Circles

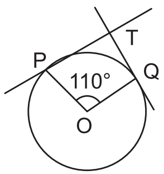
Exercise 3.1

I. Very Short Answer Type Questions **[1 Mark]**

1. **Multiple Choice Questions (MCQs)**

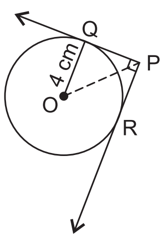
Choose the correct answer from the given options:

(1) In the given figure, if TP and TQ are tangents to a circle with centre O, so that $\angle POQ = 110°$, then $\angle PTQ$ is



(*a*) 110° (*b*) 90° (*c*) 80° (*d*) 70°

(2) In the given figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR = 90°$, then length of PQ is [CBSE Standard 2020]



(*a*) 3 cm (*b*) 4 cm (*c*) 2 cm (*d*) $2\sqrt{2}$ cm

(3) From an external point A, two tangents AB and AC are drawn to the circle with centre O. Then OA is the perpendicular bisector of

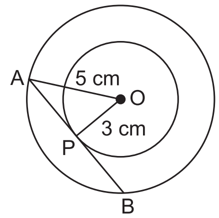
(*a*) BC (*b*) AB (*c*) AC (*d*) none of these

(4) If the radii of two concentric circles are 6 cm and 10 cm, the length of chord of the larger circle which is tangent to other is

(*a*) 14 cm (*b*) 16 cm (*c*) 18 cm (*d*) 12 cm

(5) In the given figure, the length PB is equal to

(*a*) 2 cm
(*b*) 7 cm
(*c*) 6 cm
(*d*) 4 cm



2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(*c*) Assertion (A) is true but reason (R) is false.
(*d*) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** When two tangents are drawn to a circle from an external point, they subtend equal angles at the centre.
Reason (R): A parallelogram circumscribing a circle is a rhombus.

(2) **Assertion (A):** If in a cyclic quadrilateral, one angle is 40°, then the opposite angle is 140°.
Reason (R): Sum of opposite angles in a cyclic quadrilateral is equal to 360°.

(3) **Assertion (A):** PA and PB are two tangents to a circle with centre O such that ∠AOB = 110°, then ∠APB = 90°.
Reason (R): The length of two tangents drawn from an external point are equal.

3. Answer the following:

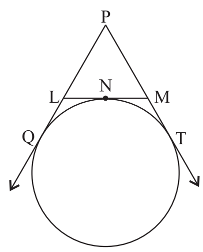
(1) Two concentric circles of radii a and b ($a > b$) are given. Find the length of the chord of the larger circle which touches the smaller circle. [AI 2019, Foreign 2015]

(2) From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB. [Delhi 2016]

(3) If the angle between two tangents drawn from an external point 'P' to a circle of radius 'r' and centre O is 60°, then find the length of OP. [CBSE Standard SP 2019-20, AI 2017]

(4) If the radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle. [CBSE Standard SP 2019-20]

(5) If PQ = 28 cm, then find the perimeter of ΔPLM. [CBSE Standard SP 2020-21]

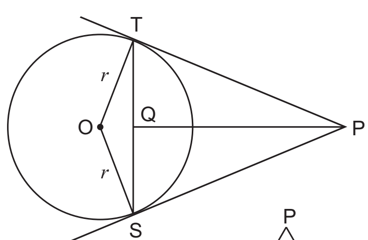


(6) PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then find ∠OQP. [CBSE Standard SP 2020-21]

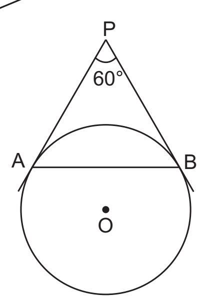
II. Short Answer Type Questions - I [2 Marks]

4. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre. [CBSE 2014]

5. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = $2r$, show that ∠OTS = ∠OST = 30°. [CBSE 2016]



6. In the given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠APB = 60°. Find the length of chord AB. [CBSE 2016]

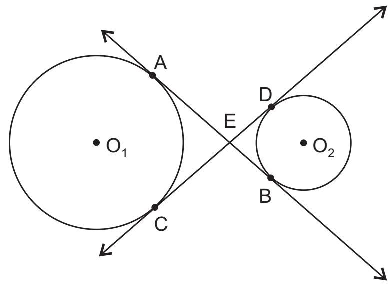


7. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 12 cm. Find the length of PQ. [NCERT]

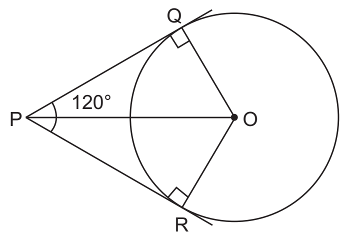
8. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle. [NCERT] **[Imp.]**

9. Prove that the tangents at the extremities of a chord of a circle make equal angles with the chord. [NCERT Exemplar][AI 2017]

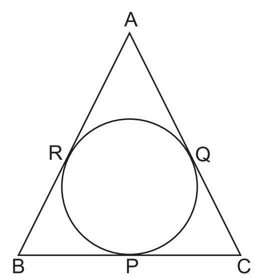
10. In the given figure, common tangents AB and CD to two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD. [AI 2015]



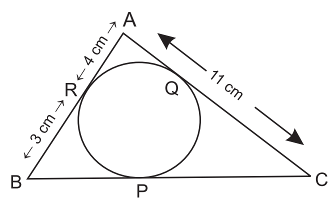
11. If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that ∠QPR = 120°, prove that 2PQ = PO. [Delhi 2014, Foreign 2016]



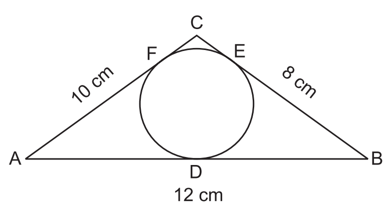
12. In the given figure, a circle inscribed in ΔABC, touches its sides BC, CA and AB at the points P, Q and R respectively. If AB = AC, then prove that BP = CP. [Foreign 2013]



13. In the given figure, ΔABC is circumscribing a circle. Find the length of BC. [CBSE 2009]

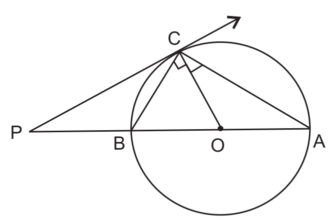


14. A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in the following figure. Find AD, BE and CF. [CBSE 2009, 2012, 2013] **[Imp.]**

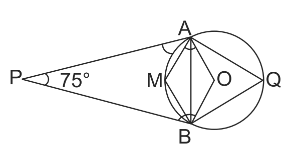


15. PA and PB are tangents from P to the circle with centre O. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that KN = AK + BN.

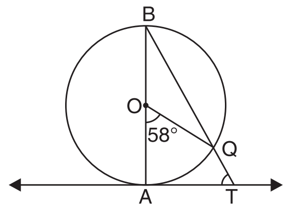
16. PC is a tangent to the circle at C. AOB is the diameter which when extended meets the tangent at P. Find ∠CBA, ∠AOC and ∠BCO, if ∠PCA = 110°. [CBSE 2013]



17. In the given figure, O is the centre of the circle. Determine ∠AQB and ∠AMB, if PA and PB are tangents and ∠APB = 75° [CBSE 2012]

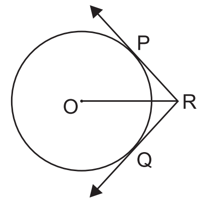


18. In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$. [Delhi 2016]

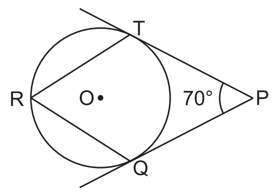


19. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of the line segment PQ. [Delhi 2016]

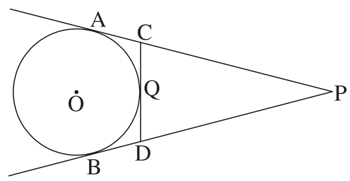
20. In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that OR = PR + RQ. [AI 2016]



21. In the given figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70°$, find $\angle TRQ$. [Foreign 2016]



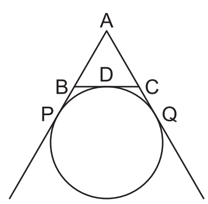
22. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD. [Delhi 2017]



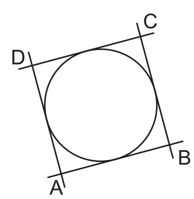
23. A circle touches all the four sides of a quadrilateral ABCD. Prove that

AB + CD = BC + DA [CBSE Standard 2020, AI 2017, CBSE 2016, NCERT]

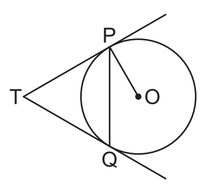
24. In the given figure, find the perimeter of ΔABC, if AP = 12 cm. [CBSE Standard 2020]



25. In the given figure, a circle touches all the four sides of a quadrilateral ABCD in which AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.



26. In the given figure, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\,\angle OPQ$. [Standard 2020]



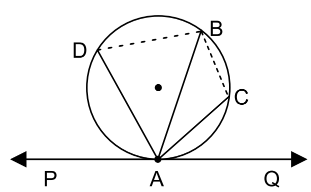
27. Using the given figure, answer the following questions:

(*i*) Name the alternate segment of circle of $\angle BAQ$

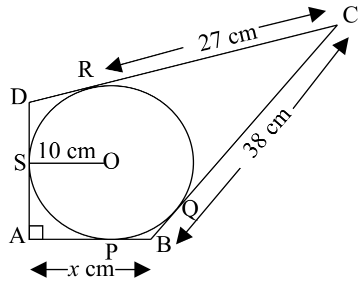
(*ii*) Name the alternate segment of circle of $\angle DAP$.

(*iii*) If B is joined with C then $\angle ACB$ is equal to which angle?

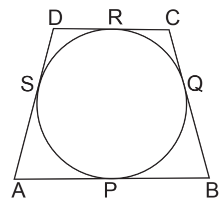
(*iv*) $\angle ABD$ and $\angle ADB$ is equal to which angles?



28. In the figure, quadrilateral ABCD is circumscribing a circle with centre O and AD ⊥ AB. If radius of incircle is 10 cm, then find the value of x. [CBSE Standard SP 2020-21]



29. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.



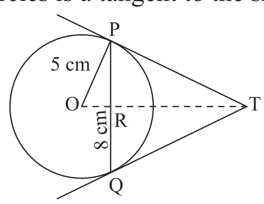
III. Short Answer Type Questions - II **[3 Marks]**

30. The incircle of a ΔABC touches the sides AB, BC and CA at P, Q, R respectively. Show that

$AP + BQ + CR = PB + QC + RA = \frac{1}{2}$(Perimeter of ΔABC) **[Imp.]**

31. If a chord AB of the larger of the two concentric circles is a tangent to the smaller circle at P, prove that PA = PB.

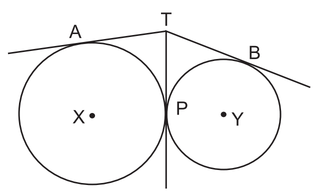
32. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP. [AI 2019]



33. The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle, touching it at D and intersecting the larger circle at P on producing. Find the length of AP. [CBSE SP 2018-19]

34. A circle is touching the side BC of ΔABC at X and touching AB and AC produced at P and Q respectively. Prove that $AP = AQ = \frac{1}{2}$(Perimeter of ΔABC). Given AP = 10 cm, find the perimeter of ΔABC. [CBSE 2001, 2002]

35. In the given figure, two circles with centres X and Y touch externally at P. If tangents AT and BT meet the common tangent at T, then prove that AT = BT. [CBSE 2013] **[Imp.]**



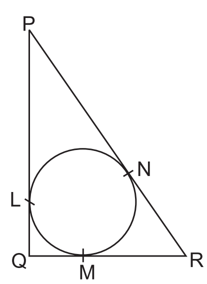
36. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre. [NCERT]

37. Prove that the tangents drawn at the ends of a diameter of a circle are parallel. [NCERT, CBSE 2014] **[Imp.]**

38. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact. [Foreign 2012]

39. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. [AI 2019, Delhi 2013]

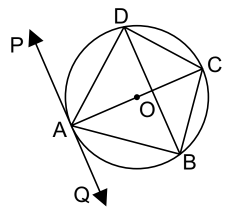
40. In the given figure, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths of QM, RN and PL. [AI 2013]



41. Two concentric circles are of radii 7 cm and r cm respectively, where $r > 7$ cm. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r. [Delhi 2009]

42. If d_1, d_2, $(d_2 > d_1)$ be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$. [Foreign 2009] [NCERT Exemplar]

43. In a cyclic quadrilateral ABCD, diagonal AC bisects the angle C (see the given figure). Then prove that diagonal BD is parallel to the tangent PQ of a circle which passes through the points A, B, C and D. [Delhi 2020]



IV. Long Answer Type Questions [5 Marks]

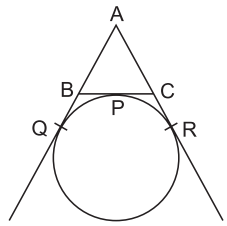
44. Two tangents PA and PB are drawn to a circle with centre O, such that ∠APB = 120°. Prove that OP = 2AP. [CBSE 2016]

45. Tangents AP and AQ are drawn to circle with centre O from an external point A. Prove that ∠PAQ = 2∠OPQ. [CBSE 2017, 2013, 2012, 2011, 2009] **[Imp.]**

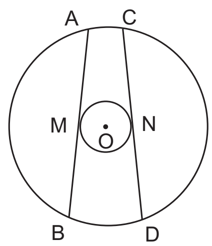
46. A circle touches the side BC of a ΔABC at P and AB and AC when produced at Q and R respectively as shown in the figure. [NCERT Exemplar] [CBSE 2012, 2011]

Show that AQ = $\frac{1}{2}$ (Perimeter of ΔABC)

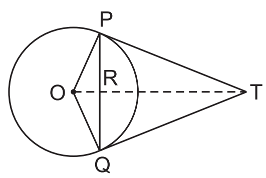
or show that AQ = $\frac{1}{2}$ (BC + CA + AB).



47. In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length (see figure). [CBSE 2015]



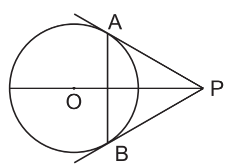
48. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ. [Foreign 2015]



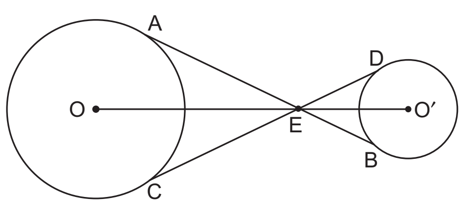
49. If a, b, c, are the sides of a right-angled triangle, where c is the hypotenuse, then prove that the radius r of the circle which touches the sides of the triangle is given by $r = \frac{a+b-c}{2}$. [NCERT Exemplar; CBSE 2012]

50. Two concentric circles of radii 13 cm and 12 cm, are given. Find the length of chord of the larger circle which touches smaller circle. [CBSE 2012, 2011]

51. In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre O. Prove that OP is the right bisector of line segment AB. [CBSE 2011]



52. In the given figure, common tangents AB and CD of two circles with centres O and O′ intersect at E. Prove that the points O, E and O′ are collinear. [NCERT Exemplar]



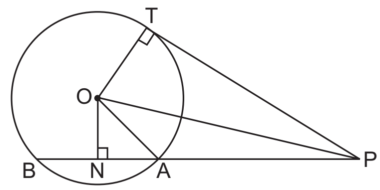
53. If a number of circles pass through the end points P and Q of a line segment PQ, then show that their centres lie on the perpendicular bisector of PQ. [NCERT Exemplar]

54. In the given figure, from an external point P, a tangent PT and a line segment PAB are drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that [NCERT Exemplar]

(*a*) $PA \cdot PB = PN^2 - AN^2$

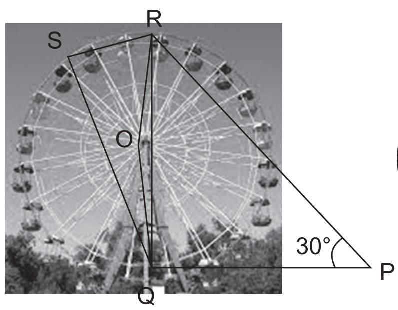
(*b*) $PN^2 - AN^2 = OP^2 - OT^2$

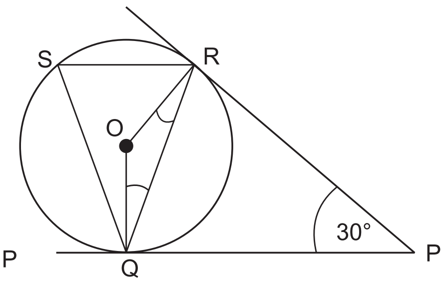
(*c*) $PA \cdot PB = PT^2$



Case Study Based Questions

I. A Ferris Wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity. After taking a ride in Ferris Wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride . She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





1. In the given figure, $\angle ROQ$ is

(*a*) 60° (*b*) 100° (*c*) 150° (*d*) 90°

2. $\angle RQP$ equals

(*a*) 75° (*b*) 60° (*c*) 30° (*d*) 90°

3. $\angle RSQ$ equals

(*a*) 60° (*b*) 75° (*c*) 100° (*d*) 30°

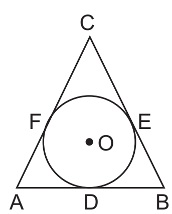
4. $\angle ORP$ equals

(*a*) 90° (*b*) 75° (*c*) 100° (*d*) 60°

5. If $\angle P = 60°$ and OQ = OR = 5 m, then the length of PQ is

(*a*) 5 m (*b*) 12 m (*c*) $5\sqrt{3}$ m (*d*) None of these

II. Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff . The logo design is given as in the figure and he is working on the fonts and different colours according to the theme. In the given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.



1. The length of AD is

(*a*) 7 cm (*b*) 8 cm (*c*) 5 cm (*d*) 9 cm

2. The length of BE is

(*a*) 8 cm (*b*) 5 cm (*c*) 2 cm (*d*) 9 cm

3. The length of CF is

(*a*) 20 cm (*b*) 5 cm (*c*) 2 cm (*d*) 3 cm

4. If the radius of the circle is 4 cm, then the area of triangle OAB is

(*a*) 20 sq cm (*b*) 36 sq cm (*c*) 24 sq cm (*d*) 48 sq cm

5. The area of triangle ABC is

(*a*) 50 sq cm (*b*) 60 sq cm (*c*) 100 sq cm (*d*) 90 sq cm

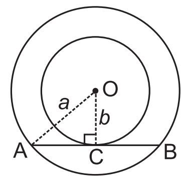
Answers and Hints

1. (1) (*d*) 70° (1) (2) (*b*) 4 cm (1)
(3) (*a*) BC (1) (4) (*b*) 16 cm (1)
(5) (*d*) 4 cm (1)

2. (1) (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (1)
(2) (*c*) Assertion (A) is true but reason (R) is false. (1)
(3) (*d*) Assertion (A) is false but reason (R) is true. (1)

3. (1) AC = BC

$\Rightarrow$ AB = 2AC



Now in ΔOCA,

$AO^2 = OC^2 + AC^2$

$\Rightarrow \quad a^2 = b^2 + AC^2$

$\Rightarrow \quad AC = \sqrt{a^2 - b^2}$

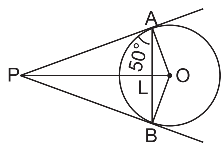
$\therefore \quad AB = 2\sqrt{a^2 - b^2} = 2AC$

Thus, length of chord $= 2\sqrt{a^2 - b^2}$ (1)

(2) Given, $\angle PAB = 50°$

$\angle PAB + \angle OAB = 90°$

[$\because$ an angle between radius OA and tangent PA is 90°]



$\Rightarrow \quad 50° + \angle OAB = 90°$

$\Rightarrow \quad \angle OAB = 90° - 50° = 40°$

$\Rightarrow \quad \angle OBA = 40°$ [$\because$ OA = OB]

Now in ΔAOB, we have

$\angle AOB + \angle ABO + \angle BAO = 180°$

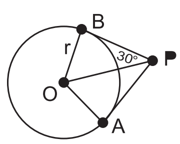
[$\because$ sum of angles in a triangle is 180°]

$\Rightarrow \angle AOB + 40° + 40° = 180°$

$\Rightarrow \quad \angle AOB = 180° - 80°$

$= 100°$ (1)

(3)



In ΔOBP, $\dfrac{OB}{OP} = \sin 30°$

$\Rightarrow \quad \dfrac{r}{OP} = \dfrac{1}{2}$

$\therefore \quad OP = 2r$ (1)

(4) Length of chord

$= 2 \times \sqrt{5^2 - 4^2}$

$= 2 \times 3$ cm $= 6$ cm (1)

(5) PQ = PT

PL + LQ = PM + MT

PL + LN = PM + MN

Perimeter (ΔPLM) = PL + LM + PM (½)

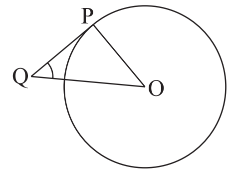
= PL + LN + MN + PM

= 2(PL + LN)

= 2(PL + LQ)

= 2 × 28 = 56 cm (½)

(6) In ΔOPQ



$\angle P + \angle Q + \angle O = 180°$

$2\angle Q + \angle P = 180°$ (½)

$2\angle Q + 90° = 180°$

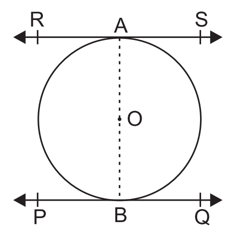
$2\angle Q = 90°$

$\angle Q = 45°$ (½)

4. **Construction:** Join OA and OB.

Proof: As we know, OB is perpendicular to PQ.

[Tangent is perpendicular to radius at the point of contact.]



Now, given, PQ $\parallel$ RS

$\Rightarrow$ BO (produced to RS) is perpendicular to RS. ...(*i*)

[A line perpendicular to one of the two parallel lines is perpendicular to other line also]

Also, OA is perpendicular to RS.

[$\because$ Tangent perpendicular to radius] ...(*ii*) (1)

From (*i*) and (*ii*), OA and OB must coincide as only one line can be drawn perpendicular to the line from a point outside the line.

$\therefore$ AOB is straight line.

$\therefore$ A, O, B are collinear.

$\Rightarrow$ AB passes through O, the centre of the circle. (1)

6. 5 cm (2)

7. Since $PQ = \sqrt{OQ^2 - OP^2} = \sqrt{12^2 - 5^2}$ (1)

$= \sqrt{144 - 25} = \sqrt{119}$ cm (1)

8. $\therefore$ Using Pythagoras theorem, we get

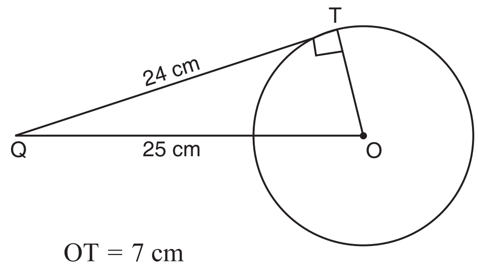
$OQ^2 = QT^2 + OT^2$

$\Rightarrow \quad OT^2 = OQ^2 - QT^2$

$= 25^2 - 24^2$

$= (25 - 24)(25 + 24)$

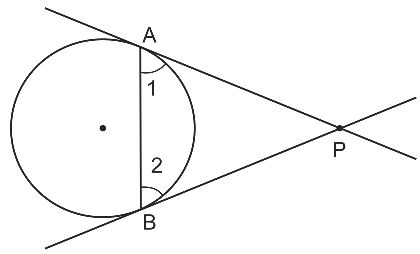
$= 1 \times 49 = 49$ (1)



$\Rightarrow$ OT = 7 cm

Thus, the required radius is 7 cm. (1)

9. We know that tangents drawn from an external point P to a circle are equal so PA = PB. (1)



$\Rightarrow \angle 2 = \angle 1$

[Angles opposite to equal sides of a triangle are equal]

Hence, tangents PA and PB make equal angles with chord AB.

Hence proved. (1)

10. AB and CD are common tangents to the two given circles with centres O_1 and O_2 respectively (refer to question for figure).

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$\therefore$ AE = EC and EB = ED (1)

$\Rightarrow$ AE + EB = CE + ED $\Rightarrow$ AB = CD (1)

11. Proof: In triangle OQP and ORP,

OQ = OR = r (say) [Radii]

OP = OP (Common)

PQ = PR

[The lengths of the tangents drawn from an external point to a circle are equal]

$\therefore \Delta OQP \cong \Delta ORP$ (by SSS)

$\therefore \angle OPQ = \angle OPR$ (CPCT) (1)

Now, given $\angle QPR = 120°$

$\Rightarrow \angle OPQ + \angle OPR = 120°$

$\Rightarrow 2\angle OPQ = 120°$

$\Rightarrow \angle OPQ = 60° = \angle OPR$

Now, in ΔOQP, $\angle Q = 90°$

[$\because$ Tangent is perpendicular to radius]

Then, $\dfrac{PQ}{OP} = \cos 60° = \dfrac{1}{2}$

$\Rightarrow$ OP = 2PQ (1)

12. AB = AC

$\Rightarrow$ AR + BR = AQ + CQ

$\Rightarrow$ AR + BR = AR + CQ [AQ = AR equal tangents]

$\Rightarrow$ BR = CQ (1)

Now, BR = BP [Length of equal tangents]

and CQ = CP

$\Rightarrow$ BP = CP (1)

13. BP = BR = 3 cm

AQ = AR = 4 cm (1)

$\Rightarrow$ CQ = 11 cm – 4 cm = 7 cm

$\Rightarrow$ CP = CQ = 7 cm

Hence, BC = BP + CP = (3 + 7) = 10 cm (1)

14. Let AD = x_1, BE = x_2 and CF = x_3;

then AF = AD = x_1, BD = BE = x_2

and CE = CF = x_3.

$\therefore x_1 + x_2 = 12;\ x_2 + x_3 = 8;\ x_1 + x_3 = 10$ (1)

Adding,

$2(x_1 + x_2 + x_3) = 30$

$\Rightarrow x_1 + x_2 + x_3 = 15$

Solve for x_1, x_2 and x_3 to get

AD = 7 cm, BE = 5 cm,

CF = 3 cm (1)

16. $\angle AOC = 140°$, $\angle CBA = 70° = \angle BCO$ (2)

17. $\angle AQB = 52.5°$, $\angle AMB = 127.5°$ (2)

18. $\because$ AT is a tangent and BA is a diameter.

So OA $\perp$ AT

$\Rightarrow \angle OAT = 90°$ or $\angle BAT = 90°$

Arc AQ subtends an angle of 58° at the circle.

$\angle AOQ = 2\angle ABQ$

So, $\angle ABQ = 29°$ (1)

In ΔABT,

$\angle A + \angle ABT + \angle ATB = 180°$

$\Rightarrow 90° + 29° + \angle ATB = 180° \Rightarrow \angle ATB = 61°$

$\Rightarrow \angle ATQ = 61°$

Hence, $\angle ATQ = 61°$. (1)

19. TP and TQ are tangents at P and Q respectively.

So, OP $\perp$ PT and OQ $\perp$ QT

$\therefore \angle OPT = \angle OQT = 90°$

In ΔOPT and ΔOQT

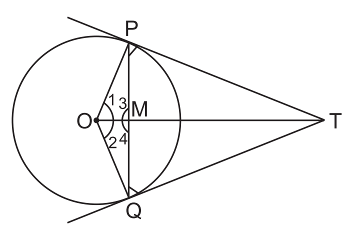
OP = OQ (Radius)

$\angle P = \angle Q$ (Each 90°)

OT = OT (Common)

So, $\Delta OPT \cong \Delta OQT$ (RHS)

$\Rightarrow \angle 1 = \angle 2$ (CPCT) (1)



Now, in ΔOMP and ΔOMQ,

OP = OQ (Radius)

$\angle 1 = \angle 2$ (Proved above)

OM = OM (Common)

So, $\Delta OMP \cong \Delta OMQ$ (SAS)

$\Rightarrow$ PM = MQ and $\angle 3 = \angle 4$ (By CPCT)

Now $\angle 3 + \angle 4 = 180°$ ($\because$ Linear Pair Axiom)

$\Rightarrow 2\angle 3 = 180° \Rightarrow \angle 3 = 90°$

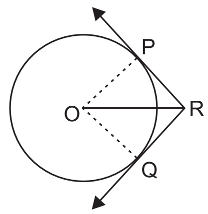
$\Rightarrow \angle OMP = 90°$

Hence, OT is the right bisector of the line segment PQ.(l)

20. OR bisects $\angle PRQ$

$\therefore \quad \angle PRO = \angle QRO = 60°$

$[\because \quad \angle PRQ = \angle ORP + \angle ORQ = 120°]$



In right ΔOPR,

$OP \perp PR$

[$\because$ radius is perpendicular to the tangent at point of contact]

$\therefore \quad \cos\angle ORP = \dfrac{PR}{OR} = \cos 60°$

$\Rightarrow \quad OR = 2PR$ ...(*i*) (1)

Similarly, in right ΔOQR,

$\dfrac{QR}{OR} = \cos 60° \Rightarrow OR = 2QR$ ...(*ii*)

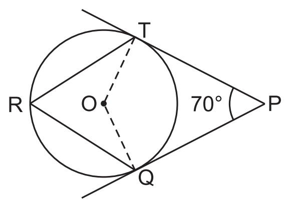
Adding (*i*) and (*ii*), we get

$2OR = 2PR + 2QR$

$\Rightarrow \quad OR = PR + RQ$ (1)

21. We know that tangent is perpendicular to radius. Hence,

$\angle OTP = \angle OQP = 90°$



(1)

In quadrilateral PQOT,

$\angle QOT + \angle OTP + \angle TPQ + \angle OQP = 360°$

$\Rightarrow \angle TOQ + \angle TPQ = 180° \Rightarrow \angle TOQ = 110°$

Also $\quad \angle TOQ = 2\angle TRQ$

$\Rightarrow \quad 110° = 2\angle TRQ$

$\Rightarrow \quad \angle TRQ = 55°$ (1)

22. $PA = PC + CA = PC + CQ$

$\Rightarrow \quad 12 = PC + 3$

$\Rightarrow \quad PC = 9$ cm (1)

Similarly, $\quad PD = 9$ cm

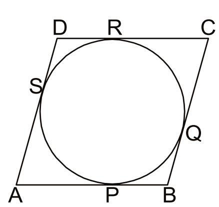
$\therefore \quad PC + PD = 18$ cm (1)

23. Here $\quad AP = AS$

$BP = BQ$

$CR = CQ$

$DR = DS$ (1)

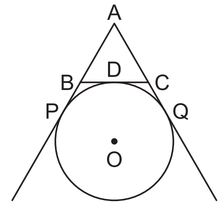


Adding $(AP + PB) + (CR + RD)$

$= (AS + SD) + (BQ + QC)$

$\Rightarrow \quad AB + CD = AD + BC$ (1)

24. Since AP and AQ are two tangents drawn from common external point A.



$\therefore \quad AP = AQ = 12$ cm (Given)

Similarly, $\quad BP = BD$ ...(*i*)

and $\quad CQ = CD$ ...(*ii*) (1)

Now perimeter of

$\Delta ABC = AB + BC + AC$

$= AB + BD + CD + AC$

$= AB + BP + CQ + AC$

[From (*i*) and (*ii*)]

$= AP + AQ = 12 + 12$

$= 24$ cm (1)

25. 3 cm (2)

27. (*i*) $\angle ADB$ (*ii*) $\angle ABD$

(*iii*) $\angle BAP$ (*iv*) $\angle DAP$ and $\angle BAQ$ (2)

28. $\angle A = \angle OPA = \angle OSA = 90°$ (½)

Hence, $\quad \angle SOP = 90°$

Also, $\quad AP = AS$

Hence, OSAP is a square

$AP = AS = 10$ cm (½)

$CR = CQ = 27$ cm

$BQ = BC - CQ$

$= 38 - 27 = 11$ cm (½)

$BP = BQ = 11$ cm

$x = AB = AP + BP$

$= 10 + 11 = 21$ cm (½)

29. **To prove:** $AB + CD = AD + BC$

Proof: $AS = AP$ (Length of tangents from an external point to a circle are equal)

$BQ = BP$

$CQ = CR$

$DS = DR$ (1)

$AS + BQ + CQ + DS = AP + BP + CR + DR$

$(AS + DS) + (BQ + CQ)$

$= (AP + BP) + (CR + DR)$

$AD + BC = AB + CD$ (1)

32. **Given:** $\quad PQ = 8$ cm and $PO = 5$ cm

Now, $\quad PR = RQ$ (Perpendicular from the centre bisects the chord)

$= \dfrac{8}{2} = 4$ cm (1)

In ΔOPR, $\quad OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$

$= \sqrt{25 - 16} = \sqrt{9} = 3$ cm

Let $\angle POR$ be θ.

In ΔPOR, $\tan\theta = \dfrac{PR}{RO} = \dfrac{4}{3}$ (1)

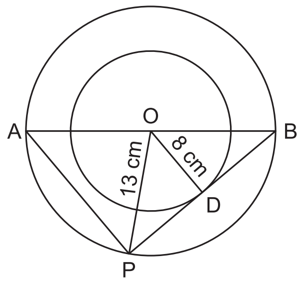
We know, OP ⊥ TP as point of contact of a tangent is perpendicular to the line from the centre.

In ΔOTP, $\tan\theta = \frac{OP}{TP} \Rightarrow \frac{4}{3} = \frac{5}{TP} \Rightarrow TP = \frac{15}{4}$

$\Rightarrow$ TP = 3.75 cm (1)

33. $\angle APB = 90°$ (Angle in semi-circle) (½)

$\angle ODB = 90°$ (Radius is perpendicular to tangent)



(1)

$\Delta ABP \sim \Delta OBD$

$\Rightarrow \frac{AB}{OB} = \frac{AP}{OD} \Rightarrow \frac{26}{13} = \frac{AP}{8}$

$\Rightarrow$ AP = 16 cm (1½)

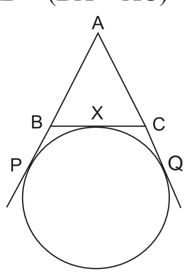
34. In the given figure,

AP = AQ, BP = BX and CX = CQ

$\Rightarrow$ AB + BX = AC + CX ...(*i*) (1)

∴ Perimeter of

ΔABC = AB + BC + CA
= AB + (BX + XC) + CA



= (AB + BX) + (XC + CA)
= 2(AB + BX), using (*i*). (1)

Hence, AP = $\frac{1}{2}$ × (perimeter of ΔABC) = AQ.

If AP = 10 cm, then perimeter of ΔABC = 20 cm. (1)

36. In right Δ OAP and right Δ OBP, we have

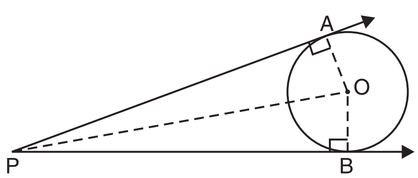
PA = PB [Tangents to circle from an external point P]

OA = OB [Radii of the same circle]

OP = OP [Common]

∴ By SSS congruency,

$\Delta OAP \cong \Delta OBP$ (1)



∴ Their corresponding parts are equal.

∴ $\angle OPA = \angle OPB$

and $\angle AOP = \angle BOP$

$\Rightarrow$ $\angle APB = 2\angle OPA$

and $\angle AOB = 2\angle AOP$ (1)

But $\angle AOP = 90° - \angle OPA$

$\Rightarrow$ $2\angle AOP = 180° - 2\angle OPA$

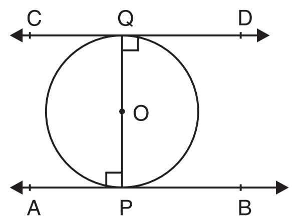
$\Rightarrow$ $\angle AOB = 180° - \angle APB$

$\Rightarrow$ $\angle AOB + \angle APB = 180°$. (1)

37. In the figure, we have:

PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.



Since the tangent at a point to a circle is perpendicular to the radius through the point. (1)

∴ PQ ⊥ AB $\Rightarrow$ $\angle APQ = 90°$

and PQ ⊥ CD $\Rightarrow$ $\angle PQD = 90°$ (1)

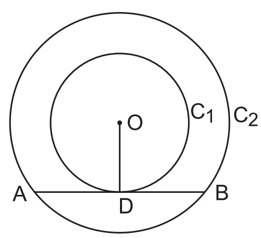
$\Rightarrow$ $\angle APQ = \angle PQD$

But they form a pair of alternate angles.

∴ AB || CD. (1)

38. **Given:** Let O be the centre of two concentric circles C_1 and C_2.

Let AB be the chord of larger circle C_2 which is a tangent to the smaller circle C_1 at D.



To prove: Now we have to prove that the chord AB is bisected at D that is AD = BD. (1)

Construction: Join OD.

Proof: Now since OD is the radius of the circle C_1 and AB is the tangent to the circle C_1 at D.

So, OD ⊥ AB

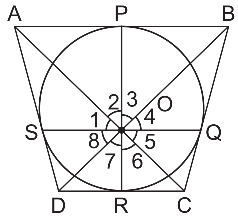
[Radius of the circle is perpendicular to tangent at any point of contact] (1)

Since AB is the chord of the circle C_2 and OD ⊥ AB.

∴ AD = DB

[Perpendicular drawn from the centre to the chord always bisects the chord] (1)

39. Given: ABCD is a quadrilateral circumscribing the circle with centre O touching it at P, Q, R, S.



To prove:

$\angle AOB + \angle DOC = 180°$

$\angle AOD + \angle BOC = 180°$

Construction: Join AO, PO, BO, QO, CO, RO, DO, SO. (1)

Proof: In ΔAOS and ΔAOP,

$AO = AO$ (Common)

$AS = AP$ (Tangents from external point)

$OS = OP$ (Radii of same circle)

By SSS congruence rule,

$\Delta AOS \cong \Delta AOP$

$\angle 1 = \angle 2$ ...(*i*) (by CPCT) (1)

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$ ...(*ii*)

Now,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ [$\because$ ASP of quadrilateral]

$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360°$ [By (*i*) and (*ii*)]

$\Rightarrow 2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360°$

$\angle AOB + \angle COD = 180°$

Similarly, $\angle AOD + \angle BOC = 180°$ (1)

40. Let $QM = x = QL$

$MR = y = RN$

and $PL = z = PN$

Now $PQ = 10$ cm,

$QR = 8$ cm,

$PR = 12$ cm

$\Rightarrow x + y = 8, y + z = 12, z + x = 10$ (1)

$\Rightarrow 2x + 2y + 2z = 8 + 12 + 10 = 30$

$\Rightarrow x + y + z = 15 \Rightarrow 8 + z = 15$

$\Rightarrow z = 7$ (1)

$\Rightarrow x + 12 = 15 \Rightarrow x = 3$

$\Rightarrow y + 10 = 15 \Rightarrow y = 5$

Hence, QM = 3 cm, RN = 5 cm and PL = 7 cm. (1)

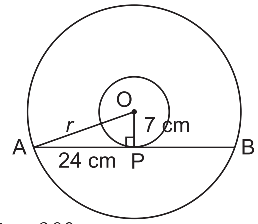
41. Given: $OP = 7$ cm, $OA = r$ cm

$AB = 48$ cm

Now $OP \perp AB$ (as radius makes an angle of 90° with the tangent at point of contact)

Also, $AP = PB$ (Perpendicular drawn from centre to the chord bisects the chord)

So, $AP = 24$ cm (1)



In ΔOPA, $\angle P = 90°$ (1)

By Pythagoras theorem in ΔOPA,

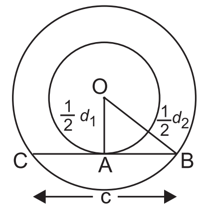
$$OA^2 = AP^2 + OP^2$$

$$r^2 = 24^2 + 7^2$$

$$= 576 + 49 = 625$$

$\Rightarrow r = 25$ cm (1)

42. $\because$ Diameter of bigger circle $= d_2$



So, radius of bigger circle $= \frac{1}{2}d_2 = OB$

Diameter of smaller circle $= d_1$ (1)

So, radius of smaller circle $= \frac{1}{2}d_1 = OA$

$AB = \frac{c}{2}$

In right ΔOAB,

$\angle A = 90°$ [$\because$ Radius is perpendicular to the tangent at point of contact] (1)

By Pythagoras theorem

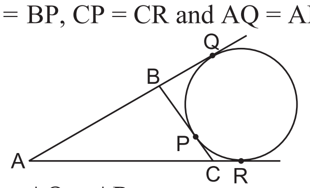
$$OB^2 = AB^2 + OA^2$$

$$\Rightarrow \left(\frac{1}{2}d_2\right)^2 = \left(\frac{1}{2}c\right)^2 + \left(\frac{1}{2}d_1\right)^2$$

$$\Rightarrow \frac{1}{4}d_2^2 = \frac{1}{4}c^2 + \frac{1}{4}d_1^2$$

$\Rightarrow d_2^2 = c^2 + d_1^2$ (1)

46. We have $BQ = BP$, $CP = CR$ and $AQ = AR$ (1)



Now, $2AQ = AQ + AR$

$= (AB + BQ) + (AC + CR)$ (1)

$= AB + BP + AC + CP$

$= (BP + CP) + AC + AB$ (1)

$= BC + CA + AB$ (1)

i.e. $AQ = \frac{1}{2}(BC + CA + AB)$ (1)

48. Join OT intersecting PQ at R.

OT bisects $\angle PTQ$

$\therefore \angle PTO = \angle QTO$

$\therefore \angle PTR = \angle QTR$ ...(*i*) (1)

In ΔPTR and ΔQTR,

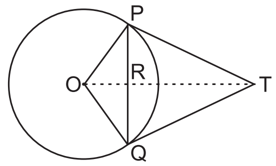
$PT = QT$ [Length of tangents drawn from common external point are equal]

$RT = RT$ [common] (1)

$\angle PTR = \angle QTR$ [$\because$ from (*i*)]

$\therefore$ $\Delta PTR \cong \Delta QTR$ [By SAS]

$\Rightarrow$ $PR = RQ$ [$\because$ By CPCT]



(1)

$\Rightarrow$ R is mid-point of PQ

$OR \perp PQ$

In right triangle ORP,

$OP^2 = PR^2 + OR^2$ [$\because$ Given OP = 5 cm, PQ = 8 cm, PR = QR = 4 cm]

$\Rightarrow$ $25 = 16 + OR^2$

$OR = 3$ cm (1)

In ΔORQ and ΔOQT

$\angle ORQ = \angle OQT$ (Each 90°)

$\angle ROQ = \angle ROQ$ (Common)

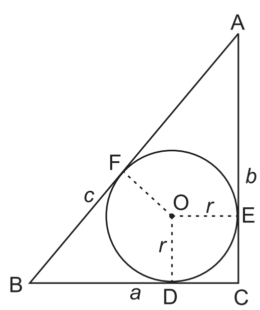
$\Delta ORQ \sim \Delta OQT$ (By AA criterion)

$\Rightarrow$ $\dfrac{OR}{OQ} = \dfrac{RQ}{QT}$ (By C.P. of similar triangles)

$\Rightarrow$ $\dfrac{3}{5} = \dfrac{4}{QT}$ $\Rightarrow$ $QT = \dfrac{20}{3}$ cm

Also $PT = QT$ $\Rightarrow$ $PT = \dfrac{20}{3}$ cm (1)

49. Let the circle touches the sides BC, CA, AB of the right triangle ABC at D, E and F respectively, where BC = a, CA = b and AB = c (see figure). Then AE = AF and BD = BF. Also CE = CD = r. (2)



(1)

i.e. $b - r = AF$, $a - r = BF$

or $AB = c = AF + BF = b - r + a - r$ (1)

This gives $r = \dfrac{a+b-c}{2}$. (1)

50. 10 cm (5)

51. Join OA and OB.

In ΔPAO and ΔPBO,

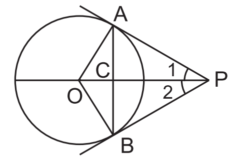
$OA = OB$ [Radii]

$OP = OP$ [Common]

and $AP = BP$ [Tangents from P]

$\because$ $\Delta PAO \cong \Delta PBO$ (SSS congruence rule) (2)

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]



In ΔAPC and ΔBPC,

$\angle 1 = \angle 2$ [Proved]

$AP = BP$ and $PC = PC$ (1)

$\therefore$ $\Delta APC \cong \Delta BPC$ (SSS congruence rule)

$\Rightarrow$ $AC = BC$ [CPCT]

and $\angle ACP = \angle BCP$ [CPCT] (1)

Also, $\angle ACP + \angle BCP = 180°$

$\Rightarrow$ $\angle ACP = \angle BCP = 90°$

$\therefore$ OP is the right bisector of AB. (1)

52. In ΔAEO and ΔCEO,

$OE = OE$ [Common]

$OA = OC$ [Radii of same circle]

$EA = EC$ [Tangents from an external point to a circle are equal in length]

$\therefore$ $\Delta OEA \cong \Delta OEC$

[By SSS criterion of congruence] (1)

$\Rightarrow$ $\angle OEA = \angle OEC$ [CPCT]

$\therefore$ $\angle 1 = \angle 2$ [CPCT] (1)

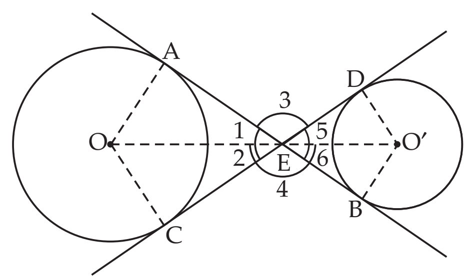
Similarly, $\angle 5 = \angle 6$

and $\angle 3 = \angle 4$ [Vertically opposite angles] (1)

Since sum of angles at a point = 360°

$\therefore$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$

$\Rightarrow$ $2(\angle 1 + \angle 3 + \angle 5) = 360°$



(1)

$\Rightarrow$ $\angle 1 + \angle 3 + \angle 5 = 180°$

$\Rightarrow$ $\angle OEO' = 180°$

$\therefore$ OEO′ is a straight line.

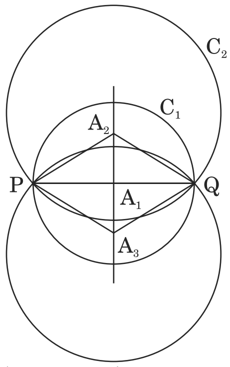
Hence, O, E and O′ are collinear. (1)

53. Centre of any circle passing through the end points P and Q of a line segment are equidistant from P and Q.

$\therefore$ $A_1P = A_1Q$

$A_2P = A_2Q$

$A_3P = A_3Q$ (2)



(1)

As we know that any point on perpendicular bisector of a segment is equidistant from the end points of the segment. Hence, A_1, A_2, A_3 points are the centres of circles passing through the end points P and Q of a segment PQ or the centres of circles lie on the perpendicular bisector of PQ. (2)

54. (*a*) $PA \cdot PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$ (As AN = BN)

$\Rightarrow PA.PB = PN^2 - AN^2$ ...(*i*) (1)

(*b*) $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$ (As $ON \perp PN$)

$= OP^2 - (ON^2 + AN^2)$ (1)

$= OP^2 - OA^2$ (As $ON \perp AN$)

$\Rightarrow PN^2 - AN^2 = OP^2 - OT^2$ (As OA = OT) ...(*ii*) (1)

(*c*) From (*i*) and (*ii*)

$PA \,.\, PB = OP^2 - OT^2 = PT^2$ (As $\angle OTP = 90°$) (1)

Case Study Based Questions

I. 1. (*c*) 150° **2.** (*a*) 75°

3. (*b*) 75° **4.** (*a*) 90°

5. (*c*) $5\sqrt{3}$ m

II. 1. (*a*) 7 cm **2.** (*b*) 5 cm

3. (*d*) 3 cm **4.** (*c*) 24 sq cm

5. (*b*) 60 sq cm