Sample Paper 11

Class- X Exam - 2022-23

Mathematics - Standard

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A

20 marks

(Section A consists of 20 questions of 1 mark each.)

1. A prime number greater than 91 but less than 100 is :

(a) 94 (b) 97

(c) 96 (d) 95 1

2. Find a zero of the polynomial $x^3 - 8$.

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) 3 (d) 2 1

3. Write a quadratic polynomial whose sum of zeroes is $-\frac{1}{4}$ and product of zeroes is $\frac{1}{4}$.

(a) $4x^2 + x + 1$ (b) $x^2 + 4x - 1$

(c) $2x^2 + 3x - 1$ (d) $x^2 - 2x + 1$ 1

4. Determine the roots of the equation $\sqrt{3}\,x^2 - 2x - \sqrt{3} = 0$

(a) $\sqrt{3}, -\frac{1}{\sqrt{3}}$ (b) $1, -\frac{1}{2}$

(c) $\sqrt{3}, -1$ (d) $\frac{1}{\sqrt{3}}, -1$ 1

5. Find the 15^{th} term of the AP: $x - 7$, $x - 2$, $x + 3$...

(a) $x + 67$ (b) $x + 4$

(c) $x + 36$ (d) $x + 63$ 1

6. Find The discriminant of the equation $(x + 1)^3 = 4 - x + x^3$

(a) 52 (b) 53

(c) 64 (d) 72 1

7. The next term of the A.P.: $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}$,... is:

(a) 0 (b) $3 + 4\sqrt{2}$

(c) $3 - 4\sqrt{2}$ (d) 1 1

8. The value of x and y : $x + 2y = 9$, $2x - y = 8$ is:

(a) 0, 0 (b) 3, 5

(c) 5, 2 (d) 0, 1 1

9. The condition for the points $(a, 0)$, $(0, b)$ and $(1, 1)$ to be collinear, is:

(a) $\frac{1}{a} - \frac{1}{b} = 1$ (b) $\frac{1}{ab} = 1$

(c) 0 (d) $\frac{1}{a} + \frac{1}{b} = 1$ 1

10. The length of the altitude of an isosceles triangle of sides 6 cm, 6 cm and 4 cm is:

(a) $4\sqrt{2}$ cm (b) $5\sqrt{2}$ cm

(c) $6\sqrt{2}$ cm (d) $2\sqrt{2}$ cm 1

11. Find the coordinates of a point on y-axis which is equidistant from the points (6, 5) and (–4, 3).

(a) (4, 2) (b) (3, 2)

(c) (0, 9) (d) (9, 2) 1

12. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

(a) 7 cm (b) 4 cm

(c) 9 cm (d) 10 cm 1

13. If 3cos A = 1, then find the value of cosec A.

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{5}{\sqrt{2}}$

(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{3}{2\sqrt{2}}$ 1

14. The perimeter of a quadrant of a circle of radius '*r*' is:

(a) $\frac{r^2}{2}$ (b) $\pi + 4$

(c) $\frac{r}{2}(\pi+4)$ (d) $\frac{r}{2}$ 1

15. The total surface area of a quadrant of a sphere of radius '*r*' is:

(a) πr^2 (b) $2\pi r^2$

(c) $2\pi r$ (d) $\frac{\pi r^2}{2}$ 1

16. The probability of drawing at random a green coloured ball from a bag containing 6 red and 5 black balls is:

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{3}{5}$ (d) 1 1

17. The median of the first 50 even natural numbers is:

(a) 48 (b) 49

(c) 50 (d) 51 1

18. The value of $\frac{1+\tan^2\theta}{1+\cot^2\theta}$ is :

(a) $\tan^2\theta$ (b) $\cos^2\theta$

(c) $1 + \sin^2\theta$ (d) 0 1

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).
Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. **Statement A (Assertion):** There are infinite number of lines which passes through (1, 13).

Statement R (Reason): A linear equation in two variables has infinitely many solutions. 1

20. **Statement A (Assertion):** Two similar triangles are always congruent.

Statement R (Reason): Two similar triangles are said to be congruent if their areas are equal. 1

SECTION - B

10 marks

(Section B consists of 5 questions of 2 marks each.)

21. Write the prime factorisation of 8190.

OR

Find the HCF of 2205, 5145 and 4410. 2

22. In an A.P., a = 5, d = 2 and n = 50, find an. 2

23. If Q (0, 1) is equidistant from P(5, –3) and R (x, 6), find the values of 'x'. Also, find the distances of QR and PR.

OR

Find the ratio in which P(4, *p*) divides the line segment joining the points A(2, 3) and B(6, 3). Hence find the value of *p*. 2

24. A tree casts a shadow of $4\sqrt{3}$ m. on ground. If the sun's elevation is 60°. then find the height of the tree. 2

25. Find the mode of the following frequency distribution:

Class	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	3	8	9	10	3	2

2

SECTION - C

18 marks

(Section C consists of 6 questions of 3 marks each.)

26. Prove that $2\sqrt{3} - 4$ is an irrational number, using the fact that $\sqrt{3}$ is an irrational number. 3

27. The sum of two numbers, as well as, the difference of their squares is 9. Find the numbers.

OR

Find the values of k for which the following equations have an infinite number of solutions:

$2x + 3y = 7$; $(k - 1)x + (k + 2)y = 3k$ 3

28. Show that Δ ABC with vertices A(–2, 0), B(2, 0) and C(0, 2) is similar to Δ PQR with vertices P(–4, 0), Q(4, 0) and R(0, 4) 3

29. What is the ratio between the areas of the circle and the square when a square is inscribed in a circle? 3

30. If sum and product of quadratic polynomial are 2 and –8 respectively, then find a quadratic polynomial and zeroes of the polynomial so obtained.

OR

The area of a sector of a circle of radius 36 cm is 54 π sq cm. Find the length of the corresponding arc of the sector. 3

31. Find the mean of the following frequency distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60	Below 70	Below 80	Below 90	Below 100
Number of students	12	22	35	50	70	86	97	104	109	115

3

SECTION - D

20 marks

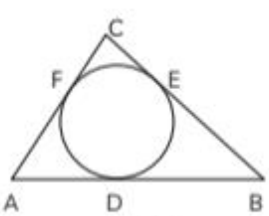
(Section D consists of 4 questions of 5 marks each.)

32. If tan q + sin q = m and tan q – sin q = n, show that: $m2 - n2 = 4\sqrt{mn}$

OR

The angle of elevation of the top of a building from the foot of a tower is 30°, and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building. 5

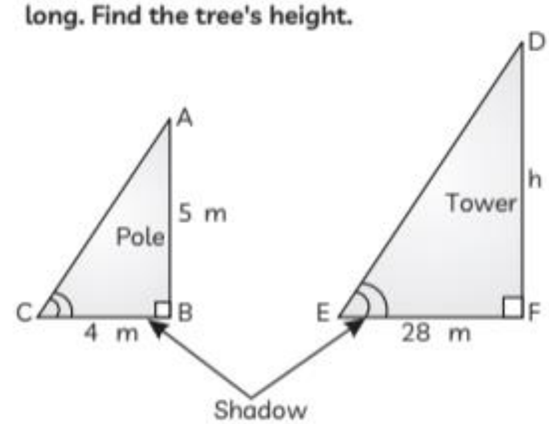
33. A circle is inscribed in a DABC having sides 8 cm, 10 cm and 12 cm as shown in the figure. Find AD, BE and CF.



OR

The 6th term of an AP is five times the 1st term and the 11th term exceeds twice the 5th term by 3. Find the 8th term of the AP. 5

34. On the ground, a tree with a length of 6 m creates a shadow that is 4 m long, while another tree creates a shadow that is 28 m long. Find the tree's height.



5

35. Two dice have the following numbers: 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3. The total of the numbers on them is calculated once they are thrown. Calculate the probability of getting each sum from 2 to 6 individually. 5

SECTION - E

12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *On the roadway, Points A and B, which stand in for Chandigarh and Kurukshetra, respectively, are located nearly 90 kilometres apart. At the same time, a car departs from Kurukshetra and one from Chandigarh. These cars will collide in 9 hours if they are travelling in the same direction, and in 9/7 hours if they are travelling in the other direction. Let X and Y be two cars that are travelling at x and y kilometres per hour from places A and B, respectively.*

On the basis of the above information, answer the following questions:

(A) When both cars move in the same direction, then find the situation can be represented algebraically.

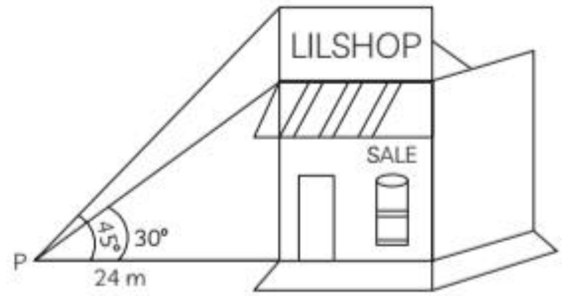
OR

When both cars move in the opposite direction, then find the situation can be represented algebraically. 2

(B) Find the speed of car *x*. 1

(C) Find the speed of car *y*. 1

37. *Eshan purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.*



From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45°. The point P is at a distance of 24 m from the base of the building.

On the basis of the above information, answer the following questions:

(A) Find the height of the building (without the sign board).

OR

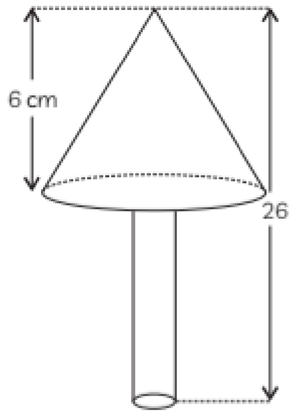
The height of the building (with the sign board) 2

(B) Find the height of the sign board. 1

(C) Find the distance of the point P from the top of the sign board. 1

38. *In a toys manufacturing company, wooden parts are assembled and painted to prepare a toy. One specific toy is in the shape of a cone mounted on a cylinder.*

For the wood processing activity center, the wood is taken out of storage to be sawed, after which it undergoes rough polishing, then is cut, drilled and has holes punched in it. It is then fine polished using sandpaper.



For the retail packaging and delivery activity center, the polished wood sub-parts are assembled together, then decorated using paint.

The total height of the toy is 26 cm and the height of its conical part is 6 cm. The diameters of the base of the conical part is 5 cm and that of the cylindrical part is 4 cm.

On the basis of the above information, answer the following questions:

(A) If its cylindrical part is to be painted yellow, find the surface area need to be painted. 1

(B) If its conical part is to be painted green, find the surface area need to be painted.

OR

Find the volume of the wood used in making this toy. 2

(C) If the cost of painting the toy is 3 paise per sq cm, then find the cost of painting the toy. (Use $\pi = 3.14$) 1

SOLUTION

SECTION - A

1. *(b) 97*

Explanation: prime number greater than 91 but less than 100 is 97.

2. *(d) 2*

Explanation: Here, $x^3 - 8 = 0$ gives $x^3 = 8$ *i.e.*

$x = \sqrt[3]{8} = 2$

3. *(a) $4x^2 + x + 1$*

Explanation: Sum of zeroes = $\frac{-1}{4}$

Product of zeroes = $\frac{1}{4}$

$\therefore$ Quadratic Polynomial is

x^2 – (sum of zeroes) x + product of zeroes = 0

$$\therefore \quad x^2 - \left(\frac{-1}{4}\right)x + \frac{1}{4} = 0$$

$$\Rightarrow \quad 4x^2 + x + 1 = 0$$

4. *(a) $\sqrt{3}, -\frac{1}{\sqrt{3}}$*

Explanation: The given equation has roots, whose sum is $\frac{2}{\sqrt{3}}$ and product is –1. This is possible only with $\sqrt{3}$ and $-\frac{1}{\sqrt{3}}$.

⚠ Caution

➥ *Practicing of these types of problems, helps to increase the accuracy in such problems.*

5. *(d) $x + 63$*

Explanation: Here, $a = x - 7$, $d = 5$

So, 15^{th} term = $a + 14d = (x - 7) + 14(5)$

$= x + 63$

⚠ Caution

➥ *Remember that first term of an A.P. is only a not 'a + d'.*

6. *(a) 52*

Explanation: Given, equation is :

$$(x + 1)^3 = 4 - x + x^3$$

$$\Rightarrow \quad x^3 + 1 + 3x(x + 1) = 4 - x + x^3$$

$$\Rightarrow \quad 1 + 3x^2 + 3x = 4 - x$$

$$\Rightarrow \quad 3x^2 + 4x - 3 = 0$$

On comparing this equation with $ax^2 + bx + c = 0$, we get $a = 3$, $b = 4$, $c = -3$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= 4^2 - 4 \times 3 \times (-3)$

$= 16 + 36$

$= 52$

7. (b) $3+4\sqrt{2}$

Explanation: Here,

$$d = \sqrt{2}$$

So, next term is $(3+3\sqrt{2})+\sqrt{2}$ *i.e.* $3 + 4\sqrt{2}$.

8. (c) 5, 2

Explanation: Here, $x + 2y = 9$ ...(i)

$2x - y = 8$ ...(ii)

Multiply equation (ii) by 2 and add both the equations.

$$x + 2y = 9$$
$$4x - 2y = 16$$
$$5x = 25$$
$$x = \frac{25}{5} = 5$$

Then, $y = \frac{9-x}{2} = \frac{9-5}{2} = 2$

9. (d) $\frac{1}{a}+\frac{1}{b}=1$

Explanation: As the given points are collinear,

$a(b - 1) + 0\,(1 - 0) + 1\,(0 - b) = 0$

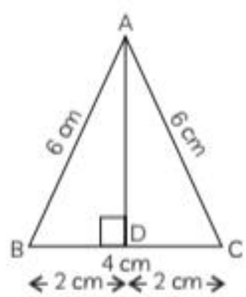
$\Rightarrow \quad ab - a - b = 0$

$\Rightarrow \quad ab = a + b$

$\Rightarrow \quad \frac{1}{a}+\frac{1}{b} = 1$

10. (a) $4\sqrt{2}$ cm

Explanation: Since, AD is an altitude in isosceles ΔABC.



So, it will bisect the base.

Therefore, in ΔADB, by Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$
$\Rightarrow \quad 6^2 = AD^2 + 2^2$

$\Rightarrow \quad AD^2 = 36 - 4 = 32$

$\Rightarrow \quad AD = \sqrt{32} = 4\sqrt{2}$ cm

11. (c) 0, 9

Explanation: Let, the coordinate on y axis be P(0, y)

The given points are A(6, 5) and B(–4, 3)

$\therefore \quad PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (0 - 6)^2 + (y - 5)^2 = (0 + 4)^2 + (y - 3)^2$

$\Rightarrow \quad 36 + y^2 + 25 - 10y = 16 + y^2 + 9 - 6y$

$\Rightarrow \quad -4y = -36$

$\Rightarrow \quad y = 9$

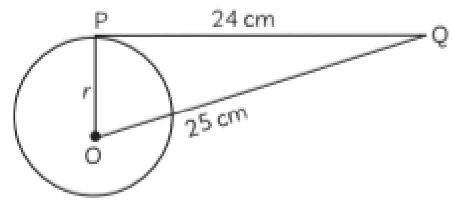
Then, coordinate on y-axis is (0, 9).

Caution

The points given in a ordered pair represent the first point as x-coordinate and the second point as y-coordinate.

12. (a) 7 cm

Explanation: Here, OP is a tangent to a circle with radius 'r' of length 24 cm.



And $\quad OQ = 25$ cm

Now, $\angle QPO = 90°$, as tangent at any point to a circle is $\perp r$ to the radius.

$\therefore$ In ΔOPQ by pythagoras theorem

$$OQ^2 = OP^2 + PQ^2$$
$$25^2 = r^2 + 24^2$$
$\Rightarrow \quad r^2 = 625 - 576$

$= 49$

$\Rightarrow \quad r = 7$ cm

Caution

Remember that the point where tangent touches the circle is perpendicular to the radius.

13. (d) $\frac{3}{2\sqrt{2}}$

Explanation: Here, $3 \cos A = 1$

$$\cos A = \frac{1}{3}$$

Then, $\cos^2 A = \frac{1}{9}$

Then, $\sin^2 A = 1 - \cos^2 A$

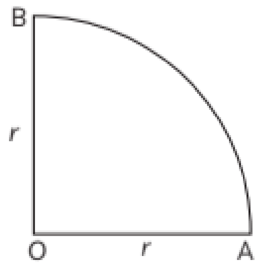
$= 1 - \frac{1}{9} = \frac{8}{9}$

$$\sin A = \frac{2\sqrt{2}}{3}$$

Then, $\text{cosec } A = \frac{1}{\sin A} = \frac{3}{2\sqrt{2}}$

14. *(c)* $\frac{r}{2}(\pi+4)$

Explanation: Perimeter of quadrant,



Perimeter = OB + OA + $\widehat{AB}$

$$= r + r + \frac{2\pi r}{4}$$

$$= 2r+\frac{\pi}{2}r$$

$$= \frac{r}{2}(\pi + 4)$$

15. *(b)* $2\pi r^2$

Explanation: Total surface area of quadrant

$$= \frac{4\pi r^2}{4} + \frac{\pi r^2}{2} + \frac{\pi r^2}{2}$$

$$= \pi r^2 + \pi r^2$$

$$= 2\pi r^2$$

16. *(a)* *0*

Explanation: From the 11 (6 + 5) balls in the bag, no ball is of green colour.

P(a green ball) = $\frac{0}{11}$ i.e. 0

17. *(d)* *51*

Explanation: First 50 even natural number are:

2, 4, 6, ..., 98, 100.

As median is the middle-most value,

median = $\frac{50+52}{2}$ = 51

18. *(a)* $tan^2\theta$

Explanation: $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta}$

$$= \frac{1 + \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\cos^2 \theta}{\sin^2 \theta}} = \frac{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2\theta$$

Caution

➥ *Apply deduction of trigonometric identities, wherever necessary.*

19. *(b)* *Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)*

Explanation: Through a point infinite lines can be drawn. Through (1,13), infinite number of lines can be drawn.

Also a line has infinite points on it, hence a linear equation representing a line has infinite solutions.

20. *(d)* *Assertion (A) is false but reason(R) is true.*

Explanation: Two similar triangles are not congruent generally.

If the area of two similar triangles are equal then the triangles are congruent.

SECTION - B

21. The prime factorisation of 8190 is:

$$8190 = 2 \times 3 \times 3 \times 5 \times 7 \times 13.$$

2	8190
3	4095
3	1365
5	455
7	91
	13

Caution

➥ *While calculating prime factors, start with the lowest prime number.*

OR

HCF of 2205, 5145 and 4410

3	2205
3	735
5	245
7	49
7	7
	1

3	5145
5	1715
7	343
7	49
7	7
	1

2	4410
5	2205
3	441
7	147
7	21
3	3
	1

$$2205 = 3 \times 3 \times 5 \times 7 \times 7$$

$$5145 = 3 \times 5 \times 7 \times 7 \times 7$$

$$4410 = 2 \times 3 \times 3 \times 5 \times 7 \times 7$$

$$\therefore \quad HCF = 3 \times 5 \times 7 \times 7$$

$$= 735$$

22. In given A.P.

Where a is 1st term, d is common difference and n is number of terms.

$$\begin{aligned} a_n &= a + (n-1)d \\ &= 5 + (50-1) \times 2 \\ &= 5 + 49 \times 2 \\ &= 5 + 98 \\ &= 103 \end{aligned}$$

23. Since, Q(0, 1) is equidistant from P(5, −3) and R(x, 6),

$$PQ = QR$$

$\Rightarrow$ $$PQ^2 = QR^2$$

i.e. $$(5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$$

i.e. $$25 + 16 = x^2 + 25$$

i.e. $$x^2 = 16$$

or $$x = \pm 4$$

Thus, R(4, 6) or R (−4, 6)

For R (4, 6),

$$QR = \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$$

and $$PR = \sqrt{(4-5)^2 + (6+3)^2} = \sqrt{1+81} = \sqrt{82}$$

For R(−4, 6),

$$QR = \sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$$

$$PR = \sqrt{(-4-5)^2 + (6+3)^2} = \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$$

OR

Given, point P(4, p) and line segment A(2, 3) and B(6, 3)

Let, the point P divide given line segment AB in the ratio of k : 1.

Then, $$P(4, p) = \left[\left(\frac{k \times 6 + 2}{k+1}\right), \left(\frac{k \times 3 + 3}{k+1}\right)\right]$$

∴ On comparing, we get

$$\frac{6k+2}{k+1} = 4$$

$\Rightarrow$ $$6k + 2 = 4k + 4$$

$\Rightarrow$ $$k = 1$$

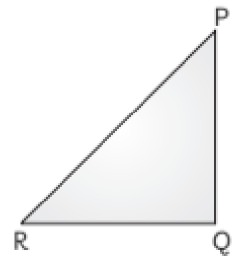
Then point P divides line AB in the ratio of 1 :

Now, $$p = \frac{1 \times 3 + 3}{1+1} = \frac{6}{2}$$

$\Rightarrow$ $$p = 3$$

Hence, value of p is 3.

24.



Let, PQ be the tree and RQ be its shadow.

∴ In ΔPQR

$$\tan 60° = \frac{PQ}{QR}$$

$$\sqrt{3} = \frac{PQ}{4\sqrt{3}}$$

$$PQ = 12 \text{ m.}$$

Hence, the height of the tree is 12 m.

25. In the given frequency distribution, modal class is 30-35, with maximum frequency 10.

Here, lower limit of modal class, $l = 30$

frequency of class preeceding modal class, $f_0 = 9$

frequency of modal class, $f_1 = 10$

frequency of class succeeding modal class, $f_2 = 3$

$$\begin{aligned} M_0 &= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \\ &= 30 + \frac{10-9}{20-9-3} \times 5 \\ &= 30 + \frac{5}{8} = 30 + 0.625 \\ &= 30.625 \end{aligned}$$

SECTION - C

26. Let us assume on the contrary, that $2\sqrt{3} - 4$ be a rational number.

Then,

$2\sqrt{3} - 4 = \frac{p}{q}$, where p and q are co-primes and $q \neq 0$.

$\Rightarrow$ $$\sqrt{3} = \frac{1}{2}\left(\frac{p}{q} + 4\right)$$

Since p and q are integers, $\frac{1}{2}\left(\frac{p}{q} + 4\right)$ is rational and so $\sqrt{3}$ is rational. But, this contradicts the fact that $\sqrt{3}$ is irrational.

Hence, $2\sqrt{3} - 4$ is an irrational number.

27. Let the two numbers be x and y. $(x > y)$.

Then, $x + y = 9$ and $x^2 - y^2 = 9$

$\Rightarrow$ $x + y = 9$ and $(x - y)(x + y) = 9$

$\Rightarrow$ $x + y = 9$ and $x - y = 1$

Solving the two equations, we get $x = 5$ and $y = 4$

Thus, the two numbers are 5 and 4.

OR

Given, equations are

$$2x + 3y = 7$$

$$(k - 1)x + (k + 2)y = 3k$$

Here, $a_1 = 2, b_1 = 3, c_1 = 7$

$a_2 = k - 1, b_2 = k + 2, c_2 = 3k$

Since, given system of equations has infinite solutions.

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$

On comparing the two ratios

$\Rightarrow$ $2(k + 2) = 3(k - 1)$

$\Rightarrow$ $2k + 4 = 3k - 3$

$\Rightarrow$ $-k = -7$

$\Rightarrow$ $k = 7$

and, $\frac{3}{k+2} = \frac{7}{3k}$

$\Rightarrow$ $9k = 7k + 14$

$\Rightarrow$ $2k = 14$

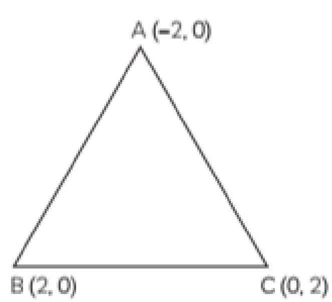
$\Rightarrow$ $k = 7$

Hence, the value of k is 7.

28. Here, vertices of ΔABC are A(–2, 0), B(2, 0) and C(0, 2)

Another Δ PQR, with vertices P(–4, 0), Q(4, 0) and R(0, 4).

In ΔABC,



Then, length of AB $= \sqrt{(2+2)^2 + (0-0)^2}$

$= 4$ units

length of BC $= \sqrt{(0-2)^2 + (2-0)^2}$

$= \sqrt{4+4} = \sqrt{8}$

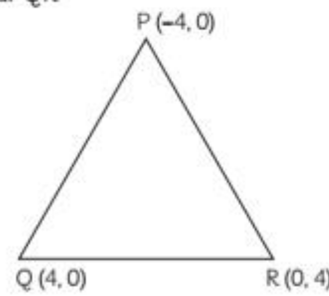
$= 2\sqrt{2}$ units

length of AC $= \sqrt{(-2-0)^2 + (0-2)^2}$

$= \sqrt{4+4}$

$= 2\sqrt{2}$

Now, in ΔPQR



Length of PQ $= \sqrt{(4+4)^2 + (0-0)^2}$

$= \sqrt{8^2} = 8$ units

Length of QR $= \sqrt{(4-0)^2 + (0-4)^2}$

$= \sqrt{4^2 + 4^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$ units

Length of PR $= \sqrt{(-4)^2 + (4)^2} = 4\sqrt{2}$ units

$\therefore$ $$\frac{AB}{PQ} = \frac{4}{8} = \frac{1}{2},$$

$$\frac{BC}{QR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

and $$\frac{AC}{PR} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$$

Now, $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{2}$$

Hence, $\Delta ABC \sim \Delta PQR$ (by SSS – Similarity)

29. Let,

radius of the circle be r units.

Then, diagonal of the square $= 2r$

$\Rightarrow$ Side of the square $= \frac{2r}{\sqrt{2}} = \sqrt{2}r = \frac{\pi}{2}$

$$\therefore \frac{\text{Area of the circle}}{\text{Area of the square}} = \frac{\pi r^2}{(\sqrt{2}r)^2} = \frac{\pi r^2}{2r^2}$$

$= \pi : 2$

30. Let α and β be the zeroes of the polynomial.

Given: Sum of zeroes, $\alpha + \beta = 2$

Product of zeroes, $\alpha \times \beta = -8$

Equation of polyomial:

x^2 – (sum of zeroes)x + product of zeroes

$\therefore \quad x^2 - (2)x + (-8)$

$\Rightarrow \quad x^2 - 2x - 8$

Zeores of polynomial

Let $\quad P(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

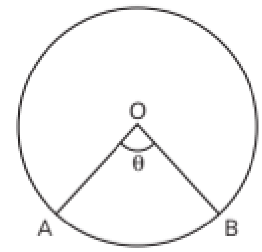
$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

Hence, the zeroes of the polynomial are 4 and –2.

OR

Let θ be the angle of the sector. Then,



$$\frac{\theta}{360}\pi(36)^2 = 54\pi$$

$$\Rightarrow \qquad \theta = \frac{54 \times 360}{36 \times 36} = 15°$$

$\therefore$ Length of the corresponding arc

$$= \left[\frac{15}{360} \times 2\pi(36)\right] \text{cm}$$

$$= \frac{66}{7} \text{ cm or } 9\frac{3}{7} \text{ cm}$$

31. Re-writing the distribution in the form of the grouped distribution with each class interval as 10 and taking assumed mean to be 55, we get the following table:

Class	Mid-value $\left(x_i = \frac{l+u}{2}\right)$	$d_i = x_i - A$ (A = 55)	$u_i = \frac{d_i}{h}$	Number of students (f_i)	$f_i u_i$
0–10	5	–50	–5	12	–60
10–20	15	–40	–4	10	–40
20–30	25	–30	–3	13	–39
30–40	35	–20	–2	15	–30
40–50	45	–10	–1	20	–20
50–60	55 = A	0	0	16	0
60–70	65	10	1	11	11
70–80	75	20	2	7	14
80–90	85	30	3	5	15
90–100	95	40	4	6	24
				$\Sigma f_i = 115$	$\Sigma f_i u_i = -125$

$$\text{Mean} = A + \frac{f_i u_i}{f_i} \times h = \qquad 55 + \frac{-125}{115} \times 10, \quad i.e. \quad 44.13 \text{ (Approx.)}$$

SECTION - D

32. We have

$$\text{LHS} = m^2 - n^2$$

$= (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= (\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta)$

$\quad - (\tan^2\theta + \sin^2\theta - 2\tan\theta\sin\theta)$

$= 4\tan\theta\sin\theta$ ...(i)

and $\qquad \text{RHS} = 4\sqrt{mn}$

$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$

$= 4\sin\theta\sqrt{\frac{1 - \cos^2\theta}{\cos^2\theta}} = 4\sin\theta\,\frac{\sin\theta}{\cos\theta}$

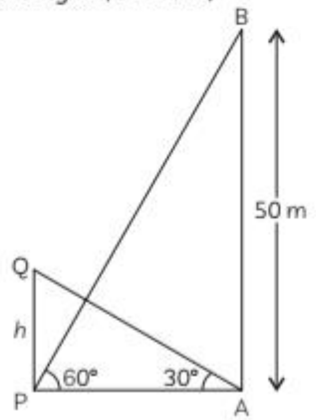
$= 4\tan\theta\sin\theta$ ...(ii)

From (i) and (ii), we have:

$$m^2 - n^2 = 4\sqrt{mn}$$

OR

From the figure, in ΔPAB,



$$\frac{AB}{PA} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{50}{PA} = \sqrt{3}$$

$$\Rightarrow \quad PA = \frac{50}{\sqrt{3}}, \text{ or } \frac{50}{3}\sqrt{3}\, m \qquad \text{....(i)}$$

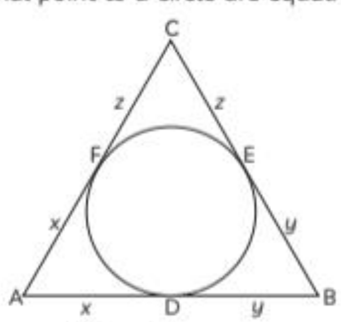
Also, in ΔAPQ,

$$\frac{PQ}{PA} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{\frac{50}{3}\sqrt{3}} = \frac{1}{\sqrt{3}} \qquad \text{[from (i)]}$$

$$\Rightarrow \quad h = \frac{50\sqrt{3}}{3\sqrt{3}} = \frac{50}{3} m \text{ i.e. } 16.67 \text{ m}$$

33. We know that the tangents drawn from an external point to a circle are equal. Therefore,



$$AD = AF = x, \text{ say}$$

$$BD = BE = y, \text{ say}$$

and $CE = CF = z$, say

Now,

AB = 12 cm, BC = 8 cm and CA = 10 cm

$\Rightarrow \quad x + y = 12, \; y + z = 8$ and $z + x = 10$

$\Rightarrow 2(x + y + z) = 30$, or $x + y + z = 15$

Now, $x + y = 12$ and $x + y + z = 15$

gives $z = 3$

Again, $y + z = 8$ and $x + y + z = 15$

gives $x = 7$

Also, $z + x = 10$ and $x + y + z = 15$

gives $y = 5$

Hence, AD = x = 7 cm, BE = y = 5 cm and CF = z = 3 cm.

OR

Let a and d be the first term and the common difference of the AP respectively. Then,

$$a_6 = a + 5d = 5a \qquad \text{...(i)}$$

and $\quad a_{11} = 2a_5 + 3$

$$\Rightarrow \quad a + 10d = 2(a + 4d) + 3 \qquad \text{...(ii)}$$

Simplifying (i) and (ii), we get

$$4a = 5d \quad \text{and} \quad a - 2d + 3 = 0$$

Solving these equations, we get $d = 4$ and $a = 5$

Thus, $\quad a_8 = a + 7d = 5 + 7(4) = 33$

34. Given,

Length of the vertical pole = 6 m

Shadow of the pole = 4 m

Let the height of the tower be h m.

Length of the shadow of the tower = 28 m

In ΔABC and ΔDFE,

$\angle C = \angle E$ (angle of elevation)

$\angle B = \angle F = 90°$

By AA similarity criterion,

$\Delta ABC \sim \Delta DFE$

We know that the corresponding sides of two similar triangles are proportional.

$$\frac{AB}{DF} = \frac{BC}{EF}$$

$$\frac{6}{h} = \frac{4}{28}$$

$$h = \frac{6 \times 28}{4}$$

$$h = 6 \times 7$$

$$h = 42$$

Hence, the height of the tower = 42 m.

35. Number of total outcome = $n(S) = 36$

(i) Let E_1 be the event 'getting sum 2'

Favourable outcomes for the event $E_1 = \{(1,1),(1,1)\}$

$$n(E_1) = 2$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

(ii) Let E2 be the event 'getting sum 3'

Favourable outcomes for the event E_2

$$= \{(1,2),(1,2),(2,1),(2,1)\}$$

$$n(E_2) = 4$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(iii) Let E_3 be the event 'getting sum 4'

Favourable outcomes for the event E_3

$$= \{(2,2)(2,2),(3,1),(3,1),(1,3),(1,3)\}$$

$n(E_3) = 6$

$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(iv) Let E4 be the event 'getting sum 5'

Favourable outcomes for the event E_4

$= \{(2,3),(2,3),(4,1),(4,1),(3,2),(3,2)\}$

$n(E_4) = 6$

$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(v) Let E_5 be the event 'getting sum 6'

Favourable outcomes for the event E_5

$= \{(3,3),(3,3),(4,2),(4,2),(5,1),(5,1)\}$

$n(E_5) = 6$

$P(E_5) = \frac{n(E_5)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

SECTION - E

36. (A) Suppose two cars meet at point Q.

Then, Distance travelled by car X = AQ,

Distance travelled by car Y = BQ.

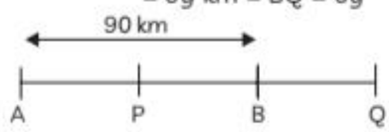
It is given that two cars meet in 9 hours.

∷ Distance travelled by car X in 9 hours

$= 9x$ km $= AQ = 9x$

Distance travelled by car Y in 9 hours

$= 9y$ km $= BQ = 9y$



Clearly, $AQ - BQ = AB$

$= 9x - 9y = 90$

$= x - y = 10$

OR

Suppose two cars meet at point P.

Then Distance travelled by car X = AP

and Distance travelled by car Y = BP.

In this case, two cars meet in 9/7 hours.

∴ Distance travelled by car X in $\frac{9}{7}$ hours

$= \frac{9}{7}x$ km

$\Rightarrow \quad AP = \frac{9}{7}x$

Distance travelled by car Y in $\frac{9}{7}$ hours

$= \frac{9}{7}y$ km

Clearly, $AP + BP = AB$

$\Rightarrow \quad \frac{9}{7}x + \frac{9}{7}y = 90$

$\Rightarrow \quad \frac{9}{7}(x+y) = 90$

$\Rightarrow \quad x + y = 70$

(B) We have $x - y = 10$

$\Rightarrow \quad x + y = 70$

Adding equations (i) and (ii), we get

$2x = 80$

$\Rightarrow \quad x = 40$

Hence, speed of car X is 40 km/hr.

(C) We have $x - y = 10$

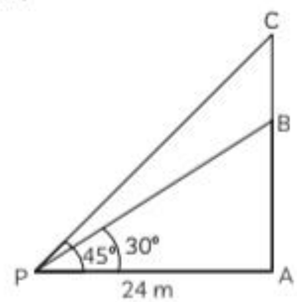
$\Rightarrow \quad 40 - y = 10$

$\Rightarrow \quad y = 30$

Hence, speed of car y is 30 km/hr

37. (A) Without the sign board, the height of the shop is AB.

In ΔPAB,



$\tan 30° = \frac{AB}{PA}$

$\frac{1}{\sqrt{3}} = \frac{AB}{24}$

$AB = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3}$ m

$= 13.85 \simeq 14$ m

OR

Considering, the diagram in the above question, AC as the new height of the shop including the sign-baord.

∴ In ΔAPC,

$\tan 45° = \frac{AC}{AP}$

$1 = \frac{AC}{24}$

$\Rightarrow \quad AC = 24$ m

(B) From Q (i) and Q (ii).

Length of sign board,

$$BC = AC - AB$$
$$= 24 - 14$$
$$= 10 \text{ m}$$

(C) In ΔAPC,

$$\cos 45° = \frac{AP}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{24}{PC}$$

$$\Rightarrow \quad PC = 24\sqrt{2} \text{ m}$$

38. (A) C.S.A. of cylinder $= 2\pi rH + \pi r^2$

$= \pi r(2H + r)$

$= 2\pi (2 \times 20 + 2) \qquad [\because H = 26 - 6 = 20]$

$= 84\pi$

(B) C.S.A. of cone $= \pi rl + \pi(R^2 - r^2)$

$= \pi\left[r\sqrt{r^2 + h^2} + (R^2 - r^2)\right]$

$= \pi\left[2.5\sqrt{2.5^2 + 6^2} + (2.5^2 - 2^2)\right]$

$= \pi [2.5 \times 6.5 + 0.5 \times 4.5]$

$= \pi [16.25 + 2.25]$

$= 18.5\pi$ sq units

OR

Volume of toy

= Volume of cone + Volume of cylinder

$= \frac{1}{3}\pi r^2 h + \pi R^2 H$

$= \pi\left[\frac{1}{3} \times 2.5 \times 2.5 \times 6 + 2 \times 2 \times 20\right]$

$= \pi [12.5 + 80]$

$= 92.5\pi \text{ cm}^3$

(C) Surface area = S.A. of cone + S. A. of cylinder

$= 84\pi + 18.5\pi$

$= 102.5\pi$

$\therefore$ Cost of painting

$= 0.03 \times 102.5\pi$

$=$ ₹ 9.65

⚠ Caution

➥ *In calculating the surface area, sometimes the surface areas of some parts of the solids are disappeared. So, think wisely before calculating surface areas of the combination of solids.*