CHAPTER 14

Prestressed Concrete

14.1 Introduction

According to **ACI Committee of Pre-stressed Concrete,** it is defined as "*Pre-stressed concrete is the one in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from the external loading are counter balanced to a desired degree.*" Pre-stressed concrete got developed along with reinforced concrete. Basically in pre-stressed concrete, internal stresses are developed in a predetermined manner such that the stresses induced due to external loading gets nullified with these internal stresses of concrete. This is generally achieved by tensioning the reinforcing steel. Some earlier examples of pre-stressing include the force fitting of metal bands for construction of wooden barrels and the shrink fitting of metal tyres on wooden wheels. The principle of prestressing of concrete lies in the fact that stresses are induced in a concrete member in a planned manner so that when external loads are imposed then stresses due to external loading gets modified by stresses induced due to prestressing.

Do You Know?

What is the Need of prestressing a Concrete Member?

Concrete is quite strong in compression and can take compressive stresses of upto 50 N/mm^2 or more for standard 150 mm cubes. However, concrete is very weak in tension and it is very difficult to have concrete of tensile strength greater than 2.5 N/mm^2. Thus in usual reinforced concrete, it is reinforced with steel bars. The bars are provided where tensile strength of concrete has to be increased. Due to the assumption of perfect bond between steel and concrete, cracks get developed in concrete as the steel yields i.e. undergoes strain. Thus under usual conditions of working loads, concrete develops minute cracks present in the tension zone of concrete.

The prestressing of concrete reduces these defects. A prestressed concrete member is a usual concrete member but tendons are provided to supply the necessary prestressing force. For getting the maximum advantage of prestressing, it is beneficial to have high strength concrete along with high tensile steel tendons.

14.2 Need of High Strength Concrete in Prestressing

High strength concrete for prestressing is required due to following reasons:

1. Due to large prestressing force to be applied by tendons, large bearing stresses get developed in concrete at the ends by anchoring devices.

2. Bursting stresses at the ends cannot be effectively resisted by low strength concrete.
3. Stress transfer from tendons to concrete takes place due to bond between concrete and steel. High bond strength can be achieved by the use of high strength concrete.
4. With the use of high strength concrete, shrinkage cracks are low.
5. High strength concrete will have high modulus of elasticity of concrete $(= 5000\sqrt{f_{ck}})$ and hence elastic and crack strains are very low resulting in smaller loss of prestressing force.
6. Use of high strength concrete reduces the size of concrete members thereby resulting in reduction of self weight of the member.

14.3 Need of High Tensile Steel in Prestressing

High tensile steel is used for the following reasons:

With the use of low tensile steel (e.g. mild steel), due to creep and shrinkage of concrete, loss of prestress is large. Thus, the net tensile stress which is offering the required prestressing force will be very small.

Generally losses due to creep and shrinkage is in the range of 0.0008 (= 0.08%)

$$\therefore \quad \text{Strain lost} = 0.0008$$

$$\Rightarrow \quad \text{Stress lost} = 0.0008 \times E_s = 0.0008 \times 2 \times 10^5 = 160 \text{ N/mm}^2$$

Total loss may be in the range of about 200 to 300 N/mm².

$$\text{But losses} \not> 10 - 20\%$$

So initial prestress in steel should be in the range of 1000 to 2000 N/mm².

14.4 Relative Comparison of Prestressed and Reinforced Concrete Beam

Table 14.1: *Comparison of RC beam with PSC beam*

Reinforced Concrete Beam	Prestressed Beam
Only the concrete in compression side is useful.	The whole concrete section is useful since whole section is in compression i.e., uncracked section.
Reinforced concrete beams are generally larger in size and require additional shear reinforcement to resist shear forces.	Prestressed beams are smaller in size and due to prestressed tendons, a considerable amount of shear reinforcement is avoided.
Use of high strength concrete is not a prime requirement.	High strength concrete is required for prestressed concrete members.
Reinforced concrete beams are quite heavy and large in size.	Prestressed beams are light in weight and smaller in size. These are suitable for long spans carrying heavy loads.
It does not require auxillary units much for construction.	Auxillay units are required like anchoring devices, equipments for prestressing etc.

14.5 Terminologies

Given below are some of the terms that are frequently used in pre-stressed concrete.

14.5.1 Tendon

It is a stretched element in a concrete member to pre-stress to the concrete. In general, high tensile steel wires are used as tendons.

14.5.2 Anchorage

Once the tendon has been stretched to the desired degree, anchorages are provided so that the tendons can impart compressive force to concrete and they also maintain the required pre-stress in tendons. Commonly adopted anchorages are **Freyssinet, Magnel Blaton, Gifford-Udall, Leonhardt-Baur, Dywidag, LeeMcCall and Roebling system.**

14.5.3 Pre-tensioning

It is a method of pre-stressing in which tendons are stretched/tensioned before the placement of concrete. In this method, pre-stress is imparted to concrete through **bond** between steel and concrete.

14.5.4 Post-tensioning

In this method of pre-stressing, tension is applied to tendons after the concrete has been set and hardened. Here the pre-stress is imparted to concrete through **bearing.**

14.5.5 Bonded pre-stressed concrete

Here the pre-stress is imparted to concrete through bond between the steel and surrounding concrete. **Pre-tensioned members** are classed into this group of concrete.

14.5.6 Non-bonded pre-stressed concrete

Here the pre-stress is imparted to concrete NOT by bond between the steel and the surrounding concrete but by bearing. Here the tendons are placed in ducts formed in concrete or they may be placed outside the concrete section. **Post-tensioned members** are classed into this group of concrete.

14.5.7 Full pre-stressing

Here the tensile stresses in concrete due to the applied loads are completely nullified by the existing pre-stress in the members.

14.5.8 Partial/limited pre-stressing

Here only a limited amount of tensile stresses are allowed in concrete under the working load conditions. Here in addition to tension steel, a good quantity of un-tensioned reinforcement is also used to limit the cracks developed at service loads.

14.5.9 Axial pre-stressing

Members in which the entire cross-section of concrete has a uniform compressive pre-stress. Here the centroid of the tendons coincide with that of the concrete section.

14.5.10 Eccentric pre-stressing

Here the tendons are eccentric to the centroid of the section thereby resulting in a triangular or trapezoidal compressive stress distribution.

14.5.11 Concordant pre-stressing

In this method of pre-stressing, the cable follows a concordant profile. In indeterminate structures, concordant pre-stressing does not cause any change in the support reaction.

14.5.12 Relaxation in steel

The decrease in stress in steel at a constant strain is called as relaxation in steel.

14.5.13 Creep coefficient

It is the ratio of creep strain to the elastic strain.

14.5.14 Degree of pre-stressing

It is the measure of magnitude of pre-stressing force that is being related to the resultant stress occurring in structural member at working load.

14.515 Debonding

It is the prevention of bond between the steel tendons and the surrounding concrete.

14.5.16 Transmission length

It is the length of bond anchorage of pre-stressing wire from the end of pre-tensioned member to the point of full steel stress.

14.5.17 Transfer

It represents the stage corresponding to the transfer of pre-stress to concrete. For pre-tensioned members, transfer takes place at the release of pre-stress from the bulk heads; and for post-tensioned members, it takes place after the completion of the process of tensioning.

14.5.18 Circular pre-stressing

It refers to the process of pre-stressing in circular members like pipes, circular tanks etc.

14.5.19 Proof stress

It refers to the tensile stress in steel which produces a residual strain of 0.002 or 0.2% of the original gauge length upon unloading.

14.5.20 Cap cable

It is the short curved cable arranged at the interior supports of a continuous member. Here the anchors are in compression zone while the curved portion is in tension zone.

14.5.21 Uniaxial, bi-axial and tri-axial pre-stressing

These refer to the situations where concrete is pre-stressed (a) in only one direction; (b) in two mutually perpendicular directions and (c) in three mutually perpendicular directions, respectively.

14.6 Advantages of Prestressed Concrete

Pre-stressed concrete offers more advantages than the reinforced concrete which are as under:

1. In fully pre-stressed concrete members which are free from tensile stresses under working loads, there the cross-section is utilized more efficiently as compared to reinforced concrete which gets cracked after the cracking moment at service loads.
2. The permanent dead load can be nullified by increasing the eccentricity of pre-stressing tendon in a pre-stressed concrete member thereby reducing the self-weight of the concrete member.
3. Pre-stressed concrete members offer improved resistance against shear forces due to the compressive pre-stressing force. Use of curved tendons especially in long span members helps to decrease the shearing forces at the supports.
4. For the same depth of concrete member, a pre-stressed concrete member is stiffer than the reinforced concrete member under working loads. However, once the cracks appear in pre-stressed member, then the behavior of pre-stressed concrete member is very similar to the reinforced concrete member.
5. The use of high strength concrete and high strength steel in pre-stressed concrete member results in smaller sections as compared to reinforced concrete.
6. Pre-stressing the concrete members improve the ability of concrete member to absorb impact loads more efficiently. Moreover the ability to resist repeated working loads has found to be very good in pre-stressed concrete member as compared to reinforced concrete member.
7. In long span structures, pre-stressed concrete is more economical than the reinforced concrete owing to smaller pre-stressed concrete sections.
8. Pre-stressed concrete members offer better resilience due to the capacity of completely recovering from the effects of overloading.
9. Fatigue strength of pre-stressed concrete is better than other materials due to the fact that there is small stress variation in steel which is recommended for dynamically loaded structures.
10. Owing to the fact that concrete in the tension zone is also useful, there is considerable savings in concrete (about 15-30 %) as compared to reinforced concrete.

Do You Know?

Some remarkable Prestressed Concrete Structures in India.

Below mentioned are some of the structures in India built on pre-stressed concrete:

1. **Ganga Bridge, Patna:** It is the longest pre-stressed concrete bridge in the world with a total length of 5575 m consisting of continuous spans of 121.65 m long pre-stressed concrete girders of variable depth.
2. **Ball tank, Trombay, Mumbai:** It is a pre-stressed concrete tank of 4 x 10^6 liters capacity being constructed for the Department of Atomic Energy.
3. **Gomti Aqueduct, U.P.:** It is the longest and the biggest aqueduct of India consisting of 9.9 m deep pre-stressed concrete girders with weight of each girder being 550 tons (or 5500 kN) over a span of 31.8 m.
4. **Boeing Hanger, Santa Cruz Airport:** The roof of this hanger consists of barrel shells supported on pre-stressed concrete edge beams of span 45.73 m.
5. **Zuari Bridge, Goa:** This 807 m long bridge consists of pre-stressed concrete cantilever box girders with main spans of four in number of 122 m each; two end spans of 69.5 m and a via-duct with five spans of 36 m each.
6. **Pre-stressed concrete bridge on river Brahmaputra, Jogighopra, Assam:** It has a span of 286 m between the two towers with side spans of 114 m, consisting of single cell pre-stressed concrete box girder.

14.7 Design of High Strength Concrete Mixes

Pre-stressing requires the use of high strength concrete mix. High strength concrete mix can be designed by any of the following ways:

1. Empirical method given by Erntroy and Shacklock.
2. Mix design procedure as given by the **American Concrete Institute** for no slump concrete.
3. **British method** based on the works of Franklin, Erntroy and Teychenne.
4. **IS Code** method.

14.8 High Tensile Steel

14.8.1 Types of high tensile steel

In pre-stressed concrete, we require high strength steel wires for pre-stressing. High strength steel generally consists of wires, bars or strands. **The higher tensile strength is generally achieved by slightly increasing the carbon content in steel as compared to mild steel. High tensile steel generally contains 0.6 to 0.85% carbon, 0.7 to 1% manganese, 0.05% sulphur and phosphorus with traces of silicon.**

The process of cold drawing through dies decreases the durability of wires and thus is subsequently tempered to improve their properties. Tempering/ageing/stress relieving is done by heat treatment of wires at 150 to 420°C which results in enhanced tensile strength. Cold drawn stress relieved wires are available generally in the diameters of 2.5, 3, 4, 5, 7 and 8 mm and they should coform to **IS 1785 (Part-I): 1983.** Because of superior bond characteristics, hard drawn steel wires are preferred for pre-tensioned elements and they should conform to **IS 6003: 1983.** Strands generally comprises of 2 to 5 mm steel wires with number of wires ranging from two, three or seven. **Helical form of wires in strands improves the bond strength.**

The ultimate tensile strength of **bars** generally do not vary much with diameter since the high strength of bars is achieved by alloying and NOT by cold working which is done for **wires.**

14.8.2 Strength requirement of high tensile steel

The ultimate tensile strength of plain hard drawn **wires** varies with the diameter. The tensile strength of wire decreases with increase in the wire diameter.

The **Indian Standard Code** prescribes a minimum percentage elongation of 2.5% for wires and 10% for bars. For strands, the percentage elongation measured on a gauge length of 600 mm should not be less than 3.5% just prior to the fracture of wire. **IS 1343** specifies the moduli of elasticity of high tensile wires as 210 kN/mm^2, high tensile bars as 200 kN/mm^2 and for strands as 195 kN/mm^2.

Table14.2 : *Tensile strength and elongation characteristics of cold drawn stress relieved wires (**IS 1785 (Part-I):1983**)*

Nominal Diameter (mm)	Min. Tensile Strength (N/mm^2)	Elongation (%)
2.5	2010	2.5
3.0	1865	2.5
4.0	1715	3.0
5.0	1570	4.0
7.0	1470	4.0
8.0	1375	4.0

14.8.3 Stress relaxation in steel

When a high tensile steel wire is stretched and thereafter maintained at the constant strain, then the initial pre-stressing force does not remain constant but decreases with time. This is called as **stress relaxation** which is the decrease in the stress in steel at constant strain. In pre-stressed members, the high tensile wires between the anchorages are more or less in the state of constant strain but the actual state of stress relaxation will be less than that indicated by test of wire at constant length since there will be shortening of the member due to other reasons. The various codal provisions on stress relaxation are based on the results of a 1000 hour relaxation test on wires. The **Indian Standard Code** prescribes the 1000 hour relaxation test with no relaxation exceeding 5% of the initial stress.

14.8.4 Stress Corrosion

Stress corrosion results in sudden brittle failure and is thus more perilous. The combined action of corrosion and the static tensile stress results in stress corrosion. Heat treated wires are more prone to stress corrosion than the cold drawn wires. Also, if the ducts of the post tensioned members are not grouted, then there is a possibility of stress corrosion leading to sudden failure of the structure. Other common types of corrosion encountered in pre-stressed concrete are **pitting corrosion** and **chloride corrosion.**

14.9 Cover Requirements in Prestressed Concrete Members

It is important to note that not only the thickness of cover but also the density of concrete in the cover is crucial to provide protection to steel. **IS 1343** specifies a minimum clear cover of **20 mm for protected pre-tensioned members** and **30 mm (or the cable diameter, whichever is higher) for protected post-tensioned members.** If the pre-stressed concrete members are exposed to aggressive environments then this cover requirement is increased by 10 mm.

14.10 Protection of Prestressing Steel

In order to prevent the pre-stressing steel from corrosion, un-bonded tendons should be coated by non-reactive materials like **epoxy** or **zinc/zinc aluminium.** Provision of non-corroding sheathing material like high density polyethylene (HDPE) is advantageous. The space between sheathing and the duct should be filled with corrosion resisting materials like grease, wax or the petroleum jelly.

14.11 Prestressing System

The various methods available to impart pre-compression in concrete are classified as follows:

1. Generation of compressive force between the structural element and its abutments using the flat jacks.
2. Imparting hoop compression in round/cylindrically shaped structures by winding the wire circumferentially.
3. By the use of longitudinally tensioned steel embedded in concrete.
4. By the use of deflected structural steel sections embedded in concrete until the hardening of concrete.

The most widely used method of pre-stressing is the longitudinally tensioning of steel by the tensioning devices. Development of expansive cements has now made possible the pre-stressing in concrete due to chemical pre-stressing.

14.12 Tensioning Devices

The various types of devices for tensioning are classified into the following four groups viz.:

1. **Mechanical devices**: These types of devices generally consists of weights with or without lever transmission, geared transmission along with pulley blocks, screw jacks with or without gears and the wire winding machines. These devices are generally used in factories where pre-stressed structural concrete components are produced in a large number.
2. **Hydraulic devices:** These are the simplest means of producing the pre-stressing force. There are several patented hydraulic jacks due to the works of Freyssinet, Magnel, Gifford Udall and Baur-Leonhardt for the range of 5 to 100 tons.

3. **Electrical/thermal devices:** These devices were used in USSR in 1950s for imparting tension to steel wires and deformed bars. The steel wires are electrically heated and anchored before placing the concrete in moulds. This method is also called as **thermo-electric pre-stressing.**
4. **Chemical devices:** Here the pre-stressing is imparted due to the expanding cements by controlled variation of curing conditions. Due to prevention of expansion of cement on setting, it induces tensile forces in tendons and compressive forces in concrete.

14.13 Pre-tensioning and Post-tensioning Systems

14.13.1 Pre-tensioning systems

In pre-tensioning system, the tendons are first prestressed/tensioned between the rigid anchor blocks. These blocks are casted on ground.

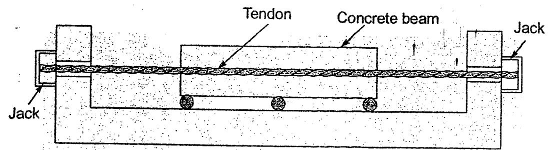


Fig.14.1 *Beam with straight tendon*

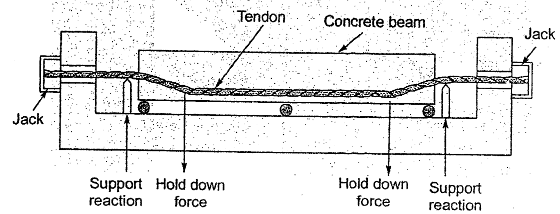


Fig.14.2 *Beam with variable tendon eccentricity*

High strength concrete is used for this purpose when concrete attains the sufficient strength, the pursue from the jacks is released. Due to elasticity of tendons, they tend to come to its original state thereby inducing compressive stress in concrete. Thus the stress gets transformed from tendon to concrete through bond.

For the production of pre-tensioned elements on a large scale, HOYER system is adopted. In the HOYER system of pre-tensioning, the tendons are stretched between the two bulk heads several hundred meters apart so that a number of pre-tensioned units can be casted on the same group of pre-tensioned tendons. The tension is applied through hydraulic jacks.

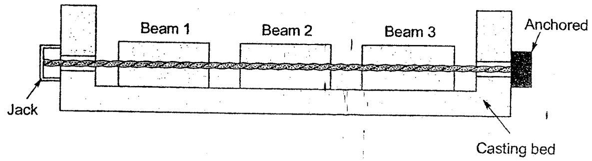


Fig.14.3 *Hoyer's system of pre-tensioning*

The transfer of prestress to concrete is achieved through bond and while releasing the tension in the wires, all wires are simultaneously released. The bond between the concrete and the tendon can be improved by having surface indentations on protusions by helical crimping of wires. Strands have better bond characteristics than plain wires of same cross-sectional area.

14.13.2 Post-tensioning systems

In this system, the concrete units casted first and having a provision of ducts incorporation a grooves to house the tendons. When concrete attains the required strength, the high tensile wires are tensioned by means of jack. The force is transferred to concrete by end anchorages and when the prestressing tendon is curved then this force transfer is achieved through radial pressures between the duct and the cable. The space between the tendon and the duct is grouted after the tensioning process. Most of the patented and commercially available pre-tensioning systems are based on following principles:

(i) Wedge action which produces a friction grip on the wire.

(ii) Direct bearing from rivet or bolt heads formed at the end of the wires.

(iii) Looping the wires around the concrete.

Post-tensioning anchorages

At present, there are about 64 patented post tensioning systems. The post tensioning systems based on wedge action are FREYSSINET, GIFFORD-UDALL, MAGNEL-BLATON etc.

Freyssinet System consists of a cylinder with a cone in tension through which high tensile wires pass Fig. (14.4). The major advantage of freyssinet system is that a large number of wires or strands can be tensioned simultaneously.

Gifford-Udall system was developed in the United Kingdom and consists of a steel split cone and cylindrical female cone anchorages to house the high tensile wires bearing against the steel plates. Here in this system, each wire is tensioned separately and anchored by pushing a sleeve wedge into the cylindrical grip resting against a bearing plate Fig (14.5).

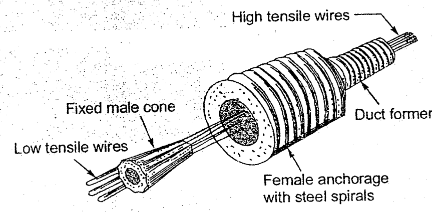


Fig.14.4 *Freyssinet Anchorage*

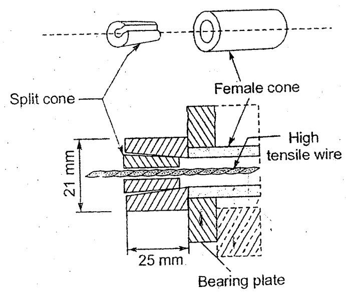


Fig.14.5 *Gifford-Udall System*

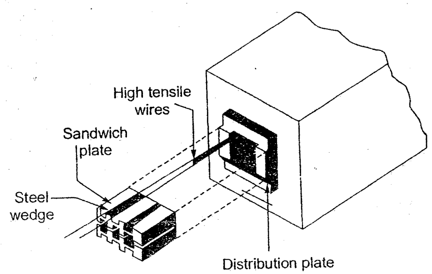
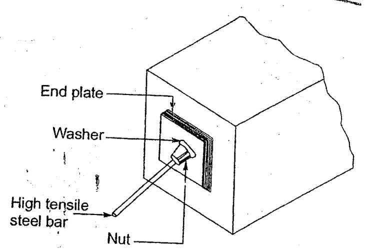


Fig.14.6 *Magnel-Blaton System*

Magnel Blaton system of post tensioning employs flat wedges, metallic sandwich plates, distribution plate for anchoring the wires. Each sandwich plate can house upto four pair of wires. The number of wires vary from 2 to 24 Fig. (14.6).

Lee-McCall System: This system uses bars having threads at end as tendons. Anchoring is done by screwing a nut and washer tightly against the end plates. In this system forces are transmitted by bearing action at end plates. Only one bar at a time can be tensioned. Disadvantage of this method is that curved tendons cannot be used.



Lee-McCall System

14.13.3 Thermo-electrical and chemical prestressing

Themo Electric Prestressing: The prestressing is achieved by heating the tendons by passing an electric current through the high tensile wires. The wires are heated to a temperature of 300-400°C for about 3-5 minutes. Due to this, wires undergo an elongation of about 0.4-0.5%. These wires on cooling get shorter but are prevented in doing so by anchors fixed at the two ends.

This process induces a prestress of about 500-600 N/mm². Concrete is placed in the moulds only after the temperature of wires goes below 90°C.

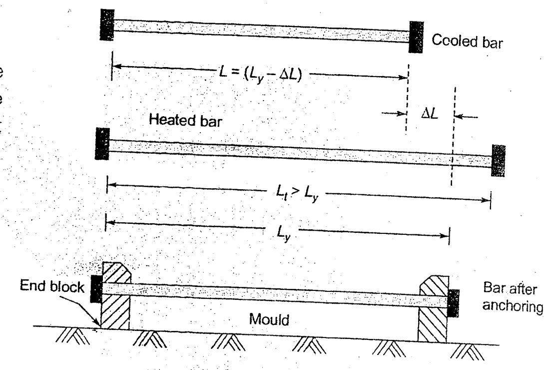


Fig.14.7 *Electro-Thermal Prestressing*

Chemical Prestressing: This is made possible by the development of expanding cements which consists of about 75% Portland cement 15% high alumina cement and 10% Gypsum. This results in the formation of calcium sulpho-aluminate. The linear expansion of this cement is about 3-4%. By varying the curing conditions, degree of expansion can be controlled.

Now the expansion of cement is restrained by high tensile steel wires thereby developing a compressive stress is concrete and tensile stress in wires. A tensile stress of about 850 N/mm² has been achieved in steel by the expansion of concrete. Structural elements suitable for chemical prestressing are shells, slabs, walls, pipes, composite columns etc.

14.14 Assumptions in the Analysis and Design of Prestressed Concrete Members

The following assumptions are made while analysing and designing a prestressed concrete member:

1. Plane sections remains plane after the bending.
2. It is assumed that Hooke's law is applicable within the limits of elastic deformation.
3. Stress in the reinforcement remain constant along the length of reinforcement i.e.. stress variation along the length of reinforcement is ignored. Variation of stress takes place in concrete only.
4. Change of stress in reinforcement due to external loads is also ignored.

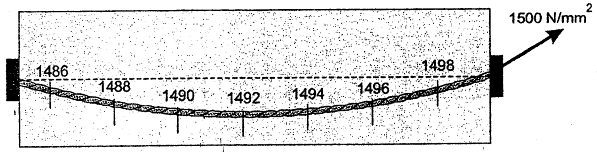


Fig.14.8 *Jacking from one end*

14.15 Analysis of Prestress

14.15.1 Concentric tendon

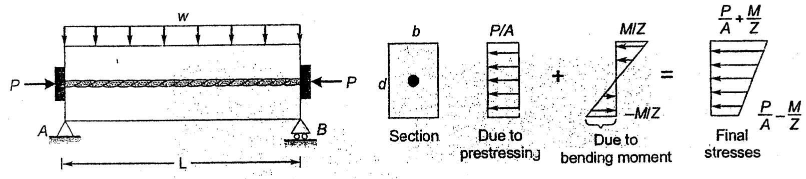


Fig.14.9 *Concentric Prestressing*

Due to concentric tendon, eccentricity is zero and thus a uniform pre-stress induced in concrete is (P/A) which is compressive in nature along the whole depth of the beam section. Consider a simply supported beam *AB* of span *L* which is prestressed by a tendon passing through the centroidal axis of beam.

Let P = Prestressing force provided by the tendon.
A = Cross-sectional area of the beam.
M = BM at a section due to dead and external loads.

Direct Stresses

Compressive stress induced in concrete = $f_c = \dfrac{P}{A}$

Bending/Flexural stresses due to bending moment $M = f_b = \pm\dfrac{M}{Z}\left(= \pm\dfrac{M}{I}\cdot y\right)$

where Z = section modulus of beam section.

$\therefore$ Extreme fibre stresses are obtained by superimposing the stresses due to prestress and flexural stresses.

Final Stress

$\therefore$ Extreme fibre stresses = $f_c \pm f_b = \dfrac{P}{A} \pm \dfrac{M}{Z}$

i.e. extreme fibre stress at top = $\dfrac{P}{A} + \dfrac{M}{Z}$

and extreme fibre stress at bottom = $\dfrac{P}{A} - \dfrac{M}{Z}$

Thus the action of external loads induce tensile stresses in the lower portion of neutral axis which is more effectively nullified by providing eccentric tendons.

14.15.2 Eccentric tendon

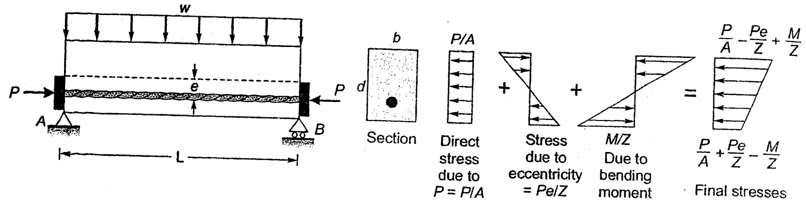


Fig.14.10 *Concentric Prestressing*

As shown in the figure above, the beam is subjected to an eccentric pre-stressing force P acting at an eccentricity of '*e*' from the neutral axis. The stresses induced at the top and bottom fibers of the beam are:

$$f_{top} = \left(\frac{P}{A}-\frac{Pe}{Z}\right); \quad f_{bottom} = \left(\frac{P}{A}+\frac{Pe}{Z}\right)$$

Due to eccentric prestressing force *P*, additional bending/flexural stresses will get induced in the beam. Thus,

1. Direct stress due to axial load $(P) = f_a = \frac{P}{A}$
2. Flexural stresses due to eccentricity of prestressing force $(P) = f_e = \mp\frac{Pe}{Z} = \mp\frac{Pe}{I}\cdot y$

   Here *Pe* = Moment induced in beam due to eccentric prestressing force.
3. Flexural stresses due to self weight and external loading i.e. due to moment *M* at any section.

$$f_b = \pm\frac{M}{Z} = \pm\frac{M}{I}.Y$$

14.15.3 Final Stress/Resultant Stresses

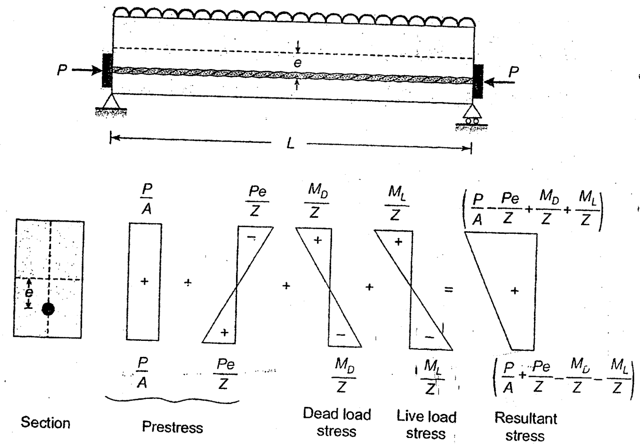


Fig.14.11 *Stress Distribution due to Eccentric Prestressing, Dead and Live Loads*

As shown in the **Fig. 14.11**, the beam is subjected to a uniformly distributed live load and dead load of w_L and w_D respectively. The beam is pre-stressed by a straight tendon at an eccentricity '*e*' from the neutral axis. The resultant stress at any section of the beam is obtained by super-imposing the effects of pre-stressing, dead and live loads. If M_L and M_D are respectively the live load moment and dead load moment for a simply supported beam of span '*l*' then,

$$M_L = \frac{w_L L^2}{8}; \quad M_D = \frac{w_D L^2}{8}$$

The resultant stresses at the top and bottom fibers are:

$$f_{top} = \left(\frac{P}{A}-\frac{Pe}{Z}\right)+\frac{M_D}{Z}+\frac{M_L}{Z} = \left(\frac{P}{A}-\frac{Pe}{I}y_t\right)+\frac{M_D}{I}y_t+\frac{M_L}{I}y_t$$

$$f_{bottom} = \left(\frac{P}{A}+\frac{Pe}{Z}\right)-\frac{M_D}{Z}-\frac{M_L}{Z} = \left(\frac{P}{A}+\frac{Pe}{I}y_b\right)-\frac{M_D}{I}y_b-\frac{M_L}{I}y_b$$

where, y_t and y_b are the distances of extreme top and bottom fibers respectively from the neutral axis of the beam.

Remember

- Comparing the final stresses of the above two cases, it is clear that tendon placed at an eccentricity offers additional compressive stress along with axial prestressing stress which improves the load carrying capacity of the beam.
- In pre-stressed concrete members, usually the cross-sectional area of high tensile steel wires is very small as compared to the gross area of concrete and thus stress computations are done based on gross concrete area. Use of equivalent concrete section is indispensable for interpreting and analyzing the test results but this does not significantly alter the stresses computed from the gross concrete area.

Example 14.1 A PSC beam of size 200 mm × 350 mm is prestressed by 5 high tensile steel wires of 6 mm diameter. The wires are stressed to 1350 N/mm² and located at an eccentricity of 75 mm. Find the percentage difference in stresses developed at the soffit of beam by considering nominal concrete and equivalent concrete.

Solution:

Applied prestressing force $(P) = 5\times\frac{\pi}{4}\times 6^2\times 1350\text{ N} = 190.85\text{ kN}$

For nominal concrete section,

Cross-sectional area, $(A) = 200\times 350 = 70000\text{ mm}^2$

Moment of inertia $(I) = 200\times\frac{350^3}{12} = 714.583\times 10^6\text{ mm}^4$

∴ Axial compressive stress at the soffit of the section

$$= \frac{P}{A} = \frac{190.85\times 10^3}{70000} = 2.73\text{ N/mm}^2$$

∴ Final stress at the soffit of section $= \left(\frac{P}{A}+\frac{Pe}{I}y\right) = \left(2.73+\frac{190.85\times 10^3\times 75}{714.583\times 10^6}\times 175\right)$

$= 6.24\text{ N/mm}^2$

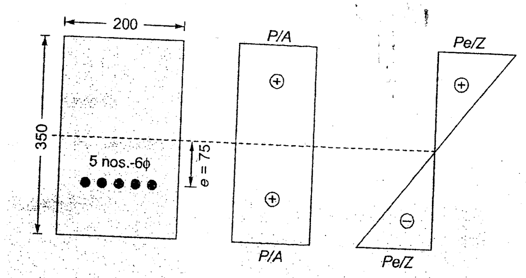
For equivalent concrete section,

Let modulur ratio (m) = 6

$\therefore$ Equivalent concrete area (A_e) = $70000+(6-1)\times 5\times\frac{\pi}{4}\times 6^2 = 70706.86\ \text{mm}^2$

Centroid of section from soffit = $100\ \text{mm}\left(=\frac{350}{2}-75\ \text{mm}\right)$

Location of Neutral Axis



$\therefore$ Stress at soffit of section = $\frac{190.85\times 10^3}{70706.86}+\frac{190.85\times 10^3\times 75}{718.52\times 10^6}\times 174.25$

= $2.7 + 3.47 = 6.17\ \text{N/mm}^2$

$\therefore$ Percentage difference in stresses at soffit

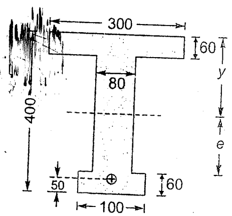
= $\frac{6.24-6.17}{6.24}\times 100 = 1.12\%$

Example 14.2 An unsymmetrical I-section is required to support an imposed load of 1.8 kN/m over a span of 7.8 m. Top flange is 300 mm wide and 60 mm thick, bottom flange is 100 mm wide and 60 mm thick, web thickness is 80 mm with overall beam depth as 400 mm. An effective prestressing force of 150 kN is there at 50 mm from soffit of beam at mid-span. What are the stress at the centre of span for:

(i) Prestress + self weight?

(ii) Prestress + self weight + imposed load??

Solution:



Prestress force (P) = 150 kN

Area of concrete = $(300\times 60)+(100\times 60)+(80\times 280)$

= $46400\ \text{mm}^2$

Let y = Distance of centroid from top

$\therefore$ $y = \frac{300\times 60\times 30+280\times 80\times 200+100\times 60\times 370}{46400}$

= 156 mm

$\therefore$ $e = (400-50)-y = 350-156 = 194\ \text{mm}$

$$I = 300\times\frac{60^3}{12}+300\times 60\times 126^2+80\times\frac{280^3}{12}+80\times 280\times 44^2$$

$$+100\times\frac{60^3}{12}+100\times 60\times 214^2$$

$= 5.4\times 10^6 + 285.768\times 10^6 + 146.35\times 10^6 + 43.37\times 10^6 + 1.8\times 10^6$

$+ 274.776\times 10^6$

$= 757.464\times 10^6\ \text{mm}^4$

$\therefore$ $z_t = \frac{I}{y} = \frac{757.464\times 10^6}{156} = 4.8555\times 10^6\ \text{mm}$

$z_b = \left(\frac{I}{400-y}\right) = \frac{757.464\times 10^6}{400-156} = 3.104\times 10^6\ \text{mm}^3$

Self weight of beam = $2.5\times 46400\times 10^{-6} = 1.16\ \text{kN/m}$

$\therefore$ Moment due to self weight of beam (M_1) = $1.16\times\frac{7.8^2}{8} = 8.82\ \text{kNm}$

Moment due to imposed load (M_2) = $1.8\times\frac{7.8^2}{8} = 12.69\ \text{kNm}$

Stress type	Top fibre	Bottom fibre
Prestress	$\frac{P}{A}=\frac{150\times 1000}{46400}=+3.23\ \text{N/mm}^2$	$\frac{P}{A}=\frac{150\times 1000}{46400}=+3.23\ \text{N/mm}^2$
	$\frac{P.e}{z_b}=\frac{150\times 1000\times 194}{4.8555\times 10^6}=-5.99\ \text{N/mm}^2$	$\frac{P.e}{z_b}=\frac{150\times 1000\times 194}{3.104\times 10^6}=+9.375\ \text{N/mm}^2$
Stress due to self weight of beam	$\frac{M_1}{z_t}=\frac{8.82\times 10^6}{4.8555\times 10^6}=41.82\ \text{N/mm}^2$	$\frac{M_1}{z_b}=\frac{8.82\times 10^6}{3.104\times 10^6}=-2.84\ \text{N/mm}^2$
Stress due to imposed load	$\frac{M_2}{z_t}=\frac{13.69\times 10^6}{4.8555\times 10^6}=+2.82\ \text{N/mm}^2$	$\frac{M_2}{z_b}=\frac{13.69\times 10^6}{3.104\times 10^6}=-4.41\ \text{N/mm}^2$

Thus, stress due to (prestress + self weight)

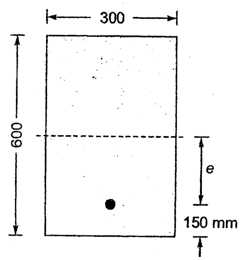
= $-0.94\ \text{N/mm}^2$ (at top), $9.765\ \text{N/mm}^2$ (at bottom)

Stress due to (prestress + self weight + imposed load)

= $1.88\ \text{N/mm}^2$ (at top), $5.355\ \text{N/mm}^2$ (at bottom)

Example 14.3 A 300 mm × 600 mm rectangular concrete beam is prestressed by four 14 mm diameter bars of high tensile steel located at 150 mm from the beam soffit. The net stress in the bars is 750 MPa. What is the maximum BM that can be applied at the section with no tension in the beam soffit?

Solution:



Area of beam section = $A = 300\times 600 = 18\times 10^4\ \text{mm}^2$

Section modulus (z) = $\frac{300\times 600^2}{6} = 18\times 10^6\ \text{mm}^3$

Area of steel bars (A_s) = $\frac{300\times 600^2}{6} = 18\times 10^6\ \text{mm}^3$

Eccentricity (e) = $\frac{600}{2}-150 = 150\ \text{mm}$

$\therefore$ Prestressing force (P) = 750×615.75 N = 461.81 kN

$$\therefore \quad \frac{P}{A} = \frac{461.81 \times 1000}{18 \times 10^4} = 2.57 \text{ N/mm}^2$$

$$\frac{P.e}{z} = \frac{461.81 \times 1000 \times 150}{18 \times 10^6} = 3.85 \text{ N/mm}^2$$

$\therefore$ Total stress the beam soffit = $2.57 + 3.85 = 6.42$ N/mm²

Let M = maximum BM for zero stress at the beam soffit

$$\therefore \quad \frac{M}{z} = 6.42$$

$$\Rightarrow \quad M = 6.42 \times 18 \times 10^6 = 115.56 \text{ kNm}$$

14.16 Prestress Pressure Distribution in Beams

From the above BMD for a simply supported beam subjected to a uniformly distributed load (w) including the self weight of beam, it is evident that maximum flexural stress occurs at mid-span and these flexural stresses go on reducing as distance from the mid-span increases.

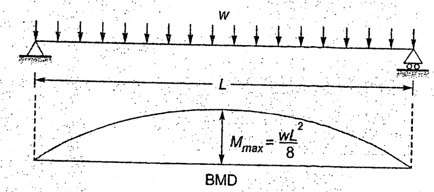


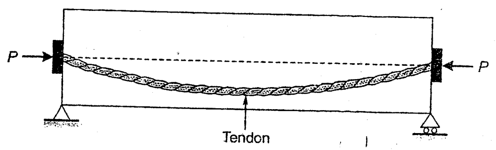
Fig.14.12 *BMD for a simply supported uniformly loaded beam*

In order to counter act the external flexural stresses as per the BMD above, it will be beneficial to have maximum prestressing stress at mid-span and this prestressing stress can be reduced on either side of the mid-span. This type of prestress distribution is called as non-uniform prestress distribution. Thus we have following types of prestress presure distribution viz.:

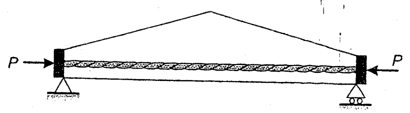
1. Uniform prestress pressure distribution
2. Non-uniform prestress pressure distribution.

Non-uniform prestress pressure distribution is achieved by following ways:

1. By varying the tendon/cable profile



2. By varying the beam cross-section



14.17 Effect of Loading on Stresses in Tendons

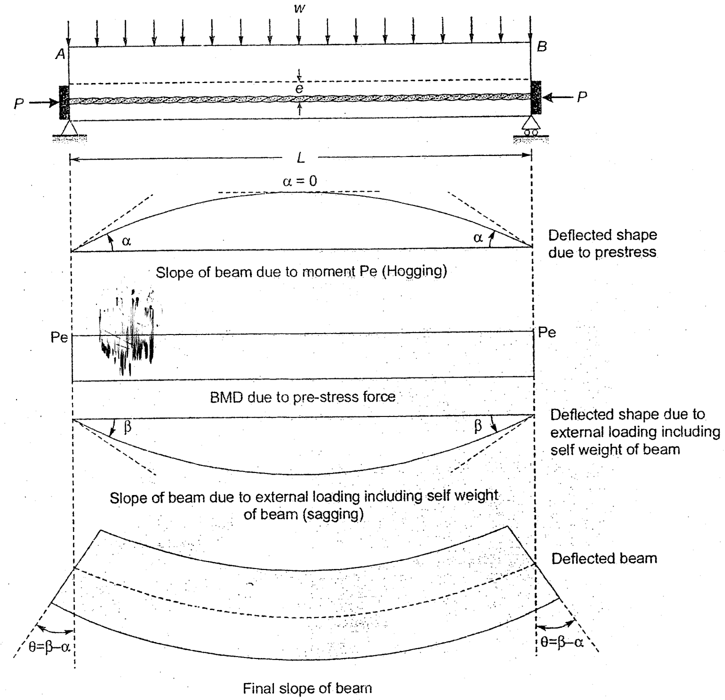


Fig.14.13 *Variation in tendon stress*

Consider a simply supported beam AB of span L prestressed by a force P acting at an eccentricity 'e' from the centroidal axis of beam.

Hogging moment due to prestress = Pe

$$\therefore \quad \text{Slope at the beam ends} = \alpha - 0 = \frac{\text{Area of BMD}}{EI}$$

$$\therefore \quad \alpha = \frac{PeL}{2EI}$$

Similarly slope at the beam ends due to uniformly distributed load (w) including the self weight of beam

$$\beta = \frac{wL^3}{24EI}$$

Case $\beta > \alpha$

If $\beta > \alpha$ then net slope will be in the downward direction

$$\therefore \text{Net slope at each end of beam} = \theta = \beta - \alpha = \frac{wL^3}{24E_cI} - \frac{P_eL}{2E_cI}$$

Due to slope in the downward direction there will be an increase in the length of tendon.

Total increase in length = $\delta l_1 = e\theta + e\theta = 2e\theta$

Increase in tendon strain = $\frac{2e\theta}{L}$

$\therefore$ Gain in stress in steel reinforcement = $\frac{\delta l_1}{L} \times E_s = \frac{2e\theta}{L} \cdot E_s = \frac{2e\theta E_s}{L}$

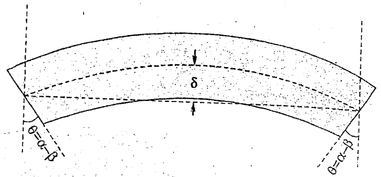
Case $\beta < \alpha$

There will be a net slope in upward direction

$$= \theta = \alpha - \beta = \frac{P_e L}{2E_c I} - \frac{wL^3}{24E_c I}$$

reduction in length $= 2e\theta = \delta l_2$

$\therefore$ Loss in stress $= \frac{\delta l_2}{L} \times E_s = \frac{2e\theta}{L} E_s = \frac{2e\theta E_s}{L}$



In general, any increase of loading on the pre-stressed member within the elastic range does not result in significant change in the tendon stress. Alternatively, it can be said that the stress in the tendon remains more or less constant provided the transverse loading is within the elastic limits.

14.18 Prestressed Beam with Parabolic Tendon Profile

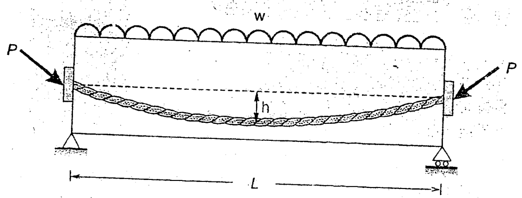
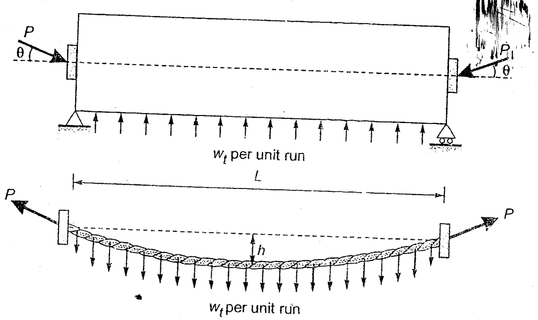


Fig.14.14 *Prestressed beam with parabolic tendon profile*

Consider a simply supported beam of span L with parabolic tendon profile as shown above. The stressed tendon will exert a uniform pressure upwards (w_t) in the beam and in return receive a downward uniform pressure.



From structural analysis of cables and suspension bridges, horizontal reaction at each end is given by:

$$H = \frac{w_t L^2}{8h}$$

Let the inclination of tendon at the ends in very small so that it can be conveniently assumed that,

$$H \simeq P$$

$$\therefore \quad P = \frac{w_t L^2}{8h} \Rightarrow w_t = \frac{8Ph}{L^2}$$

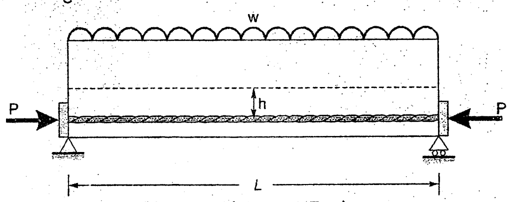
= Upward uniformly distributed pressure to a parabolic presented tendon carrying a prestressing force P.

Thus a part of downward acting load will be counteracted by this uniform upward pressure (w_T). This gives rise to the concept of LOAD BALANCING which is described in later sections.

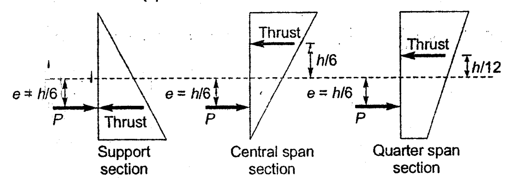
14.19 C-line or Pressure Line

As seen in earlier sections, the resultant stress at any section of a pre-stressed concrete member is the combined effect of the pre-stressing force and the stresses induced due to external loads which results in the distribution of concrete stresses that can be resolved into a single force. The **locus of point of application of this resultant force in any structure is called as the pressure line or the thrust line.**

In pre-stressed concrete members, obviously the location of pressure line depends on the magnitude and direction of applied moments at the section combined with the magnitude and direction of the effect of stresses due to the pre-stressing force.



(a) *Beam with Eccentric Tendons*



(b) *Distribution of Stresses at Various Sections along the Span*

Fig. 14.15

As shown in the figure above, the load is of such magnitude that the bottom fibers at mid span section have zero stresses. At the support section, the stresses due to the applied load (and thus applied moment) are zero and thus the pressure line coincides with the centroid of steel located at an eccentricity of (h/6) from the neutral axis. It is quite evident that at the mid span section of the beam, the bottom stresses are zero and the pressure line has shifted towards the top by an amount of (h/3) from the initial position. At the quarter span section, the externally applied moment is small and so does the shift in pressure line is also small, being equal to (h/4) from the initial position. Similarly, the pressure line location at other sections can also be determined. The locus of all such points gives the pressure line as shown in the **Fig. 14.16**.

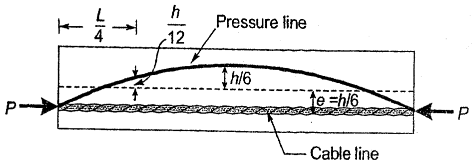


Fig.14.16 *Location of Pressure Line in the Prestressed Beam*

NOTE

A change in the external moments in the elastic range of pre-stressed concrete beam results in a shift of pressure line rather than an increase in the resultant force of the beam. This is in contrast to the reinforced concrete beam where increase in the externally applied moments results in an increase in the tensile stresses and compressive stresses. This increase in the stresses in reinforced concrete is due to the **constant lever arm**. In pre-stressed concrete sections, the load carrying mechanism is the **constant force with changing lever arm** and in reinforced concrete; it is the **constant lever arm with changing force**. But as a pre-stressed concrete member gets cracked then it behaves just like reinforced concrete section.

Apart from direct method of analysis of stresses at a section as described in earlier sections, the method based on **pressure line or the thrust line** can also be adopted for analysis of stresses. This method is generally called as **internal resisting couple method** or the **C-line method**. In this method, the pre-stressed concrete beam is analyzed just like plain concrete elastic beam with the usual principles of statics. The pre-stressing force is treated as external compressive force with a constant tensile force (T) in the tendons throughout the span.

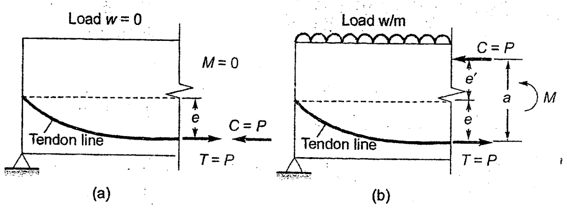


Fig.14.17 *Free-Body Diagram of Forces and Moments at a Section of Prestressed Concrete Beam*

As shown in the figure above, when gravity loads are zero then the *C* and *T* lines coincide because there is no moment anywhere at the section. Under the influence of transverse loads, the *C* line or the center of pressure line/thrust line is a varying distance '*a*' from the T line.

Let M = Flexural moment at any section due to dead and live loads

e = Eccentricity of the pre-stressing tendon

$T = P$ = Pre-stressing force in the tendon

Thus from moment equilibrium,

$$M = Ca = Ta = Pa$$

or

$$a = \frac{M}{P}$$

The shift of pressure line 'e'' is measured from the centroidal axis as:

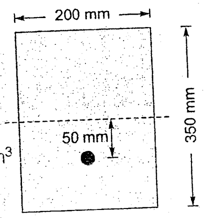
$$e' = a - e = \left(\frac{M}{P}\right) - e$$

The resultant stresses at the top and bottom fibers of the section are:

$$f_{top} = \frac{P}{A} + \frac{Pe'}{Z}; \quad f_{bottom} = \frac{P}{A} - \frac{Pe'}{Z}$$

Example 14.4 A prestressed concrete beam of size 200 mm × 350 mm is used for an effective span of 5.5 m in order to support a uniformly distributed load of 5 kN/m (including self weight of the beam). The beam is prestressed by a straight cable carrying a force of 200 kN and located at an eccentricity of 50 mm. Determine the location of thrust line in the beam.

Solution:



Eccentricity, $e = 50$ mm

Gross cross-sectional area of beam, $A = 200 \times 350 = 70000 \text{ mm}^2$

Sectional modulus of the beam cross-section

$$Z = 200 \times \frac{350^2}{6} = 4.083 \times 10^6 \text{ mm}^3$$

Prestressing force, $P = 200$ kN

Direct stress due to prestressing force $= \frac{P}{A} = \frac{200 \times 1000}{70000} = 2.875 \text{ N/mm}^2$

Flexural stresses due to prestressing force $= \mp\frac{Pe}{Z} = \mp\frac{200 \times 10^3 \times 50}{4.083 \times 10^6} = \mp 2.45 \text{ N/mm}^2$

Maximum BM, $M = 5 \times \frac{5.5^2}{8} = 18.91$ kNm

Flexural stress due to $M = \pm\frac{18.91 \times 10^6}{4.083 \times 10^6} = \pm 4.63 \text{ N/mm}^2$

Thus resultant stress at mid-span section $= \begin{cases} 2.857 - 2.45 + 4.63 = 5.037 \text{ N/mm}^2 \text{ (at top)} \\ 2.857 + 2.45 - 4.63 = 0.677 \text{ N/mm}^2 \text{ (at bottom)} \end{cases}$

$\therefore$ Shift of pressure line from cable line $= \frac{M}{P} = \frac{18.91 \times 10^6}{200 \times 10^3} = 94.55$ mm

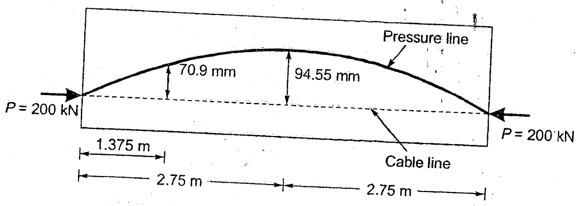
BM at quarter span section, $M_1 = R\frac{L}{4} - \frac{wL}{4}\frac{L}{8} = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{3wL^2}{32}$

$$= \frac{3}{32} \times 5 \times 5.5^2 = 14.18 \text{ kNm}$$

$\therefore$ Flexural stresses due to $M_a = \pm\frac{14.18 \times 10^6}{4.083 \times 10^6} = \pm 3.47 \text{ N/mm}^2$

Resultant stresses at quarter span $= \begin{cases} 2.857 - 2.45 + 3.47 = 3.877 \text{ N/mm}^2 \text{ (at top)} \\ 2.857 + 2.45 - 3.47 = 1.837 \text{ N/mm}^2 \text{ (at bottom)} \end{cases}$

$\therefore$ Shift of pressure line from cable line $= \dfrac{M_1}{P} = \dfrac{14.18 \times 10^6}{200 \times 10^3} = 70.9$ m



Locus of pressure line

Example 14.5 A prestressed concrete beam of size 120 mm × 300 mm is prestressed concentrically by a prestressing force of 180 kN. The effective span of the beam is 6 m and it supports a uniformly distributed load of 5 kN/m including the self weight of beam. Find the locus of pressure line in the beam.

Solution:

Prestressing force, $P = 180$ kN

$\because$ Beam is concentrically prestressed.

$\therefore$ Eccentricity, $e = 0$

Gross cross-sectional area of beam, $A = 120 \times 300 = 36000$ mm²

Sectional modulus, $Z = \dfrac{120 \times 300^2}{6} = 1.8 \times 10^6$ mm³

Mid-span flexural moment in beam, $M = 5 \times \dfrac{6^2}{8} = 22.5$ kNm

Direct stress due to prestressing force $= \dfrac{P}{A} = \dfrac{180 \times 10^3}{36000} = 5$ N/mm²

Flexural stress due to moment $= \dfrac{M}{Z} = \dfrac{22.5 \times 10^6}{1.8 \times 10^6} = 12.5$ N/mm²

Stresses at the mid-span section

At top, stress $= 5 + 12.5 = 17.5$ N/mm² (compression)

At bottom, stress $= 5 - 12.5 = -7.5$ N/mm² (tensile)

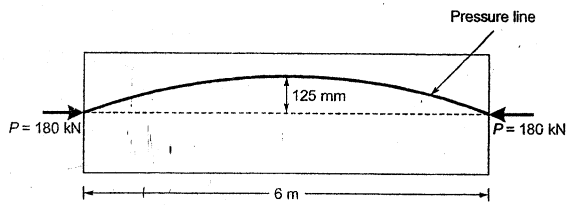
Thus shift of pressure line is given by,

$$\frac{180 \times 10^3}{36000} + \frac{180 \times 10^3 e}{1.8 \times 10^6} = 17.5$$

$\Rightarrow$ $5 + 0.1e = 17.5$

$\Rightarrow$ $e = 125$ mm

Alternatively, $e = \dfrac{M}{P} = \dfrac{22.5 \times 10^6}{180 \times 10^3} = 125$ mm



Pressure line

Example 14.6 A Prestressed concrete beam is 120 mm × 300 mm and is used for an effective span of 6 m. It supports a uniformly distributed load of 5 kN/m including its own weight. The beam is prestressed by a prestressing force of 180 kN at an eccentricity of 30 mm. Find the locus of pressure line.

Solution:

Prestressing force, $P = 180$ kN

Eccentricity, $e = 30$ mm

Gross area of cross-section, $A = 120 \times 300 = 36000$ mm²

Sectional modulus, $Z = 120 \times \dfrac{300^2}{6} = 1.8 \times 10^6$ mm³

Direct stress $= \dfrac{P}{A} = \dfrac{180 \times 1000}{36000} = 5$ N/mm²

Flexural stress due to eccentricity $= \dfrac{Pe}{Z} = \dfrac{180 \times 1000 \times 30}{1.8 \times 10^6} = 3$ N/mm²

Mid span moment, $M = 5 \times \dfrac{6^2}{8} = 22.5$ kNm

Flexural stress due to moment $= \dfrac{M}{Z} = \dfrac{22.5 \times 10^6}{1.8 \times 10^6} = 12.5$ N/mm²

Stresses at mid-span section

At top, stress $= 5 - 3 + 12.5 = 14.5$ N/mm² (compression)

At bottom, stress $= 5 + 3 - 12.5 = -4.5$ N/mm² (tension)

Shift of pressure line is given by,

$$\frac{180 \times 10^3}{36000} + \frac{180 \times 10^3 e}{1.8 \times 10^6} = 14.5$$

$\Rightarrow$ $5 + 0.1e = 14.5$

$\Rightarrow$ $e = 95$ mm

$\therefore$ Pressure line gets shifted from original pressure line by $95 + 30 = 125$ mm

Alternatively:

This shift of pressure line can be computed as,

Shift $= \dfrac{M}{P} = \dfrac{22.5 \times 10^6}{180 \times 10^3} = 125$ mm (which is same as above)

At quarter span

Moment at quarter span, $M_1 = \left(\dfrac{wL}{2}\right)\dfrac{L}{4} - \dfrac{wL}{4}\dfrac{L}{8} = \dfrac{wL^2}{8} - \dfrac{wL^2}{32} = \dfrac{3wL^2}{32}$

$= \dfrac{3\times5\times6^2}{32} = 16.875$ kNm

$\therefore$ Flexural stress due to moment, $M_1 = \dfrac{M_1}{Z} = \dfrac{16.875\times10^6}{1.8\times10^6} = 9.375$ N/mm²

$\therefore$ At top, stress $= 5-3+9.375 = 11.375$ N/mm² (compression)

$\therefore$ At bottom, stress $= 5+3-9.375 = -1.375$ N/mm² (tension)

$\therefore$ Shift of pressure line is,

$$\frac{180\times10^3}{36000} + \frac{180\times10^3 e}{1.8\times10^6} = 11.375$$

$\Rightarrow$ $e = 63.75$ mm

Thus pressure line gets shifted from original location by

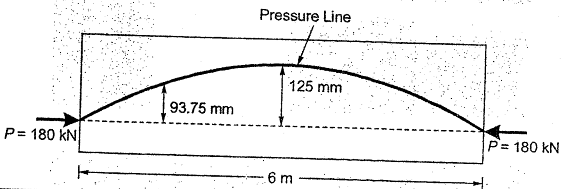
$63.75 + 30 = 93.75$ mm

Alternatively:

This shift of pressure line can be computed as,

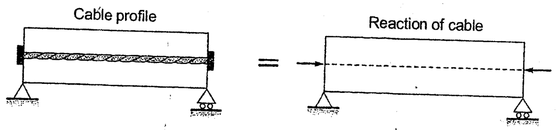
$$\text{Shift} = \frac{M_1}{P} = \frac{16.875\times10^6}{180\times10^3} = 93.75 \text{ mm}$$

(same as calculated above)

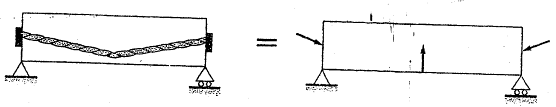


14.20 The Concept of Load Balancing

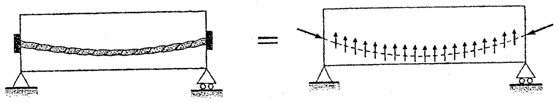
In a pre-stressed concrete member, it is always possible to have a suitable cable profile such that the transverse component of the cable force balances the external transverse loading.



(a) Straight tendon



(b) Bent tendon



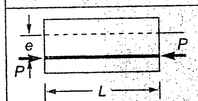
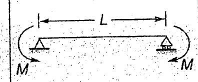
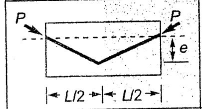
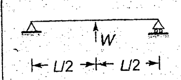
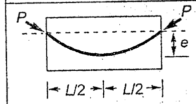
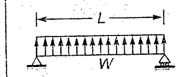
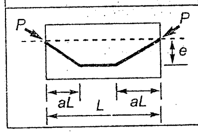
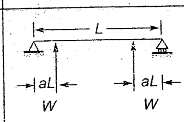
(c) Curved tendon

Fig.14.18 *Reactions of cable on beam*

The reaction of the cable on the concrete member depends on the shape of the cable profile. Straight cable profiles do not induce any reactions except at the supports but the curved cables result in uniformly distributed loads. Furthermore, sharp cable profile induce concentrated load on the concrete member.

In general, the requirement of load balancing will be satisfied if the cable profile in the pre-stressed member corresponds to the shape of the bending moment diagram due to the external loads. For a beam supporting two concentrated loads, the cable profile must have two bends i.e. its profile should be like a trapezium; similarly for a beam supporting uniformly distributed load, the cable profile should be parabolic.

Table 14.1 *Tendon Profiles and Equivalent Loads in Prestressed Concrete Beams*

Tendon profile	Equivalent moment or Load	Equivalent loading	Camber
	$M = Pe$		$\dfrac{ML^2}{8EI}$
	$W = \dfrac{4Pe}{L}$		$\dfrac{WL^2}{48EI}$
	$W = \dfrac{8Pe}{L^2}$		$\dfrac{5WL^2}{384EI}$
	$W = \dfrac{Pe}{aL}$		$\dfrac{a(3-4a^2)WL^2}{24EI}$

Example 14.7 A rectangular prestressed concrete beam of size 150 × 300 mm is to be used for an effective span of 9 m. The prestressing cable is having zero eccentricity at supports and eccentricity varies linearly upto 40 mm at the centre. The effective prestressing force is 550 kN. Find the concentrated load at the mid-span location is the load nullifies the flexural effect of prestressing force (excluding the self weight of beam)

Solution:

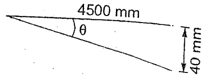
Gross cross-sectional area of beam, $A = 150\times300 = 45000$ mm²

Sectional modulus, $Z = 150\times\dfrac{300^2}{6} = 225\times10^4$ mm³

Prestressing force, $P = 550$ kN

Self weight of beam $= 0.15 \times 0.3 \times 25 = 1.125$ kN/m

$$\tan\theta = \frac{40}{4500}$$



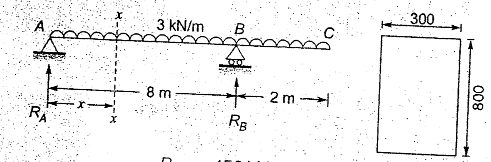
∴ Concentrated load, $Q = 2P\sin\theta \approx 2P\tan\theta$

$$= 2\times550\left(\frac{40}{4500}\right) = 9.78 \text{ kN}$$

Thus concentrated load required at mid-span section = 9.78 kN

Example 14.8 A single overhang prestressed concrete beam is simply supported over a span of 8 m with overhang of 2 m. The section of beam is 300 × 800 mm. The beam supports a uniformly distributed live load of 3 kN/m over the entire span. Find the cable profile of prestressing cable for an effective prestress of 450 kN which can counteract the effect of dead and live load.

Solution:



Prestressing force, $P = 450$ kN

Self weight of beam $= 0.3 \times 0.8 \times 25 = 6$ kN/m

Live load on beam $= 3$ kN/m

∴ Total load $= 6 + 3 = 9$ kN/m

$R_A + R_B = 9 \times 10 = 90$ kN

and taking moment about A is zero

$R_B(8) - 9 \times 10 \times 5 = 0$

⇒ $R_B = 56.25$ kN

∴ $R_A = 90 - R_B = 33.75$ kN

Moment at section x-x, distant x from A $= R_A x - 9x.\frac{x}{2}$

$= 33.75x - 4.5x^2$

For maximum moment, $\frac{dM_x}{dx} = 0$

⇒ $33.75 - 9x = 0$

⇒ $x = 3.75$ m

∴ $M_{max} = 33.75(3.75) - 4.5(3.75)^2 = 63.28$ kNm

⇒ $e = \frac{M_{max}}{P} = = 140.62$ mm

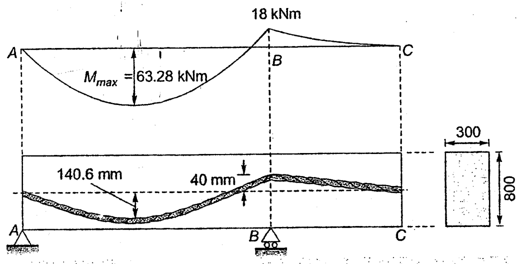
When $M_x = 0$

⇒ $33.75x - 4.5x^2 = 0$

⇒ $x = 0, 7.5$m (from support A)

Eccentricity of cable at support B $= \frac{M_B}{P} = \frac{18\times10^6}{450\times1000} = 40$ mm

At support A, moment is zero and thus eccentricity is zero at A.



Cable profile

Example 14.9 A prestressed concrete beam of effective span 5.5 m is 150 mm wide and 300 mm deep. The tendon is located at a constant eccentricity of 45 mm having a prestressing stress of 850 N/mm². What is the percentage increase of stress in tendons if the beam supports a line load of 4.5 kN/m. Take cross-sectional area of tendons as 90 mm², modulus of elasticity of concrete as 36 kN/mm² and that of steel as 210 kN/mm².

Solution:

Prestressing force, $P = 850 \times 90 = 76.5$ kN

Moment of inertia, $I = 150\times\frac{300^3}{12} = 337.5\times10^6$ mm⁴

Rotation due to prestress, $\theta_1 = \frac{PeL}{2E_cI} = \frac{(76.5\times1000)45(5500)}{2\times36000\times337.5\times10^6}$

$= 0.0007792$ radians (Hogging)

Self weight of beam $= 0.15 \times 0.3 \times 25 = 1.125$ kN/m

Live load $= 4.5$ kNm

∴ Total load $= 5.625$ kN/m $= 5.625$ N/mm

Rotation due to loads, $\theta_2 = \frac{wL^3}{24E_cI} = \frac{5.625(5500)^3}{24\times36\times1000\times337.5\times10^6}$

$= 0.00321$ radians (Sagging)

∴ Net rotation $= 0.00321 - 0.0007792$

$= 0.0024308$ radians (Sagging)

∴ Cable elongation $= 2e\theta$

$= 2 \times 45 \times 0.0024308 = 0.218772$ mm

∴ Increase in stress $= \left(\frac{2e\theta}{L}\right)E_s = \left(\frac{0.218772}{5500}\right)210\times10^3 = 8.353$ N/mm²

$$\text{Initial prestress} = 850 \text{ N/mm}^2$$

$$\therefore \quad \text{Percentage increase in stress} = \frac{8.353}{850} \times 100 = 0.983\%$$

14.21 Stresses in Beam at Different Stages of Loading

Stresses get induced in beam at the following stages of loading viz.

1. At transfer
2. At final stage of loading

14.21.1 Stress at transfer stage

It gets induced just after the transfer of prestressing force in beam. Some important points must be considered in this regard:

1. Total prestressing force (initial value) is considered
2. No loss of prestress is considered
3. Dead loads may be considered. If the beam is still kept on ground or still supported on supports then no dead load should be considered.
4. No live load is considered

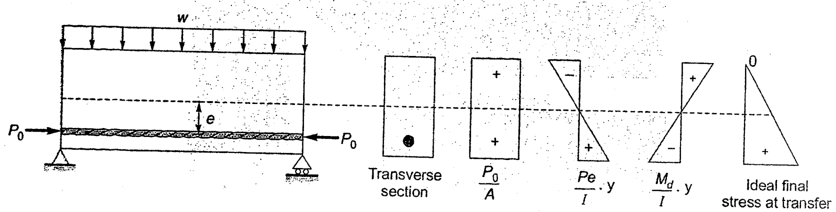


Fig.14.19 *Variation of stresses in the transverse section of an eccentrically prestressed beam*

$$\text{Stress at top} = \frac{P_0}{A} - \frac{Pe}{I} \cdot y + \frac{M_d}{I} \cdot y$$

$$\text{Stress of bottom} = \frac{P_0}{A} + \frac{Pe}{A} \cdot y - \frac{M_d}{I} \cdot y$$

These two forces i.e. the *P*-force and the *C*-force are equal and opposite. Now external moment is zero. No level difference between the *C*-force and P-force i.e. *P*-force and *C*-force acts at the same level. Thus the *p*-line and *C*-line coincide in the absence of external loading.

Now let the beam is subjected to an external moment (M). The c-line will shift upwards to counteract the external moment. Let this shift of *c*-line from *p*-line is *a*,

$$\therefore \quad a = \frac{M}{P}$$

$$\therefore \quad \text{Final stress in concrete} = \frac{C}{A} \pm \frac{C \times \text{eccentricity of } C}{Z}$$

Ideal stress diagram at transfer should have zero stress at top and maximum stress at bottom at shown above.

14.21.2 Stress at final stage of loading

It occurs after the application of line loads.

1. Loss of prestress is considered

   Prestressing force = kP_0

   where k = loss factor = $\left(1 - \frac{p_L}{100}\right)$ where p_L is in percentage.

2. All loads i.e. dead loads and live loads are considered.

$$\text{Final stress} = \frac{kP_0}{A} \mp \frac{kP_0 e}{I} \cdot y \pm \frac{M_d}{I} \cdot y \pm \frac{M_l}{I} \cdot y$$

Ideal stress diagram at final stage of loading should have zero stress at bottom and maximum stress at the top.

14.22 Prestress Losses

The initial pre-stress in concrete does not remain constant with time but in fact decreases with time at transfer stage due to many possible reasons. This gradual decrease in pre-stress is called as **loss of pre-stress.** It is to be noted that different types of pre-stress losses are encountered in the two types of pre-stressed concrete viz. pre-tensioned and post-tensioned concrete.

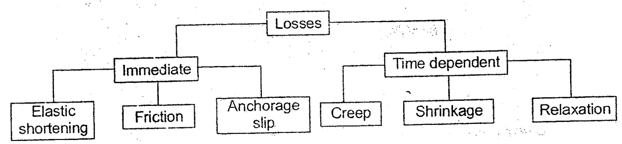


Fig.14.20 *Losses in prestressed concrete*

Table 14.4: *Types of pre-stress losses in pre-stressed concrete*

Pre-tensioned concrete	Post-tensioned concrete
Loss due to elastic deformation of concrete.	If wires are tensioned simultaneously, then no loss due to elastic deformation of concrete but however if the wires are tensioned successively, then loss due to elastic deformation occurs.
Loss due to stress relaxation in steel.	Loss due to stress relaxation in steel.
Loss due to creep and shrinkage of concrete.	Loss due to creep and shrinkage of concrete.
No loss due anchorage slip and friction.	Loss due to anchorage slip and friction.

Apart from the above losses, there are other types of losses also like loss due to temperature, particularly in steam curing of pre-tensioned concrete members.

14.22.1 Loss due to elastic deformation of concrete

This type of pre-stress loss in pre-stressed concrete members depends on the modular ratio and average stress in concrete at the level of centroid of steel tendons/wires.

Let f_c = Pre-stress in concrete at the level of centroid of tendons/wires

E_s = Modulus of elasticity of steel

E_c = Modulus of elasticity of concrete

Then modular ratio $(m) = \frac{E_s}{E_c}$

So, strain in concrete at the level of steel $= \frac{f_c}{E_c}$

Corresponding to the above strain, stress in steel $= \left(\frac{f_c}{E_c}\right)E_s = mf_c$

$\therefore$ Loss of stress in steel $= mf_c$.

Example 14.10 A pre-stressioned concrete beam of size 150 × 300 mm is prestressed by straight wires with an initial prestressing force of 150 kN. The eccentricity of the wire is 60 mm. What is the percentage loss of stress in steel due to elastic deformation of concrete? Take E_s = 210 kN/mm², E_c = 35 kN/m², area of wires as 180 mm².

Solution:

$$\text{Initial prestressing force } (P) = 150 \text{ kN}$$

$$\text{Gross cross-sectional area of beam } (A) = 150 \times 300 = 45000 \text{ mm}^2$$

$$\text{Moment of inertia } (I) = 150 \times \frac{300^3}{12} = 337.5 \times 10^6 \text{ mm}^4$$

$$\text{Initial stress in steel wires} = \frac{150 \times 1000}{180} = 833.33 \text{ N/mm}^2$$

$$\text{Stress in concrete at the level of prestress wire} = \frac{P}{A} + \frac{Pe}{I}e = \frac{450 \times 1000}{45000} + \frac{(150 \times 1000)60}{337.5 \times 10^6} \times 60$$

$$= 3.33 + 1.6 = 4.93 \text{ N/mm}^2$$

$$\text{Modular ratio } (\alpha) = \frac{E_s}{E_c} = \frac{210}{35} = 6$$

$\therefore$ Loss of stress due to elastic deformation of concrete = 6 × 4.93 = 29.58 N/mm²

$$\therefore \quad \text{Percentage loss of stress in steel} = \frac{29.58}{833.33} \times 100 = 3.55\%$$

14.22.2 Loss due to shrinkage of concrete

It is well known fact that concrete shrinks with age. This shrinkage of concrete in a pre-stressed concrete member results in the shortening of prestressed wires thereby resulting in the loss of pre-stress. Obviously, the shrinkage of concrete is influenced by the type of cement used and the type of aggregates (both coarse and fine aggregates) along with the water-cement ratio and also the method of curing employed. Thus use of high strength concrete is preferred with low water cement ratio which results in low shrinkages and thus the resulting loss due to concrete shrinkage reduces.

For **pre-tensioned members,** shrinkage of concrete is prevented by moist curing of members till the time of stress transfer. Thus, the total residual shrinkage strain is large in pre-tensioned members after the stress transfer in comparison to post tensioned members, where a part of shrinkage strain has already occurred at the stage of stress transfer.

For **pre-tensioned members,** the total residual shrinkage strain is 3×10^{-4}.

For **post-tensioned members,** total residual shrinkage strain is given by,

$$\frac{2 \times 10^{-4}}{\log_{10}(t+2)}$$

where t = age of concrete at the time of transfer in days.

This value can be enhanced by 50% in dry atmospheric conditions but must not exceed 3×10^{-4}.

Example 14.11 A concrete beam is prestressed by a cable with a prestressing force of 450 kN. Area of cable wires is 150 mm². Find the percentage loss of stress due to shrinkage of concrete assuming the beam to be (a) Pre-tensioned (b) Post-tensioned. Take E_s = 210 kN/mm² and age of concrete at transfer = 8 days.

Solution:

$$\text{Initial stress in wires} = \frac{450 \times 1000}{150} = 3000 \text{ N/mm}^2$$

(a) Pre-tensioned

For pre-tensioned beam total residual shrinkage strain = 3×10^{-4}

$$\therefore \quad \text{Loss of stress} = E_s \times 3 \times 10^{-4} = 210 \times 10^3 \times 3 \times 10^{-4} = 63 \text{ N/mm}^2$$

$$\therefore \quad \text{Percentage loss of stress} = \frac{63}{3000} \times 100 = 2.1\%$$

(b) Post-tensioned beam

For post-tensioned beam, total residual shrinkage strain

$$= \frac{2 \times 10^{-4}}{\log(t+2)} = \frac{2 \times 10^{-4}}{\log(8+2)} = 2 \times 10^{-4}$$

$$\therefore \quad \text{Loss of stress} = 210 \times 10^3 \times 2 \times 10^{-4} = 42 \text{ N/mm}^2$$

$$\therefore \quad \text{Percentage loss of stress} = \frac{42}{3000} \times 100 = 1.4\%$$

14.22.3 Loss due to creep of concrete

It is well known fact that sustained loading on concrete results in creep of concrete. Due to this reason only, the sustained pre-stress in a pre-stressed concrete member results in creep of concrete. The loss of stress in steel due to creep of concrete can be judged if the ultimate creep coefficient is known. Thus we have two methods, viz.:

1. **Creep coefficient method:** As described earlier, we define creep coefficient as:

$$\text{Creep coefficient } (\phi) = \frac{\text{Creep strain}(\varepsilon_c)}{\text{Elastic strain }(\varepsilon_e)}$$

Thus,

$$\varepsilon_c = \varepsilon_e \cdot \phi = \phi\left(\frac{f_c}{E_c}\right)$$

Thus loss of stress in steel = $\varepsilon_c E_c \phi E_s = \phi E_s \left(\frac{f_c}{E_c} \right) = m\phi f_c$

The magnitude of creep coefficient (ϕ) varies with humidity of the atmosphere, quality of concrete and the duration of applied loading along with the age of concrete at the time of loading. Generally the creep coefficient varies from 1.5 for wet situations to 4.0 for dry situations with relative humidity of about 35%.

2. **Ultimate creep strain method:** If ε_{cc} is the ultimate creep strain for a sustained stress of magnitude unity, then the loss of pre-stress due to creep in concrete is equal to $\varepsilon_{cc} f_c E_s$ where f_c is the compressive stress in concrete at the level of centroid of steel and E_s is the modulus of elasticity of steel.

Example 14.12 **A prestressed rectangular concrete beam of size 150 × 450 mm is prestressed by 5 wires of 6 mm diameter located at an eccentricity of 45 mm. The initial prestress in the wires is 1350 N/mm². What is the loss of stress in steel due to creep of concrete? Take E_s = 210 kN/mm², E_c = 35 kN/mm², ultimate creep strain = 41 × 10⁻⁶ mm/mm per N/mm², creep coefficient ϕ = 1.6.**

Solution:

Gross cross-sectional area of beam, $A = 150 \times 450 = 67500 \text{ mm}^2$

Moment of inertia, $I = 150 \times \frac{450^3}{12} = 11.3906 \times 10^8 \text{ mm}^4$

Prestressing force, $P = 1350 \times 5 \times \frac{\pi}{4} \times 6^2 = 190.85 \text{ kN}$

Modular ratio, $m = \frac{E_s}{E_c} = \frac{210}{35} = 6$

Creep coefficient, $\phi = 1.6$

Stress in concrete at the level of steel $= \frac{190.85 \times 10^3}{67500} + \frac{190.85 \times 10^3 \times 45 \times 45}{11.3906 \times 10^8}$

$= 2.83 + 0.34 = 3.17 \text{ N/mm}^2$

Loss of stress from ultimate creep strain method

$= (41 \times 10^{-6}) \times 3.17 \times 210 \times 1000 = 27.29 \text{ N/mm}^2$

Loss of stress from creep coefficient method

$= 1.6 \times 3.17 \times 6 = 30.432 \text{ N/mm}^2$

Example 14.13 **A post-tensioned prestressed concrete beam of size 150 × 450 mm prestressed using a parabolic cable with zero eccentricity at supports. The eccentricity at the centre of span is 50 mm. Initial prestress in the cable is 1300 N/mm² and area of cable is 175 mm². If ultimate creep strain is 3 × 10⁻⁵ mm/mm per N/mm² and E_s = 210 kN/m², find the loss of stress in steel due to creep.**

Solution:

Gross cross-sectional area of beam, $A = 150 \times 450 = 67500 \text{ mm}^2$

Moment of inertia of beam, $I = 150 \times \frac{450^3}{12} = 11.3906 \times 10^8 \text{ mm}^4$

$P = 1300 \times 175 = 227.5 \text{ kN}$

Stress in concrete at the level of steel,

at support $= \frac{227.5 \times 1000}{67500} = 3.37 \text{ N/mm}^2$

at mid-span $= \frac{227.5 \times 1000}{67500} + \frac{227.5 \times 1000 \times 50 \times 50}{11.3906 \times 10^8}$

$= 3.37 + 0.5 = 3.87 \text{ N/mm}^2$

∴ Average stress at the level of steel $= 3.37 + \frac{2}{3} \times (3.87 - 3.37)$ 3.703 N/mm^2

∴ Loss of stress due to creep $= 3 \times 10^{-5} \times 3.703 \times 210 \times 1000 = 23.33 \text{ N/mm}^2$

14.22.4 Loss due to stress relaxation in steel

In order to account for loss due to relaxation in steel, most of the codes all over the world describe the loss of stress due to stress relaxation as a percentage of initial stress in steel. In line with this only, the IS Code recommends a value ranging from 0 to 90 N/mm² for stress in wires varying from $0.5f_{pu}$ to $0.8f_{pu}$.

Loss of stress due to stress relaxation in steel as per IS 1343 at 1000 hrs. at 27°C is as given in the table.

In order to reduce this type of pre-stress loss, a temporary over stressing by about 5 to 10% for a period of 2 min is usually recommended for drawn wires.

Table 14.5: IS 1343 recommendations for relaxation loss at 1000 hrs. at 27°C

Initial Stress	Relaxation loss (N/mm²)
$0.5f_{pu}$	0
$0.6f_{pu}$	35
$0.7f_{pu}$	70
$0.8f_{pu}$	90

14.22.5 Loss of stress due to friction

This type of loss occurs in post-tensioned members. In post-tensioned members, the tendons are passed through the ducts. These ducts are either straight or follow a curved profile as per the requirements. Thus on applying the pre-stressing force to these tendons, loss of stress occurs due to the friction between the tendons and the surrounding concrete. Friction loss is of two types viz.:

1. Loss of stress due to the effect of curvature which in fact depends on the alignment of the tendon.
2. Loss of stress due to wobble effect. It depends on the local variations in the alignment of the cable. The wobble or the wave effect occurs due to accidental or unavoidable misalignment in the cable profile because the ducts or the sheaths cannot be perfectly placed in order to follow exactly the cable profile throughout the length of the member.

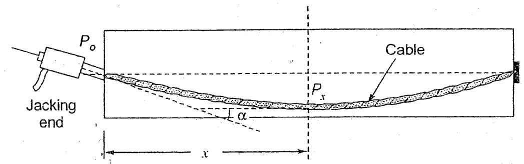


Fig.14.21 *Loss of Stress due to Friction*

The magnitude of the pre-stressing force (P_x) at a distance x from the tensioning end follows the following exponential function:

$$P_x = P_o e^{-(\mu\alpha + kx)}$$

where, P_o = Pre-stressing force at the jacking end

μ = Friction coefficient between the cable and the duct

α = Angle (in radians) through which the tangent to the cable profile has turned between the two points under consideration

k = Coefficient of friction for wave effect

Typical values of coefficient of friction (μ):

0.55 for steel moving on smooth concrete

0.35 for steel moving on steel fixed to duct

0.25 for steel moving on steel fixed to concrete

0.25 for steel moving on lead

Value of friction coefficient for wave effect (k) is taken as 0.15 per 100 m for normal conditions.

This value can be taken as zero where clearance between the duct and the cable is large enough to nullify the wave effect.

The coefficient of friction (μ) can be considerably reduced by using various types of lubricants like grease, oils, graphite, paraffin wax etc. Experimentally, it is found that the graphite gives the lowest value of coefficient of friction. For small values of ($\mu\alpha + kx$), the above expression can be simplified as,

$$P_x = P_o e^{-(\mu\alpha + kx)}$$

$$\Rightarrow \quad P_x = P_o(1-\mu\alpha-kx)$$

$$\Rightarrow \quad \frac{P_o - P_x}{A} = (\mu\alpha + kx)\frac{P_o}{A}$$

$$\Rightarrow \quad \text{Prestress loss} = (\mu\alpha + kx) \times \text{Initial stress}$$

The above simplified expression shows that for a cable of single curvature, the pre-stress varies linearly with distance from the stretching end.

14.22.6 Loss due to anchorage slip

It is observed mostly in post-tensioning system wherein the cable is tensioned and the jack is released to transfer the pre-stress to concrete, the friction wedges that are used to grip the wires, get slipped for a small distance before the wires are anchored firmly between the edges. When anchor plates are used, it is quite essential to allow for a small settlement of plate into the end of the concrete member.

Let, Δ = Slip of the anchorage (in mm)

L = Length of the cable (in mm)

A = Cross-sectional area of the cable (in mm²)

E_s = Modulus of elasticity of steel (in N/mm²)

P = Pre-stressing force in the cable (in N)

Thus,
$$\Delta = \frac{PL}{AE_s}$$

Table 14.6: *Some typical values of anchorage slip*

Anchorage System	Anchorage Slip (Δ)
Freyssinet	
12.5 mm diameter strands	4 mm
12.8 mm diameter strands	6 mm
Magnel	8 mm
Dywidag System	1 mm

Thus loss of stress due to anchorage slip = $\left(\frac{P}{A}\right) = \frac{\Delta E_s}{L}$

The loss during anchoring is usually accounted for (at site) by over extending the tendon in pre-stressing process. But the momentary over-stress must not exceed 80-85% of ultimate tensile strength of wires.

In long line pre-tensioning system, the slip at the anchorages is usually very small as compared to the length of the tensioned wire and thus is generally neglected. However, while pre-stressing a short member, due allowance should be made for loss of pre-stress due to anchorage slip which in fact forms a major portion of the total loss of pre-stress.

Example 14.14 **A concrete beam is post-tensioned by a cable carrying a pre-stress of 900 N/mm². At the jacking end, the slip was observed to be 8 mm. Find the percentage loss of stress due to anchorage slip if length of beam is (a) 20 m and (b) 5 m. Take E_s = 210 kN/mm².**

Solution:

Loss of stress due to anchorage slip = $E_s = \frac{\Delta}{L}$

(a) For 20 m beam, loss of stress = $210 \times 1000 \times \frac{8}{20000} = 84$ N/mm²

$\therefore$ Percentage loss of stress = $\frac{84}{900} \times 100 = 9.33\%$

(b) For 5 m, beam, loss of stress = $210 \times 1000 \times \frac{8}{5000} = 336$ N/mm²

$\therefore$ Percentage loss of stress = $\frac{336}{900} \times 100 = 37.33$ N/mm²

Example 14.15 **A prestressed concrete beam of size 250 mm x 400 mm has a span of 10 m. The beam is prestressed by steel wires of area 360 mm² provided at a uniform eccentricity of 50 mm with an initial prestress of 1150 N/mm². Find the percentage loss of stress in the wires of (a) the beam is pre-tensioned (b) the beam is post tensioned. Take E_s = 210 kN/mm², E_c = 35 kN/m², Ultimate creep stress = 45 x 10^{-6} mm/mm per N/mm² for pre-tensioned beam and 22 x 10^{-6} mm/mm per N/mm² for post tensioned beam, shrinkage of concrete = 3 x 10^{-4} for pre-tensioned beam = 2.15 x 10^{-4} for post tensioned beam, steel stress relaxation = 5% of initial stress, anchorage slip = 1.25 mm, friction coefficient of wave effect = k = 0.0013.**

Solution:

Gross cross-sectional area of beam (A) = 250 x 400 = 1 x 10^5 mm²

Moment of inertia of the beam section (I) = $\frac{250 \times 400^3}{12} = 13.33 \times 10^8$ mm⁴

Initial prestressing force (P) = 1150 x 360 = 414 kN

$$m = \frac{E_s}{E_c} = \frac{210}{35} = 6$$

Stress in concrete at the level of steel,

$$f_c = \frac{P}{A} + \frac{Pe}{I}e = \frac{414000}{10^5} + \frac{414000(50)^2}{13.33 \times 10^8}$$
$$= 4.14 + 0.7764 = 4.9164 \text{ N/mm}^2$$

(a) Loss of stress in pre-tensioned beam

(i) Loss of stress due to elastic shortening of concrete

$$= mf_c = 6 \times 4.964 = 29.4984 \text{ N/mm}^2 = 29.5 \text{ N/mm}^2$$

(ii) Loss of stress due to creep of concrete

$$= 45 \times 10^{-6} \times 4.9164 \times 210 \times 10^3 = 46.5 \text{ N/mm}^2$$

(iii) Loss of stress due to shrinkage of concrete

$$= 3 \times 10^{-4} \times 210 \times 10^3 = 63 \text{ N/mm}^2$$

(iv) Loss of stress due to relaxation $= 0.05 \times 1150 = 57.5$ N/mm²

(v) Loss of stress due to anchorage slip = 0

(vi) Loss of stress due to friction = 0

Total loss of stress = 196.5 N/mm²

$$\therefore \quad \text{Percentage loss of stress} = \frac{196.5}{1150} \times 100 = 17.09\%$$

(b) **Loss of stress in post-tensioned beam**

(i) Loss of stress due to elastic shortening of concrete = 0

(ii) Loss of stress due to creep of concrete $= 22 \times 10^{-6} \times 4.9164 \times 210 \times 10^3$

$$= 22.7 \text{ N/mm}^2$$

(iii) Loss of stress due to shrinkage of concrete $= 2.15 \times 10^{-4} \times 210 \times 10^3$

$$= 45.15 \text{ N/mm}^2$$

(iv) Loss of stress due to relaxation $= 0.05 \times 1150 = 57.5$ N/mm²

(v) Loss of stress due to anchorage slip $= \dfrac{1.25}{10 \times 1000} \times 210 \times 1000 = 26.25$ N/mm²

(vi) Loss of stress due to friction $= 1150 \times 0.0013 \times 10 = 14.95$ N/mm²

Total loss of stress = 166.55 N/mm²

$$\therefore \quad \text{Percentage loss of stress} = \frac{166.55}{1150} \times 100 = 14.48\%$$

14.22.7 Loss due to successive tensioning of curved cables

As explained in the section of load balancing, the tendon profile is selected in such a way that it counteract the superimposed transverse loads. Mostly in bridges, the tendons are curved having maximum eccentricity at the center of span. For such cases, the loss of pre-stress due to elastic deformation of concrete is computed by taking average stress in concrete at the level of centroid of steel.

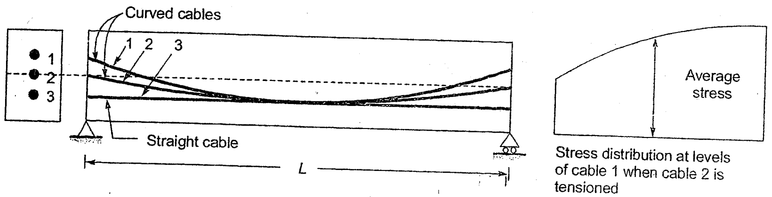


Fig.14.22 *Successive Tensioning of Curved Cables*

The above figure shows the three parabolic cables that are post-tensioned. The stress distribution in concrete at the level of cable no. 1 is also shown when cable no. 2 is tensioned. For calculating the loss of stress, we take into account the average stress.

14.22.8 Total loss of prestress

In normal design practice of pre-stressed concrete members, it is quite convenient to express the total loss of stress as a percentage of the initial stress and make suitable provisions in the design to account for these stress losses. Now the loss of pre-stress depends on various factors like the properties of concrete and steel, method of curing, the method of pre-stressing used and so on. Thus, it is quite difficult to have an exact idea of the total amount of loss of pre-stress.

Let, f_{pe} = Effective stress in tendons after loses

f_{pi} = Stress in tendons at transfer

η = Prestress loss reduction factor

$$\therefore \quad \eta = \left(\frac{f_{pe}}{f_{pi}}\right)$$

This η is generally taken as 0 .75 for pre-tensioned members and 0.80 for post-tensioned members.

Table 14.7: *Percentage loss of prestress*

Cause of prestress loss	Post-tensioned members	Pre-tensioned members
Shrinkage of concrete	6%	7%
Creep of concrete	5%	6%
Creep of steel	3%	2%
Elastic shortening of concrete	1%	3%
Total	15%	18%

Further different types of prestress loss can be summarized as:

Table 14.8: *Summary of prestress losses*

Types of loss	Equation
1. Curvature and friction effect	$(\mu\alpha + kx)P_0$
2. Slip at anchorages	$(\Delta/L)E_s$
3. Loss due to shrinkage	$\epsilon_s E_s$
4. Loss due to creep of concrete	$m\phi f_c$
5. Loss due to elastic shortening of concrete	mf_c
6. Loss due to stress relaxation	2-5%

14.23 Cracking Moment

It is the moment at which visible cracks will appear in the section and this moment is called as cracking moment. Cracks appear when tensile stresses developed in the section are more than the permissible tensile strength of concrete. Let cracking moment = M_c

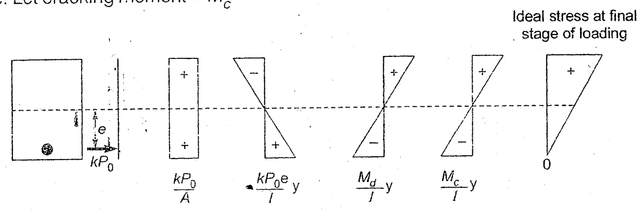


Fig.14.23 *Concept of cracking moment*

$$\text{Final stress at bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_c}{Z} = f_t$$

Load corresponding to cracking moment = w_c

$$\therefore \quad M_c = \frac{w_c l^2}{8}$$

$$\text{Load factor against cracking} = \frac{\text{Cracking Load}}{\text{Working Load}}$$

Here, Load = Dead load + Live load

Example 14.16 A prestressed concrete beam of 8 m span is prestressed by three cables each of area 150 mm² and initial prestress of 1100 N/mm². Cable is parabolic with eccentricities of 40 mm each at support and mid-span. Cable 2 is also parabolic but with zero eccentricity at supports and 40 mm at mid-span. Cable 3 is straight with uniform eccentricity of 40 mm. Calculate the loss of stress in each cable due to friction is cables are stretched from one end only. Take $k = 0.0015$ per m and $\mu = 0.35$.

Solution:

For parabolic cable profile, $y = \frac{4e}{L^2}x(L-x)$

$$\therefore \quad \text{Slope} = \frac{dy}{dx} = \frac{4e}{L^2}(L-2x)$$

$\therefore$ Slope at support for cable 1

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{4eL}{L^2} = \frac{4e}{L} = \frac{4\times 40}{8000} = 0.025 \text{ radians}$$

$$\therefore \quad \alpha = 2\times 0.025 = 0.04 \text{ radians}$$

Slope at support for cable 2

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{4e}{L} = \frac{4\times 20}{8000} = 0.01 \text{ rad}$$

$$\therefore \quad a = 2\times 0.01 = 0.02 \text{ radian}$$

Initial prestressing force (P_x) = $1100 \times 150 = 165$ kN

$$\therefore \quad P_x = P_o e^{-(\mu\alpha + kx)} = P_o(1 - \mu\alpha - kx)$$

$\therefore$ Loss of prestress $= P_o - P_x = P_o(\mu\alpha + kx)$

For cable 1, prestress loss $= P_o(0.35 \times 0.04 + 0.0015 \times 8)$

$= 0.026\,P_o = 0.026 \times 1100 = 28.6$ N/mm²

For cable 2, prestress loss $= P_o(0.35 \times 0.02 + 0.0015 \times 8)$

$= 0.019P_o = 0.019 \times 1100 = 20.9$ N/mm²

For cable 3, prestress loss $= P_o(0.35 \times 0 + 0.0015 \times 8)$

$= 0.012P_o = 0.012 \times 1100 = 13.2$ N/mm²

14.24 Design of Prestressed Concrete Beam Members

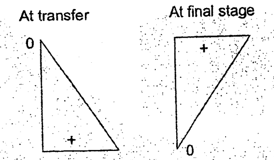
For design of prestressed concrete beam/slab, the following cases are considered:

Case I:

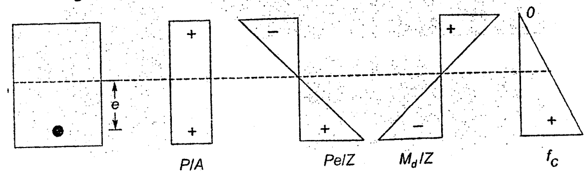
Given:
1. Permissible stress of concrete
   (i) in compression = f_c
   (ii) in tension = 0
2. Permissible stress in steel
3. Live load
4. Loss of prestress = 0

Find out:
1. Size of beam/slab requested
2. Prestressing force
3. Eccentricity

For design purposes, use following stress diagrams:



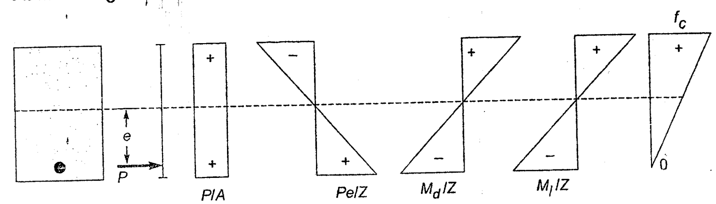
1. At transfer stage:



$$\text{Stress at top} = \frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z} = 0 \qquad \text{...(i)}$$

$$\text{Stress at bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} = f_c \qquad \text{...(ii)}$$

2. At final stage:



$$\text{Stress at top} = \frac{P}{A} - \frac{Pe}{Z} + \frac{M_d}{Z} + \frac{M_l}{Z} = f_c \qquad \text{...(iii)}$$

$$\text{Stress at bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M_d}{Z} - \frac{M_l}{Z} = 0 \qquad \text{(iv)}$$

Design Formula

Let

$$M_l = \text{BM due to live load}$$

The beam should be so designed that resultant stress will be as shown above i.e.

$$\frac{M_l}{Z} = f_c \quad \text{...(v)}$$

from (ii)

$$\frac{P}{A}+\frac{Pe}{Z}-\frac{M_d}{Z} = f_c = \frac{M_l}{Z} \quad \text{...(from v)}$$

$\Rightarrow$

$$\frac{Pe}{Z} = \frac{M_d+M_l}{Z}-\frac{P}{A} \quad \text{...(vi)}$$

But from (i),

$$\frac{P}{A}+\frac{M_d}{Z} = \frac{Pe}{Z} \quad \text{...(vii)}$$

Also from (v)

$$\frac{M_l}{Z} = f_c$$

$\Rightarrow$

$$Z = \frac{M_l}{f_c} = \text{Section modulus required}$$

Select suitable value of beam width (B)

$\therefore$ Section modulus, $Z = \frac{BD^2}{6}$

$\Rightarrow$

$$D = \sqrt{\frac{6Z}{B}}$$

$\therefore$ Cross-sectional area, $A = B \times D$

Now either from (i) + (ii) or (iii) + (iv)

$$\frac{2P}{A} = f_c$$

$\Rightarrow$

$$P = \frac{Af_c}{2}$$

from (i),

$$\frac{P}{A}+\frac{M_d}{Z} = \frac{Pe}{Z}$$

from (iv),

$$\frac{-P}{A}+\frac{M_d}{Z}+\frac{M_l}{Z} = \frac{Pe}{Z}$$

Adding,

$$\frac{2M_d}{Z}+\frac{M_l}{Z} = \frac{2Pe}{Z}$$

$\Rightarrow$

$$e = \frac{2M_d+M_l}{2P} = \text{Eccentricity of prestressing tendons}$$

Design Steps for design of prestressed concrete beam member

1. Calculate live load moment $(M_l) = \frac{w_l l^2}{8}$ (for simply supported beam)
2. Calculate section modulus, $Z = \frac{M_l}{f_c}$
3. Select any size, say B (width), $Z = \frac{BD^2}{6} \Rightarrow D = \sqrt{\frac{6Z}{B}}$
4. Area, $(A = BD)$

   Prestressing force $(P) = \frac{Af_c}{2}$
5. Calculate area of steel, $A_s = \frac{P}{f_s}$
6. Calculate dead load, $w_d = BD \times 1 \times \gamma_c (\gamma_c = 25 \text{ kN/m}^3)$
7. Calculate moment due to dead load, $M_d = \frac{w_d l^2}{8}$
8. Eccentricity, 'e' $= \frac{2M_d+M_l}{2P}$

Case II:

Given:	Find out:
Permissible stress of concrete compression = f_c	Size of beam/slab requested
Permissible stress of concrete in tension = 0	Prestressing force
	Eccentricity

Loss of prestress = P_L%

$$k = \text{Loss factor} = \left(1-\frac{P_L}{100}\right)$$

Stress at transfer stage:

$$\text{Stress at top} = \frac{P}{A}-\frac{Pe}{Z}+\frac{M_d}{Z}=0 \quad \text{....(i)}$$

$$\text{Stress at bottom} = \frac{P}{A}-\frac{Pe}{Z}-\frac{M_d}{Z}=f_c \quad \text{...(ii)}$$

Stress at final stage:

$$\text{Stress at top} = \frac{kP}{A}-\frac{kPe}{Z}+\frac{M_d}{Z}+\frac{M_l}{Z}=f_c \quad \text{....(iii)}$$

$$\text{Stress at bottom} = \frac{kP}{A}+\frac{kPe}{Z}-\frac{M_d}{Z}-\frac{M_l}{Z}=0 \quad \text{...(iv)}$$

Design Formula

From equation (iii) – k. equation (i),

$$\frac{M_d}{Z} = \frac{M_l}{Z}-\frac{kM_d}{Z}=f_c$$

$\Rightarrow$

$$Z = \frac{(1-k)\,M_d+M_l}{f_c}$$

From equation (iii) + equation (iv)

$$\frac{2kP}{A} = f_c$$

$$\Rightarrow \qquad P = \frac{f_c \cdot A}{2k} = \text{Prestressing force}$$

Multiplying equation (i) by k and rearranging,

$$\frac{kPe}{Z} = \frac{kP}{A} + \frac{kM_d}{Z}$$

From equation (iv)

$$\frac{kPe}{Z} = \frac{-kP}{A} + \frac{M_d}{Z} + \frac{M_l}{Z}$$

Adding the two above equations

$$\frac{2kPe}{Z} = \frac{(1+k)M_d + M_l}{kP}$$

$$e = \frac{(1+k)M_d + M_l}{2kP}$$

Design Steps

Assume size of beam/slab i.e. width (B) and depth (D).
Calculate

$$A = BD$$

$$\therefore \qquad M_d = \frac{w_d l^2}{8} = \text{BM due to dead load}$$

$$M_l = \frac{w_l l^2}{8} = \text{BM due to live load}$$

Section modulus required.

$$Z = \frac{(1+k)M_d + M_l}{f_c}$$

Now $\qquad Z = \dfrac{BD^2}{6}$

$$\Rightarrow \qquad D = \sqrt{\frac{6Z}{B}}$$

Decide B and D

$\therefore \qquad A = BD$

Prestressing force

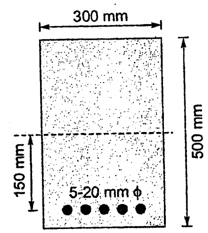
$$P = \frac{Af_c}{2k}$$

Eccentricity $\qquad e = \dfrac{(1+k)M_d + M_l}{2kP}$

Calculate area of steel $\qquad A_s = \dfrac{P}{f_c}$

Example 14.17 A prestressed concrete beam of size 300 × 500 mm is provided with a straight cable of 20 wires of 5 mm of bars stressed at 1600 N/mm² prestress eccentricity = 150 mm. Beam is subjected an UDL of 30 kN/m over a span of 7 m. Calculate the stress in beam at (i) At ends. (ii) At mid span. (Neglect losses)

Solution:



Beam width, $\qquad B = 300$ mm

Beam depth, $\qquad D = 500$ mm

Area of tendons, $\qquad A_s = 20 \times \frac{\pi}{4} \times (5)^2 = 392.7$ mm²

Prestressing force, $\qquad P = A_s \times f_s = 392.7 \times 1600$

$= 628318.53$ N $= 628.32$ kN

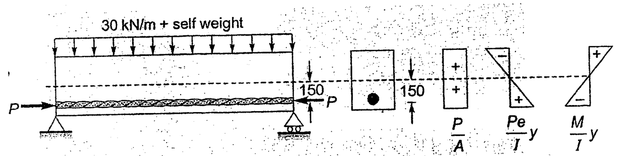
Eccentricity, $\qquad e = 150$ m

Loads $\qquad$ Live load $= 30$ kNm

Dead load $= 0.3 \times 0.5 \times 1 \times 25 = 3.75$ kN/m

Total load $= 33.75$ kN/m

Bending moment at ends $\qquad M = 0$



Bending moment at mid span $\qquad = \dfrac{wl^2}{8} = \dfrac{33.75 \times 7^2}{8} = 206.72$ kNm

Stresses in beam at ends:

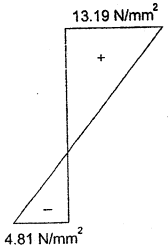
Stress at top, $\qquad f_{top} = \dfrac{P}{A} - \dfrac{Pe}{Z} = \dfrac{628.32 \times 10^3}{300 \times 500} - \dfrac{628.32 \times 10^3 \times 150}{300 \times \frac{500^2}{6}} = -3.35$ N/mm²

Stress at bottom, $\qquad f_{bottom} = \dfrac{P}{A} + \dfrac{Pe}{Z}$

$$= \frac{628.32 \times 10^3}{300 \times 500} + \frac{628.32 \times 10^3 \times 150}{300 \times \frac{500^2}{6}} \qquad (11.73 - 3.35 = 8.88)$$

$= 11.73$ N/mm²

Stresses in beam at mid span:



Stress at top, $\qquad f_{top} = \dfrac{P}{A} - \dfrac{Pe}{Z} + \dfrac{M}{Z} = 4.19 - 7.54 + \dfrac{206.72 \times 10^6}{300 \times \frac{500^2}{6}}$

$= 4.19 - 7.54 + 16.54 = 13.19$ N/mm²

Stress at bottom, $\qquad f_{bottom} = \dfrac{P}{A} + \dfrac{Pe}{Z} - \dfrac{M}{Z}$ $\quad (13.19 - 4.81 = 8.38)$

$= 4.19 + 7.54 - 16.54 = -4.81$ N/mm²

Example 14.18 Determine the profile of a load balancing cable for a beam of size 300 × 800 mm subjected to a live load of 10 kN/m over a span of 8 m. If the beam is

(i) simply supported at ends.

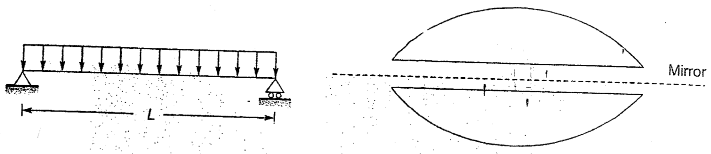
(ii) cantilever.

In both case prestressing force that can be applied = 1500 kN. Find out the shape of cable and show the position in beam. Also find the equation of cable profile in both cases.

Solution:

(i) If it is simply supported

(a) The shape of cable shall be mirror image of BM diagram as shown in figure.



(b) For complete load balancing

$$w = w_p = \frac{8Ph}{l^2}$$

Load,

$$\text{self weight} = 0.3 \times 0.8 \times 25 \times 1 = 6.0 \text{ kNm}$$

$$\text{Live load} = 10 \text{ kNm}$$

$$w = 16 \text{ kNm}$$

Eccentricity at centre,

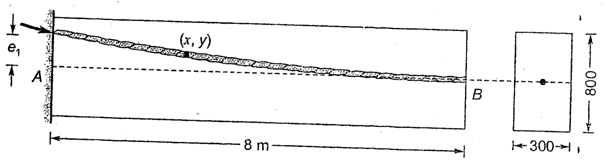
$$h = \frac{w_p l^2}{8P} = \frac{16\times8^2}{8\times1500} = 0.08533 \text{ m} = 85.33 \text{ mm}$$

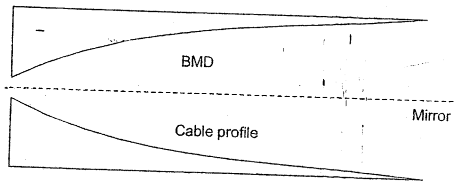
Equation at cable (with origin at ends)

$$y = \frac{4h}{l^2}x(l-x) = \frac{4\times0.085333}{8^2}x(l-x) = \frac{x(8-x)}{187.5}m$$

(ii) Cantilever

Shape of the cable shall be as shown in figure





BM at

$$A = \frac{wl^2}{2}$$

$$Pe_1 = \frac{wl^2}{2}$$

$$e_1 = \frac{wl^2}{2P} = \frac{16\times8^2}{2\times1500} = 0.34133 \text{ m} = 341.33 \text{ mm}$$

Consider any point (x, y) keeping origin at free end,

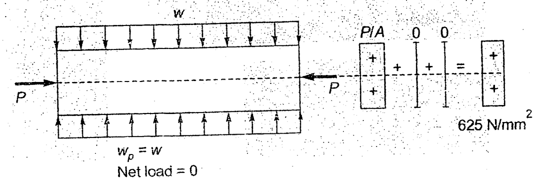
BM at point (x, y)

$$M = \frac{wx^2}{2}$$

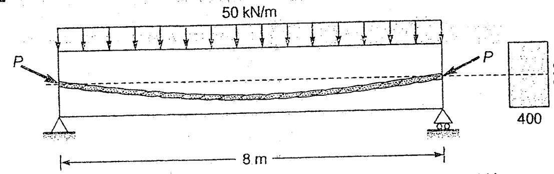
$$Py = \frac{wx^2}{2}$$

$$y = \frac{wx^2}{2P} = \frac{16x^2}{2\times1500} = \frac{x^2}{187.5} \quad \text{...(i)}$$

Stresses in the beam in both cases $= \dfrac{P}{A} = \dfrac{1500\times10^3}{300\times800} = 625 \text{ N/mm}^2$



Example 14.19 A PSC beam as shown in figure.



P = 1500 kN. The beam is subjected to a live load moment of 50 kN/m over a span = 8 m. Calculate stresses in the beam at quarter span and calculate value by three method.

Solution:

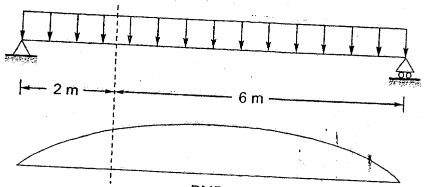
1. Eccentricity at quarter span:

$$y = \frac{4h}{l^2}x(l-x)$$

At $x = 2$

$$y = \frac{4\times0.2}{8^2}\times2(8-2) = 0.15 \text{ m} = 150 \text{ mm}$$

2. Moment at quarter span:



BMD

Total weight,

$$\text{Live load} = 50 \text{ kN/m}$$

$$\text{Self weight} = 0.40 \times 0.60 \times 1 \times 25 = 6 \text{ kN/m}$$

$$w = 56 \text{ kN/m}$$

$$R_1 = R_2 = \frac{wl}{2} = \frac{56 \times 8}{2} = 224 \text{ kN}$$

$$M = R_1 x - \frac{wx^2}{2} = 224 \times 2 - \frac{56 \times 2^2}{2} = 336 \text{ kNm}$$

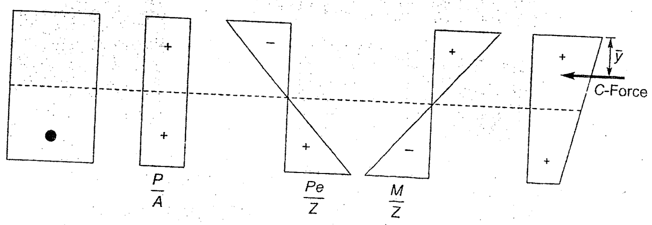
3. Cross-sectional area,

$$A = 400 \times 600 = 24 \times 10^4 \text{ mm}^2$$

Section modulus,

$$Z = \frac{BD^2}{6} = \frac{400 \times 600^2}{6} = 24 \times 10^6 \text{ mm}^3$$

A. By stress concept method



$$\frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$$

$$\tan\theta = \frac{4 \times 0.2}{8^2} \times (8 - 2 \times 2) = \frac{1}{20}$$

$$\theta = 2.86°$$

$$\cos\theta = 0.99875$$

$\Rightarrow$

$$P\cos\theta \approx P$$

Stress at top

$$f_{top} = \frac{P}{A} - \frac{Pe_p}{(Z)} + \frac{M}{Z}$$

$$= \frac{1500 \times 10^3}{400 \times 600} - \frac{1500 \times 10^3 \times 150}{24 \times 10^6} + \frac{336 \times 10^6}{24 \times 10^6}$$

$$= 6.25 - 9.375 + 14 = 10.875 \text{ N/mm}^2$$

Stress at bottom

$$f_{bottom} = \frac{P}{A} + \frac{Pe_p}{Z} - \frac{M}{Z} = 6.25 + 9.375 - 14 = 1.625 \text{ N/mm}^2$$

$$\text{C-force} = B \times D\left(\frac{f_1 + f_2}{2}\right) = 400 \times 600\left(\frac{10.875 + 1.625}{2}\right) \times \frac{1}{10^3}$$

$$= 1500 \text{ kN}$$

$\bar{y}$ value

$$\bar{y} = \left(\frac{f_1 + 2f_2}{f_1 + f_2}\right) \times \frac{D}{3} = \left(\frac{10.875 + 2 \times 1.625}{10.875 + 1.625}\right) \times \frac{600}{3}$$

$$= 226 \text{ mm}$$

Eccentricity of C-force

$$e_c = \frac{D}{2} - \bar{y} = 300 - 226 = 74 \text{ mm}$$

B. By strength concept method (based on F-force)

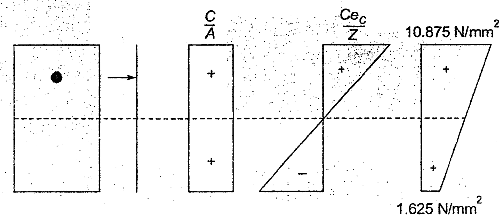
Shift of C-line w.r.t. P-line

$$x = \frac{M}{P} = \frac{336 \times 10^6}{1500 \times 10^3} = 224 \text{ mm}$$

Eccentricity of C-line

$$e_c = x - e_p = 224 - 150 = 74 \text{ mm}$$

Stresses



Stress at top,

$$f_{top} = \frac{C}{A} + \frac{C.e_c}{Z}$$

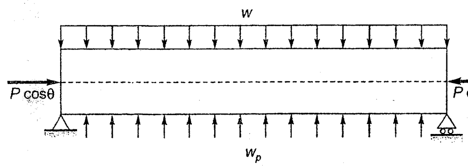
$$= \frac{1500 \times 10^3}{24 \times 10^4} + \frac{1500 \times 10^3 \times 74}{24 \times 10^6}$$

$$= 6.25 + 4.625 = 10.875 \text{ N/mm}^2$$

Stress at bottom,

$$f_{bottom} = 6.25 - 4.625 = 1.625 \text{ N/mm}^2$$

C. By load balancing method



$$w_p = \frac{8Ph}{l^2} = \frac{8 \times 1500 \times 0.2}{8^2} = 37.5 \text{ kN/m}$$

$$\text{Net UDL} = w - w_p = 56 - 37.5 = 18.5 \text{ kN/m}$$

Reaction,

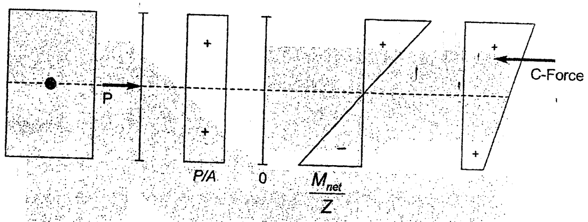
$$R_1 = R_2 = \frac{18.5 \times l}{2} = \frac{18.5 \times 8}{2} = 74 \text{ kN}$$

Flexural moment,

$$M_{net} = R_1 x - \frac{wx^2}{2}$$

$\Rightarrow$

$$M_{net} = 74 \times 2 - \frac{18.5 \times 2^2}{2} = 111 \text{ kNm}$$



Stress at top

$$f_{top} = \frac{P}{A} + \frac{M_{net}}{Z} = \frac{1500 \times 10^3}{400 \times 600} + \frac{111 \times 10^6}{24 \times 10^6}$$

$$= 6.25 + 4.625 = 10.875 \text{ N/mm}^2$$

Stress at bottom

$$f_{bottom} = \frac{P}{A} - \frac{M_{net}}{Z} = 6.25 - 4.625 = 1.625 \text{ N/mm}^2$$

Example 14.20 For a prestress beam for pretensioned, reinforcement is provided as shown in figure.

Initial prestress = 1600 N/mm² in both wires. Calculate loss of prestress only due to elastic shortening is each wire. $E_s = 2 \times 10^5$ N/mm², $E_C = 0.32 \times 10^5$ N/mm².

Solution:

$$P_1 = 12 \times \frac{\pi}{4} \times (5)^2 \times 1600 \times \frac{1}{10^3} = 377 \text{ kN}$$

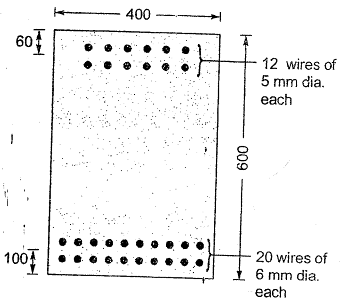
$$e_1 = 240 \text{ mm}$$

$$P_2 = 20 \times \frac{\pi}{4} \times 6^2 \times 1600 \times \frac{1}{10^3} = 904.8 \text{ kN}$$

$$e_2 = 200 \text{ mm}$$

$$A = 400 \times 600 = 24 \times 10^4 \text{ mm}^2$$

$$I = \frac{400 \times 600^3}{12} = 72 \times 10^8 \text{ mm}^4$$



Stress in concrete at the location of top steel

$$f_{ct} = \frac{P_1}{A} + \frac{P_1 e_1}{I} e_1 + \frac{P_2}{A} - \frac{P_2 e_2}{I} e_1$$

$$= \frac{377 \times 10^3}{24 \times 10^4} + \frac{377 \times 10^3 \times 240^2}{72 \times 10^8} + \frac{904.8 \times 10^3}{24 \times 10^4} - \frac{904.8 \times 10^3 \times 200 \times 240}{72 \times 10^8}$$

$$= 1.57 + 3.02 + 3.77 - 6.03 = 2.33 \text{ N/mm}^2$$

$$\text{Loss of stress} = f_{ct} \times \frac{E_s}{E_c} = 2.33 \times \frac{2.0 \times 10^5}{0.32 \times 10^5} = 14.56 \text{ N/mm}^2$$

Stress in concrete at the location of bottom steel

$$f_{cb} = \frac{P_1}{A} - \frac{P_1 e_1}{I} e_2 + \frac{P_2}{A} + \frac{P_2 e_2}{I} e_2$$

$$= \frac{377 \times 10^3}{24 \times 10^4} - \frac{377 \times 10^3 \times 240 \times 200}{72 \times 10^8} + \frac{904.8 \times 10^3}{24 \times 10^4} + \frac{904.8 \times 10^3 \times 200 \times 200}{72 \times 10^8}$$

$$= 1.57 - 2.51 + 3.77 + 5.02 = 7.85 \text{ N/mm}^2$$

$$\text{Prestress loss} = 7.855 \frac{2 \times 10^5}{0.32 \times 10^5} = 49.06 \text{ N/mm}^2$$

Alternatively

$$P_1 = 377 \text{ kN}$$

$$P_2 = 904.8 \text{ kN}$$

Total force,

$$P_R = P_1 + P_2 = 1281.8 \text{ kN}$$

$$\bar{y} = \frac{P_1 y_1 + P_2 y_2}{P_1 + P_2} = \frac{377 \times 60 + 904.8 \times 500}{1281.8} = 370.6 \text{ mm}$$

$$e_R = 370.6 - 300 = 70.6 \text{ mm}$$

$$f_{ct} = \frac{P_R}{A} - \frac{P_R e_R}{I} e_1 = \frac{1281.8 \times 10^3}{24 \times 10^4} - \frac{1281.8 \times 10^3 \times 70.6 \times 240}{72 \times 10^8}$$

$$= 5.34 - 3.01 = 2.33 \text{ N/mm}^2$$

$$\therefore \text{ Prestress loss} = 2.33 \times \frac{2}{0.32} = 14.56 \text{ N/mm}^2$$

Stress in concrete at the location of bottom steel

$$f_{cb} = \frac{P_R}{A} + \frac{P_R e_R}{I} e_2 = 5.34 + \frac{1281.8 \times 10^3 \times 70.60 \times 200}{72 \times 10^8} = 5.34 + 2.51 = 7.85 \text{ N/mm}^2$$

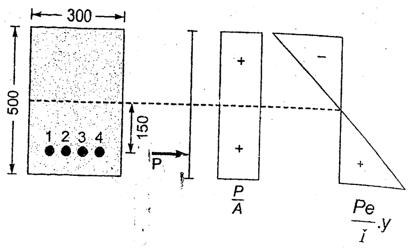
$$\text{Prestress loss} = 7.85 \times \frac{2}{0.32} = 49.05 \text{ N/mm}^2$$

Example 14.21 A post-tensioned concrete beam of size 300 × 500 mm is prestressed by 4 tendons of 400 mm² each provided at an eccentricity of 150 mm. Cable are straight and initially prestress in each tendon is 1500 N/mm². Calculate loss of prestress only due to elastic shortening in wire. If

1. All wires tensioned at same time.
2. Each wires are tensioned one by one, m = 6

Solution:

1. **Case-1:** All wires tensioned at same time
   Loss due to elastic shortening = 0



2. **Case-2:** If wires are tensioned one after another

(a) In wire no. 4 (last reinforcement)

$$\text{loss} = 0$$

(b) In wire no. 3, Prestressed loss is due to P_4

Stress in concrete at the location of wire no. 3

$$f_c = \frac{P_4}{A} + \frac{P_4 e}{I} e$$

$$= \frac{400 \times 1500}{300 \times 500} + \frac{400 \times 1500 \times 150 \times 150}{\left(300 \times \frac{500^3}{12}\right)}$$

$$= 4.0 + 4.32 = 8.32 \text{ N/mm}^2$$

Loss of stress $= mf_c$

$$= 6 \times 8.32 = 49.92 \text{ N/mm}^2$$

(c) In wire no. 2

Loss of stress in steel will be due to P_3 and P_4.

Stress in concrete at the location of wire no. 2

$$f_c = \frac{P_3 + P_4}{A} + \frac{(P_3 + P_4)e}{I} e$$

$$= 2 \times 8.32 = 16.64 \text{ N/mm}^2$$

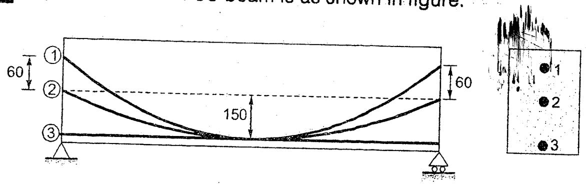
Loss of prestress

$$= 2 \times 49.92 = 99.84 \text{ N/mm}^2$$

(d) In wire no. 1

Prestress loss is due to $(P_2 + P_3 + P_4) = 3 \times 49.92 = 149.76 \text{ N/mm}^2$

Example 14.22 A post-tensioned PSC beam is as shown in figure.



Size of beam = 800 × 550 mm

Area of each wire = 500 mm^2

Initial prestress = 1500 N/mm^2

$E_s = 2 \times 10^5$ N/mm^2

$E_C = 0.35 \times 10^5$ N/mm^2

Wire No. 1 tensioned first

Wire no. 3 tensioned last

Calculate loss of stress due to elastic shortening of concrete in each wire.

Solution:

In wire no. 3,

Loss of stress = 0

Wire no. 2

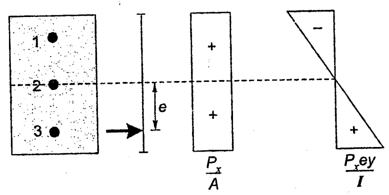
Loss of stresses in wire no. 2 is due to prestressing force in wire no. 3

Stress in concrete at the location of wire no. 2 due to prestressing force in wire no. 3

At ends:

$$f_{c1} = \frac{P_3}{A} = \frac{500 \times 10^3}{300 \times 550}$$

$$= 4.54 \text{ N/mm}^2$$



At mid span,

$$f_{c2} = \frac{P_3}{A} + \frac{P_3 e_3}{I} \times e_2$$

$$= 4.54 + \frac{500 \times 1500 \times 150 \times 150}{\frac{(300 \times 500)^3}{12}}$$

$$= 4.54 + 4.05 = 8.59 \text{ N/mm}^2$$

Average stress $\quad f_c = f_{c1} + \frac{2}{3}(f_{c2} - f_{c1}) = 4.54 + \frac{2}{3}(8.59 - 4.54) = 7.24 \text{ N/mm}^2$

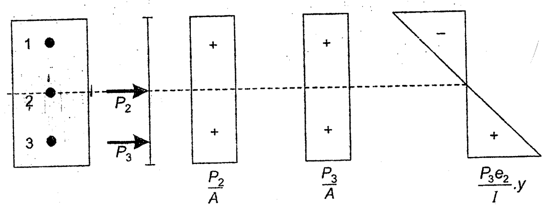
Loss of stress $\quad = mf_c = \frac{2 \times 10^5}{0.35 \times 10^5} \times 7.24 = 41.37 \text{ N/mm}^2$

In wire no. 1

Loss of stress in wire no. 1 will be due to P_3 and P_2.

∴ Stress in concrete at the location of wire no. 1 is due to prestressing force in wire 2 and 3.

At ends

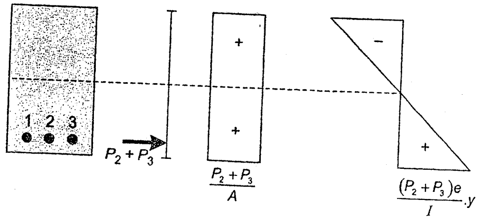


$$f_{c1} = \frac{P_2}{A} + \frac{P_3}{A} - \frac{P_3 e_3}{I} e_1$$

$$= \frac{500 \times 1500}{300 \times 550} + \frac{500 \times 1500}{300 \times 550} - \frac{500 \times 1500 \times 150 \times 60}{\frac{(300 \times 500)^3}{12}}$$

$$= 4.54 + 4.54 - 1.62 = 7.46 \text{ N/mm}^2$$

At mid span



$$f_{c2} = \frac{P_2+P_3}{A} + \frac{(P_2+P_3)}{I}e_1$$

$$= \frac{2\times500\times1500}{550\times300} + \frac{2\times500\times1500\times150^2}{\frac{300\times550^2}{12}} = 9.09 + 8.11 = 17.2\text{ N/mm}^2$$

Average stress

$$f_c = f_{c1} + \frac{2}{3}(f_{c2} - f_{c1}) = 7.46 + \frac{2}{3}(17.2 - 7.45) = 13.95\text{ N/mm}^2$$

Loss of stress

$$= \frac{2\times10^5}{0.35\times10^5}\times13.95 = 79.73\text{ N/mm}^2$$

Example 14.23 **Design a beam simply supported over a span of 20 m subjected to a load of 80 kN/m (live load). The permissible stress in concrete in compression is 16 N/mm² and in tension it is zero. Loss of prestress is 15%, permissible stress in steel is 1500 N/mm². Design PSC beam and show the reinforcement. Use straight cable.**

Solution:

Step-1: Assume size of beam

Width = 500 mm

Depth = 800 mm

Self weight $\quad w_d = 0.5\times0.8\times1\times25 = 10$ kN/m

$$M_d = \frac{10\times20^2}{8} = 500\text{ kNm}$$

Live load moment $\quad M_l = \frac{w_l\times l^2}{8} = \frac{30\times20^2}{8} = 1500$ kNm

Step-2: Section modulus required

$$Z = \frac{(1-k)M_d + M_l}{f_c}$$

$$k = \left(1-\frac{P_L}{100}\right) = \left(1-\frac{15}{100}\right) = 0.85$$

$$Z = \frac{(1-0.85)500\times10^6 + 1500\times10^6}{16} = 98.44\times10^6\text{ mm}^3$$

Depth required, $\quad D = \sqrt{\frac{6Z}{B}} = \sqrt{\frac{6\times98.44\times10^6}{500}} = 1086.86$ mm

Consider $\quad D = 1100$ mm

$B = 500$ mm

$w_d = 0.5\times1.1\times1\times25 = 13.75$ kN/m

$$M_d = \frac{13.75\times20^2}{8} = 687.5\text{ kNm}$$

Section modulus (Z) required.

$$Z = \frac{(1-0.85)687.5\times10^6 + 1500\times10^6}{16} = 100.2\times10^6\text{ mm}^3$$

$$D = \sqrt{\frac{6Z}{B}} = 1096\text{ mm}$$

So, $\quad D = 1100$ mm, B = 500 mm

Step-3: Cross-sectional area, $\quad A = B\times D = 500\times1106 = 55\times10^4$ mm²

Prestressing force

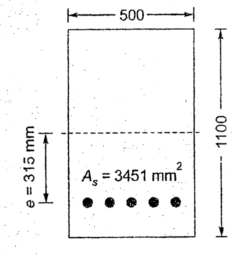
$$P = \frac{A\times f_c}{2k} = \frac{55\times10^4\times16}{2\times0.85} = 5176.5\text{ kN}$$

Step-4: Area of steel, $\quad A_s = \frac{P}{f_s} = \frac{5176.5\times10^3}{1500} = 3451\text{ mm}^2$

Step-5: Eccentricity, $\quad e = \frac{(1+k)M_d + M_l}{2kP}$

$$= \frac{(1+0.85)\times687.5\times10^6 + 1500\times10^6}{2\times0.85\times5176.5\times10^3}$$

$= 315$ mm



Example 14.24 A prestressed concrete rectangular beam of 250 mm x 600 mm is prestressed by four 16 mm dia. high tensile bars located at 225 mm from the soffit of beam. If the effective stress in the bars is 690 N/mm². What is the maximum moment that can be applied without inducing any tension in the beam?

Solution:

Area cross-section of beam $= 250\times60 = 15\times10^4$ mm²

Section modulus of the beam cross-section (z) $= \frac{250\times600^2}{6} = 15\times10^6$ mm⁴

Area of prestressing bars (A_s) $= 4\times\frac{\pi}{4}\times16^2 = 804.25$ mm²

Eccentricity (e) $= 300 - 225 = 75$ mm

∴ Prestressing force (P) $= 690\times804.25$ N $= 554.93$ kN $\simeq 555$ kN

Direct stress $= \frac{P}{A} = \frac{555\times10^3}{15\times10^4} = 3.7$ N/mm²

Flexural stress $= \mp2.775$ N/mm²

∴Maximum stress at the bottom of beam $= \frac{P}{A} + \frac{Pe}{Z} = 3.7 + 2.775 = 6.475$ N/mm²

External loading will induce tensile stresses in the bottom of beam.

Let M = Moment induced due to external loading

$\therefore$ To have zero stress (no tension) any where in the beam cross-section,

$$\frac{M}{Z} = 6.475$$

$$\Rightarrow \quad M = 6.475\,Z = 6.475 \times 15 \times 10^6 \text{ Nmm} = 97.125 \text{ kNm}$$

= Max. moment that can be induced due to external loading

Example 14.25 A prestressed concrete beam of size 225 mm × 325 mm is used for an effective span of 5.5 m. It supports a super-imposed load of 4.2 kN/m. At the mid-span section of the beam find.

(a) the concentric prestressing force necessary to have zero fibre stress at the bottom of beam when the beam is fully loaded.

(b) the eccentric prestressing force located at 115 mm from the beam bottom which will totally counteract the stresses due to external loading.

Solution:

Cross-section of beam cross-section (A) = $225 \times 325 \text{ mm}^2 = 73125 \text{ mm}^2$

Section modulus of the beam cross-section $(Z) = 225 \times \frac{325^2}{6} = 3.961 \times 10^6 \text{ mm}^3$

Self weight of beam = $0.225 \times 0.325 \times 25 \text{ kN/m} = 1.828 \text{ kN/m}$

$\therefore$ Moment due to $DL = M_D$ = $1.828 \times \frac{5.5^2}{8} = 6.91 \text{ kNm}$

Moment due to line load $= M_L$ = $4.2 \times \frac{5.5^2}{8} = 15.88 \text{ kNm}$

$\therefore$ Tensile stress at the bottom of beam due to dead and live load $= f_{bt}$

$$= \frac{M_D}{Z} + \frac{M_L}{Z} = \frac{(6.91 + 15.88)10^6}{3.961 \times 10^6} = 5.75 \text{ N/mm}^2$$

(a) Let P_1 = concentric prestressing force for zero stresses at the beam bottom

$$= \frac{P_1}{A} = 5.75 \text{ N/mm}^2$$

$$\Rightarrow \quad P_1 = 5.75 \times 73125 \text{ N} = 420.5 \text{ kN}$$

(b) Let P_2 = eccentric prestressing force for zero stresses at the beam bottom

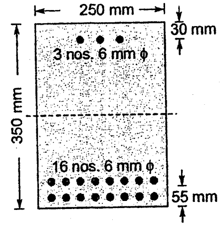
$$\text{Eccentricity } (e) = \frac{325}{2} - 115 = 47.5 \text{ mm}$$

$$\therefore \quad \frac{P_2}{A} + \frac{P_2 e}{Z} = 5.75 \text{ N/mm}^2$$

$$\Rightarrow \quad P_2\left[\frac{1}{73125} + \frac{47.5}{3.961 \times 10^6}\right] = 5.75$$

$$\Rightarrow \quad P_2 = 224 \text{ kN}$$

Example 14.26 A prestressed concrete beam of size 250 × 350 mm is prestressed by 16 wires of 6 mm diameter located at a distance of 55 mm from the bottom of beam, and 3 wires of 6 mm diameter at a distance of 30 mm from the top. If prestress in the steel is 850 MPa, find the stresses at the extreme fibres of mid-span section if the beam spans over a distance of 6 m. The imposed live load on the beam is 5 kN/m.



Solution:

Gross area of beam cross-section (A) = $250 \times 350 = 87500 \text{ mm}^2$

Section modulus of the beam cross-section $(Z) = 250 \times \frac{350^2}{6}$

$= 5.104 \times 10^6 \text{ mm}^3$

Moment of inertia of the beam cross-section $(I) = 250 \times \frac{350^3}{12}$

$= 893.33 \times 10^6 \text{ mm}^4$

Self weight of beam = $0.25 \times 0.35 \times 25 = 2.1875 \text{ kN/m}$

Moment due to self weight of beam = $2.1875 \times \frac{6^2}{8} = 9.84 \text{ kNm}$

Moment due to line load on beam = $5 \times \frac{6^2}{8} = 22.5 \text{ kNm}$

Distance at point of application of prestressing force from the bottom of beam

$$= \frac{(16 \times 55) + (3 \times 320)}{19} = 96.842 \text{ mm}$$

$\therefore$ Eccentricity, $e = \frac{350}{2} - 96.842 = 78.158 \text{mm}$

Pre-stressing force (P) = $(850 \text{ N/mm}^2) \times (19 \text{ wires}) \times \left(\frac{\pi}{4} \times 6^2\right) = 456.63 \text{ kN}$

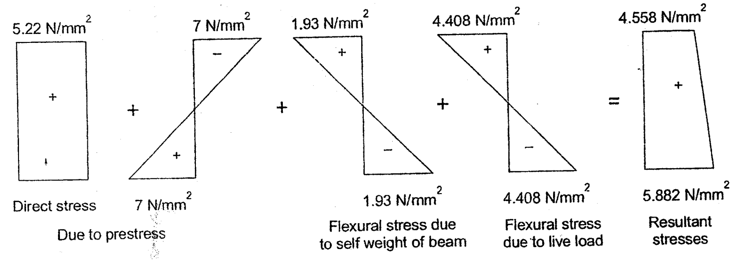
Direct stress = $\frac{P}{A} = \frac{456.63 \times 1000}{87500} = 5.22 \text{ N/mm}^2$

Flexural stress = $\mp \frac{456.63 \times 1000 \times 78.158}{5.104 \times 10^6} \mp 6.99 \text{ N/mm}^2$

Flexural stress due to prestressing = $\mp 6.99 \text{ N/mm}^2 \simeq \mp 7 \text{ N/mm}^2$

Flexural stresses due to beams self weight = $\pm \frac{9.84 \times 10^6}{5.104 \times 10^6} = \pm 1.93 \text{ N/mm}^2$

Flexural stresses due to line load = $\pm \frac{22.5 \times 10^6}{5.10^4 \times 10^6} = \pm 4.408 \text{ N/mm}^2$



Example 14.27 **Design a prestressed concrete beam of rectangular section for a span of 12 m to support a total UDL of 280 kN excluding the self weight of beam. The permissible stresses for concrete and steel are 15 N/mm² and 1100 N/mm² respectively.**

Solution:

Assume zero loss of prestress i.e. $k = 1$

Flexural moment due to superimposed load $(M_l) = 280 \times \frac{12}{8} = 420$ kNm

$\therefore$ Section modulus required $(Z_{reqd.}) = \frac{M_l}{f_c} = \frac{420 \times 10^6}{15} = 28 \times 10^6$ mm³

Let depth of beam $= \frac{1}{20}$ for span $= \frac{12000}{20} = 600$ mm

and let width of beam $= b$

$\therefore$ $\frac{b(600)^2}{6} = 28 \times 10^6$

$\Rightarrow$ $b = 466.67$ mm $\simeq 470$ mm (say)

$\therefore$ Beam size is 470 mm × 600 mm

Prestressing force, $P = \frac{f_c}{2} \cdot A = \frac{15}{2}(470 \times 600) = 2115$ kN

$\therefore$ Area of tendons required, $A_s = \frac{2115 \times 10^3}{1100} = 1922.7$ mm²

If 8 mm dia. bars are used.

Then number of bars required $= \frac{1922.7}{\frac{\pi}{4}(8)^2} = 38.25 \simeq 40$ bars (say)

$\therefore$ Provide 40 bars of 8 mm diameter.

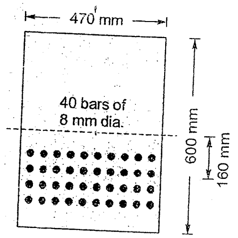
Self weight of beam $= 0.47 \times 0.6 \times 25 = 7.05$ kN/m

$\therefore$ Dead load moment $(M_d) = 7.05 \times \frac{12^2}{8} = 126.9$ kNm

$\therefore$ Eccentricity, $e = \frac{2M_d + M_l}{2P} = \frac{2(126.9) + 420}{2 \times 2115}$

$= 0.15929$ m $= 159.29$ mm

$\simeq 160$ mm (say)



Objective Brain Teasers

Q.1 While going for a pre-tensioning system
(a) the member is prestressed by external anchors
(b) member is casted prior to tensioning of high tensile tendons/wires.
(c) the high tensile wires are tensioned prior to placing of concrete
(d) None of the above

Q.2 The advantage of concentric tendons over eccentric tendons are that
(a) it induces uniform compressive stress over the entire section of beam
(b) it induces uniform tensile stress over the entire section of beam
(c) in induces linearly varying stress
(d) it induces parabolically varying stress

Q.3 Loss of prestress due to concrete shrinkage depends on
(a) shrinkage strain
(b) modulus of elasticity of concrete
(c) modular ratio
(d) concrete stress at the level of steel

Q.4 Which of the following cable profile effectively counter balances the uniformly distributed load on the beam?
(a) eccentric cable (b) parabolic cable
(c) concentric cable (d) All of the above

Q.5 High tensile bars with thread at ends are employed in which of the following prestressing system?
(a) Gifford-Udall system
(b) Lee-McCall system
(c) Freyssinet system
(d) Mangel-Blaton system

Q.6 In the system of pre-stressing by post-tensioning
(a) the tensioning wires are stretched prior to placement of concrete
(b) the wire are tensioned against the hardened concrete
(c) concrete is compressed after setting
(d) None of the above

Q.7 Loss of prestress due to stress relaxation in steel depends on
(a) initial prestress in steel
(b) coefficient of friction in between the steel and concrete
(c) amount of shrinkage in concrete
(d) amount of elongation in steel

Q.8 Which of the following prestressed beam has the highest moment of resistance?
(a) post-tensioned with unbounded wires
(b) pre-tensioned beam
(c) post-tensioned with bonded wires
(d) data insufficient

Q.9 The advantages of providing concrete beam with sloping a curved cables is that
(a) it increases the moment of resistance of the beam
(b) it decreases the shrinkage cracks
(c) it increases the torsional strength of beam
(d) it increases the shear strength of the beam

Q.10 The tensile stress in steel at the end face of a pre-tensioned beam is
(a) the minimum (b) the maximum
(c) zero (d) can't say

Q.11 Transmission length as per IS 1343 is
(a) 100ϕ (b) 30ϕ
(c) 60ϕ (d) 45ϕ
where ϕ = dia. of strand.

Q.12 Short term deflection of prestressed concrete beam is estimated by
(a) Mohr's theory (b) Rankine's theory
(c) Euler's theory (d) Hooke's law

Q.13 Stress transfer in pre-tensioned beam occurs dueto
(a) shear resistance
(b) flexural resistance
(c) bearing resistance
(d) bond between steel and concrete

Answers

1. (c) 2. (a) 3 (a) 4. (b) 5. (b)
6. (b) 7. (a) 8. (b) 9. (d) 10. (c)
11. (b) 12. (a) 13. (d)

Conventional Practice Questions

Q.1 A pre-tensioned beam of size 250 mm x 300 mm is pre-stressed by twelve wires each of 7 mm diameter. All the wires are stressed initially to 1200 N/mm² with their centroids located at 100 mm from the soffit of beam. Compute the final percentage loss in stress due to elastic deformation, creep, shrinkage and relaxation. Take relaxation of steel stress as 90 N/mm², E_s as 210 kN/mm², E_c as 35 kN/mm², creep coefficient as 1.6 and residual shrinkage strain as 0.0003.

Ans. [22%]

Q.2 A double over hang concrete beam has a mid-span of 10 m with over hangs of 2.5 m each. Find the profile of the pre-stressing cable for an effective pre-stressing force of 250 kN which can nullify the load effect of 8 kN/m uniformly distributed on the beam.

Ans. [$e_{support}$ = 100 mm, $e_{mid\text{-}span}$ = 300 mm]

Q.3 A rectangular concrete beam of 350 mm x 700 mm size and length 10 m supports a uniformly distributed load of 20 kN/m excluding the self-weight of beam. If a 200 mm eccentricity is to be maintained at the mid span portion of the beam, find the pre-stressing force and cable profile to support dead and live loads.

Ans. [Prestressing force = 1632 kN]

Q.4 A post tensioned cable of a beam with 24 parallel cables of total area 800 mm² is tensioned with two wires at a time. The cable is a circular curve with zero eccentricity at the supports and 150 mm at mid span. The span and size of the beam respectively are 10 m and 200 mm x 450 mm. The wires are stressed to a value of f_1 to overcome friction and then released to a value of f_2 such that immediately after anchoring, the initial pre stress is 840 N/mm². Find f_1 and f_2 and final design stress in steel after all losses. Take coefficient of friction for curvature as 0.6, friction coefficient for wave effect as 0.003 per m, deformation and anchorage slip as 1.25 mm, E_s as 210 kN/mm², concrete shrinkage as 0.0002, and stress relaxation of steel as 3%.

Ans. [f_1 = 954 N/mm², f_2 = 866 N/mm², 668 N/mm²]