Sample Paper – 4
Mathematics
Class XI Session 2022-23

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A

(Multiple Choice Questions)
Each question carries 1 mark.

1. If A = {1, 2, 3}, B = {1, 4, 6, 9} and R is a relation from A to B defined by x is less than y. The range of R is:
(a) {1, 2, 3} (b) {4, 6, 9}
(c) {13} (d) None of these 1

2. $(\sqrt{-4})(\sqrt{-2})$ is equal to:
(a) $\sqrt{8}$ (b) $-\sqrt{8}$
(c) $i\sqrt{8}$ (d) None of these 1

3. The largest coeffcient in the expansion of $(a+b)^{18}$ is:
(a) $^{18}C_{18}$ (b) $^{18}C_{12}$
(c) $^{18}C_{9}$ (d) $^{18}C_{6}$ 1

4. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$ is:
(a) $\frac{3}{4}$ (b) $\frac{7}{5}$
(c) $\frac{2}{5}$ (d) $\frac{3}{5}$ 1

5. Let S be an infinite set and $S_1, S_2, S_3, \ldots, S_n$ be sets such that $S_1 \cup S_2 \cup S_3 \cup S_n = S$ then
(a) at least one of the sets S_i is a finite set
(b) not more than one of the set S_i can be infinite
(c) at least one of the sets S_i is an infinite set
(d) none of these 1

6. The mean deviation from the mean of the set of observations –1, 0 and 4 is:
(a) 3 (b) 1
(c) –2 (d) 2 1

7. If $(a + ib)^{1/3} = p + iq$, then $\frac{a}{p} + \frac{b}{q}$ is:
(a) 0 (b) 1
(c) –1 (d) None of these 1

8. Let $f(x) = \begin{cases} 3-2x, & x<1 \\ 3x+5, 1 & x \le 2, \\ 5x-2, 3x & x \ge 3 \end{cases}$
then $2f(0) + f(3) =$
(a) 17 (b) 19
(c) 24 (d) 31 1

9. Let L, M, N be the feet of the perpendiculars drawn from a point P(7.2, 9.3, 3.4) on the x, y and z-axes respectively. Find the coordinates of L, M and N respectively.
(a) (7.2, 0, 0), (0, 9.3, 0), (0, 0, 3.4)
(b) (7, 0, 0), (0, 0, 9), (0, 4, 0)
(c) (0, 7, 0), (0, 0, 9), (4, 0, 0)
(d) (0, 0, 7), (0, 9, 0), (4, 0, 0) 1

10. Given the integers $r > 1$, $n > 2$, and coefficient of $(3r)^{th}$ and $(r+2)^{nd}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal, then:
(a) $n = 2r$ (b) $n = 3r$
(c) $n = 2r + 1$ (d) None of these 1

11. The symmetric difference of A = {1, 2, 3} and B = {3, 4, 5} is:
(a) {1, 2} (b) {1, 2, 4, 5}
(c) {4, 3} (d) {2, 5, 1, 4, 3} 1

12. If $-8 \le 5x - 3 < 7$, then $x \in$:
(a) $(-1, 2)$ (b) $[-1, 2)$
(c) $[-2, \infty)$ (c) $[-2, 0)$ 1

13. A pair of dice is rolled, if the outcome is such that the sum of numbers is 12, a coin is tossed. Then, the total number of outcomes for this experiment is:

(a) 40 (b) 37

(c) 41 (d) 43 1

14. The angle of a triangle are in A.P. and the ratio of angle in degree of the least to the angle in radians of the greatest is 60 : π, find the angles in degrees.

(a) 30°, 60°, 90° (b) 40°, 50°, 90°

(c) 30°, 30°, 120° (d) 20°, 130°, 30° 1

15. 18^{th} term from the end of the sequence 3, 6, 12,...25 terms is:

(a) 393216 (b) 393206

(c) 313216 (d) 303216 1

16. Consider the following data, the mean deviation about median for the data is:

r_n	15	21	27	30	35
f_i	3	5	6	7	8

(a) 5 (b) 5.3

(c) 5.1 (d) 5.2 1

17. If S = { x | x is a positive multiple of 3 less than 100} and P ={ x | x is a prime number less than 20}. Then, n(S) + n(P) is equal to:

(a) 34 (b) 31

(c) 33 (d) 41 1

18. The number of six digit numbers whose all digits are odd is:

(a) 6^5 (b) 5^6

(c) $\frac{6!}{2!}$ (d) None of these 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. Assertion (A): The foot of perpendicular drawn from the point A(1, 2, 8) on the xy-plane is (1, 2, 0).

Reason (R): Equation of xy-plane is y = 0. 1

20. Assertion (A): A coin is tossed and then a die is rolled only in case a head is shown on the coin. The sample space for the experiment is

S = {H1, H2, H3, H4, H5, H6, T}.

Reason (R): 2 boys and 2 girls are in room X, and 1 boy and 3 girls are in room Y. Then, the sample space for the experiment in which a room is selected and then a person, is

S = {XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5}.

where B_i, denote the boys and G_j, denote the girls. 1

SECTION - B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. Find the value of tan 22° 30′.

OR

Convert following radian measure into degree measures $\frac{-2}{9}$. 2

22. Find $x^2 + 4ax + 4 > 0$ for all x. 2

23. Evaluate $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$ 2

24. Expand $\left(y^2 + \frac{2}{y}\right)^5, y \neq 0.$

OR

Which is larger $(1.01)^{1000000}$ or 10,000. 2

25. Find the sum to n terms of the G.P., whose k^{th} term is 5^k. 2

SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x – 1}. Write R in roster form. Write down the domain, co-domain and range of R. 3

27. A boy has 4 movie tickets and 9 movie of his interest in the theater. Of these 9, he does not want to see Marvels Part II, unless Marvels Part I is also seen. In how many ways can he choose the 4 movies to be seen?

OR

Evaluate:

(A) $^{6}C_2$ (B) $^{40}C_{40}$

(C) $^{13}C_3$ 3

28. Evaluate: $\lim_{x \to 0}\left[\dfrac{\sin 5x - \sin 3x}{\tan 7x}\right]$. 3

29. Six new employees, two to whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?

OR

A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that:

(A) all the three balls are white.

(B) all the three balls are red.

(C) one ball is red and two balls are white. 3

30. If $\dfrac{2\sin a}{1+\cos a+\sin a} = y$, then prove that $\dfrac{1-\cos a+\sin a}{1+\sin a}$ is also equal to y. 3

31. Find the derivative of $\dfrac{3x+4}{5x^2-7x+9}$.

OR

Find the derivation of $\sqrt{\dfrac{1-\cos x}{1+\cos x}}$. 3

SECTION - D

(This section comprises of long answer-type questions (LA) of 5 marks each.)

32. Prove that $\cos\theta\cos\dfrac{\theta}{2} - \cos 3\theta\cos\dfrac{9\theta}{2} = \sin 4\theta\sin\dfrac{7\theta}{2}$. 5

33. If $x + iy = \sqrt{\dfrac{p+iq}{a+ib}}$, prove that $(x^2 + y^2)^2 = \dfrac{p^2+q^2}{a^2+b^2}$. 5

34. Find the equation of the parabola whose vertex is the point (0, 2) and the directrix is the line $x + y - 3 = 0$.

OR

Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant. 5

35. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test:

Marks	0	1	2	3	4	5
Frequency	$x-2$	x	x^2	$(x+1)^2$	$2x$	$x+1$

Where x is a positive integer. Determine the mean and standard deviation of the marks.

OR

Determine mean and standard deviation of first n terms of an A.P. whose first term is a and common difference is d. 5

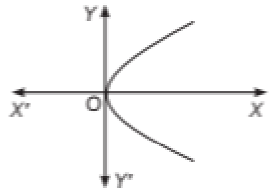
SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (A), (B), (C) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:**

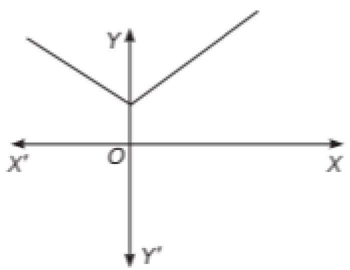
A relation from a non-empty set A to a non-empty set B is said to be a function if every element of set A has one and only one image in set B. In other words, we can say that a function f is a relation from a non-empty set A *to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element or component. If f is a function from a set A to a set B, then we write $f: A \rightarrow B$ and it is read as f is a function from A to B or f maps A to B.*

(A) Justify that the given curve represents a relation.



1

(B) Justify that the given curve represents a function.



1

(C) If $f(x) = x^2 + 2x + 3$, then among $f(1)$, $f(2)$ and $f(3)$, which one gives the maximum value.

OR

If $f(1+x) = x^2 + 1$, then find the value of $f(2 - h)$. 2

37. Case-Study 2:

A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant. Rahul, being a plant lover, decides to open a nursery and he bought few plants and pots. He wants to place pots in such a way that the number of pots in the first row is 2, in the second row is 4, in the third row is 8 and so on....

(A) What the constant multiple by which the number of pots is increasing in every row? 1

(B) Find the difference in number of pots placed in 7^{th} row and 5^{th} row. 1

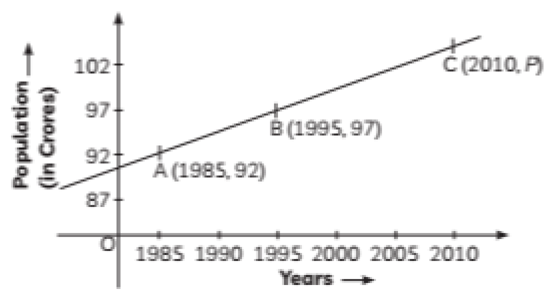
(C) Find the total number of pots upto 10^{th} row.

OR

If Rahul wants to place 510 pots in total, then how many the total number of rows formed in this arrangement? 2

38. Case-Study 3:

India faces a major population crisis due to the growing population. If we ware to estimate, we can say that almost 17% of the population of the world lives in India alone. India ranks second in the list of most populated countries. Population vs year graph given below:



Based on the above information, answer the following questions.

(A) Find the population in the year 2010 (in crores). 2

(B) Find the equation of line perpendicular to line AB and passing through (1995, 97). 2

SOLUTION

SECTION - A

1. (b) {4, 6, 9}

Explanation: The relation from A to B is x is less than y.

Hence, Range is {4, 6, 9}

2. (b) $-\sqrt{8}$

Explanation: $(\sqrt{-4})(-\sqrt{2}) = i\sqrt{4}.i\sqrt{2}$

$= i^2\sqrt{8} = -\sqrt{8}$

3. *(c)* $^{18}C_9$

Explanation: We know that if n is even then ^nC_r is greatest for $r = \frac{n}{2}$. Therefore $^{18}C_r$ is greatest for $r = 9$. Hence, the greatest coefficient is $^{18}C_9$.

4. *(c)* $\frac{2}{5}$

Explanation: Given that,

$$a = 3, b = -4, c_1 = 7, c_2 = 5$$

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|7-5|}{\sqrt{(3)^2 + (-4)^2}}$$

$$= \frac{2}{\sqrt{9+16}}$$

$$= \frac{2}{\sqrt{25}}$$

$$= \frac{2}{5}.$$

5. *(c) at least one of the sets S_i is an infinite set*

Explanation: $S_1 \cup S_2 \cup S_3 \cup \cdots \cup S_n$. For S to be an infinite set, at least one of sets S_i must be infinite, since if all S_i were finite, then S will also be finite.

6. *(d)* 2

Explanation: $\bar{x} = \frac{-1+0+4}{3} = 1$

$$\text{M.D } (\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}$$

$$= \frac{|-1-1| + |0-1| + |4-1|}{3} = 2$$

7. *(d) None of these*

Explanation: $a + ib = (p + iq)^3$

$\Rightarrow \quad a + ib = p^3 + (iq)^3 + 3\,p^2 iq + 3\,p(iq)^2$

$= p^3 - iq^3 + 3\,p^2 qi - 3\,pq^2$

$\Rightarrow \quad a + ib = (p^3 - 3\,pq^2) + i\,(3\,p^2 q - q^3)$

Thus, $\quad a = p^3 - 3\,pq^2,\ b = 3\,p^2 q - q^3$

So,

$$\frac{a}{p} + \frac{b}{q} = \frac{p(p^2 - 3q^2)}{p} + \frac{q(3p^2 - q^2)}{q}$$

$$= p^2 - 3\,q^2 + 3p^2 - q^2$$

$$= 4\,(p^2 - q^2)$$

8. *(b)* 19

Explanation: Here,

$$f(0) = 3 - 2\,(0) = 3$$

$$f(3) = 5 \times 3 - 2 = 13$$

So,

$$2\,f(0) + f(3) = 2 \times 3 + 13$$

$$= 6 + 13$$

$$= 19$$

9. *(a) (7.2, 0, 0), (0, 9.3, 0), (0, 0, 3.4)*

Explanation: Since L is the foot of perpendicular from P on the x-axis, so its y and z-coordinates are zero. So, the coordinates of L is (7.2, 0, 0). Similarly, the coordinates of M and N are (0, 9.3, 0) and (0, 0, 3.4), respectively.

10. *(a)* $n = 2r$

Explanation: Given $(1 + x)^{2n}$

$T_{3r} = T_{(3r-1)+1} = {}^{2n}C_{3r-1}\, x^{3r-1}$

and $\quad T_{r+2} = T_{(r+1)+1} = {}^{2n}C_{r+1}\, x^{r+1}$

Given, $\quad {}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow \quad 3r - 1 + r + 1 = 2n$

$[\because \ {}^nC_x = {}^nC_y \Rightarrow x + y = n]$

Or, $\quad n = 2r$

11. *(b)* {1, 2, 4, 5}

Explanation: $A \Delta B = (A - B) \cup (B - A)$

$A - B = \{1, 2\}$

$B - A = \{4, 5\}$

So, $(A - B) \cup (B - A) = \{1, 2, 4, 5\}$

12. *(b)* $[-1, 2)$

Explanation: We have, $-8 \le 5x - 3 < 7$

$\Rightarrow \quad -5 \le 5x < 10$

$\Rightarrow \quad -1 \le x < 2 \therefore x \in [-1, 2)$

13. *(b)* 37

Explanation: The sample space associated with the given random experiment is

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6,H), (6,6,T)}

Hence, total number of sample points =37

14. *(a)* $30°, 60°, 90°$

Explanation: Let the angles of the triangle be $(a - d)°, a°, (a + d)°$, where $d \ge 0$ ...(i)

Then $(a - d) + a + (a + d) = 180° \Rightarrow a = 60$

$\therefore$ From (i), the angles are $(60 - d)°, 60°, (60 + d)°$

Now, the least angle = $(60 - d)°$

And the greatest angle = $(60 + d)°$

$= (60 + d)^\circ \times \frac{\pi}{180}$ radian

By the given condition, we have

$$\frac{60-d}{\frac{\pi}{180}(60+d)} = \frac{60}{\pi}$$

$$\Rightarrow \frac{180(60-d)}{(60+d)} = 60$$

$\Rightarrow 180 - 3d = 60 + d$

$\Rightarrow 4d = 120 \Rightarrow d = 30$

$\therefore$ From (i), the angles are

$(60 - 30)^\circ, 60^\circ, (60 + 30)^\circ$ *i.e.*, $30^\circ, 60^\circ, 90^\circ$

15. *(a) 393216*

Explanation: Here $a = 3, r = 2$

$m = 18$ and $n = 25$

We know that if a sequence has n term then m^{th} term from end is equal to $(n - m + 1)$ term from beginning

$$a_8 = ar^{8-1} = 3(2)^{8-1}$$
$$= 3\,(2)^7$$
$$= 3 \times 128$$

$$= 384$$

16. *(c) 5.1*

Explanation:

x_i	f_i	c.f	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
15	3	3	$\lvert 15-30 \rvert = 15$	45
21	5	8	$\lvert 21-30 \rvert = 9$	45
27	6	14	$\lvert 27-30 \rvert = 3$	18
30	7	21	$\lvert 30-30 \rvert = 0$	0
35	8	29	$\lvert 35-30 \rvert = 5$	40
	29			148

N = 29 (odd)

c.f. = cumulative frequency

$$\therefore \quad \text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{term}$$
$$= \left(\frac{29+1}{2}\right)^{th} \text{term}$$
$$= \left(\frac{30}{2}\right)^{th} \text{term}$$
$$= 15^{th} \text{ term}$$

As, 15^{th} term lies in c.f of 21, so Median = 30.

$$\text{M.D (M)} = \frac{\sum f_i(x_i - M)}{\sum f_i}$$
$$= \frac{148}{29} = 5.1$$

17. *(d) 41*

Explanation: S = {x: x is a positive multiple of 3 less than 100}

$\Rightarrow$ S = { 3,6,9,12,. . . ,99}

$\Rightarrow n(S) = 33$

P = { x: x is a prime number less than 20}

$\Rightarrow$ P = {2,3,5,7,11,13,17,19}

$\Rightarrow n(P) = 8$

$\therefore n(S) + n(P) = 33 + 8 = 41$

18. *(b) 5^6*

Explanation: Since, six-digit numbers whose all digits are odd are to be formed, suggests that repetition of digits is a must as available digit are 1, 3, 5, 7 and 9.

Hence, the required number is 5^6.

19. *(c) A is true but R is false.*

Explanation: We know that in *xy*-plane, *z*-coordinte is 0. So, coordinate of foot of perpendicular drawn from point A(1, 2, 8) on *xy*-plane is (1, 2, 0).

Equation of *xy*-plane is $z = 0$

$\therefore$ Reason is wrong.

20. *(b) Both A and R are true but R is not the correct explanation of A.*

Explanation: The sample space is

$S = \{H^1, H^2, H^3, H^4, H^5, H^6, T\}$

Where, H and T represents head and tail respectively of a coin.

When the room is selected, then

X	Y
B_1, B_2 G_1, G_2	B_3 G_3, G_4, G_5

There are four possibilities for selection of a person from room x, which are B_1, B_2, G_1, G_2, similarly, there will be four possibilities for room Y.

So, the sample space is

$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$

SECTION - B

21. Let, $22° 30' = \frac{\theta}{2}$

$\therefore \theta = 45°$

$$\tan 22° 30' = \tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\theta}{1+\cos\theta}$$

$$\therefore = \frac{\sin 45°}{1+\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}+1}$$

$$= \frac{1\times(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sqrt{2}-1$$

Hence, $\tan 22° 30' = \sqrt{2}-1$

OR

$$\frac{-2}{9}\text{ radian} = \frac{-2}{9}\times\frac{180}{\pi}$$

$$= -\left(2\times\frac{20}{\pi}\right)$$

$$= -\left(2\times\frac{20\times 7}{22}\right)$$

$$= -\left(\frac{20\times 7}{11}\right)$$

$$= -\left(\frac{140}{11}\right) = -12.7272°$$

22. Given, $x^2 + 4ax + 4 > 0$

$\Rightarrow x^2 + 4ax + 4a^2 + 4 - 4a^2 > 0$

$\Rightarrow (x+2a)^2 + (4-4a^2) > 0$

$\Rightarrow 4 - 4a^2 > 0$

$\Rightarrow 4 > 4a^2$

$\Rightarrow 1 > a^2$

$\Rightarrow |a| < 1 \Rightarrow -1 < a < 1$

23. LHL at $x \to 0$

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0}\frac{|-h|}{-h}$$

$$= \lim_{h\to 0}\frac{h}{-h}$$

$$= \lim_{h\to 0} -1$$

$$= -1$$

RHL at $x \to 0$

$$D\lim_{h\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} f(h)$$

$$= \lim_{h\to 0}\frac{|h|}{h}$$

$$= \lim_{h\to 0}\frac{h}{h}$$

$$= \lim_{h\to 0} 1$$

$$= 1$$

Since Left Hand Limit (LHL) ≠ Right Hand Limit (RHL)

$\therefore \lim_{h\to 0} f(x)$ doesn't exist.

24. By using binomial theorem, we have

$$\left(y^2+\frac{2}{y}\right)^5$$

$$= {}^5C_0(y^2)^5 + {}^5C_1(y^2)^4\left(\frac{2}{y}\right)^1 + {}^5C_2(y^2)^3\left(\frac{2}{y}\right)^2$$

$$+ {}^5C_3(y^2)^2\left(\frac{2}{y}\right)^3 + {}^5C_4(y^2)^1\left(\frac{2}{y}\right)^4 + {}^5C_5(y^2)^0\left(\frac{2}{y}\right)^5$$

$$= y^{10} + 5y^8\left(\frac{2}{y}\right) + 10y^6\left(\frac{2}{y}\right)^2 + 10y^4\left(\frac{2}{y}\right)^3$$

$$+ 5y^2\left(\frac{2}{y}\right)^4 + \left(\frac{2}{y}\right)^5$$

$$= y^{10} + 10y^7 + 40y^4 + 80y + \frac{80}{y^2} + \frac{32}{y^5}$$

OR

$(1.01)^{1000000} = (1 + 0.01)^{1000000}$

$= {}^{1000000}C_0 + {}^{1000000}C_1\,(0.01)$

+ other positive terms

$= 1 + 1000000 \times 0.01 + \ldots$

$= 10001 + \ldots$

Hence, $(1.01)^{10,00000} > 10,000$.

25. Given that k^{th} term of a G.P. is 5^k.

$\therefore \quad a_k = 5^k$ ...(i)

Putting $k = 1$ in (1) we get

$a_1 = 5$ ...(2)

Putting $k = 2$ in (1), we get

$$a_2 = 25$$

Then, the first term is $a = a_1 = 5$ and the common ratio is $r = \frac{a_2}{a_1} = \frac{25}{5} = 5$

Hence, the sum of n terms of the G.P. is given by

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{(5)(5^n - 1)}{5 - 1} = \frac{5}{4}(5^n - 1)$$

SECTION - C

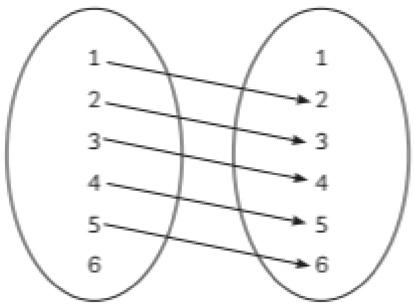
26. Given A = {1, 2, 3, 4, 5, 6} and

$R = \{(x, y) : y = x - 1\}$.

The given relation in roster form can be written as

R = {(2, 1),(3, 2), (4, 3), (5, 4), (6, 5)}

The given relation can be represented with the following arrow diagram:



So, domain of R = {1, 2, 3, 4, 5, 6}, co-domain of R = {0, 1, 2, 3, 4, 5} and range of R = {2, 3, 4, 5, 6}.

27. To choose 4 movies from 9 movies. Marvel's Part II is not to be seen, unless Marvels Part I is also seen.

Here, the order is not important.

So, each selection is a combination.

Case I: Marvels Part II is seen.

In this case, Marvels Part I is also seen.

Number of ways of selecting 2 movies from

$$7 \text{ movies} = {}^7C_2 = \frac{7!}{2!5!} = 21$$

Case II: Marvels Part II is not seen.

In this case, Marvels Part I may or may not be seen.

Number of ways of selecting 4 movies from

$$8 \text{ movies} = {}^8C_4 = \frac{8!}{4!4!} = 70$$

Hence, the required number of ways

$$= 21 + 70 = 91.$$

OR

(A)
$$^6C_2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2!4!} = \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} = 15$$

(B)
$$^{40}C_{40} = \frac{40!}{40!(40-40)!} = \frac{40!}{40!0!} = 1$$

(C)
$$^{13}C_3 = \frac{13!}{3!(13-3)!} = \frac{13!}{3!10!} = \frac{13 \times 12 \times 11 \times 10!}{3 \times 2 \times 1 \times 10!} = 286.$$

28. $$\lim_{x\to 0}\left[\frac{\sin 5x - \sin 3x}{\tan 7x}\right] = \lim_{x\to 0}\left[\frac{\frac{\sin 5x}{x} - \frac{\sin 3x}{x}}{\frac{\tan 7x}{x}}\right]$$

[on dividing numerator and denominator by x]

$$\lim_{x\to 0}\left[\frac{5\frac{\sin 5x}{5x} - 3\frac{\sin 3x}{3x}}{7\frac{\tan 7x}{7x}}\right]$$

$$= \frac{5\lim_{x\to 0}\left[\frac{\sin 5x}{5x}\right] - 3\lim_{x\to 0}\left[\frac{\sin 3x}{3x}\right]}{7\lim_{x\to 0}\left[\frac{\tan 7x}{7x}\right]}$$

$$= \frac{5\lim_{5x\to 0}\left[\frac{\sin 5x}{5x}\right] - 3\lim_{3x\to 0}\left[\frac{\sin 3x}{3x}\right]}{7\lim_{7x\to 0}\left[\frac{\tan 7x}{7x}\right]}$$

[$\because$ $5x \to 0, 3x \to 0, 7x \to 0$ as $x \to 0$]

$$= \frac{5(1) - 3(1)}{7(1)} = \frac{2}{7}$$

29. Let the couple occupied adjacent desks, consider those two as 1.

There are, (4 + 1) i.e., 5 persons to be assigned.

$\therefore$ Number of ways of assigning these five persons = 5! × 2!

Total number of ways of assigning 6 persons =6!

$\therefore$ Probability that the couple has adjacent desks $= \frac{5! \times 2!}{6!} = \frac{1}{3}$

Probability that the married couple will have non-adjacent desks $= 1 - \frac{1}{3} = \frac{2}{3}$

OR

$\because$ Number of red balls = 8

And number of white balls = 5

(A) P (all the three balls are white)

$$= \frac{^5C_3}{^{13}C_3} = \left(\frac{\frac{5\times4}{2}}{\frac{13\times12\times11}{3\times2}}\right)$$

$$= \frac{5\times4\times3}{13\times12\times11} = \frac{5}{143}$$

(B) P (all the three balls are red)

$$= \frac{^8C_3}{^{13}C_3} = \left(\frac{\frac{8\times7\times6}{3\times2\times1}}{\frac{13\times12\times11}{3\times2\times1}}\right)$$

$$= \frac{8\times7\times6}{13\times12\times11} = \frac{28}{143}$$

(C) P (one ball is red and two balls are white)

$$= \frac{^8C_1\times{}^5C_2}{^{13}C_3} = \frac{8\times10}{13\times2\times11} = \frac{40}{143}$$

30. Given, $\frac{2\sin a}{1+\cos a+\sin a} = y$

Now, $\frac{1-\cos a+\sin a}{1+\sin a}$

$$= \frac{(1-\cos a+\sin a)}{1+\sin a}\cdot\frac{(1+\cos a+\sin a)}{(1+\cos a+\sin a)}$$

$$= \frac{\{(1+\sin a)-\cos a\}}{1+\sin a}\cdot\frac{\{(1+\sin a)+\cos a\}}{1+\cos a+\sin a}$$

$$= \frac{(1+\sin a)^2-\cos^2 a}{(1+\sin a)(1+\cos a+\sin a)}$$

$$= \frac{(1+\sin^2 a+2\sin a)-\cos^2 a}{(1+\sin a)(1+\cos a+\sin a)}$$

$$= \frac{(1+\sin^2 a+2\sin a)-(1-\sin^2 a)}{(1+\sin a)(1+\cos a+\sin a)}$$

$$= \frac{2\sin^2 a+2\sin a}{(1+\sin a)(1+\cos a+\sin a)}$$

$$= \frac{2\sin a(\sin a+1)}{(1+\sin a)(1+\cos a+\sin a)}$$

$$= \frac{2\sin a}{1+\cos a+\sin a} = y$$

Hence, proved.

31. Let $y = \frac{3x+4}{5x^2-7x+9}$

Applying quotient rule of differentiation

$$\frac{d}{dx}\left(\frac{t}{s}\right) = \frac{s\cdot\frac{dt}{dx}-t\cdot\frac{ds}{dx}}{s^2}$$

Applying the rule

$$\Rightarrow \frac{dy}{dx} = \frac{(5x^2-7x+9)\frac{d}{dx}(3x+4)-(3x+4)\frac{d}{dx}(5x^2-7x+9)}{(5x^2-7x+9)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3(5x^2-7x+9)-(3x+4)(10x-7)}{(5x^2-7x+9)^2}$$

$$= \frac{(15x^2-21x+27-30x^2+21x-40x+28)}{(5x^2-7x+9)^2}$$

$$= \frac{(-15x^2-40x+55)}{(5x^2-7x+9)^2}$$

$$= \frac{(55-40x-15x^2)}{(5x^2-7x+9)^2}$$

OR

Let $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

$$\Rightarrow y = \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$\left(\because 1-\cos x = 2\sin^2\frac{x}{2},\ 1+\cos x = 2\cos^2\frac{x}{2}\right)$

$$\Rightarrow y = \sqrt{\tan^2\frac{x}{2}}$$

$$\Rightarrow y = \tan\frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \sec^2\frac{x}{2}\cdot\left(\frac{1}{2}\right).$$

SECTION - D

32. L.H.S. $= \cos\theta\cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2}$

$$= \frac{1}{2}\left[2\cos\theta\cos\frac{\theta}{2} - 2\cos 3\theta\cos\frac{9\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(\theta+\frac{\theta}{2}\right)+\cos\left(\theta-\frac{\theta}{2}\right)-\left\{\cos\left(3\theta+\frac{9\theta}{2}\right)+\cos\left(3\theta-\frac{9\theta}{2}\right)\right\}\right]$$

[$\because$ 2 cos A cos B = cos (A + B) + cos (A – B)]

$$= \frac{1}{2}\left(\cos\frac{3\theta}{2}+\cos\frac{\theta}{2}-\cos\frac{15\theta}{2}-\cos\frac{3\theta}{2}\right)$$

$$= \frac{1}{2}\left(\cos\frac{\theta}{2}-\cos\frac{15\theta}{2}\right)$$

$$= -\frac{1}{2}\left[2\sin\left(\frac{\theta+15\theta}{2}\right)\sin\left(\frac{2-152}{2}\right)\right]$$

$$\left[\because \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$$

$= -\{\sin 4\theta \sin(-\frac{7\theta}{2})\}$

$= \sin 4\theta.\ \sin\frac{7\theta}{2}$ [$\because$ sin (– θ) = – sin θ]

= R.H.S.

Hence, proved.

33. Given, $x + iy = \sqrt{\frac{p+iq}{a+ib}}$

$\Rightarrow (x+iy)^2 = \left(\frac{p+iq}{a+ib}\right)$ [on squaring both sides]

$\Rightarrow |(x+iy)|^2 = \left|\frac{p+iq}{a+ib}\right|$

[on taking modulus on both sides]

$\Rightarrow |\sqrt{x^2+(y)^2}|^2$

$= \frac{|p+iq|}{|a+ib|}$

$\Rightarrow (x^2+y^2)^2 = \frac{p^2+q^2}{a^2+b^2}$ [on squaring both sides]

⚠ Caution

➥ $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, *where a, b are two non-zero complex numbers.*

Hence proved.

34. Given, vertex of parabola is (0, 2) and directrix of parabola is

$$x + y - 3 = 0 \quad ...(i)$$

Then, slope of directrix = –1 = m_1 (say)

Since, axis of parabola is perpendicular to the directrix.

So, slope of axis $= -\frac{1}{m_1} = 1 = m_2$ (say)

Then, the equation of axis is of the form

$y = m_2x + c$, i.e., $y = x + c$.

Since, axis passes through the vertex (0, 2).

$\therefore \quad 2 = 0 + c$

$\Rightarrow \quad c = 2$

So, the equation of axis of parabola is

$$y = x + 2. \quad ...(ii)$$

On solving (i) and (ii), we get $x = 1/2, y = 5/2$

The point of intersection of the directrix and axis is D(1/2, 5/2).

Let, F(α, β) be the coordinates of the focus.

The mid-point of FD is $\left(\frac{\alpha+1/2}{2}, \frac{\beta+5/2}{2}\right)$

We know that the vertex V is the mid-point of FD.

$\therefore \frac{\alpha+1/2}{2}=0$ and $\frac{\beta+5/2}{2}=2$ i.e., $\alpha = -1/2$ and $\beta = 3/2$.

So, the coordinates of focus of parabola are (–1/2, 3/2).

Let, P(x, y) be any point on the parabola.

Then, by definition of parabola, we have

Distance of P(x, y) from (–1/2, 3/2) = Perpendicular distance of P(x, y) from $x + y - 3 = 0$

$$\Rightarrow \sqrt{(x+1/2)^2+(y-3/2)^2} = \left|\frac{x+y-3}{\sqrt{(1)^2+(1)^2}}\right|$$

$$\Rightarrow \sqrt{(x+1/2)^2+(y-3/2)^2} = \left|\frac{x+y-3}{\sqrt{2}}\right|$$

$$\Rightarrow (x+1/2)^2+(y-3/2)^2 = \frac{(x+y-3)^2}{2}$$

$$\Rightarrow 2(x^2 + x + 1/4 + y^2 - 3y + 9/4) = x^2 + y^2 + 9 + 2xy - 6x - 6y$$

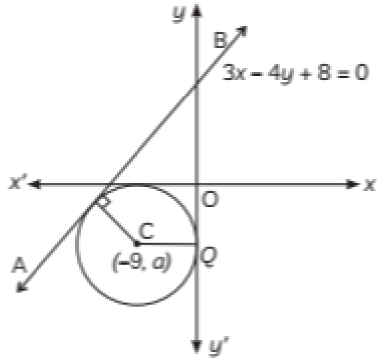
$$\Rightarrow x^2 - 2xy + y^2 + 8x - 4 = 0$$

Which is the required equation of parabola.

OR

Let a be the radius of the circle. Then, the coordinates of centre of the circle are $(-a, -a)$.

Now, perpendicular distance from C to the line AB = Radius of the circle.



$$d = \left|\frac{-3a+4a+8}{\sqrt{9+16}}\right| = \left|\frac{a+8}{5}\right|$$

$\Rightarrow \quad a = \left|\dfrac{a+8}{5}\right|$

$\Rightarrow \quad a = \dfrac{a+8}{5}$

$\Rightarrow \quad 5a = a + 8$

$\Rightarrow \quad 4a = 8$

$\Rightarrow \quad a = 2$

Or $\quad a = \dfrac{-(a+8)}{5}$

$\Rightarrow \quad 5a = -a - 8$

$\Rightarrow \quad 6a = -8$

$\Rightarrow \quad a = -8/6$

But $\quad a \neq -8/6$

($\because$ Radius can't be negative)

So, the equation of circle is

$$(x+2)^2 + (y+2)^2 = 2^2$$

$\Rightarrow \quad x^2 + 4x + 4 + y^2 + 4y + 4 = 4$

$\Rightarrow \quad x^2 + y^2 + 4x + 4y + 4 = 0$

35. Given: There are 60 students in a class. The frequency distribution of the marks obtained by the students in a test is also given.

Now, we have to find the mean and standard deviation of the marks.

It is given there are 60 students in the class, so

$\Sigma f_i = 60$

$\Rightarrow (x-2) + x + x^2 + (z+1)^2 + 2x + x + 1 = 60$

$\Rightarrow 5x - 1 + x^2 + x^2 + 2x + 1 = 60$

$\Rightarrow 2x^2 + 7x = 60$

$2x^2 + 7x - 60 = 0$

Splitting the middle term, we get

$\Rightarrow 2x^2 + 15x - 8x - 60 = 0$

$\Rightarrow x(2x + 15) - 4(2x + 15) = 0$

$\Rightarrow (2x + 15)(x - 4) = 0$

$\Rightarrow 2x + 15 = 0$ or $x - 4 = 0$

$\Rightarrow 2x = -15$ or $x = 4$

Given x is a positive number, so x can take 4 as the only value.

And let assumed mean, $a = 3$

Now put $x = 4$ and $a = 3$ in the frequency distribution table and add other columns after calculation, we get

Marks (x_i)	Frequency (f_i)	$d_i = x_i - a$	$f_i d_i$	$f_i d_i^2$
0	$x - 2 = 2$	−3	−6	18
1	$x = 4$	−2	−8	16
2	$x^2 = 4^2 = 16$	−1	−16	16
3	$(x+1)^2 = 25$	0	0	0
4	$2x = 8$	1	8	8
5	$x + 1 = 5$	2	10	20
Total	**60**		**−12**	**78**

And we know standard deviation is

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

Substituting values from the above table, we get

$$\sigma = \sqrt{\frac{78}{60} - \left(\frac{-12}{60}\right)^2}$$

$\Rightarrow \sigma = \sqrt{1.3 - 0.04}$

$\Rightarrow \sigma = 1.26$

Hence, the standard deviation is 1.26.

Now mean is

$$\bar{x} = A + \frac{\Sigma f_i d_i}{N} = 3 + \left(-\frac{12}{60}\right)$$

$$= 3 - \frac{1}{5} = \frac{14}{5} = 2.8$$

Hence, the mean and standard deviation of the marks are 2.8 and 1.26 respectively.

OR

Given first n terms of an A.P. whose first term is a and common difference is d.

Now, we have to find mean and standard deviation.

The given A.P. in tabular form is as shown below,

x_i	$d_i = x_i - a$	d_i^2
a	0	0
$a+d$	d	d^2
$a+2d$	$2d$	$4d^2$
$a+3d$	$3d$	$9d^2$
⋮	⋮	⋮
$a+(n-1)d$	$(n-1)d$	$(n-1)^2d^2$

Here we have assumed a as mean.

Given the A.P. have n terms, and we know the sum of all the terms of AP can be written as

$$\Sigma x_i = \frac{n}{2}[2a+(n-1)d]$$

Now we will calculate the actual mean,

$$\bar{x} = \frac{\Sigma x_i}{n}$$

Substituting the corresponding values, we get

$$\bar{x} = \frac{\frac{n}{2}[2a+(n-1)d]}{n}$$

The above equation can be written as

$$\bar{x} = \frac{[2a+(n-1)d]}{2}$$

$$\Rightarrow \bar{x} = a+\frac{[(n-1)d]}{2}$$

$$\Rightarrow \bar{x} = a+\frac{(n-1)}{2}d$$

We also have,

$$\Sigma d_i = \Sigma(x_i - a) = d[1+2+3+\ldots+(n-1)]$$

$$= d\left(\frac{n(n-1)}{2}\right)$$

Also, $\Sigma d_i^2 = d^2\,[1^2+2^2+3^2+\ldots+(n-1)^2]$

$$= d^2\left(\frac{n(n-1)(2n-1)}{6}\right)$$

Now we know standard deviation is given by

$$\sigma = \sqrt{\frac{\sum(x_i-a)^2}{n} - \left(\frac{\sum(x_i-a)}{n}\right)^2}$$

Substituting the corresponding values, we get

$$\sigma = \sqrt{\frac{d^2\left(\frac{n(n-1)(2n-1)}{6}\right)}{n} - \left(\frac{d\left(\frac{n(n-1)}{2}\right)}{n}\right)^2}$$

$$\Rightarrow \sigma = \sqrt{d^2\left(\frac{n(n-1)(2n-1)}{6n}\right) - d^2\left(\frac{n^2(n-1)^2}{4n^2}\right)}$$

$$\Rightarrow \sigma = \sqrt{d^2\left(\frac{(n-1)(2n-1)}{6}\right) - d^2\left(\frac{(n-1)^2}{4}\right)}$$

Taking out common terms we get

$$\sigma = \sqrt{\left(\frac{d^2(n-1)}{2}\right)\left(\frac{(2n-1)}{3} - \frac{n-1}{2}\right)}$$

By taking the LCM, we get

$$\sigma = \sqrt{\left(\frac{d^2(n-1)}{2}\right)\left(\frac{2(2n-1)-3(n-1)}{6}\right)}$$

$$\Rightarrow \sigma = \sqrt{\left(\frac{d^2(n-1)}{2}\right)\left(\frac{n+1}{6}\right)}$$

$$\Rightarrow \sigma = d\sqrt{\frac{(n^2-1)}{12}}$$

Hence, the mean and standard deviation of the given A.P. is $a+\frac{(n-1)}{2}d$ and $d\sqrt{\frac{(n^2-1)}{12}}$ respectively.

SECTION - E

36. (A) If we draw a vertical line, then it will intersect the curve at two points. It shows that a given curve is a relation.

(B) If we draw a vertical line, then it will intersect the curve at only one point. It shows that a given curve is a function.

(C) $f(1) = 1+2+3 = 6$, $f(2) = 4+4+3 = 11$ and $f(3) = 9+6+3 = 18$. Here 18 is the maximum value. So, $f(3)$ gives the maximum value.

OR

We have, $f(1+x) = x^2+1$ ...(i)

On substituting $x = (1-h)$ in eq. (i), we get

$$f(1+1-h) = (1-h)^2+1$$

$$\Rightarrow f(2-h) = 1+h^2-2h+1 = h^2-2h+2$$

37. (A) The number of pots in the successive rows are 2, 4, 8... .

∴ This forms a geometric progression,

where $a = 2$, $r = \frac{4}{2} = 2$

Hence, the constant multiple by which the number of pots is increasing in every row is 2.

(B) Number of pots in 7^{th} row,

$$a_7 = 2(2)^{7-1} = 2 \cdot 2^6 = 2^7 = 128$$

Number of pots in 5th row,

$$a_5 = 2(2)^{5-1} = 2 \cdot 2^4 = 2^5 = 32$$

Required answer = 128 − 32 = 96

(C) Total number of pots upto 10th row is

$$S_{10} = \frac{a(r^{10}-1)}{r-1} = \frac{2(2^{10}-1)}{2-1}$$

$$= \frac{2(1024-1)}{1} = 2046$$

OR

Let there be n number of rows.

$$\therefore \quad S_n = 510 = \frac{2(2^n-1)}{2-1}$$

$$\Rightarrow \quad \frac{510}{2} = 2^n - 1$$

$$\Rightarrow \quad 255 = 2^n - 1$$

$$\Rightarrow \quad 256 = 2^n$$

$$\Rightarrow \quad 2^8 = 2^n$$

$$\Rightarrow \quad n = 8$$

Caution

- $S_n = \frac{a(r^n - 1)}{r - 1}$ *if* $r > 1$ *only.*

38. (A) Let the population in year 2010 is P.

Since, A, B, C are collinear

$\therefore$ Slope of AB = Slope of BC

$$\Rightarrow \quad \frac{1}{2} = \frac{P-97}{2010-1995}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{P-97}{15}$$

$$\Rightarrow \quad 7.5 = P - 97$$

$$\Rightarrow \quad P = 97 + 7.5 = 104.5 \text{ crores}$$

(B) $\because$ Slope of AB $= \frac{1}{2}$

Slope of line perpendicular to AB and passing through (1995, 97) is

$$y - 97 = -2(x - 1995)$$

$$\Rightarrow \quad y - 97 = -2x + 3990$$

$$\Rightarrow \quad 2x + y = 4087$$