

Parabola

Exercise 22

Q. 1 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

$$y^2 = 12x$$

Answer :

Given equation : $y^2 = 12x$

Comparing given equation with parabola having equation,

$$y^2 = 4ax$$

$$4a = 12$$

- $a = 3$

Focus :

$$F(a,0) = F(3,0)$$

Vertex :

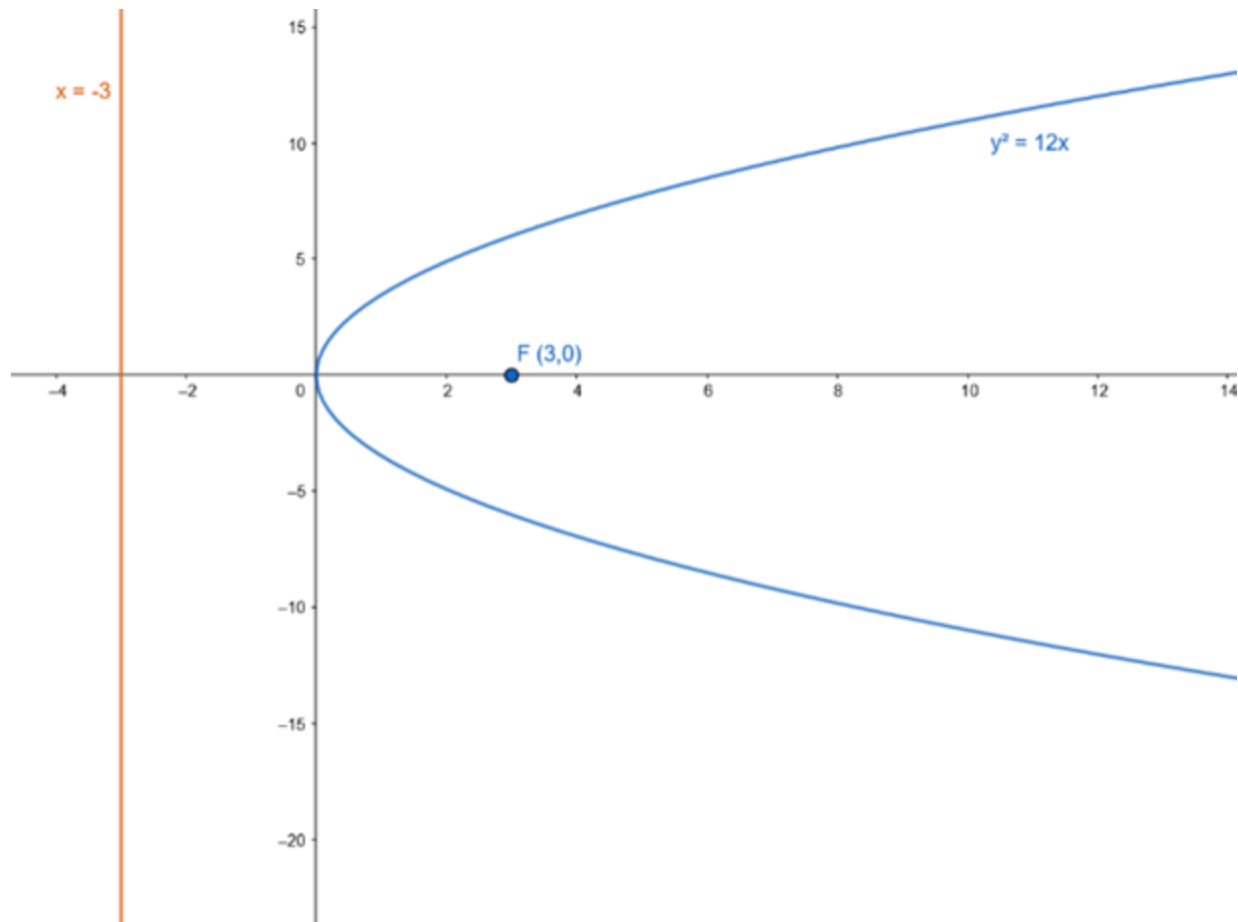
$$A(0,0) = A(0,0)$$

Equation of the directrix : $x+a=0$

- $x+3=0$

- $x = -3$

Length of latusrectum : $4a = 4.(3) = 12$



Q. 1 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

$$y^2 = 10x$$

Answer : Given equation : $y^2 = 10x$

Comparing given equation with parabola having equation,

$$y^2 = 4ax$$

$$4a = 10$$

$$\bullet a = 2.5$$

Focus : $F(a,0) = F(2.5,0)$

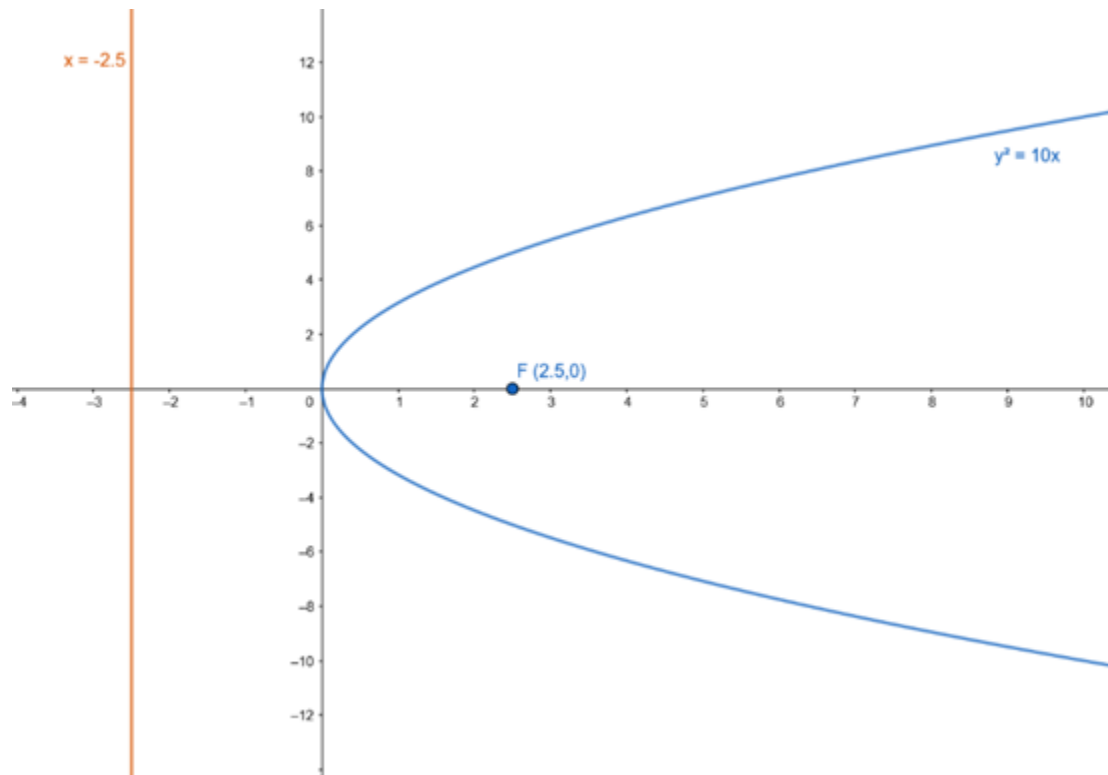
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $x+a=0$

- $x+2.5=0$

- $x = -2.5$

Length of latusrectum : $4a = 4.(2.5) = 10$



Q. 1 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

$$3y^2 = 8x$$

Answer : Given equation :

$$3y^2 = 8x$$

- $y^2 = \frac{8}{3}x$

Comparing the given equation with parabola having equation,

$$y^2 = 4ax$$

$$4a = \frac{8}{3}$$

- $a = \frac{2}{3}$

Focus : $F(a,0) = F\left(\frac{2}{3}, 0\right)$

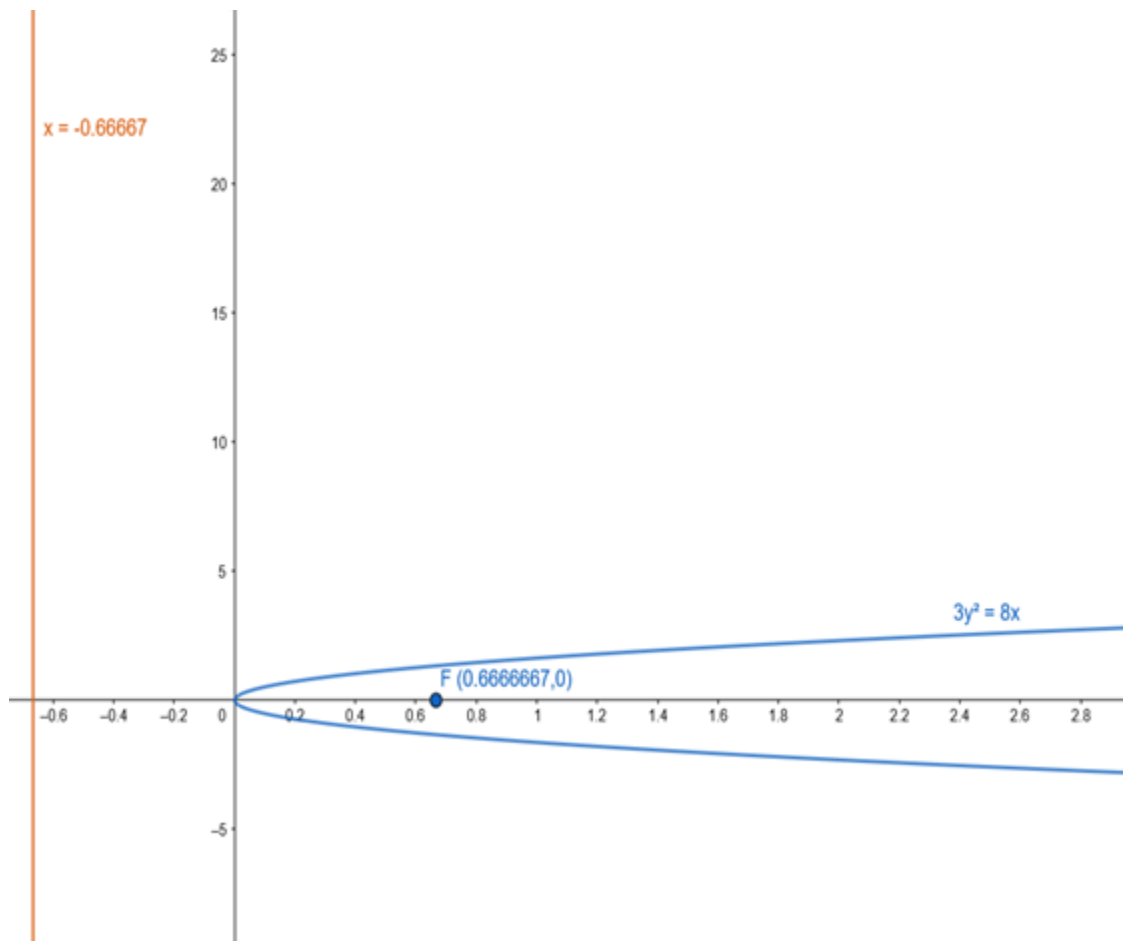
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $x+a=0$

- $x + \frac{2}{3} = 0$

- $x = -\frac{2}{3}$

Length of latusrectum : $4a = \frac{8}{3}$



Q. 2 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$y^2 = -8x$$

Answer : Given equation :

$$y^2 = -8x$$

Comparing given equation with parabola having equation,

$$y^2 = -4ax$$

$$4a = 8$$

$$\bullet a = 2$$

$$\text{Focus : } F(-a,0) = F(-2,0)$$

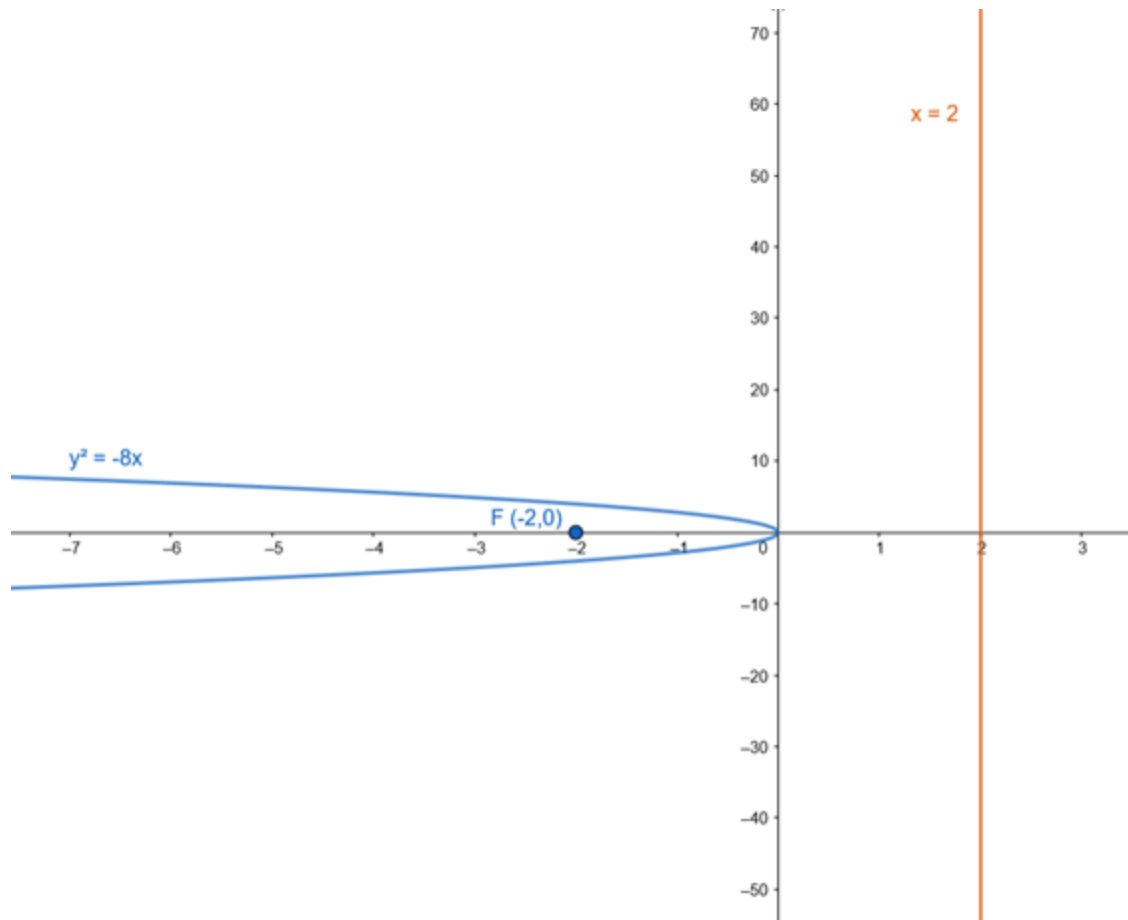
$$\text{Vertex : } A(0,0) = A(0,0)$$

$$\text{Equation of the directrix : } x - a = 0$$

$$\bullet x - 2 = 0$$

$$\bullet x = 2$$

$$\text{Length of latusrectum : } 4a = 8$$



Q. 2 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$y^2 = -6x$$

Answer : Given equation :

$$y^2 = -6x$$

Comparing given equation with parabola having equation,

$$y^2 = -4ax$$

$$4a = 6$$

$$\bullet a = \frac{3}{2}$$

$$\text{Focus : } F(-a,0) = F\left(-\frac{3}{2},0\right)$$

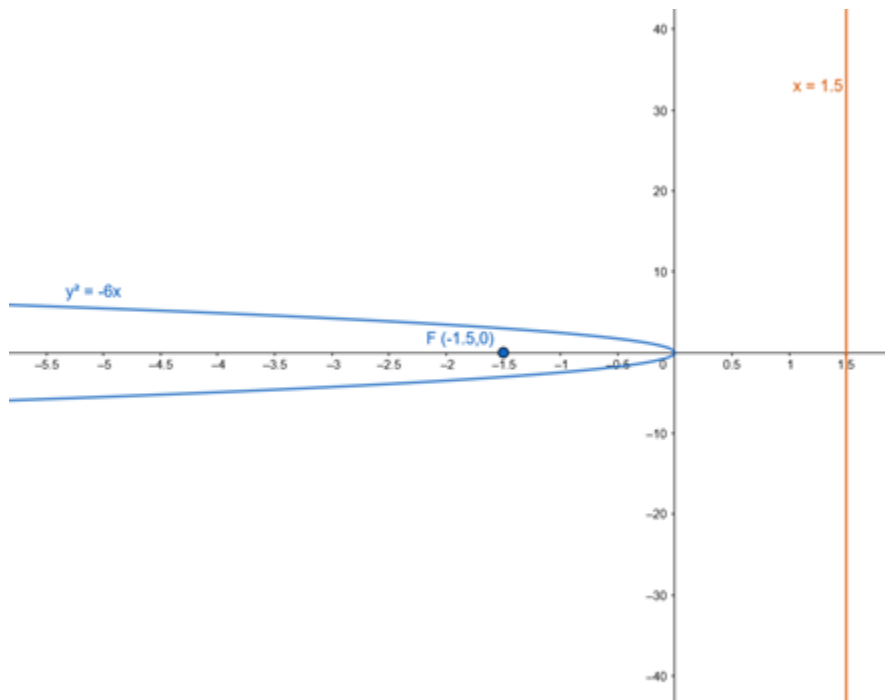
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $x - a = 0$

- $x - \frac{3}{2} = 0$

- $x = \frac{3}{2}$

Length of latusrectum : $4a = 6$



Q. 2 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$5y^2 = -16x$$

Answer : Given equation :

$$5y^2 = -16x$$

$$\bullet y^2 = -\frac{16}{5}x$$

Comparing the given equation with parabola having an equation,

$$y^2 = -4ax$$

$$\bullet 4a = \frac{16}{5}$$

$$\bullet a = \frac{4}{5}$$

Focus : $F(-a,0)$

$$= F\left(-\frac{4}{5}, 0\right)$$

Vertex :

$$A(0,0) = A(0,0)$$

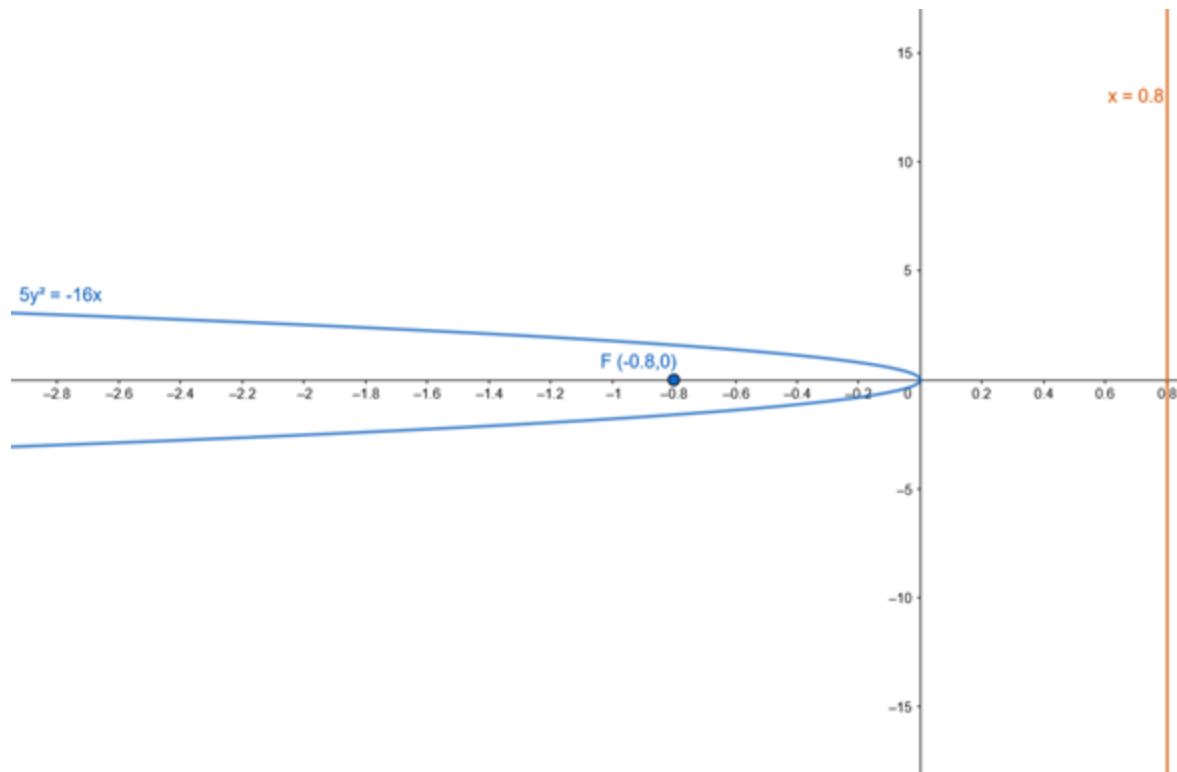
Equation of the directrix :

$$x - a = 0$$

$$\bullet x - \frac{4}{5} = 0$$

$$\bullet x = \frac{4}{5}$$

$$\text{Length of latusrectum : } 4a = \frac{16}{5}$$



Q. 3 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$x^2 = 16y$$

Answer : Given equation : $x^2 = 16y$

Comparing given equation with parabola having equation,

$$x^2 = 4ay$$

$$4a = 16$$

- $a = 4$

Focus : $F(0,a) = F(0,4)$

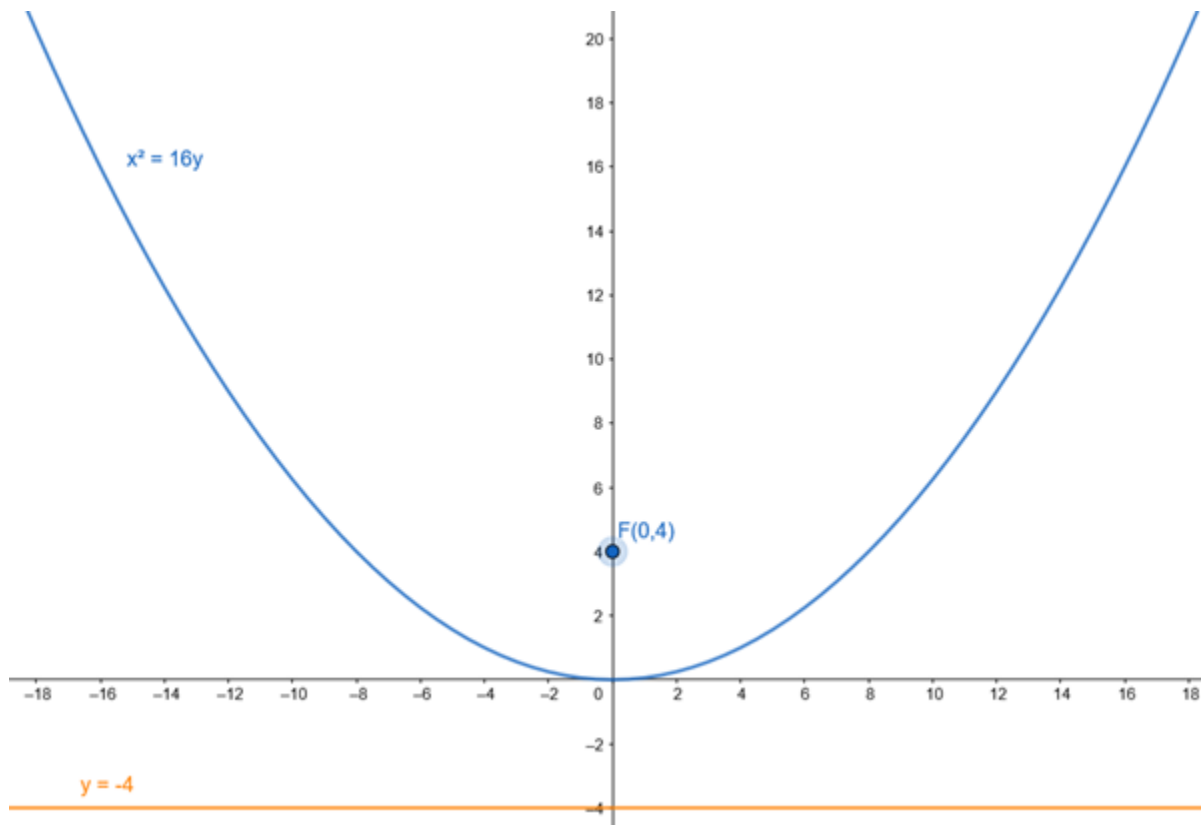
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $y+a=0$

- $y + 4=0$

- $y = -4$

Length of latusrectum : $4a = 16$



Q. 3 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$x^2 = 10y$$

Answer : Given equation : $x^2 = 10y$

Comparing given equation with parabola having equation,

$$x^2 = 4ay$$

$$4a = 10$$

$$\bullet a = 2.5$$

Focus : $F(0,a) = F(0,2.5)$

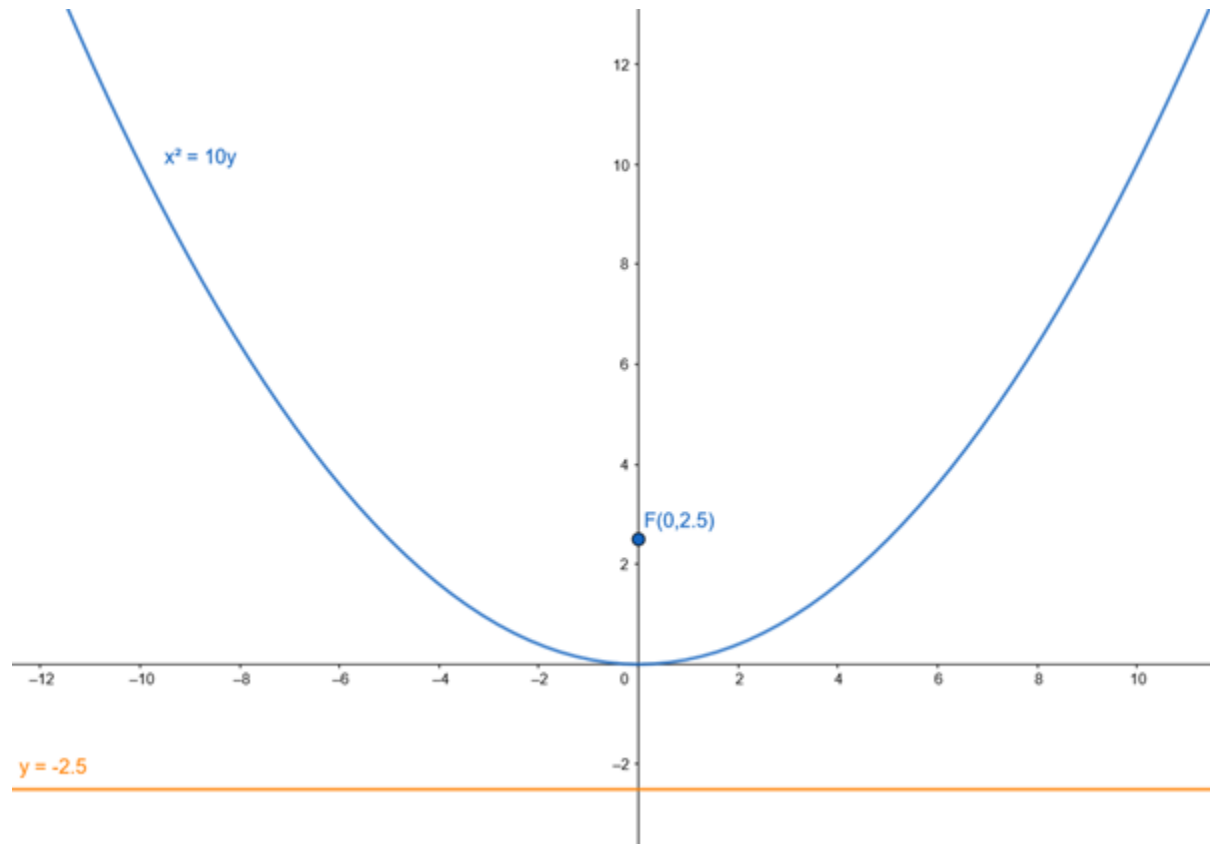
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $y+a=0$

- $y + 2.5 = 0$

- $y = -2.5$

Length of latus rectum : $4a = 10$



Q. 3 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$3x^2 = 8y$$

Answer : Given equation :

$$3x^2 = 8y$$

- $x^2 = \frac{8}{3}y$

Comparing the given equation with parabola having an equation,

$$x^2 = 4ay$$

- $4a = \frac{8}{3}$

- $a = \frac{2}{3}$

Focus : $F(0,a) = F\left(0, \frac{2}{3}\right)$

Vertex : $A(0,0) = A(0,0)$

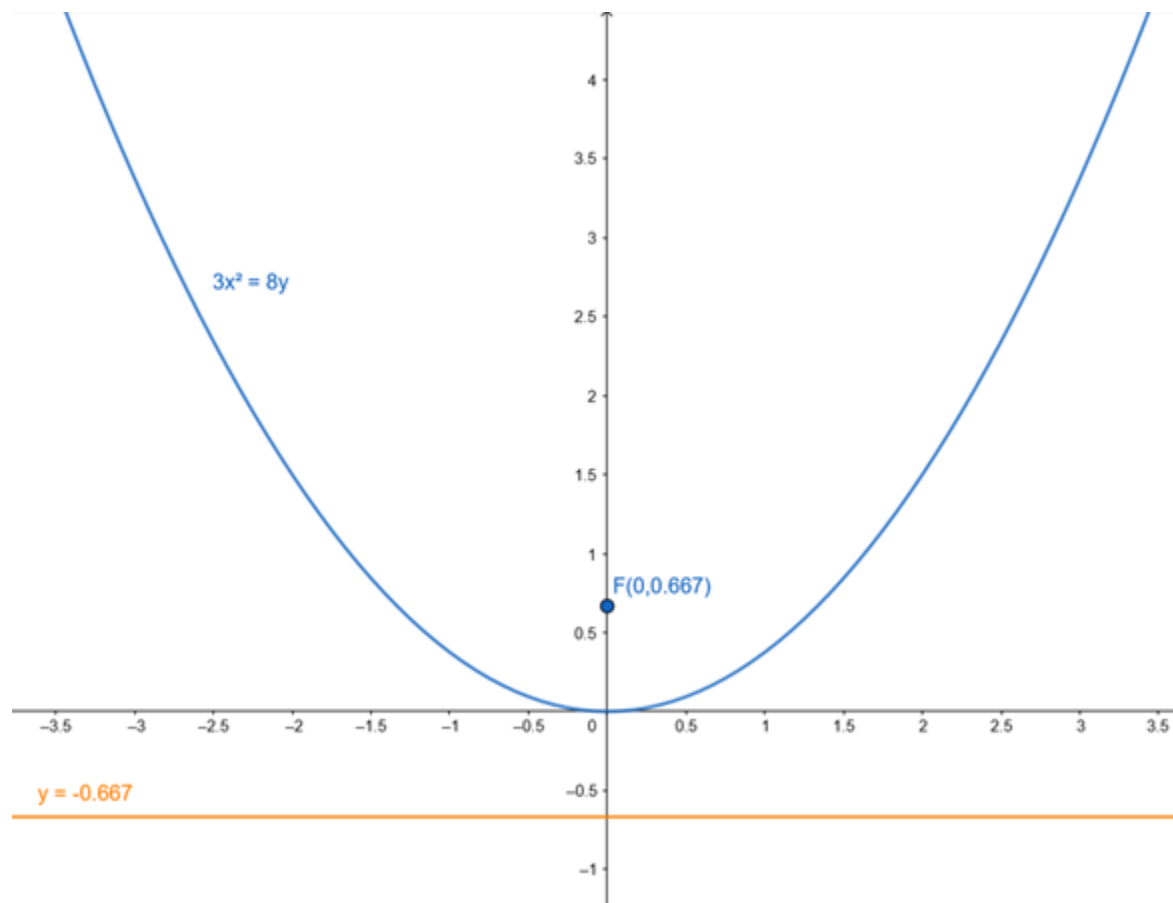
Equation of the directrix : $y + a = 0$

- $y + \frac{2}{3} = 0$

- $y = -\frac{2}{3}$

Lenth of latusrectum :

$$4a = \frac{8}{3}$$



Q. 4 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$x^2 = -8y$$

Answer : Given equation : $x^2 = -8y$

Comparing given equation with parabola having equation,

$$x^2 = -4ay$$

$$4a = 8$$

$$\bullet a = 2$$

$$\text{Focus : } F(0, -a) = F(0, -2)$$

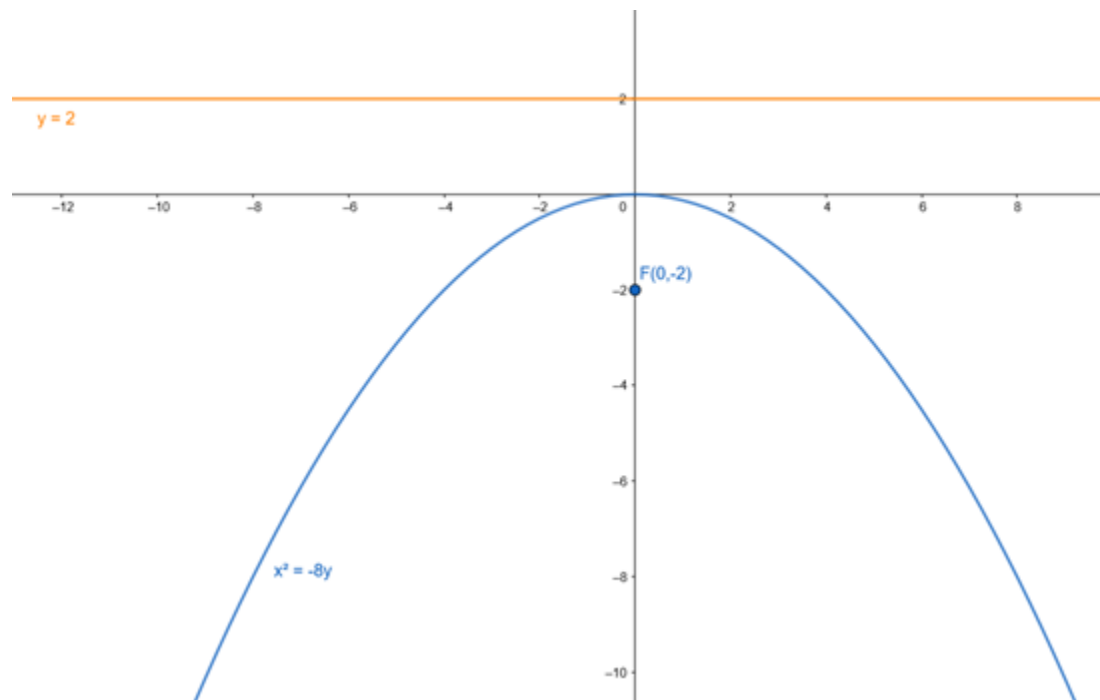
$$\text{Vertex : } A(0, 0) = A(0, 0)$$

$$\text{Equation of the directrix : } y - a = 0$$

$$\bullet y - 2 = 0$$

$$\bullet y = 2$$

$$\text{Length of latus rectum : } 4a = 8$$



Q. 4 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$x^2 = -18y$$

Answer :

Given equation : $x^2 = -18y$

Comparing given equation with parabola having equation,

$$x^2 = -4ay$$

$$4a = 18$$

- $a = \frac{9}{2}$

Focus : $F(0, -a) = F\left(0, -\frac{9}{2}\right)$

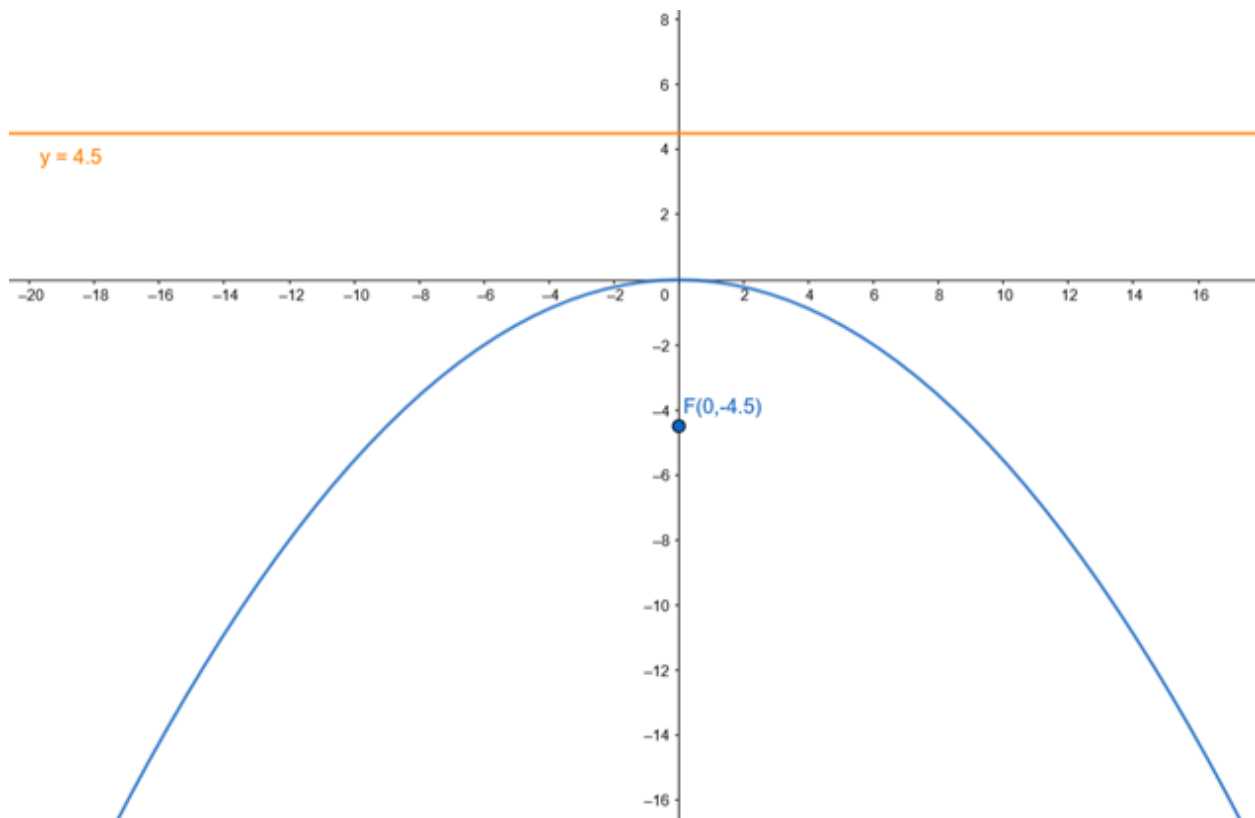
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $y - a = 0$

- $y - \frac{9}{2} = 0$

- $y = \frac{9}{2}$

Length of latus rectum : $4a = 18$



Q. 4 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$$3x^2 = -16y$$

Answer : Given equation :

$$3x^2 = -16y$$

- $x^2 = -\frac{16}{3}y$

Comparing the given equation with parabola having an equation,

$$x^2 = 4ay$$

- $4a = \frac{16}{3}$

- $a = \frac{4}{3}$

Focus : $F(0,-a) = F\left(0, -\frac{4}{3}\right)$

Vertex : $A(0,0) = A(0,0)$

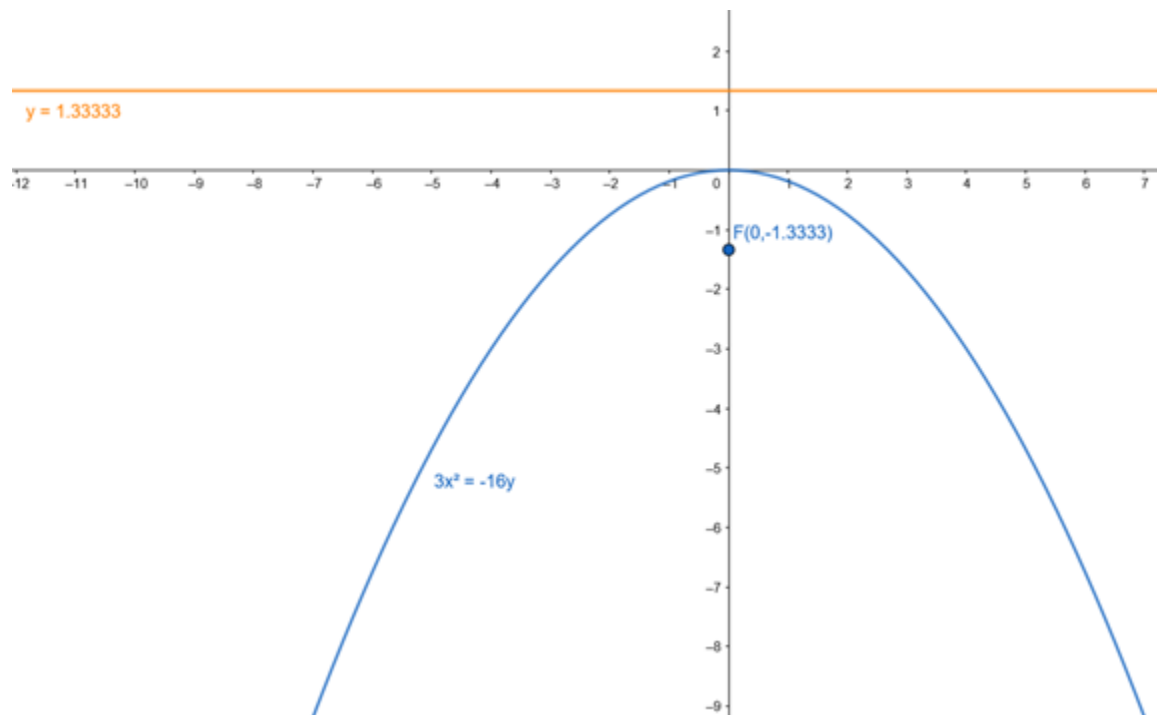
Equation of the directrix : $y - a = 0$

- $y - \frac{4}{3} = 0$

- $y = \frac{4}{3}$

Lenth of latusrectum :

$$4a = \frac{16}{3}$$



Q. 5. Find the equation of the parabola with vertex at the origin and focus at F(-2, 0).

Answer : Vertex : A (0,0)

Given focus F(-2,0) is of the form F(-a,0)

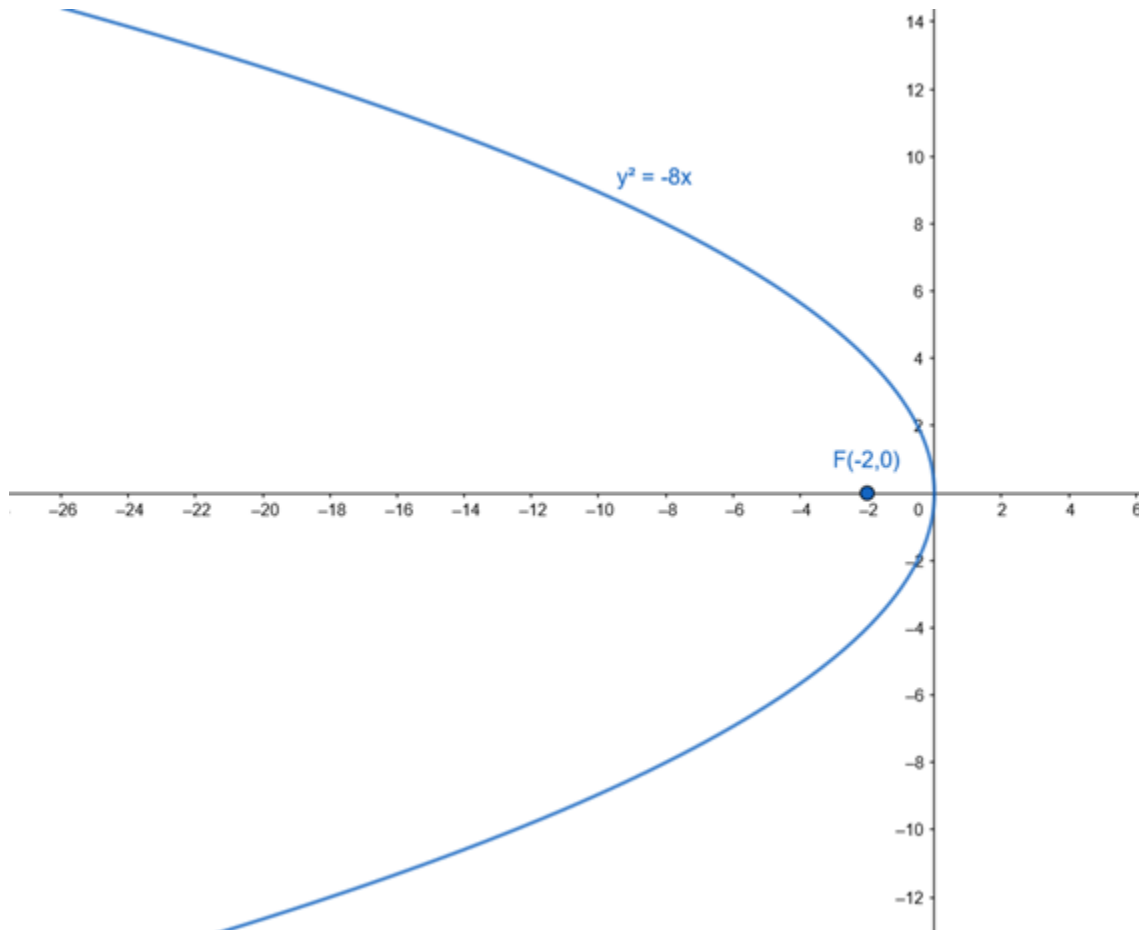
For Vertex A(0,0) and Focus F(-a,0), equation of parabola is

$$y^2 = -4ax$$

Here, $a = 2$

Therefore, equation of parabola,

$$y^2 = -8x$$



Q. 6. Find the equation of the parabola with focus F(4, 0) and directrix $x = -4$.

Answer :

Given equation of directrix : $x = -4$

- $x + 4 = 0$

Above equation is of the form, $x + a = 0$

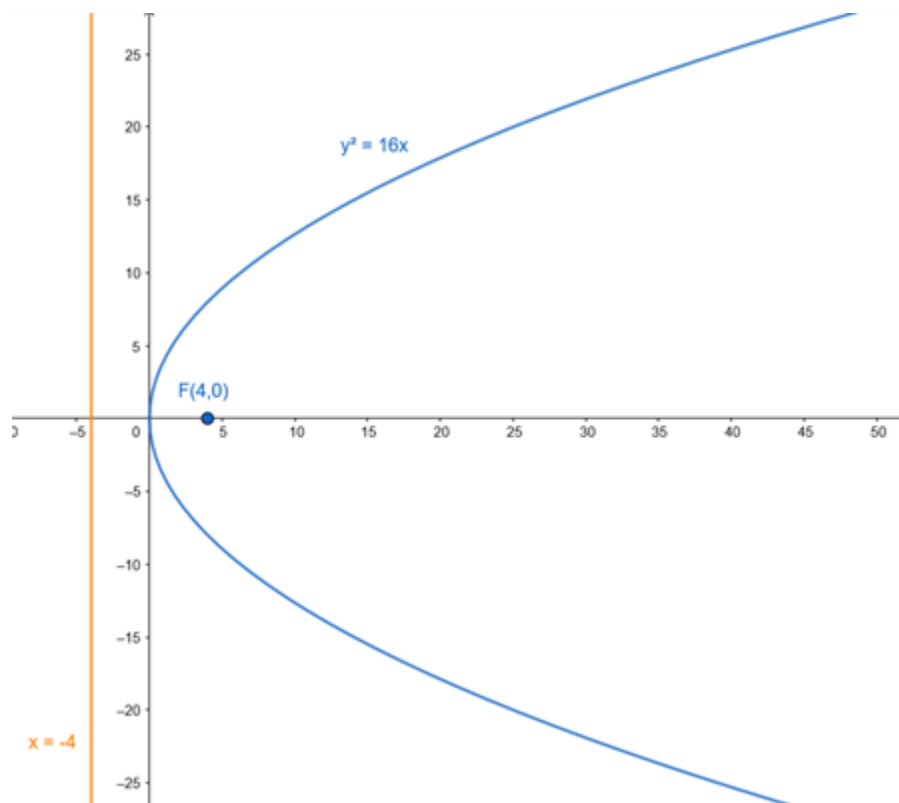
Focus of the parabola $F(4,0)$ is of the form $F(a,0)$

Therefore, $a = 4$

For directrix with equation $x+a=0$ and focus $(a,0)$, equation of the parabola is,

$$y^2 = 4ax$$

- $y^2 = 16x$



Q. 7. Find the equation of the parabola with focus $F(0, -3)$ and directrix $y = 3$.

Answer : Given equation of directrix : $y = 3$

- $y - 3 = 0$

Above equation is of the form, $y - a = 0$

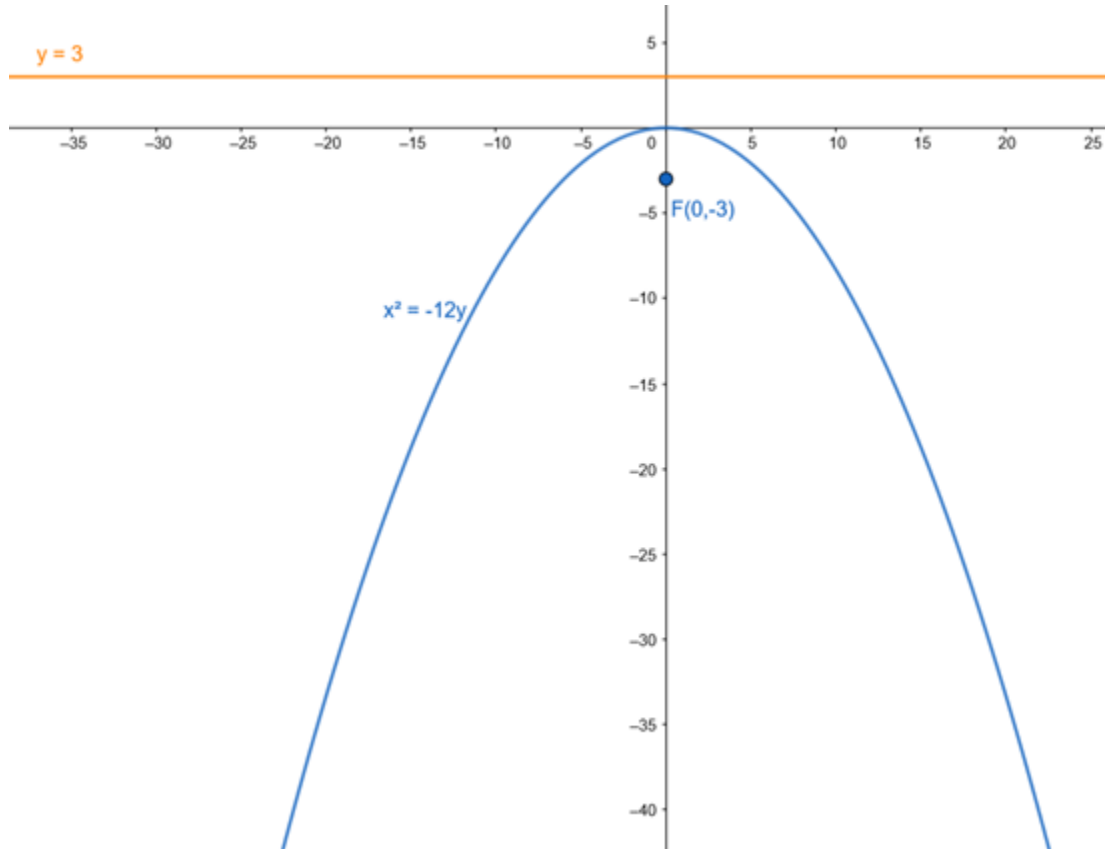
Focus of the parabola $F(0,-3)$ is of the form $F(0,-a)$

Therefore, $a = 3$

For directrix with equation $y-a=0$ and focus $(0,-a)$, equation of the parabola is,

$$x^2 = -4ay$$

$$\bullet x^2 = -12y$$



Q. 8. Find the equation of the parabola with vertex at the origin and focus $F(0, 5)$.

Answer : Vertex : A (0,0)

Given focus $F(0,5)$ is of the form $F(0,a)$

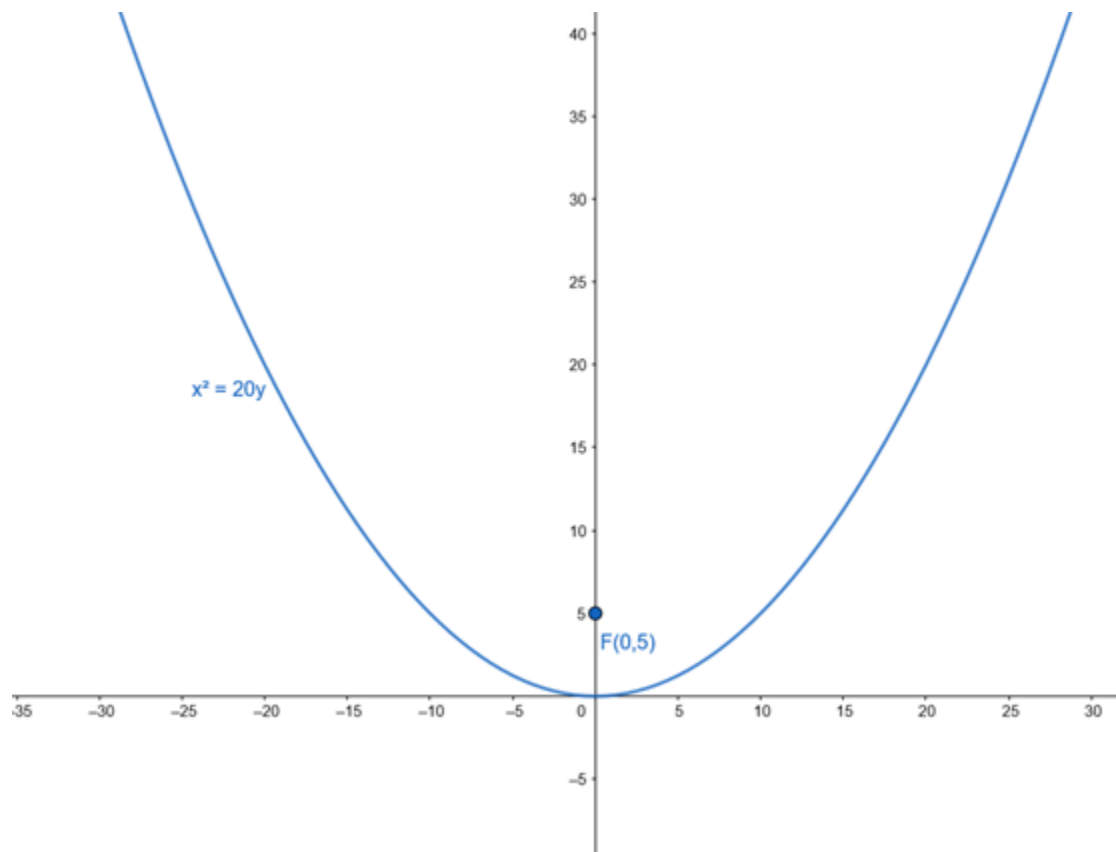
For Vertex A(0,0) and Focus $F(0,a)$, equation of parabola is

$$x^2 = 4ay$$

Here, $a = 5$

Therefore, equation of parabola,

$$x^2 = 20y$$



Q. 9. Find the equation of the parabola with vertex at the origin, passing through the point P(5, 2) and symmetric with respect to the y-axis.

Answer : The equation of a parabola with vertex at the origin and symmetric about the y-axis is

$$x^2 = 4ay$$

Since point P(5,2) passes through above parabola we can write,

$$5^2 = 4a(2)$$

$$\bullet 25 = 8a$$

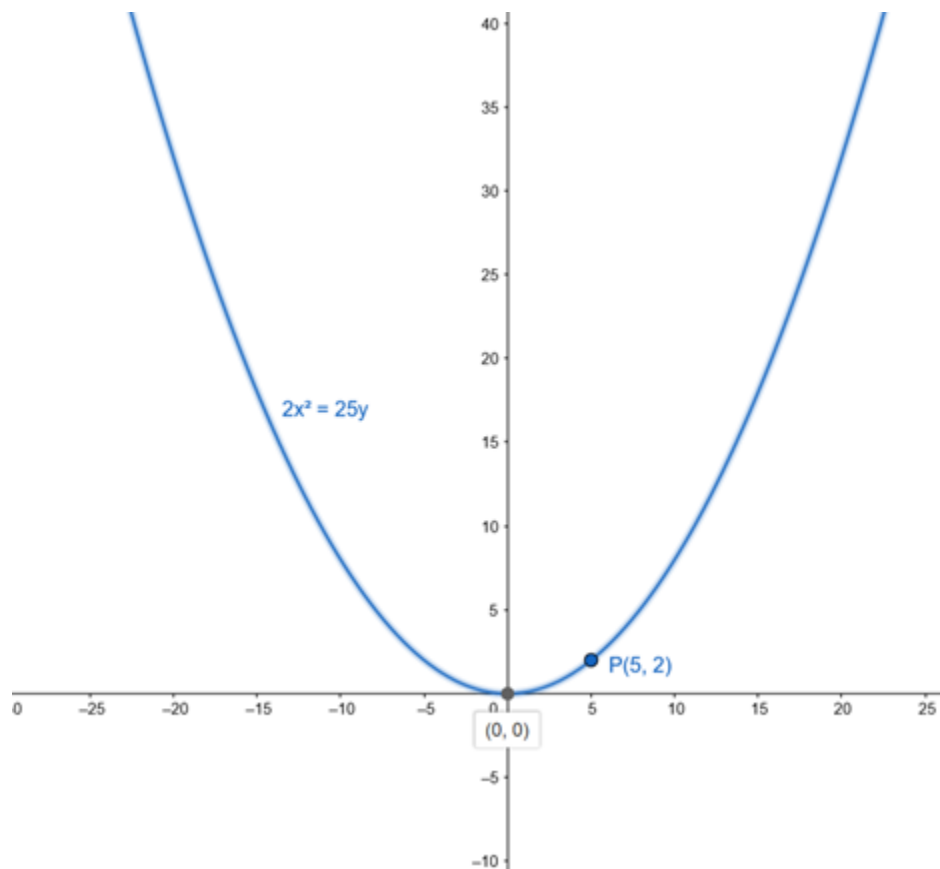
$$\bullet a = \frac{25}{8}$$

Therefore, the equation of a parabola is

- $x^2 = 4 \cdot \frac{25}{8}y$

- $x^2 = \frac{25}{2}y$

- $2x^2 = 25y$



Q. 10. Find the equation of the parabola, which is symmetric about the y-axis and passes through the point P(2, -3).

Answer : The equation of a parabola with vertex at the origin and symmetric about the y-axis is

$$x^2 = 4ay$$

Since point P(2,-3) passes through above parabola we can write,

$$2^2 = 4a(-3)$$

- $4 = -12a$

- $a = -\frac{1}{3}$

Therefore, the equation of a parabola is

- $x^2 = 4 \cdot \left(-\frac{1}{3}\right)y$

- $x^2 = -\frac{4}{3}y$

- $3x^2 = -4y$

