# CBSE Sample Paper -05 (solved)
## SUMMATIVE ASSESSMENT –I
## Class – X Mathematics

Time allowed: 3 hours | Maximum Marks: 90

**General Instructions:**

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

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### SECTION – A

1. Prove that $\cos 1°\cos 2°\cos 3°……\cos 180° = 0$.

2. If two zeros of the polynomial $f(x) = x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

3. Evaluate: $\tan 5°\tan 25°\tan 30°\tan 65°\tan 85°$

4. Find the mode of the following data:
120, 110, 130, 110, 120, 140, 130, 120, 140, 120

### SECTION – B

5. The perimeters of two similar triangles are 30 cm and 20 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.

6. Prove that the polynomial $x^2 + 2x + 5$ has no zero.

7. The areas of two similar triangles ABC and PQR are 64 $cm^2$ and 121 $cm^2$ respectively. If QR = 15.4 cm, find BC.

8. For any positive real number x, prove that there exists an irrational number $y$ such that $0 < y < x$.

9. Given that sin(A + B) = sinAcosB + cosAsinB, find the value of sin75°.

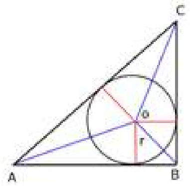
10. Find the values of $\alpha$ and $\beta$ for which the following system of linear equations has infinite number of solutions. $2x + 3y = 7$, $2\alpha x + (\alpha + \beta)y = 28$

### SECTION – C

11. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

12. Find the condition that the zeros of the polynomial $f(x)=x^3-px^2+qx-r$ may be in arithmetic progression.

13. ABC is a right-angled triangle right angled at A. A circle is inscribed in it the lengths of two sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.



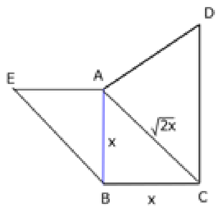

14. Find the four angles of a cyclic quadrilateral ABCD in which ∠A = $(2x-5)°$, ∠B = $(y+5)°$, ∠C = $(2y+15)°$ and ∠D = $(4x-7)°$.

15. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data.

| Number of cars | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 7 | 14 | 13 | 12 | 20 | 11 | 15 | 8 |

16. In a ΔABC, right angled at B, if AB = 4 and BC = 3, find all the six trigonometric ratios of ∠A.

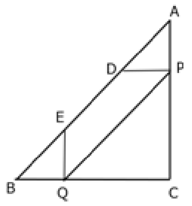
17. ABC is an isosceles triangle right-angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of ΔABE and ΔACD.




18. I am 3 times as old as my son. 5 years later, I shall be two and a half times as old as my son. How old am I and how old is my son?

19. Prove $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$.

20. Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB.




**SECTION – D**

21. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.

22. If cosecA = 2, find the value of $\frac{1}{\tan A}+\frac{\sin A}{1+\cos A}$.

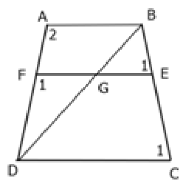
23. If $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$ and $x\sin\theta=y\cos\theta$, prove that $x^2+y^2=1$.

24. A frequency distribution of the life times of 400 T.V. picture tubes tested in a company is given below. Find the average life of a tube.

| Life time (in hours) | Frequency | Life time (in hours) | Frequency |
|---|---|---|---|
| 300-399 | 14 | 800-899 | 62 |
| 400-499 | 46 | 900-999 | 48 |
| 500-599 | 58 | 1000-1099 | 22 |
| 600-699 | 76 | 1100-1199 | 6 |
| 700-799 | 68 | | |

25. What must be added to $f(x)=4x^4+2x^3-2x^2+x-1$ so that the resulting polynomial is divisible by $g(x)=x^2+2x-3$?

26. In trapezium ABCD, AB || DC and DC = 2AB. A line EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC}=\frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7FE = 10AB.




27. Solve the following system of linear equations graphically.

$x - y = 1$

$2x + y = 8$

Shade the area bounded by these two lines and $y$-axis. Also, determine this area.

28. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

29. Following is the age distribution of a group of students. Draw the cumulative frequency polygon, cumulative frequency curve (less than type) and hence obtain the median value.

| Age | Frequency | Age | Frequency |
|---|---|---|---|
| 5-6 | 40 | 11-12 | 92 |
| 6-7 | 56 | 12-13 | 80 |
| 7-8 | 60 | 13-14 | 64 |
| 8-9 | 66 | 14-15 | 44 |
| 9-10 | 84 | 15-16 | 20 |
| 10-11 | 96 | 16-17 | 8 |

30. Prove $\dfrac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}=\sin^2 A\cos^2 A$

31. In a housing society, people decided to do rainwater harvesting. Rainwater is collected in the underground tank at the rate of 30 $cm^3$/sec. Taking volume of water collected in $x$ seconds as $y cm^3$.

a. Form a linear equation.

b. Write it in standard form as $ax + by + c = 0$.

c. Which values are promoted by the members of this society?

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## SECTION – A

1. **Solution:**

We have

$$\begin{aligned} \text{LHS} &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots\ldots \cos 180^\circ \\ &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 89^\circ \cos 90^\circ \cos 91^\circ \ldots \cos 180^\circ \\ &= \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots 0 \times \cos 90^\circ \cos 91^\circ \ldots \cos 180^\circ = 0 \end{aligned} \quad = \text{R.H.S}$$

2. **Solution:**

Let $\alpha = \sqrt{3}$ and $\beta = -\sqrt{3}$ be the given zeros and $\gamma$ be the third zero. Then,

$$\alpha + \beta + \gamma = -\left(\frac{-4}{1}\right) \qquad \left[\text{Using } \alpha + \beta + \gamma = \frac{\text{Coeff. of } x^2}{\text{Coeff. of } x^3}\right]$$

$\Rightarrow \quad \sqrt{3} - \sqrt{3} + \gamma = 4$

$\Rightarrow \quad \gamma = 4$

Hence, third zero is 4.

3. **Solution:**

We have,

$$\begin{aligned} &\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ \\ &= (\tan 5^\circ \tan 85^\circ)(\tan 25^\circ \tan 65^\circ)\tan 30^\circ \\ &= (\tan 5^\circ \cot 5^\circ)(\tan 25^\circ \cot 25^\circ)\tan 30^\circ \end{aligned} \qquad \left[\begin{aligned} \because \tan 85^\circ &= \tan(90^\circ - 5^\circ) = \cot 5^\circ \\ \tan 65^\circ &= \tan(90^\circ - 25^\circ) = \cot 25^\circ \end{aligned}\right]$$

$$= 1 \times 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

4. **Solution:**

Let us first form the frequency table for the given data as given below:

| Value ($x_i$) | 110 | 120 | 130 | 140 |
|---|---|---|---|---|
| Frequency ($f_i$) | 2 | 4 | 2 | 2 |

We observe that the value 120 has the maximum frequency.
Thus, the mode is 120.

## SECTION – B

5. **Solution:**

Let ΔABC and ΔDEF be two similar triangles of perimeters $P_1$ and $P_2$ respectively. Also, let AB = 12 cm, $P_1$ = 30 cm and $P_2$ = 20 cm. Then,

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{P_1}{P_2}$$

$$\Rightarrow \quad \frac{AB}{DE}=\frac{P_1}{P_2}$$

$$\Rightarrow \quad \frac{12}{DE}=\frac{30}{20} \quad \Rightarrow \quad DE = \frac{12\times 20}{30}=8 \text{ cm}$$

Thus, the corresponding side of the second triangle is 8 cm.

6. **Solution:**

Let f(x) = $x^2+2x+5$

= $x^2+2x+1+4$

= $(x+1)^2+4$

Now, for every real value of x, $(x+1)^2 \geq 0$

$\Rightarrow$ For every real value of x, $(x+1)^2+4\geq 4$

$\therefore$ For every real value of $x$, $f(x)\geq 4$ and hence it has no zero.

7. **Solution:**

Given that ΔABC ~ΔPQR

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{64}{121}=\frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \quad \frac{8\times 15.4}{11}=11.2 \text{ cm}$$

$$\Rightarrow \quad BC = \frac{8}{11}=\frac{BC}{15.4}$$

8. **Solution:**

If $x$ is irrational, then $y = \frac{x}{2}$ is also an irrational number such that $0 < y < x$.

If $x$ is rational, then $\frac{x}{\sqrt{2}}$ is an irrational number such that $\frac{x}{\sqrt{2}} < x$ as $\sqrt{2} > 1$.

$\therefore \quad y = \frac{x}{\sqrt{2}}$ is an irrational number such that $0 < y < x$.

9. **Solution:**

Putting A = 45° and B = 30° in sin(A + B) = sinAcosB + cosAsinB, we get

$$\sin(45° + 30°) = \sin45°\cos30° + \cos45°\sin30°$$

$$\Rightarrow \quad \sin75° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

10. **Solution:**

The given system of equations will have infinite number of solutions, if

$$\frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{7}{28}$$

$$\Rightarrow \quad \frac{1}{\alpha} = \frac{3}{\alpha+\beta} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{\alpha} = \frac{1}{4} \text{ and } \frac{3}{\alpha+\beta} = \frac{1}{4}$$

$$\Rightarrow \quad \alpha = 4 \text{ and } \alpha + \beta = 12$$

$$\Rightarrow \quad \alpha = 4 \text{ and } \beta = 12 - 4 = 8$$

**SECTION – C**

11. **Solution:**

It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is exactly divisible by the required number. In other words, required number is a factor of 391.

Similarly, required positive integer is a factor of 436 – 11 = 425 and 542 – 15 = 527.

Clearly, required number is the HCF of 391, 425 and 527.

Using the factor tree, the prime factorisations of 391, 425 and 527 are as follows:

$391 = 17 \times 23$

$425 = 5^2 \times 17$

$527 = 17 \times 31$

$\therefore$ HCF of 391, 425 and 527 is 17.

Thus, the required number is 17.

12. **Solution:**

Let $a - d$, $a$ and$a + d$ be the zeros of the polynomial $f(x)$. Then,

Sum of the zeros = $\frac{Coefficient\ of\ x^2}{Coefficient\ of\ x^3}$

$\Rightarrow \quad (a-d)+a+(a+d) = -\frac{(-p)}{1}$

$\Rightarrow \quad 3a = p \qquad \Rightarrow \qquad a = \frac{p}{3}$

Since $a$ is a zero of the polynomial $f(x)$. Therefore,

$f(a) = 0$

$\Rightarrow \quad a^3 - pa^2 + qa - r = 0$

$\Rightarrow \quad \left(\frac{p}{3}\right)^3 - p\left(\frac{p}{3}\right)^2 + q\left(\frac{p}{3}\right) - r = 0 \qquad \left[\because a = \frac{p}{3}\right]$

$\Rightarrow \quad p^3 - 3p^3 + 9pq - 27r = 0$

$\Rightarrow \quad 2p^3 - 9pq + 27r = 0$

13. **Solution**

Using Pythagoras theorem in ΔBAC, we have

$BC^2 = AB^2 + AC^2$

$\Rightarrow \quad BC^2 = 6^2 + 8^2 = 100$

$\Rightarrow \quad BC = 10$ cm

Now,

Area of ΔABC = Area of ΔOAB + Area of ΔOBC + Area of ΔOCA

$\Rightarrow \quad \frac{1}{2}\ AB \times AC = \frac{1}{2}\ AB \times r + \frac{1}{2}\ BC \times r + \frac{1}{2}\ CA \times r$

$\Rightarrow \quad \frac{1}{2}\ 6 \times 8 = \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r$

$\Rightarrow \quad 48 = 24r$

$\Rightarrow \quad r = 2$ cm

14. **Solution:**

We know that the sum of the opposite angles of a cyclic quadrilateral is 180°. In the cyclic quadrilateral ABCD, angles A and C and angles B and D form pairs of opposite angles.

$\therefore \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$\Rightarrow$ $2x - 1 + 2y + 15 = 180°$ and $y + 5 + 4x - 7 = 180°$

$\Rightarrow$ $2x + 2y = 166°$ and $4x + y = 182°$

$\Rightarrow$ $x + y = 83°$ ...(i)

And, $4x + y = 182°$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$3x = 99 \Rightarrow x = 33$

Substituting $x = 33$ in equation (i), we get $y = 50$

Hence, $\angle A = (2x-1)° = (2\times33-1)° = 65°$, $\angle B = (y+5)° = (50+5)° = 55°$

$\angle C = (2y+15)° = (2\times50+15)° = 115°$ and $\angle D = (4x-7)° = (4\times33-7)° = 125°$

15. **Solution:**

The modal class is 40-50, since it has the maximum frequency.

$l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\times h$$

$$= 40 + \left(\frac{20-12}{2\times20-12-11}\right)\times10$$

$$= 40 + \left(\frac{8}{17}\right)\times10$$

$= 40 + 4.71$

$= 44.71$ cars

16. **Solution** :

We have AB = 4 and BC = 3

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow$ $AC = \sqrt{4^2 + 3^2}$

$\Rightarrow$ $AC = \sqrt{25} = 5$

When we consider the t-ratios of $\angle A$, we have

Base = AB = 4, Perpendicular = BC = 3 and Hypotenuse = AC = 5

$\therefore$ $\sin A = \frac{BC}{AC} = \frac{3}{5}$, $\cos A = \frac{AB}{AC} = \frac{4}{5}$, $\tan A = \frac{BC}{AB} = \frac{3}{4}$

cosecA = $\frac{AC}{BC} = \frac{5}{3}$, secA = $\frac{AC}{AB} = \frac{5}{4}$ and cotA = $\frac{AB}{BC} = \frac{4}{3}$

17. **Solution:**

Let AB = BC = $x$.

It is given that ΔABC is a right-angled at B.

∴ $AC^2 = AB^2 + BC^2$

⇒ $AC^2 = x^2 + x^2$

⇒ AC = $\sqrt{2}x$

It is given that

ΔABE ~ΔACD

⇒ $\frac{\text{Area}(\Delta ABE)}{\text{Area}(\Delta ACD)} = \frac{AB^2}{AC^2}$

$= \frac{x^2}{\left(\sqrt{2}x\right)^2}$

$= \frac{1}{2}$

18. **Solution**

Suppose my age is $x$ years and my son's age is $y$ years. Then,

$x = 3y$ ...(i)

5 years later, my age will be $(x + 5)$ years and my son's age will be $(y + 5)$ years.

∴ $x + 5 = \frac{5}{2}(y+5)$

⇒ $2x - 5y - 15 = 0$ ...(ii)

Putting $x = 3y$ in equation (ii), we get

$6y - 5y - 15 = 0$ ⇒ $y = 15$

Putting $y = 15$ in equation (i), we get

$x = 45$

19. **Solution**

We have,

LHS $= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$= \sqrt{\left(\frac{1-\sin\theta}{\cos\theta}\right)^2}$

$= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\sec\theta-\tan\theta$ = RHS

20. **Solution**

In ΔABC, we have

DP || BC and EQ || AC

∴ $\frac{AD}{DB}=\frac{AP}{PC}$ and $\frac{BE}{EA}=\frac{BQ}{QC}$

⇒ $\frac{AD}{DB}=\frac{AP}{PC}$ and $\frac{AD}{DB}=\frac{BQ}{QC}$ [∵ EA = ED + DA = ED + BE = BD, ∴ AD = BE]

⇒ $\frac{AP}{PC}=\frac{BQ}{QC}$

⇒ In aΔABC, P and Q divide sides CA and CB respectively in the same ratio.

⇒ PQ || AB.

**SECTION – D**

21. **Solution**

Let the numerator and denominator of the fraction be *x* and *y* respectively.

Then,

Fraction = $\frac{x}{y}$

It is given that

Denominator = 2(Numerator) + 4

$\Rightarrow \quad y = 2x + 4$

$\Rightarrow \quad 2x - y + 4 = 0$

According to the given condition, we have

$y - 6 = 12(x - 6)$

$\Rightarrow \quad y - 6 = 12x - 72$

$\Rightarrow \quad 12x - y - 66 = 0$

Thus, we have the following system of equations

$2x - y + 4 = 0$ ...(i)

$12x - y - 66 = 0$ ...(ii)

Subtracting equation (i) from equation (ii), we get

$10x - 70 = 0$

$\Rightarrow \quad x = 7$

Putting $x = 7$ in equation (i), we get

$14 - y + 4 = 0$

$\Rightarrow \quad y = 18$

Hence, required fraction = $\frac{7}{18}$

22. **Solution:**

We have,

$$\text{cosecA} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{2}{1}$$

So, we draw a right triangle, right angled at B such that

Perpendicular = BC = 1, Hypotenuse = AC = 2

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 2^2 = AB^2 + 1^2$

$\Rightarrow \quad AB^2 = 4 - 1$

$\Rightarrow \quad AB^2 = 3$

$\Rightarrow \quad AB = \sqrt{3}$

Now,

$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}, \ \sin A = \frac{BC}{AC} = \frac{1}{2} \text{ and } \cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$\therefore \quad \frac{1}{\tan A}+\frac{\sin A}{1+\cos A}=\frac{1}{\frac{1}{\sqrt{3}}}+\frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$

$= \frac{\sqrt{3}}{1}+\frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$

$= \frac{\sqrt{3}}{1}+\frac{1}{2+\sqrt{3}}$

$= \sqrt{3}+\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \sqrt{3}+\frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$

$= \sqrt{3}+\frac{2-\sqrt{3}}{4-3}$

$= \sqrt{3}+\left(2-\sqrt{3}\right) = 2$

23. **Solution:**

We have,

$x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta\left(\sin^2\theta\right)+y\cos\theta\left(\cos^2\theta\right)=\sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta\left(\sin^2\theta\right)+x\sin\theta\left(\cos^2\theta\right)=\sin\theta\cos\theta \qquad [\because x\sin\theta=y\cos\theta]$

$\Rightarrow \quad x\sin\theta\left(\sin^2\theta+\cos^2\theta\right)=\sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta=\sin\theta\cos\theta$

$\Rightarrow \quad x=\frac{\sin\theta\cos\theta}{\sin\theta}=\cos\theta$

Now,

$x\sin\theta=y\cos\theta$

$\Rightarrow \quad \cos\theta\sin\theta=y\cos\theta \qquad [\because x=\cos\theta]$

$\Rightarrow \quad y=\frac{\cos\theta\sin\theta}{\cos\theta}=\sin\theta$

∴ $x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$

24. **Solution:**

Here, the class intervals are formed by exclusive method. If we make the series an inclusive, one of the mid-values remain same. So, there is no need to convert the series into an inclusive form.

Let the assumed mean be A = 749.5 and $h$ = 100.

Calculation of mean

| Life time (in hours) | Frequency $f_i$ | Mid-values $x_i$ | $d_i = x_i - A$ $= x_i - 749.5$ | $u_i = \frac{x_i - A}{h}$ $u_i = \frac{x_i - 749.5}{100}$ | $f_i u_i$ |
|---|---|---|---|---|---|
| 300-399 | 14 | 349.5 | –400 | –4 | –56 |
| 400-499 | 46 | 449.5 | –300 | –3 | –138 |
| 500-599 | 58 | 549.5 | –200 | –2 | –116 |
| 600-699 | 76 | 649.5 | –100 | –1 | –76 |
| 700-799 | 68 | 749.5 | 0 | 0 | 0 |
| 800-899 | 62 | 849.5 | 100 | 1 | 62 |
| 900-999 | 48 | 949.5 | 200 | 2 | 96 |
| 1000-1099 | 22 | 1049.5 | 300 | 3 | 66 |
| 1100-1199 | 6 | 1149.5 | 400 | 4 | 24 |
| $N = \sum f_i = 400$ $\sum f_i u_i = -138$ | | | | | |

We have N = 400, A = 749.5, $h$ = 100 and $\sum f_i u_i = -138$

$$\therefore \quad \overline{X} = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$$

$$= 749.5 + 100 \times \left(\frac{-138}{400}\right)$$

$$= 749.5 - \frac{138}{4}$$

$$= 749.5 - 34.5 = 715$$

Thus, the average life time of a tube is 715 hours.

25. **Solution:**

By division algorithm, we have

$$f(x) = g(x) \times q(x) + r(x)$$

$$\Rightarrow \quad f(x) - r(x) = g(x) \times q(x)$$

$$\Rightarrow \quad f(x) + \{-r(x)\} = g(x) \times q(x)$$

Clearly, RHS is divisible by $g(x)$. Therefore, LHS is also divisible by $g(x)$. Thus, if we add $-r(x)$ to $f(x)$, then the resulting polynomial is divisible by $g(x)$. Let us now find the remainder when $f(x)$ is divided by $g(x)$.

$$\begin{array}{r|l} & 4x^2 - 6x + 22 \\ \hline x^2+2x-3 & 4x^4 + 2x^3 - 2x^2 + x - 1 \\ & 4x^4 + 8x^3 - 12x^2 \\ & - \quad - \quad + \\ \hline & -6x^3 + 10x^2 + x - 1 \\ & -6x^3 - 12x^2 + 18x \\ & + \quad + \quad - \\ \hline & 22x^2 - 17x - 2 \\ & 22x^2 + 44x - 66 \\ & - \quad - \quad + \\ \hline & -61x + 65 \end{array}$$

$\therefore \quad r(x) = -61x + 65$

Thus, we should add $-r(x) = 61x - 65$ to $f(x)$ so that the resulting polynomial is divisible by $g(x)$.

26. **Solution:**

In $\Delta$DFG and $\Delta$DAB, we have

$\angle 1 = \angle 2$ [$\because$ AB || DC || EF, $\therefore \angle 1$ and $\angle 2$ are corresponding angles]

$\angle$FDG = $\angle$ADB [Common]

So, by AA-criterion of similarity, we have

$\Delta DFG \sim \Delta DAB \quad \Rightarrow \quad \dfrac{DF}{DA} = \dfrac{FG}{AB}$ ...(i)

In trapezium ABCD, we have

EF || AB || DC

$$\therefore \quad \frac{AF}{DF} = \frac{BE}{EC}$$

$$\Rightarrow \quad \frac{AF}{DF} = \frac{3}{4} \qquad \left[\because \frac{BE}{EC} = \frac{3}{4} (\text{Given})\right]$$

$$\Rightarrow \quad \frac{AF}{DF} + 1 = \frac{3}{4} + 1 \qquad [\text{Adding 1 on both sides}]$$

$\Rightarrow \quad \frac{AF + DF}{DF} = \frac{7}{4}$

$\Rightarrow \quad \frac{AD}{DF} = \frac{7}{4} \quad \Rightarrow \quad \frac{DF}{AD} = \frac{4}{7}$ ...(ii)

From (i) and (ii), we get

$\frac{FG}{AB} = \frac{4}{7} \quad \Rightarrow \quad FG = \frac{4}{7}AB$ ...(iii)

In ΔBEG and ΔBCD, we have

$\angle BEG = \angle BCD$ [Corresponding angles]

$\angle B = \angle B$ [Common]

$\therefore \quad \Delta BEG \sim \Delta BCD$ [By AA-criterion of similarity]

$\Rightarrow \quad \frac{BE}{EC} = \frac{EG}{CD}$

$\Rightarrow \quad \frac{3}{7} = \frac{EG}{CD}$ $\left[\because \frac{BE}{EC} = \frac{3}{4} \Rightarrow \frac{EC}{BE} = \frac{4}{3} \Rightarrow \frac{EC}{BE} + 1 = \frac{4}{3} + 1 \Rightarrow \frac{BC}{BE} = \frac{7}{3}\right]$

$\Rightarrow \quad EG = \frac{3}{7}CD \quad = \frac{3}{7} \times 2AB \quad = \frac{6}{7}AB$ ...(iv)

Adding (iii) and (iv), we get

$FG + EG = \frac{4}{7}AB + \frac{6}{7}AB$

$\Rightarrow \quad EF = \frac{10}{7}AB$

$\Rightarrow \quad 7EF = 10AB$

27. **Solution**:

We have,

$x - y = 1$

$2x + y = 8$

Graph of the equation $x - y = 1$:

We have,

$x - y = 1 \quad \Rightarrow \quad y = x - 1$ and $x = y + 1$

Putting $x = 0$, we get $y = -1$

Putting $y = 0$, we get $x = 1$

Thus, we have the following table for the points on the line $x - y = 1$:

| | | |
|---|---|---|
| $x$ | 0 | 1 |
| $y$ | −1 | 0 |

Graph of the equation $2x + y = 8$:

We have,

$$2x + y = 8 \quad \Rightarrow \quad y = 8 - 2x \text{ and } x = \frac{8 - y}{2}$$

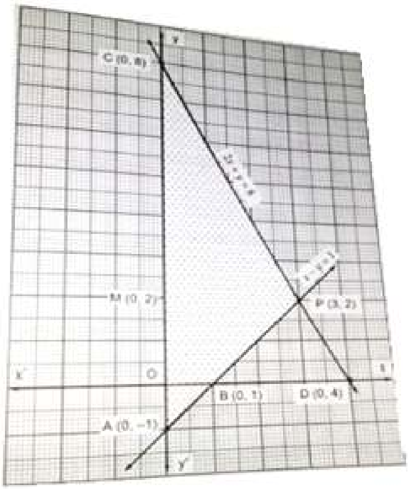
Putting $x = 0$, we get $y = 8$

Putting $y = 0$, we get $x = 4$

Thus, we have the following table for the points on the line $2x + y = 8$:

| | | |
|---|---|---|
| $x$ | 0 | 8 |
| $y$ | 8 | 0 |

Plotting points A(0, −1), B(1, 0) on the graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $x - y = 1$.




Plotting points C(0, 8), D(4, 0) on the same graph paper and drawing a line passing through them, we obtain the graph of the line represented by the equation $2x + y = 8$.

Clearly, the two lines intersect at P(3, 2). The area enclosed by the lines represented by the given equations and the $y$-axis is shaded.

Now, required area = Area of the shaded region

$$= \text{Area of } \Delta PAC$$

$$= \frac{1}{2}(\text{Base} \times \text{Height})$$

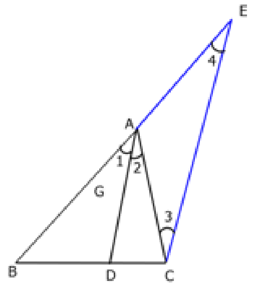
$= \frac{1}{2}(AC \times PM)$ [$\because$ PM = $x$-coordinate of P = 3]

$= \frac{1}{2}(9 \times 3)$ = 13.5 sq. units

28. **Solution**:

Given: A$\Delta$ABC in which AD is the internal bisector of $\angle$A and meets BC in D.

To prove: $\frac{BD}{DC} = \frac{AB}{AC}$




Construction: Draw CE || DA to meet BA produced in E.

Proof: Since CE || DA and AC cuts them,

$\therefore$ $\angle 2 = \angle 3$ [Alternate angles] ...(i)

And, $\angle 1 = \angle 4$ [Corresponding angles] ...(ii)

But, $\angle 1 = \angle 2$ [$\because$ AD is the bisector of $\angle$A]

From (i) and (ii), we get

$\angle 3 = \angle 4$

Thus, in $\Delta$ACE, we have

$\angle 3 = \angle 4$

$\Rightarrow$ AE = AC [Sides opposite to equal angles are equal] ...(iii)

Now, in $\Delta$BCE, we have

DA || CE

$\Rightarrow$ $\frac{BD}{DC} = \frac{BA}{AE}$ [Using Basic Proportionality Theorem]

$\Rightarrow$ $\frac{BD}{DC} = \frac{AB}{AC}$ [$\because$ BA = AB and AE = AC (From (iii)]

Thus, $\frac{BD}{DC} = \frac{AB}{AC}$

29. **Solution**:

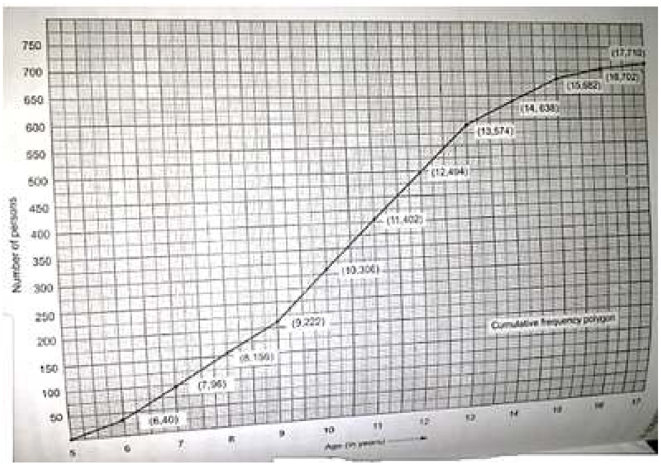
We first prepare the cumulative frequency table by less than method as given below:

| Age | Frequency | Age less than | Cumulative frequency |
|---|---|---|---|
| 5-6 | 10 | 6 | 40 |
| 6-7 | 56 | 7 | 96 |
| 7-8 | 60 | 8 | 156 |
| 8-9 | 66 | 9 | 222 |
| 9-10 | 84 | 10 | 306 |
| 10-11 | 96 | 11 | 402 |
| 11-12 | 92 | 12 | 494 |
| 12-13 | 80 | 13 | 574 |
| 13-14 | 64 | 14 | 638 |
| 14-15 | 44 | 15 | 682 |
| 15-16 | 20 | 16 | 702 |
| 16-17 | 8 | 17 | 710 |

Other than the given class intervals, we assume a class 4-5 before the first class-interval 5-6 with zero frequency.

Now, we mark the upper class limits (including the imagined class) along $x$-axis on a suitable scale and the cumulative frequencies along $y$-axis on a suitable scale.

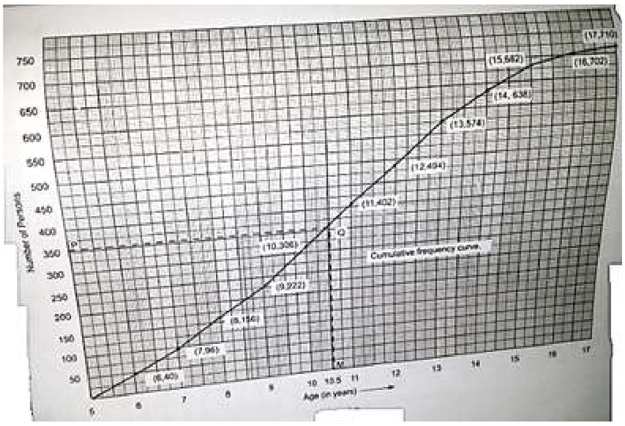
Thus, we plot the points (5, 0), (6, 40), (7, 96), (8, 156), (9, 222), (10, 306), (11, 402), (12, 494), (13, 574), (14, 638), (15, 682), (16, 702) and (17, 710). These points are marked and joined by line segments to obtain the cumulative frequency polygon as shown in the figure.



In order to obtain the cumulative frequency curve, we draw a smooth curve passing through the points discussed above.

The graph given below shows the total number of students as 710. The median is the age corresponding to $\frac{N}{2} = \frac{710}{2}$ = 355 students. In order to find the median, we first locate the point corresponding to 355th student on $y$-axis. Let the point be P. From this point, draw a line parallel to the $x$-axis cutting the curve at Q. From this point Q, draw a line parallel to $y$-axis and meeting $x$-axis at the point M. The $x$-coordinate of M is 10.5.

Hence, median is 10.5.




30. **Solution**:

We have,

$$\text{LHS} = \frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A-\operatorname{cosec}^3 A}$$

$$= \frac{\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)}{\left(\frac{1}{\cos^3 A}-\frac{1}{\sin^3 A}\right)}$$

$$= \frac{\left(1+\frac{\cos^2 A+\sin^2 A}{\sin A\cos A}\right)(\sin A-\cos A)}{\left(\frac{\sin^3 A-\cos^3 A}{\sin^3 A\cos^3 A}\right)}$$

$$= \frac{\sin A\cos A+\cos^2 A+\sin^2 A}{\sin A\cos A}\times\frac{\sin^3 A\cos^3 A}{\sin^3 A-\cos^3 A}(\sin A-\cos A)$$

$$= \frac{\sin A\cos A+1}{\sin^3 A-\cos^3 A}\times(\sin^2 A\cos^2 A)(\sin A-\cos A)$$

$$= \frac{(\sin A\cos A+1)(\sin^2 A\cos^2 A)(\sin A-\cos A)}{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)} \qquad \left[\because a^3-b^3=(a-b)(a^2+b^2+ab)\right]$$

$$= \frac{(\sin A\cos A+1)\sin^2 A\cos^2 A}{1+\sin A\cos A}$$

$= \sin^2 A\cos^2 A$ = RHS

31. **Solution**:

Rate at which rainwater is collected in the tank = 30 $cm^3$/sec

Time for which water is collected = $x$ seconds

Total amount of water collected = $y$cm3

a. According to the given condition, linear equation formed is $y = 30x$

b. The equation in standard form is $30x - y + 0 = 0$

c. Values promoted by the members of the society are environmental protection and co-operation.