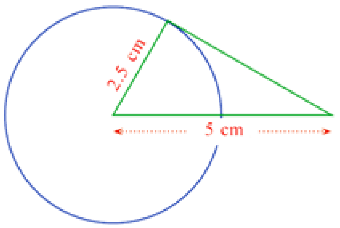
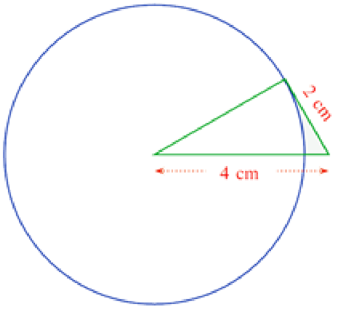
7. Tangents

Questions Pg-163

1. Question

In each of the two pictures below, a triangle is formed by a tangent to a circle, the radius though the point of contact and a line through the centre:





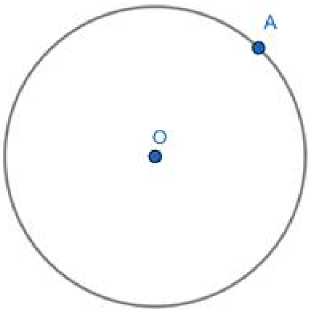
Draw these in your note book.

Answer

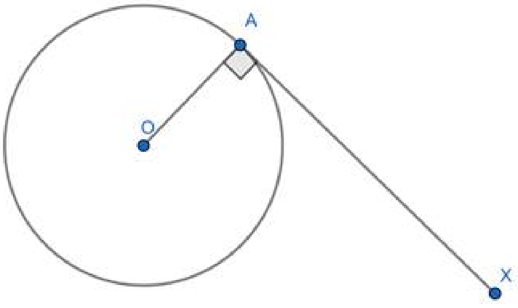
Part 1:

Steps of constructions:

1. Draw a circle of radius 2.5 cm with center O and take any point A on the circumference of circle.

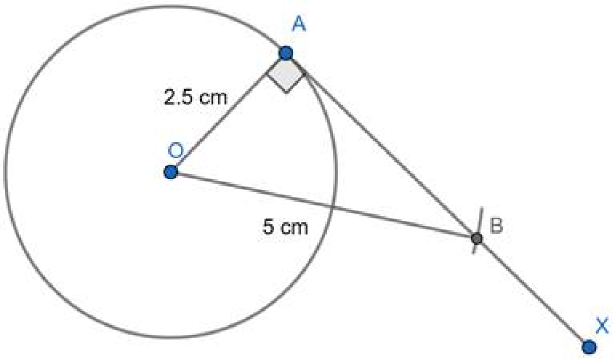


2. Join OA, and draw AX ⊥ OA



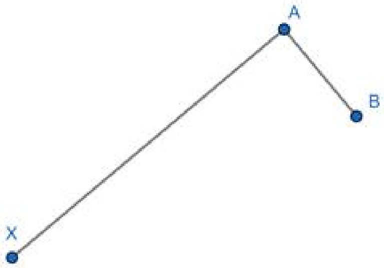
3. Taking O as center draw an arc of 5 cm, that cuts AX at B, Join OB.

PART - 2

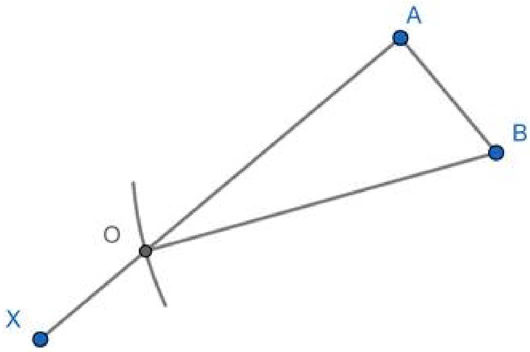


Steps of constructions:

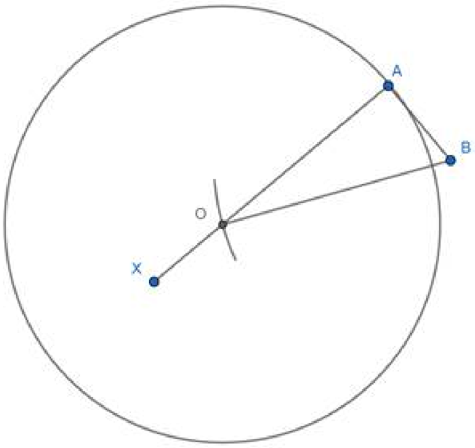
1. Draw a line AB = 2 cm and Draw ∠BAX = 90°.



2. Taking B as center, draw an arc of radius 4 cm, which intersects AX at O, Join OB.

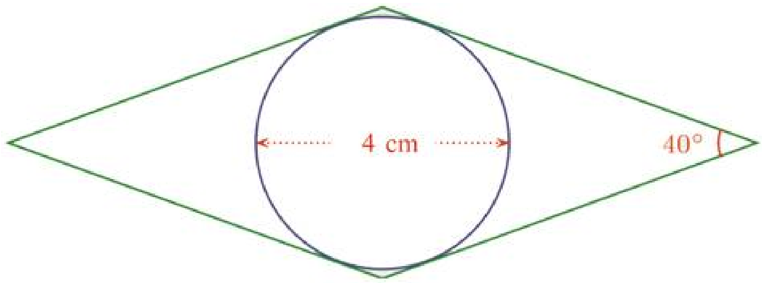


3. Taking O as center, and OA as radius, draw a circle and the required diagram is drawn.



2. Question

In the picture, all sides of a rhombus are tangents to a circle.

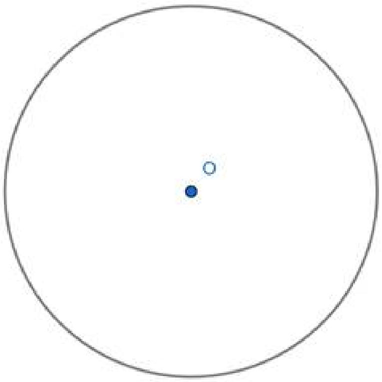


Draw this picture in your notebook.

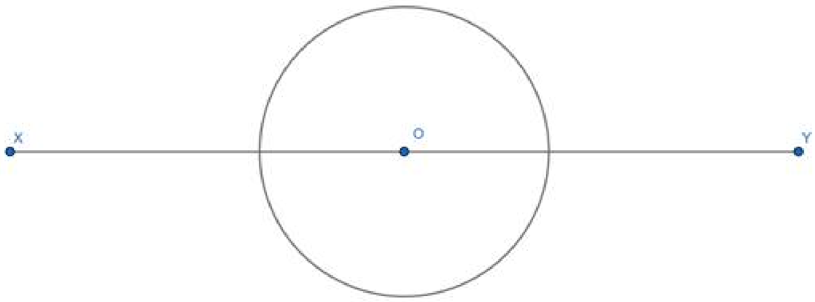
Answer

Steps of construction:

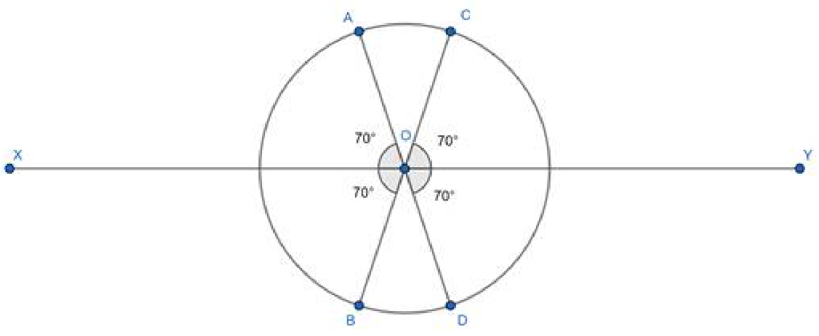
1. Draw a circle of radius 2 cm [as diameter is 4 cm] with center O.



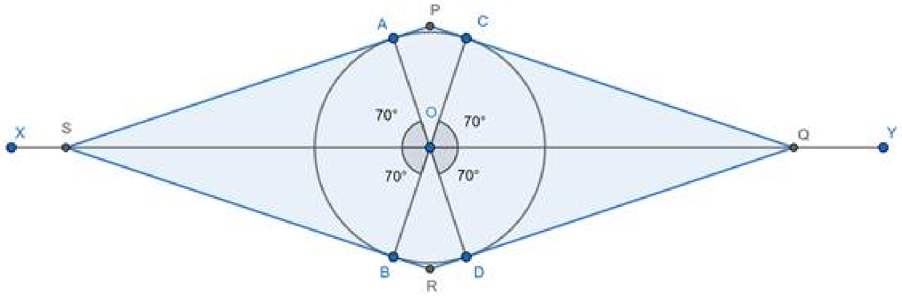
2. Draw a horizontal line XOY, through O



3. Draw ∠XOA = 70°, ∠XOB = 70° , ∠YOC = 70° and ∠YOD = 70°, such that points A, B, C and D lie on circumference.



4. Draw tangents from points A, B, C and D on the circle, such that they form a rhombus PQRS



Now, let's verify ∠CQD = 40°

In ΔCOQ and ΔDOQ

∠COQ + ∠CQO + ∠OCQ = 180° [By angle sum property of triangle]

As, ∠COQ = 70°(By construction)

And ∠ OCQ = 90° [Tangent at any point is perpendicular to the radius at the point of contact]

⇒ ∠CQO = 20°

Similarly, ∠DQQ = 20°

⇒ ∠CQO + ∠DQO = 40°

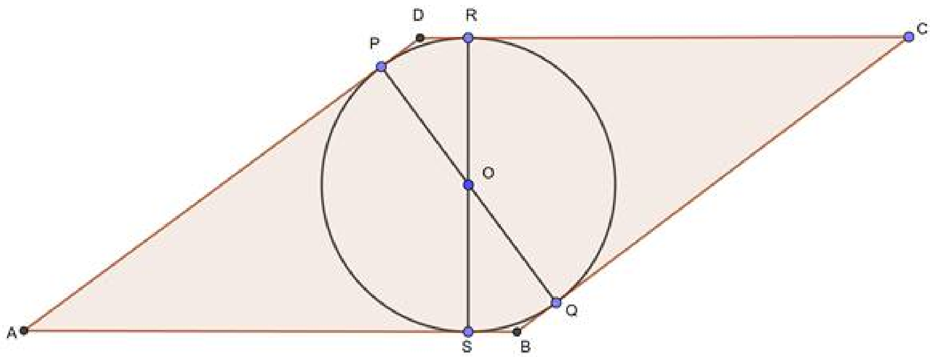
⇒ ∠CQD = 40

3. Question

What sort of a quadrilateral is formed by the tangents at the ends of two diameters of a circle?

Answer

Parallelogram



Let PQ and RS be two diagonals of a circle with center as O, and tangents through their end points make a quadrilateral ABCD.

Now, we know that Tangent at any point is perpendicular to the radius at the point of contact

OR ⊥ CD and OS ⊥ AB

⇒ ∠ORD = ∠OSB [Both 90°]

⇒ AB || CD [If two lines are cut by a transversal and the alternate interior angles are equal, then the lines are parallel]

Also,

OP ⊥ AD and OQ ⊥ BC

⇒ ∠APQ = ∠CQP [Both 90°]

⇒ AD || BC [If two lines are cut by a transversal and the alternate interior angles are equal, then the lines are parallel]

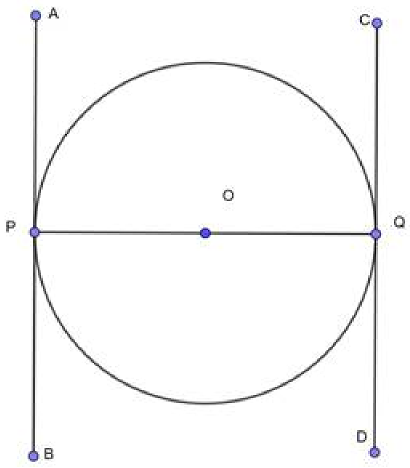
⇒ In quadrilateral ABCD, opposite sides are parallel

⇒ ABCD is a parallelogram.

4. Question

Prove that the tangents drawn to a circle at the two ends of a diameter are parallel.

Answer



Let PQ be a diameter of a circle with center O, AB and CD are two tangents drawn at the ends P and Q respectively.

To Show: AB || CD

Now, we know

Tangent at any point is perpendicular to the radius at the point of contact

⇒ OP ⊥ AB and OQ ⊥ CD

⇒ ∠OPA = ∠OQD [Both 90°]

⇒ AB || CD [If two lines are cut by a transversal and the alternate interior angles are equal, then the lines are parallel]

Hence, Proved.

Questions Pg-166

1. Question

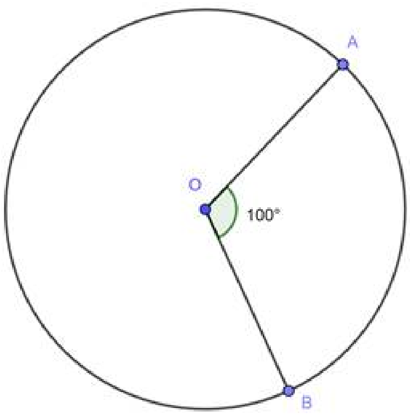
Draw a circle of radius 2.5 centimetres. Draw a triangle of angles 40°, 60°, 80° with all its sides touching the circle.

Answer

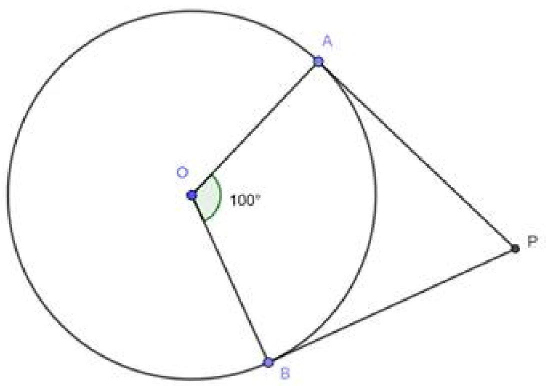
Steps of construction:

1. Draw a circle of radius = 2.5 cm with center O, take any point A on circumference and Join OB

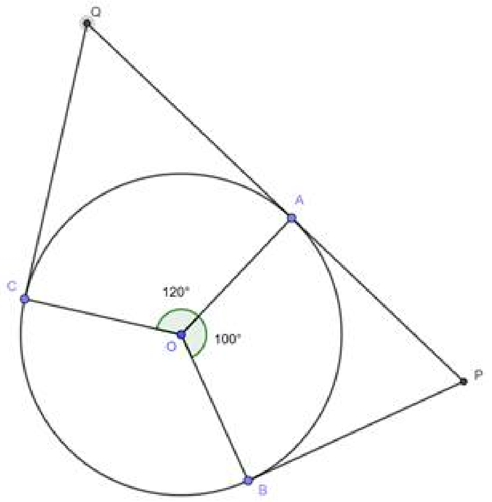
2. Draw ∠AOB = 100°, such that point B lies on circumference.



3. Draw Perpendiculars from point A and Point B such that they intersect each other at P.

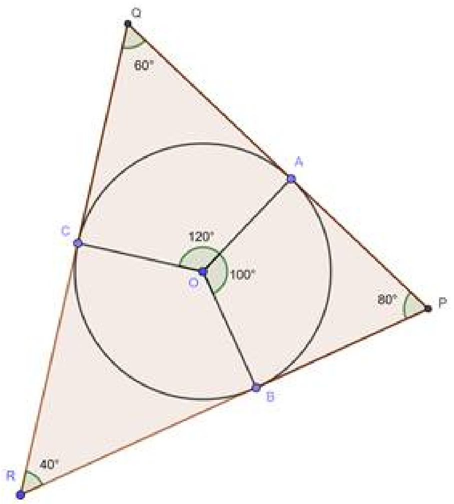


4. Draw ∠AOC = 120° in the opposite direction of ∠AOB and draw perpendiculars from point A and point C such that they intersect at Q



5. Draw perpendiculars from point B and point C such that they intersect at P.

And PQR is the required triangle.



Verification of angles:

In quadrilateral AOBP

∠AOB + ∠OAP + ∠OBP + ∠APB = 360°

As,

∠AOB = 100° [By construction]

∠OAB = ∠OBP = 90° [tangent at any point on the circle is perpendicular to the radius through point of contact]

⇒ 100 + 90 + 90 + ∠APB = 360°

⇒ ∠APB = 80°

⇒ ∠P = 80°

Similarly, ∠Q = 60°

And by angle sum property of ΔPQR,

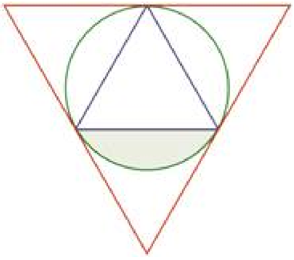
∠P + ∠Q + ∠R = 180°

⇒ 80 + 60 + ∠R = 180

⇒ ∠R = 100°

2. Question

In the picture, the small (blue) triangle is equilateral. The sides of the large (red) triangle are tangents to the circumcircle of the small triangle at its vertices.



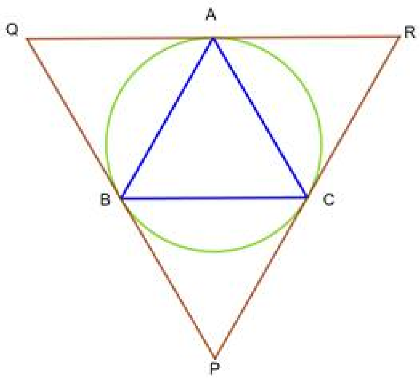
i) Prove that the large triangle is also equilateral and its sides are double that of the small triangle.

ii) Draw this picture, with sides of the smaller triangle 3 centimetres.

iii) Instead of an equilateral triangle, if we draw the tangents to the circumcircle of any other triangle at its vertices, do we get a similar triangle with double the sides? Justify.

Answer

i) Let us label the diagram as shown below,



We know, By alternate segment theorem, angle between chord and tangent is equal to the angle in the other segment.

⇒ ∠RAC = ∠ABC and ∠RCA = ∠ABC [AC is a chord]

As, ΔABC is equilateral

⇒ ∠ABC = 60°

⇒ ∠RAC = ∠RCA = ∠ABC = 60°

Also,

In ΔARC, By angle sum property

∠RAC + ∠RCA + ∠ARC = 180°

⇒ 60° + 60°+ ∠ARC = 180°

⇒ ∠ARC = 60°

⇒ ARC is equilateral triangle and ∠R = 60°

Similarly, we can show

⇒ BPC is equilateral triangle and ∠P = 60°

⇒ AQB is equilateral triangle and ∠Q = 60°

As, ∠P = ∠Q = ∠R = 60°

⇒ ΔPQR is equilateral.

Let, the side of smaller triangle be 'a', i.e. AB = BC = CA = 'a'

But, as ΔARC and ΔAQB are equilateral

⇒ AR = AC = 'a'

And AQ = AB = 'a'

⇒ AQ + AR = a + a

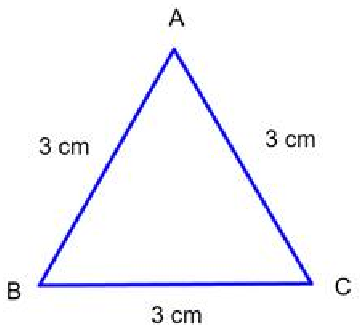
⇒ QR = 2a

As, ΔPQR is equilateral, PQ = QR = PR = '2a'

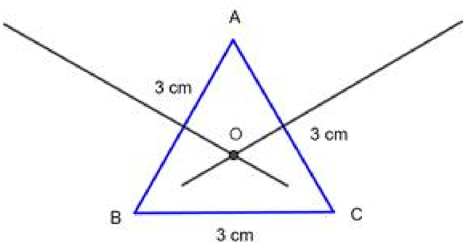
⇒ Side of larger triangle is double of side of smaller triangle.

ii) **Steps of constructions:**

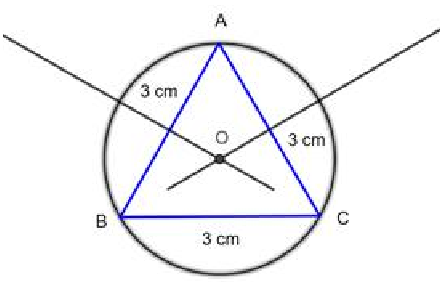
1. Draw an equilateral triangle ABC of 3 cm.



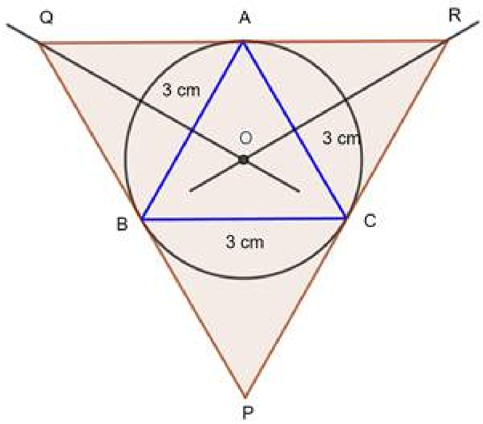
2. Draw perpendicular bisectors of sides AB and BC which intersect each other at O.



3. Taking O as center, draw a circle with radius as OA.



4. Draw tangents at Points A, B and C, which makes a triangle PQR.

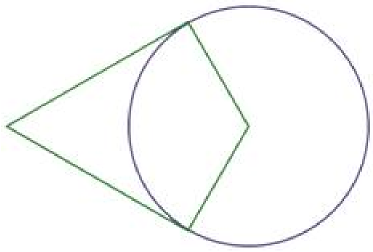


(iii) The student show try themselves.

3 A. Question

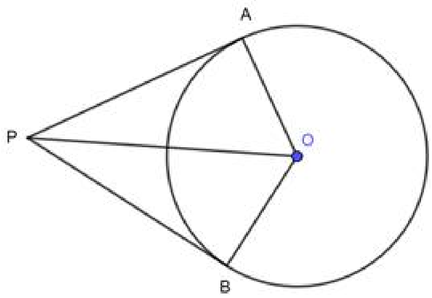
The picture shows the tangents at two points on a circle and the radii through the points of contact.

Prove that the tangents have the same length.



Answer

Construction: Label the diagram and Join OP



In ΔAOP and ΔBOP

OA = OB [Radii of same circle]

OP = OP [Common]

∠OAB = ∠OBP = 90° [Tangent at any point is perpendicular to the radius through point of contact]

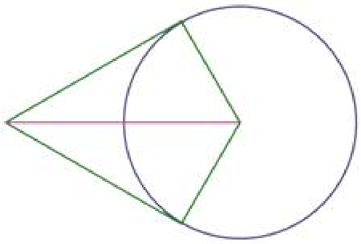
ΔAOP ≅ ΔBOP [By Right Angle - Hypotenuse - Side Criteria]

⇒ AP = BP [Corresponding parts of congruent triangle are equal]

3 B. Question

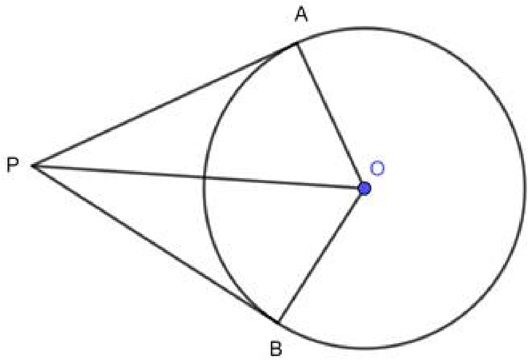
The picture shows the tangents at two points on a circle and the radii through the points of contact.

Prove that the line joining the centre and the points where the tangents meet bisects the angle between the radii.



Answer

Construction: Label the diagram.



In ΔAOP and ΔBOP

OA = OB [Radii of same circle]

OP = OP [Common]

∠OAB = ∠OBP = 90° [Tangent at any point is perpendicular to the radius through point of contact]

ΔAOP ≅ ΔBOP [By Right Angle - Hypotenuse - Side Criteria]

⇒ ∠AOP = ∠BOP [Corresponding parts of congruent triangle are equal]

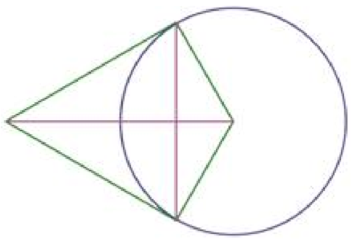
OP bisects ∠AOB.

i.e. the line joining the centre and the points where the tangents meet bisects the angle between the radii.

3 C. Question

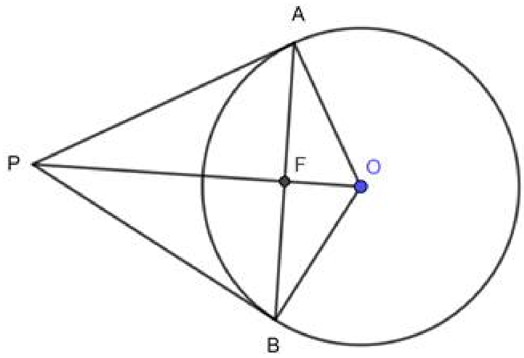
The picture shows the tangents at two points on a circle and the radii through the points of contact.

Prove that this line is the perpendicular bisector of the chords joining the points of contact.



Answer

Let us label the diagram



To Prove : OP is a perpendicular bisector of AB.

In ΔAOF and ΔBOF

OA = OB [Radii of same circle]

∠AOF = ∠BOF [i.e. the line joining the centre and the points where the tangents meet bisects the angle between the radii]

OF = OF [Common]

⇒ ΔAOF ≅ ΔBOF [By Side-Angle-Side Criterion]

⇒ AF = BF and ∠AFO = ∠BFO [Corresponding parts of congruent triangles are equal]

Also,

∠AFO + ∠BFO = 180° [Linear Pair]

⇒ ∠AFO + ∠AFO =180°

⇒ ∠AFO = 90°

⇒ OP ⊥ AB

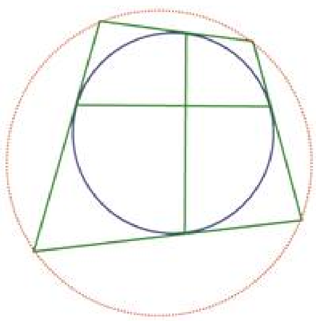
And OP bisects AB

⇒ OP is perpendicular bisector of AB.

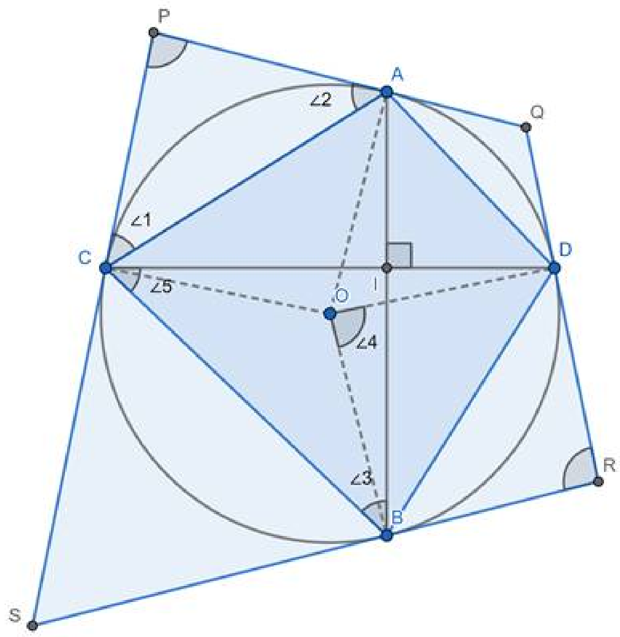
4. Question

Prove that the quadrilateral with sides as the tangents at the ends of a pair of perpendicular chords of a circle is cyclic.

What sort of a quadrilateral do we get if one chord is a diameter? And if both chords are diameters?



Answer



Let's label the diagram, AB and CD are parallel chords, and tangents from points A, B, C and D make a quadrilateral PQRS

Construction: Join OA, OB, OC and OD where O is radius

To Prove: PQRS is cyclic i.e. $\angle P + \angle R = 180°$

Proof:

We know, By alternate segment theorem

angle between chord and tangent is equal to the angle in the other segment.

Therefore,

$\angle 1 = \angle 3$ and $\angle 2 = \angle 3$ [As AC is a chord]

$\Rightarrow \angle 1 = \angle 2 = \angle 3$

Also, in ΔACP, By angle sum property

$\angle 1 + \angle 2 + \angle P = 180°$

$\Rightarrow \angle 3 + \angle 3 + \angle P = 180°$

$\Rightarrow \angle P = 180° - 2\ \angle 3$ ...[1]

In Δ BIC, By angle sum property

$\angle 3 + \angle 5 + \angle CIB = 180°$

$\Rightarrow \angle 3 + \angle 5 = 90°$ [$\angle CIB = 90°$, Since $AB \perp CD$]

$\Rightarrow \angle 3 = 90° - \angle 5$ ...[2]

From [1] and [2]

$\Rightarrow \angle P = 180 - 2(90 - \angle 5)$

$\Rightarrow \angle P = 2\ \angle 5$ ...[A]

Also, Considering arc BD

$\Rightarrow \angle 4 = 2\ \angle 5$ ...[3]

[The angle subtended by an arc at the center of the circle is double the angle subtended by arc at any point on circumference of the circle]

Also, In Quadrilateral OBRD, By angle sum property

∠4 + ∠OBD + ∠R + ∠ODR = 360°

[∠OBD = ∠ODR = 90, as tangent at any point on circle is perpendicular to the radius through point of contact]

⇒ ∠4 + 90° + 90° + ∠R = 360°

⇒ ∠R = 180° - ∠4

⇒ ∠R = 180° - 2 ∠5 ...[B] [From 3]

Adding [A] and [B], we get

⇒ ∠P + ∠R = 180° - 2 ∠5 + 2 ∠5

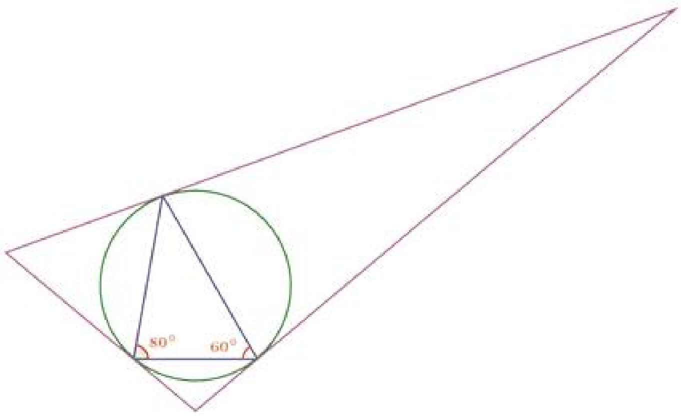
⇒ ∠P + ∠R = 180°

Hence Proved.

Questions Pg-172

1. Question

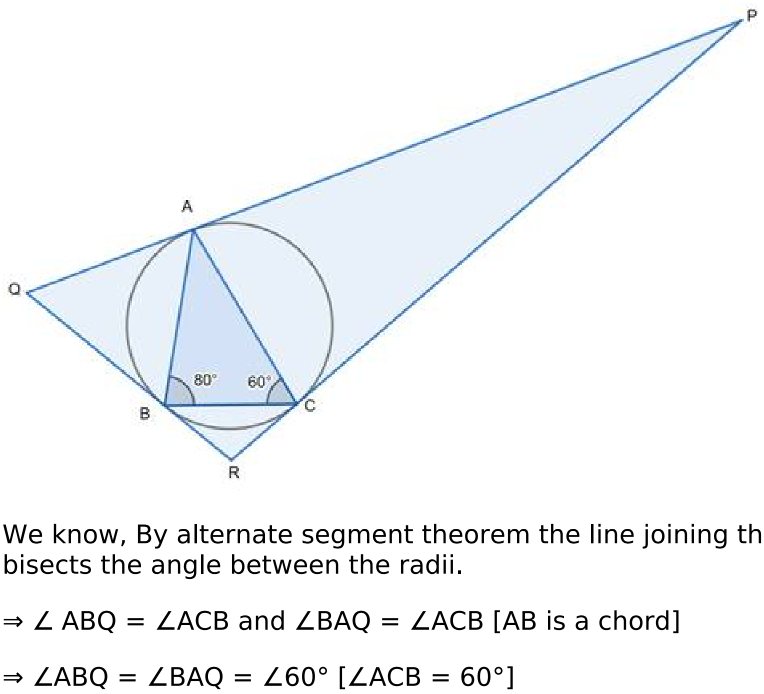
In the picture, the sides of the large triangle are tangents to the circumcircle of the small triangle, through its vertices.



Calculate the angles of the large triangle.

Answer

Let us label the diagram.



We know, By alternate segment theorem the line joining the centre and the points where the tangents meet bisects the angle between the radii.

⇒ ∠ ABQ = ∠ACB and ∠BAQ = ∠ACB [AB is a chord]

⇒ ∠ABQ = ∠BAQ = ∠60° [∠ACB = 60°]

In ΔABQ, By triangle sum property

⇒ ∠ABQ + ∠BAQ + ∠AQB = 180°

⇒ 60° + 60° + ∠Q = 180°

⇒ ∠Q = 60°

Also, By alternate segment theorem

∠ACP = ∠ ABC and ∠CAP = ∠ABC [AC is a chord]

⇒ ∠ACP = ∠CAP = 80° [∠ABC = 80°]

In ΔACP, By triangle sum property

⇒ ∠ACP + ∠CAP + ∠APC = 180°

⇒ 80° + 80° + ∠P = 180°

⇒ ∠P = 20°

In ΔPQR, By angle sum property

∠P + ∠Q + ∠R = 180°

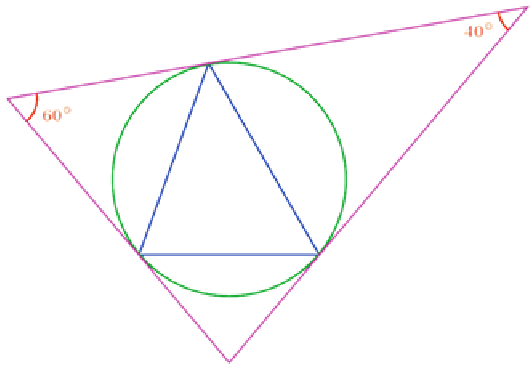
⇒ 20° + 60° + ∠R = 180°

⇒ ∠R = 100°

Hence, three angles are 20°, 60° and 100°.

2. Question

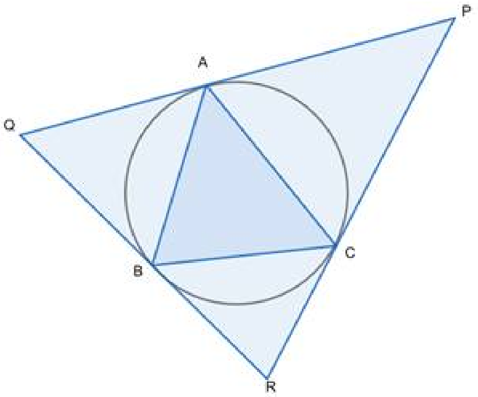
In the picture, the sides of the large triangle are tangents of the circum-circle of the smaller triangle, through its vertices.



Calculate the angles of the smaller triangle.

Answer

we first label the diagram



We know, By alternate segment theorem the line joining the centre and the points where the tangents meet bisects the angle between the radii.

⇒ ∠ ABQ = ∠ACB and ∠BAQ = ∠ACB [AB is a chord]

In ΔABQ, By triangle sum property

⇒ ∠ABQ + ∠BAQ + ∠AQB = 180°

⇒ ∠ACB + ∠ ACB + 60° = 180°

⇒ 2∠ACB = 120°

⇒ ∠ACB = 60°

Also, By alternate segment theorem

∠ACP = ∠ ABC and ∠CAP = ∠ABC [AC is a chord]

In ΔACP, By triangle sum property

⇒ ∠ACP + ∠CAP + ∠APC = 180°

⇒ ∠ABC + ∠ABC + 40° = 180°

⇒ 2∠ABC = 140°

⇒ ∠ABC = 70°

In ΔABC, By angle sum property

∠ABC + ∠ACB + ∠BAC = 180°

⇒ 70° + 60° + ∠BAC = 180°

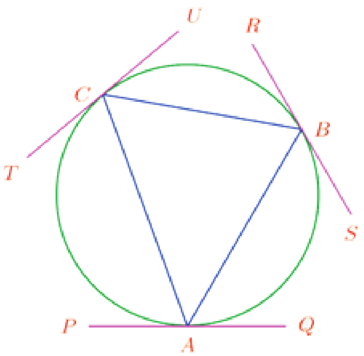
⇒ ∠BAC = 50°

Hence, three angles are 20°, 60° and 100°.

3. Question

In the picture, PQ, RS, TU are tangents to the circumcircle of Δ ABC.

Sort out the equal angles in the picture.



Answer

We know, By alternate segment theorem angle between chord and tangent is equal to the angle in the other segment.

As, AB is a chord and PQ is tangent at A.

⇒ ∠BAQ = ∠ACB …[1]

and ∠CAP = ∠ABC …[2]

As, AC is a chord and PQ is tangent at A.

⇒ ∠ACT = ∠ABC …[3]

and ∠BCU = ∠BAC …[4]

As, AB is a chord and PQ is tangent at A.

⇒ ∠ABS = ∠ACB ...[5]

and ∠CBR = ∠BAC ...[6]

From [1] and [5]

∠BAQ = ∠ABS = ∠ACB

From [2] and [3]

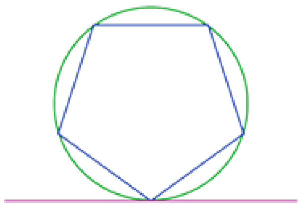
∠CAP = ∠ACT = ∠ABC

From [4] and [6]

∠BCU = ∠CBR = ∠BAC

4. Question

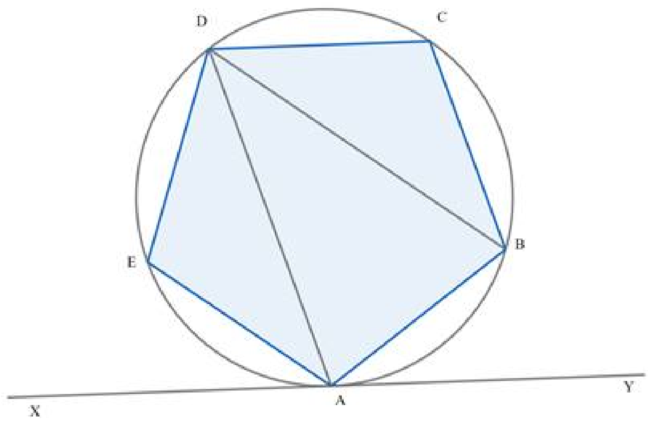
In the picture, the tangent to the circumcircle of a regular pentagon through a vertex is shown.



Calculate the angle when the tangent makes with the two sides of the pentagon through the point of contact.

Answer

Let us first label the diagram.



To find: the angle when the tangent makes with the two sides of the pentagon through the point of contact i.e. ∠EAX and ∠BAY

Construction: Join AD and BD

In ΔAED

AE = DE [Sides of regular polygon are equal]

∠EAD = ∠EDA [Angles opposite to equal sides are equal]

Also, In ΔAEC, By angle sum property

∠AED + ∠EAD + ∠EDA = 180°

⇒ 108° + ∠EDA + ∠EDA = 180° [∠AED = 108°, Each angle in regular pentagon is 108°]

⇒ 2 ∠EDA = 72°

⇒ ∠EDA = 36° ...[1]

Similarly,

∠BDC = 36° ...[2]

Also,

∠CDE = 108° [Each angle in regular pentagon is 108°]

⇒ ∠EDA + ∠ADB + ∠BDC = 108°

⇒ 36° + ∠ADB + 36° = 108° [From 1 and 2]

⇒ ∠ADB = 36°

Also, AB is a chord

⇒ ∠BAY = ∠ADB [By alternate segment theorem i.e. angle between chord and tangent is equal to the angle in the other segment]

⇒ ∠BAY = 36°

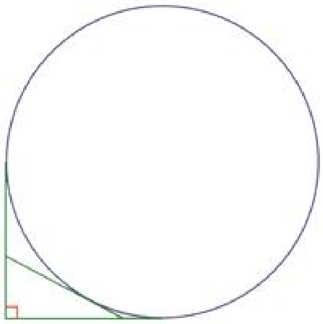
And By symmetry,

∠EAX = ∠BAY = 36°

Questions Pg-179

1. Question

In the picture, a triangle is formed by two mutually perpendicular tangents to a circle and a third tangent.



Prove that the perimeter of the triangle is equal to the diameter of the circle.

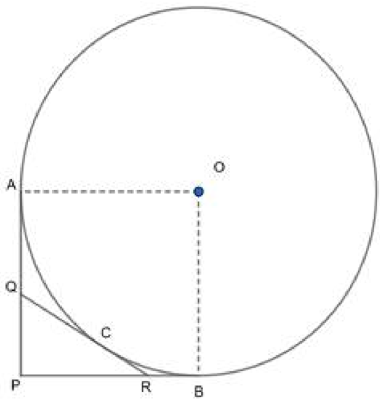
Answer

Let us label the diagram and let the radius be 'r'.

To Prove : Perimeter of triangle PQR = diameter of circle

i.e. PQ + QR + PR = 2r

Construction: Join OA and OB



Proof:

In Quadrilateral OAPB

∠OAB + ∠APB + ∠OBP + ∠AOB = 360°

Also,

∠OAB = 90° [OA ⊥ AP, as tangent at any point on the circle is perpendicular to the radius through point of contact]

∠OBP = 90° [OB ⊥ BP, as tangent at any point on the circle is perpendicular to the radius through point of contact]

∠APB = 90° [AP ⊥ BP, Given]

⇒ 90 + 90 + 90 + ∠AOB = 360

⇒ ∠AOB = 90°

Also,

OA = OB [radii of same circle]

AP = BP [Tangents drawn from same point to a circle are equal]

And we know, if in a quadrilateral

i) All angles are 90° and

ii) Adjacent sides are equal

then the quadrilateral is square.

⇒ OAPB is a square

⇒ AP = BP = OA = OB = 'r' [radius] ...[1]

Also, ⇒ as tangents drawn from same points to a circle are equal

⇒ AQ = CQ [tangents from Q] ...[2]

⇒ BR = CR [tangents from R] ...[3]

Also,

Perimeter of triangle PQR

= PQ + QR + PR

= PQ + QC + CR + PR

= PQ + AQ + BR + PR [From 2 and 3]

= AP + BP

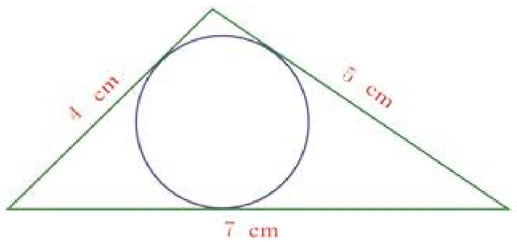
= r + r [From 1]

= 2r

Hence Proved.

2. Question

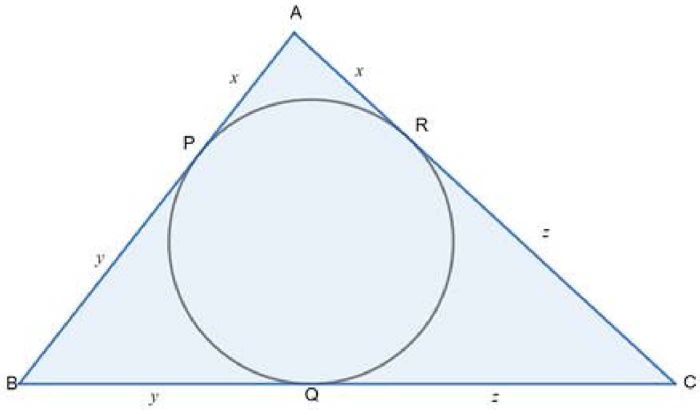
The picture shows a triangle formed by three tangents to a circle.



Calculate the length of each tangent from the corner of the triangle to the point of contact.

Answer

Let us label the diagram.



As we know that

Tangents from an external point to a circle are equal,

In given Figure we have

AP = AR = x [Tangents from point A]

BP = BQ = y [Tangents from point B]

CQ = CR = z [Tangents from point C]

Now, Given

AB = 4 cm

⇒ AP + BP = 4

⇒ x + y = 4

⇒ y = 4 - x ...[1]

and BC = 7 cm

⇒ BQ+ QC = 7

⇒ y + z = 7

⇒ 4 - x + z = 7 [From 1]

⇒ z = x + 3 ...[2]

and

AC = 5 cm

⇒ AR + CR = 5

⇒ x + z = 5 [From 2]

⇒ x + x + 3 = 5

⇒ 2x = 2

⇒ x = 1 cm

Putting value of x in [1] and [2]

y = 4 - 1 = 3 cm

z = 1 + 3 = 4 cm

So, we have

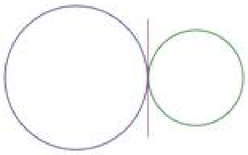
AP = AR = 1 cm

BP = BQ = 3 cm

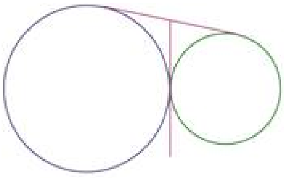
CQ = CR = 4 cm

3. Question

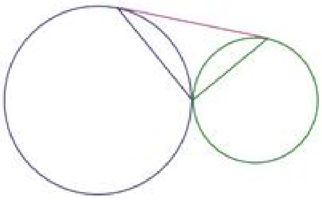
In the picture, two circles touch at a point and the common tangent at this point is drawn.



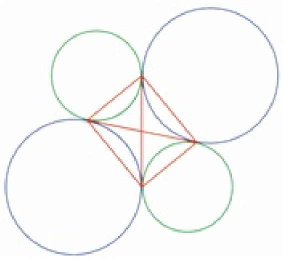
i) Prove that this tangent bisects another common tangent of these circles.



ii) Prove that the points of contact of these two tangents form the vertices of a right triangles.

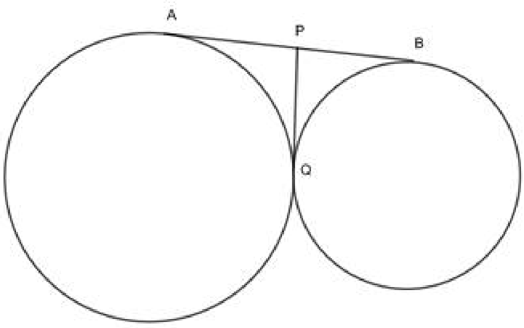


iii) Draw the picture on the right in your notebook, using convenient lengths.



Answer

(i) Let us label the diagram.



To show: PQ bisects AB

We know that, tangents drawn from an external point to a circle are equal therefore we have

AP = PQ [For bigger circle]

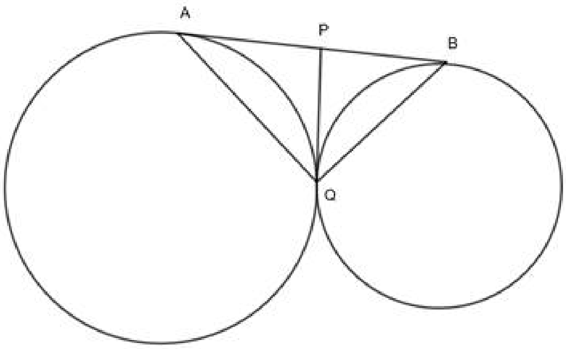
BP = PQ [For smaller circle]

⇒ AP = BP = PQ

⇒ P is the mid-point of AB

⇒ PQ bisects AB.

(ii) Let us label the diagram and draw a tangent at point Q which intersects AB at P



To show: ∠AQB = 90°

Proof:

AP = PQ [tangents drawn from an external point to a circle are equal]

∠PAQ = ∠AQP [Angles opposite to equal sides are equal]

In ΔAPQ, By angle sum property

∠APQ + ∠AQP + ∠PAQ = 180°

⇒ ∠APQ + ∠AQP + ∠AQP = 180°

⇒ 2∠AQP = 180° - ∠APQ ...[1]

Similarly, In ΔBPQ

2∠PQB = 180° - ∠BPQ ...[2]

Adding [1] and [2]

⇒ 2∠AQP + 2∠PQB = 360° - (∠APQ + ∠BPQ)

⇒ 2(∠AQP + ∠PQB) = 360° - 180° [∠APQ + ∠BPQ = 180° , linear pair]

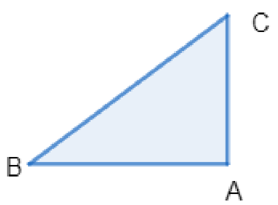
⇒ 2∠AQB = 180°

⇒ ∠AQB = 90°

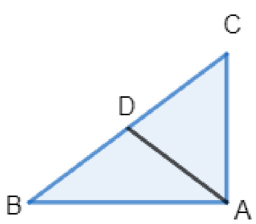
Hence, Proved.

(iii) Steps of construction:

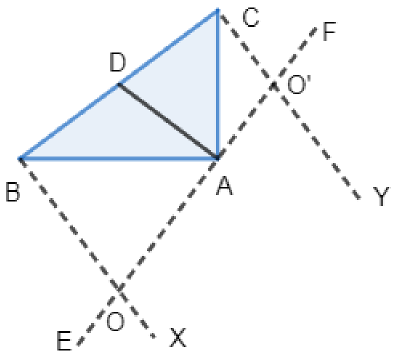
1. Draw a right-angled triangle ABC such that AB = 3 cm, AC = 4 cm and BC = 5 cm (3-4-5 is a Pythagorean triplet).



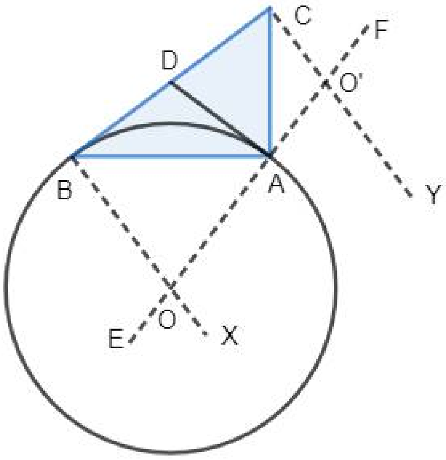
2. Draw a median AD from A to BC.



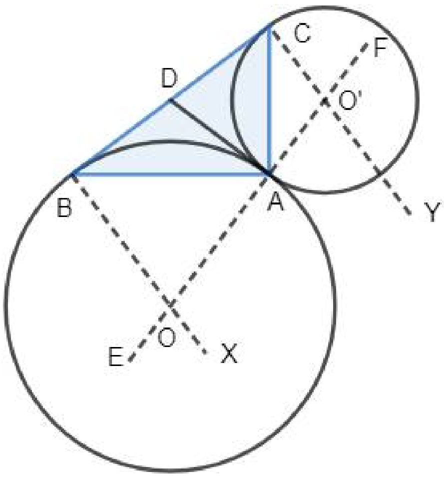
3. Draw BX ⊥ BC, CY ⊥ BC and EF ⊥ AD through A such that BX and AE intersect at O and CY and AF intersect each other at O’



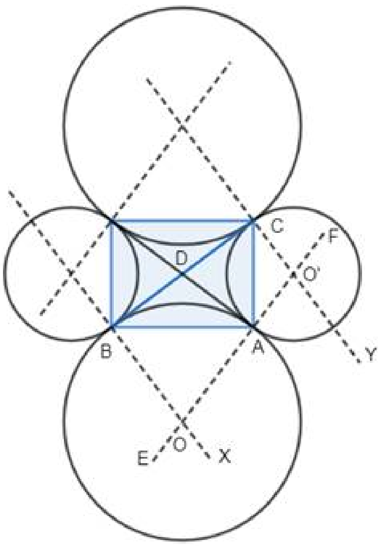
4. Taking O as center and OB as radius, draw a circle



5. Taking O’ as center and OC as radius draw another circle

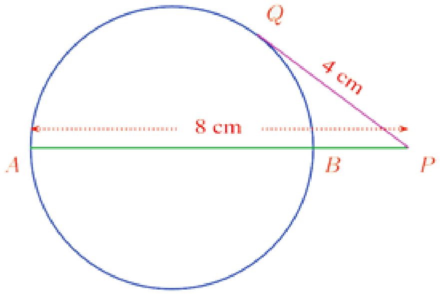


6. We can make the upper half of the figure by repeating the above steps.



4. Question

In the picture below, AB is a diameter and P is a point on AB extended. A tangent from P touches the circle at Q. What is the radius of the circle?



Answer

Let the radius of circle be 'x' cm and O be the center.

To find: radius of circle = x

Construction: Join OQ

Now,

AP = AB + BP

As, AB is diameter

AB = 2 × radius

AB = 2x

⇒ 8 = 2x + BP

⇒ BP = 8 - 2x

Now, OQ ⊥ PQ , POQ is a right angled triangle.

By Pythagoras theorem

$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$

⇒ $OP^2 = QP^2 + OQ^2$

⇒ $(OB + BP)^2 = 4^2 + OQ^2$

Since, OB = OQ = x [radii]

⇒ $(x + 8 - 2x)^2 = 16 + x^2$

⇒ $(8 - x)^2 = 16 + x^2$

⇒ $64 + x^2 - 16x = 16 + x^2$

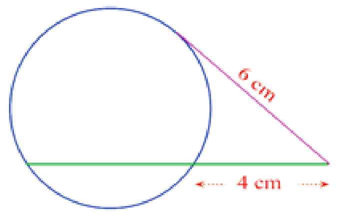
⇒ 16x = 48

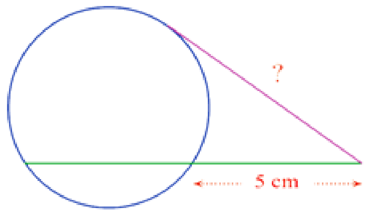
⇒ x = 3 cm

Hence, radius of circle is 3 cm.

5. Question

In the first picture below, the line joining two points on a circle is extended by 4 centimetres and a tangent is drawn from this point. Its length is 6 centimetres, as shown:

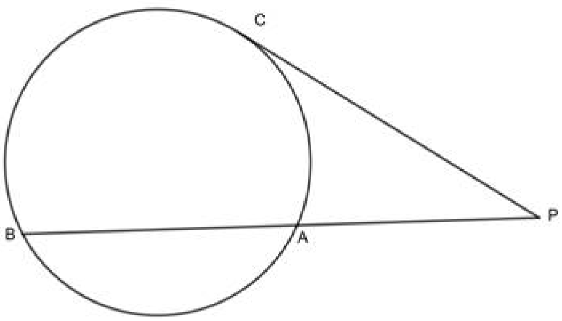




The second picture shows the same line extended by 1 centimetre more and a tangent drawn from this point. What is the length of this tangent?

Answer

Let us label the diagram



We know that,

The product of an intersecting line and its part outside the circle is equal to the square of tangent.

i.e.

$AB \times AP = CP^2$

In first case,

$AP = 4$ cm

$CP = 6$ cm

$\Rightarrow AB \times 4 = 6^2$

$\Rightarrow 4AB = 36$

$\Rightarrow AB = 9$ cm

In second case,

$AP = 5$ cm

To find : CP

As, length of AB is not changed we have,

$AB \times AP = CP^2$

$\Rightarrow CP^2 = 5(9)$

$\Rightarrow CP^2 = 45$

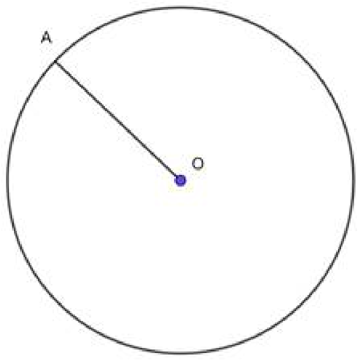
$\Rightarrow CP = 3\sqrt{5}$ cm

6. Question

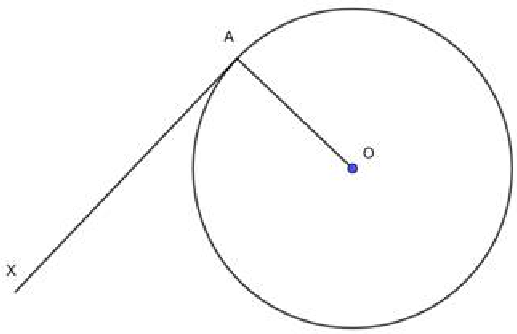
Draw a rectangle of one side 6 centimetres and area equal to that of a square of side 5 centimetres.

Answer

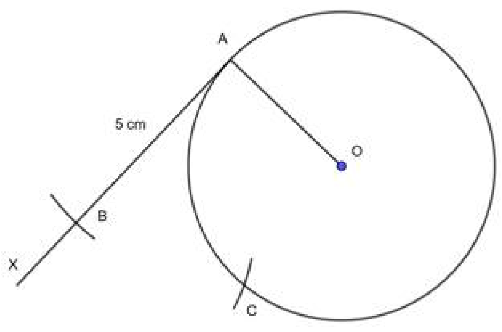
1. Draw a circle of any radius with center as O, take any point A on its circumference and Join OA.



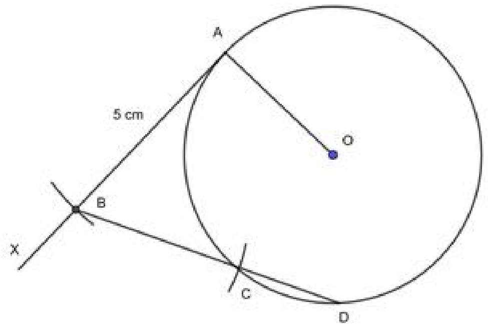
2. Draw AX perpendicular to OA, such that AX is tangent to circle.



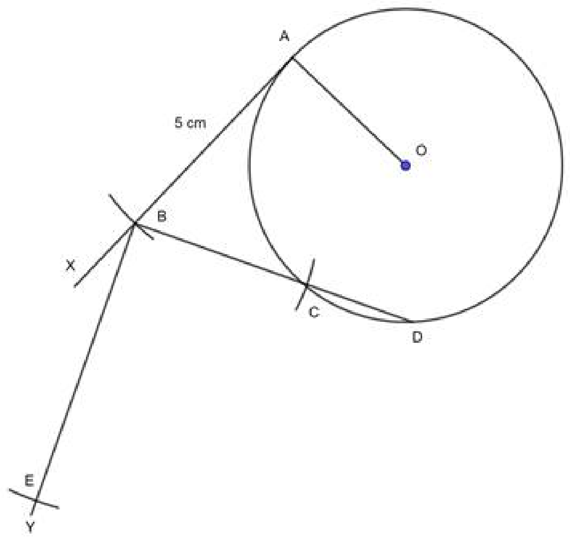
3. Draw an arc of radius 5 cm taking A as center which intersects AX at B and the draw and arc of radius 4 cm taking B as center which intersects circle at C.



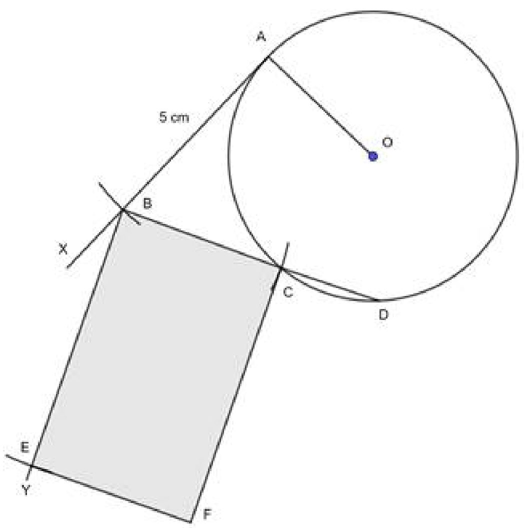
4. Join BC and extend it to D, such that point D lies on the circle.



5. Draw BY ⊥ BD and taking B as center and BD as radius draw an arc which intersect BY at E



6. Taking BC as breadth and BE as length draw a rectangle BCFE, which is required rectangle.



Verification of area:

We know that,

The rectangle with the intersecting line and its part outside the circle as sides and square with tangent as side have equal area.

i.e. area of rectangle with sides as BD and BC = area of square of side AB

now,

AB = 5 cm, By construction and BC = 4 cm By construction

Also, BD = BE

So, area of rectangle having sides as 4 cm and BE = area of square of side as 5 cm.

Questions Pg-187

1. Question

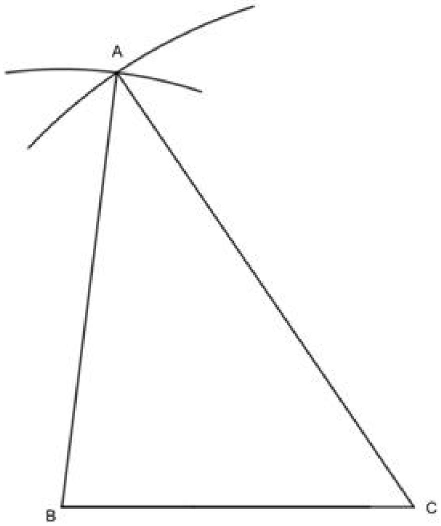
Draw a triangle of sides 4 centimetres, 5 centimetres, 6 centimetres and draw its incircle. Calculate its radius.

Answer

Steps of construction:

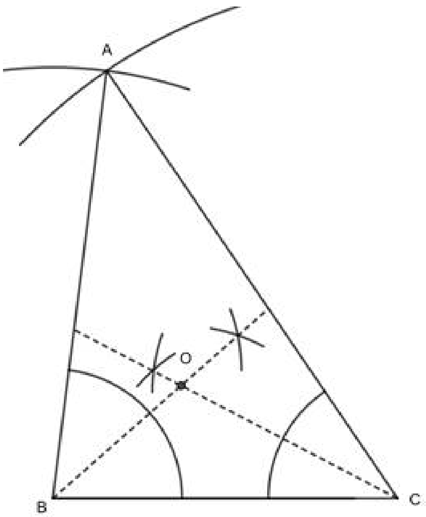
1) Draw a line BC = 4 cm

2) Taking B as center, draw an arc of radius 5 cm and then taking C as center, draw an arc of 6 cm which intersects previous arc at A.

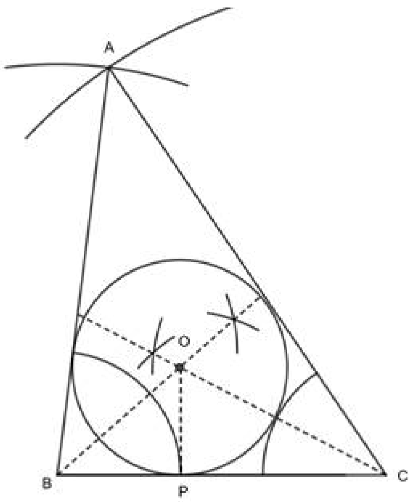


ABC is the required triangle.

3) Draw the angle bisector of ∠ABC and ∠ACB, which intersect each other at O.



4) Draw OP ⊥ BC, and taking O as center, draw a circle with radius as OP.



Calculation of radius:

We know, radius of incircle $= \frac{A}{s}$

Where, A = area of triangle

And s = semi-perimeter of triangle

Also, We know, area of triangle = √(s(s - a)(s - b)(s - c))

Where, a, b and c are sides and s is the semi-perimeter calculated as

$$s = \frac{a+b+c}{2}$$

$$\therefore s = \frac{4+5+6}{2} = \frac{15}{2} = 7.5$$

⇒ area = √(7.5(7.5 - 4)(7.5 - 5)(7.5-6)

= √(7.5 × 3.5 × 2.5 × 1.5)

$$= \sqrt{\frac{15}{2} \times \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2}} = \frac{15}{4}\sqrt{7}\text{cm}$$

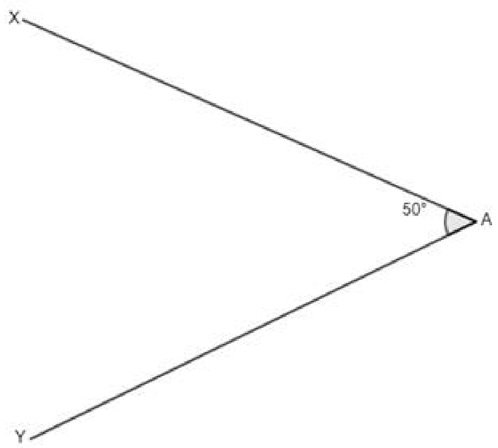
2. Question

Draw a rhombus of sides 5 centimetres and one angle 50° and draw its incircle.

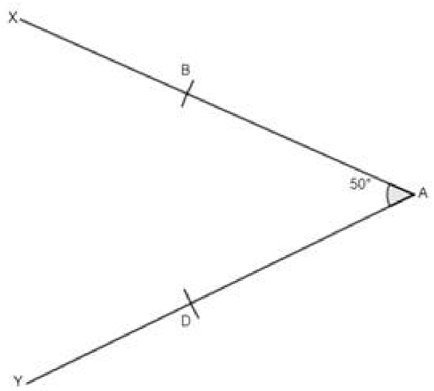
Answer

Steps of construction:

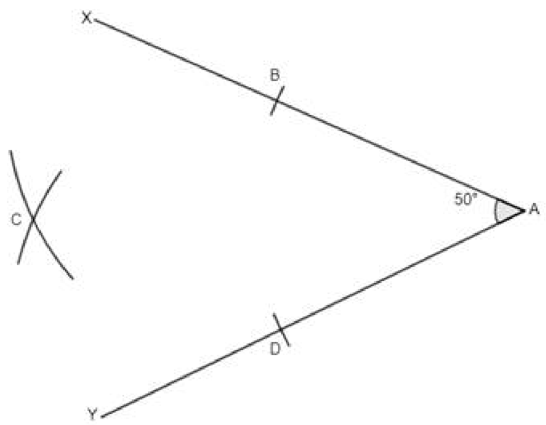
1. Draw an angle ∠XAY = 50°



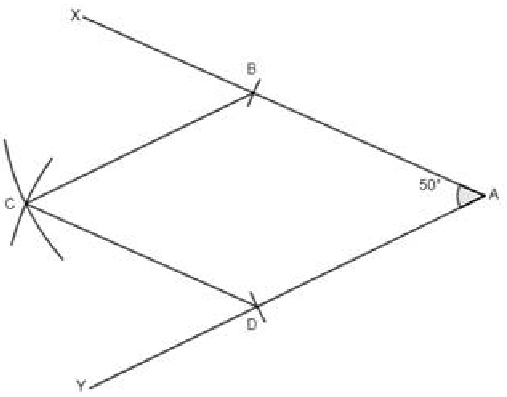
2. Taking radius as 5 cm, draw arcs on lines AX and AY such that AX is intersected by arc on B and AY is intersected by arc on D



3. From B and D draw arcs of 5 cm, which intersect at C.

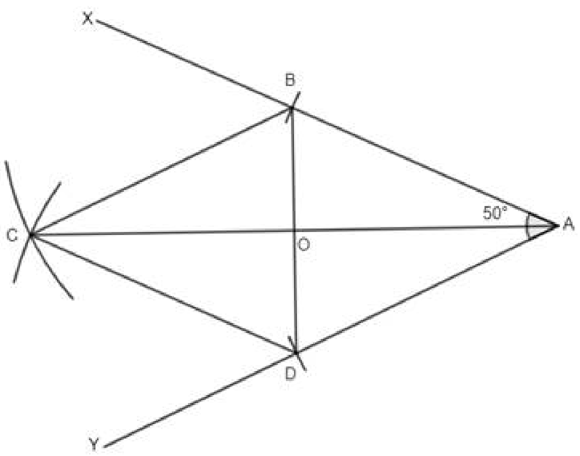


4. Join BC and CD to get the required rhombus

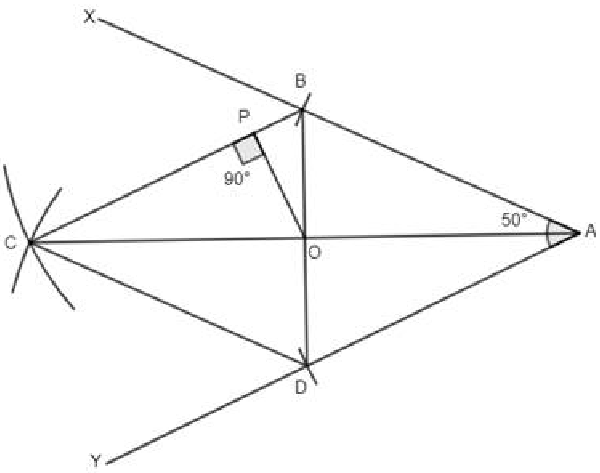


For Incircle

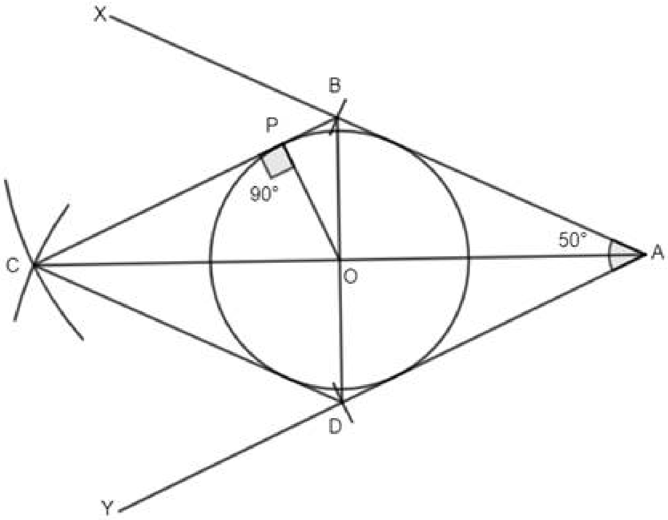
5. Join AC and BD, such that AC and BD intersect each other at O.



6. Draw OP perpendicular to any of the sides, [we have chosen BC]

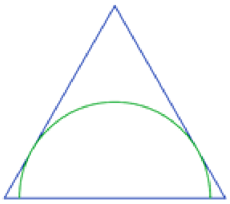


7. Taking O as center, and OP as radius draw the required incircle.



3. Question

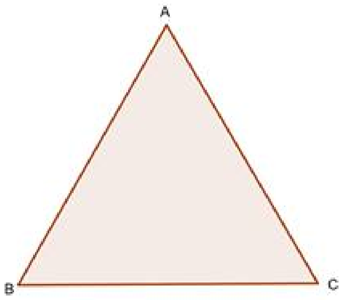
Draw an equilateral triangle and a semicircle touching its two sides, as in the picture.



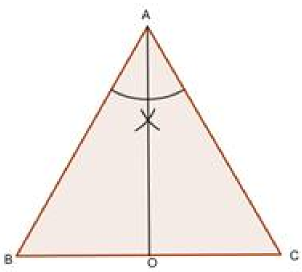
Answer

Steps of construction:

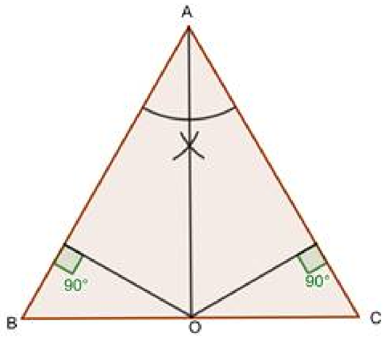
1) Draw an equilateral triangle ABC of any side.



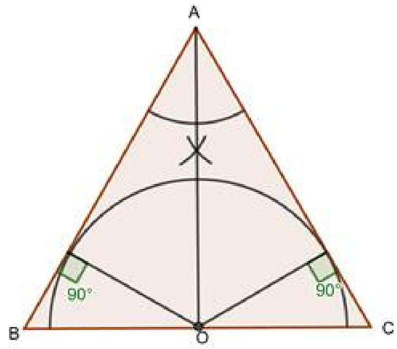
2) Draw the angle bisector of angle ∠BAC which intersects BC at O.



3) Draw OP ⊥ AB and OQ ⊥ AC



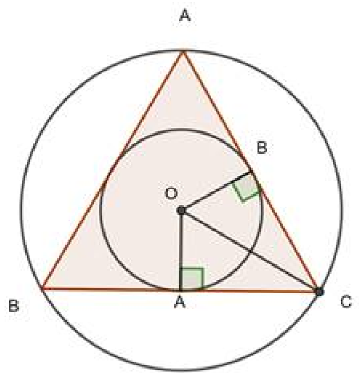
4)Taking O as center, draw a circle with radius as OP or OQ.



4. Question

Prove the radius of the incircle of an equilateral triangle is half the radius of its circumcircle.

Answer



Let ABC be an equilateral triangle, its incircle and circumcircle are drawn with center O, let the radius of incircle be 'r' and circumcircle be 'R'

To prove: R = 2r

Construction: Draw OA ⊥ BC and OB ⊥ AC

Proof:

In ΔOAC and ΔOBC

∠OAC = ∠OBC [Both 90°]

OC = OC [Common]

OA = OB [Radii of incircle]

⇒ ΔOAC ≅ ΔOBC [By Right Angle - Hypotenuse - Side Criterion]

⇒ ∠OCA = ∠OCB [Corresponding parts of congruent triangles are equal]

But,

∠C = 60° [Angle of equilateral triangle]

$\Rightarrow \angle OCA + \angle OCB = 60°$

$\Rightarrow \angle OCA + \angle OCA = 60°$

$\Rightarrow 2\angle OCA = 60°$

$\Rightarrow \angle OCA = 30°$

Now, In right-angled triangle OAC

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\Rightarrow \sin\angle OCA = \frac{OA}{OC}$$

$$\Rightarrow \sin 30° = \frac{r}{R}$$

[As, OA = radius of incircle = r and OC = radius of circumcircle = R]

$$\Rightarrow \frac{1}{2} = \frac{r}{R}$$

$\Rightarrow R = 2r$

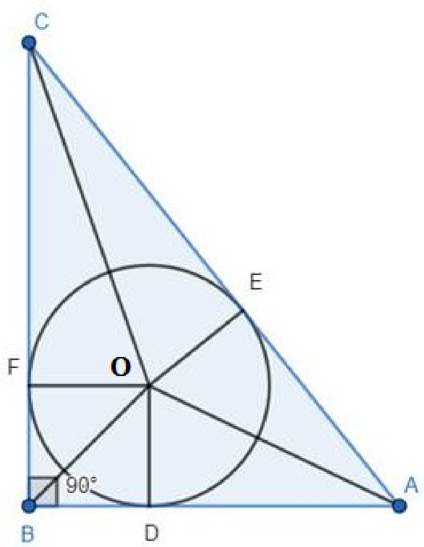
Hence Proved.

5. Question

Prove that if the hypotenuse of a right triangle is h and the radius of its incircle is r, then its area is r(h + r).

Answer

The figure is given below:



Let ABC be a right-angled triangle, right angle at B such that

AB = base = ‘a’ units

BC = perpendicular = ‘b’ units

And AC = hypotenuse = ‘h’ units

Also, Incircle of ΔABC is drawn of radius r

To Prove: area(ΔABC) = r(h + r)

Construction: Draw OD⊥AB, OF⊥BC and OE⊥AC

Proof:

Consider quadrilateral OFBD,

OF = OD = r [radii of incircle]

and ∠OFB = ∠ODB = ∠DBF = 90°

⇒ ∠DOF = 90° [Angle sum property of quadrilateral]

Hence, OFBD is a square [adjacent sides are equal and all angles are 90°]

⇒ OF = OD = BF = BD = 'r'

Also,

AB = 'a'

⇒ AD + BD = a

⇒ AD + r = a

⇒ AD = a – r

Now, AD = AE [tangents drawn from an external point to a circle are equal]

⇒ AE = a – r ...[1]

Similarly,

⇒ CE = b - r ...[2]

Adding [1] and [2], we get

AE + CE = a + b - 2r

⇒ AC + 2r = a + b

⇒ a + b = h + 2r ....[3]

Now,

area(ΔABC) = area(ΔOAB) + area(ΔOAC) + area(ΔOBC)

$\text{we know, area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow \text{ area}(\Delta ABC) = \left(\frac{1}{2} \times AB \times OD\right) + \left(\frac{1}{2} \times AC \times OE\right) + \left(\frac{1}{2} \times BC \times OF\right)$

$\Rightarrow \text{ area}(\Delta ABC) = \left(\frac{1}{2} \times a \times r\right) + \left(\frac{1}{2} \times h \times r\right) + \left(\frac{1}{2} \times b \times r\right) \Rightarrow \text{ area}(\Delta ABC) = \frac{1}{2} r(a + b + h)$

$\Rightarrow \text{ area}(\Delta ABC) = \frac{1}{2} r(h + 2r + h) = \frac{r}{2}(2r + 2h) \ldots \text{[From 3]}$

⇒ area(ΔABC) = r(h + r)

Hence Proved.

6. Question

Calculate the area of a triangle of sides 13 centimetres, 14 centimetres, 15 centimetres.

Answer

Given sides, a = 13 cm, b = 14 cm, c = 15 cm

We know, area of triangle = √(s(s - a)(s - b)(s - c))

Where, a, b and c are sides and s is the semi-perimeter calculated as

$$s = \frac{a + b + c}{2}$$

$\therefore s = \frac{13+14+15}{2} = 21$

⇒ area = $\sqrt{21(21 - 13)(21 - 14)(21 - 15)}$

= $\sqrt{21 \times 8 \times 7 \times 6}$ = $\sqrt{7 \times 3 \times 4 \times 2 \times 7 \times 3 \times 2}$

= $7 \times 3 \times 2 \times 2$

= 84 cm^2