

## PUZZLER

Have you ever wondered why a tennis ball is fuzzy and why a golf ball has dimples? A “spitball” is an illegal baseball pitch because it makes the ball act too much like the fuzzy tennis ball or the dimpled golf ball. What principles of physics govern the behavior of these three pieces of sporting equipment (and also keep airplanes in the sky)? (George Semple)



## chapter

# 15

## Fluid Mechanics

### Chapter Outline

- 15.1** Pressure
- 15.2** Variation of Pressure with Depth
- 15.3** Pressure Measurements
- 15.4** Buoyant Forces and Archimedes's Principle
- 15.5** Fluid Dynamics
- 15.6** Streamlines and the Equation of Continuity
- 15.7** Bernoulli's Equation
- 15.8** (Optional) Other Applications of Bernoulli's Equation

**M**atter is normally classified as being in one of three states: solid, liquid, or gas. From everyday experience, we know that a solid has a definite volume and shape. A brick maintains its familiar shape and size day in and day out. We also know that a liquid has a definite volume but no definite shape. Finally, we know that an unconfined gas has neither a definite volume nor a definite shape. These definitions help us picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long periods of time they tend to flow like liquids. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these), depending on the temperature and pressure. In general, the time it takes a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, as a liquid, or as a gas.

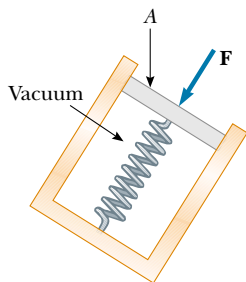
A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

In our treatment of the mechanics of fluids, we shall see that we do not need to learn any new physical principles to explain such effects as the buoyant force acting on a submerged object and the dynamic lift acting on an airplane wing. First, we consider the mechanics of a fluid at rest—that is, *fluid statics*—and derive an expression for the pressure exerted by a fluid as a function of its density and depth. We then treat the mechanics of fluids in motion—that is, *fluid dynamics*. We can describe a fluid in motion by using a model in which we make certain simplifying assumptions. We use this model to analyze some situations of practical importance. An analysis leading to *Bernoulli's equation* enables us to determine relationships between the pressure, density, and velocity at every point in a fluid.

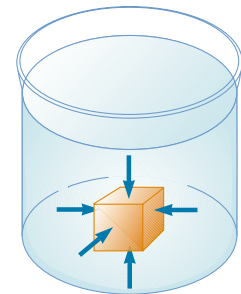
## 15.1 PRESSURE

Fluids do not sustain shearing stresses or tensile stresses; thus, the only stress that can be exerted on an object submerged in a fluid is one that tends to compress the object. In other words, the force exerted by a fluid on an object is always perpendicular to the surfaces of the object, as shown in Figure 15.1.

The pressure in a fluid can be measured with the device pictured in Figure 15.2. The device consists of an evacuated cylinder that encloses a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses on the top of the piston and compresses the spring until the inward force exerted by the fluid is balanced by the outward force exerted by the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. If  $F$  is the magnitude of the force exerted on the piston and  $A$  is the surface area of the piston,



**Figure 15.2** A simple device for measuring the pressure exerted by a fluid.



**Figure 15.1** At any point on the surface of a submerged object, the force exerted by the fluid is perpendicular to the surface of the object. The force exerted by the fluid on the walls of the container is perpendicular to the walls at all points.

## Definition of pressure



Snowshoes keep you from sinking into soft snow because they spread the downward force you exert on the snow over a large area, reducing the pressure on the snow's surface.

## QuickLab

Place a tack between your thumb and index finger, as shown in the figure. Now very gently squeeze the tack and note the sensation. The pointed end of the tack causes pain, and the blunt end does not. According to Newton's third law, the force exerted by the tack on the thumb is equal in magnitude and opposite in direction to the force exerted by the tack on the index finger. However, the pressure at the pointed end of the tack is much greater than the pressure at the blunt end. (Remember that pressure is force per unit area.)



then the **pressure**  $P$  of the fluid at the level to which the device has been submerged is defined as the ratio  $F/A$ :

$$P \equiv \frac{F}{A} \quad (15.1)$$

Note that pressure is a scalar quantity because it is proportional to the magnitude of the force on the piston.

To define the pressure at a specific point, we consider a fluid acting on the device shown in Figure 15.2. If the force exerted by the fluid over an infinitesimal surface element of area  $dA$  containing the point in question is  $dF$ , then the pressure at that point is

$$P = \frac{dF}{dA} \quad (15.2)$$

As we shall see in the next section, the pressure exerted by a fluid varies with depth. Therefore, to calculate the total force exerted on a flat wall of a container, we must integrate Equation 15.2 over the surface area of the wall.

Because pressure is force per unit area, it has units of newtons per square meter ( $\text{N}/\text{m}^2$ ) in the SI system. Another name for the SI unit of pressure is **pascal** (Pa):

$$1 \text{ Pa} \equiv 1 \text{ N}/\text{m}^2 \quad (15.3)$$

## Quick Quiz 15.1

Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were a professional basketball player wearing sneakers or a petite woman wearing spike-heeled shoes? Explain.

## Quick Quiz 15.2

After a long lecture, the daring physics professor stretches out for a nap on a bed of nails, as shown in Figure 15.3. How is this possible?

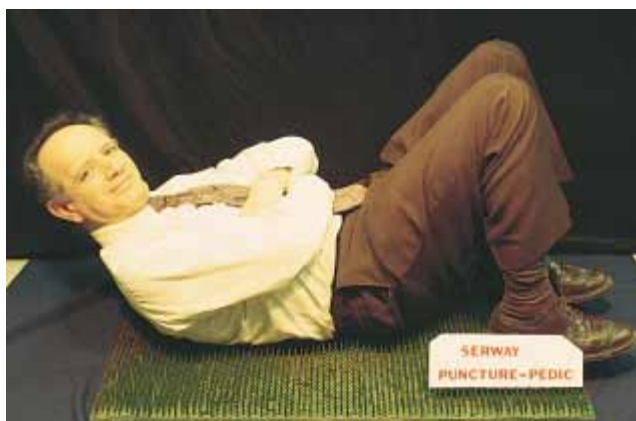


Figure 15.3

**EXAMPLE 15.1** The Water Bed

The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep. (a) Find the weight of the water in the mattress.

**Solution** The density of water is  $1\,000\text{ kg/m}^3$  (Table 15.1), and so the mass of the water is

$$M = \rho V = (1\,000\text{ kg/m}^3)(1.20\text{ m}^3) = 1.20 \times 10^3\text{ kg}$$

and its weight is

$$Mg = (1.20 \times 10^3\text{ kg})(9.80\text{ m/s}^2) = 1.18 \times 10^4\text{ N}$$

This is approximately 2 650 lb. (A regular bed weighs approx-

imately 300 lb.) Because this load is so great, such a water bed is best placed in the basement or on a sturdy, well-supported floor.

(b) Find the pressure exerted by the water on the floor when the bed rests in its normal position. Assume that the entire lower surface of the bed makes contact with the floor.

**Solution** When the bed is in its normal position, the cross-sectional area is  $4.00\text{ m}^2$ ; thus, from Equation 15.1, we find that

$$P = \frac{1.18 \times 10^4\text{ N}}{4.00\text{ m}^2} = 2.95 \times 10^3\text{ Pa}$$

**TABLE 15.1** Densities of Some Common Substances at Standard Temperature ( $0^\circ\text{C}$ ) and Pressure (Atmospheric)

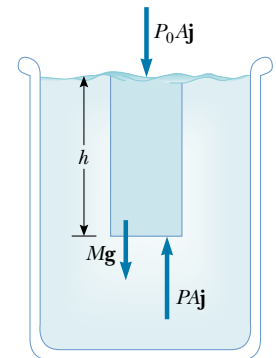
Substance	$\rho\text{ (kg/m}^3\text{)}$	Substance	$\rho\text{ (kg/m}^3\text{)}$
Air	1.29	Ice	$0.917 \times 10^3$
Aluminum	$2.70 \times 10^3$	Iron	$7.86 \times 10^3$
Benzene	$0.879 \times 10^3$	Lead	$11.3 \times 10^3$
Copper	$8.92 \times 10^3$	Mercury	$13.6 \times 10^3$
Ethyl alcohol	$0.806 \times 10^3$	Oak	$0.710 \times 10^3$
Fresh water	$1.00 \times 10^3$	Oxygen gas	1.43
Glycerine	$1.26 \times 10^3$	Pine	$0.373 \times 10^3$
Gold	$19.3 \times 10^3$	Platinum	$21.4 \times 10^3$
Helium gas	$1.79 \times 10^{-1}$	Seawater	$1.03 \times 10^3$
Hydrogen gas	$8.99 \times 10^{-2}$	Silver	$10.5 \times 10^3$

**15.2** VARIATION OF PRESSURE WITH DEPTH

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude; it is for this reason that aircraft flying at high altitudes must have pressurized cabins.

We now show how the pressure in a liquid increases linearly with depth. As Equation 1.1 describes, the *density* of a substance is defined as its mass per unit volume:  $\rho \equiv m/V$ . Table 15.1 lists the densities of various substances. These values vary slightly with temperature because the volume of a substance is temperature dependent (as we shall see in Chapter 19). Note that under standard conditions (at  $0^\circ\text{C}$  and at atmospheric pressure) the densities of gases are about  $1/1\,000$  the densities of solids and liquids. This difference implies that the average molecular spacing in a gas under these conditions is about ten times greater than that in a solid or liquid.

Now let us consider a fluid of density  $\rho$  at rest and open to the atmosphere, as shown in Figure 15.4. We assume that  $\rho$  is constant; this means that the fluid is incompressible. Let us select a sample of the liquid contained within an imaginary cylinder of cross-sectional area  $A$  extending from the surface to a depth  $h$ . The



**Figure 15.4** How pressure varies with depth in a fluid. The net force exerted on the volume of water within the darker region must be zero.

### QuickLab

Poke two holes in the side of a paper or polystyrene cup—one near the top and the other near the bottom. Fill the cup with water and watch the water flow out of the holes. Why does water exit from the bottom hole at a higher speed than it does from the top hole?

Variation of pressure with depth

pressure exerted by the outside liquid on the bottom face of the cylinder is  $P$ , and the pressure exerted on the top face of the cylinder is the atmospheric pressure  $P_0$ . Therefore, the upward force exerted by the outside fluid on the bottom of the cylinder is  $PA$ , and the downward force exerted by the atmosphere on the top is  $P_0A$ . The mass of liquid in the cylinder is  $M = \rho V = \rho Ah$ ; therefore, the weight of the liquid in the cylinder is  $Mg = \rho Ahg$ . Because the cylinder is in equilibrium, the net force acting on it must be zero. Choosing upward to be the positive  $y$  direction, we see that

$$\sum F_y = PA - P_0A - Mg = 0$$

or

$$PA - P_0A - \rho Ahg = 0$$

$$PA - P_0A = \rho Ahg$$

$$P = P_0 + \rho gh \quad (15.4)$$

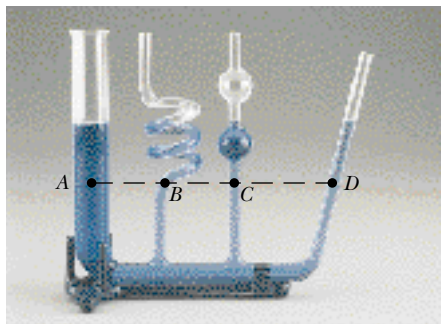
That is, **the pressure  $P$  at a depth  $h$  below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount  $\rho gh$ .** In our calculations and working of end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Equation 15.4 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

### Quick Quiz 15.3

In the derivation of Equation 15.4, why were we able to ignore the pressure that the liquid exerts on the sides of the cylinder?



This arrangement of interconnected tubes demonstrates that the pressure in a liquid is the same at all points having the same elevation. For example, the pressure is the same at points A, B, C, and D.

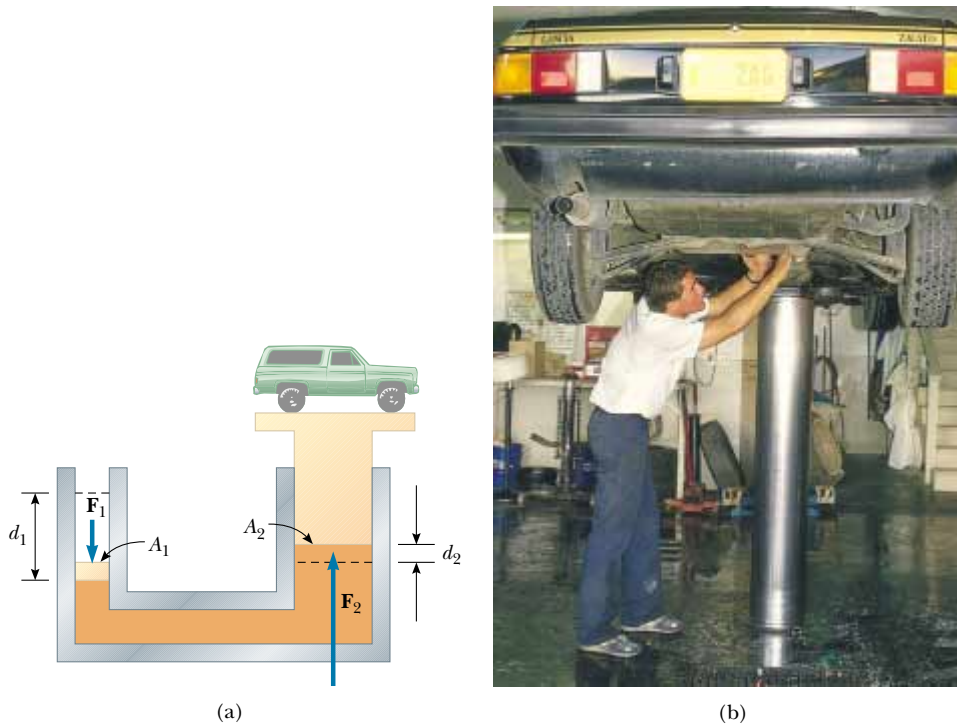
In view of the fact that the pressure in a fluid depends on depth and on the value of  $P_0$ , any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by the French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.**

An important application of Pascal's law is the hydraulic press illustrated in Figure 15.5a. A force of magnitude  $F_1$  is applied to a small piston of surface area  $A_1$ . The pressure is transmitted through a liquid to a larger piston of surface area  $A_2$ . Because the pressure must be the same on both sides,  $P = F_1/A_1 = F_2/A_2$ . Therefore, the force  $F_2$  is greater than the force  $F_1$  by a factor  $A_2/A_1$ , which is called the *force-multiplying factor*. Because liquid is neither added nor removed, the volume pushed down on the left as the piston moves down a distance  $d_1$  equals the volume pushed up on the right as the right piston moves up a distance  $d_2$ . That is,  $A_1d_1 = A_2d_2$ ; thus, the force-multiplying factor can also be written as  $d_1/d_2$ . Note that  $F_1d_1 = F_2d_2$ . Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle (Fig. 15.5b).

### Quick Quiz 15.4

A grain silo has many bands wrapped around its perimeter (Fig. 15.6). Why is the spacing between successive bands smaller at the lower portions of the silo, as shown in the photograph?





**Figure 15.5** (a) Diagram of a hydraulic press. Because the increase in pressure is the same on the two sides, a small force  $\mathbf{F}_1$  at the left produces a much greater force  $\mathbf{F}_2$  at the right. (b) A vehicle undergoing repair is supported by a hydraulic lift in a garage.



**Figure 15.6**

### EXAMPLE 15.2 The Car Lift

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm. What force must the compressed air exert to lift a car weighing 13 300 N? What air pressure produces this force?

**Solution** Because the pressure exerted by the compressed air is transmitted undiminished throughout the liquid, we have

$$F_1 = \left( \frac{A_1}{A_2} \right) F_2 = \frac{\pi(5.00 \times 10^{-2} \text{ m})^2}{\pi(15.0 \times 10^{-2} \text{ m})^2} (1.33 \times 10^4 \text{ N}) \\ = 1.48 \times 10^3 \text{ N}$$

The air pressure that produces this force is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3 \text{ N}}{\pi(5.00 \times 10^{-2} \text{ m})^2} = 1.88 \times 10^5 \text{ Pa}$$

This pressure is approximately twice atmospheric pressure.

The input work (the work done by  $\mathbf{F}_1$ ) is equal to the output work (the work done by  $\mathbf{F}_2$ ), in accordance with the principle of conservation of energy.

### EXAMPLE 15.3 A Pain in the Ear

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of a pool that is 5.0 m deep.

**Solution** First, we must find the unbalanced pressure on

the eardrum; then, after estimating the eardrum's surface area, we can determine the force that the water exerts on it.

The air inside the middle ear is normally at atmospheric pressure  $P_0$ . Therefore, to find the net force on the eardrum, we must consider the difference between the total pressure at

the bottom of the pool and atmospheric pressure:

$$\begin{aligned} P_{\text{bot}} - P_0 &= \rho gh \\ &= (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.0 \text{ m}) \\ &= 4.9 \times 10^4 \text{ Pa} \end{aligned}$$

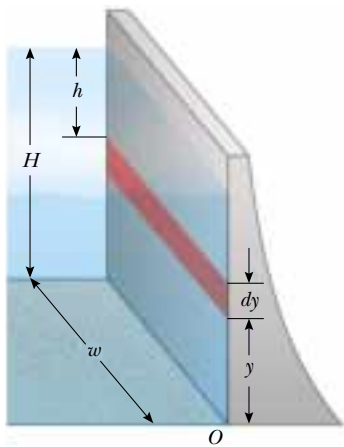
We estimate the surface area of the eardrum to be approximately  $1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$ . This means that the force on it

is  $F = (P_{\text{bot}} - P_0)A \approx 5 \text{ N}$ . Because a force on the eardrum of this magnitude is extremely uncomfortable, swimmers often “pop their ears” while under water, an action that pushes air from the lungs into the middle ear. Using this technique equalizes the pressure on the two sides of the eardrum and relieves the discomfort.

### EXAMPLE 15.4 The Force on a Dam

Water is filled to a height  $H$  behind a dam of width  $w$  (Fig. 15.7). Determine the resultant force exerted by the water on the dam.

**Solution** Because pressure varies with depth, we cannot calculate the force simply by multiplying the area by the pressure. We can solve the problem by finding the force  $dF$  ex-



**Figure 15.7** Because pressure varies with depth, the total force exerted on a dam must be obtained from the expression  $F = \int P dA$ , where  $dA$  is the area of the dark strip.

erted on a narrow horizontal strip at depth  $h$  and then integrating the expression to find the total force. Let us imagine a vertical  $y$  axis, with  $y = 0$  at the bottom of the dam and our strip a distance  $y$  above the bottom.

We can use Equation 15.4 to calculate the pressure at the depth  $h$ ; we omit atmospheric pressure because it acts on both sides of the dam:

$$P = \rho gh = \rho g(H - y)$$

Using Equation 15.2, we find that the force exerted on the shaded strip of area  $dA = w dy$  is

$$dF = P dA = \rho g(H - y)w dy$$

Therefore, the total force on the dam is

$$F = \int P dA = \int_0^H \rho g(H - y)w dy = \frac{1}{2}\rho g w H^2$$

Note that the thickness of the dam shown in Figure 15.7 increases with depth. This design accounts for the greater and greater pressure that the water exerts on the dam at greater depths.

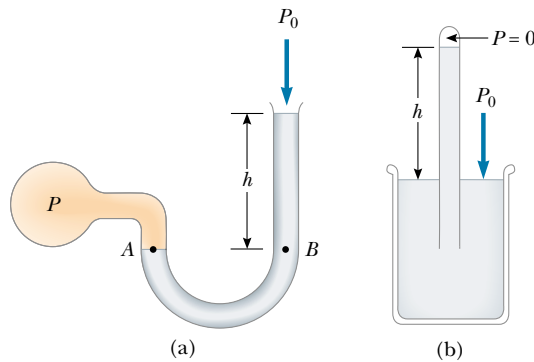
**Exercise** Find an expression for the average pressure on the dam from the total force exerted by the water on the dam.

**Answer**  $\frac{1}{2}\rho gH$ .

## 15.3 PRESSURE MEASUREMENTS

One simple device for measuring pressure is the open-tube manometer illustrated in Figure 15.8a. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure  $P$ . The difference in pressure  $P - P_0$  is equal to  $\rho gh$ ; hence,  $P = P_0 + \rho gh$ . The pressure  $P$  is called the **absolute pressure**, and the difference  $P - P_0$  is called the **gauge pressure**. The latter is the value that normally appears on a pressure gauge. For example, the pressure you measure in your bicycle tire is the gauge pressure.

Another instrument used to measure pressure is the common *barometer*, which was invented by Evangelista Torricelli (1608–1647). The barometer consists of a



**Figure 15.8** Two devices for measuring pressure: (a) an open-tube manometer and (b) a mercury barometer.

long, mercury-filled tube closed at one end and inverted into an open container of mercury (Fig. 15.8b). The closed end of the tube is nearly a vacuum, and so its pressure can be taken as zero. Therefore, it follows that  $P_0 = \rho gh$ , where  $h$  is the height of the mercury column.

One atmosphere ( $P_0 = 1 \text{ atm}$ ) of pressure is defined as the pressure that causes the column of mercury in a barometer tube to be exactly 0.760 0 m in height at  $0^\circ\text{C}$ , with  $g = 9.806\,65 \text{ m/s}^2$ . At this temperature, mercury has a density of  $13.595 \times 10^3 \text{ kg/m}^3$ ; therefore,

$$\begin{aligned} P_0 &= \rho gh = (13.595 \times 10^3 \text{ kg/m}^3)(9.806\,65 \text{ m/s}^2)(0.760\,0 \text{ m}) \\ &= 1.013 \times 10^5 \text{ Pa} = 1 \text{ atm} \end{aligned}$$

### Quick Quiz 15.5

Other than the obvious problem that occurs with freezing, why don't we use water in a barometer in the place of mercury?

## 15.4 BUOYANT FORCES AND ARCHIMEDES'S PRINCIPLE

Have you ever tried to push a beach ball under water? This is extremely difficult to do because of the large upward force exerted by the water on the ball. The upward force exerted by water on any immersed object is called a **buoyant force**. We can determine the magnitude of a buoyant force by applying some logic and Newton's second law. Imagine that, instead of air, the beach ball is filled with water. If you were standing on land, it would be difficult to hold the water-filled ball in your arms. If you held the ball while standing neck deep in a pool, however, the force you would need to hold it would almost disappear. In fact, the required force would be zero if we were to ignore the thin layer of plastic of which the beach ball is made. Because the water-filled ball is in equilibrium while it is submerged, the magnitude of the upward buoyant force must equal its weight.

If the submerged ball were filled with air rather than water, then the upward buoyant force exerted by the surrounding water would still be present. However, because the weight of the water is now replaced by the much smaller weight of that volume of air, the net force is upward and quite great; as a result, the ball is pushed to the surface.

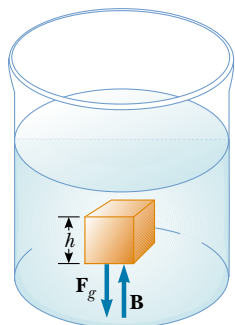


## Archimedes's principle

**Archimedes** (c. 287–212 B.C.)

Archimedes, a Greek mathematician, physicist, and engineer, was perhaps the greatest scientist of antiquity. He was the first to compute accurately the ratio of a circle's circumference to its diameter, and he showed how to calculate the volume and surface area of spheres, cylinders, and other geometric shapes. He is well known for discovering the nature of the buoyant force.

Archimedes was also a gifted inventor. One of his practical inventions, still in use today, is Archimedes's screw—an inclined, rotating, coiled tube originally used to lift water from the holds of ships. He also invented the catapult and devised systems of levers, pulleys, and weights for raising heavy loads. Such inventions were successfully used to defend his native city Syracuse during a two-year siege by the Romans.



**Figure 15.9** The external forces acting on the cube of liquid are the force of gravity  $\mathbf{F}_g$  and the buoyant force  $\mathbf{B}$ . Under equilibrium conditions,  $B = F_g$ .

The manner in which buoyant forces act is summarized by **Archimedes's principle**, which states that **the magnitude of the buoyant force always equals the weight of the fluid displaced by the object**. The buoyant force acts vertically upward through the point that was the center of gravity of the displaced fluid.

Note that Archimedes's principle does not refer to the makeup of the object experiencing the buoyant force. The object's composition is not a factor in the buoyant force. We can verify this in the following manner: Suppose we focus our attention on the indicated cube of liquid in the container illustrated in Figure 15.9. This cube is in equilibrium as it is acted on by two forces. One of these forces is the gravitational force  $\mathbf{F}_g$ . What cancels this downward force? Apparently, the rest of the liquid in the container is holding the cube in equilibrium. Thus, the magnitude of the buoyant force  $\mathbf{B}$  exerted on the cube is exactly equal to the magnitude of  $\mathbf{F}_g$ , which is the weight of the liquid inside the cube:

$$B = F_g$$

Now imagine that the cube of liquid is replaced by a cube of steel of the same dimensions. What is the buoyant force acting on the steel? The liquid surrounding a cube behaves in the same way no matter what the cube is made of. Therefore, **the buoyant force acting on the steel cube is the same as the buoyant force acting on a cube of liquid of the same dimensions**. In other words, the magnitude of the buoyant force is the same as the weight of the *liquid* cube, not the steel cube. Although mathematically more complicated, this same principle applies to submerged objects of any shape, size, or density.

Although we have described the magnitude and direction of the buoyant force, we still do not know its origin. Why would a fluid exert such a strange force, almost as if the fluid were trying to expel a foreign body? To understand why, look again at Figure 15.9. The pressure at the bottom of the cube is greater than the pressure at the top by an amount  $\rho gh$ , where  $h$  is the length of any side of the cube. The pressure difference  $\Delta P$  between the bottom and top faces of the cube is equal to the buoyant force per unit area of those faces—that is,  $\Delta P = B/A$ . Therefore,  $B = (\Delta P)A = (\rho gh)A = \rho gV$ , where  $V$  is the volume of the cube. Because the mass of the fluid in the cube is  $M = \rho V$ , we see that

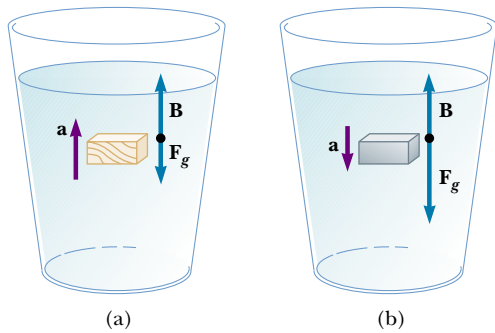
$$B = F_g = \rho Vg = Mg \quad (15.5)$$

where  $Mg$  is the weight of the fluid in the cube. Thus, the buoyant force is a result of the pressure differential on a submerged or partly submerged object.

Before we proceed with a few examples, it is instructive for us to compare the forces acting on a totally submerged object with those acting on a floating (partly submerged) object.

**Case 1: Totally Submerged Object** When an object is totally submerged in a fluid of density  $\rho_f$ , the magnitude of the upward buoyant force is  $B = \rho_f V_o g$ , where  $V_o$  is the volume of the object. If the object has a mass  $M$  and density  $\rho_o$ , its weight is equal to  $F_g = Mg = \rho_o V_o g$ , and the net force on it is  $B - F_g = (\rho_f - \rho_o) V_o g$ . Hence, if the density of the object is less than the density of the fluid, then the downward force of gravity is less than the buoyant force, and the unconstrained object accelerates upward (Fig. 15.10a). If the density of the object is greater than the density of the fluid, then the upward buoyant force is less than the downward force of gravity, and the unsupported object sinks (Fig. 15.10b).

**Case 2: Floating Object** Now consider an object of volume  $V_o$  in static equilibrium floating on a fluid—that is, an object that is only partially submerged. In this



**Figure 15.10** (a) A totally submerged object that is less dense than the fluid in which it is submerged experiences a net upward force. (b) A totally submerged object that is denser than the fluid sinks.

case, the upward buoyant force is balanced by the downward gravitational force acting on the object. If  $V_f$  is the volume of the fluid displaced by the object (this volume is the same as the volume of that part of the object that is beneath the fluid level), the buoyant force has a magnitude  $B = \rho_f V_f g$ . Because the weight of the object is  $F_g = Mg = \rho_o V_o g$ , and because  $F_g = B$ , we see that  $\rho_f V_f g = \rho_o V_o g$ , or

$$\frac{\rho_o}{\rho_f} = \frac{V_f}{V_o} \quad (15.6)$$

Under normal conditions, the average density of a fish is slightly greater than the density of water. It follows that the fish would sink if it did not have some mechanism for adjusting its density. The fish accomplishes this by internally regulating the size of its air-filled swim bladder to balance the change in the magnitude of the buoyant force acting on it. In this manner, fish are able to swim to various depths. Unlike a fish, a scuba diver cannot achieve neutral buoyancy (at which the buoyant force just balances the weight) by adjusting the magnitude of the buoyant force  $B$ . Instead, the diver adjusts  $F_g$  by manipulating lead weights.

### Quick Quiz 15.6

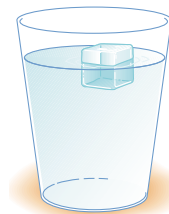
Steel is much denser than water. In view of this fact, how do steel ships float?

### Quick Quiz 15.7

A glass of water contains a single floating ice cube (Fig. 15.11). When the ice melts, does the water level go up, go down, or remain the same?

### Quick Quiz 15.8

When a person in a rowboat in a small pond throws an anchor overboard, does the water level of the pond go up, go down, or remain the same?



**Figure 15.11**

### EXAMPLE 15.5 Eureka!

Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. Legend has it that he solved this problem by weighing the crown first in air and then in water, as shown in Figure 15.12. Suppose the

scale read 7.84 N in air and 6.86 N in water. What should Archimedes have told the king?

**Solution** When the crown is suspended in air, the scale



Hot-air balloons. Because hot air is less dense than cold air, a net upward force acts on the balloons.

reads the true weight  $T_1 = F_g$  (neglecting the buoyancy of air). When it is immersed in water, the buoyant force  $\mathbf{B}$  reduces the scale reading to an apparent weight of  $T_2 = F_g - B$ . Hence, the buoyant force exerted on the crown is the difference between its weight in air and its weight in water:

$$B = F_g - T_2 = 7.84 \text{ N} - 6.86 \text{ N} = 0.98 \text{ N}$$

Because this buoyant force is equal in magnitude to the weight of the displaced water, we have  $\rho_w g V_w = 0.98 \text{ N}$ , where  $V_w$  is the volume of the displaced water and  $\rho_w$  is its density. Also, the volume of the crown  $V_c$  is equal to the volume of the displaced water because the crown is completely submerged. Therefore,

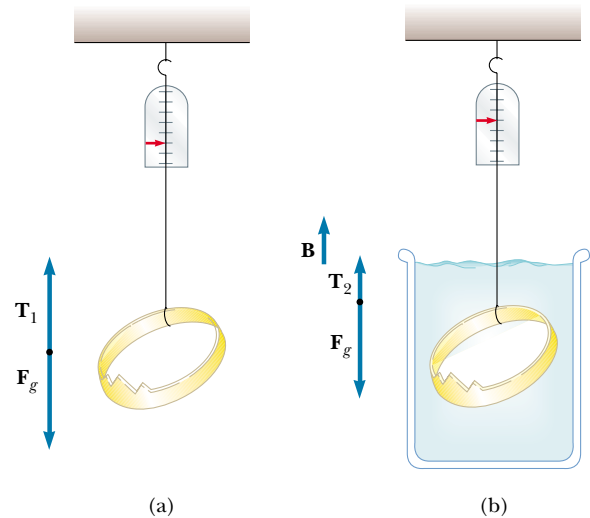
$$\begin{aligned} V_c = V_w &= \frac{0.98 \text{ N}}{g \rho_w} = \frac{0.98 \text{ N}}{(9.8 \text{ m/s}^2)(1000 \text{ kg/m}^3)} \\ &= 1.0 \times 10^{-4} \text{ m}^3 \end{aligned}$$

Finally, the density of the crown is

$$\begin{aligned} \rho_c &= \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84 \text{ N}}{(1.0 \times 10^{-4} \text{ m}^3)(9.8 \text{ m/s}^2)} \\ &= 8.0 \times 10^3 \text{ kg/m}^3 \end{aligned}$$

From Table 15.1 we see that the density of gold is  $19.3 \times 10^3 \text{ kg/m}^3$ . Thus, Archimedes should have told the king that

he had been cheated. Either the crown was hollow, or it was not made of pure gold.



**Figure 15.12** (a) When the crown is suspended in air, the scale reads its true weight  $T_1 = F_g$  (the buoyancy of air is negligible). (b) When the crown is immersed in water, the buoyant force  $\mathbf{B}$  reduces the scale reading to the apparent weight  $T_2 = F_g - B$ .

### EXAMPLE 15.6 A Titanic Surprise

An iceberg floating in seawater, as shown in Figure 15.13a, is extremely dangerous because much of the ice is below the surface. This hidden ice can damage a ship that is still a considerable distance from the visible ice. What fraction of the iceberg lies below the water level?

**Solution** This problem corresponds to Case 2. The weight of the iceberg is  $F_{gi} = \rho_i V_i g$ , where  $\rho_i = 917 \text{ kg/m}^3$  and  $V_i$  is the volume of the whole iceberg. The magnitude of the up-

ward buoyant force equals the weight of the displaced water:  $B = \rho_w V_w g$ , where  $V_w$ , the volume of the displaced water, is equal to the volume of the ice beneath the water (the shaded region in Fig. 15.13b) and  $\rho_w$  is the density of seawater,  $\rho_w = 1030 \text{ kg/m}^3$ . Because  $\rho_i V_i g = \rho_w V_w g$ , the fraction of ice beneath the water's surface is

$$f = \frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917 \text{ kg/m}^3}{1030 \text{ kg/m}^3} = 0.890 \quad \text{or} \quad 89.0\%$$



(a)



(b)

**Figure 15.13** (a) Much of the volume of this iceberg is beneath the water. (b) A ship can be damaged even when it is not near the exposed ice.

## 15.5 FLUID DYNAMICS

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluids in motion. Instead of trying to study the motion of each particle of the fluid as a function of time, we describe the properties of a moving fluid at each point as a function of time.

### Flow Characteristics

When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady**, or **laminar**, if each particle of the fluid follows a smooth path, such that the paths of different particles never cross each other, as shown in Figure 15.14. In steady flow, the velocity of the fluid at any point remains constant in time.

Above a certain critical speed, fluid flow becomes **turbulent**; turbulent flow is irregular flow characterized by small whirlpool-like regions, as shown in Figure 15.15.

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or *viscous force*, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

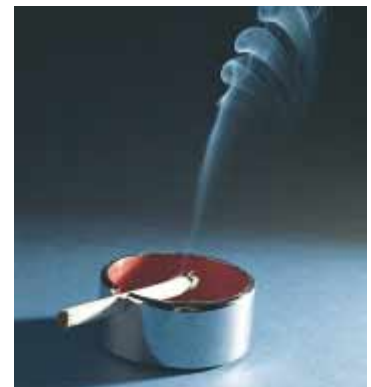
Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of an **ideal fluid**, we make the following four assumptions:

1. **The fluid is nonviscous.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady.** In steady (laminar) flow, the velocity of the fluid at each point remains constant.

Properties of an ideal fluid



**Figure 15.14** Laminar flow around an automobile in a test wind tunnel.

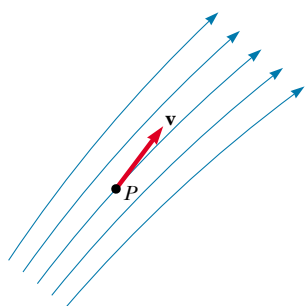


**Figure 15.15** Hot gases from a cigarette made visible by smoke particles. The smoke first moves in laminar flow at the bottom and then in turbulent flow above.



3. **The fluid is incompressible.** The density of an incompressible fluid is constant.
4. **The flow is irrotational.** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

## 15.6 STREAMLINES AND THE EQUATION OF CONTINUITY



**Figure 15.16** A particle in laminar flow follows a streamline, and at each point along its path the particle's velocity is tangent to the streamline.

Equation of continuity

The path taken by a fluid particle under steady flow is called a **streamline**. The velocity of the particle is always tangent to the streamline, as shown in Figure 15.16. A set of streamlines like the ones shown in Figure 15.16 form a *tube of flow*. Note that fluid particles cannot flow into or out of the sides of this tube; if they could, then the streamlines would cross each other.

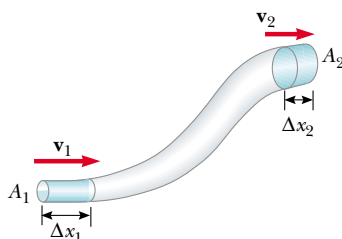
Consider an ideal fluid flowing through a pipe of nonuniform size, as illustrated in Figure 15.17. The particles in the fluid move along streamlines in steady flow. In a time  $t$ , the fluid at the bottom end of the pipe moves a distance  $\Delta x_1 = v_1 t$ . If  $A_1$  is the cross-sectional area in this region, then the mass of fluid contained in the left shaded region in Figure 15.17 is  $m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 t$ , where  $\rho$  is the (nonchanging) density of the ideal fluid. Similarly, the fluid that moves through the upper end of the pipe in the time  $t$  has a mass  $m_2 = \rho A_2 v_2 t$ . However, because *mass is conserved* and because the flow is steady, the mass that crosses  $A_1$  in a time  $t$  must equal the mass that crosses  $A_2$  in the time  $t$ . That is,  $m_1 = m_2$ , or  $\rho A_1 v_1 t = \rho A_2 v_2 t$ ; this means that

$$A_1 v_1 = A_2 v_2 = \text{constant} \quad (15.7)$$

This expression is called the **equation of continuity**. It states that

the product of the area and the fluid speed at all points along the pipe is a constant for an incompressible fluid.

This equation tells us that the speed is high where the tube is constricted (small  $A$ ) and low where the tube is wide (large  $A$ ). The product  $Av$ , which has the dimensions of volume per unit time, is called either the *volume flux* or the *flow rate*. The condition  $Av = \text{constant}$  is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.



**Figure 15.17** A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area  $A_1$  in a time interval  $t$  must equal the volume flowing through area  $A_2$  in the same time interval. Therefore,  $A_1 v_1 = A_2 v_2$ .

### Quick Quiz 15.9

As water flows from a faucet, as shown in Figure 15.18, why does the stream of water become narrower as it descends?



**Figure 15.18**

**EXAMPLE 15.7** Niagara Falls

Each second,  $5\,525\text{ m}^3$  of water flows over the 670-m-wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2 m deep as it reaches the cliff. What is its speed at that instant?

**Solution** The cross-sectional area of the water as it reaches the edge of the cliff is  $A = (670\text{ m})(2\text{ m}) = 1\,340\text{ m}^2$ . The flow rate of  $5\,525\text{ m}^3/\text{s}$  is equal to  $Av$ . This gives

$$v = \frac{5\,525\text{ m}^3/\text{s}}{A} = \frac{5\,525\text{ m}^3/\text{s}}{1\,340\text{ m}^2} = 4\text{ m/s}$$

Note that we have kept only one significant figure because our value for the depth has only one significant figure.

**Exercise** A barrel floating along in the river plunges over the Falls. How far from the base of the cliff is the barrel when it reaches the water 49 m below?

**Answer**  $13\text{ m} \approx 10\text{ m}$ .

**15.7** BERNOLLI'S EQUATION

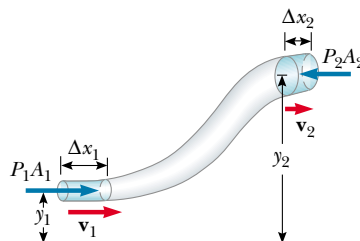
When you press your thumb over the end of a garden hose so that the opening becomes a small slit, the water comes out at high speed, as shown in Figure 15.19. Is the water under greater pressure when it is inside the hose or when it is out in the air? You can answer this question by noting how hard you have to push your thumb against the water inside the end of the hose. The pressure inside the hose is definitely greater than atmospheric pressure.

The relationship between fluid speed, pressure, and elevation was first derived in 1738 by the Swiss physicist Daniel Bernoulli. Consider the flow of an ideal fluid through a nonuniform pipe in a time  $t$ , as illustrated in Figure 15.20. Let us call the lower shaded part section 1 and the upper shaded part section 2. The force exerted by the fluid in section 1 has a magnitude  $P_1A_1$ . The work done by this force in a time  $t$  is  $W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V$ , where  $V$  is the volume of section 1. In a similar manner, the work done by the fluid in section 2 in the same time  $t$  is  $W_2 = -P_2A_2\Delta x_2 = -P_2V$ . (The volume that passes through section 1 in a time  $t$  equals the volume that passes through section 2 in the same time.) This work is negative because the fluid force opposes the displacement. Thus, the net work done by these forces in the time  $t$  is

$$W = (P_1 - P_2)V$$



**Figure 15.19** The speed of water spraying from the end of a hose increases as the size of the opening is decreased with the thumb.



**Figure 15.20** A fluid in laminar flow through a constricted pipe. The volume of the shaded section on the left is equal to the volume of the shaded section on the right.

**Daniel Bernoulli (1700–1782)**

Daniel Bernoulli, a Swiss physicist and mathematician, made important discoveries in fluid dynamics. Born into a family of mathematicians, he was the only member of the family to make a mark in physics.

Bernoulli's most famous work, *Hydrodynamica*, was published in 1738; it is both a theoretical and a practical study of equilibrium, pressure, and speed in fluids. He showed that as the speed of a fluid increases, its pressure decreases.

In *Hydrodynamica* Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases, a topic we study in Chapter 21. (Corbis—Bettmann)



### QuickLab

Place two soda cans on their sides approximately 2 cm apart on a table. Align your mouth at table level and with the space between the cans. Blow a horizontal stream of air through this space. What do the cans do? Is this what you expected? Compare this with the force acting on a car parked close to the edge of a road when a big truck goes by. How does the outcome relate to Equation 15.9?

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy. If  $m$  is the mass that enters one end and leaves the other in a time  $t$ , then the change in the kinetic energy of this mass is

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The change in gravitational potential energy is

$$\Delta U = mgy_2 - mgy_1$$

We can apply Equation 8.13,  $W = \Delta K + \Delta U$ , to this volume of fluid to obtain

$$(P_1 - P_2)V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

If we divide each term by  $V$  and recall that  $\rho = m/V$ , this expression reduces to

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gy_2 - \rho gy_1$$

Rearranging terms, we obtain

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2 \quad (15.8)$$

This is **Bernoulli's equation** as applied to an ideal fluid. It is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant} \quad (15.9)$$

This expression specifies that, in laminar flow, the sum of the pressure ( $P$ ), kinetic energy per unit volume ( $\frac{1}{2}\rho v^2$ ), and gravitational potential energy per unit volume ( $\rho gy$ ) has the same value at all points along a streamline.

When the fluid is at rest,  $v_1 = v_2 = 0$  and Equation 15.8 becomes

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho gh$$

This is in agreement with Equation 15.4.

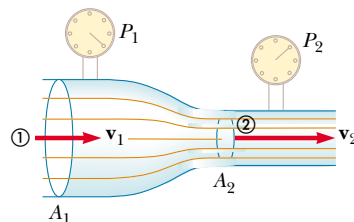
Bernoulli's equation

### EXAMPLE 15.8 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 15.21, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Let us determine the flow speed at point 2 if the pressure difference  $P_1 - P_2$  is known.

**Solution** Because the pipe is horizontal,  $y_1 = y_2$ , and applying Equation 15.8 to points 1 and 2 gives

$$(1) \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$



**Figure 15.21** (a) Pressure  $P_1$  is greater than pressure  $P_2$  because  $v_1 < v_2$ . This device can be used to measure the speed of fluid flow. (b) A Venturi tube.



(a)

(b)

From the equation of continuity,  $A_1 v_1 = A_2 v_2$ , we find that

$$(2) \quad v_1 = \frac{A_2}{A_1} v_2$$

Substituting this expression into equation (1) gives

$$P_1 + \frac{1}{2}\rho \left( \frac{A_2}{A_1} v_2 \right)^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

We can use this result and the continuity equation to obtain an expression for  $v_1$ . Because  $A_2 < A_1$ , Equation (2) shows us that  $v_2 > v_1$ . This result, together with equation (1), indicates that  $P_1 > P_2$ . In other words, the pressure is reduced in the constricted part of the pipe. This result is somewhat analogous to the following situation: Consider a very crowded room in which people are squeezed together. As soon as a door is opened and people begin to exit, the squeezing (pressure) is least near the door, where the motion (flow) is greatest.

### EXAMPLE 15.9 A Good Trick

It is possible to blow a dime off a table and into a tumbler. Place the dime about 2 cm from the edge of the table. Place the tumbler on the table horizontally with its open edge about 2 cm from the dime, as shown in Figure 15.22a. If you blow forcefully across the top of the dime, it will rise, be caught in the airstream, and end up in the tumbler. The

mass of a dime is  $m = 2.24 \text{ g}$ , and its surface area is  $A = 2.50 \times 10^{-4} \text{ m}^2$ . How hard are you blowing when the dime rises and travels into the tumbler?

**Solution** Figure 15.22b indicates we must calculate the upward force acting on the dime. First, note that a thin stationary layer of air is present between the dime and the table. When you blow across the dime, it deflects most of the moving air from your breath across its top, so that the air above the dime has a greater speed than the air beneath it. This fact, together with Bernoulli's equation, demonstrates that the air moving across the top of the dime is at a lower pressure than the air beneath the dime. If we neglect the small thickness of the dime, we can apply Equation 15.8 to obtain

$$P_{\text{above}} + \frac{1}{2}\rho v_{\text{above}}^2 = P_{\text{beneath}} + \frac{1}{2}\rho v_{\text{beneath}}^2$$

Because the air beneath the dime is almost stationary, we can neglect the last term in this expression and write the difference as  $P_{\text{beneath}} - P_{\text{above}} = \frac{1}{2}\rho v_{\text{above}}^2$ . If we multiply this pressure difference by the surface area of the dime, we obtain the upward force acting on the dime. At the very least, this upward force must balance the gravitational force acting on the dime, and so, taking the density of air from Table 15.1, we can state that

$$F_g = mg = (P_{\text{beneath}} - P_{\text{above}})A = \left(\frac{1}{2}\rho v_{\text{above}}^2\right)A$$

$$v_{\text{above}} = \sqrt{\frac{2mg}{\rho A}} = \sqrt{\frac{2(2.24 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{(1.29 \text{ kg/m}^3)(2.50 \times 10^{-4} \text{ m}^2)}}$$

$$v_{\text{above}} = 11.7 \text{ m/s}$$

The air you blow must be moving faster than this if the upward force is to exceed the weight of the dime. Practice this trick a few times and then impress all your friends!

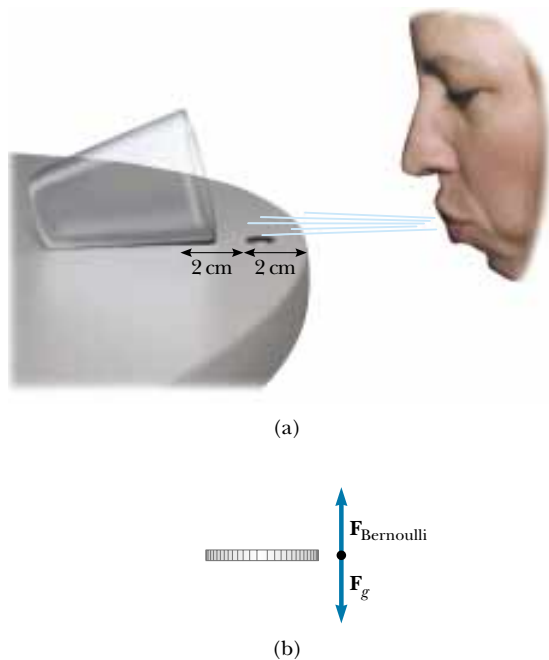
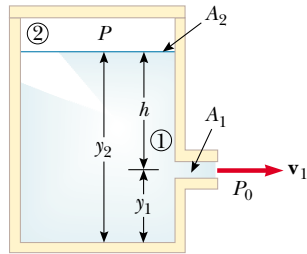


Figure 15.22

**EXAMPLE 15.10** Torricelli's Law

An enclosed tank containing a liquid of density  $\rho$  has a hole in its side at a distance  $y_1$  from the tank's bottom (Fig. 15.23). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure  $P$ . Determine the speed at



**Figure 15.23** When  $P$  is much larger than atmospheric pressure  $P_0$ , the liquid speed as the liquid passes through the hole in the side of the container is given approximately by  $v_1 = \sqrt{2(P - P_0)/\rho}$ .

which the liquid leaves the hole when the liquid's level is a distance  $h$  above the hole.

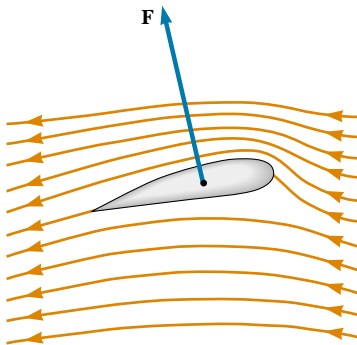
**Solution** Because  $A_2 \gg A_1$ , the liquid is approximately at rest at the top of the tank, where the pressure is  $P$ . Applying Bernoulli's equation to points 1 and 2 and noting that at the hole  $P_1$  is equal to atmospheric pressure  $P_0$ , we find that

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

But  $y_2 - y_1 = h$ ; thus, this expression reduces to

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

When  $P$  is much greater than  $P_0$  (so that the term  $2gh$  can be neglected), the exit speed of the water is mainly a function of  $P$ . If the tank is open to the atmosphere, then  $P = P_0$  and  $v_1 = \sqrt{2gh}$ . In other words, for an open tank, the speed of liquid coming out through a hole a distance  $h$  below the surface is equal to that acquired by an object falling freely through a vertical distance  $h$ . This phenomenon is known as **Torricelli's law**.

*Optional Section***15.8 OTHER APPLICATIONS OF BERNOULLI'S EQUATION**

**Figure 15.24** Streamline flow around an airplane wing. The pressure above the wing is less than the pressure below, and a dynamic lift upward results.

The lift on an aircraft wing can be explained, in part, by the Bernoulli effect. Airplane wings are designed so that the air speed above the wing is greater than that below the wing. As a result, the air pressure above the wing is less than the pressure below, and a net upward force on the wing, called *lift*, results.

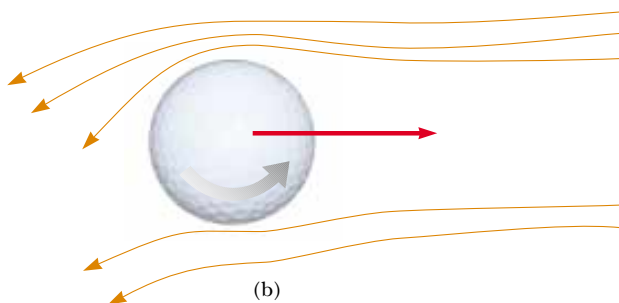
Another factor influencing the lift on a wing is shown in Figure 15.24. The wing has a slight upward tilt that causes air molecules striking its bottom to be deflected downward. This deflection means that the wing is exerting a downward force on the air. According to Newton's third law, the air must exert an equal and opposite force on the wing.

Finally, turbulence also has an effect. If the wing is tilted too much, the flow of air across the upper surface becomes turbulent, and the pressure difference across the wing is not as great as that predicted by Bernoulli's equation. In an extreme case, this turbulence may cause the aircraft to stall.

In general, an object moving through a fluid experiences lift as the result of any effect that causes the fluid to change its direction as it flows past the object. Some factors that influence lift are the shape of the object, its orientation with respect to the fluid flow, any spinning motion it might have, and the texture of its surface. For example, a golf ball struck with a club is given a rapid backspin, as shown in Figure 15.25a. The dimples on the ball help "entrain" the air to follow the curvature of the ball's surface. This effect is most pronounced on the top half of the ball, where the ball's surface is moving in the same direction as the air flow. Figure 15.25b shows a thin layer of air wrapping part way around the ball and being deflected downward as a result. Because the ball pushes the air down, the air must push up on the ball. Without the dimples, the air is not as well entrained,



(a)

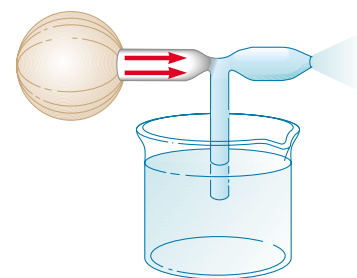


(b)

**Figure 15.25** (a) A golf ball is made to spin when struck by the club.  
(b) The spinning ball experiences a lifting force that allows it to travel much farther than it would if it were not spinning.

and the golf ball does not travel as far. For the same reason, a tennis ball's fuzz helps the spinning ball "grab" the air rushing by and helps deflect it.

A number of devices operate by means of the pressure differentials that result from differences in a fluid's speed. For example, a stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure 15.26. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this so-called atomizer is used in perfume bottles and paint sprayers. The same principle is used in the carburetor of a gasoline engine. In this case, the low-pressure region in the carburetor is produced by air drawn in by the piston through the air filter. The gasoline vaporizes in that region, mixes with the air, and enters the cylinder of the engine, where combustion occurs.



**Figure 15.26** A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

### Quick Quiz 15.10

People in buildings threatened by a tornado are often told to open the windows to minimize damage. Why?

### QuickLab

You can easily demonstrate the effect of changing fluid direction by lightly holding the back of a spoon against a stream of water coming from a faucet. You will see the stream "attach" itself to the curvature of the spoon and be deflected sideways. You will also feel the third-law force exerted by the water on the spoon.

### SUMMARY

The **pressure**  $P$  in a fluid is the force per unit area exerted by the fluid on a surface:

$$P \equiv \frac{F}{A} \quad (15.1)$$

In the SI system, pressure has units of newtons per square meter ( $\text{N/m}^2$ ), and  $1 \text{ N/m}^2 = 1 \text{ pascal (Pa)}$ .

The pressure in a fluid at rest varies with depth  $h$  in the fluid according to the expression

$$P = P_0 + \rho gh \quad (15.4)$$

where  $P_0$  is atmospheric pressure ( $= 1.013 \times 10^5 \text{ N/m}^2$ ) and  $\rho$  is the density of the fluid, assumed uniform.

**Pascal's law** states that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container.

When an object is partially or fully submerged in a fluid, the fluid exerts on the object an upward force called the **buoyant force**. According to **Archimedes's principle**, the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object. Be sure you can apply this principle to a wide variety of situations, including sinking objects, floating ones, and neutrally buoyant ones.

You can understand various aspects of a fluid's dynamics by assuming that the fluid is nonviscous and incompressible and that the fluid's motion is a steady flow with no rotation.

Two important concepts regarding ideal fluid flow through a pipe of nonuniform size are as follows:

1. The flow rate (volume flux) through the pipe is constant; this is equivalent to stating that the product of the cross-sectional area  $A$  and the speed  $v$  at any point is a constant. This result is expressed in the **equation of continuity**:

$$A_1 v_1 = A_2 v_2 = \text{constant} \quad (15.7)$$

You can use this expression to calculate how the velocity of a fluid changes as the fluid is constricted or as it flows into a more open area.

2. The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This result is summarized in **Bernoulli's equation**:

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant} \quad (15.9)$$

### QUESTIONS

1. Two drinking glasses of the same weight but of different shape and different cross-sectional area are filled to the same level with water. According to the expression  $P = P_0 + \rho gh$ , the pressure at the bottom of both glasses is the same. In view of this, why does one glass weigh more than the other?
2. If the top of your head has a surface area of  $100 \text{ cm}^2$ , what is the weight of the air above your head?
3. When you drink a liquid through a straw, you reduce the

pressure in your mouth and let the atmosphere move the liquid. Explain why this is so. Can you use a straw to sip a drink on the Moon?

4. A helium-filled balloon rises until its density becomes the same as that of the surrounding air. If a sealed submarine begins to sink, will it go all the way to the bottom of the ocean or will it stop when its density becomes the same as that of the surrounding water?
5. A fish rests on the bottom of a bucket of water while the

bucket is being weighed. When the fish begins to swim around, does the weight change?

6. Does a ship ride higher in the water of an inland lake or in the ocean? Why?
7. Lead has a greater density than iron, and both metals are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume?
8. The water supply for a city is often provided by reservoirs built on high ground. Water flows from the reservoir, through pipes, and into your home when you turn the tap on your faucet. Why is the flow of water more rapid out of a faucet on the first floor of a building than it is in an apartment on a higher floor?
9. Smoke rises in a chimney faster when a breeze is blowing than when there is no breeze at all. Use Bernoulli's equation to explain this phenomenon.
10. If a Ping-Pong ball is above a hair dryer, the ball can be suspended in the air column emitted by the dryer. Explain.
11. When ski jumpers are airborne (Fig. Q15.11), why do they bend their bodies forward and keep their hands at their sides?



**Figure Q15.11**

12. Explain why a sealed bottle partially filled with a liquid can float.
13. When is the buoyant force on a swimmer greater—after exhaling or after inhaling?
14. A piece of unpainted wood barely floats in a container partly filled with water. If the container is sealed and then pressurized above atmospheric pressure, does the wood rise, sink, or remain at the same level? (*Hint: Wood is porous.*)
15. A flat plate is immersed in a liquid at rest. For what orientation of the plate is the pressure on its flat surface uniform?
16. Because atmospheric pressure is about  $10^5 \text{ N/m}^2$  and the area of a person's chest is about  $0.13 \text{ m}^2$ , the force of the atmosphere on one's chest is around  $13\,000 \text{ N}$ . In view of this enormous force, why don't our bodies collapse?
17. How would you determine the density of an irregularly shaped rock?

18. Why do airplane pilots prefer to take off into the wind?
19. If you release a ball while inside a freely falling elevator, the ball remains in front of you rather than falling to the floor because the ball, the elevator, and you all experience the same downward acceleration  $\mathbf{g}$ . What happens if you repeat this experiment with a helium-filled balloon? (This one is tricky.)
20. Two identical ships set out to sea. One is loaded with a cargo of Styrofoam, and the other is empty. Which ship is more submerged?
21. A small piece of steel is tied to a block of wood. When the wood is placed in a tub of water with the steel on top, half of the block is submerged. If the block is inverted so that the steel is underwater, does the amount of the block submerged increase, decrease, or remain the same? What happens to the water level in the tub when the block is inverted?
22. Prairie dogs (Fig. Q15.22) ventilate their burrows by building a mound over one entrance, which is open to a stream of air. A second entrance at ground level is open to almost stagnant air. How does this construction create an air flow through the burrow?



**Figure Q15.22**

23. An unopened can of diet cola floats when placed in a tank of water, whereas a can of regular cola of the same brand sinks in the tank. What do you suppose could explain this phenomenon?
24. Figure Q15.24 shows a glass cylinder containing four liquids of different densities. From top to bottom, the liquids are oil (orange), water (yellow), salt water (green), and mercury (silver). The cylinder also contains, from top to bottom, a Ping-Pong ball, a piece of wood, an egg, and a steel ball. (a) Which of these liquids has the lowest density, and which has the greatest? (b) What can you conclude about the density of each object?





Figure Q15.24

25. In Figure Q15.25, an air stream moves from right to left through a tube that is constricted at the middle. Three Ping-Pong balls are levitated in equilibrium above the vertical columns through which the air escapes. (a) Why is the ball at the right higher than the one in the middle?



Figure Q15.25

- (b) Why is the ball at the left lower than the ball at the right even though the horizontal tube has the same dimensions at these two points?
26. You are a passenger on a spacecraft. For your comfort, the interior contains air just like that at the surface of the Earth. The craft is coasting through a very empty region of space. That is, a nearly perfect vacuum exists just outside the wall. Suddenly a meteoroid pokes a hole, smaller than the palm of your hand, right through the wall next to your seat. What will happen? Is there anything you can or should do about it?

## PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

WEB = solution posted at <http://www.saunderscollege.com/physics/> = Computer useful in solving problem = Interactive Physics

= paired numerical/symbolic problems

### Section 15.1 Pressure

1. Calculate the mass of a solid iron sphere that has a diameter of 3.00 cm.
2. Find the order of magnitude of the density of the *nucleus* of an atom. What does this result suggest concerning the structure of matter? (Visualize a nucleus as protons and neutrons closely packed together. Each has mass  $1.67 \times 10^{-27}$  kg and radius on the order of  $10^{-15}$  m.)
3. A 50.0-kg woman balances on one heel of a pair of high-heeled shoes. If the heel is circular and has a radius of 0.500 cm, what pressure does she exert on the floor?
4. The four tires of an automobile are inflated to a gauge pressure of 200 kPa. Each tire has an area of  $0.0240 \text{ m}^2$  in contact with the ground. Determine the weight of the automobile.
5. What is the total mass of the Earth's atmosphere? (The radius of the Earth is  $6.37 \times 10^6$  m, and atmospheric pressure at the Earth's surface is  $1.013 \times 10^5 \text{ N/m}^2$ .)

### Section 15.2 Variation of Pressure with Depth

6. (a) Calculate the absolute pressure at an ocean depth of 1 000 m. Assume the density of seawater is  $1\,024 \text{ kg/m}^3$  and that the air above exerts a pressure of 101.3 kPa. (b) At this depth, what force must the frame around a circular submarine porthole having a diameter of 30.0 cm exert to counterbalance the force exerted by the water?
7. The spring of the pressure gauge shown in Figure 15.2 has a force constant of 1 000 N/m, and the piston has a diameter of 2.00 cm. When the gauge is lowered into water, at what depth does the piston move in by 0.500 cm?
8. The small piston of a hydraulic lift has a cross-sectional area of  $3.00 \text{ cm}^2$ , and its large piston has a cross-sectional area of  $200 \text{ cm}^2$  (see Fig. 15.5a). What force must be applied to the small piston for it to raise a load of 15.0 kN? (In service stations, this force is usually generated with the use of compressed air.)

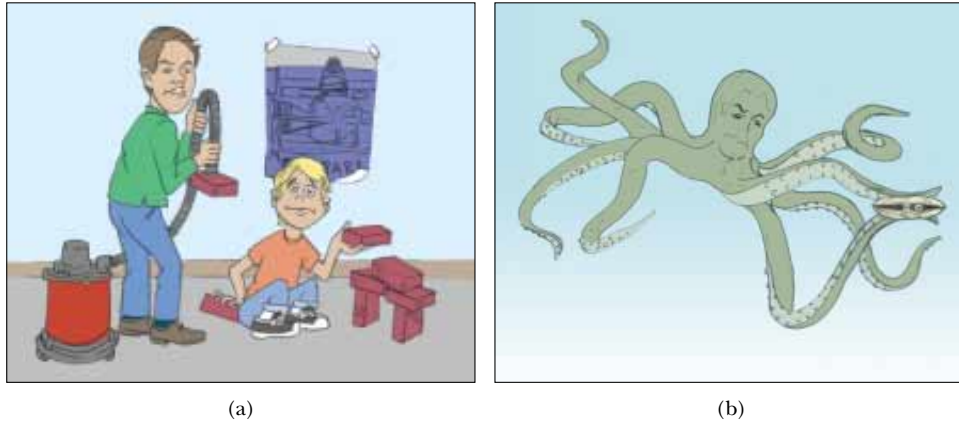


Figure P15.10

- WEB 9.** What must be the contact area between a suction cup (completely exhausted) and a ceiling if the cup is to support the weight of an 80.0-kg student?
- 10.** (a) A very powerful vacuum cleaner has a hose 2.86 cm in diameter. With no nozzle on the hose, what is the weight of the heaviest brick that the cleaner can lift (Fig. P15.10)? (b) A very powerful octopus uses one sucker of diameter 2.86 cm on each of the two shells of a clam in an attempt to pull the shells apart. Find the greatest force that the octopus can exert in salt water 32.3 m in depth. (*Caution:* Experimental verification can be interesting, but do not drop a brick on your foot. Do not overheat the motor of a vacuum cleaner. Do not get an octopus mad at you.)
- 11.** For the cellar of a new house, a hole with vertical sides descending 2.40 m is dug in the ground. A concrete foundation wall is built all the way across the 9.60-m width of the excavation. This foundation wall is 0.183 m away from the front of the cellar hole. During a rain-storm, drainage from the street fills up the space in front of the concrete wall but not the cellar behind the wall. The water does not soak into the clay soil. Find the force that the water causes on the foundation wall. For comparison, the weight of the water is given by
- $$2.40 \text{ m} \times 9.60 \text{ m} \times 0.183 \text{ m} \times 1\,000 \text{ kg/m}^3 \times 9.80 \text{ m/s}^2 = 41.3 \text{ kN}$$
- 12.** A swimming pool has dimensions 30.0 m  $\times$  10.0 m and a flat bottom. When the pool is filled to a depth of 2.00 m with fresh water, what is the force caused by the water on the bottom? On each end? On each side?
- 13.** A sealed spherical shell of diameter  $d$  is rigidly attached to a cart that is moving horizontally with an acceleration  $a$ , as shown in Figure P15.13. The sphere is nearly filled with a fluid having density  $\rho$  and also contains one small bubble of air at atmospheric pressure. Find an expression for the pressure  $P$  at the center of the sphere.

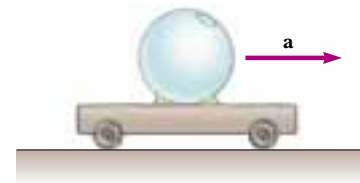


Figure P15.13

- 14.** The tank shown in Figure P15.14 is filled with water to a depth of 2.00 m. At the bottom of one of the side walls is a rectangular hatch 1.00 m high and 2.00 m wide. The hatch is hinged at its top. (a) Determine the force that the water exerts on the hatch. (b) Find the torque exerted about the hinges.

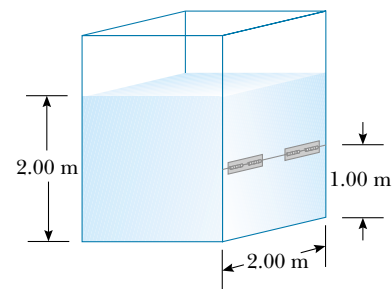


Figure P15.14

- 15. Review Problem.** A solid copper ball with a diameter of 3.00 m at sea level is placed at the bottom of the ocean (at a depth of 10.0 km). If the density of seawater is  $1\,030 \text{ kg/m}^3$ , by how much (approximately) does the diameter of the ball decrease when it reaches bottom? Take the bulk modulus of copper as  $14.0 \times 10^{10} \text{ N/m}^2$ .

### Section 15.3 Pressure Measurements

16. Normal atmospheric pressure is  $1.013 \times 10^5$  Pa. The approach of a storm causes the height of a mercury barometer to drop by 20.0 mm from the normal height. What is the atmospheric pressure? (The density of mercury is  $13.59 \text{ g/cm}^3$ .)
- WEB 17. Blaise Pascal duplicated Torricelli's barometer, using a red Bordeaux wine, of density  $984 \text{ kg/m}^3$ , as the working liquid (Fig. P15.17). What was the height  $h$  of the wine column for normal atmospheric pressure? Would you expect the vacuum above the column to be as good as that for mercury?

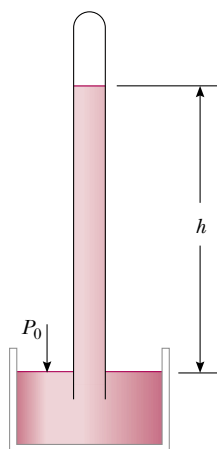


Figure P15.17

18. Mercury is poured into a U-tube, as shown in Figure P15.18a. The left arm of the tube has a cross-sectional area  $A_1$  of  $10.0 \text{ cm}^2$ , and the right arm has a cross-sectional area  $A_2$  of  $5.00 \text{ cm}^2$ . One-hundred grams of water are then poured into the right arm, as shown in Figure P15.18b. (a) Determine the length of the water column

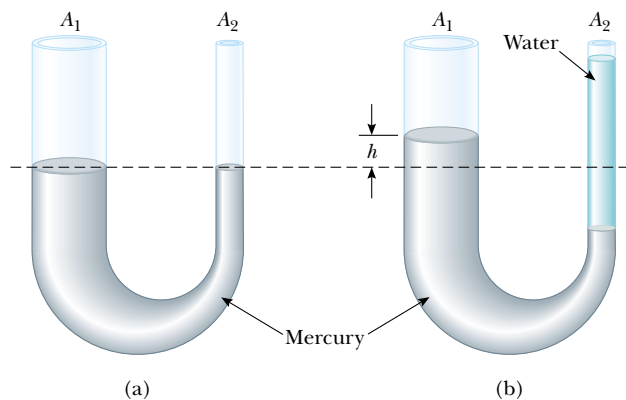


Figure P15.18

in the right arm of the U-tube. (b) Given that the density of mercury is  $13.6 \text{ g/cm}^3$ , what distance  $h$  does the mercury rise in the left arm?

19. A U-tube of uniform cross-sectional area and open to the atmosphere is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in Figure P15.19, with  $h_2 = 1.00 \text{ cm}$ , determine the value of  $h_1$ .

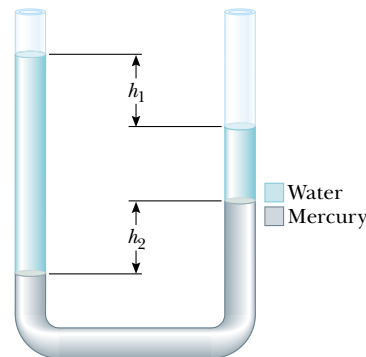
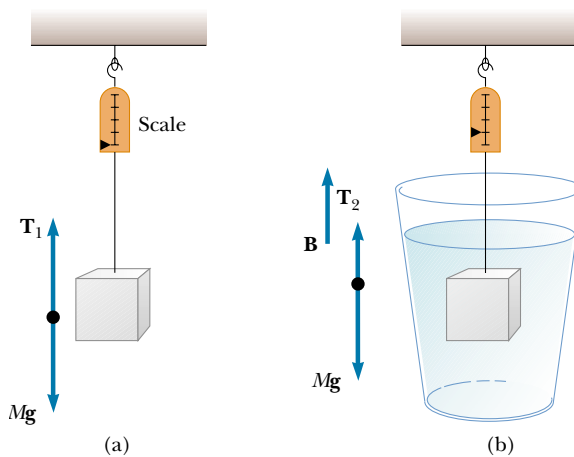


Figure P15.19

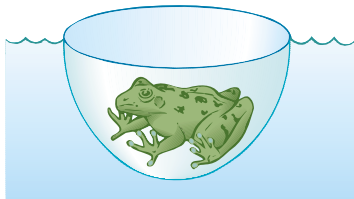
### Section 15.4 Buoyant Forces and Archimedes's Principle

20. (a) A light balloon is filled with  $400 \text{ m}^3$  of helium. At  $0^\circ\text{C}$ , what is the mass of the payload that the balloon can lift? (b) In Table 15.1, note that the density of hydrogen is nearly one-half the density of helium. What load can the balloon lift if it is filled with hydrogen?
21. A Styrofoam slab has a thickness of  $10.0 \text{ cm}$  and a density of  $300 \text{ kg/m}^3$ . When a  $75.0\text{-kg}$  swimmer is resting on it, the slab floats in fresh water with its top at the same level as the water's surface. Find the area of the slab.
22. A Styrofoam slab has thickness  $h$  and density  $\rho_s$ . What is the area of the slab if it floats with its upper surface just awash in fresh water, when a swimmer of mass  $m$  is on top?
23. A piece of aluminum with mass  $1.00 \text{ kg}$  and density  $2700 \text{ kg/m}^3$  is suspended from a string and then completely immersed in a container of water (Fig. P15.23). Calculate the tension in the string (a) before and (b) after the metal is immersed.
24. A  $10.0\text{-kg}$  block of metal measuring  $12.0 \text{ cm} \times 10.0 \text{ cm} \times 10.0 \text{ cm}$  is suspended from a scale and immersed in water, as shown in Figure P15.23b. The  $12.0\text{-cm}$  dimension is vertical, and the top of the block is  $5.00 \text{ cm}$  from the surface of the water. (a) What are the forces acting on the top and on the bottom of the block? (Take  $P_0 = 1.0130 \times 10^5 \text{ N/m}^2$ .) (b) What is the reading of the spring scale? (c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.



**Figure P15.23** Problems 23 and 24.

- WEB 25.** A cube of wood having a side dimension of 20.0 cm and a density of  $650 \text{ kg/m}^3$  floats on water. (a) What is the distance from the horizontal top surface of the cube to the water level? (b) How much lead weight must be placed on top of the cube so that its top is just level with the water?
- 26.** To an order of magnitude, how many helium-filled toy balloons would be required to lift you? Because helium is an irreplaceable resource, develop a theoretical answer rather than an experimental answer. In your solution, state what physical quantities you take as data and the values you measure or estimate for them.
- 27.** A plastic sphere floats in water with 50.0% of its volume submerged. This same sphere floats in glycerin with 40.0% of its volume submerged. Determine the densities of the glycerin and the sphere.
- 28.** A frog in a hemispherical pod finds that he just floats without sinking into a sea of blue-green ooze having a density of  $1.35 \text{ g/cm}^3$  (Fig. P15.28). If the pod has a radius of 6.00 cm and a negligible mass, what is the mass of the frog?



**Figure P15.28**

- 29.** How many cubic meters of helium are required to lift a balloon with a 400-kg payload to a height of 8 000 m? (Take  $\rho_{\text{He}} = 0.180 \text{ kg/m}^3$ .) Assume that the balloon

maintains a constant volume and that the density of air decreases with the altitude  $z$  according to the expression  $\rho_{\text{air}} = \rho_0 e^{-z/8\,000}$ , where  $z$  is in meters and  $\rho_0 = 1.25 \text{ kg/m}^3$  is the density of air at sea level.

- 30. Review Problem.** A long cylindrical tube of radius  $r$  is weighted on one end so that it floats upright in a fluid having a density  $\rho$ . It is pushed downward a distance  $x$  from its equilibrium position and then released. Show that the tube will execute simple harmonic motion if the resistive effects of the water are neglected, and determine the period of the oscillations.
- 31.** A bathysphere used for deep-sea exploration has a radius of 1.50 m and a mass of  $1.20 \times 10^4 \text{ kg}$ . To dive, this submarine takes on mass in the form of seawater. Determine the amount of mass that the submarine must take on if it is to descend at a constant speed of 1.20 m/s, when the resistive force on it is 1 100 N in the upward direction. Take  $1.03 \times 10^3 \text{ kg/m}^3$  as the density of seawater.
- 32.** The United States possesses the eight largest warships in the world—aircraft carriers of the *Nimitz* class—and it is building one more. Suppose that one of the ships bobs up to float 11.0 cm higher in the water when 50 fighters take off from it at a location where  $g = 9.78 \text{ m/s}^2$ . The planes have an average mass of 29 000 kg. Find the horizontal area enclosed by the waterline of the ship. (By comparison, its flight deck has an area of  $18\,000 \text{ m}^2$ .)

## Section 15.5 Fluid Dynamics

### Section 15.6 Streamlines and the Equation of Continuity

### Section 15.7 Bernoulli's Equation

- 33.** (a) A water hose 2.00 cm in diameter is used to fill a 20.0-L bucket. If it takes 1.00 min to fill the bucket, what is the speed  $v$  at which water moves through the hose? (Note:  $1 \text{ L} = 1\,000 \text{ cm}^3$ .) (b) If the hose has a nozzle 1.00 cm in diameter, find the speed of the water at the nozzle.
- 34.** A horizontal pipe 10.0 cm in diameter has a smooth reduction to a pipe 5.00 cm in diameter. If the pressure of the water in the larger pipe is  $8.00 \times 10^4 \text{ Pa}$  and the pressure in the smaller pipe is  $6.00 \times 10^4 \text{ Pa}$ , at what rate does water flow through the pipes?
- 35.** A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is  $2.50 \times 10^{-3} \text{ m}^3/\text{min}$ , determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.
- 36.** Through a pipe of diameter 15.0 cm, water is pumped from the Colorado River up to Grand Canyon Village, located on the rim of the canyon. The river is at an elevation of 564 m, and the village is at an elevation of 2 096 m. (a) What is the minimum pressure at which the water must be pumped if it is to arrive at the village?

(b) If  $4\,500\text{ m}^3$  are pumped per day, what is the speed of the water in the pipe? (c) What additional pressure is necessary to deliver this flow? (Note: You may assume that the acceleration due to gravity and the density of air are constant over this range of elevations.)

37. Water flows through a fire hose of diameter  $6.35\text{ cm}$  at a rate of  $0.012\,0\text{ m}^3/\text{s}$ . The fire hose ends in a nozzle with an inner diameter of  $2.20\text{ cm}$ . What is the speed at which the water exits the nozzle?
38. Old Faithful Geyser in Yellowstone National Park erupts at approximately 1-h intervals, and the height of the water column reaches  $40.0\text{ m}$  (Fig. P15.38). (a) Consider the rising stream as a series of separate drops. Analyze the free-fall motion of one of these drops to determine the speed at which the water leaves the ground. (b) Treating the rising stream as an ideal fluid in streamline flow, use Bernoulli's equation to determine the speed of the water as it leaves ground level. (c) What is the pressure (above atmospheric) in the heated underground chamber if its depth is  $175\text{ m}$ ? You may assume that the chamber is large compared with the geyser's vent.



Figure P15.38

(Optional)

### Section 15.8 Other Applications of Bernoulli's Equation

39. An airplane has a mass of  $1.60 \times 10^4\text{ kg}$ , and each wing has an area of  $40.0\text{ m}^2$ . During level flight, the pressure on the lower wing surface is  $7.00 \times 10^4\text{ Pa}$ . Determine the pressure on the upper wing surface.
40. A Venturi tube may be used as a fluid flow meter (see Fig. 15.21). If the difference in pressure is  $P_1 - P_2 = 21.0\text{ kPa}$ , find the fluid flow rate in cubic meters per second, given that the radius of the outlet tube is  $1.00\text{ cm}$ , the radius of the inlet tube is  $2.00\text{ cm}$ , and the fluid is gasoline ( $\rho = 700\text{ kg/m}^3$ ).
41. A Pitot tube can be used to determine the velocity of air flow by measuring the difference between the total pressure and the static pressure (Fig. P15.41). If the fluid in the tube is mercury, whose density is  $\rho_{\text{Hg}} = 13\,600\text{ kg/m}^3$ , and if  $\Delta h = 5.00\text{ cm}$ , find the speed of

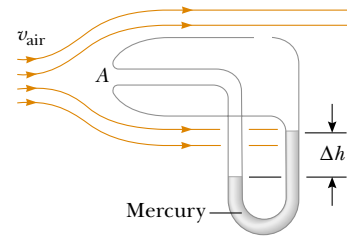


Figure P15.41

air flow. (Assume that the air is stagnant at point A, and take  $\rho_{\text{air}} = 1.25\text{ kg/m}^3$ .)

42. An airplane is cruising at an altitude of  $10\text{ km}$ . The pressure outside the craft is  $0.287\text{ atm}$ ; within the passenger compartment, the pressure is  $1.00\text{ atm}$  and the temperature is  $20^\circ\text{C}$ . A small leak occurs in one of the window seals in the passenger compartment. Model the air as an ideal fluid to find the speed of the stream of air flowing through the leak.
43. A siphon is used to drain water from a tank, as illustrated in Figure P15.43. The siphon has a uniform diameter. Assume steady flow without friction. (a) If the distance  $h = 1.00\text{ m}$ , find the speed of outflow at the end of the siphon. (b) What is the limitation on the height of the top of the siphon above the water surface? (For the flow of liquid to be continuous, the pressure must not drop below the vapor pressure of the liquid.)

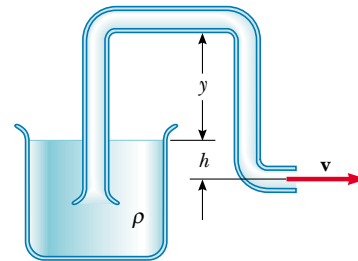


Figure P15.43

44. A hypodermic syringe contains a medicine with the density of water (Fig. P15.44). The barrel of the syringe has a cross-sectional area  $A = 2.50 \times 10^{-5}\text{ m}^2$ , and the needle has a cross-sectional area  $a = 1.00 \times 10^{-8}\text{ m}^2$ . In the absence of a force on the plunger, the pressure everywhere is  $1\text{ atm}$ . A force  $\mathbf{F}$  of magnitude  $2.00\text{ N}$  acts on the plunger, making the medicine squirt horizontally from the needle. Determine the speed of the medicine as it leaves the needle's tip.
- WEB 45. A large storage tank is filled to a height  $h_0$ . The tank is punctured at a height  $h$  above the bottom of the tank (Fig. P15.45). Find an expression for how far from the tank the exiting stream lands.



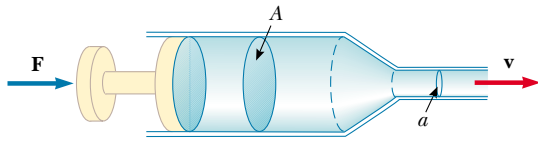


Figure P15.44

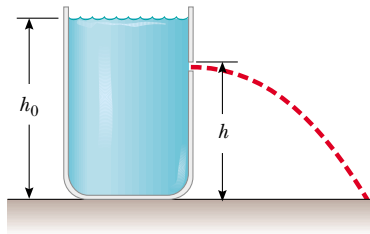


Figure P15.45 Problems 45 and 46.

46. A hole is punched at a height  $h$  in the side of a container of height  $h_0$ . The container is full of water, as shown in Figure P15.45. If the water is to shoot as far as possible horizontally, (a) how far from the bottom of the container should the hole be punched? (b) Neglecting frictional losses, how far (initially) from the side of the container will the water land?

### ADDITIONAL PROBLEMS

47. A Ping-Pong ball has a diameter of 3.80 cm and an average density of  $0.0840 \text{ g/cm}^3$ . What force would be required to hold it completely submerged under water?
48. Figure P15.48 shows a tank of water with a valve at the bottom. If this valve is opened, what is the maximum height attained by the water stream exiting the right side of the tank? Assume that  $h = 10.0 \text{ m}$ ,  $L = 2.00 \text{ m}$ , and  $\theta = 30.0^\circ$ , and that the cross-sectional area at point A is very large compared with that at point B.

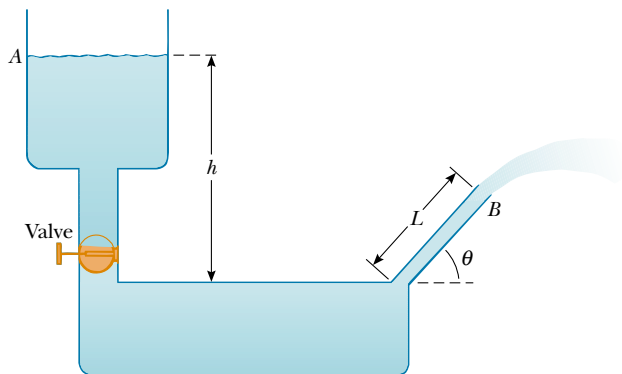


Figure P15.48

49. A helium-filled balloon is tied to a 2.00-m-long, 0.0500-kg uniform string. The balloon is spherical with a radius of 0.400 m. When released, the balloon lifts a length  $h$  of string and then remains in equilibrium, as shown in Figure P15.49. Determine the value of  $h$ . The envelope of the balloon has a mass of 0.250 kg.

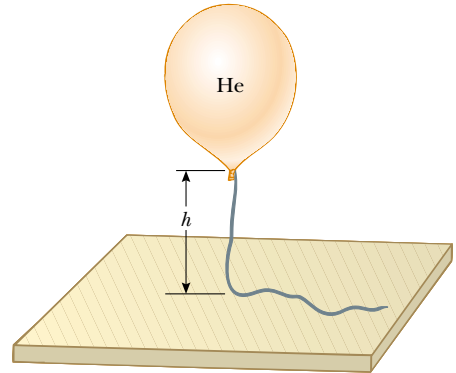


Figure P15.49

50. Water is forced out of a fire extinguisher by air pressure, as shown in Figure P15.50. How much gauge air pressure in the tank (above atmospheric) is required for the water jet to have a speed of 30.0 m/s when the water level is 0.500 m below the nozzle?

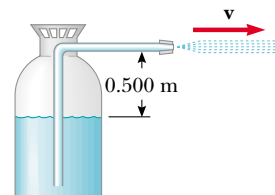


Figure P15.50

51. The true weight of an object is measured in a vacuum, where buoyant forces are absent. An object of volume  $V$  is weighed in air on a balance with the use of weights of density  $\rho$ . If the density of air is  $\rho_{\text{air}}$  and the balance reads  $F'_g$ , show that the true weight  $F_g$  is

$$F_g = F'_g + \left( V - \frac{F'_g}{\rho g} \right) \rho_{\text{air}} g$$

52. Evangelista Torricelli was the first to realize that we live at the bottom of an ocean of air. He correctly surmised that the pressure of our atmosphere is attributable to the weight of the air. The density of air at  $0^\circ\text{C}$  at the Earth's surface is  $1.29 \text{ kg/m}^3$ . The density decreases with increasing altitude (as the atmosphere thins). On the other hand, if we assume that the density is constant



( $1.29 \text{ kg/m}^3$ ) up to some altitude  $h$ , and zero above that altitude, then  $h$  would represent the thickness of our atmosphere. Use this model to determine the value of  $h$  that gives a pressure of  $1.00 \text{ atm}$  at the surface of the Earth. Would the peak of Mt. Everest rise above the surface of such an atmosphere?

53. A wooden dowel has a diameter of  $1.20 \text{ cm}$ . It floats in water with  $0.400 \text{ cm}$  of its diameter above water level (Fig. P15.53). Determine the density of the dowel.

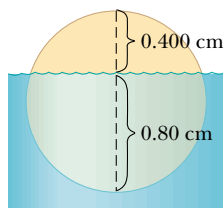


Figure P15.53

54. A light spring of constant  $k = 90.0 \text{ N/m}$  rests vertically on a table (Fig. P15.54a). A  $2.00\text{-g}$  balloon is filled with helium (density  $= 0.180 \text{ kg/m}^3$ ) to a volume of  $5.00 \text{ m}^3$  and is then connected to the spring, causing it to stretch as shown in Figure P15.54b. Determine the extension distance  $L$  when the balloon is in equilibrium.

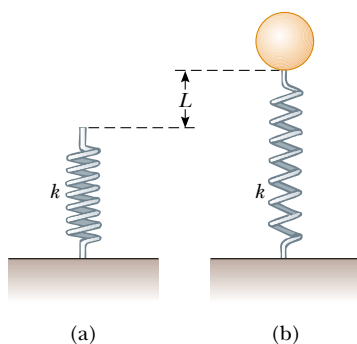


Figure P15.54

55. A  $1.00\text{-kg}$  beaker containing  $2.00 \text{ kg}$  of oil (density  $= 916.0 \text{ kg/m}^3$ ) rests on a scale. A  $2.00\text{-kg}$  block of iron is suspended from a spring scale and completely submerged in the oil, as shown in Figure P15.55. Determine the equilibrium readings of both scales.
56. A beaker of mass  $m_b$  containing oil of mass  $m_o$  (density  $= \rho_o$ ) rests on a scale. A block of iron of mass  $m_{Fe}$  is suspended from a spring scale and completely submerged in the oil, as shown in Figure P15.55. Determine the equilibrium readings of both scales.

**WEB 57. Review Problem.** With reference to Figure 15.7, show that the total torque exerted by the water behind the

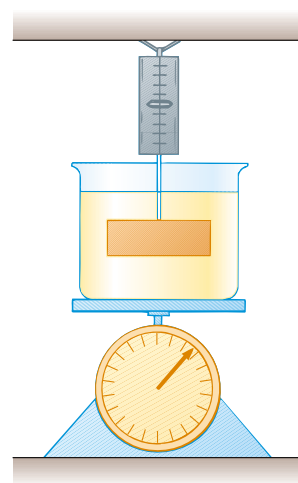


Figure P15.55 Problems 55 and 56.

dam about an axis through  $O$  is  $\frac{1}{6}\rho g w H^3$ . Show that the effective line of action of the total force exerted by the water is at a distance  $\frac{1}{3}H$  above  $O$ .

58. In about 1657 Otto von Guericke, inventor of the air pump, evacuated a sphere made of two brass hemispheres. Two teams of eight horses each could pull the hemispheres apart only on some trials, and then “with greatest difficulty,” with the resulting sound likened to a cannon firing (Fig. P15.58). (a) Show that the force  $F$

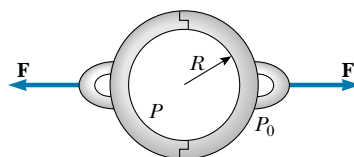


Figure P15.58 The colored engraving, dated 1672, illustrates Otto von Guericke's demonstration of the force due to air pressure as performed before Emperor Ferdinand III in 1657. (

required to pull the evacuated hemispheres apart is  $\pi R^2(P_0 - P)$ , where  $R$  is the radius of the hemispheres and  $P$  is the pressure inside the hemispheres, which is much less than  $P_0$ . (b) Determine the force if  $P = 0.100P_0$  and  $R = 0.300$  m.

59. In 1983 the United States began coining the cent piece out of copper-clad zinc rather than pure copper. The mass of the old copper cent is 3.083 g, whereas that of the new cent is 2.517 g. Calculate the percent of zinc (by volume) in the new cent. The density of copper is  $8.960 \text{ g/cm}^3$ , and that of zinc is  $7.133 \text{ g/cm}^3$ . The new and old coins have the same volume.
60. A thin spherical shell with a mass of 4.00 kg and a diameter of 0.200 m is filled with helium (density =  $0.180 \text{ kg/m}^3$ ). It is then released from rest on the bottom of a pool of water that is 4.00 m deep. (a) Neglecting frictional effects, show that the shell rises with constant acceleration and determine the value of that acceleration. (b) How long does it take for the top of the shell to reach the water's surface?
61. An incompressible, nonviscous fluid initially rests in the vertical portion of the pipe shown in Figure P15.61a, where  $L = 2.00$  m. When the valve is opened, the fluid flows into the horizontal section of the pipe. What is the speed of the fluid when all of it is in the horizontal section, as in Figure P15.61b? Assume that the cross-sectional area of the entire pipe is constant.

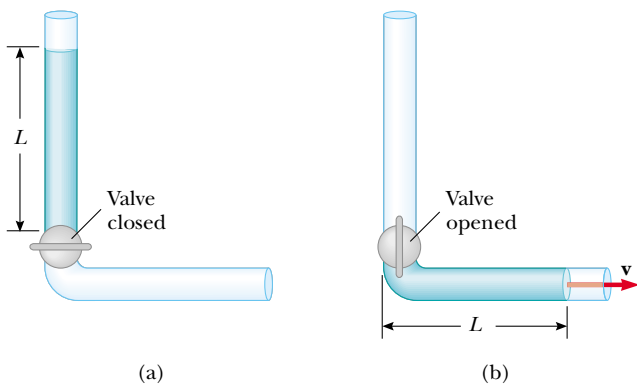


Figure P15.61

62. **Review Problem.** A uniform disk with a mass of 10.0 kg and a radius of 0.250 m spins at 300 rev/min on a low-friction axle. It must be brought to a stop in 1.00 min by a brake pad that makes contact with the disk at an average distance of 0.220 m from the axis. The coefficient of friction between the pad and the disk is 0.500. A piston in a cylinder with a diameter of 5.00 cm presses the brake pad against the disk. Find the pressure that the brake fluid in the cylinder must have.
63. Figure P15.63 shows Superman attempting to drink water through a very long straw. With his great strength,



Figure P15.63

he achieves maximum possible suction. The walls of the tubular straw do not collapse. (a) Find the maximum height through which he can lift the water. (b) Still thirsty, the Man of Steel repeats his attempt on the Moon, which has no atmosphere. Find the difference between the water levels inside and outside the straw.

64. Show that the variation of atmospheric pressure with altitude is given by  $P = P_0 e^{-\alpha h}$ , where  $\alpha = \rho_0 g / P_0$ ,  $P_0$  is atmospheric pressure at some reference level  $y = 0$ , and  $\rho_0$  is the atmospheric density at this level. Assume that the decrease in atmospheric pressure with increasing altitude is given by Equation 15.4, so that  $dP/dy = -\rho g$ , and assume that the density of air is proportional to the pressure.
65. A cube of ice whose edge measures 20.0 mm is floating in a glass of ice-cold water with one of its faces parallel to the water's surface. (a) How far below the water surface is the bottom face of the block? (b) Ice-cold ethyl alcohol is gently poured onto the water's surface to form a layer 5.00 mm thick above the water. The alcohol does not mix with the water. When the ice cube again attains hydrostatic equilibrium, what is the distance from the top of the water to the bottom face of the block? (c) Additional cold ethyl alcohol is poured onto the water's surface until the top surface of the alcohol coincides with the top surface of the ice cube (in

hydrostatic equilibrium). How thick is the required layer of ethyl alcohol?

- 66. Review Problem.** A light balloon filled with helium with a density of  $0.180 \text{ kg/m}^3$  is tied to a light string of length  $L = 3.00 \text{ m}$ . The string is tied to the ground, forming an “inverted” simple pendulum, as shown in Figure P15.66a. If the balloon is displaced slightly from its equilibrium position as shown in Figure P15.66b, (a) show that the ensuing motion is simple harmonic and (b) determine the period of the motion. Take the density of air to be  $1.29 \text{ kg/m}^3$  and ignore any energy loss due to air friction.

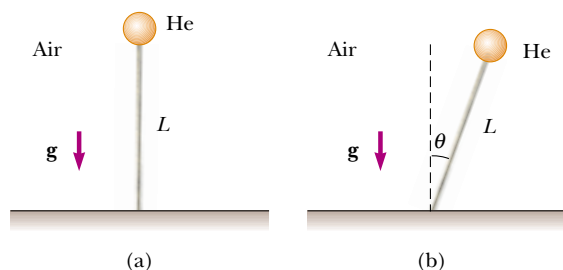


Figure P15.66

- 67.** The water supply of a building is fed through a main 6.00-cm-diameter pipe. A 2.00-cm-diameter faucet tap located 2.00 m above the main pipe is observed to fill a 25.0-L container in 30.0 s. (a) What is the speed at which the water leaves the faucet? (b) What is the gauge pressure in the 6-cm main pipe? (Assume that the faucet is the only “leak” in the building.)
- 68.** The *spirit-in-glass thermometer*, invented in Florence, Italy, around 1654, consists of a tube of liquid (the spirit) containing a number of submerged glass spheres with slightly different masses (Fig. P15.68). At sufficiently low temperatures, all the spheres float, but as the temperature rises, the spheres sink one after the other. The device is a crude but interesting tool for measuring temperature. Suppose that the tube is filled with ethyl alcohol, whose density is  $0.78945 \text{ g/cm}^3$  at  $20.0^\circ\text{C}$  and decreases to  $0.78097 \text{ g/cm}^3$  at  $30.0^\circ\text{C}$ . (a) If one of the spheres has a radius of 1.000 cm and is in equilibrium halfway up the tube at  $20.0^\circ\text{C}$ , determine its mass. (b) When the temperature increases to  $30.0^\circ\text{C}$ , what mass must a second sphere of the same radius have to be in equilibrium at the halfway point? (c) At  $30.0^\circ\text{C}$  the first sphere has fallen to the bottom of the tube. What upward force does the bottom of the tube exert on this sphere?
- 69.** A U-tube open at both ends is partially filled with water (Fig. P15.69a). Oil having a density of  $750 \text{ kg/m}^3$  is then poured into the right arm and forms a column  $L = 5.00 \text{ cm}$  in height (Fig. P15.69b). (a) Determine



Figure P15.68

the difference  $h$  in the heights of the two liquid surfaces. (b) The right arm is shielded from any air motion while air is blown across the top of the left arm until the surfaces of the two liquids are at the same height (Fig. P15.69c). Determine the speed of the air being blown across the left arm. (Take the density of air as  $1.29 \text{ kg/m}^3$ .)

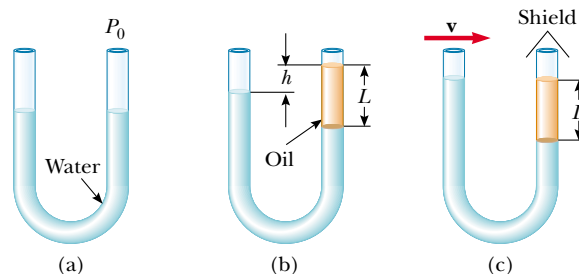


Figure P15.69

## ANSWERS TO QUICK QUIZZES

- 15.1** You would be better off with the basketball player. Although weight is distributed over the larger surface area, equal to about half of the total surface area of the sneaker sole, the pressure ( $F/A$ ) that he applies is relatively small. The woman's lesser weight is distributed over the very small cross-sectional area of the spiked heel. Some museums make women in high-heeled shoes wear slippers or special heel attachments so that they do not damage the wood floors.
- 15.2** If the professor were to try to support his entire weight on a single nail, the pressure exerted on his skin would be his entire weight divided by the very small surface area of the nail point. This extremely great pressure would cause the nail to puncture his skin. However, if the professor distributes his weight over several hundred nails, as shown in the photograph, the pressure exerted on his skin is considerably reduced because the surface area that supports his weight is now the total surface area of all the nail points. (Lying on the bed of nails is much more comfortable than sitting on the bed, and standing on the bed without shoes is definitely not recommended. Do not lie on a bed of nails unless you have been shown how to do so safely.)
- 15.3** Because the horizontal force exerted by the outside fluid on an element of the cylinder is equal and opposite the horizontal force exerted by the fluid on another element diametrically opposite the first, the net force on the cylinder in the horizontal direction is zero.
- 15.4** If you think of the grain stored in the silo as a fluid, then the pressure it exerts on the walls increases with increasing depth. The spacing between bands is smaller at the lower portions so that the greater outward forces acting on the walls can be overcome. The silo on the right shows another way of accomplishing the same thing: double banding at the bottom.
- 15.5** Because water is so much less dense than mercury, the column for a water barometer would have to be  $h = P_0/\rho g = 10.3$  m high, and such a column is inconveniently tall.
- 15.6** The entire hull of a ship is full of air, and the density of air is about one-thousandth the density of water. Hence, the total weight of the ship equals the weight of the volume of water that is displaced by the portion of the ship that is below sea level.
- 15.7** Remains the same. In effect, the ice creates a "hole" in the water, and the weight of the water displaced from the hole is the same as all the weight of the cube. When the cube changes from ice to water, the water just fills the hole.
- 15.8** Goes down because the anchor displaces more water while in the boat than it does in the pond. While it is in the boat, the anchor can be thought of as a floating object that displaces a volume of water weighing as much as it does. When the anchor is thrown overboard, it sinks and displaces a volume of water equal to its own volume. Because the density of the anchor is greater than that of water, the volume of water that weighs the same as the anchor is greater than the volume of the anchor.
- 15.9** As the water falls, its speed increases. Because the flow rate  $A v$  must remain constant at all cross sections (see Eq. 15.7), the stream must become narrower as the speed increases.
- 15.10** The rapidly moving air characteristic of a tornado is at a pressure below atmospheric pressure. The stationary air inside the building remains at atmospheric pressure. The pressure difference results in an outward force on the roof and walls, and this force can be great enough to lift the roof off the building. Opening the windows helps to equalize the inside and outside pressures.

