CBSE Sample Paper-05 (Solved) SUMMATIVE ASSESSMENT –II MATHEMATICS Class – X

Time allowed: 3 hours

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 31 questions divided into four sections A, B, C and D.
- c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
- d) Use of calculator is not permitted.

Section A

- 1. The sum of first five multiples of 4 is:

 (a) 30
 (b) 40
 (c) 50
 (d) 6
- 2. The angle of depression and the angle of elevation from an object on the ground to an object in the air are related as:
 - (a) greater than (b) less than (c) equal (d) all of them
- 3. Cards each marked with one of the numbers 6, 7, 8,, 15 are placed in a box and mixed thoroughly. One card is drawn at random from the box. The probability of getting a card with a number less than 10 is:
 - (a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$
- 4. The value of x for which the distance between the points A(2,-3) and B(x,5) is 10 units is:
 - (a) 2 (b) 4 (c) 6

Section **B**

(d) 8

- 5. For what value of *k*, are the roots of the equation $3x^2 + 2kx + 27 = 0$ are real and equal?
- 6. Two AP's have the same common difference. The first two terms of one of these is −3 and not of the other is −7. Find the difference between their 4th terms.
- 7. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If \angle PCA = 100°, then find \angle CBA.
- 8. Find the radius of the circle whose circumference is equal to the sum of circumferences of the two circles of diameter 30 cm and 24 cm.

Maximum Marks: 90

- 9. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys with the shape of a right circular cone mounted on a hemisphere of radius 3 cm. If the height of the toy is 12 cm, find the number of toys so formed.
- 10. Water flows through a circular pipe, whose internal diameter is 2 cm, at the rate of 0.7 m per second into a cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water in the cylindrical tank use in half an hour?

Section C

11. Solve the quadratic equation by using quadratic formula:

$$\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$$

- 12. Find the middle term of the AP 10, 7, 4, ..., (-62).
- 13. ABCD is a quadrilateral such that $\angle D = 90^{\circ}$. A circle C (0, *r*) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, then find *r*.



- 14. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 5 m. From a point on the plane the angles of elevation of the bottom and top of the flagstaff of height 5 m. From a point on the plane the angles of elevation of the bottom and top of the flagstaff are respectively 30° and 60°. Find the height of the tower.
- 15. Find the probability that a number selected at random from the numbers 1, 2, 3,, 35 is:
 - (i) a prime number.
 - (ii) multiple of 7.
 - (iii) multiple of 3 or 5.
- 16. If A(5,-1), B(-3,-2) and C(-1,8) are the vertices of triangle ABC, find the length of median through A and the coordinates of centroid.
- 17. If the point (x, y) is equidistant from the points (a+b,b-a) and (a-b,a+b), then prove that bx = ay.
- 18. A chord AB of a circle of radius 14 cm makes a right angle at the centre (0) of the circle. Find the area of the minor segment. $\left(\text{Use } \pi = \frac{22}{7}\right)$



- 19. The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 6 a.m. and 6.05 a.m. (Use $\pi = \frac{22}{7}$)
- 20. Find the number of coins 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Section D

- 21. Solve for x: $\frac{1}{x+4} \frac{1}{x-7} = \frac{11}{30}; \quad x \neq -4, 7$
- 22. Two years ago, a man's age was three times the square of his son's age. Three years hence, his age will be four times his son's age. Find their present ages.
- 23. Ram and Shyam have been given to find out the number of two digit numbers in between 6 and 102 which are divisible by 6. Ram calculated it by using AP while Shyam calculated it directly. Read the above passage and answer the following questions:
 - (i) How many two digits number are there in between 6 and 102 which are divisible by 6??
 - (ii) What value is depicted by Ram?

[Value Based Question]

- 24. The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.
- 25. With the vertices of a triangle ABC as centre, three circles are described, each touching the other two externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm, then find the radii of the circle.
- 26. Draw any quadrilateral ABCD. Construct another quadrilateral AB'C'D' similar to the quadrilateral ABCD with each side equal to $\frac{4}{5}$ th of the corresponding side of quadrilateral ABCD. Write the steps of construction also.

- 27. A tree breaks due to the storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 meters. Find the height of the tree.
- 28. A box contains 19 balls bearing numbers 1, 2, 3,, 19. A ball is drawn at random from the box. Find the probability that the number on the ball is:
 - (i) a prime number(ii) divisible by 3 or 5(iii) neither divisible by 5 nor by 10(iv) an even number
- 29. If P and Q are two points whose coordinates are $(at^2, 2at)$ and $(\frac{a}{t^2}, \frac{-2a}{t})$ respectively and S is

the point (a,0), then show that $\frac{1}{SP} + \frac{1}{SQ}$ is independent of *t*.

- 30. A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the volume of:
 - (i) water displaced out of the cylindrical vessel.
 - (ii) water left in the cylindrical vessel. $\left(\text{Use } \pi = \frac{22}{7} \right)$
- 31. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also find the total surface area of the remaining solid. (Use $\pi = 3.1416$)

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(Solutions)

SECTION-A

1. (d) 2. (c) 3. (c) 4. (c) 5. Here, a = 3, b = 2k, c = 27For real and equal roots, $b^2 - 4ac = 0$ $(2k)^2 - 4(3)(27) = 0$ \Rightarrow $\Rightarrow k^2 = 81$ $4k^2 = 324$ \Rightarrow $k = \pm 9$ \Rightarrow $\Rightarrow a_4 = (-3) + 3d$ 6. $a_{A} = a + 3d$(i) $A_4 = A + 3d \implies A_4 = -7 + 3d$(ii) And Difference $= A_4 - a_4$... = (-3+3d)-(-7-3d) = 4 \angle PCA = 100° and \angle BCA = 90° 7. $\angle PCB = 100^{\circ} - 90^{\circ} = 10^{\circ}$ *.*.. $\angle \text{OCP} = 90^{\circ}$ $\angle \text{OCB} = \angle \text{PCB} = 90^{\circ}$ \Rightarrow $\angle \text{OCB} + 10^\circ = 90^\circ$ \Rightarrow $\angle \text{OCB} = 80^{\circ}$ \Rightarrow ÷ OB = OC $\angle OBC = \angle OCB = 80^{\circ}$ *.*:. $\angle CBA = 80^{\circ}$ *.*:. 8. According to question, Circumference of circle = Sum of circumferences of two circles $\langle 1 \rangle$

$$\Rightarrow \qquad 2\pi r = 2\pi r_1 + 2\pi r_2 \qquad \Rightarrow \qquad 2\pi r = 2\pi \left(\frac{d_1}{2}\right) + 2\pi \left(\frac{d_2}{2}\right)$$
$$\Rightarrow \qquad 2\pi r = 2\pi \left(\frac{d_1}{2} + \frac{d_2}{2}\right) \qquad \Rightarrow \qquad r = \frac{d_1}{2} + \frac{d_2}{2}$$

$$\Rightarrow$$
 $r = \frac{30}{2} + \frac{24}{2} = \frac{54}{2} = 27 \text{ cm}$

9. Volume of solid cylinder $= \pi r^2 h$

$$=\pi \left(\frac{12}{2}\right)^2 (15) = 540\pi \text{ cm}^3$$

Volume of one toy = Volume of conical portion + Volume of hemispherical portion

$$= \frac{1}{3}\pi r^{2}h + \frac{2}{3}\pi r_{1}^{3}$$

$$= \frac{1}{3}\pi (3)^{2} + (12 - 3) + \frac{2}{3}\pi (3)^{3}$$

$$27\pi + 18\pi = 45\pi$$

:. Number of toys = $\frac{\text{Volume of Cylinder}}{\text{Volume of one toy}} = \frac{540\pi}{45\pi} = 12$

10. Water flown out through the pipe in half an hour

$$= \pi \left(\frac{2}{2}\right)^2 (0.7 \times 100) (60 \times 30) \text{ cm}^3$$

Let the water level rise by x cm. Then,

$$\pi \left(\frac{2}{2}\right)^{2} (0.7 \times 100)(60 \times 30) = \pi (40)^{2} x$$

$$\Rightarrow \quad x = 78.75 \text{ cm}$$
11. $\sqrt{2}x^{2} - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$

$$\Rightarrow \quad 2x^{2} - 3x + 1 = 0$$
Here, $a = 2, b = -3, c = 1$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$\therefore \quad x = \frac{-(-3) \pm \sqrt{(-3)^{3} - 4.(2).(1)}}{2(2)} = \frac{3 \pm \sqrt{9 - 8}}{4}$$

$$\Rightarrow \quad x = \frac{3 \pm 1}{4} \quad \Rightarrow \quad x = \frac{3 \pm 1}{4}, \frac{3 - 1}{4}$$

$$\Rightarrow \quad x = 1, \frac{1}{2}$$
12. $a = 10, d = -3, l = -62$

$$\therefore \quad l = a + (n - 1)d$$

$$\therefore \quad -62 = 10 + (n - 1)(-3)$$

$$\Rightarrow \quad n - 1 = 24$$

$$\Rightarrow \quad n = 25$$

- \therefore Number of favourable outcomes = 16
- Hence, required probability = $\frac{16}{35}$

16. Let D be the mid-point of BC. Then,

$$D \rightarrow \left(\frac{-3-1}{2}, \frac{-2+8}{2}\right) \qquad \Rightarrow \qquad D \rightarrow \left(-2, 3\right)$$
$$G \rightarrow \left(\frac{5-3-1}{3}, \frac{-1-2+8}{3}\right) \qquad \Rightarrow \qquad G \rightarrow \left(\frac{1}{3}, \frac{5}{3}\right)$$
$$\therefore \qquad AD = \sqrt{\left(-2-5\right)^2 + \left(3+1\right)^2} = \sqrt{49+16} = \sqrt{65}$$

17. According to question,

$$\begin{bmatrix} x - (a+b) \end{bmatrix}^2 + \begin{bmatrix} y - (b-a) \end{bmatrix}^2 = \begin{bmatrix} x - (a-b) \end{bmatrix}^2 + \begin{bmatrix} y - (a+b) \end{bmatrix}^2$$

$$\Rightarrow x^2 + (a+b)^2 - 2x(a+b) + y^2 + (b-a)^2 - 2y(b-a) =$$

$$x^2 + (a-b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$$

$$\Rightarrow -2ax - 2bx - 2by + 2ay = -2ax + 2bx - 2ay - 2by$$

$$\Rightarrow -2bx + 2ay = 2bx - 2ay$$

$$\Rightarrow 4ay = 4bx$$

$$\Rightarrow bx = ay$$

18. Area of the minor segment = Area of sector AOB – Area of \triangle AOB

$$= \frac{\theta}{360^{\circ}} \times \pi r^2 - \frac{1}{2} \times b \times h$$
$$= \frac{90^{\circ}}{360^{\circ}} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$$
$$= 56 \text{ cm}^2$$

19. Angle described by minute hand in 1 minute = $\frac{360^{\circ}}{60^{\circ}} = 6^{\circ}$

 \therefore Angel described by the minute hand in 5 minutes = $6^{\circ} \times 5 = 30^{\circ}$

$$\therefore \qquad \text{Required area} = \frac{22}{7} \times \left(\sqrt{21}\right)^2 \times \frac{30^\circ}{360^\circ}$$
$$= \frac{22}{7} \times 21 \times \frac{1}{12}$$
$$= 5.5 \text{ cm}^2$$
20. Number of coins
$$= \frac{\text{Volume of cylinder}}{\text{Volume of one coin}}$$
$$(4.5)^2$$

$$= \frac{\pi \left(\frac{4.5}{2}\right)^2 (10)}{\pi \left(\frac{1.5}{2}\right)^2 (0.2)}$$

$$= \frac{5.0625 \times 10}{0.5625 \times 0.2} = 450$$
21. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

 $\Rightarrow \quad \frac{(x-7) - (x+4)}{(x+4)(x-7)} = \frac{11}{30}$

 $\Rightarrow \quad -(x+4)(x-7) = 30$

 $\Rightarrow \quad -(x^2 - 7x + 4x - 28) = 30$

 $\Rightarrow \quad -(x^2 - 3x - 28) = 30$

 $\Rightarrow \quad x^2 - 3x + 2 = 0$

Here, $a = 1, b = -3, c = 2$

So, $b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1$

 $\Rightarrow \quad 1>0$

 $\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

 $\Rightarrow \quad x = \frac{-(-3) \pm \sqrt{1}}{2(1)}$

 $\Rightarrow \quad x = \frac{3 \pm 1}{2}$

 $\Rightarrow \quad x = 2, 1$

22. Let the present age of the son be *x* years.

Three years hence, age of the son = (x+3) years.

:. Three years hence, man's age = 4(x+3) years

 \therefore Present age of man = [4(x+3)-3] years = (4x+9) years

Two years ago, man's age = (4x+9-2) years = (4x+7) years

And son's age = (x-2) years

According to the question,

	$4x + 7 = 3(x - 2)^2$	\Rightarrow	$4x + 7 = 3x^2 - 12x + 12$
\Rightarrow	$3x^2 - 16x + 5 = 0$	\Rightarrow	$3x^2 - 15x - x + 5 = 0$
\Rightarrow	3x(x-5)-1(x-5)=0	\Rightarrow	(x-5)(3x-1)=0
\Rightarrow	$x = 5, \frac{1}{3}$		
\therefore	$x = \frac{1}{3}$ is inadmissible.		
. .	x = 5 and $4x + 9 = 29$		
Hence, the present ages of the man and his son are 29 years and 5 years respectively.			

23. (i) Two digit numbers between 6 and 102 which are divisible by 6 are 12, 18, 24,, 96 Which forms an AP, whose first term a = 12

Common difference (d) = 18 - 12 = 6

Last term (l) = 96

Let total number of two digit numbers between 6 and 102, is *n*.

 $\therefore \qquad l = a + (n-1)d$ $\Rightarrow \qquad 96 = 12 + (n-1)6 \qquad \Rightarrow \qquad (n-1)6 = 96 - 12$ $\Rightarrow \qquad (n-1)6 = 84 \qquad \Rightarrow \qquad n-1 = \frac{84}{6} = 14$ $\Rightarrow \qquad n = 15$

Hence, there are 15 numbers between 6 and 102 which are divisible by 6.

(ii) Ram calculated it by using AP, so time saving and seasoning are depicted by Ram.

24. \because Tangent segments from an external point to a circle are equal in length. \therefore BD = BE = 8 cm CF = CE = 6 cm Let AD = AF = x cm

:
$$s = \frac{AB + BC + CA}{2}$$

= $\frac{(x+8) + (8+6) + (x+6)}{2}$
= $(x+14)$ cm



$$\therefore \quad \text{Area of } \Delta \text{ABC}$$

$$= \text{Area of } \Delta \text{OBC} + \text{Area of } \Delta \text{OCA} + \text{Area of } \Delta \text{OAB}$$

$$\Rightarrow \quad \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2} \times \text{BC} \times \text{OE} + \frac{1}{2} \times \text{CA} \times \text{OF} + \frac{1}{2} \times \text{AB} \times \text{OD}$$

$$\Rightarrow \quad \sqrt{(x+14)\{(x+14)-14\}\{(x+14)-(x+6)\}\{(x+14)-(x+8)\}}}$$

$$= \frac{1}{2} \times (8+6) \times 4 + \frac{1}{2} \times (x+6) \times 4 + \frac{1}{2} \times (x+8) \times 4}$$

$$\Rightarrow \quad \sqrt{(x+14)(x)(8)(6)} = 28 + 2x + 12 + 2x + 16}$$

$$\Rightarrow \quad x=7$$

$$\therefore \quad \text{AB} = 15 \text{ cm}, \text{ BC} = 14 \text{ cm and } \text{AC} = 13 \text{ cm}$$

25. Let
$$AB = 9 \text{ cm}$$
, $BC = 7 \text{ cm}$ and $CA = 6 \text{ cm}$
Then, $x + y = 9$ (i)
 $y + z = 7$ (ii)
 $x + z = 6$ (iii)
Subtracting eq. (ii) from eq. (i), we get,
 $x - z = 2$ (iv)
Solving eq. (iv) and eq. (iii), we get,
 $x = 4, z = 2$
Putting the value of *x* in eq. (i), we get,
 $y = 5$



:. x = 4, y = 5, z = 2

26. Steps of construction:

(a) Draw a quadrilateral ABCD.

(b) Draw any ray AX making an acute angle with AB.

(c) Locate 5 points X_1 , X_2 , X_3 , X_4 , X_5 on AX so that $AX_1 = X_1X_2 = X_2X_3 = X_3X_4 = X_4X_5$.

(d) Join X_5B and draw a line B' X_4 parallel to X_5B .

(e) Draw a line B'C' parallel to BC and C'D' parallel to CD.

Then AB'C'D' is the required quadrilateral.



27. In right triangle ABC,





D

Height of the tree = BC + AC

$$= \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}} = \frac{30}{\sqrt{3}}$$
$$= 10\sqrt{3} = 17.32 \text{ m}$$

28. Number of all possible outcomes = 19

- (i) Prime numbers are 2, 3, 5, 7, 11, 13, 17 & 19
 - \therefore Number of favourable outcomes = 8

$$\therefore$$
 Required probability = $\frac{8}{19}$

(ii) Numbers divisible by 3 or 5 are 3, 5, 6, 9, 10, 12, 15, 18

- \therefore Number of favourable outcomes = 8
- \therefore Required probability = $\frac{8}{19}$
- (iii) Number neither divisible by 5 nor by 10 are 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18 & 19.
 - \therefore Number of favourable outcomes = 16
 - \therefore Required probability = $\frac{16}{19}$
- (iv) Even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18
 ∴ Number of favourable outcomes = 9
 - \therefore Required probability = $\frac{9}{19}$

29. SP =
$$\sqrt{(at^2 - a)^2 + (2at - 0)^2}$$

= $\sqrt{a^2t^4 + a^2 - 2a^2t^2 + 4a^2t^2}$ = $a(t^2 + 1)$
SQ = $\sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{-2a}{t} - 0\right)^2}$
= $\sqrt{\frac{a^2}{t^4} + a^2 - \frac{2a^2}{t^2} + \frac{4a^2}{t^2}}$ = $a\left(\frac{1 + t^2}{t^2}\right)$
Now $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(t^2 + 1)} + \frac{1}{\frac{a(1 + t^2)}{t^2}}$
= $\frac{1}{a(t^2 + 1)} + \frac{t^2}{a(1 + t^2)} = \frac{(1 + t^2)}{a(a + t^2)} = \frac{1}{a}$

30. For cylindrical vessel

Internal diameter = 10 cm

Internal radius
$$(r) = \frac{10}{2} = 5$$
 cm

Height (h) = 10.5 cm

Volume of water = Volume of cylindrical vessel = $\pi r^2 h$

$$= \frac{22}{7} \times 5 \times 5 \times 10.5 = 825 \text{ cm}^3$$

For solid cone

Base diameter = 7 cm

Base radius (R) = $\frac{7}{2}$ cm

Volume of solid cone = $\frac{1}{3}\pi R^2 H$ = $\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 6$ = 77 cm³

- (i) Water displaced out of the cylindrical vessel = Volume of the solid cone = 77 cm^3
- (ii) Water left in the cylindrical vessel

= Volume of cylindrical vessel - Volume of solid cone

= 825 - 77 = 748 cm³

31. For cylinder

r = 6 cm h = 8 cmVolume = $\pi r^2 h = \pi (6)^2 (8) = 288\pi \text{ cm}^3$

For cone

R = 6 cm H = 8 cm

Volume = $\frac{1}{3}\pi R^2 H = \frac{1}{3}\pi (6)^2 (8) = 96\pi \text{ cm}^3$

Volume of remaining solid = Volume of cylinder – Volume of cone

$$= 288\pi - 96\pi = 192\pi$$
$$= 192 \times 3.1416 = 603.1872 \text{ cm}^3$$

Curved Surface area of the cylinder = $2\pi rh$

$$= 2\pi(6)(8) = 96\pi \text{ cm}^2$$

Area of the base of cylinder = $\pi r^2 = \pi (6)^2 = 36\pi \text{ cm}^2$

Slant height of the cone (L) = $\sqrt{R^2 + H^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ cm}$ Curved surface area of the cone = $\pi RL = \pi(6)(10) = 60\pi \text{ cm}^2$

Now, total surface area of the remaining solid

= Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of the cone

 $= 96\pi + 36\pi + 60\pi = 192\pi$

= 192 x 3.1416

= 603.1872 cm²

