CHAPTER :- 8

Theories of failure :- (TOF)

(Imp for Interview)

Aim of this chapter is to ~~drive expression for~~ determine safe dimensions of a Component when crital point in the component is subjected to bi-axial & tri-axial state of stress.

- T.O.F. are used in the design of following machine component like.
  - (i) Power transmission shaft
  - (ii) I.C. engine crank shaft
  - (iii) spindel of a screw jack
  - (iv) Boolted joint used under eccentric loading
  - (v) welded joint ——— " ———

→ T.O.F. should be use in the design of above machine Component due to the unavailability of failure stress under similar state of stress condition. (i.e. Bi-axial & tri-axial state of stress Conditions)

- TOF are used to stablish a relationship between stress induced under Combineds state stress Condition and properties obtained from tension test like yield strength & ultimate strength

→ under uni-axial state of stress strength criterion & TOF will give same result. hence theories of Failure are optional under uni axial state of stress condition.

→ Under bi-axial & tri-axial condition all the T.O.F. will give diff. result hence appropiate TOF should be selected for Safe design of Component

Various T.O.F :-

1. Max. principal stress theory. (MPST) (Rankine's theory)

2. Max. shear stress theory (MSST) (Guest & Tresca's theory)

3. Max. principal strain theory (MPStT) (St. Venant's theory)

4. Total strain energy theory (TSET) (Haighs theory)

5. Max. Distortion energy theory (MDET) or Max. shear strain energy theory (Von-Mise's & Hencky's theory)

Ductile Material - 2 & 5

Brittle Material 1

Condition for failure! –

$$\left(\begin{array}{l}\text{max. Distortion energy per} \\ \text{unit volume at a critical} \\ \text{point in a component} \\ \text{under tri-axial state} \\ \text{of stress}\end{array}\right) > \left(\begin{array}{l}\text{Distortion energy per} \\ \text{unit volume at yield} \\ \text{point under tension} \\ \text{test}\end{array}\right).$$

- Max. principal stress theory (MPST)

⇒ best T.O.F. for brittle material under any state of stress condⁿ (effect of shear stress & $\sigma_{2,3}$ Neglected)

⇒ Also suitable for ductile material under.
- (i) uni-axial state of stress
- (ii) bi-axial state of stress when $\sigma_{1,2}$ are like in nature
- (iii) hydro static state of stress condⁿ (No shear)

For ductile material

* MDET is the best T.O.F. (i.e. safe & economic design)
* MSST will give over safe design (i.e. uneconomic design)

⇒ Both theories of failure are not suitable under hydrostatic state of stress condition.

MPST and MSST will give same result under following condition
- (i) Uni-axial state of stress condition.
- (ii) bi-axial state of stress condⁿ when principal stress are like in nature.
- (iii) For hydro-static state of stress conditions.

MPST, MPSt.T & TSE.T are suitable
↳ best T.O.F. for hydrostatic

$\Rightarrow \sigma_1 = \tau \; ; \; \sigma_2 = -\tau$

when yield shear occurs $\tau = S_{ys}$

$\sigma_1 = S_{ys} \; ; \; \sigma_2 = -S_{ys}$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = (S_{yt})^2$$

$$(S_{ys})^2 + (S_{ys})^2 + (S_{ys})^2 = (S_{yt})^2$$

$$\boxed{S_{ys} = \frac{S_{yt}}{\sqrt{3}}}$$

Conclusion

* Me (equivalent Bending Moment), Te (equavalent torque) eqns should be use for safe design of solid circular ~~shaft~~ Component. subject to both bending moment & twisting moment

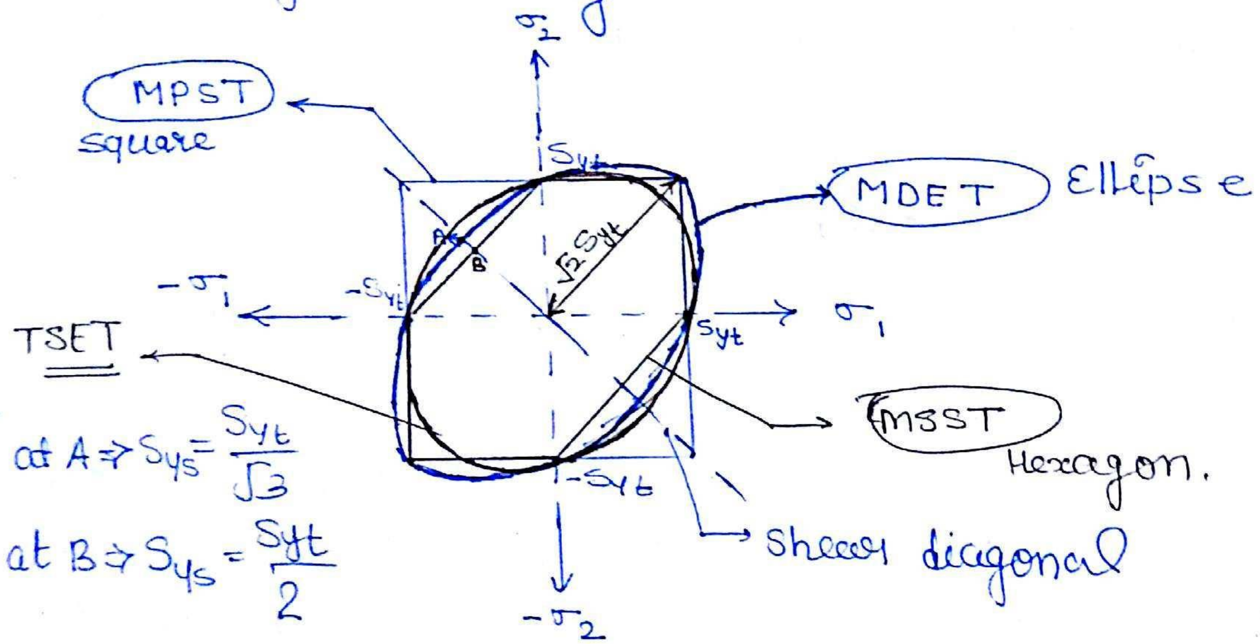
* Permissable tensile equations should be use when crictical point in a component is subjected to bi-axial state of stress in such a way that normal stress is in one of the ⊥er dirn passing through point is zero (either σ_x or $\sigma_y = 0$)

* Disign eqn are the best equation because they are valid under any state of stress condition but these eqn become simple when σ_1 & σ_2 at that point are know (i.e. principal stress know)

* Ratio of $\frac{S_{ys}}{S_{yt}}$ are obtained by substuting $\sigma_1 = S_{yt}$ & $\sigma_2 = -S_{yt}$, $N = 1$ in correspond theories of failure equations.

* ~~sh safosat~~ shape of safe boundary are valid for ductile material because $\sigma_1 = S_{yt}$, $\sigma_2 = -S_{yt}$

* safe boundary are used to check weather given dimension of a component are safe or unsafe under given loading condition.



* larger the area larger the failure

⇒ Area of the T.O.F. curve in a quadrant (↑)

⇓

Failure stress (↑) ✓

⇓

dimensions (↓) ✓

⇓

Safety & Cost (↓) ✓

In all the quadrants

(Area of MDET) > (Area of MSST)

⇓

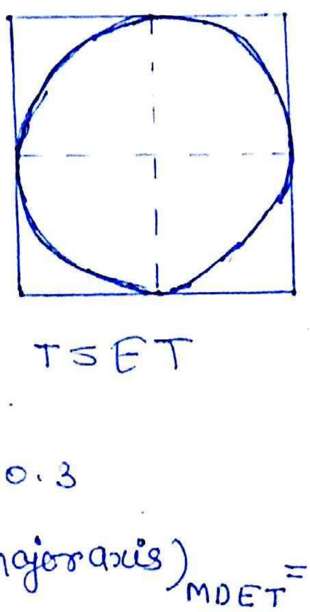
$(\text{failure stress})_{MDET} > (\text{Failure stress})_{MSST}$

⇓

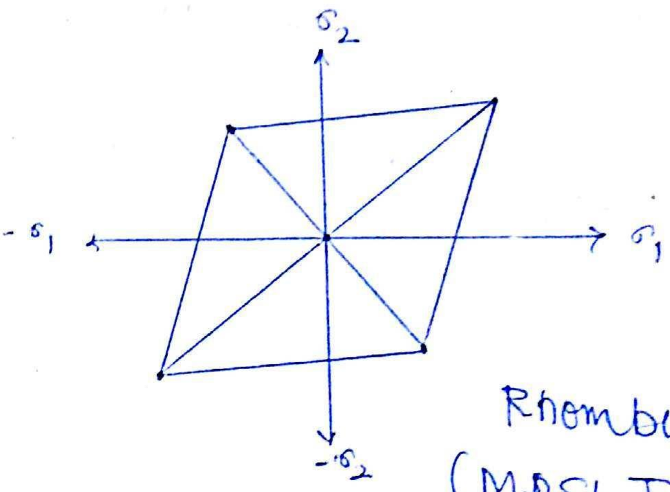
$(\text{dim}^{ns})_{MDET} < (\text{dim}^{ns})_{MSST}$

⇓

$(\text{Safety \& Cost})_{MDET} < (\text{Safety \& Cost})_{MSST}$



TSET



Rhombus (MPSt.T)

$\mu = 0.3$

$(\text{semi major axis})_{MDET} = 1.414\, S_{yt}$ $(\text{Semi } \cancel{\text{major}} \text{ minor axis})_{MDED} = 0.82\, S_{yt}$

$(\text{Semi major axis})_{TSET} = 1.2\, S_{yt}$ $(\text{Semi minor axis})_{TSET} = 0.87\, S_{yt}$

* Distortion energy does not depends on orientation of shear stress. $\boxed{\text{total S.E./Vol} = \text{Vol-S.E./Vol} + \text{DE/Vol}}$

$$\text{Total S.E./Vol} = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$$

$$\text{Vol. S.E.} = \frac{1}{2}(\sigma_{avg})\,\epsilon_v = \frac{1}{2}\left[\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right]\left[\frac{1-2\mu}{E}\right]\left[\sigma_1+\sigma_2+\sigma_3\right]$$

Q. 18 workbook
Pg. 49

$\sigma_1 = 360$ MPa

$\sigma_2 = 140$ MPa

$(\sigma_{wkg})_{MDET} = ?$ working / Permissable stress

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \left(\frac{S_{yt}}{N}\right)^2$$

$$(360)^2 + (140)^2 - (360)(140) = (\sigma_{wkg})^2$$

$$\sigma_{working} = 314.32 \text{ MPa}$$

Q.15

$(\sigma_b)_{max} = 80$ MPa $= \sigma_x$

$\tau_{max} = 30$ MPa $= \tau_{xy}$

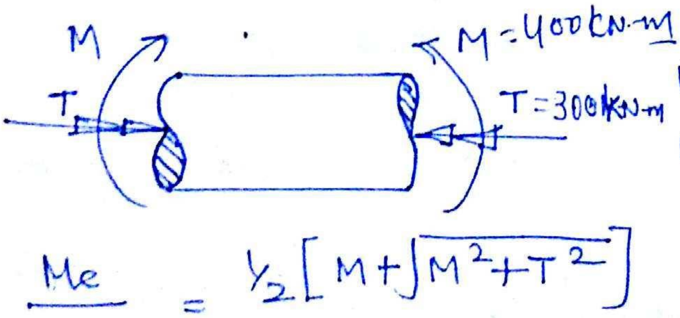
$S_{yt} = 280$ MPa. $(N)_{MSST} = ?$

$$(\sigma_t)_{per} = \frac{S_{yt}}{N} = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$\frac{280}{N} = \sqrt{(80)^2 + 4(30)^2}$$

$$N = 2.8$$

Q.16



MSST → τ

MPST → σ

$$\frac{M_e}{T_e} = \frac{\frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]}{\sqrt{M^2 + T^2}} = \frac{\frac{\pi}{32} d^3 (\sigma_t)_{per} \to \sigma}{\frac{\pi}{16} d^3 (\tau_{per}) \to \tau}$$

$$\Rightarrow \frac{1}{2}\left[\frac{9}{5}\right] = \frac{1}{2}\frac{\sigma}{\tau} \Rightarrow \boxed{\frac{\sigma}{\tau} = \frac{9}{5}}$$

Q.81

$$T_e = \sqrt{M^2 + T^2}$$

$$T_{e_P} = \sqrt{10^2 + 45^2} = \sqrt{2125} = 46.097$$

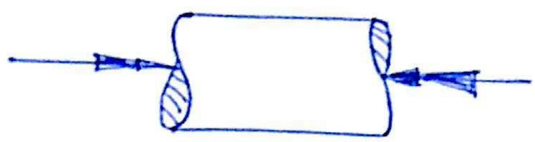
$$T_{e_Q} = \sqrt{40^2 + 30^2} = \sqrt{2500} = 50$$

✓ $$T_{e_R} = \sqrt{20^2 + 50^2} = \sqrt{2900} = 53.85$$

$$T_{e_T} = \sqrt{15^2 + 40^2} = \sqrt{1825} = 42.72$$

Q 14 Pure bending comes under uni-axial state of stress. Under uni-axial state of stress all theories give same result

Q12



MSST

$$(T)_{per} = T$$

MPST

$$T_{per} = T_1 = ?$$

$$T = \frac{\pi}{16} d^3 \left(\frac{S_{ys}}{N}\right)$$

For Given dia. & N

$$T \propto (S_{ys})$$

$$\frac{T_{MPST}}{T_{MSST}} = \frac{(S_{ys})_{MPST}}{(S_{ys})_{MSST}} = \frac{S_{yt}}{S_{yt}/2} = 2$$

$$\boxed{T_{MPST} = 2\,T_{MSST}}$$

Q.9 For a given torque

$$\frac{d_{MSST}}{d_{MPST}} = ?$$

For a given torque Under FOS (N)

$$d^3 \propto \frac{1}{S_{ys}}$$

$$\left(\frac{d_{MSST}}{d_{MPST}}\right)^3 = \frac{(S_{ys})_{MPST}}{(S_{ys})_{MSST}} = \frac{S_{yt}}{S_{yt}/2} = 2$$

$$\frac{d_{MSST}}{d_{MPST}} = (2)^{1/3} = 1.26$$

* Ratio of dia acc. to MSST & MDET

$$\left(\frac{d_{MSST}}{d_{MDET}}\right)^3 = \frac{(S_{ys})_{MDET}}{(S_{ys})_{MSST}} = \frac{S_{yt}/\sqrt{3}}{S_{yt}/2} = \frac{2}{\sqrt{3}}$$

$$\frac{d_{MSST}}{d_{MDET}} = 1.05$$

Q.25. $|\sigma_1| = \left\{ \frac{S_{yt}}{N} \right\}$ ⇒ $\frac{S_{yt}}{(1)} = |-(-100)| = 100$

σ_1 & σ_2 are like in nature $N = 1$