SET-I

DESIGN OF THE QUESTION PAPER

Mathematics

Class X

Time : 3 Hours **Maximum Marks : 80**

Weightage and the distribution of marks over different dimensions of the question shall be as follows:

(A) Weightage to Content/ Subject Units :

S.No.	Content Unit	Marks
1.	Number Systems	04
2.	Algebra	20
3.	Trigonometry	12
4.	Coordinate Geometry	08
5.	Geometry	16
6.	Mensuration	10
7.	Statistics and Probability	10
		Total : 80

(B) Weightage to Forms of Questions :

S.No.	Form of Questions	Marks for each Question	Number of Questions	Total Marks
1.	MCQ	01	10	10
2.	SAR	02	05	10
3.	SA	03	10	30
4.	LA	06	05	30
		Total	**30**	**80**

(C) Scheme of Options

All questions are compulsory, i.e., there is no overall choice. However, internal choices are provided in one question of 2 marks, three questions of 3 marks each and two questions of 6 marks each.

(D) Weightage to Difficulty Level of Questions

S.No.	Estimated Difficulty Level of Questions	Percentage of Marks
1.	Easy	20
2.	Average	60
3.	Difficult	20

Note : A question may vary in difficulty level from individual to individual. As such, the assessment in respect of each will be made by the paper setter/ teacher on the basis of general anticipation from the groups as whole taking the examination. This provision is only to make the paper balanced in its weight, rather to determine the pattern of marking at any stage.

BLUE PRINT
MATHEMATICS
CLASS X

Form of Question → / Units ↓	MCQ	SAR	SA	LA	Total
Number Systems	2(2)	2(1)	-	-	4(3)
Algebra Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Arithmatic Progressions	3(3)	2(1)	9(3)	6(1)	20(8)
Trigonometry Introduction to Trigonometry, Some Applications of Trigonometry	1(1)	2(1)	3(1)	6(1)	12(4)
Coordinate Geometry	1(1)	4(2)	3(1)	-	8(4)
Geometry Triangles, Circles, Constructions	1(1)	-	9(3)	6(1)	16(5)
Mensuration Areas related to Circles, Surface Areas and Volumes	1(1)	-	3(1)	6(1)	10(3)
Statistics & Probability	1(1)	-	3(1)	6(1)	10(3)
Total	**10(10)**	**10(5)**	**30(10)**	**30(5)**	**80(30)**

SUMMARY

Multiple Choice Questions (MCQ)	Number of Questions : 10	Marks : 10
Short Answer Questions with Resasoning (SAR)	Number of Questions : 05	Marks : 10
Short Answer Questions (SA)	Number of Questions : 10	Marks : 30
Long Answer Qustions (LA)	Number of Questions : 05	Marks : 30
Total	**30**	**80**

Mathematics
Class X

Maximum Marks : 80 **Time : 3 Hours**

General Instructions

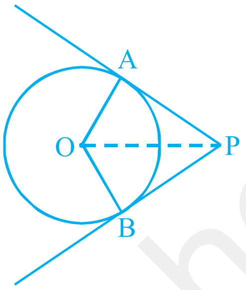
1. All questions are compulsory.
2. The question paper consists of 30 questions divided into four sections A, B, C, and D. Section A contains 10 questions of 1 mark each, Section B contains 5 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 5 questions of 6 marks each.
3. There is no overall choice. However, an internal choice has been provided in one question of 2 marks, three questions of 3 marks and two questions of 6 marks each.
4. In questions on construction, the drawing should be neat and exactly as per given measurements.
5. Use of calculators is not allowed.

SECTION A

1. After how many decimal places will the decimal expansion of the number $\frac{47}{2^3 5^2}$ terminate?
(A) 5 (B) 2 (C) 3 (D) 1
2. Euclid's division lemma states that for two positive integers *a* and *b*, there exist unique integers *q* and *r* such that $a = bq + r$, where
(A) $0 \le r \le a$ (B) $0 < r < b$ (C) $0 \le r \le b$ (D) $0 \le r < b$
3. The number of zeroes, the polynomial $p(x) = (x-2)^2 + 4$ can have, is
(A) 1 (B) 2 (C) 0 (D) 3
4. A pair of linear equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ is said to be inconsistent, if
(A) $\frac{a_1}{a_2} \ne \frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2} \ne \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \ne \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} \ne \frac{c_1}{c_2}$
5. The smallest value of *k* for which the equation $x^2 + kx + 9 = 0$ has real roots, is
(A) – 6 (B) 6 (C) 36 (D) –3
6. The coordinates of the points P and Q are (4, –3) and (–1, 7). Then the abscissa of a point R on the line segment PQ such that $\frac{PR}{PQ} = \frac{3}{5}$ is

(A) $\frac{18}{5}$ (B) $\frac{17}{5}$ (C) $\frac{17}{8}$ (D) 1

7. In the adjoining figure, PA and PB are tangents from a point P to a circle with centre O. Then the quadrilateral OAPB must be a

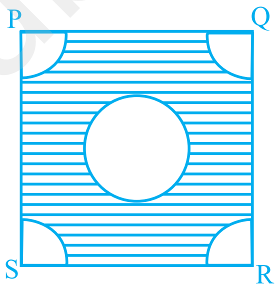


(A) square (B) rhombus
(C) cyclic quadrilateral (D) parallelogram

8. If for some angle θ, $\cot 2\theta = \frac{1}{\sqrt{3}}$, then the value of $\sin 3\theta$, where $2\theta \le 90^\circ$ is

(A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) 0 (D) $\frac{\sqrt{3}}{2}$

9. From each corner of a square of side 4 cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. The area of the remaining (shaded) portion is



(A) $(16 - 2\pi)$ cm^2 (B) $(16 - 5\pi)$ cm^2
(C) 2π cm^2 (D) 5π cm^2

10. A letter of English alphabets is chosen at random. The probability that it is a letter of the word 'MATHEMATICS' is

(A) $\frac{11}{26}$ (B) $\frac{5}{13}$ (C) $\frac{9}{26}$ (D) $\frac{4}{13}$

SECTION B

11. Is there any natural number n for which 4^n ends with the digit 0? Give reasons in support of your answer.

12. Without using the formula for the n^{th} term, find which term of the AP : 5, 17, 29, 41, ... will be 120 more than its 15^{th} term? Justify your answer.

OR

Is 144 a term of the AP : 3, 7, 11, ... ? Justify your answer.

13. The coordinates of the points P, Q and R are (3, 4), (3, –4) and (–3, 4), respectively. Is the area of ΔPQR 24 sq. units? Justify your answer.

14. The length of a line segment is 10 units. If one end is (2, –3) and the abscissa of the other end is 10, then its ordinate is either 3 or –9. Give justification for the two answers.

15. What is the maximum value of $\frac{3}{\text{cosec}}$? Justify your answer.

SECTION C

16. Find the zeroes of the polynomial $p(x) = 4\sqrt{3}x^2 - 2\sqrt{3}\,x - 2\sqrt{3}$ and verify the relationship between the zeroes and the coefficients.

OR

On dividing the polynomial $f(x) = x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, the quotient $q(x)$ and remainder $r(x)$ are $x - 3$, $-3x + 5$, respectively. Find the polynomial $g(x)$.

17. Solve the equations $5x - y = 5$ and $3x - y = 3$ graphically.

18. If the sum of the first n terms of an AP is $4n - n^2$, what is the 10^{th} term and the n^{th} term?

OR

How many terms of the AP : 9, 17, 25, ... must be taken to give a sum 636?

19. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find the values of x and y.

20. The sides AB, BC and median AD of a ΔABC are respectively propotional to the sides PQ, QR and the median PM of ΔPQR. Show that ΔABC ~ ΔPQR.

21. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 7 cm, respectively. Find the sides AB and AC.

22. Construct an isosceles triangle whose base is 6 cm and altitude 5 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the isosceles triangle.

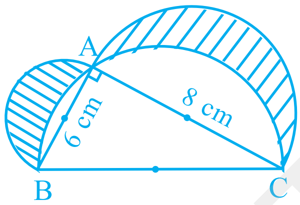
23. Prove that $\frac{\cos \quad -\sin \quad 1}{\sin \quad \cos \quad -1} \quad \frac{1}{\text{cosec} \quad -\cot}$.

OR

Evaluate:

$$\left(\frac{3\cos 43}{\sin 47}\right)^2 - \frac{\cos 37 \ \text{cosec} 53}{\tan 5 \ \tan 25 \ \tan 45 \ \tan 65 \ \tan 85}$$

24. In the figure, ABC is a triangle right angled at A. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



25. A bag contains white, black and red balls only. A ball is drawn at random from the bag. The probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$. Find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

SECTION D

26. If the price of a book is reduced by Rs 5, a person can buy 5 more books for Rs 300. Find the original list price of the book.

OR

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. Is this situation possible? If so, determine their present ages.

27. Prove that the lengths of the tangents drawn from an external point to a circle are equal.
Using the above theorem, prove that:
If quadrilateral ABCD is circumscribing a circle, then AB + CD = AD + BC.

OR

Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.
Using the above theorem, do the following :
ABC is an iscosceles triangle right angled at B. Two equilateral triangles ACD and ABE are constructed on the sides AC and AB, respectively. Find the ratio of the areas of Δ ABE and Δ ACD.

28. The angles of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 60°, respectively. Find the height of the tower and also the horizontal distances between the building and the tower.

29. A well of diameter 3 m and 14 m deep is dug. The earth, taken out of it, has been evenly spread all around it in the shape of a circular ring of width 4 m to form an embankment . Find the height of the embankment.

30. The following table shows the ages of the patients admitted in a hospital during a month:

Age (in years) :	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65
Number of patients :	6	11	21	23	14	5

Find the mode and the mean of the data given above.

MARKING SCHEME

SECTION A

MARKS

1. (C)	**2.** (D)	**3.** (C)	**4.** (C)	**5.** (A)	**6.** (D)
7. (C)	**8.** (B)	**9.** (A)	**10.** (D)		**(1 × 10 = 10)**

SECTION B

11. No $(\frac{1}{2})$

$4^n = 2^{2n}$

Therefore, 2 is the only prime number in its prime facorisation, so it cannot end with zero. $(1\frac{1}{2})$

12. 25^{th} term $(\frac{1}{2})$

120 will be added in 10 terms (since $d = 12$)

Therefore, $15 + 10 = 25$ $(1\frac{1}{2})$

OR

No $(\frac{1}{2})$

Here, $a = 3$ (odd), $d = 4$ (even)

Sum of (odd + even) = odd but 144 is even $(1\frac{1}{2})$

13. Yes $(\frac{1}{2})$

Here, PQ = 8,

PR = 6, therefore, area = $\frac{1}{2}$. 8.6 = 24 sq. units. $(1\frac{1}{2})$

14. Let ordinate of the point be y. Then $(10 - 2)^2 + (y + 3)^2 = 10^2$, i.e.,$y + 3 = \pm 6$,

i.e., $y = 3$ or -9 (1 + 1)

15. Maximum value = 3 $(\frac{1}{2})$

Since $\frac{3}{\operatorname{cosec} \theta} = 3 \sin\theta$, and $\sin\theta \leq 1$, therefore, $3 \sin\theta \leq 3$ $\left(1\frac{1}{2}\right)$

SECTION C

16. $p(x) = 4\sqrt{3}\, x^2 - 2\sqrt{3}\, x - 2\sqrt{3} = 2\sqrt{3}\,(2x^2 - x - 1)$

$= 2\sqrt{3}\,(2x + 1)(x - 1)$

Therefore, two zeroes are $-\frac{1}{2}, 1$ (1)

Here $a = 4\sqrt{3}$, $b = 2\sqrt{3}$, $c = -2\sqrt{3}$

Therefore, $\alpha + \beta = -\frac{1}{2} + 1 = \frac{1}{2}$, $-\frac{b}{a} = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}$, i.e., $\alpha + \beta = -\frac{b}{a}$ (1)

$\alpha\beta = \left(-\frac{1}{2}\right) 1 = -\frac{1}{2}$, $\frac{c}{a} = \frac{-2\sqrt{3}}{4\sqrt{3}} = -\frac{1}{2}$, i.e., $\alpha\beta = \frac{c}{a}$ (1)

OR

$f(x) = g(x)\, q(x) + r(x)$

Therefore, $x^3 - 5x^2 + 6x - 4 = g(x)\,(x - 3) + (-3x + 5)$ (1)

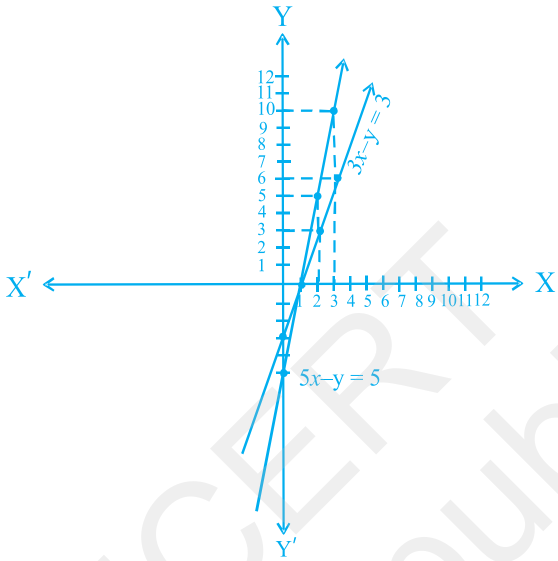
Therefore, $g(x) = \frac{x^3 - 5x^2 + 6x - 4 + 3x - 5}{x - 3} = \frac{x^3 - 5x^2 + 9x - 4}{x - 3}$ (1)

$= x^2 - 2x + 3$ (1)

17. $5x - y = 5$ $3x - y = 3$

x	1	2	3
y	0	5	10

x	1	2	3
y	0	3	6



For correct graph (2)

Solution is $x = 1, y = 0$ (1)

18. $S_n = 4n - n^2$. Therefore, $t_{10} = S_{10} - S_9 = (40 - 100) - (36 - 81)$ $(\frac{1}{2})$

$= -60 + 45 = -15$ (1)

$t_n = S_n - S_{n-1} = (4n - n^2) - [4(n-1) - (n-1)^2]$ $(\frac{1}{2})$

$= 4n - n^2 - 4n + 4 + n^2 + 1 - 2n = 5 - 2n$ (1)

OR

$a = 9, d = 8, S_n = 636$

Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we have $636 = \frac{n}{2}[18 + (n-1)8]$ $(1\frac{1}{2})$

Solving to get $n = 12$ $(1\frac{1}{2})$

19. Let A (1, 2), B (4, y) and C (x, 6) and D (3, 5) be the vertices.

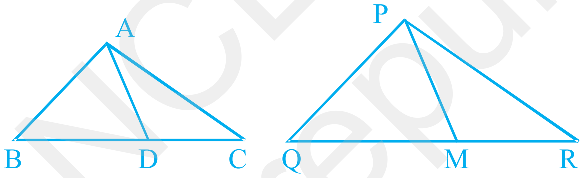
The mid-point of AC is $\left(\frac{x+1}{2}, 4\right)$ $(\frac{1}{2})$

and mid-point of BD is $\left(\frac{7}{2}, \frac{y+5}{2}\right)$ $(\frac{1}{2})$

ABCD is *a* parellologram. Therefore, $\frac{x+1}{2} = \frac{7}{2}$, i.e., $x = 6$ (1)

$\frac{y+5}{2} = 4$, i.e., $y = 3$ (1)

20.

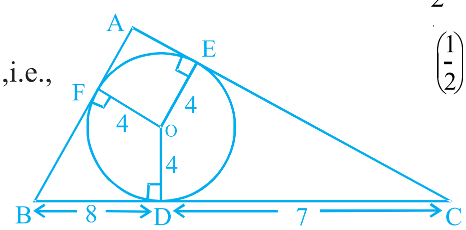


Given $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{BD}{QM} = \frac{AD}{PM}$

Therefore, Δ ABD ~ Δ PQM [SSS] $(1\frac{1}{2})$

Therefore, $\angle B = \angle Q$. Also, since $\frac{AB}{PQ} = \frac{BC}{QR}$, i.e., $(\frac{1}{2})$

Δ ABC ~ Δ PQR [SAS] $(1\frac{1}{2})$



21. Let AE (=AF) = x cm.

Area Δ ABC = $\frac{1}{2}$. 4 . (AB + BC + AC)

$= \sqrt{s(s-a)(s-b)(s-c)}$

i.e., $4\,s = \sqrt{s\,(s-a)\,(s-b)(s-c)}$ $(\frac{1}{2})$

$16\,s = (s-a)\ (s-b)\ (s-c)$ $(\frac{1}{2})$

i.e., $16\ (15 + x) = x\ .\ 8\ .\ 7$, i.e., $x = 6$ (1)

Therefore, AB = 14 cm and AC = 13 cm $(\frac{1}{2})$

22. Construction of isosceles Δ with base 6 cm and altitude 5 cm (1)

Construction of similar Δ with scale factor $\frac{7}{5}$ (2)

23. LHS $= \dfrac{\cos\theta - \sin\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{\cot\theta - 1 + \operatorname{cosec}\theta}{1 + \cot\theta - \operatorname{cosec}\theta}$ (1)

$= \dfrac{\cot\theta - 1 + \operatorname{cosec}\theta}{1 - (\operatorname{cosec}\theta - \cot\theta)} = \dfrac{\operatorname{cosec}\theta + \cot\theta - 1}{(\operatorname{cosec}^2\theta - \cot^2\theta) - (\operatorname{cosec}\theta - \cot\theta)}$ (1)

$= \dfrac{\operatorname{cosec}\theta + \cot\theta - 1}{(\operatorname{cosec}\theta - \cot\theta)\ (\operatorname{cosec}\theta + \cot\theta - 1)} = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$ (1)

OR

$\dfrac{3\cos 43}{\sin 47}^{2} - \dfrac{\cos 37\ \ \operatorname{cosec} 53}{\tan 5\ \ \tan 25\ \ \tan 45\ \ \tan 65\ \ \tan 85}$

$= \left[\dfrac{3\cos 43°}{\cos 43°}\right]^2 - \dfrac{\cos 37°.\sec 37°}{\tan 5°\ \tan 25°\ (1)\ \cot 25° \cot 5°}$ (2)

$= (3)^2 - \frac{1}{1} = 9 - 1 = 8$ (1)

24.

Required area $= \begin{cases} \text{area of semicircle with diameter AB +} \\ \text{area of semicircle with diameter AC +} \\ \text{area of right triangle ABC –} \\ \text{area of semicircle with diameter BC} \end{cases}$ (1)

Required area = $\frac{1}{2} \cdot (3)^2 \quad \frac{1}{2} \quad (4)^2 \quad \frac{1}{2} \quad 6 \quad 8 - \frac{1}{2} \quad (5)^2$ sq. units (1)

$= 24 + \frac{1}{2}\pi\,(9 + 16 - 25) = 24$ sq. units (1)

25. P(Red ball) = 1 – {P(White ball) + P(Black ball)} (1)

$= 1 - \left\{\frac{3}{10} + \frac{2}{5}\right\} = \frac{3}{10}$ $(\frac{1}{2})$

Let the total number of balls be *y*.

Therefore, $\frac{20}{y} \quad \frac{2}{5}$, i.e., $y \quad 50$ $(1\frac{1}{2})$

SECTION D

26. Let the list price of a book be Rs *x*

Therefore, number of books, for Rs 300 $= \frac{300}{x}$ $(\frac{1}{2})$

No. of books, when price is $(x - 5) = \frac{300}{x-5}$ $(\frac{1}{2})$

Therefore, $\frac{300}{x-5} - \frac{300}{x} = 5$ (2)

$300\,(x - x + 5) = 5x\,(x - 5)$

$300 = x\,(x - 5)$, i.e., $x^2 - 5x - 300 = 0$ (1)

i.e., $x = 20$, $x = -15$ (rejected) (1)

Therefore, list price of a book = Rs 20 (1)

OR

Let the present age of one of them be *x* years, so the age of the other = $(20 - x)$ years

Therefore, 4 years ago, their ages were, $x - 4$, $16 - x$ years (1)

Therefore, $(x - 4)\,(16 - x) = 48$ $(1\frac{1}{2})$

i.e., $-x^2 + 16x + 4x - 64 - 48 = 0$

$x^2 - 20x + 112x = 0$ (1)

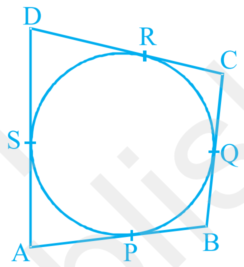
Here $B^2 - 4AC = \sqrt{(20)^2 - 4(112)} = \sqrt{-48}$ $(\frac{1}{2})$

Thus, the equation has no real solution (1)

Hence, the given situation is not possible (1)

27. For correct, given, to prove, contruction and figure (2)

For correct proof (2)



$\left.\begin{array}{l} AP = AS \\ BP = BQ \\ DR = DS \\ CR = CQ \end{array}\right\}$ (tangents to a circle from external point are equal) (1)

Adding to get (AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)

i.e., AB + CD = AD + BC (1)

OR

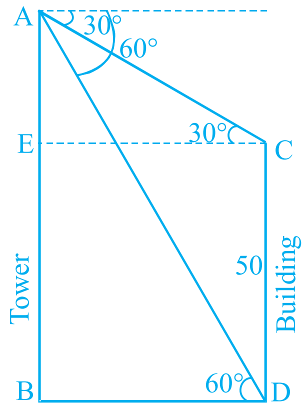
For correct,given, to prove, construction and figure (2)

For correct proof (2)

Let AB = BC = a, i.e., AC = $\sqrt{a^2 + a^2} = \sqrt{2}\,a$ $(\frac{1}{2})$

$$\frac{\text{area } \Delta ABC}{\text{area } \Delta ACD} = \frac{AB^2}{AC^2} = \frac{a^2}{2a^2} = \frac{1}{2}$$ $(1\frac{1}{2})$

28. For correct figure (1)



In Δ ABD, $\frac{AB}{BD} = \tan 60° = \sqrt{3}$ (1)

Therefore, AB = $\sqrt{3}$ BD (I)

In Δ ACE, $\frac{AE}{EC} = \frac{AE}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$ (1)

i.e., $\frac{(AB - 50)}{BD} = \frac{1}{\sqrt{3}}$, i.e., $\sqrt{3}$ (AB – 50) = BD (1)

Therefore, from (I) AB = $\sqrt{3} \cdot \sqrt{3}$ (AB – 50) ,i.e., AB = 3AB – 150 ,i.e.,
AB = 75 m (1)

BD = $\sqrt{3}$ (75 – 50) = $25\sqrt{3}$ m (1)

29. Volume of earth dug out = $\pi r^2 h = \pi (1.5)^2 \times 14 = 31.5\ \pi\ m^3$ (2)

Area of circular ring = $\pi[R^2 - r^2] = \pi[(5.5)^2 - (1.5)^2]$ (1)

$= \pi(7)\ (4) = 28\pi\ m^2$ (1)

Let height of embankment be h metres

Therefore, $28\pi \times h = 31.5\ \pi$ (1)

$h = \frac{31.5}{28} = 1.125$ m (1)

30.

Age (in years)	**5-15**	**15-25**	**25-35**	**35- 45**	**45-55**	**55-65**	**Total**	
No. of patients(f_1)	6	11	21	23	14	5	80	
Class marks(x_i)	10	20	30	40	50	60		($\frac{1}{2}$)
$f_i x_i$	60	220	630	920	700	300	2830	(1)

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{2830}{80} = 35.375$ years (1)

Modal class is (35 – 45) ($\frac{1}{2}$)

Therefore, Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$ (1)

Putting $l = 35, f_1 = 23, f_0 = 21, f_2 = 14$ and $h = 10$, we get (1)

Mode = $35 + \frac{2}{11} \times 10 = 36.81$ years (1)

Note: Full credit should be given for alternative correct solution.