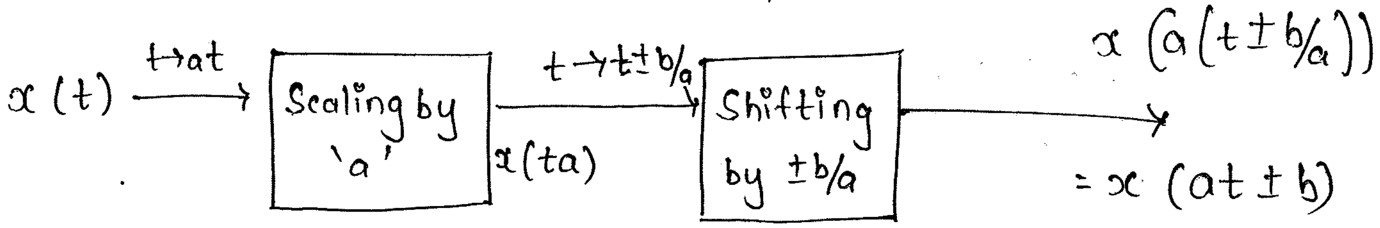


(v) $x(-t)$

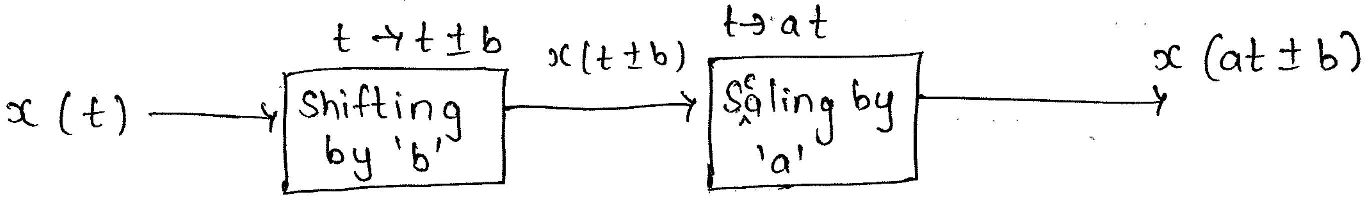


[folding w.r.t. Y-axis]

Method:-1 $\quad y(t) = x\left(a\left(t \pm b/a\right)\right)$



Method:-2

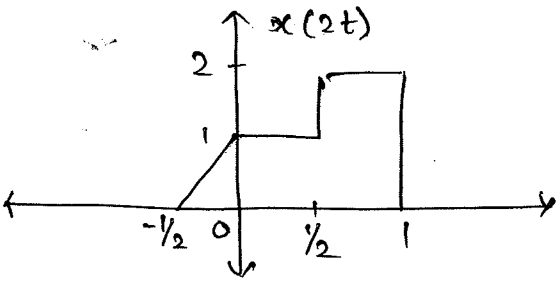


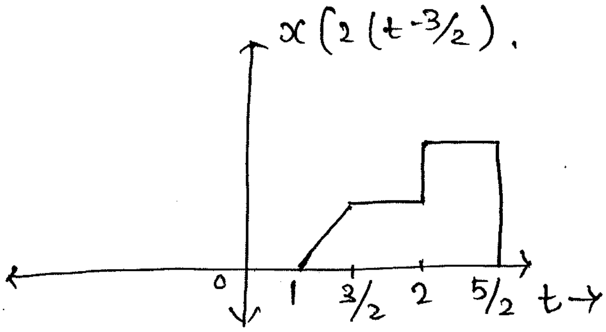
Any of the above method can be used

But for discrete time signal Method-2 is only prefered.

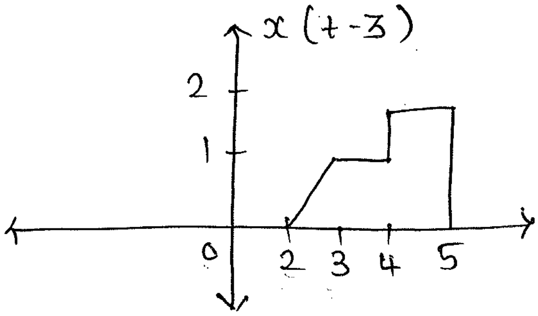
(6) $x(2t-3)$

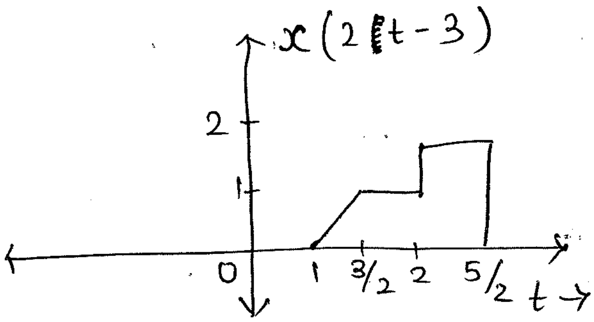
method:-1 $\quad x\left(2\left(t - 3/2\right)\right)$



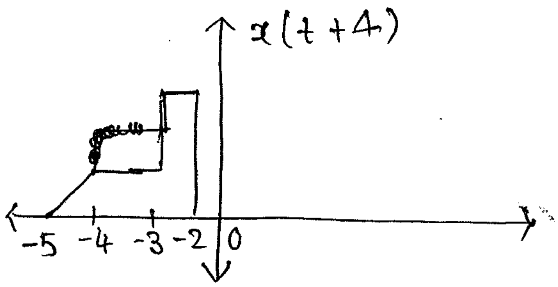


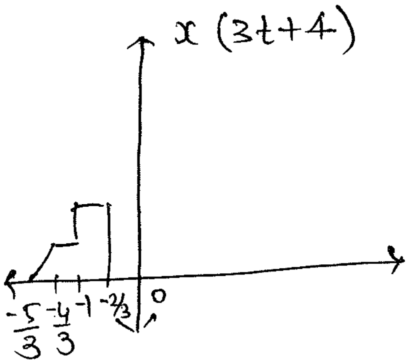
Method:-2

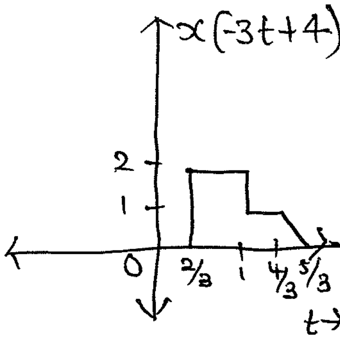




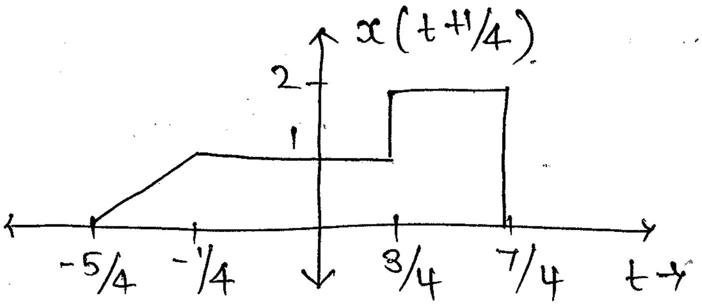
(7) $x(-3t+4)$

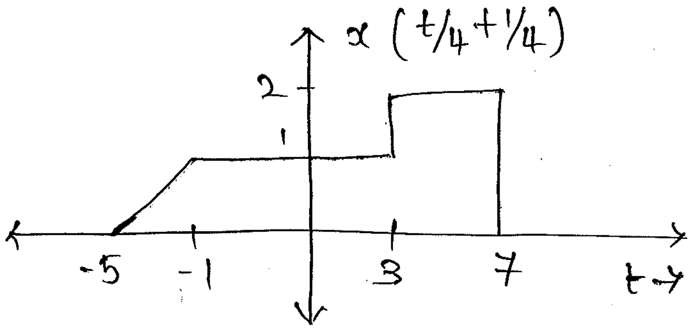


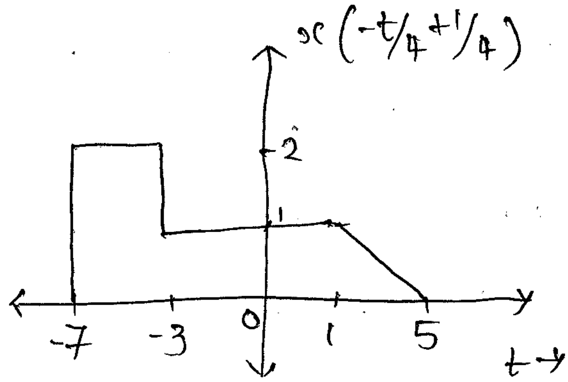




(8) $x\left(\frac{-t+1}{4}\right) = x\left(-t/4 + 1/4\right)$







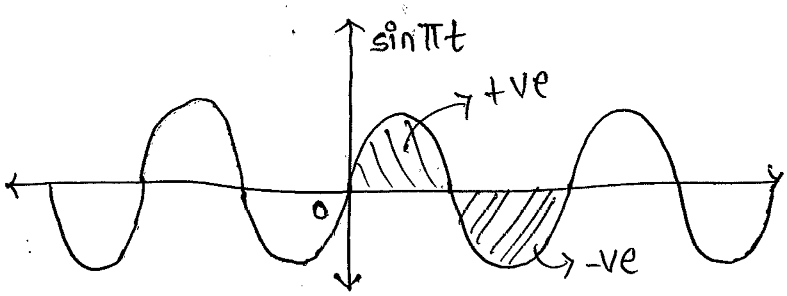
Q:- Draw $x(t) = u[\sin \pi t]$

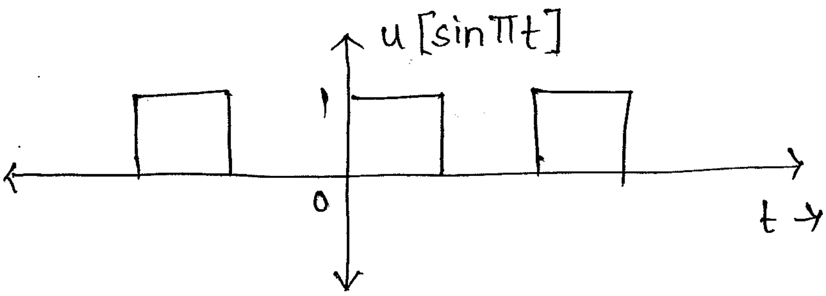
Sol:- $u[t] = 1 \ , t > 0$

$= 0 \ , t < 0$

$x(t) = 1 \ , \sin \pi t > 0$

$= 0 \ ; \sin \pi t < 0$





* Time Shifting *

$x[n \pm n_0]$

* Time Scaling *

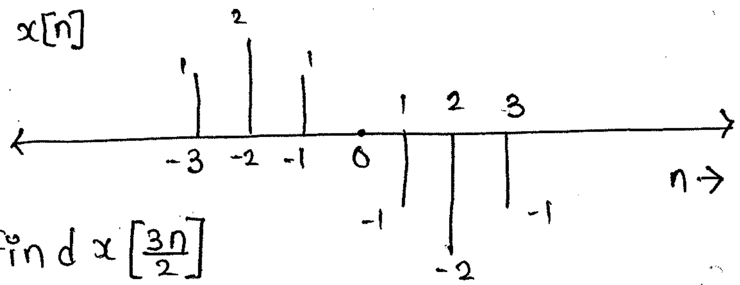
$y[n] = x[mn]$

$m > 1 \Rightarrow$ ~~Int~~ Decimation

$m < 1 \Rightarrow$ Interpolation

$\boxed{\text{First do interpolation than decimation}}$

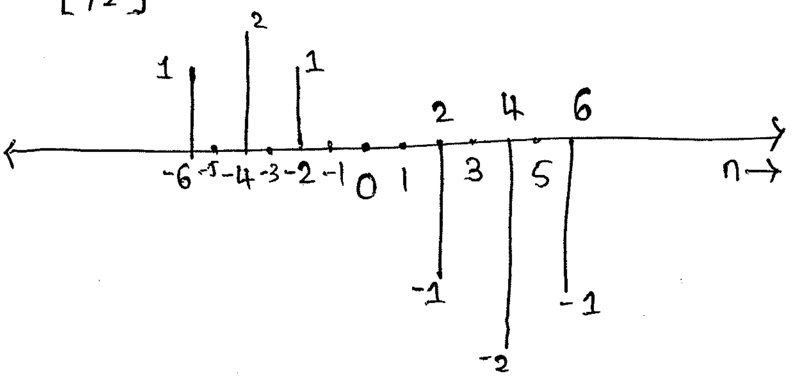
(1) $x[n]$



Find $x\left[\frac{3n}{2}\right]$

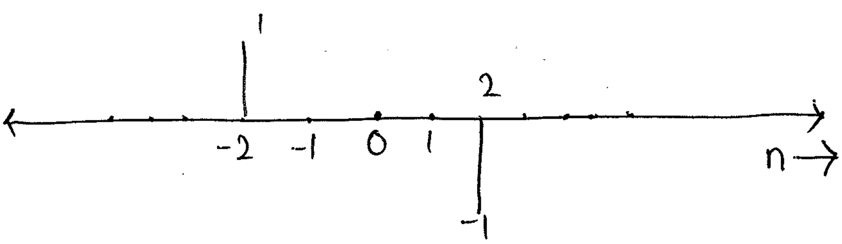
① Interpolation

$x[n/2]$



② Decimation

$x[3n/2]$



* For continuous.

① $x(t) \rightarrow$ Expansion 'α' $\xrightarrow{x(t/\alpha)}$ Compression 'α' $\xrightarrow{x(\alpha \cdot t/\alpha)}$ $x(t)$ ✓

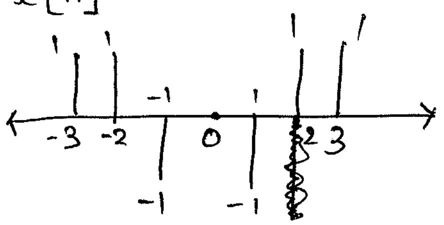
② $x(t) \rightarrow$ Compression 'α' $\xrightarrow{x(\alpha t)}$ Expansion 'α' $\xrightarrow{x(\alpha t \cdot 1/\alpha)}$ $x(t)$ ✓

* For discrete

① $x[n] \rightarrow$ Interpolation 'm' $\xrightarrow{x[n/m]}$ Decimation 'n' $\xrightarrow{x\left[\frac{n}{m} \cdot m\right]}$ $x'[n] = x[n]$ ✓

② $x[n] \rightarrow$ Decimation 'm' $\xrightarrow{x[mn]}$ Interpolation 'n' $\xrightarrow{x\left[mn \cdot \frac{1}{m}\right]}$ $x'[n] \neq x[n]$ ✗

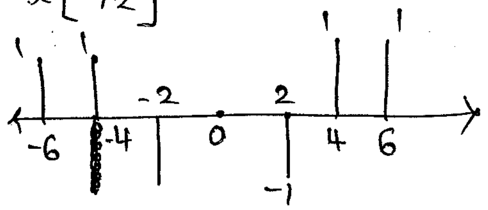
Eg:-1] $x[n]$

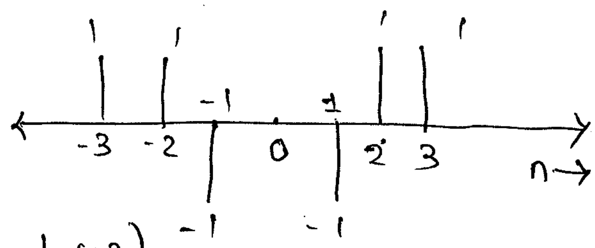


(Using method (1))

$x\left[\frac{n}{2} \cdot 2\right] = x[n]$

$x[n/2]$

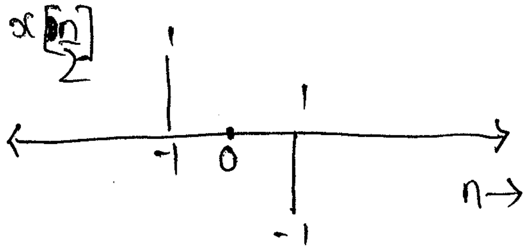




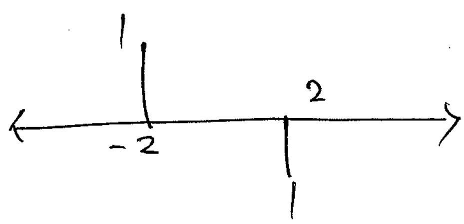
2] $x[2n]$

(Using method (2))

$x\left[\frac{2n}{2}\right]$



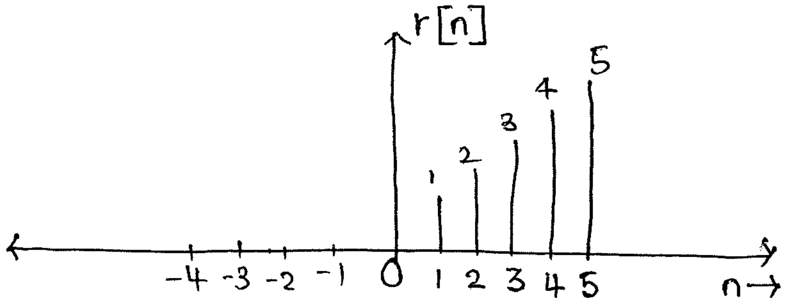
$x\left[\frac{n}{2} \cdot 2\right]$



$\neq x[n]$

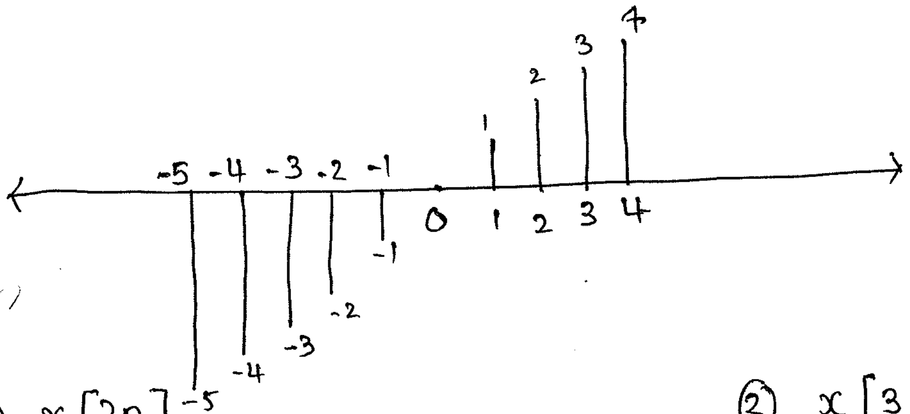
* Discrete Ramp Function *

$$r[n] = n \;,\; n \geq 0$$
$$\quad\;\; = 0 \;,\; n < 0$$



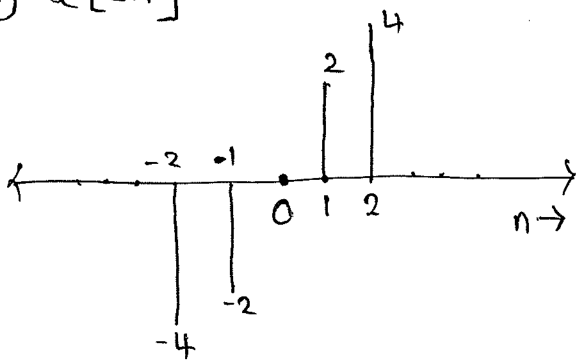
$$r[n] = n \cdot u[n]$$
$$= n \cdot 1 \;,\; n \geq 0$$
$$= 0 \quad ,\; n < 0$$

Q:- Sketch $x[n] = n \cdot \left[u[n+5] - u[n-5]\right]$



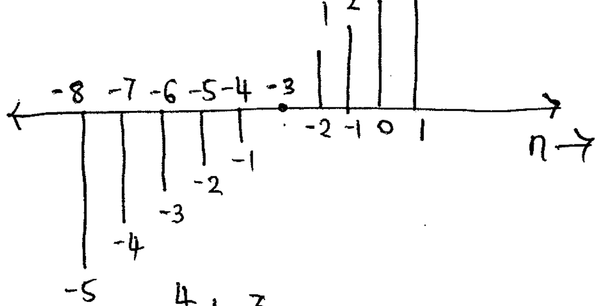
[In discrete,
first shifting
second scaling]

① $x[2n]$

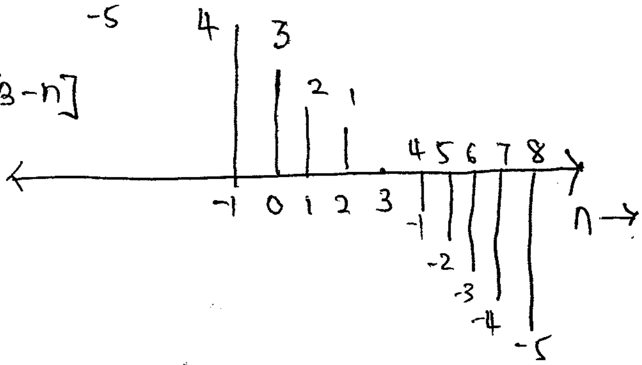


② $x[3-n]$
$= x[-n+3]$

$x[n+3]$



$x[3-n]$



(3) $x\left[\frac{3-n}{3}\right]$

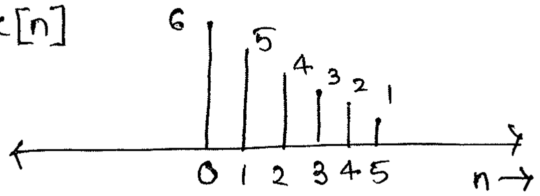
$= x\left[1-\frac{n}{3}\right] = x\left[\frac{-n}{3}+1\right]$

Q:- Given $x[n] = (6-n)\left[u[n] - u[n-6]\right]$

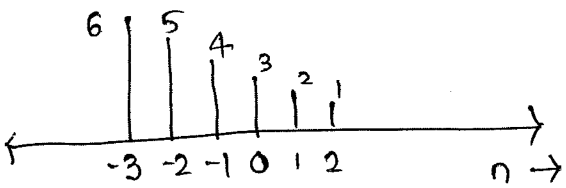
Sol^n:- $x[n] = (6-n)\cdot 1 \quad , 0 \le n \le 5$

$\qquad = 0 \quad$, otherwise

$x[n]$



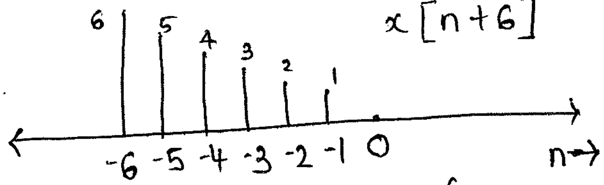
(1) $x[n+3]$



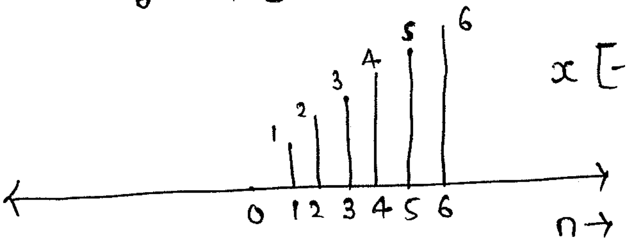
(2) $x[6-n]$

$= x[-n+6]$

$x[n+6]$

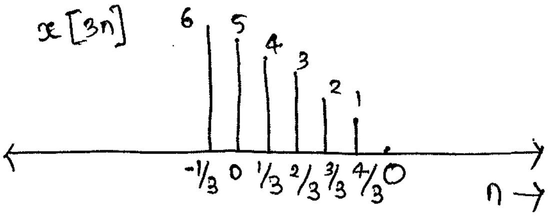


$x[-n+6]$

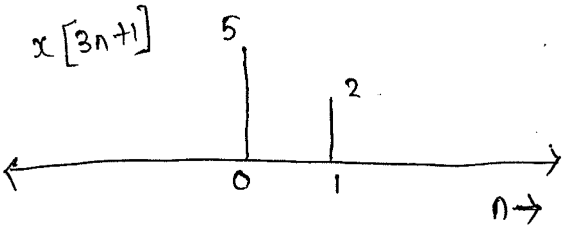


(3) $x[3n+1]$

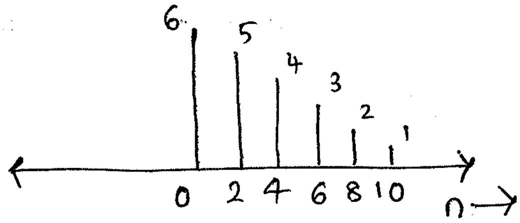
$x[3n]$



$x[3n+1]$



(4) $x[n/2]$



(5) $x[n-1]\cdot\delta[n-3]$

$= x[3-1]\cdot\delta[n-3]$

$= x[2]\cdot\delta[n-3]$

$= 4\cdot\delta[n-3]$

Q:- If $x[n] = 1 - \sum_{k=4}^{\infty} \delta[n-1-k]$ such that $x[n] = u[Mn - n_0]$

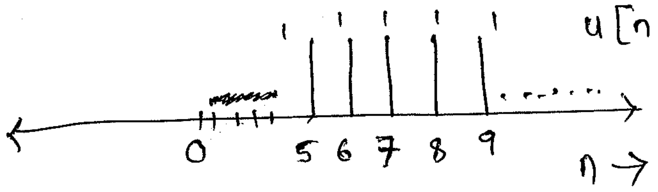
GATE

Find M and n_0.

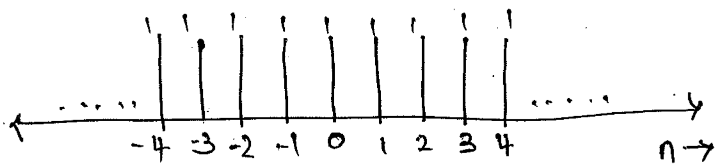
Sol^n:- $x[n] = 1 - \left[\delta[n-5] + \delta[n-6] + \delta[n-7] + \dots\right]$

$x[n] = 1 - u[n-5]$

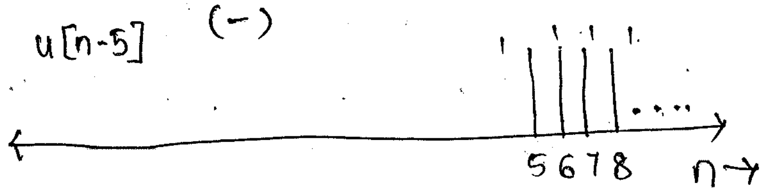
$u[n-5]$.



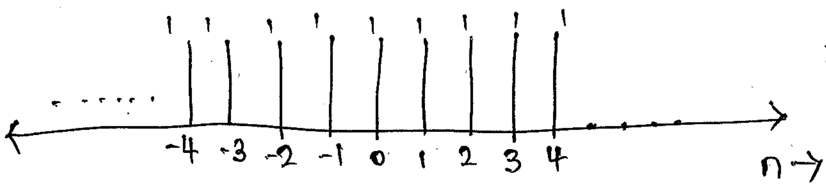
Now, 1



$u[n-5]$ $(-)$

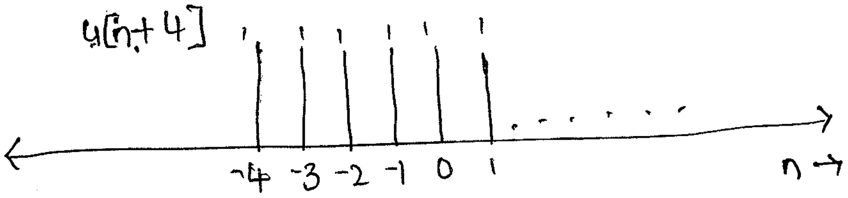


$=$

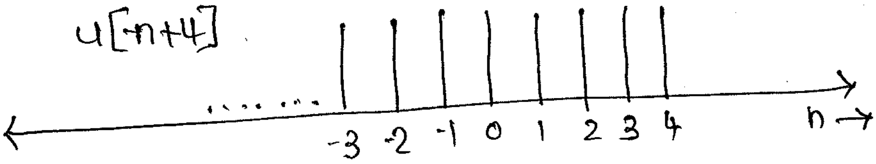


$\Rightarrow u[-n+4]$

$u[n+4]$



$u[-n+4]$



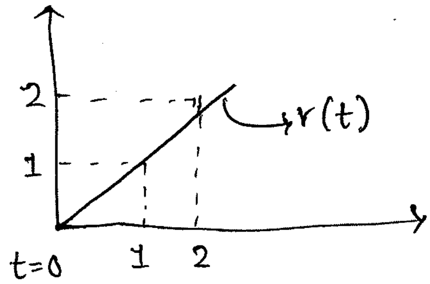
$\therefore u[-n+4] = u[Mn - n_0]$

$\boxed{M = -1}$, $\boxed{n_0 = -4}$

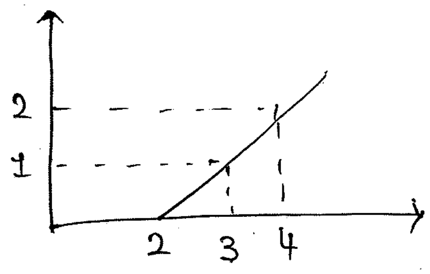
$r(t) = t \cdot u(t)$

$= t \ , \ t \geq 0$

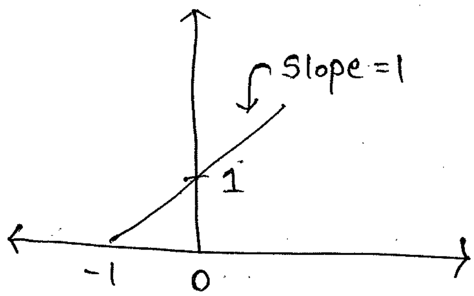
$= 0 \ , \ t < 0$



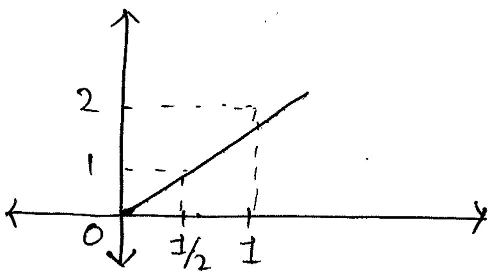
Q. Sketch (1) $r(t-2)$:-



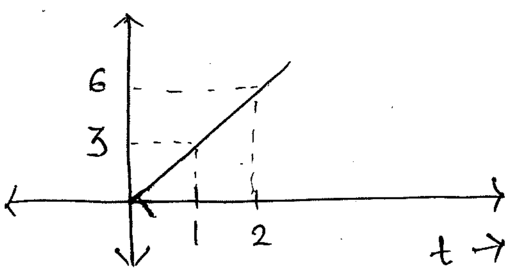
(2) $r(t+1)$



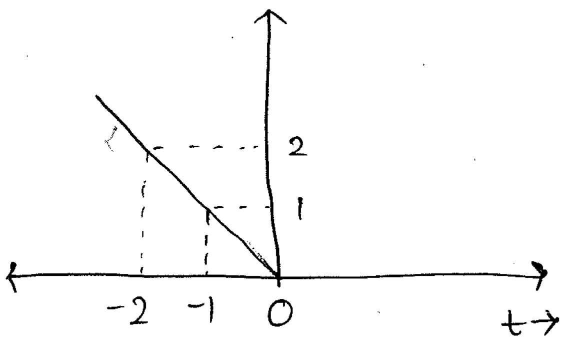
(3) $r(2t) = 2t \cdot u(2t)$



(4) $3r(t)$

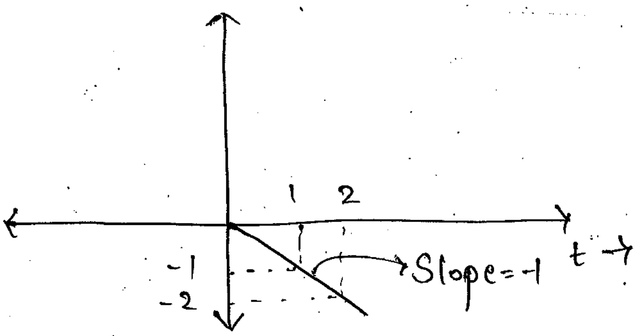


(5) $r(-t)$

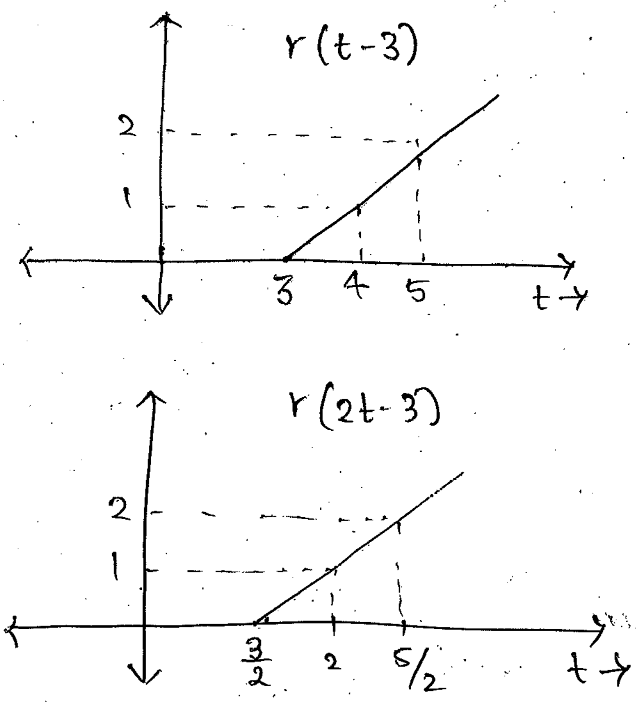


(6) $-r(t) = u(-t)\cdot(-t)$

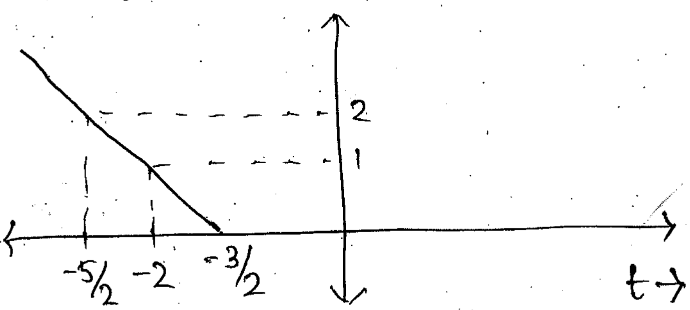
$= -t \cdot u(-t)$



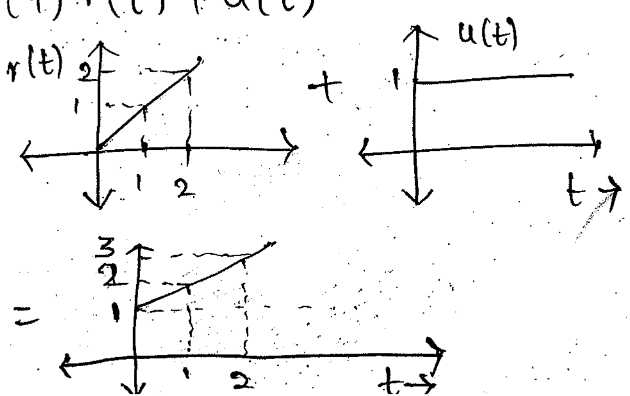
(7) $r(2t-3)$



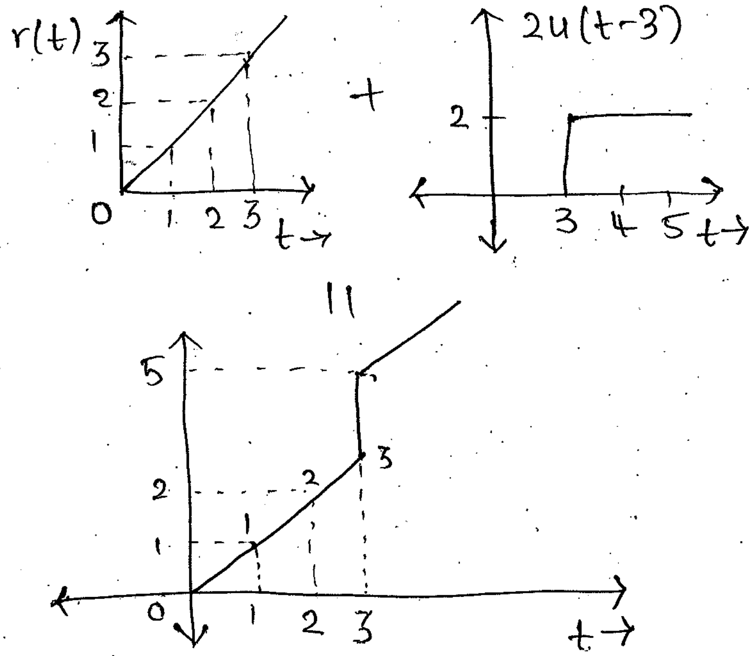
(8) $r(-2t-3)$



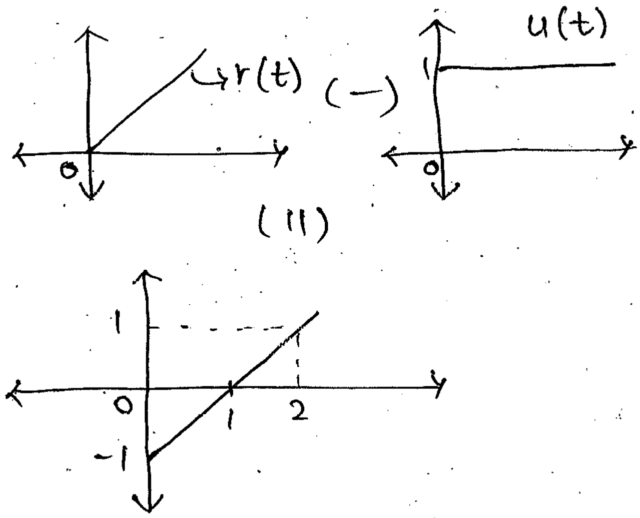
(9) $r(t)+u(t)$



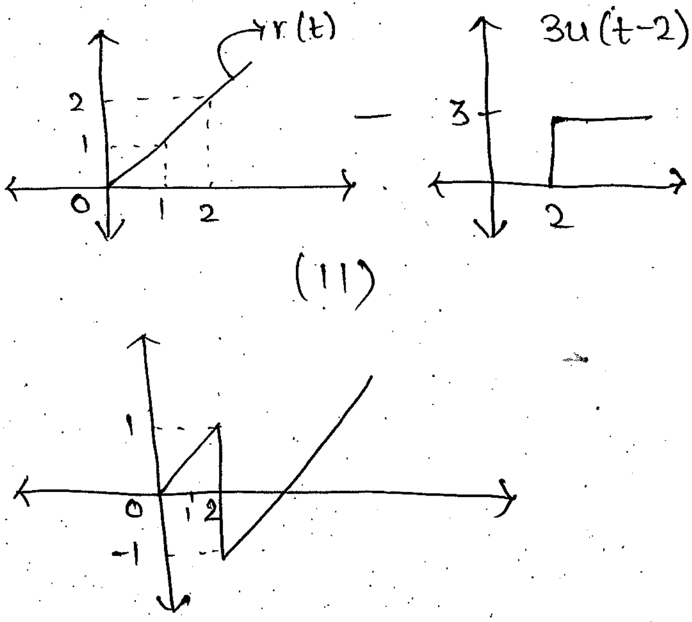
(10) $r(t)+2u(t-3)$



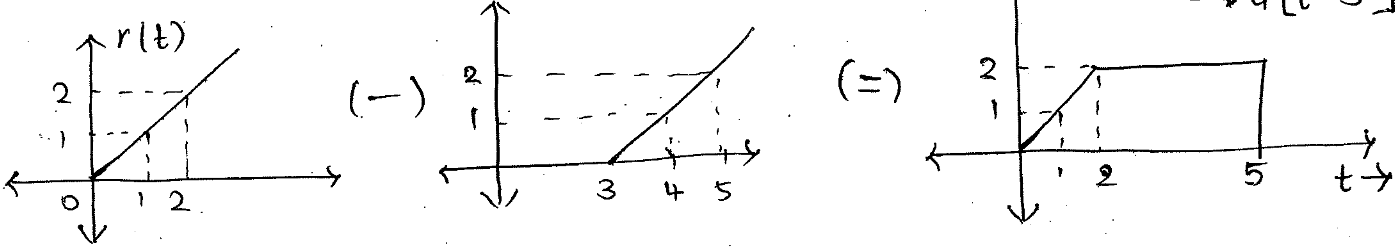
(11) $r(t)-u(t)$



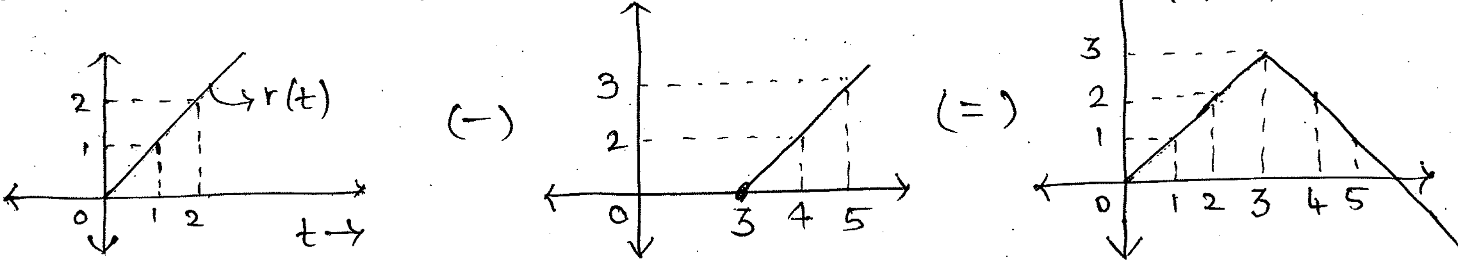
(12) $r(t)-3u(t-2)$



(13) $r(t) - r(t-3)$

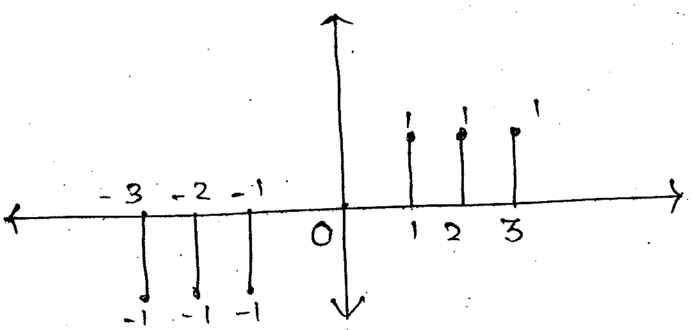


(14) $r(t) - 2r(t-3)$



* Discrete signum function *

$$\text{sgn}[n] = \begin{cases} 1, & n > 0 \\ 0, & n = 0 \\ -1, & n < 0 \end{cases}$$

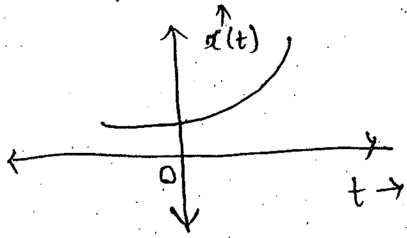


* Exponential function *

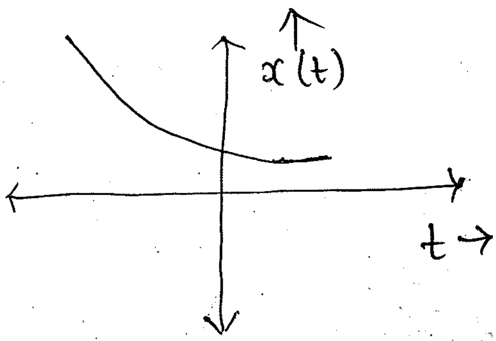
$$x(t) = e^{\alpha t}$$

(i) $\alpha > 0$

$x(t) = e^{\alpha t}$



(ii) $\alpha < 0$



Q:- Is u[n] is sample version of u[t]!

Sol^n:- No. bcz. u[t] is not defined at $t=0$ but

$u[n] = 1$, $n=0$

$\Rightarrow$ Every discrete signal is not sample version of continuous signal.

Draw $x[n] = \alpha^n \cdot u[n]$

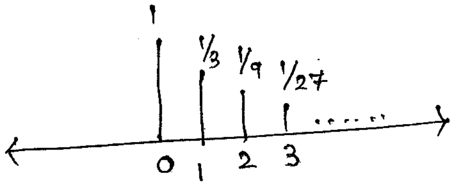
① $0 < \alpha < 1$

Let, $\alpha = 1/3$

$= \left(\frac{1}{3}\right)^n \cdot u[n]$.

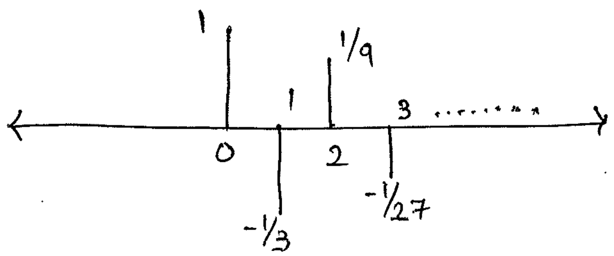
$= \left(\frac{1}{3}\right)^n \cdot 1$, $n \geq 0$

$= 0$, $n < 0$



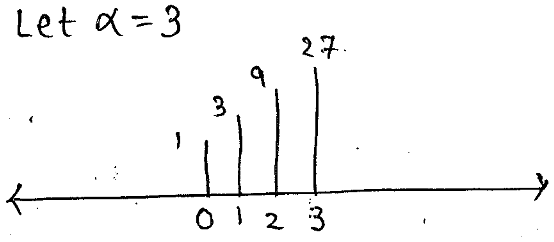
② $-1 < \alpha < 0$

Let $\alpha = -1/3$

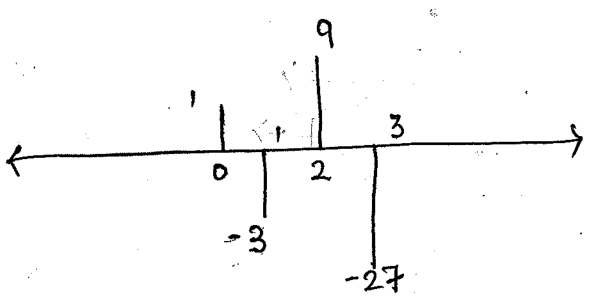


③ $\alpha > 1$

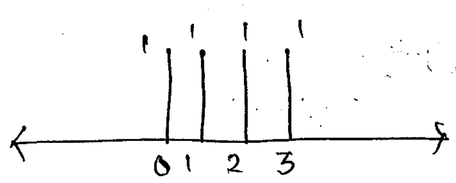
Let $\alpha = 3$



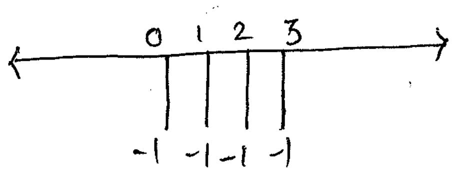
④ $\alpha < -1$, $\alpha = -3$



⑤ $\alpha = 1$

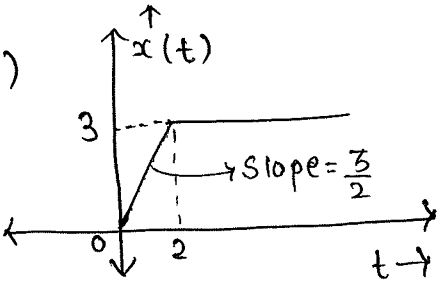


⑥ $\alpha = -1$



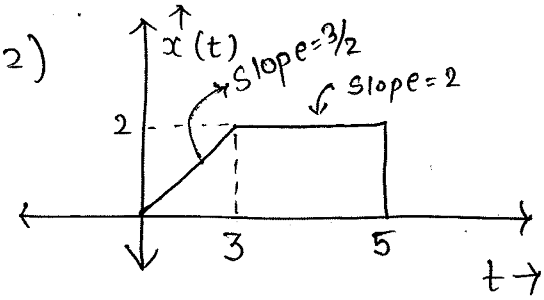
Q:- Write expression in terms of $u(t)$ & $r(t)$

(1)



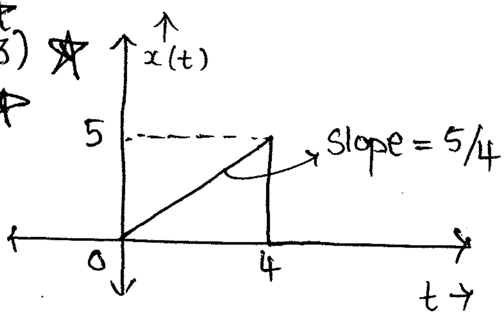
$$= \frac{3}{2} r(t) - \frac{3}{2} r(t-2)$$

(2)



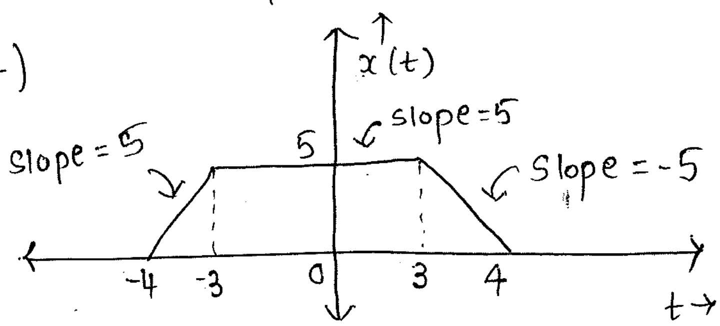
$$= \cancel{\frac{3}{2}}\ \frac{2}{3} r(t) - \frac{2}{3} r(t-3) - 2u(t-5)$$

(3)



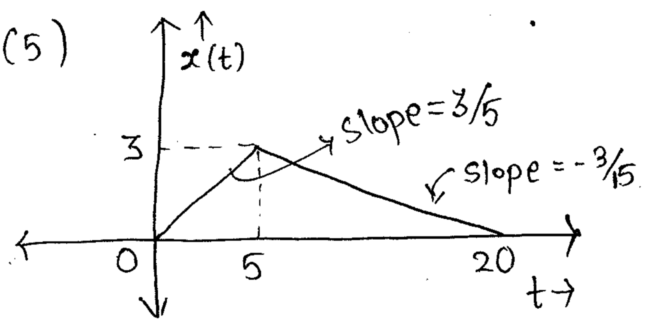
$$= \frac{5}{4} r(t) - \frac{5}{4} r(t-4) - 5u(t-4)$$

(4)



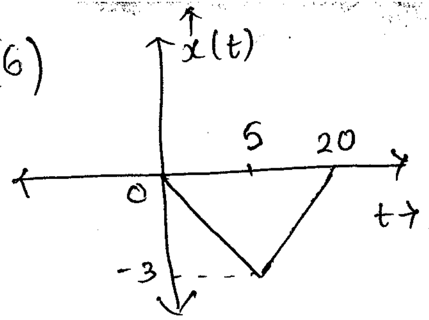
$$= 5r(t+4) - 5r(t+3) + 5r(t-4) - 5r(t-3)$$

(5)



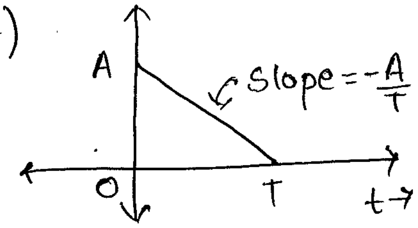
$$= \frac{3}{5} r(t) - \frac{4}{5} r(t-5) + \frac{1}{5} r(t-20)$$

(6)



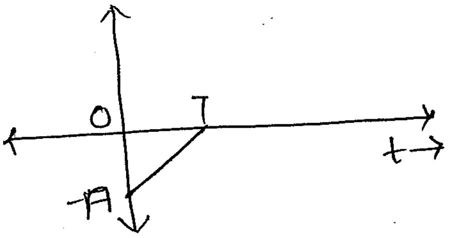
$$= -\frac{3}{5} r(t) + \frac{4}{5} r(t-5) - \frac{1}{5} r(t-20)$$

(7)



$$= A u(t) - \frac{A}{T} r(t) + \frac{A}{T} r(t-T)$$

(8)



amplitude inversal of Q.7

$$= -A u(t) + \frac{A}{T} r(t) - \frac{A}{T} r(t-T)$$

(1) $\int_{-\infty}^{\infty} x(t) \cdot dt = \text{AREA}$

(2) $\sum_{n=-\infty}^{\infty} x[n] = \text{AREA}$

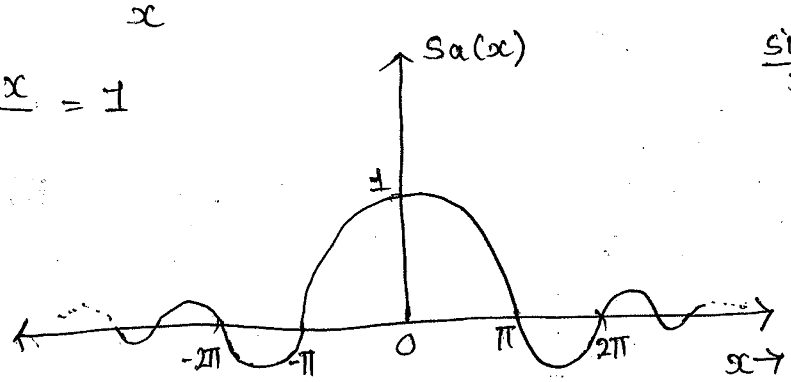
(3) $\int_{-\infty}^{\infty} |x(t)|\, dt = \text{Absolute value}$

(4) $\sum_{n=-\infty}^{\infty} |x[n]| = \text{Absolute value}$

* S.a function *

$$x(t) = Sa(x) = \frac{\sin x}{x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



$\frac{\sin x}{x} = 0$, $\sin x = 0$

$x = \pm n\pi$

$\Rightarrow n = 1, 2, 3, \ldots$

* sinc function

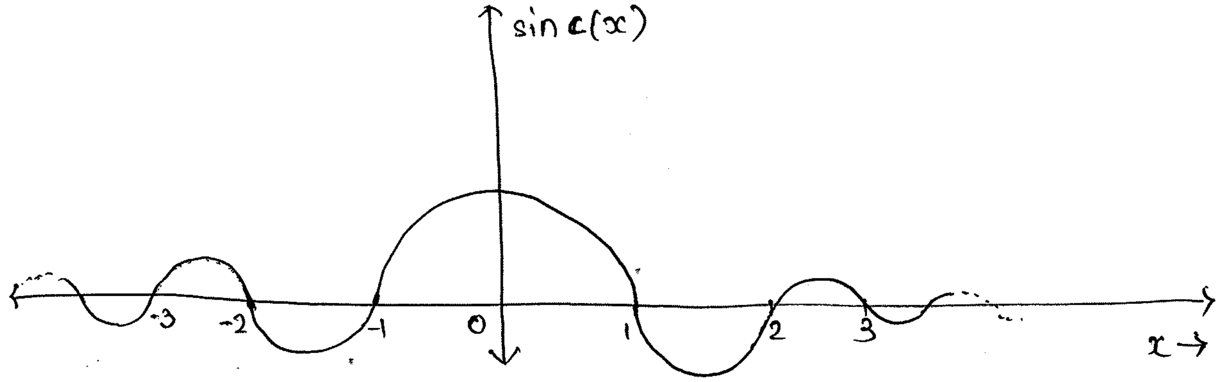
$$\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

$$\frac{\sin \pi x}{\pi x} = 0 \quad , \quad \sin \pi x = 0$$

$$\pi x = \pm n\pi$$

$$x = \pm n$$

$$\lim_{x \to 0} \text{sinc}(x) = \lim_{x \to 0} \frac{\sin \pi x}{\pi x} = 1$$



* Energy and Power of signals

① Energy signal

(i) Continuous time

$$E_x(t) = \int_{-\infty}^{\infty} (x(t))^2 \, dt$$ ← real-valued signals

$$E_x(t) = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$ ← complex-value signals

(ii) Discrete Time

$x[n]$

$$E_x[n] = \sum_{n=-\infty}^{\infty} x[n]^2$$ ← Real-value signal

$$E_x[n] = \sum_{n=-\infty}^{\infty} |x[n]|^2$$ ← Complex value signal

(2) Power signal

(i) Continuous time

(1) Periodic signal $x(t)$

$$\boxed{P_{avg.} = \frac{1}{T}\int_{T} |x(t)|^2 dt}$$

fundamental time period

(2) Non-periodic signal

$$\boxed{P_{avg.} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt}$$

(ii) Discrete signal

(1) Periodic signal $x[n]$

$$\boxed{P_{avg.} = \frac{1}{N+1}\sum_{n=0}^{N} |x(n)|^2}$$

fundamental time period

(2) Non-periodic signal

$$\boxed{P_{avg.} = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x(n)|^2}$$

$-N \quad -1 \quad 0 \quad 1 \quad N$

$-N$ to $-1 \Rightarrow N$
1 to $N \Rightarrow N$ $\Big\}$ $+ = 2N+1$
$n=0 \Rightarrow 1$ $+$

Q: Find Energy of following signal:-

(1) $e^{-at}\cdot u(t)$

(2) $e^{at}\cdot u(-t)$

(3) $(e^{-at} + e^{-bt})u(t)$

Sol^n:- (1)
$$E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$
$$= \int_{-\infty}^{\infty} (e^{-at}\cdot u(t))^2 dt$$
$$= \int_{0}^{\infty} e^{-2at}\cdot dt$$
$$E_{x(t)} = \left[\frac{e^{-2at}}{-2a}\right]_0^{\infty} = \frac{e^{-2a\infty} - e^{0}}{-2a} = \frac{1-e^{-2a\cdot\infty}}{2a}$$

(i) If $a > 0$
$$E_{x(t)} = \frac{1}{2a}$$

(ii) If $a < 0$
$$E_{x(t)} = \frac{1-e^{\infty}}{2a} = \frac{-\infty}{2a}_{\,a<0} = \infty$$

NOTE:- $e^{-at} \cdot u(t)$

$$E = \frac{1}{2a}, \quad a > 0$$

$$E = \infty, \quad a < 0$$

Eg:- (1) $e^{3t} \cdot u(-t)$

$$= \frac{1}{6}$$

(2) $e^{-3t} \cdot u(-t)$ (3) $e^{-3t} \cdot u(t)$ (4) $e^{3t} \cdot u(t)$

$E = \infty$ $E = 1/6$ $E = \infty$

(2) $e^{at} \cdot u(-t)$

$$E = \int_{-\infty}^{\infty} (e^{at} \cdot u(-t))^2 dt$$

$$= \int_{-\infty}^{0} e^{2at} \cdot dt$$

$$E = \frac{1}{2a} \left[e^{2at}\right]_{-\infty}^{0} = \frac{1}{2a} \quad [a > 0]$$

$$E = \infty \quad [a < 0]$$

(3) $x(t) = (e^{-at} + e^{-bt}) u(t)$

$$E = \int_{-\infty}^{\infty} \left((e^{-at} + e^{-bt}) \cdot u(t)\right)^2 dt$$

$$= \int_{0}^{\infty} (e^{-at} + e^{-bt})^2 dt$$

$$= \int_{0}^{\infty} (e^{-2at} + e^{-2bt} + 2 e^{-at} \cdot e^{-bt}) dt$$

$$= \int_{0}^{\infty} (e^{-2at} + e^{-2bt} + 2 e^{-(a+b)t}) dt$$

$$= \left[\frac{e^{-2at}}{-2a}\right]_{0}^{\infty} + \left[\frac{e^{-2bt}}{-2b}\right]_{0}^{\infty} + \frac{2}{-(a+b)} \left[e^{-(a+b)t}\right]_{0}^{\infty}$$

$$= \frac{e^{0} - e^{-2a \cdot \infty}}{2a} + \frac{e^{0} - e^{-2b \cdot \infty}}{2b} + \frac{2}{a+b} \left(e^{0} - e^{-(a+b) \cdot \infty}\right)$$

If $a>0$ & $b>0$	If $a<0$ & $b<0$	If $a>0$ & $b<0$ OR $a<0$ & $b>0$
$E=\frac{1}{2a}+\frac{1}{2b}+\frac{2}{a+b}$	$E=\infty$	$E=\frac{1}{2a}+\infty+\infty=\infty$

(4) A·(d.c.)

$$E=\int_{-\infty}^{\infty}A^2\,dt$$

$$E=[A^2 t]_{-\infty}^{\infty}$$

$$E=A^2(\infty-(-\infty))$$

$$E=\infty$$

Eg:- $(e^{-3t}+e^{-5t})u(t)$

$$E=\frac{1}{6}+\frac{1}{10}+\frac{2}{8}=\frac{31}{60}$$

Eg:- $(e^{-3t}+e^{5t})u(t)$

$$E=\frac{1}{6}+\infty=\infty$$

Eg:- $(e^{3t}+e^{-5t})u(t)$

$$E=\infty$$

Eg:- $(e^{3t}+e^{5t})\cdot u(t)$

$$E=\infty$$

(5) $A\cdot e^{j\omega_0 t}=x(t)$

$$E=\int_{-\infty}^{\infty}(x(t))^2\,dt$$

$$E=\int_{-\infty}^{\infty}A^2\cdot(e^{j\omega_0 t})^2\,dt \quad \left[\begin{array}{l}=|e^{j\omega_0 t}| \\ =|\cos\omega_0 t+j\sin\omega_0 t| \\ =\sqrt{\cos^2\omega_0 t+\sin^2\omega_0 t} \\ =1\end{array}\right]$$

$$\boxed{\begin{array}{l}e^{j\theta}=\cos\theta+j\sin\theta \\ e^{\theta}=\cosh\theta+\sinh\theta\end{array}}$$

$$E=\int_{-\infty}^{\infty}A^2\,dt=\infty$$

(6) $x(t)=A\sin(\omega t+\theta)$

$$E=\int_{-\infty}^{\infty}x^2(t)\,dt$$

$$=\int_{-\infty}^{\infty}A^2\sin^2(\omega t+\theta)\,dt$$

$$=\int_{-\infty}^{\infty}A^2\left[\frac{1-\cos 2(\omega t+\theta)}{2}\,dt\right]$$

$$=\int_{-\infty}^{\infty}\frac{A^2}{2}\left[t-\frac{\sin 2(\omega t+\theta)}{2\omega}\right]=\frac{A^2}{2}\left[\infty-\dots\right]=\infty$$

(7) $A\cos(\omega t+\theta) = x(t)$

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} A^2\cos^2(\omega t+\theta)\,dt$$

$$= \frac{A^2}{2}\left[t + \frac{\cos 2(\omega t+\theta)}{\omega t}\right]_{-\infty}^{\infty}$$

$$E = \frac{A^2}{2}\left[\infty + \dots\right] = \infty$$

(8) $x(t) = u(t)$

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} u^2(t)\cdot dt$$

$$= \int_{0}^{\infty} 1\,dt$$

$E = \infty$

(9) $x(t) = r(t)$

$r(t) = t,\ t \geq 0$
$\quad\ = 0,\ t < 0$

$$E = \int_0^{\infty} t^2\,dt$$

$$E = \left[\frac{t^3}{3}\right]_0^{\infty} = \infty$$

If $x(t) \Rightarrow E_x(t)$ $\alpha\cdot x(t) \Leftarrow\Rightarrow \alpha^2 E_x(t)$
Energy of any signal is depends on amplitude & width.

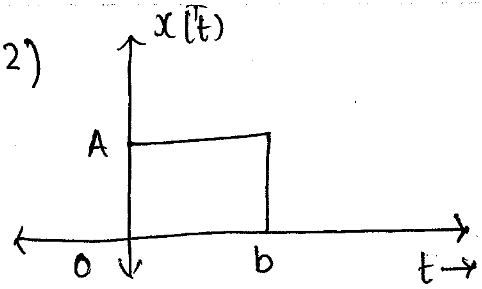
(10) -4

$E = \infty$

(11) $\sin(4t-30^\circ)$

$E = \infty$

(12)



$$x(t) = A \quad , 0 < t < b$$
$$= 0 \quad , \text{otherwise}$$

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$E = \int_0^b A^2 \cdot dt = A^2 \cdot b$$

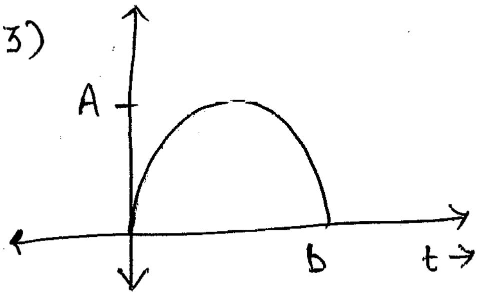
A^2 ← Amplitude, b ← width

$$E = \int_0^b A^2 dt = A^2 \cdot b.$$

If $\alpha > 1$ [time scaling factor]

then energy of signal decreases

(13)



$$x(t) = \sin \omega t$$

$$\omega = \frac{2\pi}{T} \quad [T = 2b] \leftarrow \text{full cycle time period}$$

$$\omega = \frac{2\pi}{2b} = \frac{\pi}{b}$$

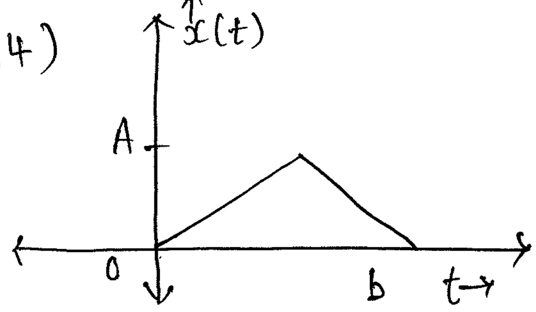
$$\therefore x(t) = A \sin \frac{\pi}{b} t \quad , 0 \le t \le b$$
$$= 0 \quad , \text{otherwise}$$

$$E_{x(t)} = \int_0^b A^2 \sin^2\left(\frac{\pi}{b} t\right) dt$$

$$= \int_0^b A^2 \left[\frac{1 - \cos \frac{2\pi t}{b}}{2}\right] dt$$

$$E_{x(t)} = \frac{A^2}{2} \int_0^b dt - \frac{A^2}{2} \int_0^b \cancel{\cos \frac{2\pi}{b} t \, dt}^{\,0} = \frac{A^2 b}{2}$$

(14)



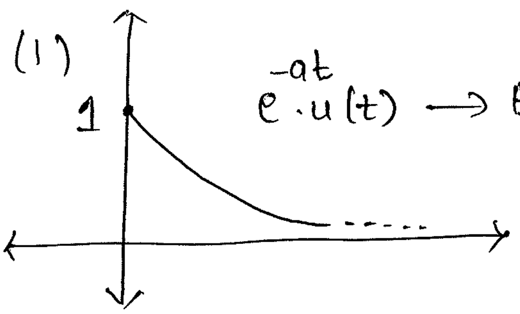
$$E_x(t) = \frac{A^2 b}{3}$$

$$\boxed{\begin{array}{l} x(t) \longrightarrow E_x(t) \\ x(\alpha t) \longrightarrow \frac{1}{|\alpha|} E_x(t) \\ x(2t) \longrightarrow \frac{1}{2} \cdot E_x(t) \\ x(t/3) \longrightarrow 3 E_x(t) \end{array}}$$

$$\boxed{\begin{array}{l} E \propto (\text{Amp.})^2 \\ E \propto \text{width} \end{array}}$$

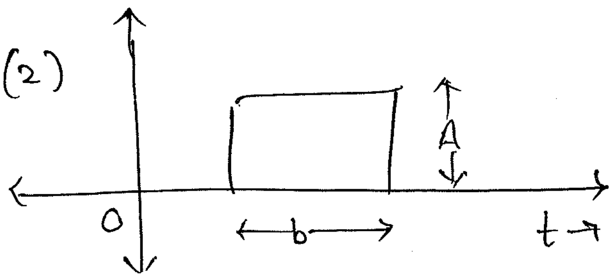
<u>NOTE:</u>
⇒ Energy is always positive.
⇒ Energy is directly proportional to square of amplitude & width of signal.

(1)



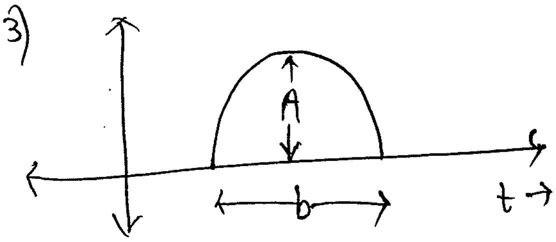
$e^{-at} \cdot u(t) \rightarrow E = \frac{1}{2a}$, $a > 0$

(2)



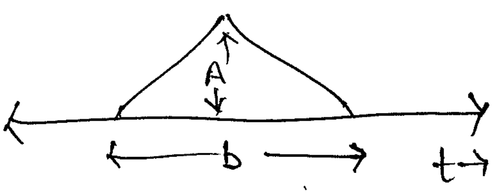
$E = A^2 b$

(3)



$$E = \frac{A^2 b}{2}$$

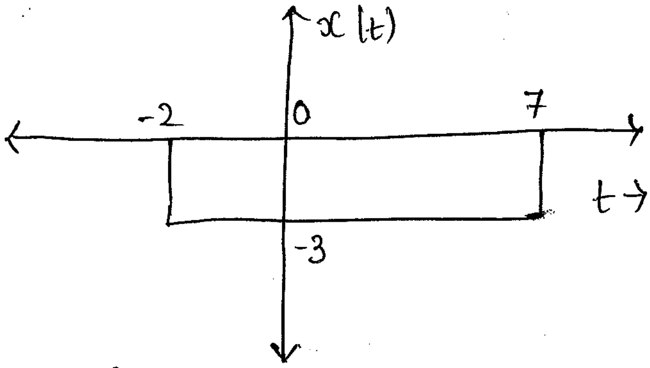
(4)



$$E = \frac{A^2 b}{3}$$

Find energy of following signals:-

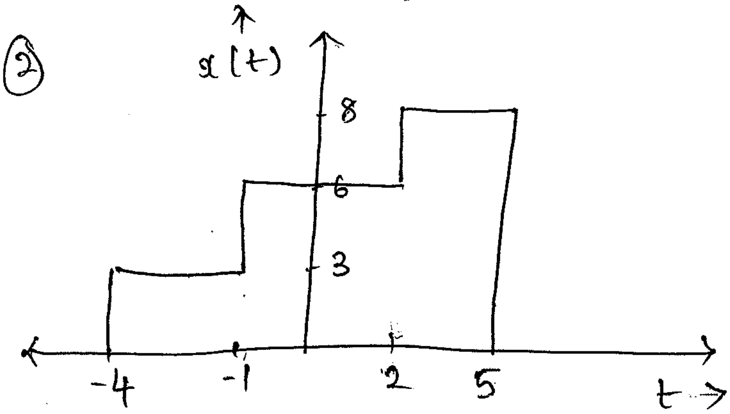
(1)



$E = A^2 \cdot b$

$E = (-3)^2 \cdot 9 = 9 \cdot 9 = 81\,J$

(2)

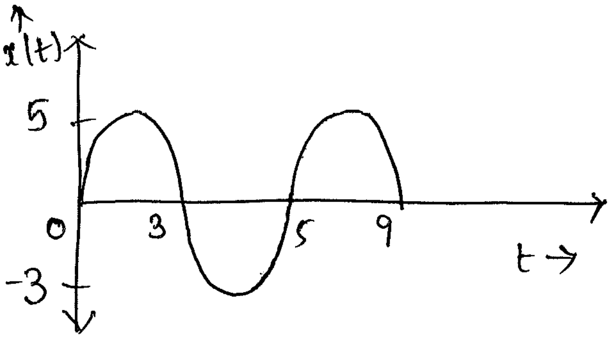


$E = 3^2 \cdot 3 + 6^2 \cdot 3 + 8^2 \cdot 3$

$E = 27 + 108 + 192$

$E = 327\,J$

(3)



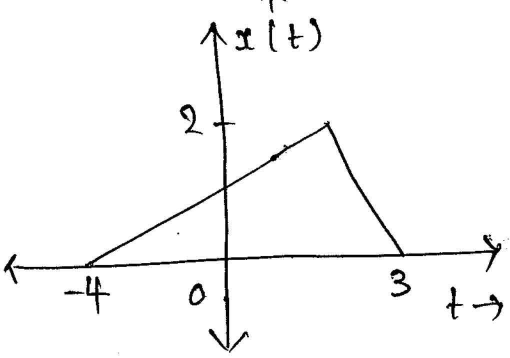
$E = \frac{A^2 b}{2} + \frac{A^2 b}{2} + \frac{A^2 b}{2} + \cancel{\frac{A^2 b}{2}}$

$E = \frac{25 \cdot 3}{2} + \frac{9 \cdot 2}{2} + \frac{25 \cdot 4}{2}$

$E = \frac{75}{2} + \cancel{\frac{18}{2}}\,9 + \cancel{\frac{100}{2}}\,50$

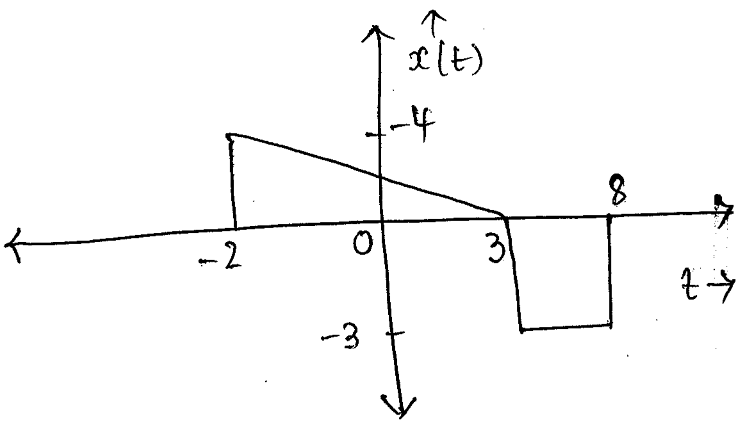
$E = \cancel{8.33\,J}\ 86.5\,J$

(4)



$E = \frac{A^2 \cdot b}{3} = \frac{4 \cdot 7}{3} = \frac{28}{3}\,J$

(5)

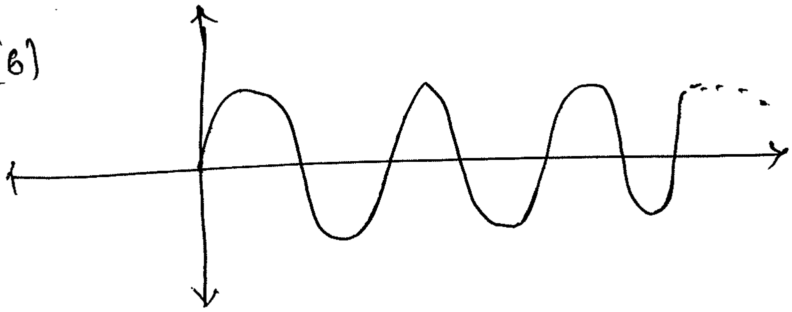


$E = \frac{A^2 \cdot b}{3} + A^2 \cdot b$

$E = \frac{16 \cdot 5}{3} + 9 \cdot 5$

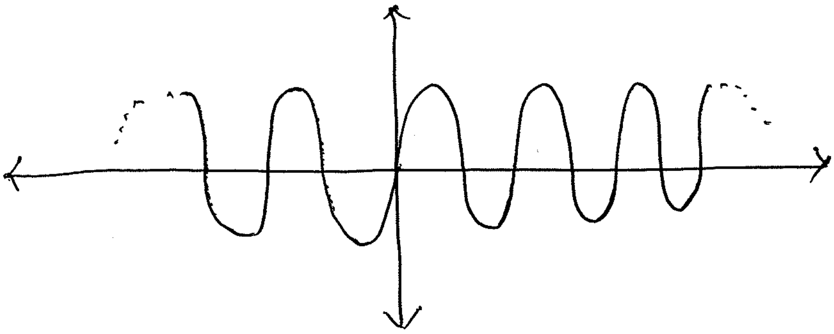
$E = \frac{80}{3} + 45 = \frac{215}{3} = 71.67\,J$

(6)



$$E = \frac{A^2 \cdot b}{2} = \infty$$

(7)



$E = \infty$

[everlasting signal]

Q:-(1) $x[n] = \{-3, 5, \underset{\uparrow}{2}, 1, 6, 7\}$

$n=0$

$$E_{x[n]} = \sum_{n=-\infty}^{\infty} |x^2[n]|$$

$$E_{x[n]} = \sum_{n=-2}^{n=3} |x^2[n]| = 9+25+1+4+36+49 = 124$$

(2) $x[n] = \left(\frac{1}{2}\right)^n \cdot u(n)$

$$E_{x[n]} = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n \cdot u(n)\right|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n} \cdot 1 = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{4} + 1 + \frac{1}{4^2} + \frac{1}{4^3} + \dots$$

$$= \frac{1}{1 - 1/4} = \frac{4}{3}$$

(3) $a^n \cdot u[n] = x[n]$

$$E = \sum_{n=0}^{\infty} a^{2n}$$

$$= a^0 + a^2 + a^4 + a^6 + \dots$$

$$= 1 + a^2 + (a^2)^2 + (a^2)^3 + \dots$$

$$E = \frac{1}{1-a^2} \quad ; |a| < 1$$

$$E = \infty \quad , |a| > 1$$

$$\frac{1}{1-a^2}$$

$|a| < 1$

Let $a = 1/2$

$$E = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots$$

$$E = \frac{1}{1-1/4} = \frac{4}{3}$$

$|a| > 1$

Let $a = 2$

$$E = 1 + (2^1)^1 + (2^2)^2 + \dots$$

$$E = \infty$$

(4) $\left[\left(\frac{1}{3}\right)^n + \left(\frac{1}{5}\right)^n\right] \cdot u[n]$

$$E = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n} + \left(\frac{1}{5}\right)^{2n} + 2\left(\frac{1}{3}\right)^n\left(\frac{1}{5}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n + \left(\frac{1}{25}\right)^n + 2\left(\frac{1}{15}\right)^n$$

$$= \frac{1}{1-1/9} + \frac{1}{1-1/25} + 2 \cdot \frac{1}{1-1/15}$$

$$[\alpha < 1 \qquad \alpha < 1 \qquad \alpha < 1]$$

$$= \frac{9}{8} + \frac{25}{24} + \frac{30}{14}$$

$$E = 4.308 \cong 4.31$$

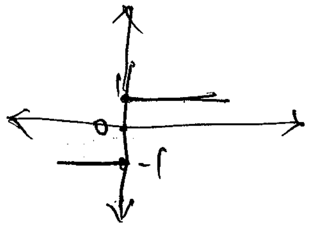
(5) $u[n]$

$$E = \sum_{n=0}^{\infty} 1 \; dt$$

$$= 1 + 1 + \dots$$

$$\boxed{E = \infty}$$

(6) $sgn[n]$



$$sgn(n) = 1 \; ; \; n > 0$$
$$= 0 \; ; \; n = 0$$
$$= -1 \; ; \; n < 0$$

$$E = \sum_{n=-\infty}^{n=-1} (-1)^2 + \sum_{n=1}^{\infty} (1)^2$$

$$\boxed{E = \infty}$$

(7) $x[n] = r[n]$

$= \infty$

(8) $x[n] = 5^n \cdot u[-n]$

$$E = \sum_{n=-\infty}^{0} (5^n)^2 dt$$

$$= \sum_{n=-\infty}^{0} (25)^n dt$$

Let $n = -m$

$$= \sum_{-m=-\infty}^{-m=0} (25)^{-m}$$

$$= (25)^0 + (25)^{-1} + (25)^{-2} + \cdots$$

$$= 1 + \frac{1}{25} + \left(\frac{1}{25}\right)^2 + \cdots$$

$$E = \frac{1}{1 - 1/25} = \frac{25}{24} = 1.041$$

$$\boxed{\begin{array}{l} E \propto (\text{Amplitude})^2 \\ E \propto \text{width} \end{array}}$$

Q:- A discrete time signal is given by $x[n] = \cos\left(\frac{\pi n}{3}\right)(u[n] - u[n$

$$E = \sum_{n=-\infty}^{\infty} \cos^2 \frac{\pi n}{3} \left[u[n] - u[n-6]\right]^2$$

$$E = \sum_{n=0}^{5} \cos^2 \frac{\pi n}{3}$$

$$E = \sum_{n=0}^{5} \frac{1 + \cos \frac{2\pi n}{3}}{2}$$

$$E = \sum_{n=0}^{5} \frac{1}{2} + \sum_{n=0}^{5} \frac{1}{2} \cos \frac{2\pi n}{3}$$

$$E = \frac{1}{2} 6 + \frac{1}{2} \cdot 6 \left[0+0+0+0+0+0\right]$$

$$\boxed{E = 3\,J}$$

Q:- If energy of signal $x(t)$ is 8 unit (J). Then find energy of following signal.

1. $x(t-2) \rightarrow 8J$
2. $x(t+1) \rightarrow 8J$
3. $x(-t) \rightarrow 8J$
4. $x(3t) \rightarrow 8/3\ J$
5. $x(-t/0.2) \rightarrow 8 \cdot (0.2) = 1.6\ J$
6. $x(-3t+2) \rightarrow 8/3\ J$
7. $x(0.2t) : \frac{1}{5} \rightarrow \frac{1}{25} \cdot 8 \cdot \frac{5}{0.2} = \frac{40}{25} = 1.6\ J$

(8) $3-x(t)$

$= \infty$ (add DC and DC ~~energy~~ power is ∞)

(9) $x\left(\frac{3-4t}{5}\right)$

$= 8 \cdot \frac{5}{4} = 10$ J.

Q:- Find power of the signal?

(1) $x(t) = A\cos(\omega t+\theta)$

(2) $x(t) = A\sin(\omega t+\theta)$

(3) $x(t) = A \cdot e^{j(\omega t+\theta)}$

periodic signal with fundamental period $= A$

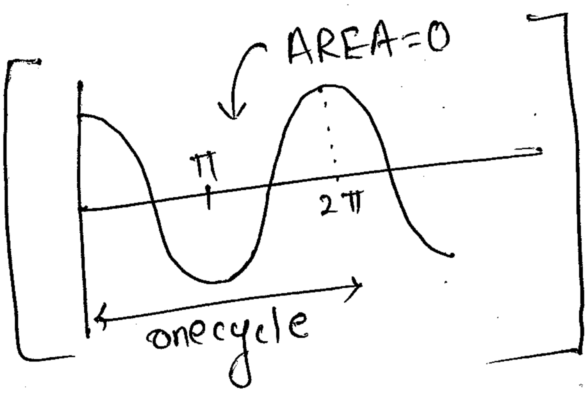
(4) $x(t) = A \cdot dc$

Solⁿ:- (1) $P_{avg.} = \frac{1}{T}\int_T |x(t)|^2 dt$

$= \frac{1}{T}\int_T A^2 \cdot \cos^2(\omega t+\theta)\, dt$

$= \frac{1}{T}\int_T \left[\frac{A^2}{2} + \frac{A^2}{2}\cos(2\omega t + 2\theta)\right] dt$

$= \frac{1}{T}\int_T \frac{A^2}{2} dt + \frac{1}{T}\int_T \frac{A^2}{2}\cos(2\omega t+2\theta)\, dt$



$= \frac{1}{T}\left[\frac{A^2}{2} t\right]_0^T$

$= \frac{1}{\cancel{T}} \cdot \frac{A^2 \cdot \cancel{T}}{2}$

$$\boxed{P_{avg.} = \frac{A^2}{2}}$$

(2) $x(t) = A\sin(\omega t+\theta)$

$$P_{avg.} = \frac{1}{T}\int_T |x(t)|^2 dt$$

$$= \frac{1}{T}\int_T A^2 \sin^2(\omega t+\theta)\, dt$$

$$= \frac{1}{T}\int_T \frac{A^2}{2} dt - \frac{1}{T}\int_T \frac{A^2}{2}\cos(2\omega t+2\theta)\, dt$$

$$\boxed{P_{avg.} = \frac{A^2}{2}}$$

Power = finite }
Energy = infinite } for periodic signal

(3) $x(t) = A \cdot e^{j(\omega t+\theta)}$

$$P_{avg.} = \frac{1}{T}\int_T |x(t)|^2 dt$$

$$= \frac{1}{T}\int_T A^2 \cdot (e^{j(\omega t+\theta)})^2 dt$$

$$\boxed{P_{avg.} = \frac{A^2 \cdot T}{T} = A^2}$$

(4) $x(t) = A \cdot (dc)$

$$P_{avg.} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$$

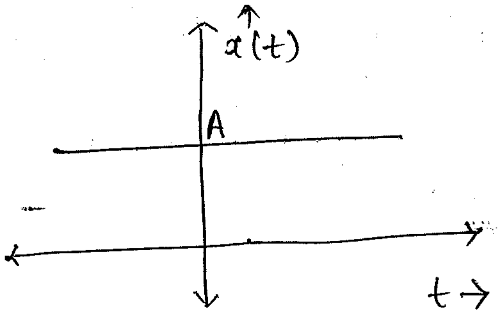
$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A^2 \cdot dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[A^2 \cdot t\right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[A^2 \cdot T + A^2 T\right]$$

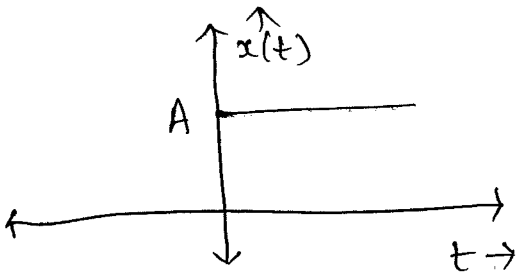
$$\boxed{P_{avg.} = \lim_{T\to\infty} \frac{1}{2\cancel{T}}\left[2A^2 \cdot \cancel{T}\right] = A^2}$$

Q:- find power



$E = \infty$

$P = A^2$



$E = \infty$

$P = \frac{A^2}{2}$

Unit step is a power signal.

(1) $x(t) = A \cdot u(t)$

$P = \frac{A^2}{2}$, $E = \infty$

(2) $x(t) = u(t)$

$P = \frac{1}{2}$, $E = \infty$

(3) $x(t) = r(t) = t \quad , t \geq 0$
$\qquad\qquad\quad = 0 \quad , t < 0$

$E = \infty$

$P = \infty$

[Neither power signal nor energy signal]

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T t^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{t^3}{3}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2\cancel{T}} \left[\frac{T^{\cancel{3}\,2}}{3}\right]$$

$$= \lim_{T\to\infty} \frac{T^2}{6}$$

$$\boxed{P = \infty}$$

Q:- The power of a signal $x[n] = (-1)^n u[n]$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[(-1)^n \cdot u[n]\right]^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (-1)^{2n} u^2[n]$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \cdot N$$

$$P = \lim_{N\to\infty} \frac{\cancel{N}}{\cancel{N}(2+1/N)}$$

$$P = \frac{1}{2} = 0.5\,W$$

$x(t) = e^{at} \cdot u(t)$

$$\begin{array}{cc} a<0 & a>0 \\ \downarrow & \downarrow \\ E = \frac{1}{2a} & E = \infty \\ \downarrow & \downarrow \\ P = 0 & P = \text{non-defined} \end{array}$$

Power of discrete time signal OR digital

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(t)|^2$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_0^T e^{2at} \cdot dt$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \left[\frac{e^{2at}}{2a} \right]_0^T$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \left[\frac{e^{2at} - 1}{2a} \right]$$

(i) $a < 0$

$$P = \lim_{T \to \infty} \frac{1}{2T} \left[\frac{e^{-2aT} - 1}{-2a} \right] = \frac{1}{\infty} \left[\frac{e^{-\infty} - 1}{-2a} \right] = \frac{1}{\infty} \left(\frac{0 - 1}{2a} \right) = 0$$

(ii) $a > 0$

$$P = \lim_{T \to \infty} \frac{1}{2T} \left[\frac{e^{-2aT} - 1}{-2a} \right] = \frac{1}{\infty} \cdot \infty = \text{undefined.}$$

NOTE: Power is always positive, power is depends only on amplitude of signal. Power is not depend on frequency and phase of given signal.

Q:- Find power of given signal?

(1) -4

$P = A^2 = 16\,W$

(2) $(3-4j)e^{-j5t} = x(t)$

$|x(t)| = |(3-4j)e^{-j5t}|$

$= \sqrt{3^2+4^2}\,|e^{-j5t}|$

$|x(t)| = 5$

$P = \frac{1}{T}\int_T |x(t)|^2 dt$

$P = \frac{1}{T}\int_T 25\,dt$

$P = \left[\frac{25t}{T}\right]_0^T = \frac{25\cdot \cancel{T}}{\cancel{T}} = 25W$

$x(t) = (3-j4)(\cos 5t - j\sin 5t)$

$x(t) = 3\cos 5t + 4\cancel{j}\sin 5t$
$-j3\sin 5t - 4\sin 5t$

$x(t) = (3\cos t - 4\sin 5t) - j(3\sin 5t$
$+ 4\cos 5t)$

$|x(t)| = \sqrt{(3\cos 5t - 4\sin 5t)^2}$
$+ (3\sin 5t + 4\cos 5t)^2$

$|x(t)| = \sqrt{9\cos^2 5t + 16\sin^2 5t - 24\cos\cancel{5t}\sin 5t}$
$+ 9\sin^2 5t + 16\cos^2 5t$
$+ 24\sin\cancel{5t}\cos 5t$

$|x(t)| = \sqrt{9+16} = 5$

(3) $5\cos(3t - 45^\circ)$

$P = \frac{A^2}{2} = \frac{25}{2} = 12.5\,W$

(4) $x(t) = 3\sin^2\left(\frac{\pi}{3}t\right)$

$x(t) = 3\left[\frac{1-\cos\frac{2\pi}{3}t}{2}\right]$

$= \frac{3}{2} - \frac{3}{2}\cos\frac{2\pi}{3}t$

$P = A^2 + \frac{A^2}{2}$

$= \frac{9}{4} + \frac{9}{4\cdot 2} = \frac{9}{4} + \frac{9}{8} = \frac{27}{8}\,W$

(5) $\cos 3t \cdot \sin 5t = x(t)$

$$x(t) = \cos 3t \cdot \sin 5t$$

$$x(t) = \frac{\sin 8t + \sin 2t}{2}$$

$$P = \frac{A^2}{2} + \frac{A^2}{2} = \frac{1}{4 \cdot 2} + \frac{1}{4 \cdot 2} = \frac{2}{8} = \frac{1}{4} W$$

(6) $x(t) = \left[\sin \frac{\pi}{3} t + \cos (3\pi t)\right]^2$

$$x(t) = \sin^2 \frac{\pi}{3} t + \cos^2 3\pi t + 2 \sin \frac{\pi}{3} t \cdot \cos 3\pi t$$

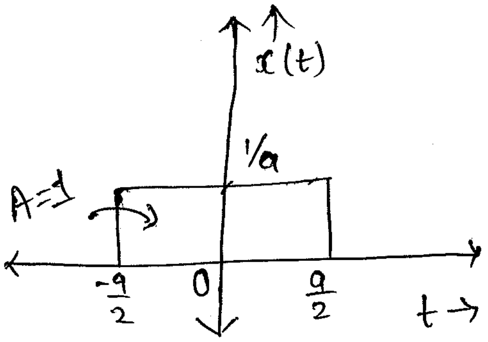
$$= \frac{1 - \cos \frac{2\pi}{3} t}{2} + \frac{1 + \cos 6\pi t}{2} + \sin \left(3\pi + \frac{\pi}{3}\right) t + \sin \left(3\pi - \frac{\pi}{3}\right) t$$

$$= \frac{1}{2} - \frac{\cos \frac{2\pi}{3} t}{2} + \frac{1}{2} + \frac{\cos 6\pi t}{2} + \sin \frac{10\pi}{3} t + \sin \frac{8\pi}{3} t$$

$$P = A^2 + \frac{A^2}{2} + A^2 + \frac{A^2}{2} + \frac{A^2}{2} + \frac{A^2}{2}$$

$$P = \frac{1}{4} + \frac{1}{4 \cdot 2} + \frac{1}{4 \cdot 2} + \frac{1}{4} + \frac{1}{2} + \frac{1}{2} = \boxed{\frac{7}{4} W}$$

NOTE:- As impulse fonction is singular function we can't find its derivative completely.



A=Area

$a \to 0 \Rightarrow A \to \infty$

$x(t) = \frac{1}{a} \cdot u\left(t+\frac{a}{2}\right) - \frac{1}{a} \cdot u\left(t-\frac{a}{2}\right)$

$\lim_{a \to 0} x(t) = \delta(t)$

$\Rightarrow \delta(t) = \lim_{a \to 0} x(t)$

$\delta'(t) = \lim_{a \to 0} x(t)$

$\delta(t) = \lim_{a \to 0} \left[\frac{1}{a} u\left(t+\frac{a}{2}\right) - \frac{1}{a} u\left(t-\frac{a}{2}\right)\right]$

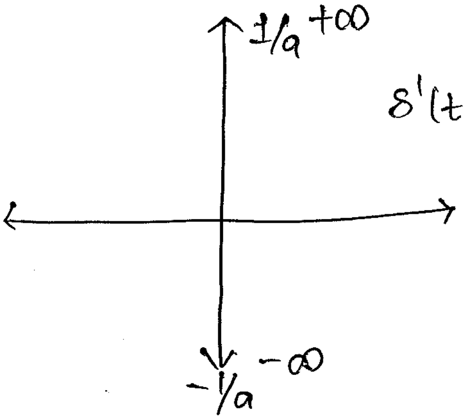
$\delta'(t) = \lim_{a \to 0} \left[\frac{1}{a} \cdot \delta\left(t+\frac{a}{2}\right) - \frac{1}{a} \cdot \delta\left(t-\frac{a}{2}\right)\right]$

$\delta'(t) = \frac{1}{0} \cdot \delta(t) - \frac{1}{0} \cdot \delta(t)$

$\downarrow$

impulse @ $t=0 \Rightarrow$ Amp $= -\infty$

impulse @ $t=0 \Rightarrow$ Amp $= \infty$



$\delta'(t)$ = DOUBLET

$$\boxed{\text{AREA under doublet} = \int_{-\infty}^{\infty} \delta'(t)\, dt = 0}$$

① $$\int_{t_1}^{t_2} x(t)\cdot\delta(t-t_0)\,dt = x(t_0), \quad t_1 \le t_0 \le t_2$$
$$= 0, \quad \text{otherwise}$$

② $$\int_{t_1}^{t_2} x(t)\cdot\delta^n(t-t_0)\,dt = (-1)^n \cdot \frac{d^n x(t_0)}{dt^n}, \quad t_1 \le t_0 \le t_2$$
$$= 0, \quad \text{otherwise}$$

where n is n^{th} order derivative

Eg:- (1) $$\int_{2}^{10} e^{-3(t-3)}\,\delta'(t-5)\,dt \qquad [\dot{\delta}(t-5) = \delta'(t-5)]$$

Soln:- Here $x(t) = e^{-3(t-3)}$

$$\frac{dx(t)}{dt} = e^{-3(t-3)}(-3)$$

$$= (-1)\frac{dx(t_0)}{dt}$$
$$= 3e^{-3(t_0-3)}$$
$$= 3e^{-3(5-3)}$$
$$= 3e^{-6}$$
$$= \frac{3}{e^6}$$

(2) $$\int_{-\infty}^{\infty} e^{-2t}\cdot\delta'(t)\,dt$$

Here $x(t_0) = e^{-2t}$

$$\frac{d'x(t_0)}{dt'} = e^{-2t}(-2)$$

$$\int_{-\infty}^{\infty} e^{-2t}\cdot\delta'(t)\,dt = (-1)^n\cdot\frac{d^n x(t_0)}{dt^n}$$
$$= (-1)^1\cdot e^{-2(0)}(-2) \qquad [\text{Here } t_0 = 0]$$
$$= 2$$

*Classification of signals

(1) Right sided and left ~~ho~~ sided signals

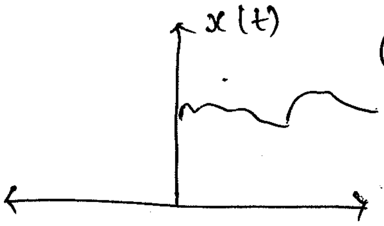
Signal $x \to +\infty$, $t \geq t_0$ $\Rightarrow$ Right sided

Signal $x \to -\infty$, $t < t_0$ $\Rightarrow$ Left sided

Double sided

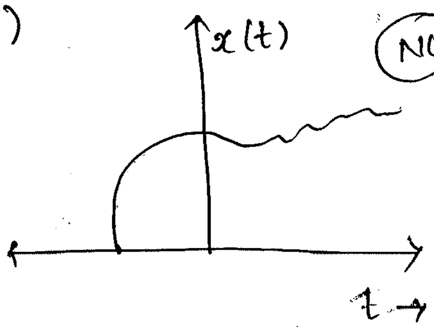
Everlasting signal $x \to -\infty$ to $+\infty$

(1) C



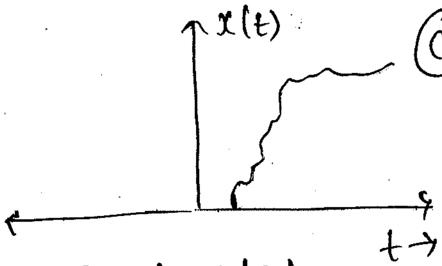
Right sided

(2) NC



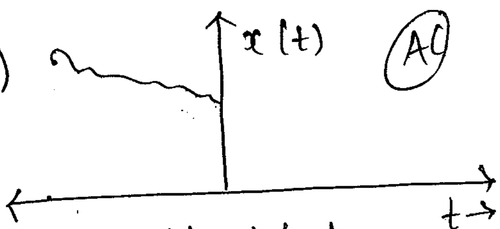
Right sided

(3) C



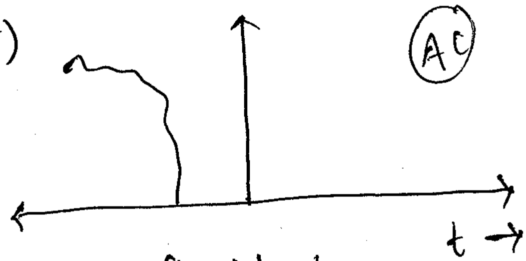
Right sided

(4) AC



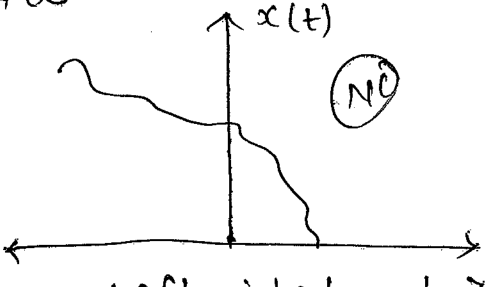
Left-sided

(5) AC



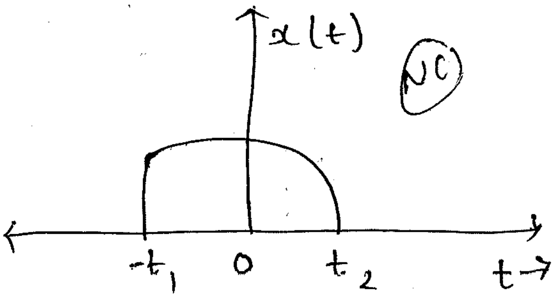
Left sided

(6) NC



Left-sided

(7) NC

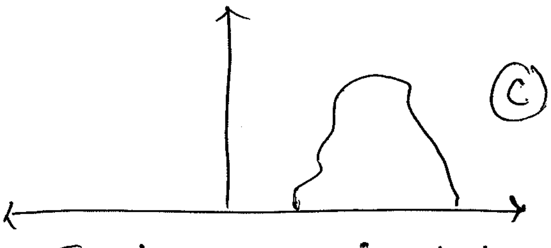


Finite double sided

(8) AC

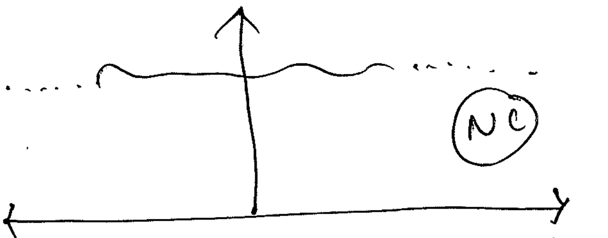
Finite left sided

(9) C



Finite right sided t→

(10) NC



Everlasting signal

(2) Causal and Non-causal signals

For every causal signal $x(t) = 0, t < 0$ and for every anticausal signal $x(t) = 0, t > 0$ and a signal which doesn't satisfies above two conditions is said to be non-causal signal.

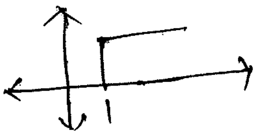
NOTE: Every causal signal is right sided signal but every right sided signal didn't be causal signal

Every non-causal signal is left handed signal but every left-sided signal didn't be non-causal signal.

Everlasting signal is always non-causal.

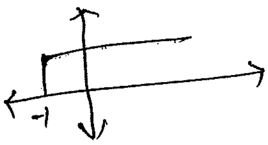
Finite double sided signal is always non-causal.

Eg:- (1) $u(t-1)$



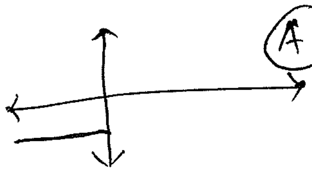
C

(2) $u(t+1)$



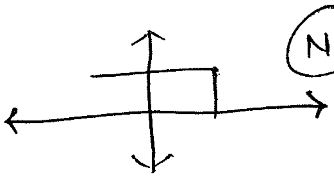
NC

(3) $u(-t)$



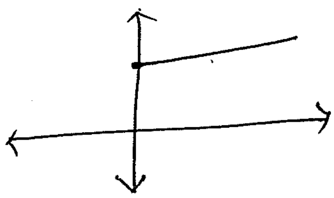
AC

(4) $u(-t+1)$



NC

(5) $u(t)$



C

[Causal & Anti-causal signals are mutually exclusive signals]

(3) Odd and Even signal

For every continuous even/symmetric signal

$$\boxed{x(t) = x(-t)}$$

For every continuous odd/anti-symmetric signal

$$\boxed{x(t) = -x(-t)}$$

For every discrete even/symmetric signal

$$\boxed{x[n] = x[-n]}$$

For every discrete odd/anti-symmetric signal

$$\boxed{x[n] = -x[-n]}$$

Those signals which do not satisfy above conditions are neither odd or even signal.

Every signal can be described in odd & even parts of that signal.

$$x(t) = x_e(t) + x_o(t) \quad \text{—— (1)}$$

$$x(-t) = x_e(-t) + x_o(-t) \quad \text{—— (2)}$$

$$x(-t) = x_e(t) - x_o(t) \quad \text{—— (3)}$$

eq^n(1) & eq^n(3)

$$x(t) + x(-t) = 2x_e(t)$$

for continuous $\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$

$$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

For discrete,

$$\boxed{x_e[n] = \frac{x[n] + x[-n]}{2}}$$

$$\boxed{x_o[n] = \frac{x[n] - x[-n]}{2}}$$

Q:- Check symmetry of given signal:-

(1) $x_1(t) = t^5$

$x_1(-t) = -t^5$

$-x_1(-t) = t^5$

$x_1(t) = -x_1(-t)$

↓

odd signal

(2) $x_2(t) = t^2$

$x_2(-t) = t^2$

$-x_2(-t) = -t^2$

$x_2(t) = x_2(-t)$

↓

even signal

1] $o \pm o = o$

2] $e \pm e = e$

3] $o \pm e$ = NENO

4] $e \pm o$ = NENO

5] $o \times o = e$

6] $o \times e = o$

7] $e \times o = o$

8] $e \times e = e$

9] $\frac{o}{o} = e$

10] $\frac{e}{e} = e$

11] $\frac{o}{e} = o$

12] $\int e = o$

13] $\int o = e$

14] $\frac{d}{dt} e = o$

15] $\frac{d}{dt} o = e$

16] $\frac{e}{o} = o$

$$\boxed{\begin{aligned} \int_T \text{even} &= 2\int_{T/2} \text{even} \\ \int_T \text{odd} &= 0 \text{ (zero)} \end{aligned}}$$

Q:- Check eq^n is even or odd? Also find odd & even part.

1] $x(t) = 1+2t+3t^2$

Trace the symmetry of following signals & find odd & even part of that signals.

$x(-t) = 1-2t+3t^2 \neq x(t)$

NENO.

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$= \frac{1+\cancel{2t}+3t^2+1-\cancel{2t}+3t^2}{2}$$

$$x_e(t) = 1+3t^2$$

$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

$$= \frac{1+2t+3t^2-(1-2t+3t^2)}{2}$$

$$x_o(t) = 2t$$

2] $x(t) = e^{-t}$

$x(-t) = e^{t}$

$-x(-t) = -e^{t}$

NENO

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$= \frac{e^{-t}+e^{t}}{2}$$

$$x_e(t) = \cosh t$$

$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

$$= \frac{e^{-t}-e^{+t}}{2}$$

$$\boxed{x_o(t) = -\sinh t}$$

3] $\cosh t \rightarrow$ even

4] $\sinh t \rightarrow$ odd

[5] $x(t) = \sin t \rightarrow$ odd

6] $x(t) = \cos t \rightarrow$ even

7] $x(t) = u(t)$

NENO

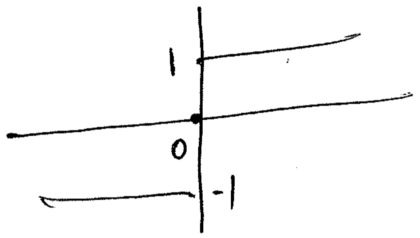
$$x_e(t) = \frac{x(t)+x(-t)}{2}$$
$$= \frac{u(t)+u(-t)}{2}$$
$$\boxed{x_e(t) = \frac{1}{2}}$$

$$x_o(t) = \frac{x(t)-x(-t)}{2}$$
$$= \frac{u(t)-u(-t)}{2}$$
$$\boxed{x_o(t) = \frac{1}{2}\,\mathrm{sgn}(t)}$$

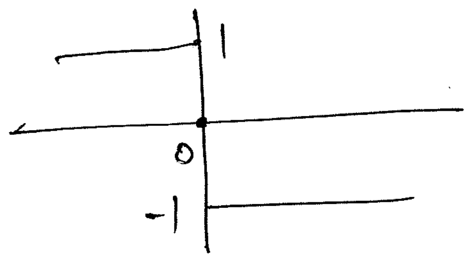
Now,
$$x(t) = x_e(t) + x_o(t)$$
$$= \frac{1}{2} + \frac{1}{2}\,\mathrm{sgn}(t)$$
$$x(t) = \frac{1}{2}\left[1+\mathrm{sgn}(t)\right] = u(t)$$

8] $x(t) = \mathrm{sgn}(t)$

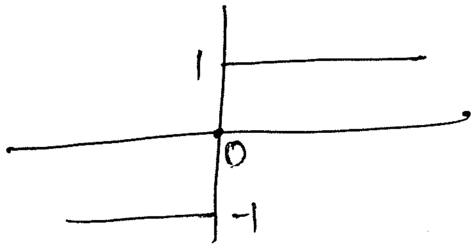
$x(t) = \mathrm{sgn}(t)$



$x(-t) = \mathrm{sgn}(-t)$



$-x(-t) = -\mathrm{sgn}(-t) = \mathrm{sgn}(t)$



It is an odd function.

9] $x(t) = e^{-3t}\cdot\cos 2t$

$x(-t) = e^{3t}\cos 2t$

NENO.

$-x(-t) = -e^{+3t}\cos 2t$

$$x_e(t) = \frac{e^{-3t}\cos 2t + e^{3t}\cos 2t}{2}$$
$$= \cos 2t \cdot \cosh 3t \quad [e \cdot e = e]$$

$x_e(t) = \text{even}$

$$x_o(t) = \frac{e^{-3t}\cos 2t - e^{3t}\cos 2t}{2}$$
$$= \cos 2t\left(\frac{e^{3t} - e^{-3t}}{2}\right)$$
$$x_o(t) = \cos 2t \sinh 3t$$
$e \cdot o = \text{odd}.$

10] $x(t) = e^{-3t} \cdot u(t)$

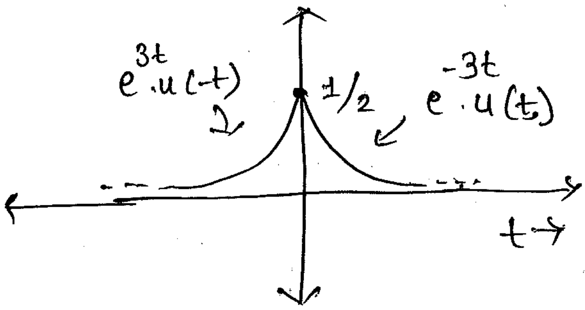
$x(-t) = e^{3t} \cdot u(-t)$ NENO

$-x(-t) = -e^{+3t} u(-t)$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{e^{-3t} \cdot u(t) + e^{3t} \cdot u(-t)}{2}$$

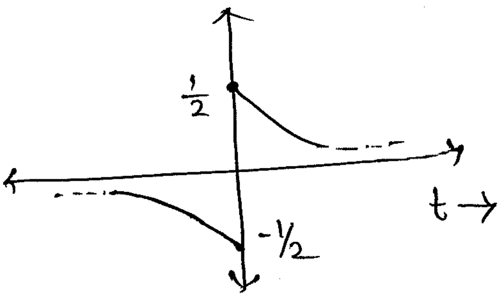
$t>0$ $\quad$ $t<0$ $\quad$ $\rightarrow t<0$



$$= \left(\frac{e^{-3t} + e^{3t}}{2}\right) (u(t) + u(-t))$$

$$x_e(t) = \frac{e^{-3|t|} \cdot u(t)}{2}$$

$$x_o(t) = \frac{e^{-3t} \cdot u(t) - e^{+3t} u(-t)}{2}$$



11] $x(t) = \sin t + \cos 3t + \sin 5t \cdot \cos 7t$

$x(-t) = \sin(-t) + \cos 3t - \sin 5t \cos 7t$

$x(-t) = -\sin t + \cos 3t - \sin 5t \cos 7t$ NENO.

$-x(-t) = +\sin t - \cos 3t + \sin 5t \cos 7t$

$x_e(t) = \cos 3t \qquad x_o(t) = \sin t + \sin 5t \cos 7t$

12] $x(t) = (t + \sin t)^2$

$x(t) = t^2 + \sin^2 t + 2t \sin t$

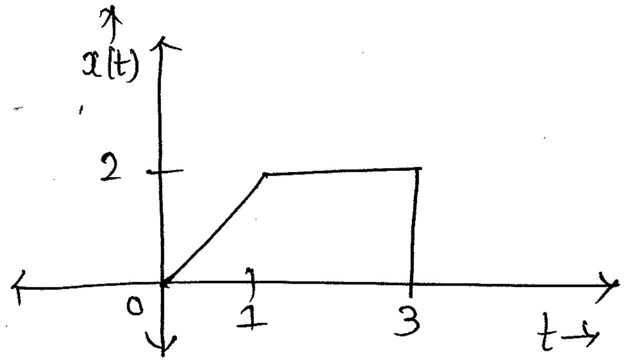
$x(-t) = t^2 + \sin^2 t + 2t \sin t$ EVEN

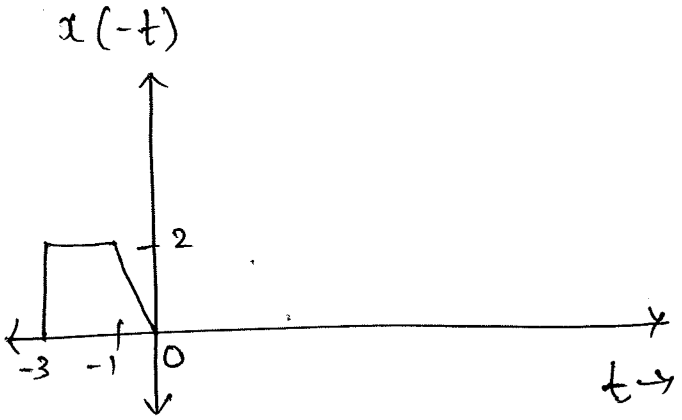
$-x(-t) = -t^2 - \sin^2 t - 2t \sin t$

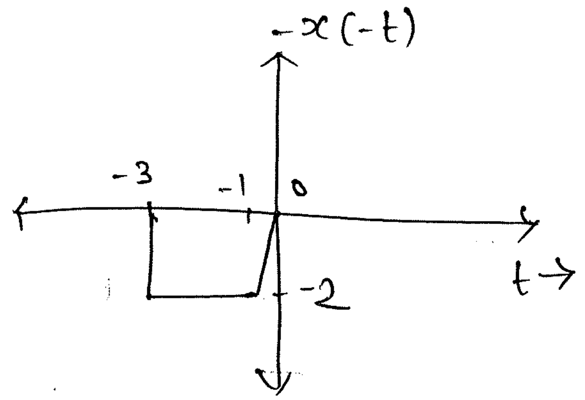
$x_e(t) = t^2 + \sin^2 t\, 2 \sin t \cdot t$

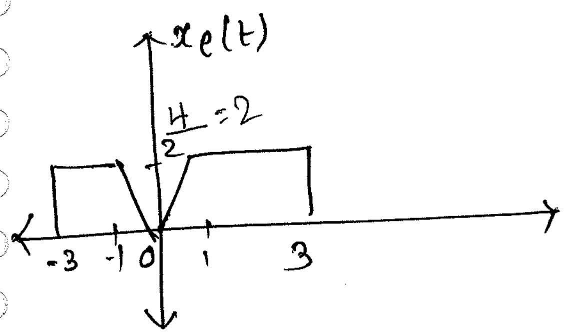
$x_o(t) = 0$

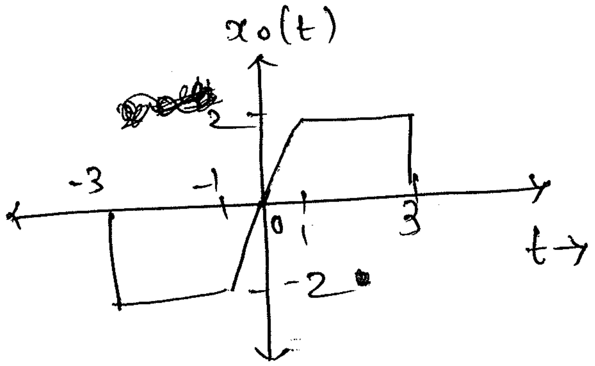
13] Sketch odd & even part of $x(t)$



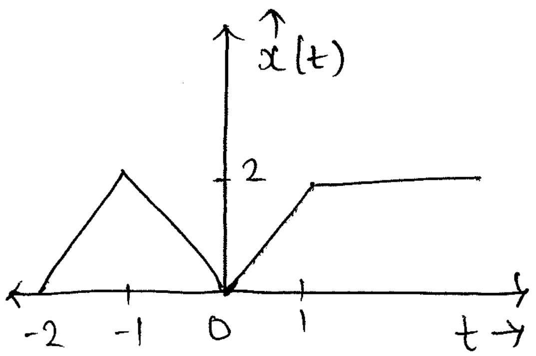


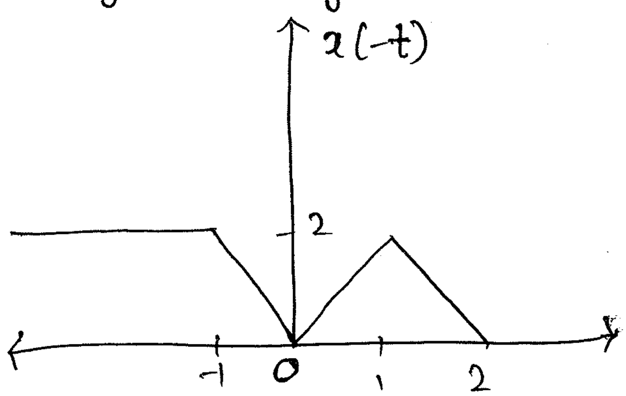


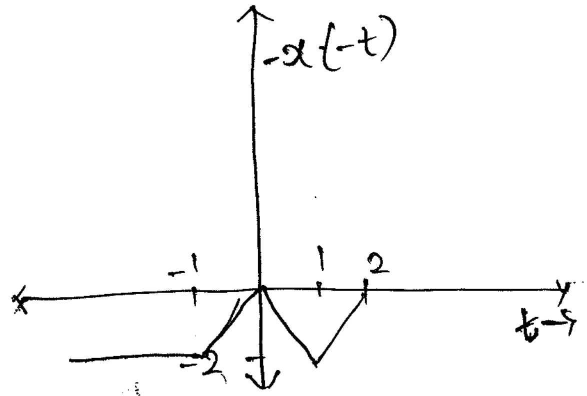


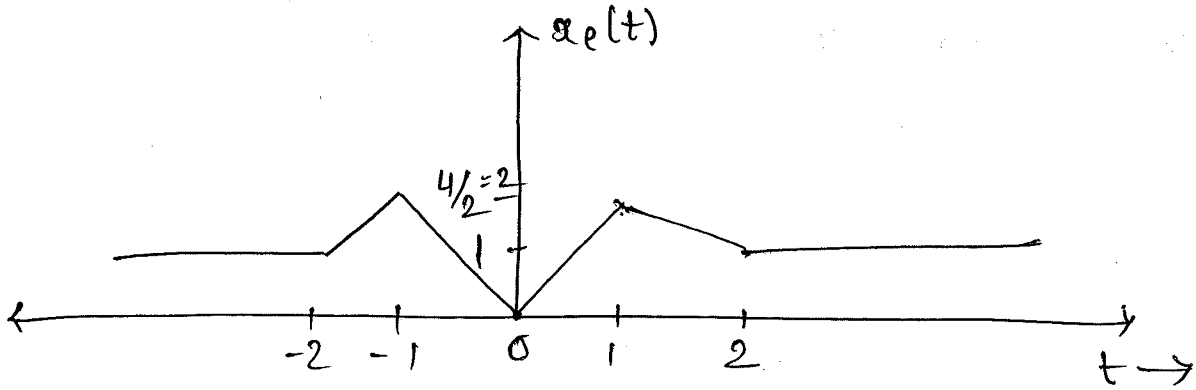


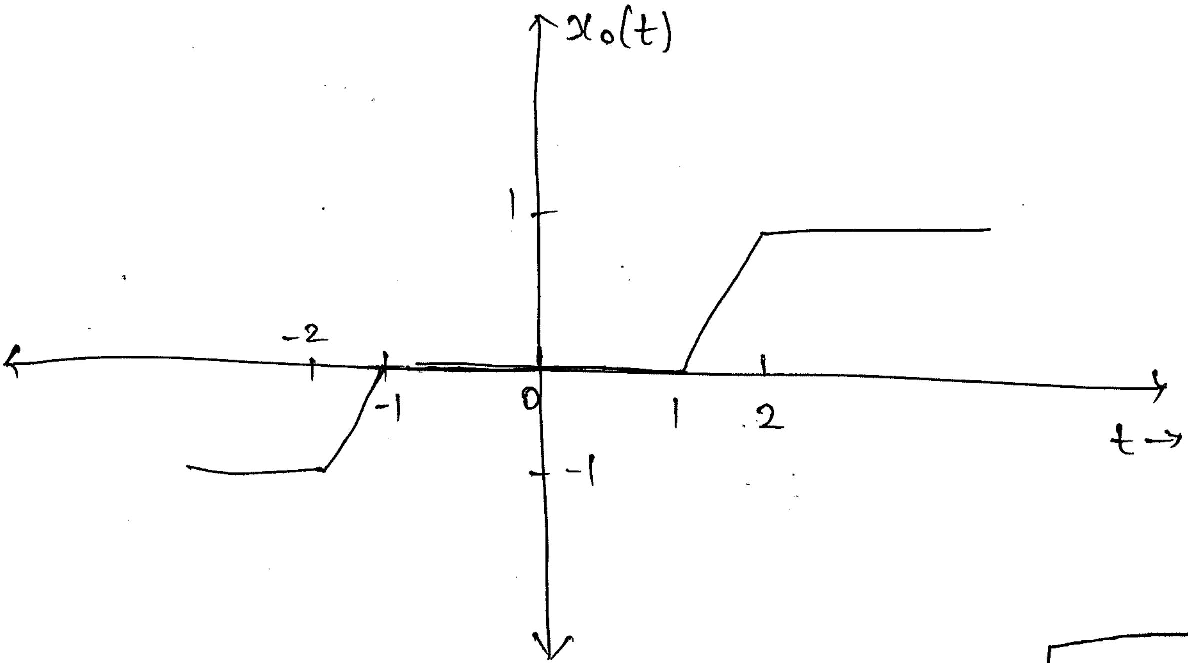
Q:- Sketch odd & even part of given signal.











Even & odd function

(i) Real & Imaginary value signal

$x(t) = x(-t) \longrightarrow$ even / symmetric

$x(t) = -x(-t) \longrightarrow$ odd / anti-symmetric

$x[n] = x[-n] \longrightarrow$ even / symmetric

$x[n] = -x[-n] \longrightarrow$ odd / anti-symmetric

$x(t) = a(t)$
↑
pure Real.

$x(t) = \pm j\, b(t)$
↑
Pure imaginary

NOTE: Symmetrical condition for real value and pure imaginary value signal remains same whereas for complex value signal it is known as conjugate even OR odd symmetry OR conjugate odd OR anti-symmetry.

(ii) Complex value signal

$$x(t) = \underset{\substack{\uparrow \\ \text{Real}}}{a(t)} \pm j\,\underset{\substack{\uparrow \\ \text{Imaginary}}}{b(t)}$$

Conjugate even / Symmetric

Conjugate odd / Anti-symmetric

(i) Conjugate even

$$x(t) = x^*(-t)$$

$$x[n] = x^*[-n]$$

(ii) Conjugate odd

$$x(t) = -x^*(-t)$$

$$x[n] = -x^*[-n]$$

$$x_e(t) = \frac{x(t) + x^*(-t)}{2}$$

$$x_e[n] = \frac{x[n] + x^*[-n]}{2}$$

$$x_o(t) = \frac{x(t) - x^*(-t)}{2}$$

$$x_o[n] = \frac{x[n] - x^*[-n]}{2}$$

Q:- Test the symmetry of following signal?

① $x(t) = jt$

$x(-t) = -jt$

$-x(-t) = jt$

ODD / anti-symmetric

② $x(t) = 3 - jt$

$x(-t) = 3 + jt$

$x^*(-t) = 3 - jt$

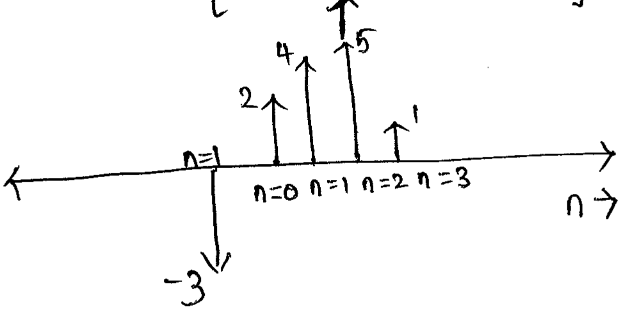
$\therefore \boxed{x(t) = x^*(-t)}$

EVEN / symmetric

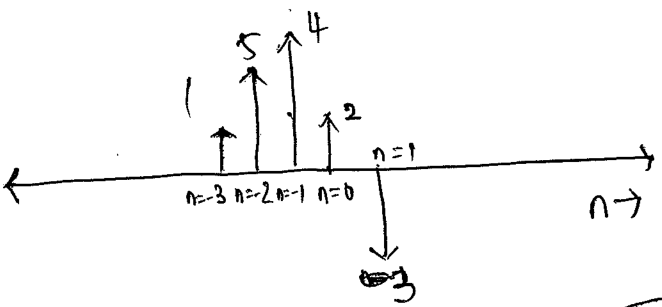
$x_e(t) = 3 \;, \; x_o(t) = 0$

Q:- Find even and odd part:-

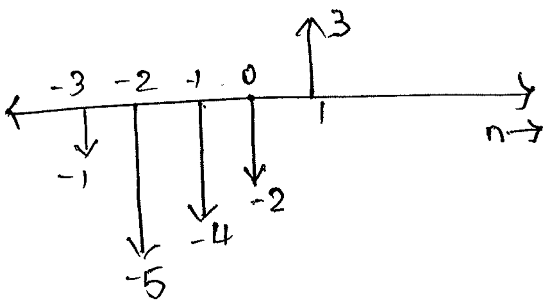
① $x[n] = \{-3, \underset{\uparrow}{2}, 4, 5, 1\}$



$x[-n]$

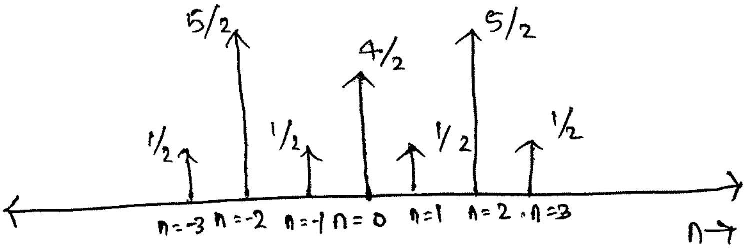


$-x[-n]$



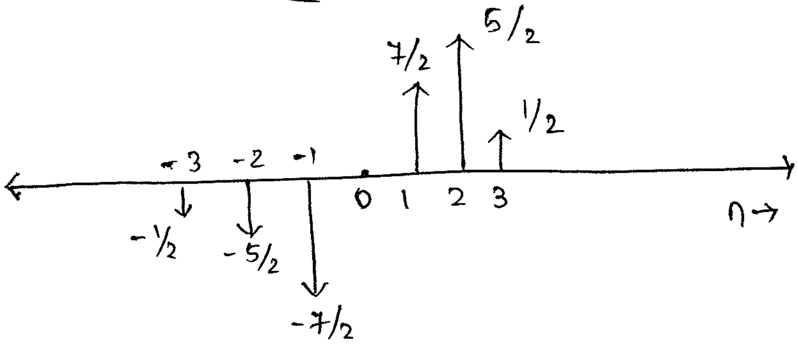
NENO

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$



$$\left\{ \frac{1}{2}, \frac{5}{2}, \frac{1}{2}, 2, \frac{1}{2}, \frac{5}{2}, \frac{1}{2} \right\}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

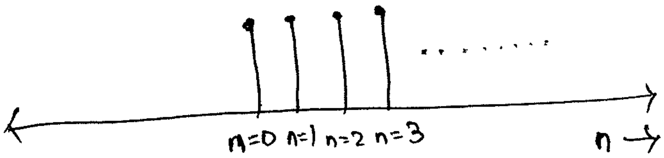


$$\left\{ -\frac{1}{2}, -\frac{5}{2}, -\frac{7}{2}, 0, \frac{7}{2}, \frac{5}{2}, \frac{1}{2} \right\}$$

(2) $x[n] = u[n]$

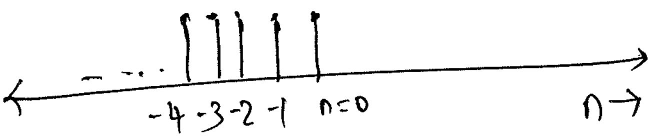
$u[n] = 1 \quad , n \geq 0$
$\quad\;\;\, = 0 \quad , n < 0$

$u[n]$

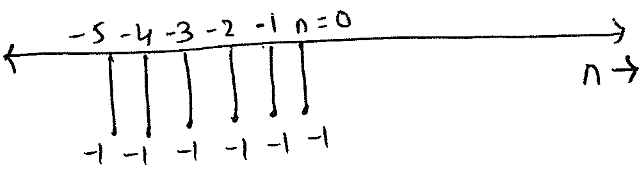


NENO

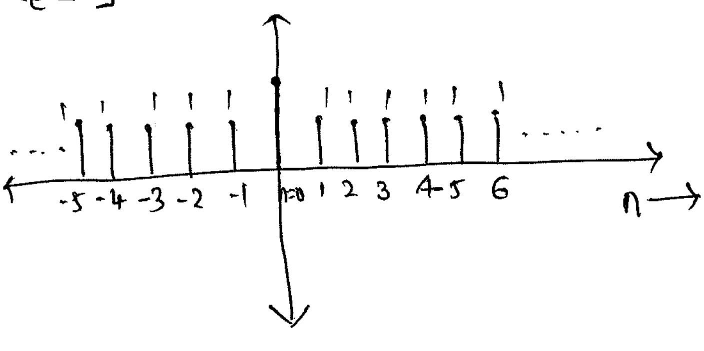
$u[-n]$



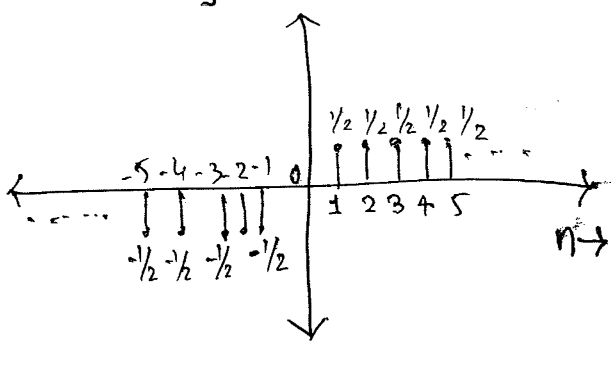
$-u[-n]$



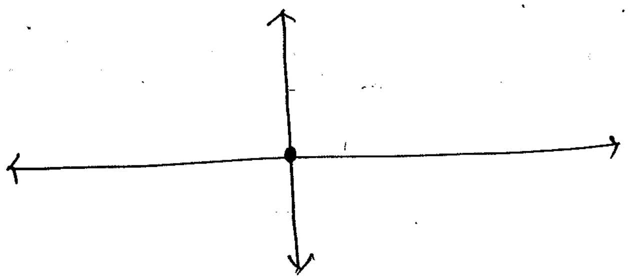
$x_e[n]$



$x_o[n]$



(3) $x[n] = sgn[n]$



$$\int_{-2}^{1} (t+t^2)\,\delta(t-3)\,dt$$

At $t=3$ is not in the interval $-2 < t < 1$

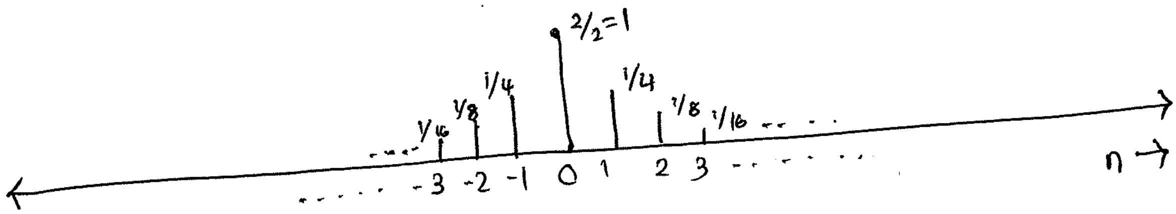
$$\therefore \int_{-2}^{1} (t+t^2)\cdot\delta(t-3)\,dt = 0 \quad \left[\int_{-\infty}^{\infty} x(t)\cdot\delta(t-t_0)\,dt = x(t_0)\right]$$

$$-\infty \leq t_0 \leq \infty$$

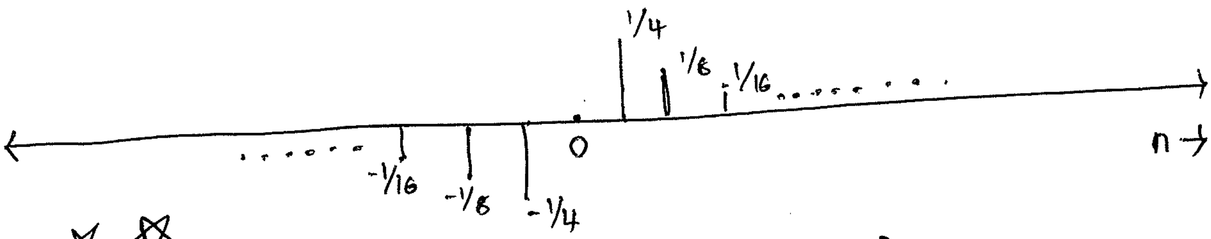
4] $x[n] = \left(\frac{1}{2}\right)^n \cdot u[n]$

$$x_e[n] = \frac{1}{2}\left[\underbrace{\left(\frac{1}{2}\right)^n \cdot u[n]}_{n>0} + \underbrace{\left(\frac{1}{2}\right)^{-n} \cdot u[-n]}_{n<0}\right]$$

$$x_e[n] = \frac{1}{2}\left(1 \cdot 1 + 1 \cdot 1\right) = 1$$



$$x_o[n] = \frac{1}{2}\left[\left(\frac{1}{2}\right)^n \cdot u(n) - \left(\frac{1}{2}\right)^{-n} \cdot u(-n)\right]$$



5] $x[n] = \{ \underset{n=0}{\underset{\uparrow}{10}}, \underset{n=1}{\underset{\uparrow}{-2+2j}}, \underset{n=2}{\underset{\uparrow}{-2}}, \underset{n=3}{\underset{\uparrow}{-2-2j}} \}$

$$x[-n] = \{ \underset{n=-3}{\underset{\uparrow}{-2-2j}}, \underset{n=-2}{\underset{\uparrow}{-2}}, \underset{n=-1}{\underset{\uparrow}{-2+2j}}, \underset{n=0}{\underset{\uparrow}{10}} \}$$

$$x^*[-n] = \{ \underset{n=-3}{\underset{\uparrow}{-2+2j}}, \underset{n=-2}{\underset{\uparrow}{-2}}, \underset{n=-1}{\underset{\uparrow}{-2-2j}}, \underset{n=0}{\underset{\uparrow}{10}} \}$$

$$-x^*[-n] = \{ \underset{n=-3}{\underset{\uparrow}{2-2j}}, \underset{n=-2}{\underset{\uparrow}{2}}, \underset{n=-1}{\underset{\uparrow}{2+2j}}, \underset{n=0}{\underset{\uparrow}{-10}} \}$$

$$x_e[n] = \frac{1}{2}\{-2+2j, -2, -2-2j, 20, -2+2j, -2, -2-2j\} = \{-1+j, -1, -1-j, 10, -1+j, -1, -1-j\}$$

$$x_o[n] = \frac{1}{2}\{2-2j, 2, 2+2j, 0, -2+2j, -2, -2-2j\}$$

Q:- Check it is even or not?

$$x_e[n] = \{-1+j, -1, -1-j, \underset{\substack{\uparrow \\ n=0}}{10}, -1+j, -1, -1-j\}$$

$$x_e[-n] = \{-1-j, -1, -1+j, 10, -1-j, -1, -1+j\}$$

$$x_e^*[-n] = \{-1+j, -1, -1-j, 10, -1+j, -1, -1-j\}$$

$\therefore x_e[n] = x_e^*[-n]$

$\therefore$ It is an even signal

* ENERGY AND POWER SIGNAL

For every energy signal

$$0 < E < \infty, \ P = 0$$

For every power signal

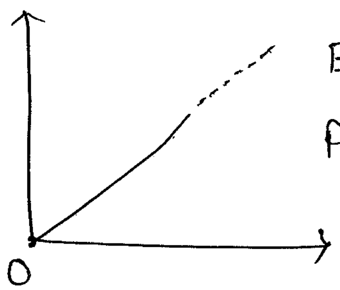
$$0 < P < \infty, \ E = \infty$$

For every energy signal, energy is finite & power is zero. ($P = 0$).

For every power signal, power is finite & energy is infinite ($E = \infty$)

- The signal which is not satisfies above two conditions is neither energy nor power signal.

For eg:- $r(t) = t$



$E = \infty$

$P = \infty$

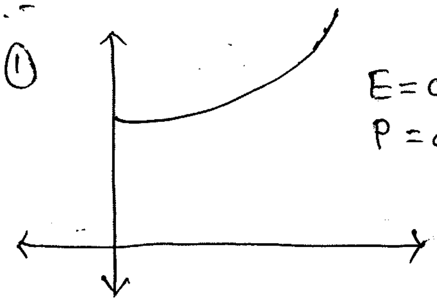
NENP

* Energy and Power Signal

Q:- Check whether the given signal is energy signal OR power signal?

Sol:-

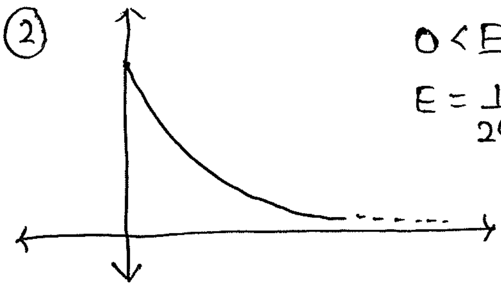
①



$E = \infty$
$P = \infty$

NENP

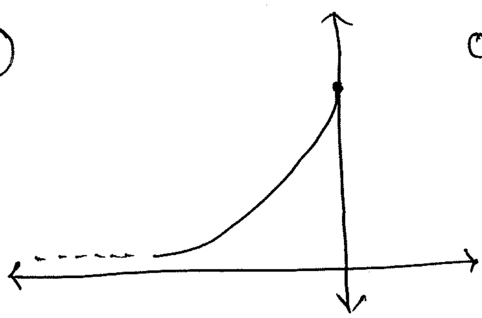
②



$0 < E < \infty$
$E = \frac{1}{2a}$, $P = 0$

Energy signal.

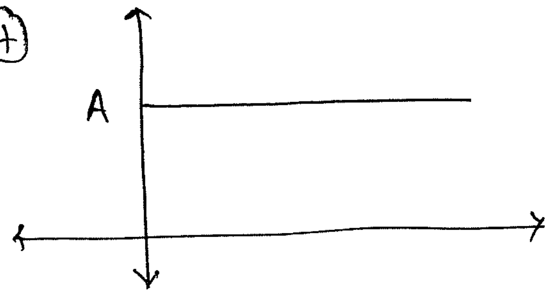
③



$0 < E < \infty$
$P = 0$

energy signal.

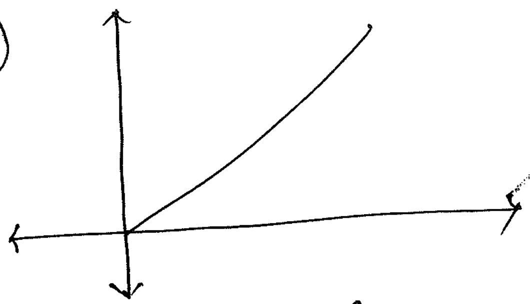
④



$E = \infty$
$P = \frac{A^2}{2}$

Power signal.

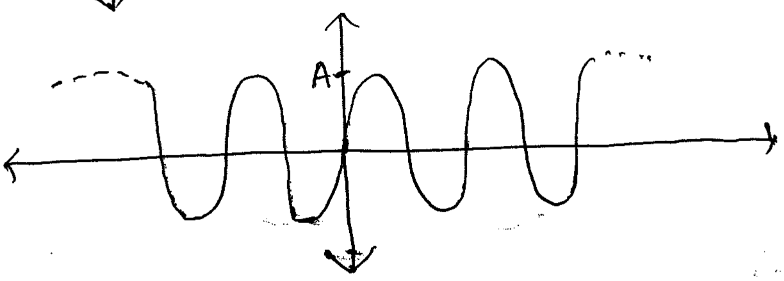
⑤



$P = \infty$
$E = \infty$

NENP

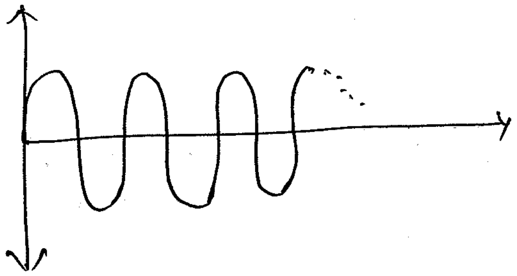
⑥



$E = \infty$
$P = \frac{A^2}{2}$, $0 < P < \infty$

∴ Power signal.

(7)



$E = \infty$

$P =$ finite

⇒ Every periodic signal is power signal but power signal didn't be periodic.

⇒ Every energy signal is non-periodic signal but every non-periodic signal didn't be energy signal.

⇒ For every energy signal,

Amp. $\rightarrow 0$ as $t \rightarrow \infty$

⇒ Energy and power signals are mutually exclusive.

⇒ A signal which maintains constant amplitude over the signal exists is said to be power signal.

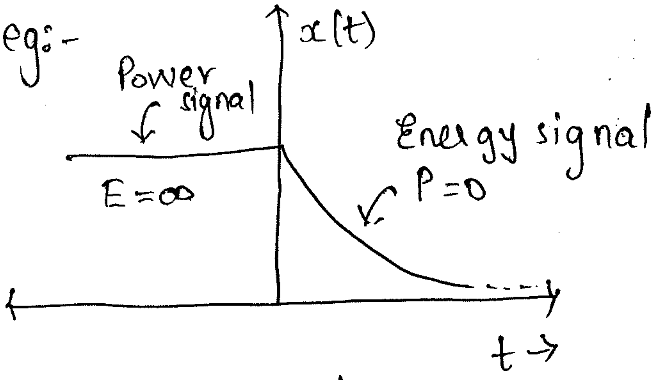
⇒ Every finite amplitude & finite duration signal is energy signal.

⇒ Every everlasting signal with constant amplitude, is a power signal but every power signal didn't be everlasting.

NOTE:-

If signal is combination of ENERGY SIGNAL + POWER SIGNAL = POWER SIGNAL

For eg:-



$$x(t) = A\cdot u(-t) + e^{-at}\cdot u(t)$$

$$P_{avg.} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A^2\cdot u^2(-t) + e^{-2at}\cdot u^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\int_{-T}^{0} A^2 dt + \int_{0}^{T} e^{-2at} dt\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[A^2\cdot T + \frac{1-e^{-2at}}{2a}\right]$$

$$= \frac{A^2}{2} + 0$$

$$\boxed{P_{avg.} = \frac{A^2}{2}} \leftarrow \text{finite}$$

Therefore above given signal is power signal bcz. it requires $E=\infty$ which is not possible.

Every practical signal is <u>energy signal</u>.

NOTE:-

→ For every even conjugate signals

Real part → even

Ima. part → odd

$x(t) = e^{jt} = \cos t + j\sin t$

$\cos t$ ↑ Real part ↓ even; $j\sin t$ ↑ Ima. part ↓ odd

$x(-t) = e^{-jt}$

$x^*(-t) = e^{jt}$

$\therefore \boxed{x(t) = x^*(-t)}$

→ For every odd conjugate signals

Real → odd

Ima. → even

$x(t) = t \cdot e^{jt}$ — (1)

$x(-t) = -t \cdot e^{-jt}$

$x^*(-t) = -te^{jt}$ — (2)

$\boxed{x(t) = -x^*(-t)}$

Eg:- $x(t) = t\cos t + jt\sin t$

$o \times e = o$ ↑ Real part odd signals

$o \times o = e$ ↑ Imag. part even signals

* PERIODIC SIGNAL & APERIODIC SIGNAL

→ In continuous time, a signal is said to be periodic if $x(t) = x(t \pm T)$ OR $x(t) = x(t \pm KT)$, K = integer and T = fundamental time period.

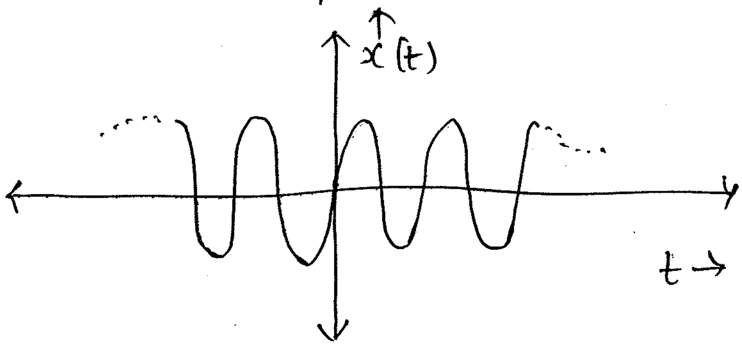
→ The value of T for above condition is satisfies is called fundamental time period OR Periodicity and given signal is said to be periodic signal.

→ Otherwise it is said to be 'non-periodic' OR Aperiodic signal.

→ All periodic signals are everlasting signals.

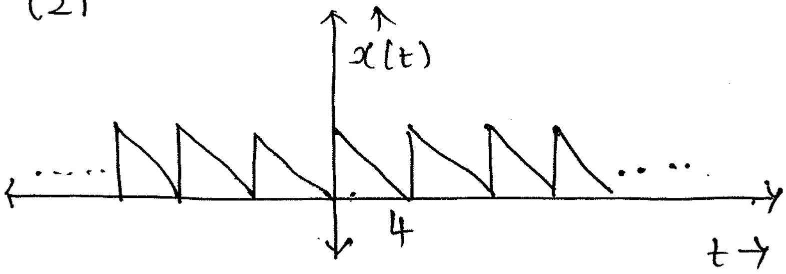
(1) $x(t) = A\sin\omega t$

$x(t) = A\sin\frac{2\pi}{T}t$



Periodic

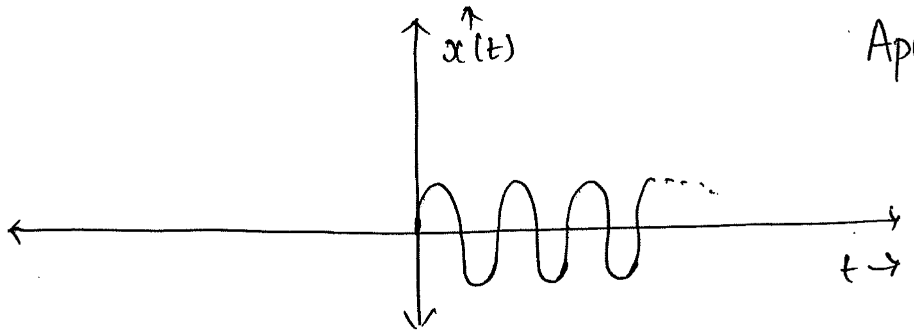
(2)



Periodic

$x(t) = x(t \pm 4k)$

(3) $x(t) = \sin\omega t \cdot u(t)$



Aperiodic

(4) $x(t) = 2\sin 150\pi t$

$x(t) = 2\sin\frac{2\pi}{T}\cdot t$

Periodic

$150\pi = \frac{2\pi}{T}$

$T = \frac{1}{75}$ sec $\quad f = 75 Hz = 75$ cycles/sec.

(5) $x(t) = 3\cos 150\pi t$

$f = 75 Hz$

PERIODIC

(6) $x(t) = 4\sin(150\pi t + \theta)$

PERIODIC

* Procedure to test periodicity of signal if signal is combination of two or more periodic signal.

$$x(t) = x(t_1) + x(t_2) + x(t_3) + \ldots\ldots$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ T_1, \omega_1 & T_2, \omega_2 & T_3, \omega_3 \end{array}$$

S_1: Find fundamental time period of each signal i.e. $T_1, T_2, T_3, \ldots\ldots$

S_2: Find ratio $\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4}, \ldots\ldots$

S_3: If all of ratio of S_2 are rational, then it is called periodic signal.

If any one of ratio of S_2 is irrational, then it is called non-periodic signal.

S_4: $T_0 = \text{LCM}(T_1, T_2, \ldots\ldots)$

$\omega_0 = \text{HCF}(\omega_1, \omega_2, \omega_3, \ldots\ldots)$; $\omega_0 = \frac{2\pi}{T}$

$\text{LCM}\left[\frac{p_1}{q_1}, \frac{p_2}{q_2}, \ldots\ldots\right]$

$$\text{LCM} = \frac{\text{LCM}\,[\text{Numerator of } T_1, T_2, \ldots\ldots]}{\text{HCF}\,[\text{Denominator of } T_1, T_2, \ldots\ldots]}$$

2^{nd} Method:

S_1: Find fundamental frequency of each signal

$\omega_1, \omega_2, \omega_3, \ldots\ldots$

S_2: Find ratio $\frac{\omega_1}{\omega_2}, \frac{\omega_1}{\omega_3}, \frac{\omega_1}{\omega_4}, \ldots\ldots$

S_3: If all ratios of S_2 are rational then it is periodic signal otherwise non-periodic.

$$\omega_0 = \frac{\text{HCF (numerator of } \omega_1, \omega_2, \omega_3, \ldots\ldots)}{\text{LCM (denominator of } \omega_1, \omega_2, \omega_3, \ldots\ldots)}$$

Q:- Identify the following signals are periodic or not. Also find their fundamental time period?

(1) $x(t) = 4\cos 3t \cdot u(t)$

Aperiodic

(2) $x(t) = -10\sin(\underbrace{5\pi t}_{\omega t} - 30^\circ)$

$\omega \cancel{t} = 5\pi \cancel{t}$

$\omega = 5\pi$

$\frac{2\cancel{\pi}}{T} = 5\cancel{\pi}$

$\boxed{T = \frac{2}{5} = 0.4\,sec}$

$x(t+T) = x(t)$

$x\left(t+\frac{2}{5}\right) = -10\sin\left(5\pi\left(t+\frac{2}{5}\right) - 30^\circ\right)$

$= -10\sin(5\pi t + 2\pi - 30^\circ)$

$= -10\sin(2\pi + (5\pi t - 30^\circ))$

$= -10\sin(5\pi t - 30^\circ)$

$= x(t)$ $\quad\therefore$ It is a periodic signal.

(3) $x(t) = 3\cos^2(0.5\pi t)$

$$= 3\left[\frac{1+\cos \pi t}{2}\right]$$

$$= \frac{3}{2} + \frac{3}{2}\cos \pi t$$

$$\omega = \pi$$

$$\frac{2\cancel{\pi}}{T} = \cancel{\pi}$$

$$\boxed{T = 2\,\text{sec}}$$

$$x(t+2) = 3\cos^2(0.5\,\pi(t+2))$$

$$= 3\cos^2(\pi + 0.5\pi t)$$

$$= 3\left[\frac{1+\cos(2\pi+\pi t)}{2}\right]$$

$$= 3\left[\frac{1+\cos \pi t}{2}\right]$$

$$= \frac{3}{2}\cos^2 0.5\pi t$$

$$= x^2(t)$$

It is periodic signal.

NOTE:- Adding constant to periodic signal will never change periodicity

(4) $x(t) = 10\, e^{j(5/7 t + 30^\circ)}$

→ Complex sinusoidal signal are always periodic.

$$x(t) = A e^{j(\omega t + \theta)}$$

$$\omega = 5/7$$

$$\frac{2\pi}{T} = 5/7$$

$$T = \frac{14\pi}{5}\ \text{sec.}$$

$$x(t+T) = 10\, e^{j(5/7(t+\frac{14\pi}{5})+30^\circ)}$$

$$= 10\, e^{j(5/7 t + 2\pi + 30^\circ)}$$

$$= 10\, e^{j(5/7 t + 30^\circ)} \cdot e^{j2\pi}$$

$$= x(t)\left[\cos 2\pi + j\sin 2\pi\right]$$

$$x\left(t+\frac{14\pi}{5}\right) = x(t)$$

(5) $x(t) = \sin\sqrt{3}t$

$$\underset{\omega}{\uparrow}$$

$$\omega = \sqrt{3}$$

$$\frac{2\pi}{T} = \sqrt{3}$$

$$\boxed{T = \frac{2\pi}{\sqrt{3}}}$$

$$\cancel{\sin}\,x\left(t+\frac{2\pi}{\sqrt{3}}\right) = \sin\sqrt{3}\left(t+\frac{2\pi}{\sqrt{3}}\right)$$

$$= \sin\left(\sqrt{3}t + 2\pi\right)$$

$$= \sin\sqrt{3}t$$

(6) $x(t) = 1+2t+3t^2$

Non-periodic

(7) $x(t) = \cos 5t \cdot \sin 7t$

$$= \frac{1}{2}\left[\sin 12t + \sin 2t\right] \qquad [\because s+s = 2sc]$$

$$\omega_1 = 12 \qquad \omega_2 = 2$$

$$\frac{2\pi}{T_1} = \cancel{12}\,6 \qquad \frac{\cancel{2}\pi}{T_2} = \cancel{2}$$

$$T_1 = \frac{\pi}{6} \qquad T_2 = \pi$$

$$\frac{T_1}{T_2} = \frac{\pi}{6\cdot\pi} = \frac{1}{6} \leftarrow \text{rational.}$$

$\boxed{\text{So periodic}}$

$\omega_0 = HCF(\omega_1, \omega_2, \omega_3, \ldots)$

$= HCF(2, 12)$

$\omega_0 = 2$

$\frac{2\pi}{T_0} = 2$

$\boxed{T_0 = \pi}$

NOTE: In continuous time if individual signals are periodic then combination of that ~~sign~~ periodic signals didn't be periodic.

(8) $x(t) = \cos \frac{\pi}{3} t - \sin(10t - 30^\circ)$

$\omega_1 = \frac{\pi}{3}$ $\qquad$ $\omega_2 = 10$

$\frac{2\pi}{T_1} = \frac{\pi}{3}$ $\qquad$ $\frac{2\pi}{T_2} = \cancel{10}\,5$

$\boxed{T_1 = 6}$ $\qquad$ $\boxed{T_2 = \frac{\pi}{5}}$

$\frac{T_1}{T_2} = \frac{6 \cdot 5}{\pi} = \frac{30}{\pi}$

$\therefore$ It is an irrational ~~signal~~ number

$\therefore$ Signal is aperiodic/non-periodic.

(9) $x(t) = 5\cos(5\pi t) - 3\sin\left(\frac{\pi}{0.3} + 45^\circ\right) + \sin\left(\frac{3\pi}{5}\right)t$

$\omega_1 = 5\pi$ $\qquad$ $\omega_2 = \frac{\pi}{0.3} = \frac{10\pi}{3}$ $\qquad$ $\omega_3 = \frac{3\pi}{5}$

$\frac{2\pi}{T_1} = 5\pi$ $\qquad$ $\frac{2\pi}{T_2} = \frac{\cancel{10}\,5\pi}{3}$ $\qquad$ $\frac{2\pi}{T_3} = \frac{3\pi}{5}$

$T_1 = \frac{2}{5}$ $\qquad$ $T_2 = \frac{3}{5}$ $\qquad$ $T_3 = \frac{10}{3}$

$\frac{T_1}{T_2} = \frac{2\cancel{5}}{\cancel{5} \cdot 3} = \frac{2}{3}$ rational.

$T_1/T_3 = \frac{2 \cdot 3}{5 \cdot \cancel{10}\,5} = \frac{\cancel{6}\,3}{25}$ $\qquad$ $\therefore$ It is a periodic signal.

$\omega_1 = \frac{5\pi}{1}$, $\omega_2 = \frac{10\pi}{3}$, $\omega_3 = \frac{3\pi}{5}$

$\omega_0 = HCF(\omega_1, \omega_2, \omega_3)$

$= HCF\left(5\pi, \frac{10\pi}{3}, \frac{3\pi}{5}\right)$

$$\omega_0 = \frac{HCF(5\pi, 10\pi, 3\pi)}{LCM(1,3,5)} = \frac{\pi}{15}$$

$$\frac{2\pi}{T} = \frac{\pi}{15}$$

$\boxed{T = 30 sec.}$

Q:- Check periodicity of signal and find time period.

$x(t) = even\{\cos 3\pi t \cdot u(t)\}$.

$$\Rightarrow x_e(t) = \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi t \cdot (u(-t))}{2}$$

$$= \cos 3\pi t \left[\frac{u(t) + u(-t)}{2}\right]$$

$$x_e(t) = \frac{\cos 3\pi t}{2} \cdot 1$$

$\downarrow$

$\omega = 3\pi$

$$\frac{2\pi}{T} = 3\pi$$

$\boxed{T = \frac{2}{3} sec}$. rational.

$\therefore$ It is periodic signal.

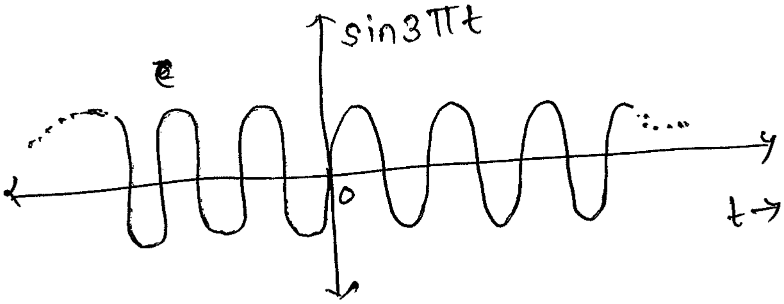
Q:- $x(t) = \text{even}\{\sin 3\pi t \cdot u(t)\}$

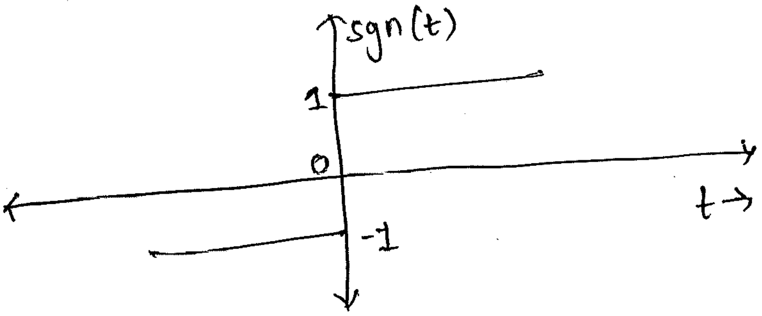
Sol:- $x(-t) = -\sin 3\pi t \cdot u(-t)$

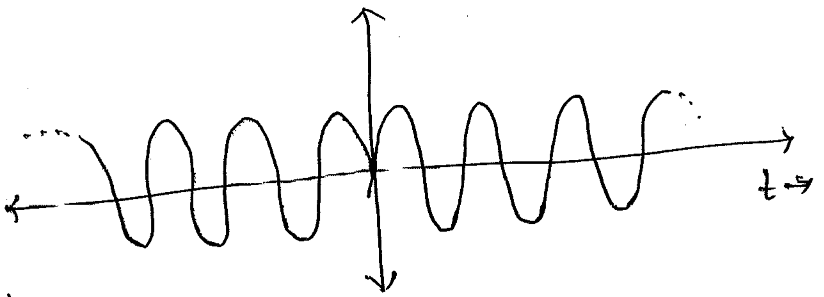
$x(t) = \sin 3\pi t \cdot u(t)$

$$x_e(t) = \frac{\sin 3\pi t \cdot u(t) - \sin 3\pi t \cdot u(-t)}{2}$$

$$= \sin 3\pi t \left(\frac{u(t) - u(-t)}{2}\right)$$







Aperiodic signal

Q:- $y(t) = je^{10t}$

Sol:- $y(t) = e^{j\pi/2} e^{j10t}$

$y(t) = e^{j(10t + \pi/2)}$

Additionally phase change of 90°

It will never change periodicity.

$\omega = 10$

$\frac{\cancel{2}\pi}{T} = \cancel{10}\,5$

$$\boxed{T = \frac{\pi}{5}}$$

$x(t) = A\cos(\omega t + \theta)$

ω → analog angular frequency

$\omega = 2\pi f$

f → Hz / cycle/sec.

$t = nT_s$ → sampling period $= T_s = \frac{1}{f_s}$

f_s → sampling frequency

$$x[n] = A\cos\left(\frac{2\pi}{T} \cdot nT_s + \theta\right)$$

$$= A\cos\left(\frac{2\pi}{T} \cdot T_s n + \theta\right)$$

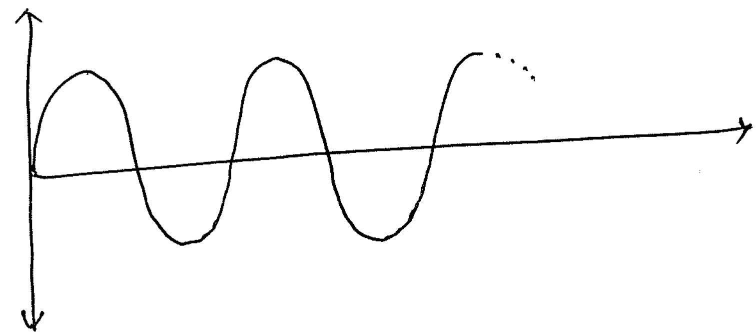
$$x[n] = A\cos(\Omega \times n + \theta)$$

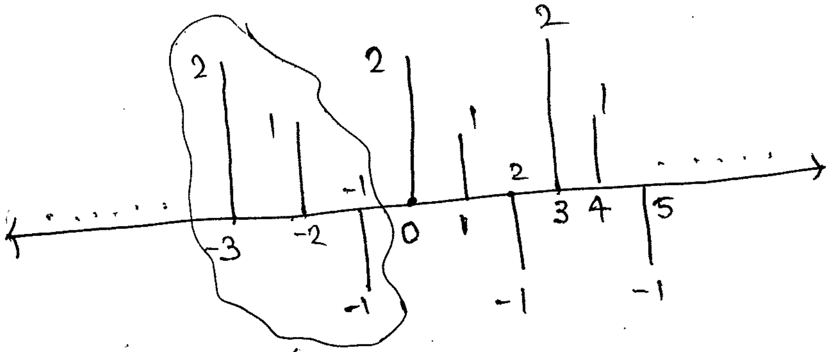
$\Omega = \frac{2\pi}{T} \cdot T_s$ → angular frequency

$\therefore \boxed{\Omega = 2\pi f \cdot T_s}$

$f \cdot T_s = \frac{f}{f_s}$ = cycles/samp.

Suppose consider a sinusoidal signal





$\therefore N = 3$

→ no. of samples.

→ After every three samples signal (discrete) is repeated.

- A discrete time signal is said to be periodic if $x[n] = x[n+N]$ OR $x[n] = x[n \pm kN]$; N = fondamental time period

- A signal which do not satisfy above condition is said to be non-periodic OR Aperiodic signal.

$x[n] = A\cos[\Omega n + \phi]$

$x[n+N] = A\cos[\Omega(n+N) + \phi]$

$= A\cos[\Omega n + \phi + \Omega N]$

$= A\cos[\Omega n + \phi]$

$\downarrow$

$\Omega n = 2\pi K$

$\Omega N = 2\pi\ (2)$ $\quad K=2 \rightarrow$ two cycle

$\Omega N = 2\pi\ (1)$ $\quad K=1 \rightarrow$ one cycle

$\Omega N = 2\pi\ (3)$ $\quad K=3 \rightarrow$ three cycle

$$\boxed{\frac{\Omega}{2\pi} = \left\{\frac{K}{N}\right.}$$

K → No. of cycles of contⁿ time system

N → fundamental time period

→ rational number

Q:- find cycles & fundamental time period of signal?

$x(t) = 2\cos(150\pi t + 45^\circ)$, $f_s = 200\text{Hz}$

Solⁿ:- $\omega \cancel{t} = 150\pi \cancel{t}$

$\omega = 150\pi$

$\frac{2\cancel{\pi}}{T} = 150\cancel{\pi}$

$T = \frac{1}{75}$ sec.

$f_s = 200\ \text{Hz}$

$T_s = \frac{1}{200}$ sec.

$$x[n] = 2\cos(150\pi(nT_s) + 45^\circ)$$

$$= 2\cos\left(\cancel{150}^{3}\pi\left(n \cdot \frac{1}{\cancel{200}_{4}}\right) + 45^\circ\right)$$

$$x[n] = 2\cos\left(\frac{3\pi n}{4} + 45^\circ\right)$$

$$\Omega \cancel{n} = \frac{3\pi \cancel{n}}{4}$$

$$\frac{\Omega}{2\pi} = \frac{3\cancel{\pi}}{4 \cdot 2\cancel{\pi}} = \frac{3}{8} \begin{cases} \to \text{cycles} \\ \to N = 8 \end{cases}$$

Now,

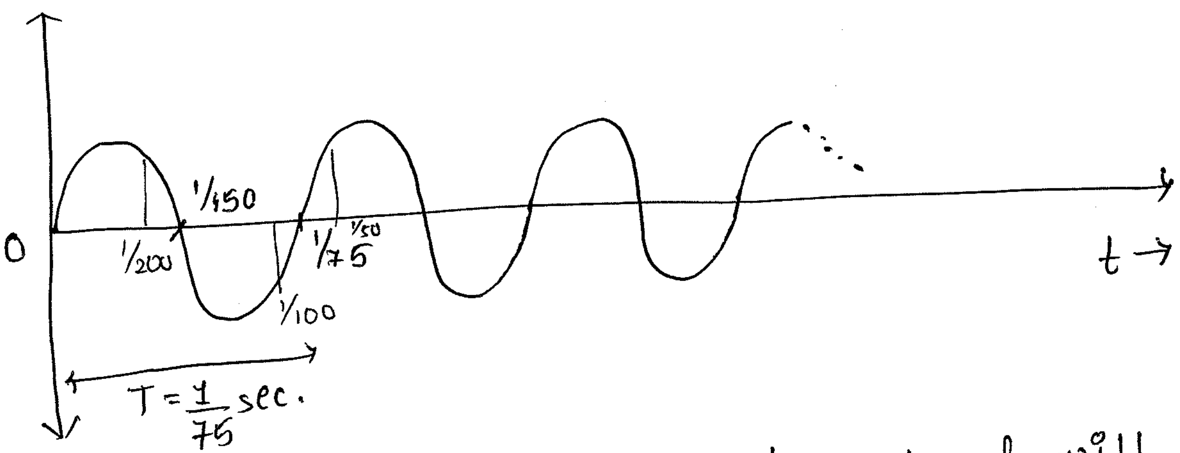
$$T = nT_s$$

$$n = \frac{T}{T_s}$$

$$= \frac{1}{75} \cdot 200$$

$$= \frac{8}{\cancel{3}} \cdot \cancel{3}$$

$$n = 8 \qquad \therefore n = \frac{3T}{T_s}$$



It means that after 3 cycles signal will be periodic & there will be 8 fundamental time period OR 8 signals (discrete).

Q:- $x[n] = \sin\left[\frac{3\pi}{5} \cdot n\right]$

$\Omega = \frac{3\pi}{5}$

$\frac{\Omega}{2\pi} = \frac{3\pi}{5} \cdot \frac{1}{2\pi} = \frac{3}{10}$ → cycles

↳ $N = 10$

NOW,

$\Omega = \frac{2\pi}{T} \cdot T_s$

$T = \frac{10}{3} t_s$

* PROCEDURE to test PERIODICITY of discrete time signal if $x[n]$ is combination of two or more periodic signals.

$x[n] = x_1[n] + x_2[n] + x_3[n] + \ldots\ldots$

<u>Step:-1</u> Find fundamental period of each samples

<u>Step:-2</u> Ratio of $\frac{N_1}{N_2}, \frac{N_1}{N_3}, \frac{N_1}{N_4}, \ldots\ldots$

<u>Step:3</u> If in step:2 all ratio are rational number then given signal is periodic and fundamental time period of signal is

$LCM(N_1, N_2, N_3, \ldots\ldots)$

- If one of the signal ratio in step:2 is irrational then signal is non-periodic <u>OR</u> Aperiodic signal.

Q:- Test the periodicity of given signal and find fondamental time period of given signal.

(1) $x[n] = 10\cos(3\pi n)\,u[n]$

$\downarrow$

$\Omega = 3\pi$

not everlasting signal bcz. of $u[n]$

$\therefore$ It is non-periodic signal.

(2) $x[n] = -4\sin^2(0.4\pi n + 45^\circ)$

$$= -4\left[\frac{1-\cos 2(0.04\pi n + 45^\circ)}{2}\right]$$

$$= -4\left[\frac{1-\cos(0.8\pi n + 90^\circ)}{2}\right]$$

$$= -2\left[1-\cos(0.8\pi n + 90^\circ)\right]$$

$$= -2 + 2\cos(0.8\pi n + 90^\circ)$$

$\downarrow$

$\Omega = 0.8\pi$

$$\frac{\Omega}{2\pi} = \frac{0.8\pi}{2\pi} = \frac{8}{20} = \frac{4}{10} = \frac{2}{5}$$

$2 \to$ cycles

$5 \to N=5$

$\therefore$ It is periodic signal.

(3) $x[n] = 4e^{-j(100n)}$

$\Omega = 100$

$$\frac{\Omega}{2\pi} = \frac{100}{2\pi} = \frac{50}{\pi}$$ $\times$ It is an irrational number

$\therefore$ It is aperiodic signal.

④ $x[n] = 15\cos\left(\frac{30}{7}n\right)$

$\Omega = \frac{30}{7}$

$\frac{\Omega}{2\pi} = \frac{\cancel{30}^{15}}{7 \cdot \cancel{2}\pi} = \frac{15}{7\pi} \rightarrow$ irrational

$\therefore$ It is aperiodic signal

⑤ $x[n] = 7\cos\left(\frac{\pi}{3}n - 30^\circ\right) - 5\sin\left(\frac{2\pi}{10} \cdot n\right)$

$\frac{\Omega}{2\pi} = \frac{\cancel{\pi}}{3} \cdot \frac{1}{2\cancel{\pi}} = \frac{1}{6}$ $N_1 = 6$

$\frac{\Omega}{2\pi} = \frac{\cancel{2\pi}}{10} \cdot \frac{1}{\cancel{2\pi}} = \frac{1}{10}$ $N_2 = 10$

$\frac{N_1}{N_2} = \frac{6}{10} = 0.6 \rightarrow$ rational.

$T = LCM(6, 10)$ Periodic Signal.

$\boxed{T = 30 sec.}$

⑥ $x[n] = 4\sin(7\pi n) - 13\cos(6n)$

$\frac{\Omega}{2\pi} = \frac{7\cancel{\pi}}{2\cancel{\pi}} = \frac{7}{2}$ $N_1 = 2$

$\frac{\Omega}{2\pi} = \frac{6}{2\pi} = \frac{3}{\pi}$ $N_2 = \pi$

$\frac{N_1}{N_2} = \frac{2}{\pi} \rightarrow$ irrational

$\therefore$ It is aperiodic signal

7] $x[n] = (j)^{n/2}$

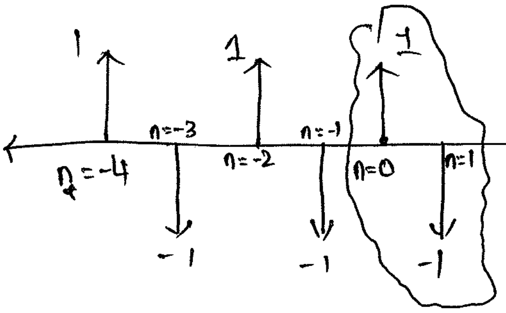
$$x[n] = e^{j\, n/2 \cdot \pi/2} = e^{\frac{j\pi n}{4}}$$

$$\Omega = \frac{\pi}{4}$$

$$\frac{\Omega}{2\pi} = \frac{\cancel{\pi}}{4 \cdot 2\cancel{\pi}} = \frac{1}{8}$$

$\boxed{N_1 = 8}$, $T_1 = 1$ sec.

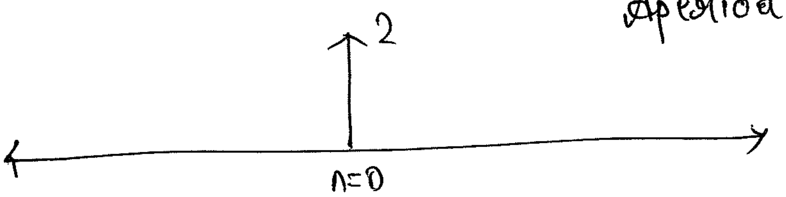
8] $x[n] = (-1)^{n^2}$



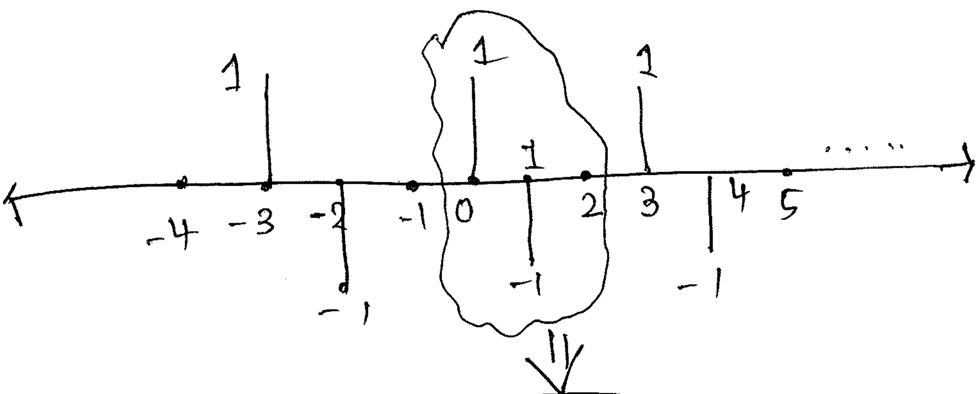
$\Rightarrow \boxed{N=2}$

9] $x[n] = u[n] + u[-n]$

Aperiodic



10] $x[n] = \sum_{k=-\infty}^{\infty} \delta(n-3k) - \delta(n-1-3k)$



$\Downarrow$

$\boxed{N=3}$

$k=-3$, $\delta(n+9)-\delta(n+8)$

$k=-2$, $\delta(n+6)-\delta(n+5)$

$k=-1$, $\delta(n+3)-\delta(n+2)$

$k=0$, $\delta(n)-\delta(n-1)$

$k=1$, $\delta(n-3)-\delta(n-4)$

$k=2$, $\delta(n-6)-\delta(n-7)$

$k=3$, $\delta(n-9)-\delta(n-10)$

11] $x[n] = \cos\left[\frac{n}{6} + \frac{\pi}{4}\right]$

Here, $\Omega = \frac{1}{6}$

$$\frac{\Omega}{2\pi} = \frac{1}{6} \cdot \frac{1}{2\pi} = \frac{1}{12\pi} \Rightarrow \text{irrational}$$

$\therefore$ Aperiodic signal.

12] $x[n] = \sin\left(\frac{\pi n}{3}\right) + \cos\left(\frac{\pi n}{4}\right)$

$$\Omega = \frac{\pi}{3}$$

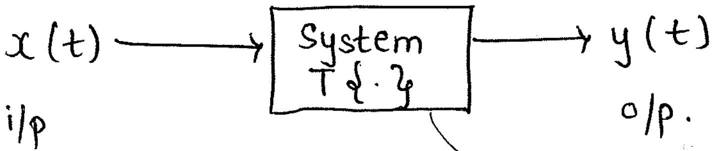
$$\frac{\Omega}{2\pi} = \frac{\cancel{\pi}}{3} \cdot \frac{1}{2\cancel{\pi}} = \frac{1}{6} \Rightarrow N_1 = 6$$

$$\Omega = \frac{\pi}{4}$$

$$\frac{\Omega}{2\pi} = \frac{\cancel{\pi}}{4} \cdot \frac{1}{2\cancel{\pi}} = \frac{1}{8} \Rightarrow N_2 = 8$$

$$\boxed{N = LCM(N_1, N_2) = 24}$$

* Introduction to Systems *



$y(t) = \ln(x(t))$ any operation

$y(t) = x^2 t$

- A system is an operator which maps relation between input and output signal by process of transformation

- A system is a set of elements arranged in such a way that it produces expected o/p signal to given i/p signal.

For eg:- Electrical sys., Mechanical sys., Electromechanical sys.

* Representation of systems *

① $y(t) = x^2(t)$

② $\frac{dy(t)}{dt} + y(t) = x(t)$

③ $y[n] + y[n-1] = x[n]$

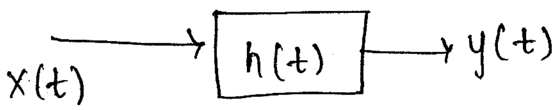
④ $x[w]$, $X[s]$, $X[z]$

$Y[w] = H[w] \cdot X[w]$

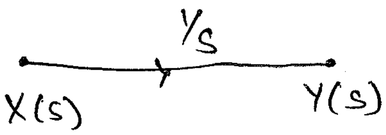
$Y[s] = H[s] \cdot X[s]$

$Y[z] = H[z] \cdot X[z]$

⑤



⑥



$Y(s) = X(s) \cdot \frac{1}{s}$

① Continuous Time System

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

$$y(t) = x(t) * h(t)$$

② Discrete Time System

$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

$$y[n] = x[n] * h[n]$$

* Classification of Systems *

- It depends upon i/p and o/p.

① Linear and Non-linear system

- A system is said to be linear if it satisfies superposition theorem. Superposition theorem is combination of addivity and homogeneity.

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

① Additivity

$x_1(t) \longrightarrow \boxed{h(t)} \longrightarrow y_1(t)$

$x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow y_2(t)$

$y(t) = y_1(t) + y_2(t)$ —— (1)

$x(t) = x_1(t) + x_2(t) \longrightarrow h(t) \rightarrow y'(t)$ — (2)

If $y(t) = y'(t)$ is satisfied $\Rightarrow$ Additivity

② Homogeneity theorem

$x(t) \longrightarrow k x(t)$

$$K \cdot x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) \quad \text{—(1)}$$

$y(t) = T\{x(t)\}$

$K \cdot y(t) = K \cdot T\{x(t)\}$ — (2)

If (1) = (2) $\Rightarrow$ Homogeneity

Q:- $y(t) = x(t)\sin\omega t$. Check linear or non-linear?

Sol:- $y_1(t) = x_1(t)\sin\omega t$

$y_2(t) = x_2(t)\sin\omega t$

$y(t) = y_1(t) + y_2(t)$

$y(t) = x_1(t)\sin\omega t + x_2(t)\sin\omega t$ —— (1)

$x(t) \longrightarrow x_1(t) + x_2(t)$

$y(t) = [x_1(t) + x_2(t)]\sin\omega t$

$y(t) = x_1(t)\sin\omega t + x_2(t)\sin\omega t$ —— (2)

(1) = (2)

Now, $x(t) \longrightarrow K \cdot x(t)$

$y(t) = (K \cdot x(t))\sin\omega t$

$y(t) = K \cdot x(t) \cdot \sin\omega t$ —(1)

$y'(t) = K(x(t)\sin\omega t)$

$y'(t) = K \cdot x(t)\sin\omega t$ —— (2)

(1) = (2)

$\therefore$ System given is LINEAR.

Q:- $y(t) = \alpha e^{x(t)}$

Sol^n:- $y_1(t) = \alpha e^{x_1(t)}$

$y_2(t) = \alpha e^{x_2(t)}$

$y(t) = y_1(t) + y_2(t)$

$y(t) = \alpha e^{x_1(t)} + \alpha e^{x_2(t)}$

$y(t) = \alpha\left(e^{x_1(t)} + e^{x_2(t)}\right)$ —— (1)

Now, $x(t) \longrightarrow x_1(t) + x_2(t)$

$y(t) = \alpha e^{(x_1(t) + x_2(t))}$ —— (2)

$(1) \neq (2)$

Non-linear.

Q:- $\int_{-\infty}^{t} x(\tau)\, d\tau = y(t)$

Sol^n: $y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$

$y_2(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau$

$y(t) = y_1(t) + y_2(t)$

$y(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau + \int_{-\infty}^{t} x_2(\tau)\, d\tau$ —— (1)

$x(t) \longrightarrow x_1(t) + x_2(t)$

$y(t) = \int_{-\infty}^{t} (x_1(\tau) + x_2(\tau))\, d\tau$ —— (2)

$(1) = (2)$

$\therefore$ System is LINEAR

$x(t) \rightarrow k \cdot x(t)$

$y(t) = \int_{-\infty}^{t} k \cdot x(\tau)\, d\tau$ —— (1)

$y'(t) = k \cdot \int_{-\infty}^{t} x(\tau)\, d\tau$ —— (2)

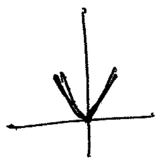
$(1) = (2)$

Q:- $y(t) = \sin(x(t))$

Non-linear

Q:- $y(t) = |m(t)|$

Non-linear



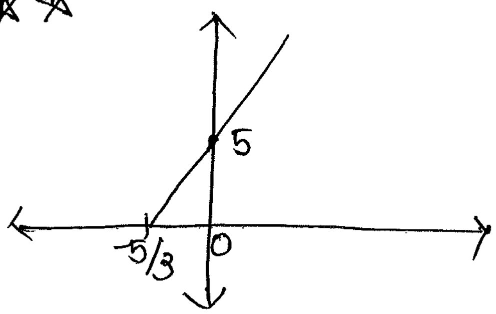
$m(t) \rightarrow -5m(t)$

$y(t) = |-5m(t)| = m(t) \cdot 5$ —(1)

$y'(t) = -5|m(t)| = -5m(t)$ —(2

(1) $\neq$ (2).

Q:- $y(t) = 3x(t) + 5$



Non-linear

But actually it is incremental linear system.

Q:- $y[n] = 3x[n] - 2x[n-1]$

Linear

Q:- $y[n] = 2^{x[n]}$

Non-linear

Q:- $y[n] = \dfrac{1}{3x[n] - 2x[n]}$

Non-linear

Q:- $y[n] = \text{sgn}[x[n]]$

Non-linear

Q:- $y[n] = x^*[n]$

Let $\alpha = 2 - j3$

$y[n] = (\alpha x[n])^*$

$= [(2-j3)x[n]]^*$

$y[n] = (2+j3)x^*[n]$

$\alpha y[n] = y'[n] = (2-j3)\, x^*(n)$

Non-linear

Q:- $y[n] = x[-n]$

Linear

Q:- $y[n] = x^2[n]$

Non-linear

Q:- $y[n] = x[2n]$

Linear

Q:- $y[n] = x[2n-k]$

Linear

Q:- $y[n] = \ln x[n]$

Non-linear

Q:- $y[n] = a^{x[n]}$

$y_1[n] = a^{x_1[n]}$

$y_2[n] = a^{x_2[n]}$

$y[n] = y_1[n] + y_2[n]$
$= a^{x_1[n]} + a^{x_2[n]}$

$y[n] = a^{[x_1[n] + x_2[n]]}$

Non-linear

Q:- $y[n] = \text{Real}[x[n]]$

o/p due to $2+j3$ $= \text{Real}\{(2+j3)x[n]\}$
$= 2 \cdot \text{Real}[x[n]]$

$(2+j3)\,\text{Real}\{x[n]\} \neq 2\,\text{Real}[x[n]]$

So, non-linear

$\boxed{\text{Real, complex part and conjugate all are non-linear.}}$

Q:- $y[n] = \text{median}[x[n]]$

Sol^n:- $o/p_1 = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$ $= 3$

$o/p_2 = \{1, 1, 2, 1, 2\}$ $= 2$

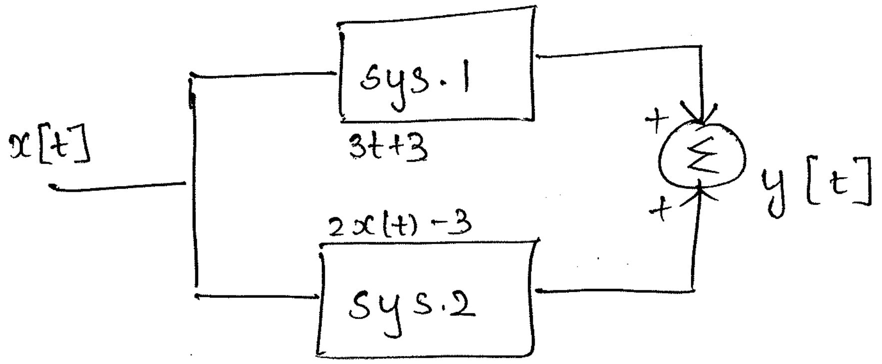
$3 + 2 = 5$

$o/p_1 + o/p_2 = \{1, 1, 1, 1, 2, 2, 2, 3, 4, 5\}$

$\frac{2+2}{2} = 2$ So, non-linear

Quantization is non-linear operation while Sampling is linear operation.

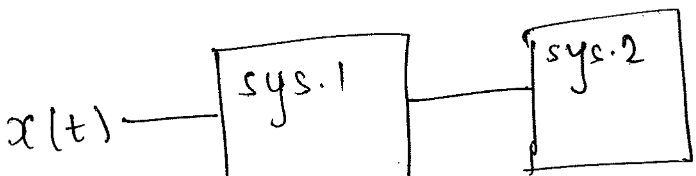
* Two-system connected in parallel. Then comment on linearity?



- Directly we can't comment on linearity bcz. we have to check both system and also mathematical operation between them. (+ OR -)

- But if sys.① = linear & sy② = linear
then sys ① + sys ② = linear.

* If two systems are connected in cascade/series



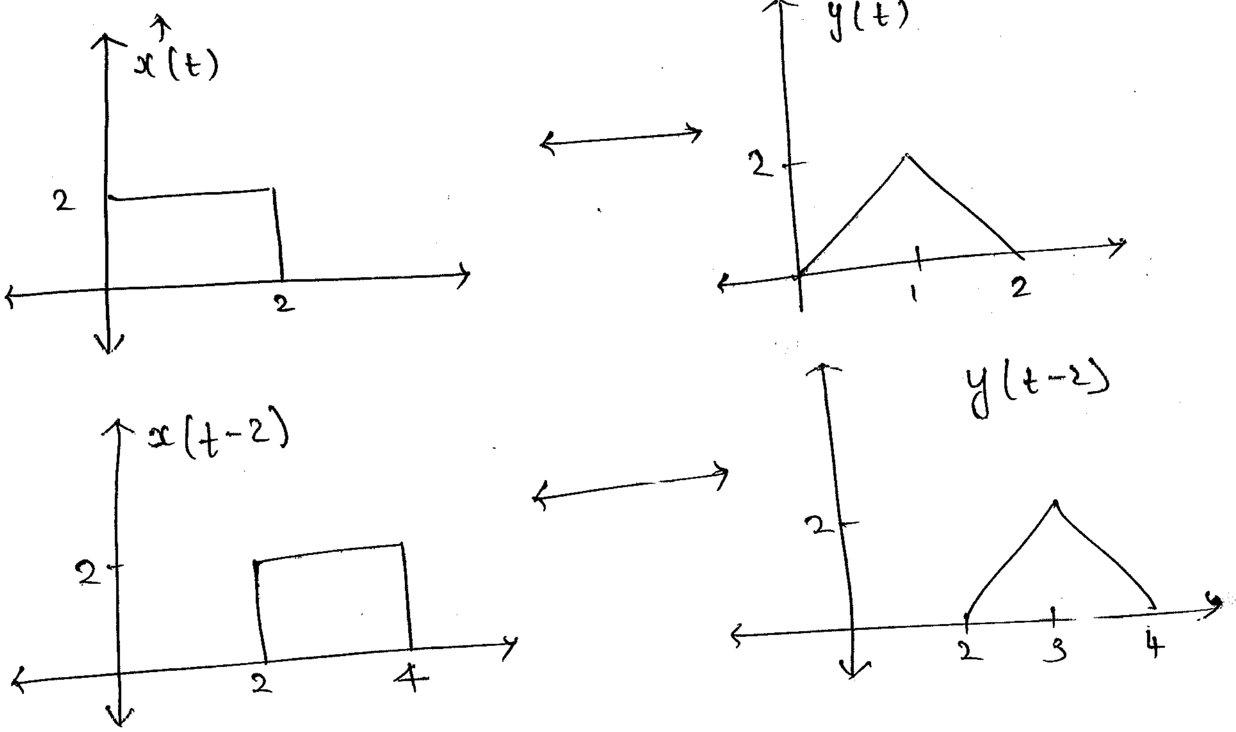
exp.	ln
$y(t) = x(t)$	$y(t) = \frac{1}{x(t)}$
$5x(t)$	$2x(t)$
$\sqrt{x(t)}$	$(x(t))^2$

Sol^n: - Directly we can't comment on linearity.

② Time Variant and Time Invariant

$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$

- A system is said to be time invariant if its input output characteristics is not changing w.r.t to time. i.e. if input is delayed OR advanced by t_0 seconds then output should also be delayed/advanced by t_0 seconds. Otherwise it is said to be Time Variant system.



Time Invariant

① $y[n,k]$ = output due to delayed i/p

$y[n-k]$ = delayed o/p

② $x(x, t_0)$: output due to delayed i/p

$y(t-t_0)$: delayed o/p.

Q:- $y(t) = x(2t)$

$y(t-t_0) = x(2(t-t_0))$

$= x(2t-2t_0)$ —— (2)

$y(t, t_0) = x(2t-t_0)$ —— (1)

$(1) \neq (2) \Rightarrow$ Time Variant

Q:- $y(t) = t \cdot x(t)$

i] $y(t, t_0) = t\, x(t-t_0)$ —— (1)

ii] $y(t-t_0) = (t-t_0)\, x(t-t_0)$ —— (2)

$(1) \neq (2)$

Time Variant

Eg:- An ideal resistor (Time invariant)

Q:- $y(t) = e^{-x(t)}$

$y(t, t_0) = e^{-x(t-t_0)}$ —— (1)

$y(t-t_0) = e^{-x(t-t_0)}$ —— (2)

$(1) = (2)$

Time Invariant

Aircraft (mass of aircraft changes as fuel is consumed)

$a(t) = \frac{F(t)}{m(t)}$ (Time Variant)

Q:- $y(t) = x^2(t)$

$y(t, t_0) = x^2(t-t_0)$ — (i)

$y(t-t_0) = x^2(t-t_0)$ — (ii)

$(i) = (ii)$

Time Invariant, Non-linear

Q:- $y(t) = x(-t)$

$y(t, t_0) = x(-t-t_0)$ — (i)

$y(t-t_0) = x(-(t-t_0)) = x(-t+t_0)$ — (ii)

$(i) \neq (ii)$ Time Variant, Non-linear

Q:- $y[n] = g[n]x[n]$

Sol^n:- $y[n, n_0] = g[n]x[n-n_0]$ — (i)

$y[n-n_0] = g[n-n_0]x[n-n_0]$ — (ii)

(i) $\neq$ (ii)

Time Variant

Q:- $y[n] = 3x[n]$

Sol^n:- Time Invariant, Linear

Q:- $y[n] = x[-n]$

Sol^n:- Time Variant, Linear

Q:- $y[n] = 3x[n] - x[n-1]$

Sol^n:- $y[n, n_0] = 3x[n-n_0] - x[n-1-n_0]$ — (i)

$y[n-n_0] = 3x[n-n_0] - x[n-n_0-1]$ — (ii)

(i) = (ii)

Time Invariant, Linear

Q:- $y[n] = x^2[n]$

Sol^n:-

$y[n, n_0] = x^2[n-n_0]$ — (i)

$y[n-n_0] = x^2[n-n_0]$ — (ii)

Time Invariant, Non-linear

Q:- $y[n] = X[n^2]$

Sol^n:- $y[n, n_0] = X[n^2 - n_0]$ — (i)

$y[n-n_0] = X[(n-n_0)^2]$ — (ii)

(i) $\neq$ (ii)

Time Variant, Linear

Q:- $y[n] = x[2n-k]$

Sol^n:- Time Variant, Linear

Q:- $y[n] = n\,x[n]$

Sol^n:- Time Variant, Linear

Q:- $y[n] = \log\{x[n]\}$

Sol^n:- Time invariant, non-linear

Q:- $y[n] = \cos\{x[n]\}$

Sol^n:- Time invariant, non-linear

Q:- $y[n] = a^{x[n]}$

Sol^n:- Time invariant, non-linear

③ Causal and Non-Causal ~~signal~~ system

A system is causal system if the present o/p depends only on present i/p and past values of the i/p but not on future values i.e. causal systems are non-anticipative

(1) $y(t) = (3t+1)\,x(t)$

Causal, ~~static~~

(2) $y(t) = \sin\{x(t)\}$

Causal

(3) $y(t) = x\{\sin(t)\}$

Non-causal

$y(-\pi) = x\{\sin(-\pi)\}$

$y(-\pi) = x(0)$

o/p is expecting future value. So, non-causal

(4) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

Causal

(5) $y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$

Non-causal

(6) $y[n] = 2x[n] + 3u[n+1]$

$y[0] = 2x[0] + 3u[1]$

$y[0] = 2x[0] + 3$

Causal

(7) $y[n] = \sum_{k=n_0}^{n} x[k]$

$n_0 = 0$

$y[n] = \sum_{k=0}^{n} x[k]$

Non-causal

$y[2] = \sum_{k=0}^{2} x[2]$

$y[2] = x[0] + x[1] + x[2]$

$y[-1] = \sum_{k=0}^{-1} x[-1]$

$y[-1] = x[0] + x[-1]$

$n_0 > n$ Eg:- $n_0 = 5$, $n = 2$

$y[n] = \sum_{k=5}^{2} x[k]$

$y[n] = x[5] + x[4] + x[3] + x[2]$

Eg:- $n_0 = 0$, $n = -1$

$y[-1] = \sum_{k=-1}^{0} x[k] = x[0] + x[-1]$

Non-causal

$n_0 < n$

$n = 5$, $n_0 = 2$

$y[n] = \sum_{k=2}^{5} x[k]$

$= x[2] + x[3] + x[4] + x[5]$

(8) $y[n] = \sum_{k=-\infty}^{n} x[k]$ → Accumulator → causal

$y[3] = \sum_{k=-\infty}^{3} x[k] = x[-\infty] + \dots + x[1] + x[2] + x[3]$

(9) $y[n] = \frac{1}{2m+1} \sum_{k=-m}^{m} x[n-k]$

→ moving average system

$$y[n] = \sum_{k=-1}^{1} x[n-k] \cdot \frac{1}{3}$$

$$= \frac{1}{3}\left[x[n+1] + x[0] + x[n-1]\right]$$

Non-causal.

Q: 10 $y[n] = 3x[n] - 2x[n-1]$

Causal

Causal : Non-anticipatory

Non-causal : Anticipatory

(11) $y[n] = x[-n]$

Non-causal

(12) $y[n] = g[n] \cdot x[n]$

Causal

Real time example :-

Causal

- Thermostat based AC
- Motor or generator

Non-causal

- Weather forecasting system
- Missile guidance system
- thinking of future (cricket, etc)

(13) $y[n] = x[2n-k]$

$2n-k \leq n$

$n \leq k$

$k \geq n$

$y[1] = x[2-3] = x[-1]$

↓

causal

$2n-k > n$

$n > k$

$k < n$

$y[2] = x[4-1] = x[3]$

↓

non-causal.

(4) Static/Memoryless and Dynamic/Memory System

- A system is said to be static if present state of output is depends on present input of system only.

- Otherwise the system is dynamic.

1. $y(t) = (3t+1)\,x(t)$ (S)

2. $y(t) = \sin\{x(t)\}$ (S)

3. $y(t) = x\{\sin t\}$ (D)

4. $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$ (D)

5. $y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$ (D)

6. $y[n] = 2x[n] + 3u[n+1]$ (S)

7. $y(t) = x(2t+1)$

   $y(0) = x(1)$ (D)

8. $y[n] = \sum_{k=n_0}^{k=n} x[k]$

   $n_0 = 0$ (D)

   $n_0 > n$ (D)

   $n_0 < n$ (D)

9. $y[n] = \sum_{k=-\infty}^{n} x[k]$ (D)

10. $y[n] = \frac{1}{2m+1} \sum_{k=-m}^{m} x[n-k]$ (D)

11. $y[n] = 3x[n] - 2x[n-1]$ (D)

12. $y(t) = e^{-(3+t)}x(t)$ (S)

13. $y[n] = g[n+3] \cdot x[n]$ (S)

$$\boxed{\text{Every static is causal but every causal didn't be static.}}$$

⑤ Stable and Unstable System

- A system is said to be stable if it produces bounded output for given bounded input.

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

B.I.   BIBO   B.O.

BI: $|x(t)| < M_x < \infty$

BO: $|y(t)| < M_y < \infty$

* In all examples take $x(t) = u(t)$

Q:- For given example, check stability?

① $y(t) = x^2(t)$

$x(t) = u(t) \rightarrow$ B.I.

$y(t) = (u(t))^2$

$y(t) = u(t) \longrightarrow$ B.O.

BIBO $\rightarrow$ stable

② $y(t) = x(2t)$

$x(2t) = u(2t)$

$y(t) = u^2(2t)$

Stable

③ $y(t) = \frac{d}{dt} \cdot x(t)$

$u(t) = x(t)$

$y(t) = \frac{d}{dt} \cdot u(t)$

$y(t) = \delta(t) \rightarrow$

Unstable.

④ $y(t) = x(t) \cdot \cos\omega_c t$

$x(t) = u(t)$

$y(t) = u(t) \cdot \cos\omega_c t$

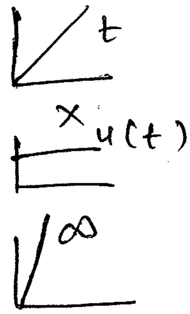
Stable

⑤ $y(t) = t \cdot x(t)$

$x(t) = u(t)$

$y(t) = t \cdot u(t)$

Unstable



⑥ $y(t) = \int_{-\infty}^{t} |x(\tau) \cos \omega_c t| \, d\tau$

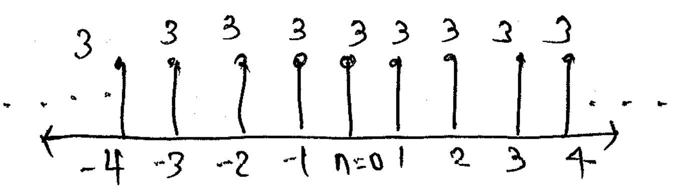
$y(t) = \int_{-\infty}^{t} |u(t) \cdot \cos \omega_c t| \, d\tau$

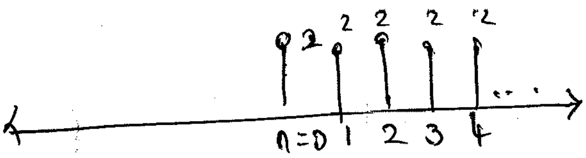
$y(t) = \int_{0}^{t} \cos \omega_c t \cdot d\tau \cdot u(t)$

$y(t) = \cos \omega_c t \, [t] \cdot u(t)$

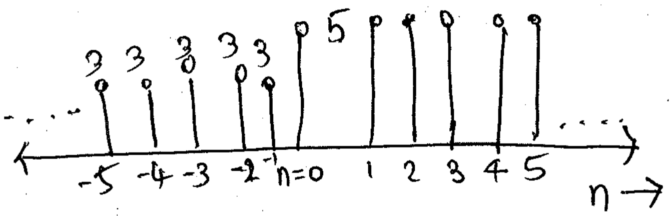
Unstable

⑦ $y[n] = 2x[n] + 3$





Stable



⑧ $y(t) = e^{x[n]}$

$x(t) = u(t) \longrightarrow$ Bounded input

$y(t) = e^{u(t)}$

$$\left.\begin{aligned} y(t) &= e^{1} \\ &= e^{2} \\ &= e^{3} \end{aligned}\right\} \text{Bounded output}$$

BIBO

∴ Stable.

6 Invertible and Non-invertible system

$$x(t) \longrightarrow \boxed{T\{\cdot\}} \xrightarrow{y(t)=T\{x(t)\}} \boxed{T^{-1}\{\cdot\}} \longrightarrow z(t)$$

- A system is said to be invertible if it leads different output on given different input otherwise it is said to be non-invertible system

- If it is invertible then $T^{-1}(T)=1$

$y(t)=T\{x(t)\}$

$z(t)=T^{-1}\{T\{x(t)\}\}$

$\boxed{z(t)=x(t)}$

Test signal

$u(t)$	$u[n]$
$-u(t)$	$-u[n]$
$\delta[n]$	$\delta(t)$
$-\delta[n]$	$-\delta(t)$

For eg:- $y(t)=x^2(t)$
i/p.

If $u(t) \longrightarrow y(t)=u(t)$

$-u(t) \longrightarrow y(t)=u(t)$

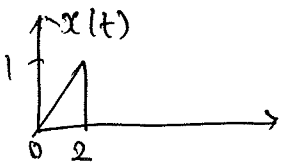
Non-invertible bcz. it leads same o/p on different i/p.

1 $y(t)=x(t-4)$

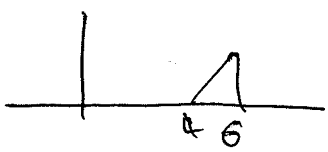
i/p
$u(t) = y(t) = u(t-4)$

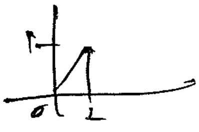
$-u(t) = y(t) = -u(t-4)$

Invertible



$\rightarrow y(t) = x(t-4) \longrightarrow \boxed{z(t)=y(t+4)}$





inverse of y(t)
$y^{-1}(t)=y(t+4)$

③ $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

Invertible

④ $y(t) = \frac{d}{dx} x(t)$

Invertible

⑤ $y[n] = x[n]\cdot x[n-3]$

$x[n] = u[n]$

$y[n] = u[n]\cdot u[n-3]$

$y[n] = -u[n]\cdot(-u[n-3])$

$y[n] = u[n]\cdot u[n-3]$

Non-invertible

⑥ $y[n] = n\cdot x[n]$

$x_1(t) = 5[u(t)+u(-t)]$

$y(t) = \frac{d}{dt} x_1(t)$

$= 5[\delta(t)+\delta(-t)(-1)]$

$= 5[\delta(t)-\delta(-t)]$

$y(t) = 0$

Non-invertible

⑦ $y[n] = \sum_{k=-\infty}^{n} x[k]$

Sol^n:- $\sum \Rightarrow \int$

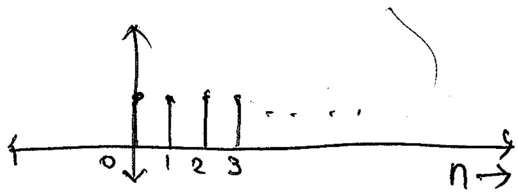
Contin. Discrete

i/p $\Rightarrow$ o/p

$\delta[n] \longrightarrow u[n]$

$-\delta[n] \longrightarrow -u[n]$

So, invertible



$\sum_{k=0}^{n} (1) = (n+1)u[n]$

So, invertible

Inverse is $y[n]-y[n-1]$

$= \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k]$

$= x[n] + \cancel{\sum_{k=-\infty}^{n-1} x[k]} - \cancel{\sum_{k=-\infty}^{n-1} x[k]}$

$= x[n]$

⑧ $y[n] = x[n]\cdot \sin\left(\frac{5\pi n}{6}\right)$

i/p $\to$ o/p

$\delta(n) \to \delta(n)\sin\left[\frac{5\pi n}{6}\right]$

$= \sin\left[\frac{5\pi n}{6}\right]\cdot\delta[n]$ ($\because n_0 = 0$)

$= 0$

$-\delta[n] \longrightarrow -\delta[n]\sin\left(\frac{5\pi n}{6}\right)$

$= 0$

So, non-invertible

* If $x[n]$ is multiply by any signal which is also depends on n. Then put $\delta[n]$ & $-\delta[n]$ and use product property. *

Ex :- $y[n] = x^2[n]$

N.L. , T.I.V. , Causal, Non invertible, Static, Stable

Ex:- $y[n] = x[n^2]$

Linear, Non-causal, Stable, Invertible, Dynamic, Time Variant