2019 **MATHEMATICS**

Full Marks: 100

Pass Marks: 33

Time: Three hours

Attempt all Questions.

The figures in the right margin indicate full marks for the questions. For Question Nos. 1-6, write the letter associated with the correct answer.

1. The value of
$$\sin \left[\frac{\pi}{3} - \sin^{-1} \left(-\frac{1}{2} \right) \right]$$
 is:

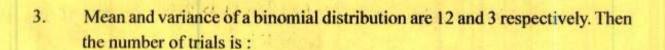
$$B. \qquad \frac{1}{2}$$

$$C. \qquad \frac{1}{\sqrt{2}}$$

2. If
$$f: R \longrightarrow R$$
 be given by $f(x) = (3-x^3)^{\frac{1}{3}}$, then $f^{-1}(x)$ equals:

$$B. \qquad \frac{1}{x^3}$$

C.
$$3-x^3$$
D. $(3-x^3)^{\frac{1}{3}}$



4.
$$\int e^x \sec x (1 + \tan x) dx$$
 equals:

A.
$$e^x \cos x + C$$

B.
$$e^x \sec x + C$$

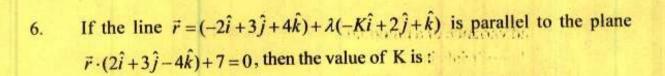
C.
$$e^x \sin x + C$$

D.
$$e^x \tan x + C$$

5. The slope of the normal to the curve
$$y = 2x^2 + 3 \sin x$$
 at $x = 0$ is:

$$B. \qquad \frac{1}{3}$$

$$D. -\frac{1}{3}$$



- A. 0
- B. 1
- C. -1
- D. -2
- 7. Show that the operation * on R₊ (set of all positive real numbers) defined by $a*b = \frac{ab}{3}$, $\forall a,b \in R_+$ is a binary operation on R₊.
- 8. Is Rolle's Theorem applicable to the function f(x) = |x| in the interval [-1, 1]?
- 9. If $\frac{dy}{dx} = \frac{y}{x}$, prove that $\frac{d^2y}{dx^2} = 0$.
- 10. Prove that the function given by $f(x) = x^3 3x^2 + 3x 5$ is increasing in R.
- 11. Evaluate: $\int_{1}^{\sqrt{3}} \frac{1}{1+x^2} dx$.
- 12. What is meant by the general solution of a differential equation?

- 13. If $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, find the angle between \vec{a} and \vec{b} .
- 14. Define position vector of a point.
- 15. The certesian equation of a line is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Write its vector form.
- 16. If α , β , γ be the angles made by a line with the coordinate axes, prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.
- 17. Show that the relation R on N × N defined by $(a,b) R (c,d) \Leftrightarrow a+d=b+c, \ \forall \ (a,b), \ (c,d) \in N \times N$ is an equivalence relation.
- 18. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2,
 - show that $I + A = (I A)\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.
- 19. Evaluate $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$.
- 20. Prove that $\int \frac{1}{\sqrt{a^2 x^2}} dx = \sin^{-1} \left(\frac{x}{a}\right) + C.$
- 21. Find the differential equation of the family of curves $y = e^x (A \cos x + B \sin x)$, where A and B are arbitrary constants.

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- Two cards are drawn simultaneously from a well shuffled pack of 52 cards. 22. Find the probability distribution of the number of aces. 3
- Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$, (xy < 1) and hence deduce that 23.

(i)
$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x - y}{1 + xy} \right), (xy > -1)$$

(ii)
$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1 - x^2} \right), (|x| < 1)$$

- If the inverse of a square matrix exists, prove that it is unique. If A and 24. B are both invertible square metrices of the same order, prove that 4 $(AB)^{-1} = B^{-1}A^{-1}$.
- If f(x) defined by 25.

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{If } x < 0 \\ c, & \text{If } x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{If } x > 0 \end{cases}$$

is continuous at x = 0, find the values of a, b and c.

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26. Find
$$\frac{dy}{dx}$$
, if $x^y + y^x = a^b$.

OR

If
$$x^y = e^{x-y}$$
, prove that $\frac{dy}{dx} = \frac{\log x}{(\log ex)^2}$.

27. Find the area of the region bounded by the triangle whose vertices are (-1, 2),(1, 5) and (3, 4).

OR

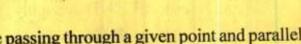
Find the area of the region bounded by the lines x + 2y = 2, y - x = 1 and 2x + y = 7.

- 28. Find the integrating factor of the linear equation $\frac{dy}{dx} + Py = Q$ and hence obtain the general solution of the equation.
- 29. Define cross product of two vectors and give the geometrical interpretation of the cross product of two vectors. If $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$ and $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$, obtain the algebraic formula for $\vec{a} \times \vec{b}$.
- 30. Prove that:

$$\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2} = \frac{\pi}{1 - a^2}; (a < 1)$$

OR

$$\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{\pi^2}{2ab}.$$



32. Derive the vector equation of a line passing through a given point and parallel to a given vector and hence obtain the cartesian equation of the line.

OR

Derive the vector equation of a plane in the normal form and hence obtain the cartesian equation of the plane.

33. Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$.

OR

Prove that the curves $y^2 = x$ and xy = k cut at right angle if $8k^2 = 1$.

34. If
$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$
 and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB and hence solve the

following system of linear equations:

$$x-y=3$$

$$2x + 3y + 4z = 17$$

$$y + 2z = 7.$$

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35. Two godowns A and B have a given storage capacity of 100 quintals and 50 quintals respectively. They supply grain to three ration shops D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from godowns to the shops are given in the table below:

To From	Transportation cost per quintal (in rupees)		
	D	Е	F
A	6	3	2.50
В	4	2	3

How should the supplies be transported in order that the transportation cost is minimum? Solve the problem graphically.