2. Laws of Indices

Let us Work Out 2

1 A. Question

Let us find out the values:

$$\left(5\sqrt{8}\right)^{5/2} \times (16)^{\frac{-3}{2}}$$

Answer

$$\left(5\sqrt{8}\right)^{5/2} \times (16)^{\frac{-3}{2}}$$

$$= \left(5 \times 8^{\frac{1}{2}}\right)^{\frac{5}{2}} \times (2^4)^{-\frac{3}{2}}$$

$$= \left(5^{\frac{5}{2}} \times 8^{\frac{5}{2}\times\frac{1}{2}}\right) \times \left(2^{4\times-\frac{3}{2}}\right)$$

$$= \left(5^{\frac{5}{2}} \times 8^{\frac{5}{4}}\right) \times (2^{2\times-3})$$

$$= \left(5^{\frac{5}{2}} \times 2^{3\times\frac{5}{4}}\right) \times (2^{-6})$$

$$= \left(5^{\frac{5}{2}} \times 2^{\frac{15}{4}} \times (2^{-6})\right)$$

$$= (5^{\frac{5}{2}} \times 2^{\frac{15}{4}-6})$$

$$= (5^{\frac{5}{2}} \times 2^{\frac{15-24}{4}})$$

$$= (5^{\frac{5}{2}} \times 2^{\frac{-9}{4}})$$

1 B. Question

Let us find out the values:

$$\left\{(125)^{-2} \times (16)^{\frac{-3}{2}}\right\}^{\frac{-1}{6}}$$

Answer

$$\left\{(125)^{-2} \times (16)^{\frac{-3}{2}}\right\}^{\frac{-1}{6}}$$

$= (5^{3^{(-2)}} \times 2^{4^{\times -\frac{3}{2}}})^{-\frac{1}{6}}$

$= (5^{-6} \times 2^{-6})^{-\frac{1}{6}}$

$= (5 \times 2)^{-6 \times -\frac{1}{6}}$

$= (10)^{1} = 10$

1 C. Question

Let us find out the values:

$$4^{\frac{1}{3}} \times \left[2^{\frac{1}{3}} \times 3^{\frac{1}{2}}\right] \div 9^{\frac{1}{4}}$$

Answer

$$4^{\frac{1}{3}} \times \left[2^{\frac{1}{3}} \times 3^{\frac{1}{2}}\right] \div 9^{\frac{1}{4}}$$

$$= 4^{\frac{1}{3}} \times \frac{\left[2^{\frac{1}{3}} \times 3^{\frac{1}{2}}\right]}{3^{2 \times \frac{1}{4}}}$$

$$= 4^{\frac{1}{3}} \times \frac{\left[2^{\frac{1}{3}} \times 3^{\frac{1}{2}}\right]}{3^{\frac{1}{2}}}$$

$$= 4^{\frac{1}{3}} \times 2^{\frac{1}{3}}$$

$$= (8)^{\frac{1}{3}}$$

$$= 2^{3^{\frac{1}{3}}} = 2$$

2 A. Question

Let us simplify:

$$\left(8a^3 \div 27x^{-3}\right)^{\frac{2}{3}} \times \left(64a^3 \div 27x^{-3}\right)^{-\frac{2}{3}}$$

Answer

$$\left(8a^3 \div 27x^{-3}\right)^{\frac{2}{3}} \times \left(64a^3 \div 27x^{-3}\right)^{-\frac{2}{3}}$$

$$= \left\{\left(\frac{(2a)^3}{(3x^{-1})^3}\right)^{\frac{2}{3}} \times \left(\frac{(4a)^3}{(3x^{-1})^3}\right)^{-\frac{2}{3}}\right\}^1$$

$$= \left\{\left(\frac{(2a)^3}{(3 \times \frac{1}{x})^3}\right)^{\frac{2}{3}} \times \left(\frac{(4a)^3}{(3 \times \frac{1}{x})^3}\right)^{-\frac{2}{3}}\right\}^1$$

$$= \left\{\left(\frac{(2a)^3 \times (x)^3}{3^3}\right) \times \left(\frac{(4a)^3 \times (x)^3}{3^3}\right)\right\}^1$$

$$= \left(\frac{2ax}{3}\right)^{3\frac{2}{3}} \times \left(\frac{4ax}{3}\right)^{3-\frac{2}{3}}$$

$$= \left(\frac{2ax}{3}\right)^2 \times \left(\frac{4ax}{3}\right)^{-2}$$

$$= \left(\frac{2ax}{3} \times \frac{3}{4ax}\right)^2$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

2 B. Question

Let us simplify:

$$\left\{\left(x^{-5}\right)^{\frac{2}{3}}\right\} - \frac{3}{10}$$

Answer

$$\left\{\left(x^{-5}\right)^{\frac{2}{3}}\right\} - \frac{3}{10}$$

$$= \left\{(x^{-5})^{\frac{2}{3} \times -\frac{3}{10}}\right\}$$

$= (x^{-5})^{-\frac{1}{5}}$

$= x^{-5\times-\frac{1}{5}} = x$

2 C. Question

Let us simplify:

$[\{2^{-1})^{-1}\}^{-1}]^{-1}$

Answer

$[\{2^{-1})^{-1}\}^{-1}]^{-1}$

$= \left[\left\{\left(\frac{1}{2}\right)^{-1}\right\}^{-1}\right]^{-1}$

$= [\{(2)^{-1}\}^{-1}]$

$= \left(\frac{1}{2}\right)^{-1}$

= 2

2 D. Question

Let us simplify:

$\sqrt[3]{a^{-2}}.\,b \times \sqrt[3]{b^{-2}}.\,c \times \sqrt[3]{c^{-2}}.\,a$

Answer

$\sqrt[3]{a^{-2}}.\,b \times \sqrt[3]{b^{-2}}.\,c \times \sqrt[3]{c^{-2}}.\,a$

$= \left(a^{-\frac{2}{3}} \times b \times b^{-\frac{2}{3}} \times c \times c^{-\frac{2}{3}} \times a\right)$

$= a^{1-\frac{2}{3}} \times b^{1-\frac{2}{3}} \times c^{1-\frac{2}{3}}$

$= a^{\frac{3-2}{3}} \times b^{\frac{3-2}{3}} \times c^{\frac{3-2}{3}}$

$= a^{\frac{1}{3}} \times b^{\frac{1}{3}} \times c^{\frac{1}{3}}$

$= (abc)^{\frac{1}{3}}$

2 E. Question

Let us simplify:

$$\left(\frac{4^{m+\frac{1}{4}} \times \sqrt{2.2^m}}{2.\sqrt{2^{-m}}}\right)^{\frac{1}{m}}$$

Answer

$$\left(\frac{4^{m+\frac{1}{4}} \times \sqrt{2.2^m}}{2.\sqrt{2^{-m}}}\right)^{\frac{1}{m}}$$

$$= \left(\frac{2^{2(m+\frac{1}{4})} \times 2^{\frac{m+1}{2}}}{2^{1-\frac{m}{2}}}\right)^{\frac{1}{m}}$$

$$= \left(\frac{2^{2m+\frac{1}{2}} \times 2^{\frac{m}{2}+\frac{1}{2}}}{2^{1-\frac{m}{2}}}\right)^{\frac{1}{m}}$$

$$= (2^{2m+\frac{1}{2}+\frac{m}{2}+\frac{1}{2}-1+\frac{m}{2}})^{\frac{1}{m}}$$

$$= (2^{2m+\frac{m}{2}+\frac{m}{2}+1-1})^{\frac{1}{m}}$$

$$= (2^{2m+m})^{\frac{1}{m}}$$

$$= (2^{3m})^{\frac{1}{m}} = 2^3 = 8$$

2 F. Question

Let us simplify:

$$9^{-3} \times \frac{16^{\frac{1}{4}}}{6^{-2}} \times \left(\frac{1}{27}\right)^{-\frac{4}{3}}$$

Answer

$$9^{-3} \times \frac{16^{\frac{1}{4}}}{6^{-2}} \times \left(\frac{1}{27}\right)^{-\frac{4}{3}}$$

$$= 3^{3\times-3} \times \frac{2^{4\times\frac{1}{4}}}{(2\times 3)^{-2}} \times \left(\frac{1}{3^3}\right)^{-\frac{4}{3}}$$

$$= 3^{-9} \times \frac{2^1}{2^{-2}\times 3^{-2}} \times (\frac{1}{3^{-4}})$$

$$= 3^{(-9+2+4)} \times 2^{(1+2)}$$

$$= (3^{-3} \times 2^3) = \left(\frac{2}{3}\right)^3$$

$$= \frac{8}{27}$$

2 G. Question

Let us simplify:

$$(\frac{x^a}{x^b})^{a^2+ab+b^2} \times (\frac{x^b}{x^c})^{b^2+bc+c^2} \times (\frac{x^c}{x^a})^{c^2+ca+a^2}$$

Answer

$$(\frac{x^a}{x^b})^{a^2+ab+b^2} \times (\frac{x^b}{x^c})^{b^2+bc+c^2} \times (\frac{x^c}{x^a})^{c^2+ca+a^2}$$

$$= (x)^{a-b^{a^2+ab+b^2}} \times (x)^{b-c^{b^2+bc+c^2}} \times (x)^{c-a^{c^2+ca+a^2}}$$

$$= (x)^{(a^2+ab+b^2)(a-b)} \times (x)^{(b^2+bc+c^2)(b-c)} \times (x)^{(c^2+ca+a^2)(c-a)}$$

$$= (x)^{(a^2+ab+b^2)(a-b)+(b^2+bc+c^2)(b-c)+(c^2+ca+a^2)(c-a)}$$

$$= (x)^{(a^3-a^2b+a^2b-ab^2+b^2a-b^3+b-b^2c+b^2c-bc^2+c^2b-c^3+ +c^3-c^2a+c^2a-ca^2+a^2c-a^3)}$$

$$= (x)^0 = 1$$

3 A. Question

Let us arrange in ascending order.

$$5^{\frac{1}{2}}, 10^{\frac{1}{4}}, 6^{\frac{1}{3}}$$

Answer

In this type of problems we try to make the powers same by taking LCM of them.

Here, $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$ are in the powers .

So, we take LCM of 2, 4 and 3.

4= 2$\times$2

2= 2$\times$1

3= 3$\times$1

LCM= 2$\times$3 = 6

Now, $5^{\frac{1}{2}} = 5^{\frac{1}{2} \times \frac{6}{6}} = 5^{\frac{6}{12}} = 15625^{\frac{1}{12}} = \sqrt[12]{15625}$

$10^{\frac{1}{4}} = 10^{\frac{1}{4} \times \frac{3}{3}} = 10^{\frac{3}{12}} = 1000^{\frac{1}{12}} = \sqrt[12]{1000}$

$6^{\frac{1}{3}} = 6^{\frac{1}{3} \times \frac{4}{4}} = 6^{\frac{4}{12}} = 1296^{\frac{1}{12}} = \sqrt[12]{1296}$

Here, all the numbers are having same power.

So, in ascending order we have

$\sqrt[12]{1000} < \sqrt[12]{1296} < \sqrt[12]{15625}$

Hence, $10^{\frac{1}{4}} < 6^{\frac{1}{3}} < 5^{\frac{1}{2}}$.

3 B. Question

Let us arrange in ascending order.

$3^{\frac{1}{3}}, 2^{\frac{1}{2}}, 8^{\frac{1}{4}}$

Answer

In this type of problems we try to make the powers same by taking LCM of them.

Here, $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}$ are in the powers .

So, we take LCM of 2, 4 and 3.

4= 2$\times$2

2= 2$\times$1

3= 3$\times$1

LCM= 2$\times$3 = 6

Now, $3^{\frac{1}{3}} = 3^{\frac{1}{3}\times\frac{4}{4}} = 3^{\frac{4}{12}} = 81^{\frac{1}{12}} = \sqrt[12]{81}$

$2^{\frac{1}{2}} = 2^{\frac{1}{2}\times\frac{6}{6}} = 2^{\frac{6}{12}} = 64^{\frac{1}{12}} = \sqrt[12]{64}$

$8^{\frac{1}{4}} = 8^{\frac{1}{4}\times\frac{3}{3}} = 8^{\frac{3}{12}} = 512^{\frac{1}{12}} = \sqrt[12]{512}$

Here, all the numbers are having same power.

So, in ascending order we have

$\sqrt[12]{64} < \sqrt[12]{81} < \sqrt[12]{512}$

Hence, $2^{\frac{1}{2}} < 3^{\frac{1}{3}} < 8^{\frac{1}{4}}$.

3 C. Question

Let us arrange in ascending order.

$2^{60}, 3^{48}, 4^{36}, 5^{24}$

Answer

We will try to make the powers of the same . We can write

$2^{60} = {2^5}^{12} = 32^{12}$

$3^{48} = {3^4}^{12} = 81^{12}$

$4^{36} = {4^3}^{12} = 64^{12}$

$3^{24} = {3^2}^{12} = 9^{12}$

Now, we can clearly see $9^{12} < 32^{12} < 64^{12} < 81^{12}$

So, $3^{24} < 2^{60} < 4^{36} < 3^{48}$.

4 A. Question

Let us prove

$$\left(\frac{a^q}{a^r}\right)^p \times \left(\frac{a^r}{a^p}\right)^q \times \left(\frac{a^p}{a^q}\right)^r = 1$$

Answer

$$\text{LHS} = \left(\frac{a^q}{a^r}\right)^p \times \left(\frac{a^r}{a^p}\right)^q \times \left(\frac{a^p}{a^q}\right)^r$$

$$= \left(a^{q-r^{p}} \times a^{r-p^{q}} \times a^{p-q^{r}}\right)$$

$$= \left(a^{pq-pr} \times a^{qr-qp} \times a^{rp-rq}\right)$$

$$= \left(a^{pq-pr+qr-qp+rp-rq}\right)$$

$$= (a^{0}) = 1 = \text{RHS}$$

Therefore, LHS = RHS

Hence, proved.

4 B. Question

Let us prove

$$\left(\frac{x^{m}}{x^{n}}\right)^{m+n}\left(\frac{x^{n}}{x^{\ell}}\right)^{n+l}\left(\frac{x^{\ell}}{x^{m}}\right)^{l+m} = 1$$

Answer

$$\text{LHS} = \left(\frac{x^{m}}{x^{n}}\right)^{m+n}\left(\frac{x^{n}}{x^{\ell}}\right)^{n+l}\left(\frac{x^{\ell}}{x^{m}}\right)^{l+m}$$

$$= \left(x^{m-n^{m+n}} \times x^{n-1^{n+1}} \times x^{1-m^{1+m}}\right)$$

$$= \left(x^{m^{2}-n^{2}} \times x^{n^{2}-1} \times x^{1-m^{2}}\right)$$

$$= \left(x^{m^{2}-n^{2}+n^{2}-1+1-m^{2}}\right)$$

$$= (x^{0}) = 1 = \text{RHS}$$

Therefore, LHS = RHS

Hence, proved .

4 C. Question

Let us prove

$$\left(\frac{x^{m}}{x^{n}}\right)^{m+n-\ell} \times \left(\frac{x^{n}}{x^{\ell}}\right)^{n+\ell-m} \times \left(\frac{x^{\ell}}{x^{m}}\right)^{\ell+m-n} = 1$$

Answer

$$\text{LHS} = \left(\frac{x^{m}}{x^{n}}\right)^{m+n-1} \times \left(\frac{x^{n}}{x^{1}}\right)^{n+1-m} \times \left(\frac{x^{1}}{x^{m}}\right)^{1+m-n}$$

$= \left(x^{m-n^{m+n-1}} \times x^{n-1^{n+1-m}} \times x^{1-m^{1+m-n}}\right)$

$= \left(x^{m^2-n^2-m+n} \times x^{n^2-1-mn+m} \times x^{1-m^2-n+mn}\right)$

$= (x^{m^2-n^2-m+n+n^2-1-mn+1+m-m^2-n+mn})$

$= (x^{m^2-m^2-m+m+n^2-n^2-n+n+mn-mn+1-1})$

$=(x^0) = 1 = RHS$

Therefore, LHS = RHS

Hence, proved .

4 D. Question

Let us prove

$$(a^{\frac{1}{x-y}})^{\frac{1}{x-2}} \times (a^{\frac{1}{y-z}})^{\frac{1}{y-x}} \times (a^{\frac{1}{z-x}})^{\frac{1}{z-y}} = 1$$

Answer

$$= \left(a^{\frac{1}{x-y}^{\frac{1}{x-z}}} \times a^{\frac{1}{y-z}^{\frac{1}{y-x}}} \times a^{\frac{1}{z-x}^{\frac{1}{z-y}}}\right)$$

$$= \left(a^{(\frac{1}{x-y} \times \frac{1}{x-z})} \times a^{(\frac{1}{y-z} \times \frac{1}{y-x})} \times a^{(\frac{1}{z-x} \times \frac{1}{z-y})}\right)$$

$$= (a^{(\frac{1}{(x-y)(x-z)} + \frac{1}{(y-z)(y-x)} + \frac{1}{(z-x)(z-y)})})$$

$$= (a^{(\frac{1}{(x-y)(x-z)} + \frac{1}{(y-z)(y-x)} - \frac{1}{(z-x)(y-z)})})$$

$$= [a^{\{\frac{1}{(x-y)(x-z)} + \frac{1}{y-z}(\frac{1}{(y-x)} - \frac{1}{z-x}\}}]$$

$$= [a^{\{\frac{1}{(x-y)(x-z)} + \frac{1}{y-z}(\frac{z-x-y+x}{(y-x)(z-x)}\}}]$$

$$= [a^{\{\frac{1}{(x-y)(x-z)} + \frac{1}{y-z}(\frac{z-y}{(y-x)(z-x)}\}}]$$

$$= [a^{\{\frac{1}{(x-y)(x-z)} + \frac{1}{y-z}(\frac{-(y-z)}{(y-x)(z-x)}\}}]$$

$$= [a^{\{\frac{1}{(x-y)(x-z)} - \frac{1}{(y-x)(z-x)}\}}]$$

$$= [a^{\{\frac{1}{(x-y)(x-z)} - \frac{1}{(x-y)(x-z)}\}}]$$

$= (a^0) = 1 = \text{RHS}$

Therefore, LHS = RHS

Hence, proved.

5. Question

If x + z = 2y and b^2 = ac, then let us show that $a^{y-z}\ b^{z-x}\ c^{x-y} = 1$.

Answer

Given that x + z = 2y and b^2 = ac

We can write from x + z = 2y

$\Rightarrow \frac{x+z}{y} = 2$ ...eq(1)

Now, b^2 = ac

$\Rightarrow b^{\frac{x+z}{y}} = ac$

$\Rightarrow b^{\frac{x}{y}+\frac{z}{y}} = ac$

$\Rightarrow b^{\frac{x}{y}} \times b^{\frac{z}{y}} = a \times c$

Now, we can write as

$a = b^{\frac{x}{y}}$ and $c = b^{\frac{z}{y}}$ ...eq(2)

Now, LHS = $a^{y-z}\ b^{z-x}\ c^{x-y}$

$= b^{\frac{x}{y}^{y-z}} \times b^{z-x} \times b^{\frac{z}{y}^{x-y}}$

$= b^{\frac{x}{y}\times y-\frac{xz}{y}} \times b^{z-x} \times b^{\frac{zx}{y}-\frac{z}{y}\times y}$

$= b^{x-\frac{xz}{y}+z-x+\frac{zx}{y}-z}$

$= b\left(x - x + z - z + \frac{xz}{y} - \frac{xz}{y}\right)$

= b^0 = 1 = RHS

Therefore, LHS = RHS

Hence, proved .

6. Question

If a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}, then let us show that $a^{q-r} b^{r-p} c^{p-q} = 1$.

Answer

Given, a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}

Now, LHS = $a^{q-r} b^{r-p} c^{p-q}$

$= (xy^{p-1})^{q-r} \times (xy^{q-1})^{r-p} \times (xy^{r-1})^{p-q}$

$= (x^{q-r}y^{(p-1)(q-r)} \times x^{r-p}y^{(q-1)(r-p)} \times x^{p-q}y^{(r-1)(p-q)})$

$= (x^{q-r+r-p+p-q} \times y^{pq-pr-q+r+qr-qp-r+p+pr-qr-p+q})$

$= (x^{q-q+p-p+r-r} \times y^{pq-pq+r-r-pr+pr+q-q-qr+qr-p-q})$

$= x^0 \times y^0 = 1 \times 1 = 1$ = RHS

Therefore, LHS = RHS

Hence, proved.

7. Question

If $x^{\frac{1}{a}} = y^{\frac{1}{b}} = z^{\frac{1}{c}}$ and xyz = 1, then let us show that a + b + c = 0.

Answer

Given $x^{\frac{1}{a}} = y^{\frac{1}{b}} = z^{\frac{1}{c}}$ and xyz = 1

Let $x^{\frac{1}{a}} = y^{\frac{1}{b}} = z^{\frac{1}{c}} = k$

Therefore, $x^{\frac{1}{a}} = k$

$x = k^a$ ...eq(1)

$y^{\frac{1}{b}} = k$

$y = kb$ ...eq(2)

$z^{\frac{1}{c}} = k$

$z = k^c$ ...eq(3)

Now, xyz = 1

$\Rightarrow (k^a \times k^b \times k^c) = 1$

(from eq(1), eq(2)and eq(3))

$\Rightarrow \ (k^{a+b+c}) = 1$

$\Rightarrow (k^{a+b+c}) = k^0$

We know that if base of the exponents are same then the powers are also equal.

Therefore, a + b + c = 0.

Hence the relation is proved.

8. Question

If $a^x = b^y = c^z$ and abc = 1, then let us show that xy + yz + zx = 0.

Answer

Given that : -

$a^x = b^y = c^z = k$ and abc = 1

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^0 = 1$

$a = k^{\frac{1}{x}}$ ....(i)

$b = k^{\frac{1}{y}}$ .... (ii)

$c = k^{\frac{1}{z}}$ .... (iii)

multiplying (i),(ii)&(iii) we get

$\Rightarrow abc = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$ using formula (a1)

$\Rightarrow 1 = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

using formula (b1)

$\Rightarrow k^0 = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

$$\Rightarrow 0 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$\Rightarrow 0 = \frac{xy + yz + xz}{xyz}$$

$\Rightarrow 0 = xy + yz + xz$ ………….proved

9 A. Question

Let us solve:

$49^x = 7^3$

Answer

Given that $49^x = 7^3$

Formula used: - (a1) if $m^a = m^b$ then a = b

Solving it

$49^x = 7^3$

We can write $49^x = 7^{2x}$

Then , $7^{2x} = 7^3$

Using the formula (a1)

$\Rightarrow 2x = 3$

$$\Rightarrow x = \frac{3}{2}$$

9 B. Question

Let us solve:

$2^{x+2} + 2^{x-1} = 9$

Answer

Given that $2^{x+2} + 2^{x-1} = 9$

Formula used: - (a1) if $m^a = m^b$ then a = b

$2^{x+2} + 2^{x-1} = 2^3 + 1$

$\Rightarrow 2^{x+2} + 2^{x-1} = 2^3 + 2^0$

Using formula (a1)

$x + 2 = 3$ and $x - 1 = 0$

$\Rightarrow$ x = 1

9 C. Question

Let us solve:

$2^{x+1} + 2^{x+2} = 48.$

Answer

Given that

$2^{x+1} + 2^{x+2} = 48$

Formula used: - (a1) if $m^a = m^b$ then a = b

$\Rightarrow 2^{x+1} + 2^{x+2} = 2^4 + 2^5$

$\Rightarrow x + 1 = 4$ and $x + 2 = 5$

$\Rightarrow x = 3$

9 D. Question

Let us solve:

$$2^{4x} \cdot 4^{3x-1} = \frac{4^{2x}}{2^{3x}}$$

Answer

Given that

$$2^{4x} \cdot 4^{3x-1} = \frac{4^{2x}}{2^{3x}}$$

Formula used: - (a1) if $m^a = m^b$ then a = b

(b1) $m^p.m^q = m^{p+q}$

$$2^{4x}.2^{6x-2}.2^{3x} = 2^{4x}$$

$$\Rightarrow 2^{4x+6x-2+3x} = 2^{4x}$$

$$\Rightarrow 13x - 2 = 4x$$

$$\Rightarrow 9x = 2$$

$$\Rightarrow x = \frac{2}{9}$$

9 E. Question

Let us solve:

$$9.81^x = 27^{2-x}$$

Answer

Given that $9.81^x = 27^{2-x}$

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

$$\Rightarrow 9.9^{2x} = 3^6 - 3^x$$

$$\Rightarrow 3^{4x+2} = 3^6 - 3^x$$

$$\Rightarrow 4x + 2 = 6 - 3x$$

$$\Rightarrow 7x = 4$$

$\Rightarrow\ x = \frac{7}{4}$

9 F. Question

Let us solve:

$2^{5x+4} - 2^9 = 2^{10}$

Answer

GIVEN THAT $2^{5x+4} - 2^9 = 2^{10}$

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p . m^q = m^{p+q}$

$\Rightarrow\ 2^{5x+4} = 512$

$\Rightarrow\ 2^{5x+4} = 2^9$

⇒ 5x + 4 = 9

⇒ x = 1

9 G. Question

Let us solve:

$6^{2x+4} = 3^{3x} . 2^{x+8}$

Answer

Given that $6^{2x+4} = 3^{3x} . 2^{x+8}$

Formula used: - (a1) if $m^a = m^b$ then $a = b$

$\Rightarrow\ 3^{2x+4} . 2^{2x+4} = 3^{3x} . 2^{x+8}$

$\Rightarrow\ 2x + 4 = 3x$ and $2x + 4 = x + 8$

$\Rightarrow\ x = 4$

10 A. Question

The value of $(0.243)^{0.2} \times (10)^{0.6}$ is

A. 0.3

B. 3

C. 0.9

D. 9

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

$(0.243)^{0.2} \times (10)^{0.6} = (0.243)^{\frac{1}{5}}.(10)^{\frac{3}{5}}$

$= (243)^{\frac{1}{5}}.(10)^{\frac{3}{5}-\frac{3}{5}}$

$= (243)^{\frac{1}{5}}$

$= 3$

10 B. Question

The value of $2^{\frac{1}{2}} \times 2^{-\frac{1}{2}} \times (16)^{\frac{1}{2}}$ is

A. 1

B. 2

C. 4

D. $\frac{1}{2}$

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

$= 2^{\frac{1}{2}} \times 2^{-\frac{1}{2}} \times (16)^{\frac{1}{2}}$

$= 2^{\frac{1}{2}}.2^{-\frac{1}{2}}.2^{\frac{4}{2}}$

$= 2^{\frac{1}{2}}.2^{\frac{1}{2}}.2^{2}$

$= 2^{\frac{1}{2}-\frac{1}{2}+2}$

$= 2^2 = 4$

Hence, option (c) is correct.

10 C. Question

If $4^x = 8^3$, then the value of x is

A. $\frac{3}{2}$

B. $\frac{9}{2}$

C. 3

D. 9

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

$4^x = 2^{2x}$ ……… (i)

And $8^3 = 2^{3.3} = 2^9)$ ……(ii)

From (i)(ii)

$\Rightarrow 2^x = 2^9$

$\Rightarrow x = \frac{9}{2}$

10 D. Question

If $20^{-x} = \frac{1}{7}$, then the value of $(20)^{2x}$ is

A. $\frac{1}{49}$

B. 7

C. 49

D. 1

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

$20^{-x} = \frac{1}{7}$

$7 = \frac{1}{20^{-x}}$

$\Rightarrow 7 = 20^x$

So, $20^{2x} = 7^2 = 49$

10 E. Question

If $4 \times 5^x = 500$, then the value of x^x is

A. 8

B. 1

C. 64

D. 27

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

$2^2.5^x = 5.10^2$

$\Rightarrow 2^2.5^x = 5.5^2.2^2$

$\Rightarrow 2^2.5^x = 5^3.2^2$

$\Rightarrow x = 3$

According to question

$\Rightarrow x^x = 3^3 = 27$

11 A. Question

If $(27)^x = (81)^y$, then let us write x : y.

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

$(27)^x = 3^{3x}$ …… (i)

$(81)^y = 3^{4y}$ …… (ii)

From (i)&(ii) we get

$3^{3x} = 3^{4y}$

$\Rightarrow 3x = 4y$

$\Rightarrow \frac{x}{y} = \frac{4}{3}$

According to question

x:y = 4:3

11 B. Question

If $(5^5 + 0.01)^2 - (5^5 - 0.01)^2 = 5^x$, then let us calculate the value of x and write it.

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

(c1)$(a + b)^2 = a^2 + b^2 + 2ab$

(d1) $(a-b)^2 = a^2 + b^2 - 2ab$

$(5^5 + 0.01)^2 - (5^5 - 0.01)^2 = 5^x$

Using formula (c1)&(d1)

$\Rightarrow 5^{5.2} + (0.01)^2 + \{2.(5^5).(0.01)\} - 5^{5.2} - (0.01)^2 + \{2.(5^5).(0.01)\} = 5^x$

Using formula (b1)

$\Rightarrow 4.(5^5).(5)^{-2}.(2)^{-2} = 5^x$

$\Rightarrow 5^3 = 5^x$ from formula (a1)

$\Rightarrow x = 3$

11 C. Question

If $3.27^x = 9^{x+4}$, then let us calculate the value of x and write it.

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

$3.27^x = 9^{x+4}$

$\Rightarrow 3.3^{3x} = 9^{x+4}$

$\Rightarrow 3^{3x+1} = 3^{2x+8}$

Using formula (a1)$\Rightarrow 3x + 1 = 2x + 8$

$\Rightarrow x = 7$

11 D. Question

Let us find out the value of $\sqrt[3]{\left(\frac{1}{64}\right)^{\frac{1}{2}}}$ and write it.

Answer

Formula used: - (a1) if $m^a = m^b$ then $a = b$

(b1) $m^p.m^q = m^{p+q}$

Solving the equation

$\sqrt[3]{\left(\frac{1}{64}\right)^{\frac{1}{2}}}$

$= \sqrt[3]{\left(\frac{1}{8.8}\right)^{\frac{1}{2}}}$

$= \sqrt[3]{\frac{1}{8}}$

$= \sqrt[3]{\frac{1}{2.2.2}}$

$= \frac{1}{2}$

11 E. Question

Let us write explaining the greater value between 3^{3^3} and $\left(3^3\right)^3$ with reason.

Answer

Given equation are 3^{3^3} and $(3^3)^3$

Let as find out the value of

$3^3 = 27$

$27^3 = 19683$ ...... (a)

But

$3^{27} = 3^{3^3}$ ...... (b)

$3^{27} = 7625597484987 = 3^{3^3}$

Now if we compare $3^{27}$with 27^3 then

$3^{27} > 27^3$

From (a) and (b)

$3^{3^3} > (3^3)^3$

Therefore are $3^{3^3} > (3^3)^3$ from comparing the exact value.