Ch-4 Time Response Domain Analysis

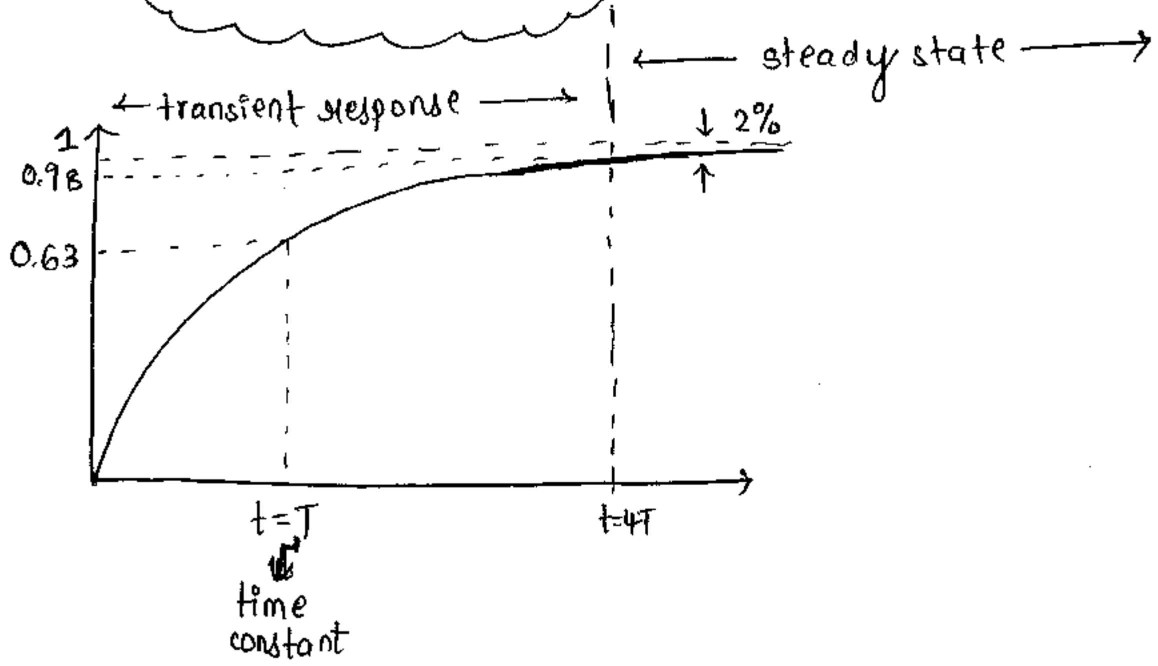
Purpose: To evaluate performance of system with respect to time.

Time Response:

If the response of system varies w.r.t time then it is said to be time response.

- Time response consists 2 response
  ① Transient response
  ② Steady state response

$$c(t) = c_{tr}(t) + c_{ss}(t)$$



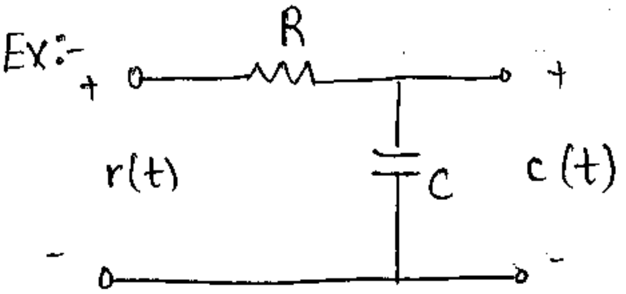
$$c(t) = \left(1 - e^{-t/T}\right) u(t)$$

$$c(t)\Big|_{t=T} = 1 - e^{-1} = 0.63$$

$$c(t)\Big|_{t=4T} = 1 - e^{-4} = 0.98$$

$$\boxed{c(t)\Big|_{t=\infty} = 1 = \text{final value} = \text{Steady state value}}$$

Ex:-



$$T.F. = \frac{c(s)}{r(s)} = \frac{1}{1+sCR}$$

Let $\tau = RC$

$$\frac{C(s)}{R(s)} = \frac{1}{1+s\tau}$$

For U.S.R.

$$R(s) = A/s$$

$$C(s) = \frac{A}{s(1+s\tau)}$$

$$= \frac{1}{\cancel{\tau}}\left[\frac{1}{s} - \frac{1}{s+1/\tau}\right]\frac{1}{1/\tau}$$

$$\boxed{c(t) = A(1-e^{-t/\tau})u(t)}$$

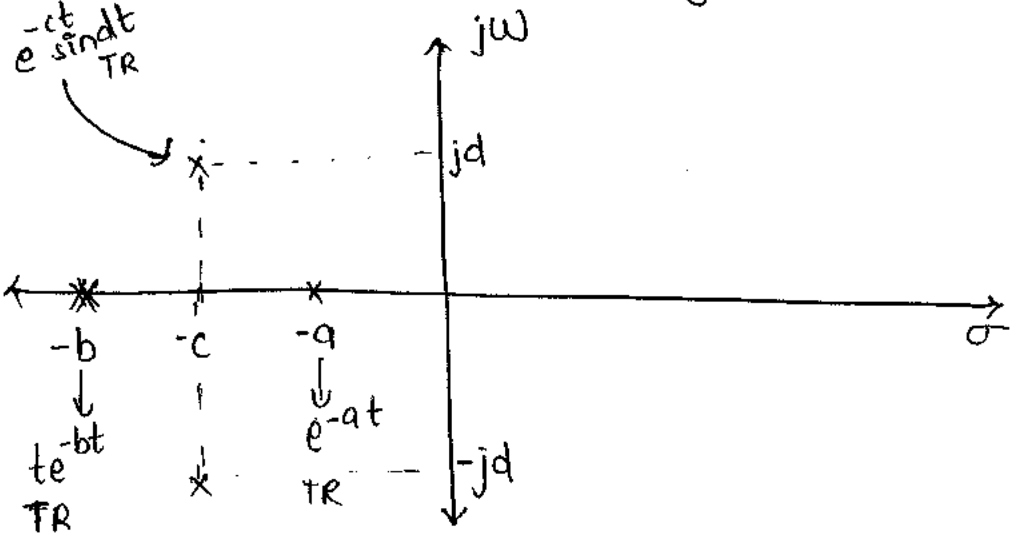
* Transient Response: It is the part of time response that becomes zero for a large value of time

$$\lim_{t\to\infty} C_{tr}(t) = 0$$

The -Transient ~~response~~ part of time constant reveals nature of the response (i.e. oscillotory, nature of damping). and also gives indication about its speed (i.e. time constant)

+ $\boxed{\lim_{t\to\infty} C_{tr}(t) = 0}$

means terms containing $e^{-t/\tau} \longrightarrow$ TR

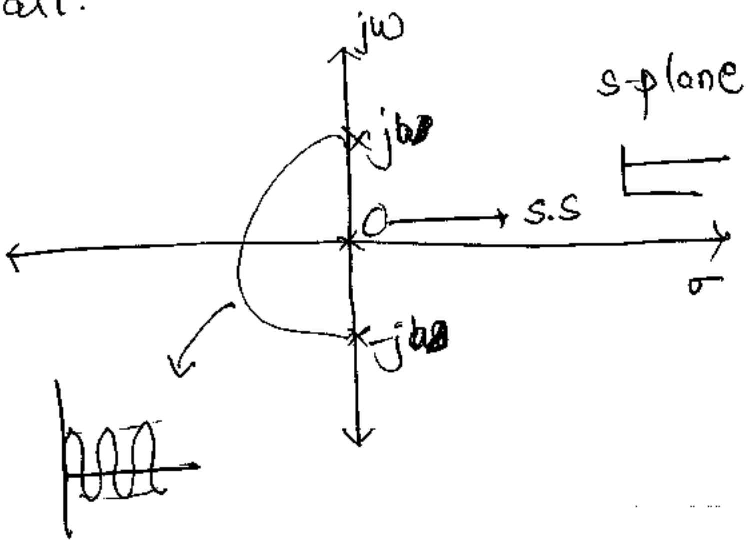


*STEADY STATE PART/STEADY STATE RESPONSE

- It is the system response that remains after the transient response becomes zero.

- The steady state part of time constant reveals the accuracy or steady state error of control system.

- Non-repeated poles lies on imaginary axis gives steady state part.



Non-repeated poles lies on imaginary axis gives steady-state part.

The poles lies on left hand side of s-plane gives transient part

The term which consists exponential decay always gives transient term.

Q:- Find transient and steady state term of given response

$$c(t) = (\underset{\underset{\text{S.S.}}{\downarrow}}{10} + \underset{\underset{\text{S.S.}}{\downarrow}}{2\sin 2t} + \underset{\underset{\text{S.S.}}{\downarrow}}{3\cos 3t} + \underset{\underset{\text{T.r}}{\downarrow}}{4te^{-4t}} + \underset{\underset{\text{Tr}}{\downarrow}}{5e^{-5t}\sin 5t} + \underset{\underset{\text{Tr}}{\downarrow}}{6te^{-6t}\cos 6t})\, u(t)$$

* Standard Test signal

$$\left.\begin{array}{l} \text{① Sudden change} \longrightarrow \text{unit step input} \\ \text{② Velocity type} \longrightarrow \text{ramp input} \\ \text{③ Acceleration type} \longrightarrow \text{parabolic input} \end{array}\right\} \text{Time Domain Analysis}$$

④ Sudden shocks ⟶ impulse input → Stability Analysis

①, ④ ⟶ bounded

②, ③ ⟶ unbounded

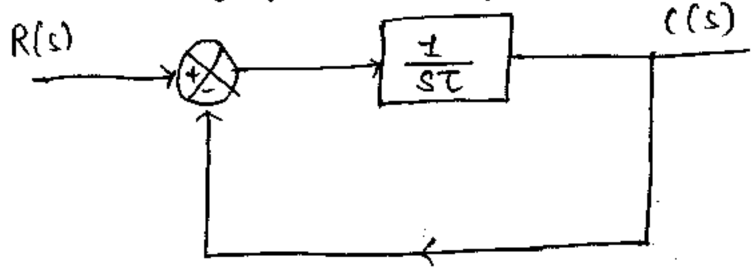
*Time response to the 1^{st} order system

$OLTF = \frac{1}{s\tau}$

Type : 1

Order : 1

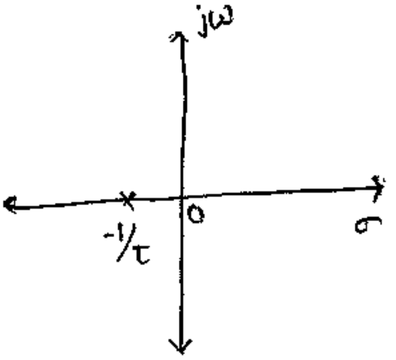
U~~B~~FS :- (unity feedback system)



CLTF :

$$\frac{C(s)}{R(s)} = \frac{1}{1+s\tau}$$

$$C(s) = \frac{R(s)}{1+s\tau}$$



① Impulse Response

⇒ For IR ⇒ $r(t) = \delta(t)$

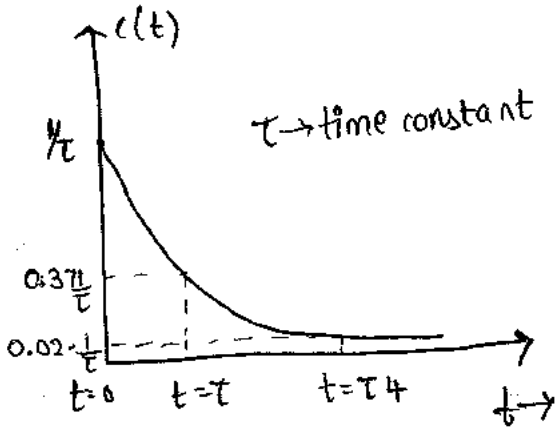
$$R(s) = 1$$

$$C(s) = \frac{1}{1+s\tau} = \frac{1}{\tau} \cdot \frac{1}{s+1/\tau}$$

$$\Big\downarrow L^{-1}$$

$$I.R. = c(t) = \underbrace{\frac{1}{\tau} e^{-t/\tau} u(t)}_{\downarrow}$$

Transient response



τ → time constant

* For stability :-

$$c(t)\big|_{t\to\infty} = \frac{1}{\tau} e^{-\infty} = 0 \longrightarrow \boxed{\text{stable}}$$

* Error :-

$$\underset{\underset{\text{Error}}{\downarrow}}{e(t)} = \underset{\underset{\text{Ref. i/p}}{\downarrow}}{r(t)} - \underset{\underset{\text{Actual o/p}}{\downarrow}}{c(t)}$$

* Error is deviation fom actual o/p to reperence i/p.

$$E[s] = R[s] - C[s]$$

* Steady state error

* Deviation ~~from~~ of the acutal o/p from reference i/p at steady state time.

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$= \lim_{t\to\infty} r(t) - c(t)$$

$$\Big\downarrow L$$

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$e_{ss} = \lim_{s\to 0} s[R(s) - C(s)]$$

For impulse response,

$$e_{ss} = \lim_{t\to\infty} r(t) - c(t)$$

$$e_{ss} = \lim_{t\to\infty} \delta(t) - \frac{1}{\tau} e^{-t/\tau} u(t)$$

$$e_{ss} = 0$$

But actually it is not possible

① Bcz. there is no steady state response in impulse response.

② Unit step Response

For U.S.R. = $u(t) = r(t)$

$$R(s) = 1/s$$

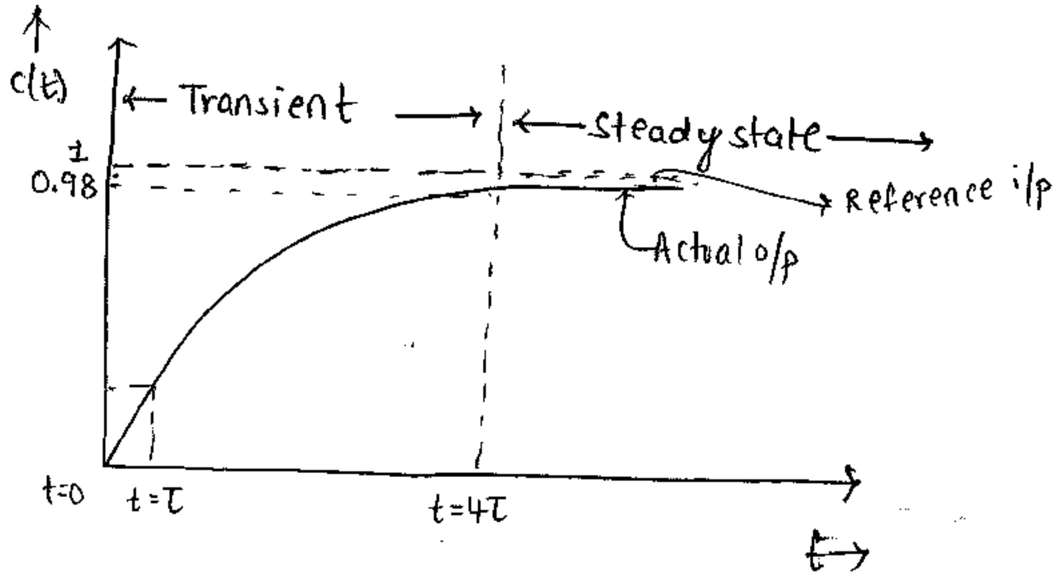
$$C(s) = \frac{1}{(1+s\tau)\cdot s}$$

$$= \frac{A}{S} + \frac{B}{(S+1/\tau)\tau}$$

$$= \frac{1}{\tau}\left[\frac{A}{s} + \frac{B}{s+1/\tau}\right]$$

$$= \frac{1}{\tau}\left[\frac{\tau}{s} - \frac{\tau}{s+1/\tau}\right]$$

$$c(t) = \left(1 - e^{-t/\tau}\right)u(t)$$



$$\left[\begin{array}{l}\text{But at } t\to\infty \\ r(t) = c(t) \;\therefore e_{ss} = 0\end{array}\right]$$

$$c(t) = \underbrace{u(t)}_{\text{S.S.}} - \underbrace{u(t)e^{-t/\tau}}_{\text{T.S.}}$$

* Steady state error

$$e_{ss} = \lim_{t\to\infty} \overset{r(t)\,-\,c(t)}{\cancel{u(t)} - \cancel{u(t)}} + e^{-t/\tau}u(t)$$

$$\boxed{e_{ss} = 0}$$

③ Unit Ramp Response

For U.R.R. $r(t) = tu(t)$

$$R(s) = \frac{1}{s^2}$$

$$c(s) = \frac{1}{s^2(s+1/\tau)\tau}$$

$$= \frac{1}{\tau}\left[\frac{B}{s^2} + \frac{A}{s} + \frac{C}{(s+1/\tau)}\right]$$

$$= \frac{1}{\tau}\left[\frac{-\tau^2}{s} + \frac{\tau}{s^2} + \frac{\tau^2}{(s+1/\tau)}\right]$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1/\tau} =$$

$$As(s+1/\tau) + B(s+1/\tau) + Cs^2 = 1$$

At $s = -1/\tau$

$$C\frac{1}{\tau^2} = 1$$

$$\boxed{C = \tau^2}$$

$$B(1/\tau) = 1$$

$$\boxed{B = \tau}$$

$$C + A = 0$$

$$\boxed{A = -\tau^2}$$

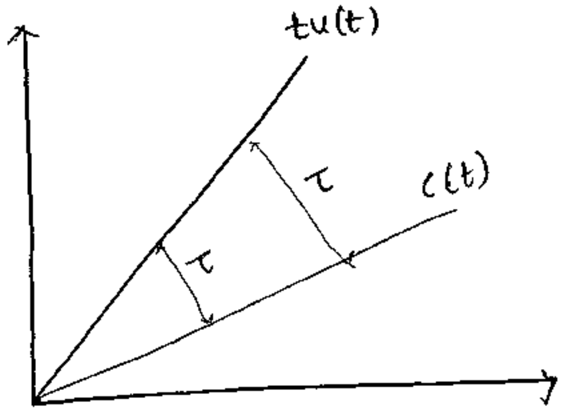
$$c(s) = -\frac{\tau}{s} + \frac{1}{s^2} + \frac{\tau}{s+1/\tau}$$

$$\Big\downarrow L^{-1}$$

$$c(t) = \left(-\tau + t + \tau e^{-t/\tau}\right)u(t)$$

t	$c(t)$	$r(t)$	$e(t)$
0	0	0	
1	$e^{-1} = 0.37$	1	
2	$1 + e^{-2} = 1.35$	2	
3	$2 + e^{-8} = 2.05$	3	
4	$3 + e^{-4} =$	4	$r(t) - c(t)$
5	$4 + e^{-5} \cong 4$	5	1
6	$5 + e^{-6} \cong 5$	6	1 $\Big\}\,\tau$
7	$6 + e^{-7} \cong 6$	7	1

Let $\tau = 1$ sec.



* Steady state error

$$e_{ss} = r(t) - c(t)$$

$$= t - \left(-\tau + t + \tau e^{-t/\tau}\right)$$

$$e_{ss} = \tau - \tau e^{-t/\tau}$$

$$\boxed{e_{ss}\big|_{t\to\infty} = \tau} = \text{constant}$$

④ Unit Parabolic response

$r(t) = A \cdot \frac{t^2}{2} u(t)$

$R(s) = \frac{A}{s^3}$

$C(s) = \frac{1}{s^3(1+s\tau)}$

$= \frac{1}{\tau s^3(s+1/\tau)}$

$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{(s+1/\tau)} = \frac{1}{\tau}$

$(s+1/\tau)As^2 + Bs(s+1/\tau) + C(s+1/\tau) + Ds^3 = \frac{1}{\tau}$

At $s = -1/\tau$

$\boxed{D = -\tau^2}$

At $s = 0$

$C(1/\tau) = \frac{1}{\tau}$

$\boxed{C = 1}$

$C(s) = \frac{\tau^2}{s} - \frac{\tau}{s^2} + \frac{1}{s^3} - \frac{\tau^2}{(s+1/\tau)}$

$\Big\downarrow L^{-1}$

$c(t) = \left(\tau^2 - t\tau + \frac{t^2}{2} - \tau^2 e^{-t/\tau}\right) u(t)$

$A + D = 0$

$\boxed{A = \tau^2}$

$\frac{1}{\tau} A + B = 0$

$B = \frac{-A}{\tau} = \frac{-\tau^2}{\tau} = -\tau$

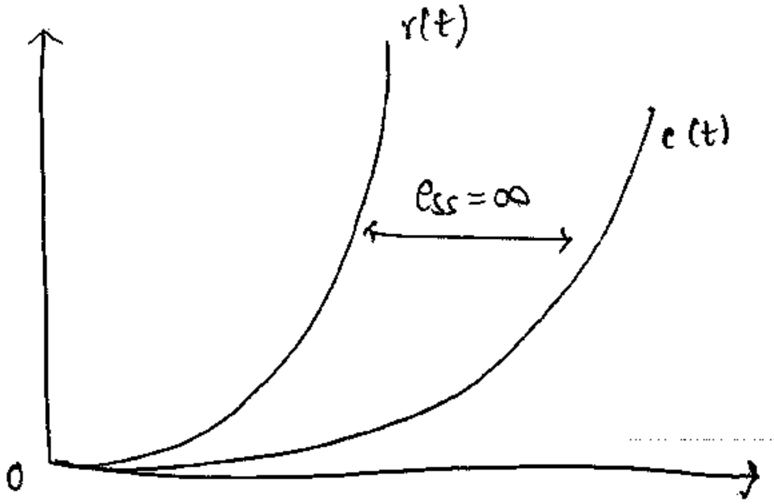
*Steady state error

$e_{ss} = \lim_{t\to\infty} r(t) - c(t)$

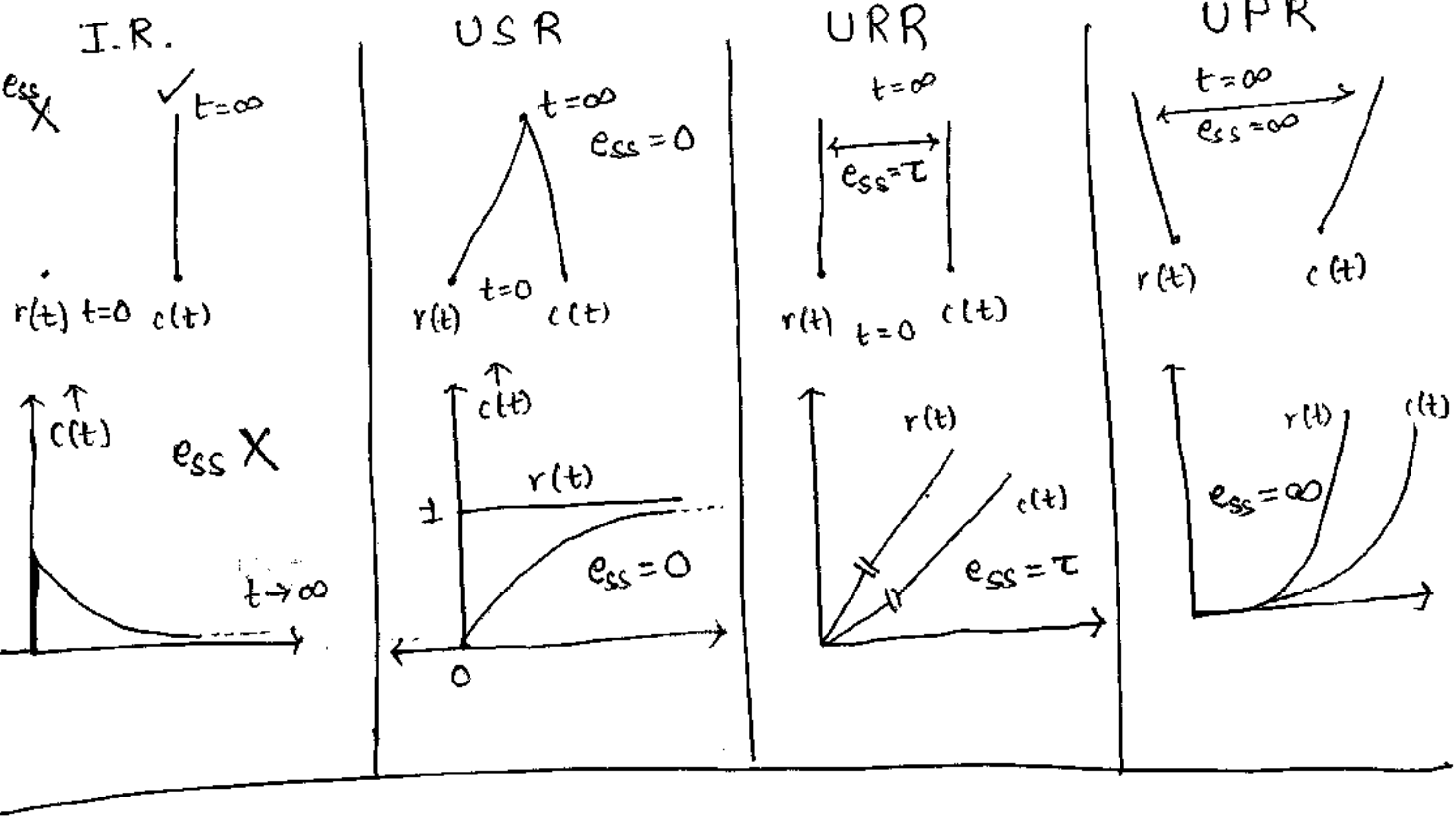
$= \lim_{t\to\infty} \cancel{\frac{t^2}{2}} \cancel{u}(t) - \left[\tau^2 - t\tau + \cancel{\frac{t^2}{2}} - \tau^2 e^{-t/\tau}\right] u(t)$

$= \lim_{t\to\infty} \left(\tau^2 + t\tau + \tau^2 e^{-t/\tau}\right) u(t)$

$\boxed{e_{ss} = \infty}$



Time Response to 1^{st} order system



- Practical ckt for first order system is simple RC or RL circuit
- The impulse response doesn't consists any steady state term Hence we can not define steady state error for impulse response. The impulse input doesn't exists at t→∞ hence we can't compare actual o/p to reference i/p at t→∞

Time Response to 2^{nd} order system

$$O.L.T.F. = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

ξ → zeta

ω_n = natural freq. of oscillation/undamped oscillation

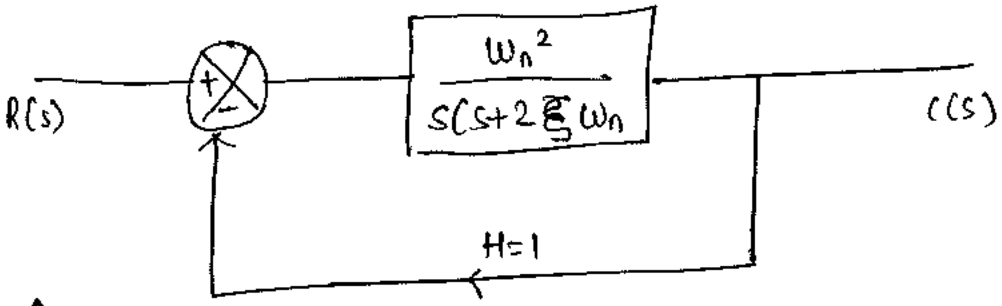
ξ = damping ratio

$\xi\omega_n$ = damping factor

Order = 2

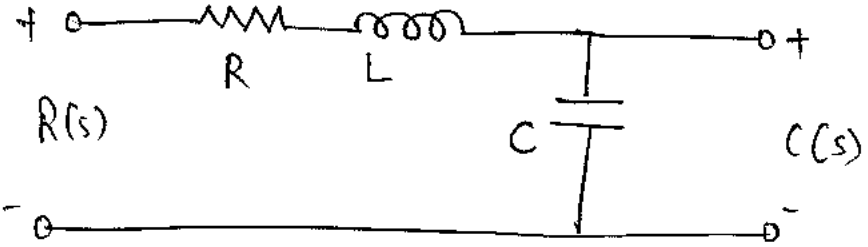
Type = 1

Unity feedback System



$$\boxed{C.L.T.F. = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{C(s)}{R(s)}}$$

Practical ckt for 2^{nd} order system is series RLC circuit.



$$\frac{C(s)}{R(s)} = \frac{1/sC}{R+sL+\frac{1}{sC}}$$

$$= \frac{1}{LCs^2+sCR+1}$$

$$\frac{C(s)}{R(s)} = \frac{1/LC}{s^2+s\frac{R}{L}+\frac{1}{LC}}$$

$$\omega_n = \frac{1}{\sqrt{LC}} \text{ rad/sec.}$$

$$2\xi\omega_n = \frac{R}{L}$$

$$\xi\omega_n = \frac{R}{2L}$$

$$\xi = \frac{R}{2} \cdot \frac{1}{L} \cdot \frac{1}{\omega_n}$$

$$\xi = \frac{R}{2} \cdot \frac{1}{L} \sqrt{LC}$$

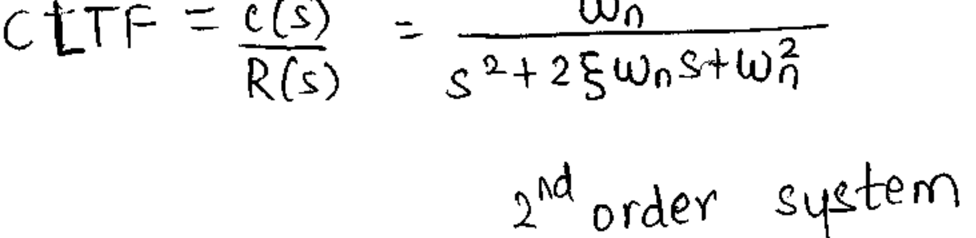
$$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

$$Q = \frac{1}{2f} = \frac{\text{energy used}}{\text{energy store}}$$

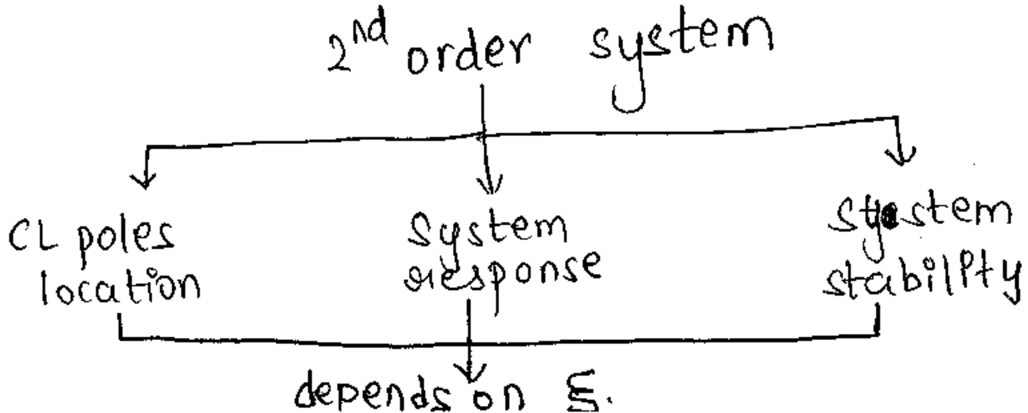
$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

quality factor

Time Response to the 2nd order system



$$\text{CLTF} = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$



The characteristics eqⁿ is:-

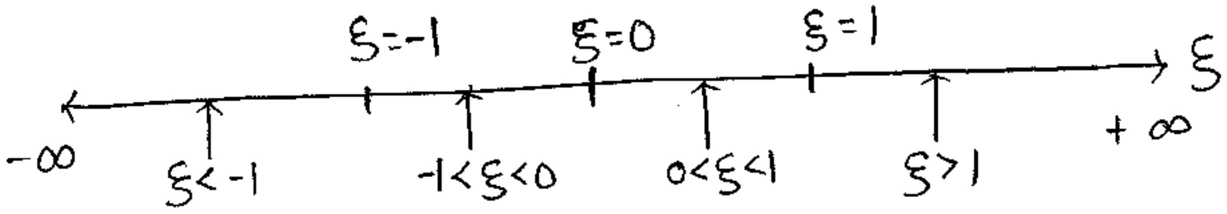
$$1+GH(s)=0$$

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

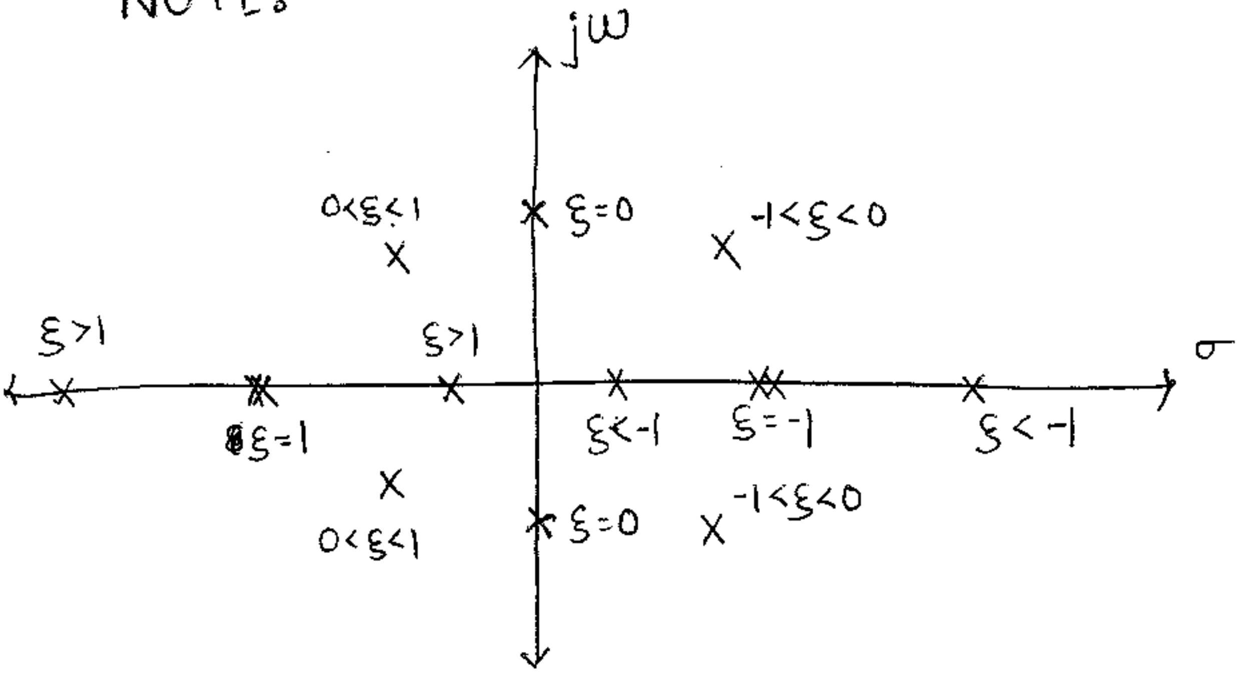
$$s_1, s_2 = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2-4\omega_n^2}}{2\cdot 1}$$

$$\boxed{s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}}$$



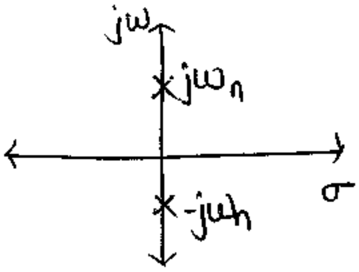
NOTE:-



① $\xi = 0$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$$s_1, s_2 = \pm j\omega_n$$



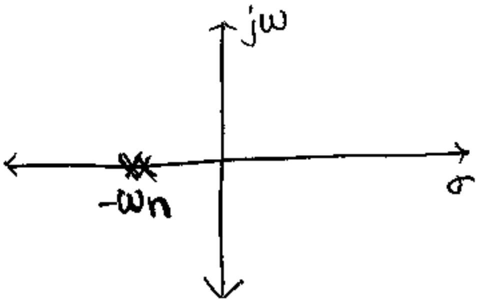
Marginal stable

② $\xi = 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$s_1, s_2 = -\omega_n, -\omega_n$$

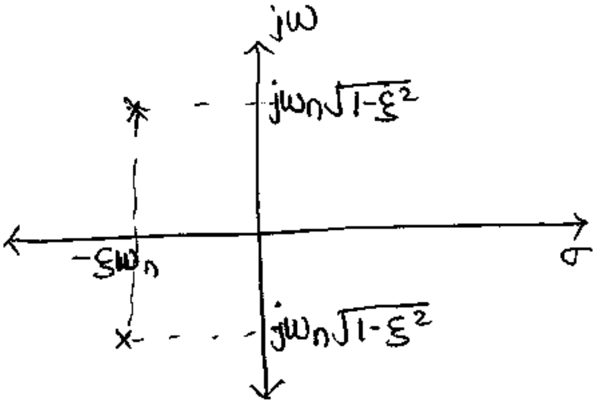


stable

③ $0 < \xi < 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$s_1, s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



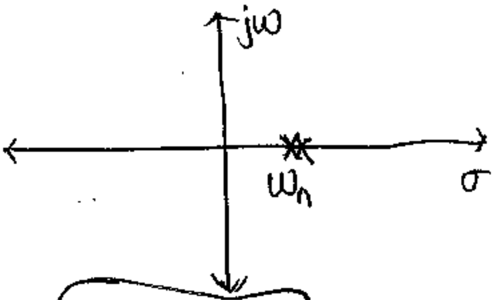
stable

④ $\xi = -1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2-2\omega_n s+\omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s-\omega_n)^2}$$

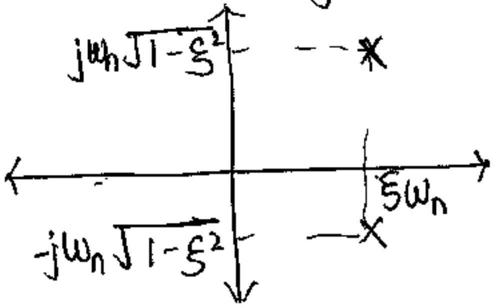
$$s_1, s_2 = +\omega_n, +\omega_n$$



Unstable

⑤ $-1 < \xi < 0$

$$s_1, s_2 = \xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

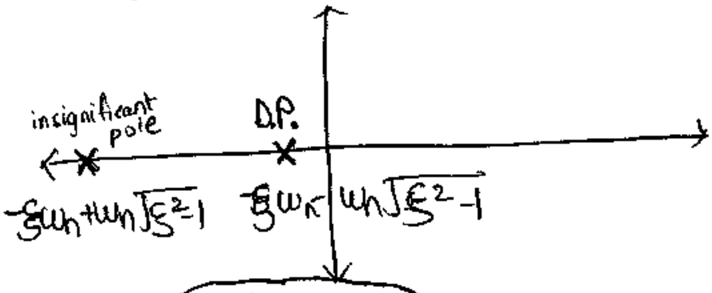


unstable

⑥ $\xi > 1$

$$s_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2-1}$$

$$s_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2-1}$$

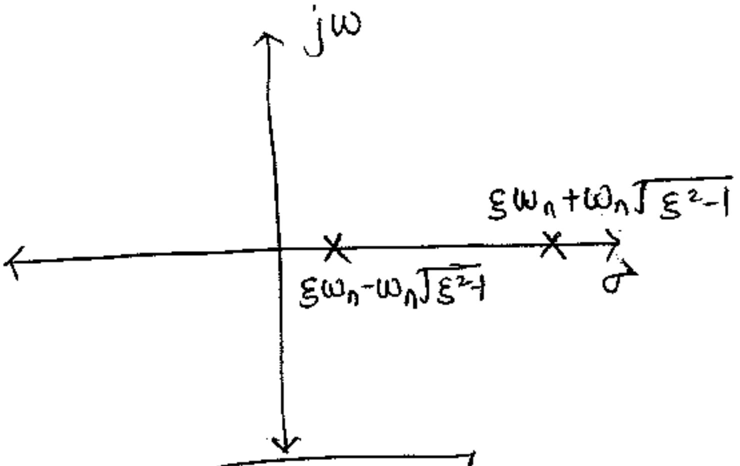


stable

(7) $\xi < -1$

$s_1 = \xi\omega_n + \omega_n\sqrt{\xi^2-1}$

$s_2 = \xi\omega_n - \omega_n\sqrt{\xi^2-1}$



$\boxed{\text{unstable}}$

① Impulse Response

For I.R. ⇒ $r(t) = \delta(t)$

$R(s) = 1$

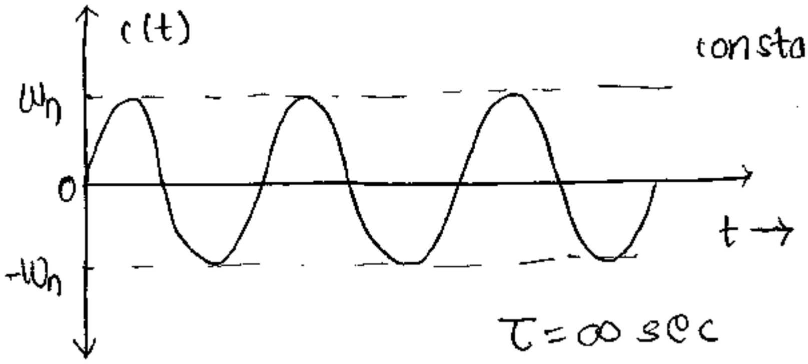
$$C(s) = \frac{\omega_n^2}{s^2+2s\xi\omega_n+\omega_n^2}$$

① $\xi = 0$ ⟶ Undamped system

$$C(s) = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$\downarrow L^{-1}$

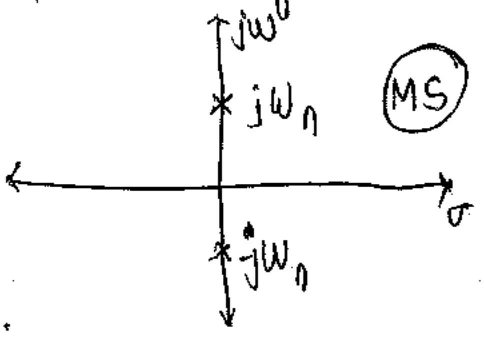
$c(t) = (\omega_n \sin\omega_n t)u(t)$



constant amplitude and f.o.o. frequency of oscillation

$\tau = \infty$ sec

$\text{foo}_{us} = \omega_n$ rad/sec.



② $0<\xi<1$ $\Rightarrow$ Underdamped system

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n)^2 + (\omega_n\sqrt{1-\xi^2})}$$

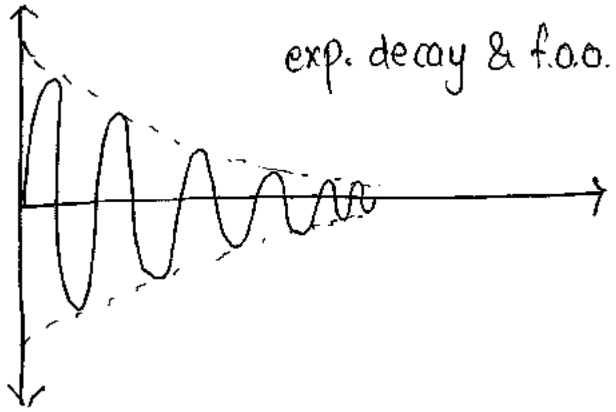
$$s_1, s_2 = -\xi\omega_n \pm j\,\underbrace{\omega_n\sqrt{1-\xi^2}}_{\omega_d}$$

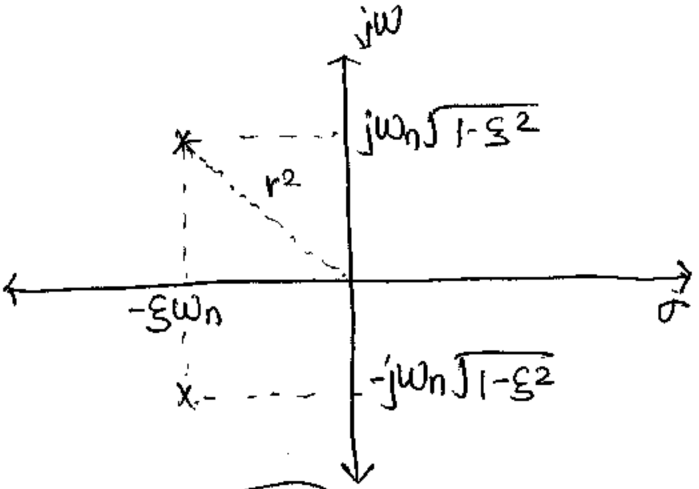
$$\omega_d = \omega_n\sqrt{1-\xi^2} \Rightarrow \text{Damped oscillation frequency} \qquad \boxed{\omega_n > \omega_d}$$

$$C(s) = \frac{\omega_n}{\sqrt{1-\xi^2}} \times \frac{\omega_n\sqrt{1-\xi^2}}{(s+\xi\omega_n)^2 + \omega_d^2}$$

$$\Big\downarrow L^{-1}$$

$$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t} \sin\omega_d t \cdot u(t)$$





stable

$\tau_{ocs} = \frac{1}{\xi\omega_n}$ sec.

$r^2 = \xi^2\omega_n^2 + \omega_n^2(1 \overset{-}{\cancel{+}} \xi^2)$

$r^2 = \omega_n^2$

$\boxed{r = \omega_n}$

$f.o.o = \omega_n\sqrt{1-\xi^2} = \omega_d$

③ $\xi = 1 \Rightarrow$ Critically damped system

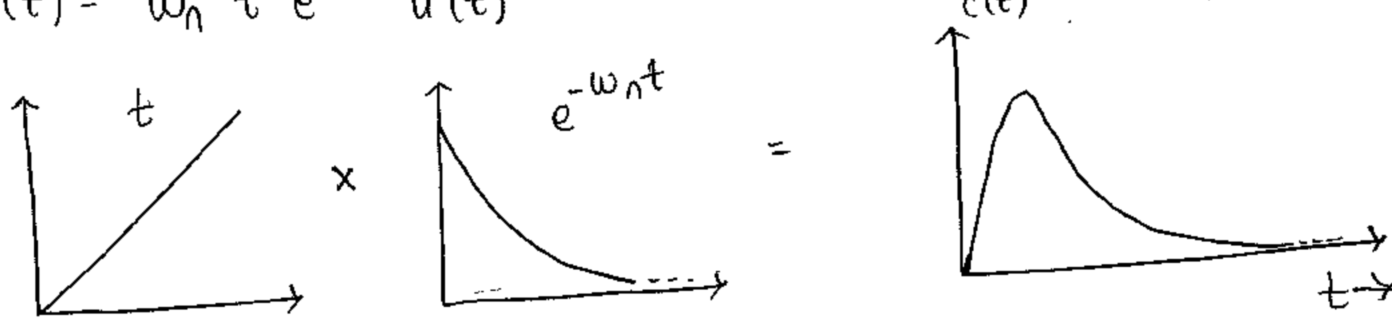
$$c(s) = \frac{\omega_n^2}{s + 2\xi\omega_n s + \omega_n^2}$$

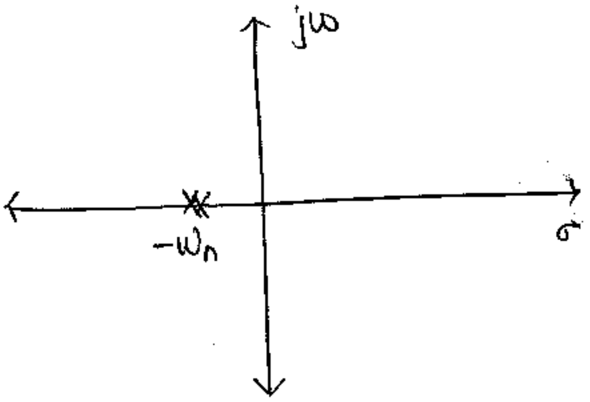
$$c(s) = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$$\Big\updownarrow L^{-1}$$

$$c(t) = \omega_n t e^{-\omega_n t} u(t)$$

$$c(t) = \omega_n \, t \, e^{-\omega_n t} u(t)$$





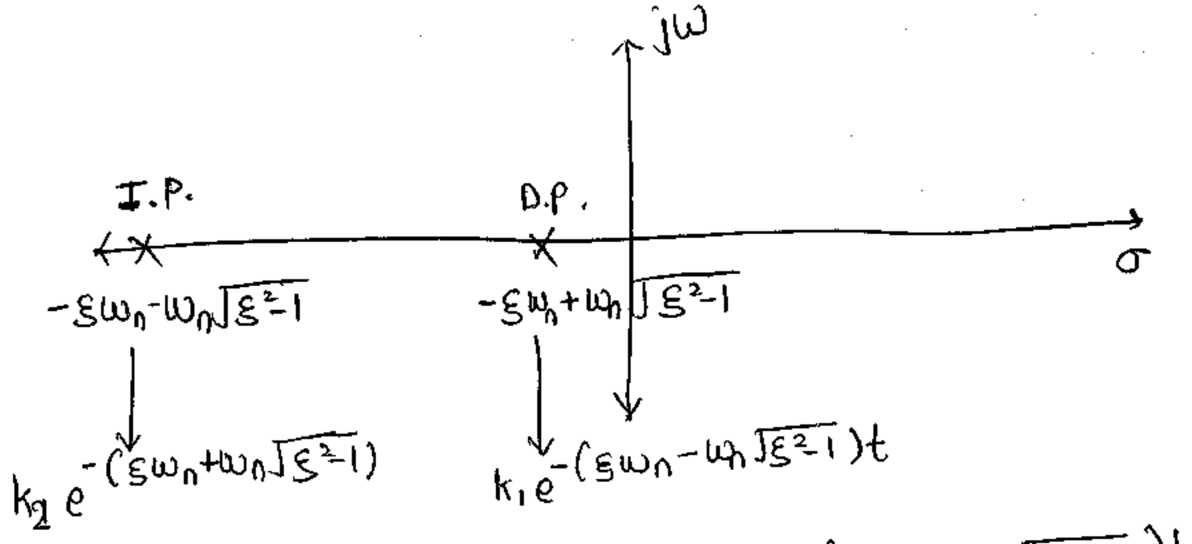
$$\boxed{\tau_{co} = \frac{1}{\omega_n}}$$

$$\boxed{f.o.o = 0}$$

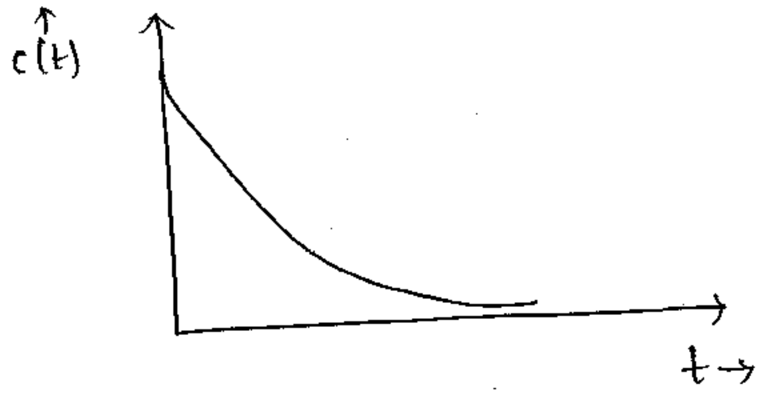
④ $\xi > 1 \Rightarrow$ Over damped system

$$c(s) = \frac{\omega_n^2}{\underbrace{(s + \xi\omega_n + \omega_n\sqrt{\xi^2-1})}_{a}\underbrace{(s + \xi\omega_n - \omega_n\sqrt{\xi^2-1})}_{b}}$$

$$= \frac{k_1}{s + \xi\omega_n + \omega_n\sqrt{\xi^2-1}} + \frac{k_2}{s + \xi\omega_n - \omega_n\sqrt{\xi^2-1}}$$



$$c(t)=K_1 e^{-(\xi\omega_n-\omega_n\sqrt{\xi^2-1})t}+k_2 e^{-(\xi\omega_n+\omega_n\sqrt{\xi^2-1})t}$$



$$\tau_{overdamped}=\frac{1}{\xi\omega_n-\omega_n\sqrt{\xi^2-1}}$$

NOTE:-

foo

$\xi=0 \rightarrow$ Undamped system $\longrightarrow$ $\tau_{undamped}=\infty$, ω_n

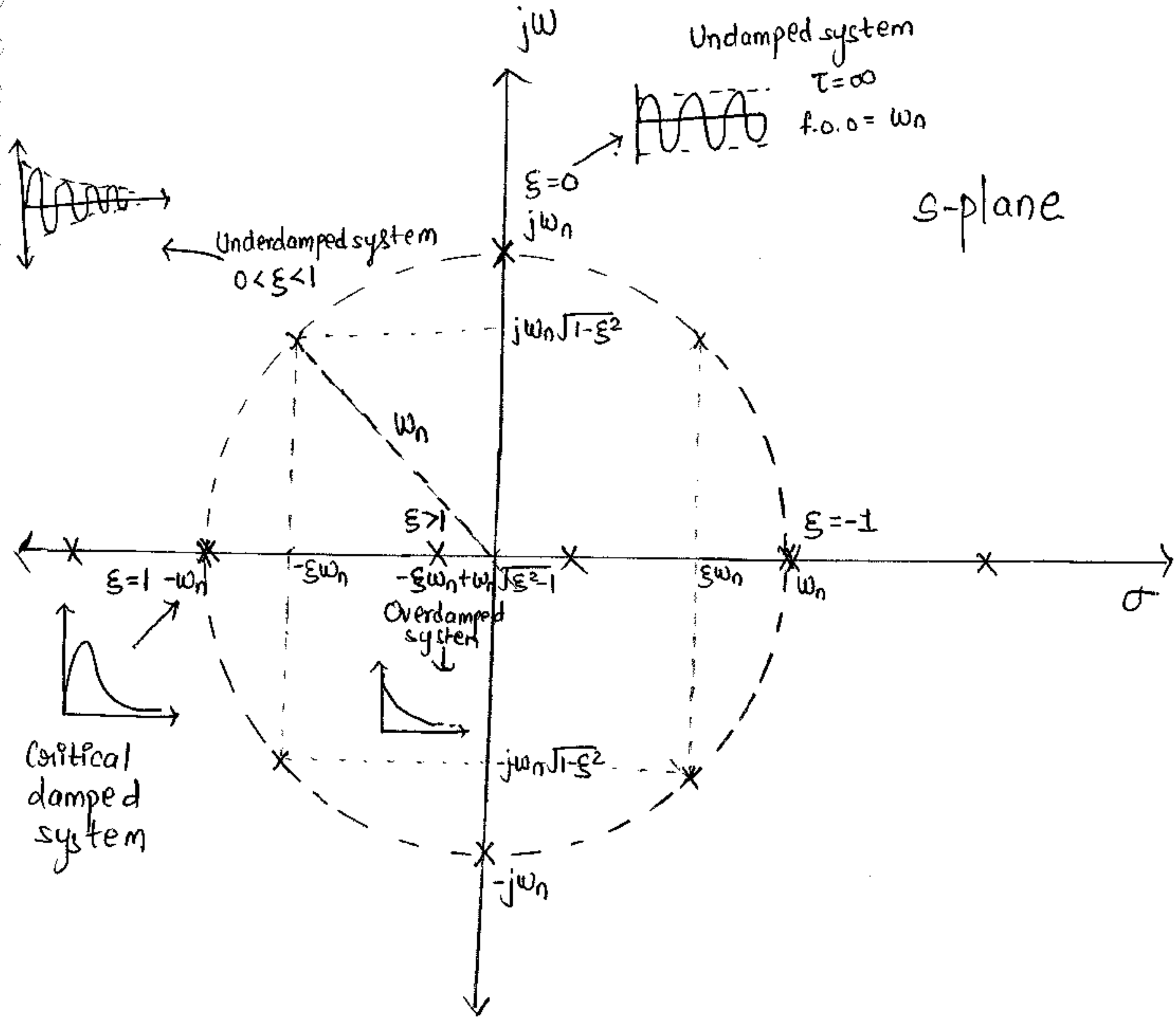
$0<\xi<1 \rightarrow$ Underdamped system $\longrightarrow$ $\tau_{underdamped}=\frac{1}{\xi\omega_n}$, $\omega_n\sqrt{1-\xi^2}$

$\xi=1 \rightarrow$ Critically damped system $\longrightarrow$ $\tau_{criticallydamped}=\frac{1}{\omega_n}$, 0

$\xi>1 \rightarrow$ Overdamped system $\longrightarrow$ $\tau_{overdamped}=\frac{1}{\xi\omega_n-\omega_n\sqrt{\xi^2-1}}$, 0

$\tau_{undamped} > \tau_{overdamped} > \tau_{underdamped} > \tau_{critical\ damped}$

NOTE:-



- As we move from $\xi = -1$ to $\xi = 1$, the pole location will make circle with radius ω_n.

② Unit-step Response

① $\xi = 0 \Rightarrow$ Undamped system

$R(s) = 1/s$

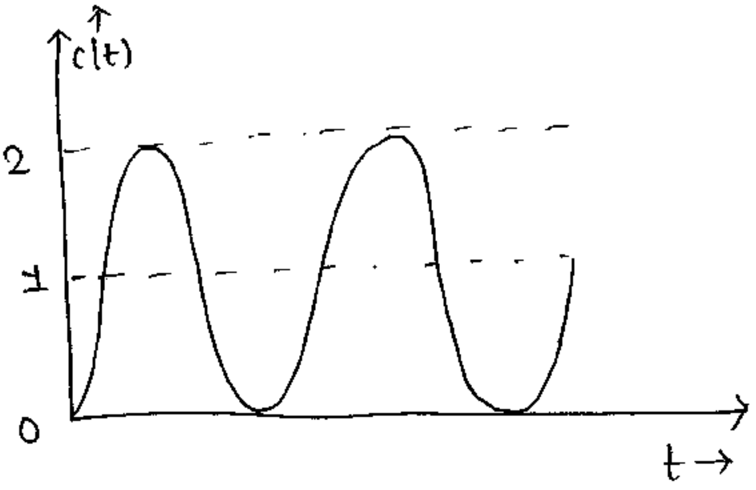
$$C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)}$$

$$C(s) = \frac{A}{s} + \frac{Bs+C}{s^2+\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}$$

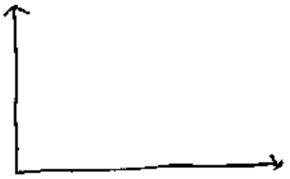
$\Big\downarrow L^{-1}$

$$c(t) = (1 - \cos\omega_n t)\,u(t)$$



③ Ramp response

④ Parabolic response



② $0 < \xi < 1 \Rightarrow$ Underdamped system

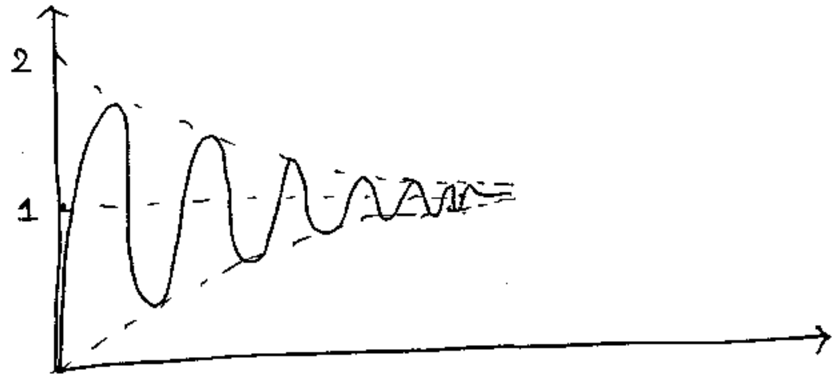
$R(s) = 1/s$

$$C(s) = \frac{1}{s\left[(s+\xi\omega_n)^2 + (\omega_n\sqrt{1-\xi^2})^2\right]}$$

$$C(s) = \frac{A}{s} + \frac{Bs+C}{(s+\xi\omega_n)^2+(\omega_n\sqrt{1-\xi^2})^2}$$

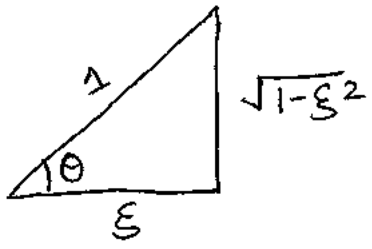
$\Big\downarrow L^{-1}$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)}$$



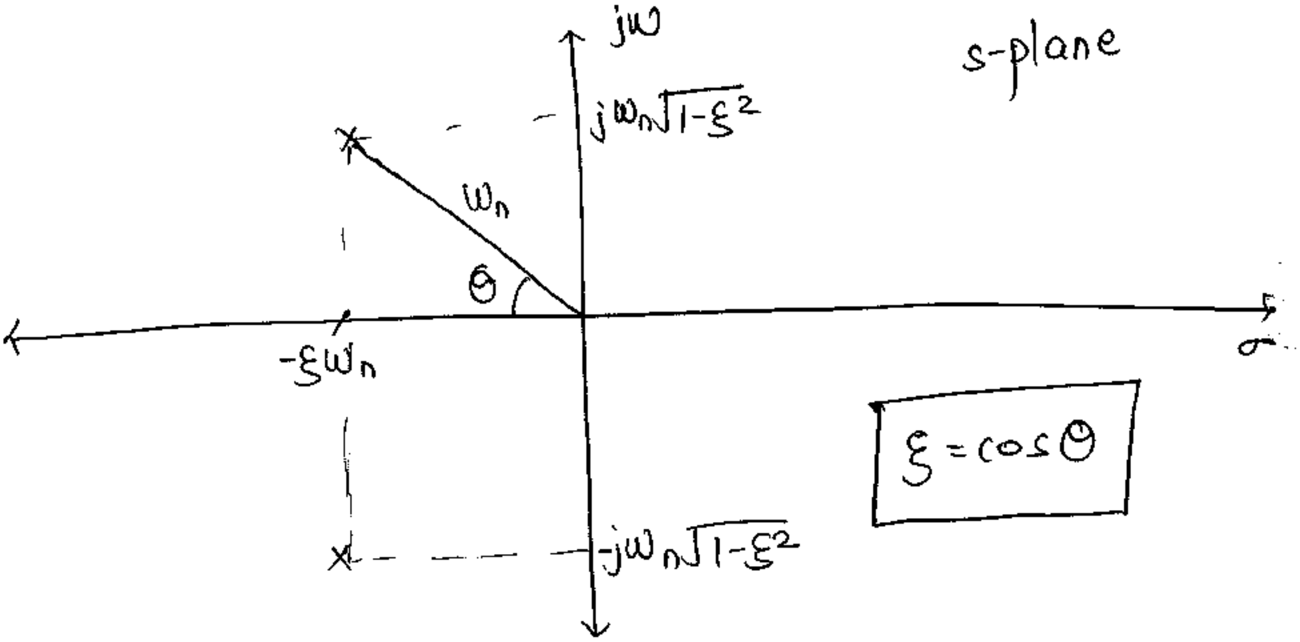
$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$$



$$\tan\theta = \frac{opp.}{adj} = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\boxed{\cos\theta = \xi}$$



s-plane

$$\boxed{\xi = \cos\theta}$$

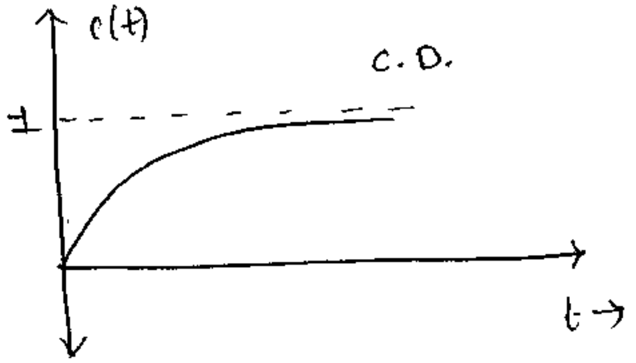
③ $\xi = 1 \Rightarrow$ critical damped system

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$C(s) = \frac{A}{s} + \frac{B}{(s+\omega_n)^2} + \frac{C}{(s+\omega_n)}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$c(t) = \left(1 - \omega_n t\, e^{-\omega_n t} + e^{-\omega_n t}\right) u(t)$$



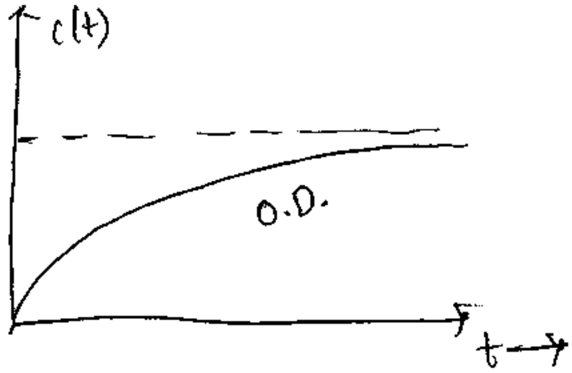
④ $\xi > 1 \Rightarrow$ overdamped system

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n\xi+\omega_n\sqrt{\xi^2-1})\cdot(s+\xi\omega_n-\omega_n\sqrt{\xi^2-1})}$$

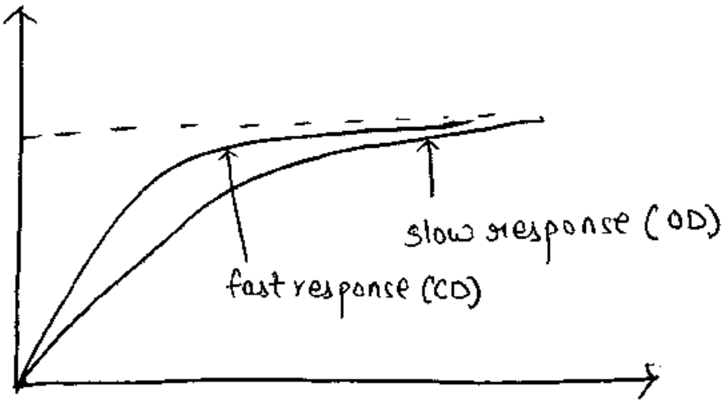
$$C(s) = \frac{A}{s} + \frac{B}{s+\xi\omega_n+\omega_n\sqrt{\xi^2-1}} + \frac{C}{s+\xi\omega_n-\omega_n\sqrt{\xi^2-1}}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$c(t) = 1 + k_1 e^{-(+\xi\omega_n+\omega_n\sqrt{\xi^2-1})t} + k_2 e^{-(+\xi\omega_n-\omega_n\sqrt{\xi^2-1})t}$$



Q:- Identify from below figure which is overdamped & critical damped system



Now, as we know

$$\tau_{OD.} > \tau_{CD}$$

* IMP points to remember

1. The second order system response completely depends on value of ξ.

2. The second order system is stable for all the value positive value of ξ bcz. poles lies on the left hand side of s-plane for $\xi > 0$.

3. The second order system is unstable for all the value negative value of ξ bcz. poles lies on the right hand side of s-plane for

4. The second order system is marginally stable for $\xi = 0$ bcz. the poles lies on imaginary axis of s-plane at $\xi = 0$.

5. When $\xi=0$, the second order system response is constant amplitude and frequency of oscillation which are called undamped oscillation.

6. Any system which produces undamped oscillation is called undamped system and system becomes marginal stable.

7. When $0<\xi<1$, the closed loop poles lies on left hand side of s-plane which are complex conjugate the system is stable and system response is exponential decay and frequency of oscillation.

8. Any system which produces damped oscillation is called underdamped system..

9. When $\xi=1$ both the poles lies on negative real axis at same location and system is stable. The system response is called critical damped response bcz. it generates critically only one damped oscillation.

10. The value of resistance use to get critical damped nature is called critical resistance.

11. When $\xi>1$ both the poles lies on left hand side of s-plane at different location on real axis and system is stable. The system response is called overdamped system | response bcz. system components overcomes the damped oscillation.

12. When ξ increases from -1 to 1, the second order system poles path is circle with radius of ω_n

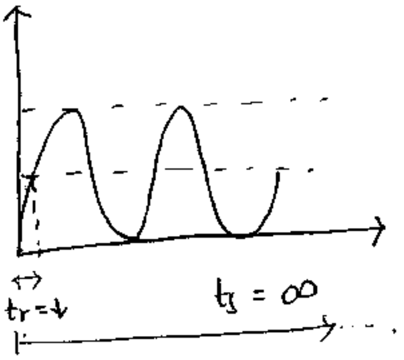
13. Radial distance ~~from~~ of complex poles from origin is W_n.

14. When ξ increases from 0 to 1, the poles moves towards left and near to real axis.

15. When ξ increases from 1 to ∞, then one poles move towards origin on real axis and second pole move towards the $-\infty$ on real axis

Input	tr	s.s.	stability
Impulse	✓	✗	✓ (Practically/bounded)
Unit step	✓	✓	✓
Unit ramp	✓	✓	✗
Unit Parabola	✓	✓	✗

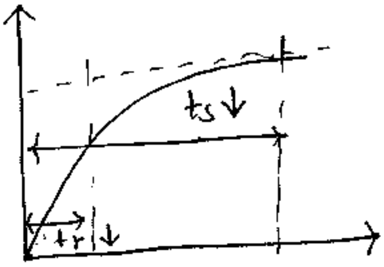
① $\xi = 0$ → Undamped system



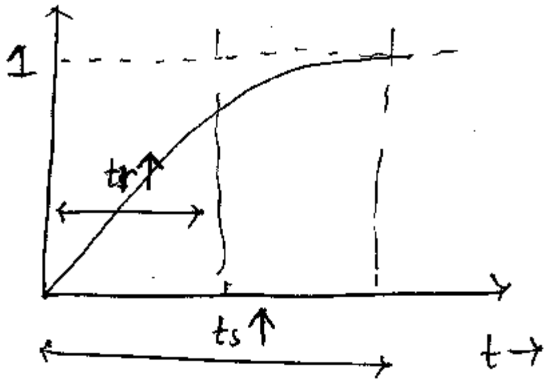
rise time is less

while setting time is infinite

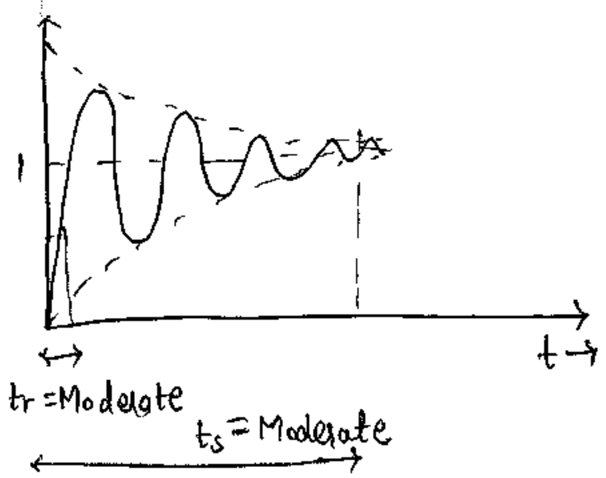
② $\xi = 1$ → Critical damped system



③ $\xi > 1$ → overdamped system



④ $0 < \xi < 1$ → underdamped system



① To find system response = impulse

② To find behaviour of the system w.r.t time = Unit step

Time Domain Specifications

- For time domain specification, select underdamped system because

(1) If we select undampedsy. then the t_r is very small and t_s is infinite.

(2) If we select critical damped system then t_r is large and t_s is very small

(3) If we select over damped system then t_r is large and t_s is also large.

- Proctically any system requires moderate t_r & t_s

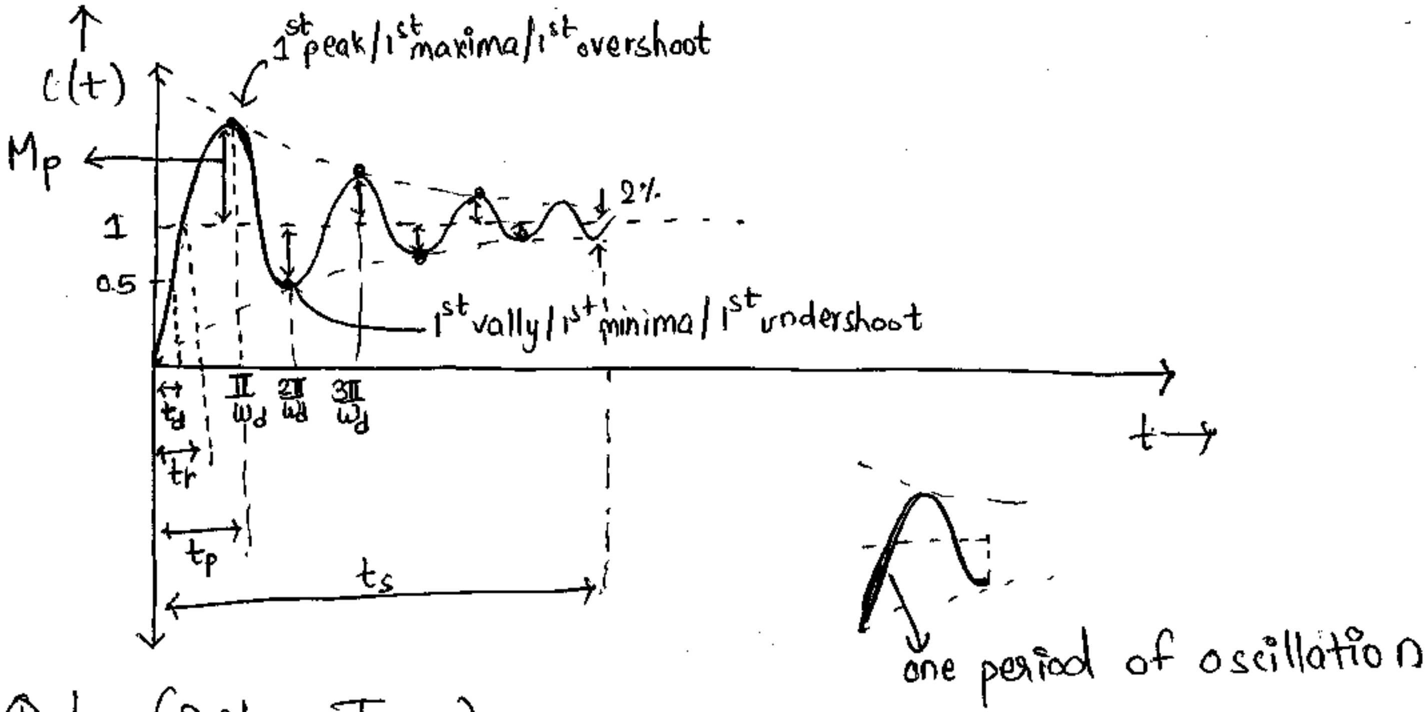
(4) In <u>underdamped system</u> we can get <u>moderate values of t_r and t_s</u>. In underdamped system the best range of ξ is $0.4 < \xi < 0.7$.

$$0.4 < \xi < 0.7$$

When $0 < \xi < 1$

The unit step response

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \cos^{-1} \xi)$$



① t_d (Delay Time)

- Time required to (rise) reach from 0% to 50% of its final value.

$$c(t)\big|_{t=t_d} = 0.5$$

$$\boxed{t_d = \frac{1+0.7\xi}{\omega_n}}$$

② t_r (Rise time)

- It is the time required to rise from
  - 0% to 100% ⟶ underdamped system
  - 5% to 95% ⟶ critical damped system
  - 10% to 90% ⟶ over damped system

$$c(t)\big|_{t=0} = 0$$

$$c(t)\big|_{t=tr} = 1$$

$$\boxed{t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}}$$

③ Peak time (t_p)

It is the time required for system response to rise from 0 to peak's of the system response is called peak time.

$$\boxed{t_p = \frac{n\pi}{\omega_d}}$$

By default, $t_p = \frac{\pi}{\omega_d}$

Now, $t_p = \frac{n\pi}{\omega_d}$

$n=1 \rightarrow$ 1st peak
$n=2 \rightarrow$ 1st vally
$n=3 \rightarrow$ 2nd peak
$n=4 \rightarrow$ 2nd vally

For 2nd peak $t_p = \frac{3\pi}{\omega_d}$

odd → peak
even → vally

④ Peak overshoot (M_p):

It is the difference betn time response peak and steady state value.

$$\boxed{M_p = c(t_p) - c(\infty)}$$

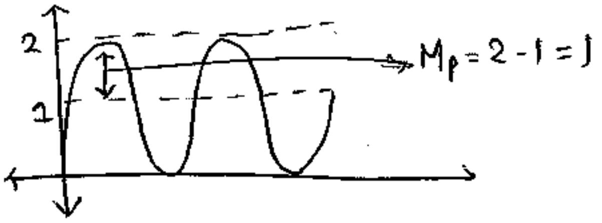
⑤ % Peak overshoot (%M_p)

$$\boxed{\%M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%}$$

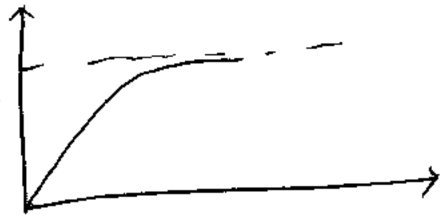
$$\boxed{\%M_p = e^{\frac{-n\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%}$$

n = odd ⇒ for peak value
n = even ⇒ for vally value

If $\xi = 0 \longrightarrow M_p = 100\%$



If $\xi = 1 \longrightarrow M_p = 0\%$



NOTE:-

As we increase ξ from 0 to 1 peak overshoot decreases 100% to 0%.

⑥ T_{osc} (Time period of one oscillation)

$$\omega_d t = \frac{2\pi}{T} t$$

$$\boxed{T_{osc} = \frac{2\pi}{\omega_d}}$$

⑦ t_s

- It is the time required for system response to rise from 0% to specified tolerance break is called settling time.

$$0\% \longrightarrow t_s = 5\tau = \frac{5}{\xi\omega_n}$$

$$\boxed{2\% \longrightarrow t_s = 4\tau = \frac{4}{\xi\omega_n}}$$ Bydefault

$$5\% \longrightarrow t_s = 3\tau = \frac{3}{\xi\omega_n}$$

$$\boxed{t_s = \frac{4}{\xi\omega_n}}$$

⑧ No. of oscillation

$$\boxed{N_{osc} = \frac{t_s}{T_{osc}}}$$

SUMMARY:-

$t_d = \frac{1+0.7\xi}{\omega_n}$ → delay time

$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$ → rise time

$t_p = \frac{n\pi}{\omega_d}$ → peak time

$\% M_p = e^{\frac{-n\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$ → Peak overshoot

$T_{osc} = \frac{2\pi}{\omega_d}$ → time period of 1 oscillation

$t_s = \frac{4}{\xi\omega_n}$ → settling time

$N_{osc} = \frac{t_s}{T_{osc}}$ → no. of oscillation

Q:- Find time domain specifications:-

$$G[s] = \frac{25}{s(s+4)}$$

$$H[s] = 1$$

$$T.F. = \frac{G(s)}{1-G(s)H(s)}$$

$$= \frac{25}{s(s+4)+25}$$

$$T.F. = \frac{25}{s^2+4s+25}$$

Compairing with $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$\boxed{\omega_n = 5}$, $2\xi\omega_n = 4$

$$\xi = \frac{4}{5 \cdot 2} = 0.4$$

$$\boxed{\xi = 0.4}$$

① $t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.28}{5} = 0.256$ seconds

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5\sqrt{1-0.16}$$

$$\boxed{\omega_d = 4.5825 \text{ rad/sec.}}$$

② $t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d}$

$= \frac{\pi - 1.159}{4.5825}$

$t_r = 0.4325$ seconds

③ $t_p = \frac{\pi}{\omega_d} = 0.6854$ seconds

④ $\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$

$= e^{-\frac{1.2564}{0.9165}} \times 100\%$

$= e^{-1.3708} \times 100\%$

$= 0.2539 \times 100\%$

$\% M_p = 25.39\%$

⑤ $T_{osc} = \frac{2\pi}{\omega_d} = 1.37$ seconds

⑥ $t_s = \frac{4}{\xi \omega_n} = 2.18$ seconds

⑦ $N_{osc} = \frac{t_s}{T_{osc}} = \frac{2.18}{1.37} = 1.59$

Q:- Repeat the above problem where y is o/p & x is input

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8X$$

$$s^2 Y(s) + 4sY(s) + 8Y(s) = 8X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{8}{s^2+4s+8}$$

Comparing $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$$\omega_n^2 = 8$$

$$\boxed{\omega_n = 2\sqrt{2} = 2.828 \text{ rad/sec.}}$$

$$2\xi\omega_n = 4$$

$$\boxed{\xi = \frac{4}{2\times 2.828} = 0.707}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\boxed{\omega_d = 1.999 \text{ rad/sec.}}$$

(1) $t_d = \frac{1+0.7\xi}{\omega_n} = 0.528$ sec.

(2) $t_p = \frac{\pi}{\omega_d} = 1.571$ sec.

(3) $t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} = 1.178$ sec.

(4) $\% M_p = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \times 100\%$

$$= e^{\frac{-2.2206}{0.707}} \times 100$$

$$= e^{-3.1408} \times 100 = 4.32\%$$

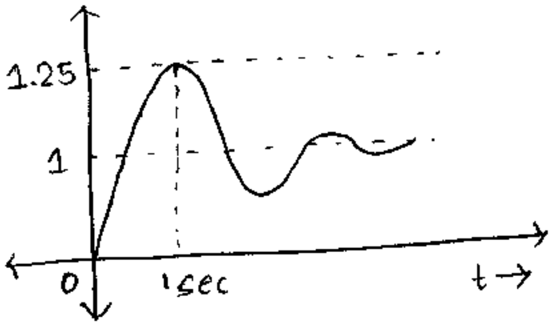
(5) $T_{osc} = \frac{2\pi}{\omega_d} = 3.142$ sec.

(6) $t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.707 \cdot 2.828} = 2.0$ sec.

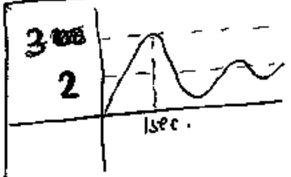
(7) $N_{osc} = \frac{t_s}{T_{osc}} = \frac{2}{3.142} = 0.636$.

Q:- Calculate ξ, ω_n, t_d, t_s, OLTF, CLTF, M_p, $\% M_p$

Assume unity feedback system.



NOTE:-



$M_p = \frac{3-2}{2} = \frac{1}{2} = 0.5$

$\% M_p = 50\%$

⇒ It is underdamped system

$\boxed{M_p = 1.25 - 1 = 0.25}$

$\% M_p = 25\%$

$M_p = e^{\frac{-n\pi\xi}{\sqrt{1-\xi^2}}}$

$e^{\frac{\pi\xi}{\sqrt{1-\xi^2}}} = \frac{1}{0.25} = 4$

$\frac{\pi\xi}{\sqrt{1-\xi^2}} = \ln(4)$

$(\pi\xi)^2 = (1.386)^2 (1-\xi^2)$

$\pi^2\xi^2 = 1.922(1) - 1.922\xi^2$

$\xi^2(11.799) = 1.992$

$\boxed{\xi = 0.411}$

$t_p = \frac{\pi}{\omega_d} = 1$

$\boxed{\omega_d = \pi \text{ rad/sec.}}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$= \omega_n\sqrt{0.8311}$

$\pi = \omega_n(0.9116)$

$\boxed{\omega_n = 3.44 \text{ rad/sec.}}$

$\boxed{t_d = \frac{1+0.7\xi}{\omega_n} = 0.37 \text{ sec.}}$

$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} = \frac{\pi - 1.159}{\pi}$

$\boxed{t_r = 0.63 \text{ sec}}$

$$t_s = 4\tau = \frac{4}{\xi\omega_n} = \frac{4}{0.4 \cdot 3.43} = 2.9\,sec.$$

$$T_{osc} = \frac{2\pi}{\omega_d} = 2\,sec.$$

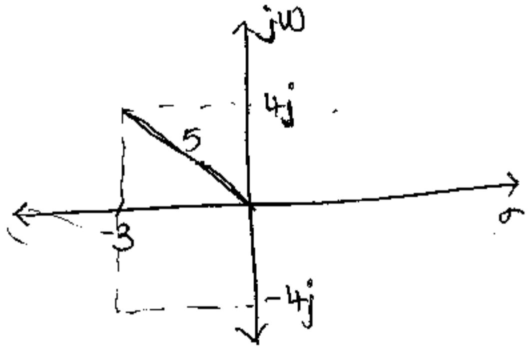
$$OLTF = \frac{\omega_n^2}{s(s+2\xi\omega_n)} = \frac{11.8}{s(s+2.77)}$$

$$CLTF = \frac{11.8}{s^2+2.77s+11.8}$$

Q:- The impulse response of system is $ke^{-3t}\sin 4t\,u(t)$

Find following factors:- $t_s, \omega_n, \xi, \%M_p, t_d, t_r, t_p$

$$c(s) = \frac{k \cdot 4}{(s+3)^2+(4)^2}$$



$$\boxed{\omega_n = 5\ rad/sec}$$

$$3 = \xi\omega_n$$

$$\xi = \frac{3}{\omega_n} = \frac{3}{5}$$

$$\boxed{\xi = 0.6}$$

$$\tau = \frac{-1}{\text{dominant pole real part}} = \frac{-1}{-3} = \frac{1}{3}$$

$$\tau = 1/3\ sec.$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5(0.8)$$

$$\omega_d = 4\ rad/sec.$$

$$\%M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

$$= e^{\frac{-1.885}{0.8}} \times 100$$

$$= e^{-2.356} \times 100$$

$$= 0.09479 \times 100$$

$$\boxed{\%M_p = 9.48\%.}$$

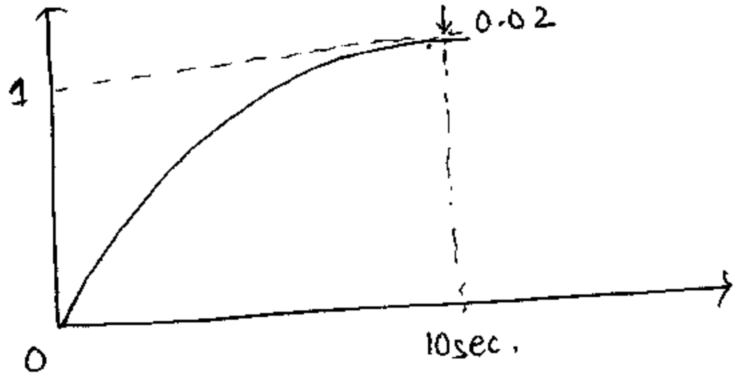
$$t_d = \frac{1+0.7\xi}{\omega_n} = 0.284sec.$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} = \frac{\pi - 0.927}{4} \approx 0.55 sec.$$

$$t_p = \frac{\pi}{\omega_d} = 0.785sec.$$

✯ ✯

Q:- The unit step response is shown in figure find following factor. Find $\tau, t_d, t_p, t_r, t_s, M_p,$



Sol:- $t_s = 10sec.$

$t_s = 4\tau$

$\tau = \frac{t_s}{4} = \frac{10}{4} = 2.5$

$\boxed{\tau = 2.5sec.}$

$\boxed{t_p = 0\ sec}$ not defined

M_p = not defined

By default, we have to go for underdamped system

$c(t) = k(1-e^{-t/\tau})\,u(t)$

$c(t) = (1-e^{-t/\tau})\,u(t)$

① t_d

$$c(t)\big|_{t=t_d} = \left(1-e^{-t_d/\tau}\right)u(t) = 0.5$$

$$e^{-t_d/\tau} = 1-0.5 = 0.5$$

$$e^{-t_d/\tau} = \frac{1}{2}$$

$$\frac{t_d}{\tau} = \ln 2$$

$$t_d = \tau \ln 2$$

$$t_d = 2.5\cdot(0.693)$$

$$\boxed{t_d = 1.73\text{sec.}}$$

[illegible]

~~$t_r = 0.693\tau$~~

② t_r

$$t_r = t_2 - t_1$$

$$c(t)\big|_{t=t_1} = 0.1 = 1-e^{-t_1/\tau}$$

$$c(t)\big|_{t=t_2} = 0.9 = 1-e^{-t_2/\tau}$$

$$e^{-t_1/\tau} = 0.9$$

$$t_1 = \tau\ln\left(\frac{1}{0.9}\right) = 0.10\tau$$

$$t_2 = \tau\ln\left(\frac{1}{0.1}\right) = 2.30\tau$$

$$t_r = t_2 - t_1 = 2.30\tau - 0.10\tau = 2.20\tau = 5.5\text{sec.}$$

Q:- Find the $\%M_p$ of following system for unit step input.

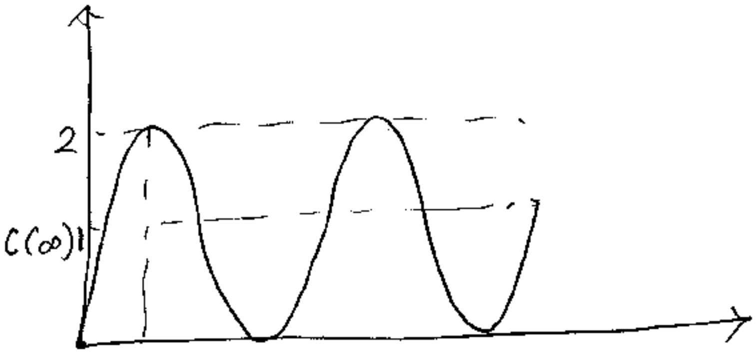
$$\frac{C(s)}{R(s)} = \frac{25}{s^2+25}$$

Sol^n:

$\xi = 0$

$\% M_p = 100\%$

$C(t) = 1 - \cos \omega_n t$

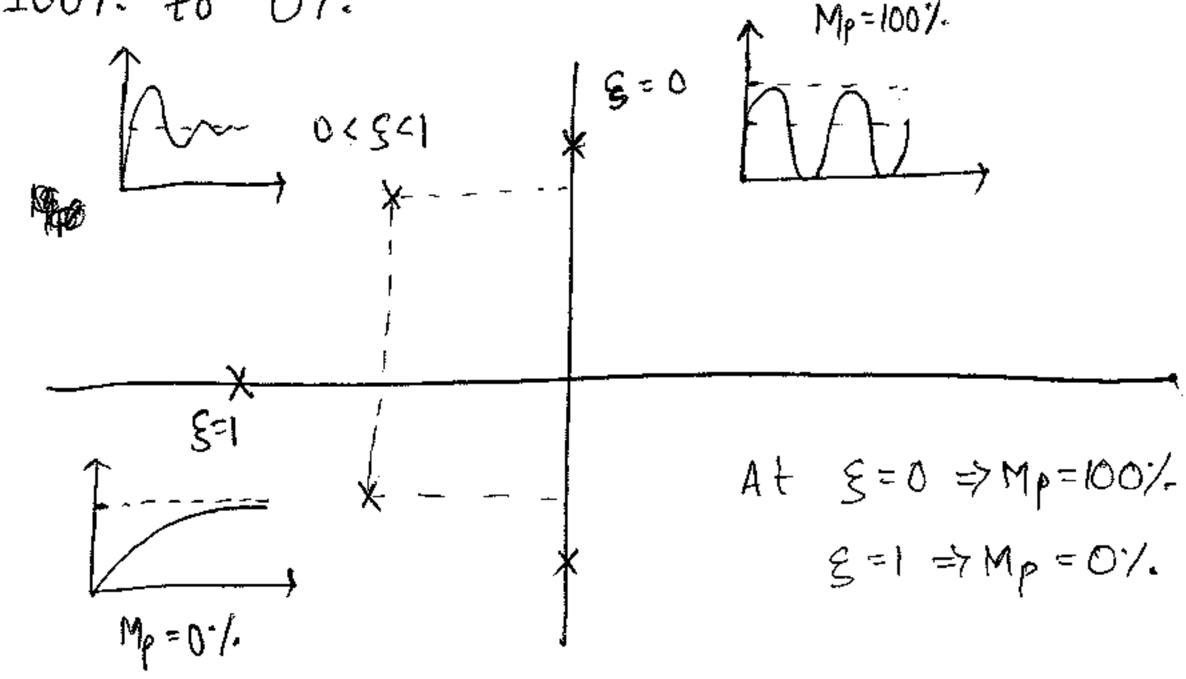


$$\% M_p = \frac{2-1}{1} \times 100$$

$$\boxed{\% M_p = 100\%}$$

NOTE:-

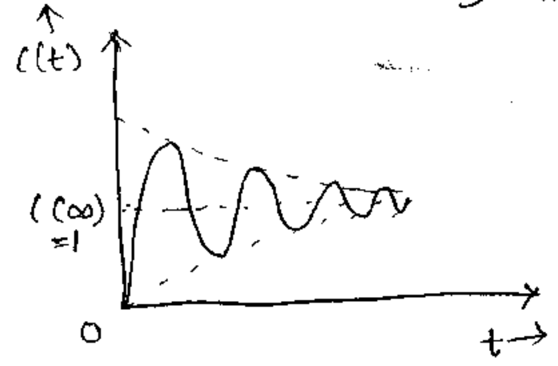
When ξ increases from 0 to 1, M_p decreases from 100% to 0%.



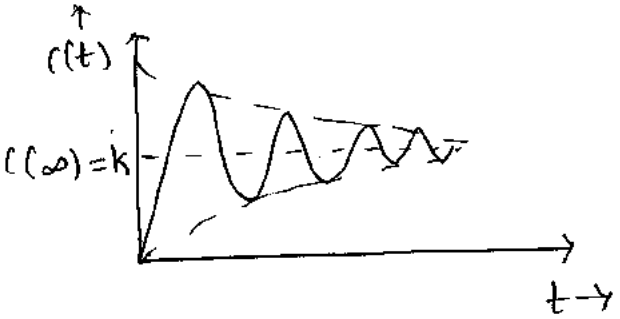
At $\xi = 0 \Rightarrow M_p = 100\%$

$\xi = 1 \Rightarrow M_p = 0\%$

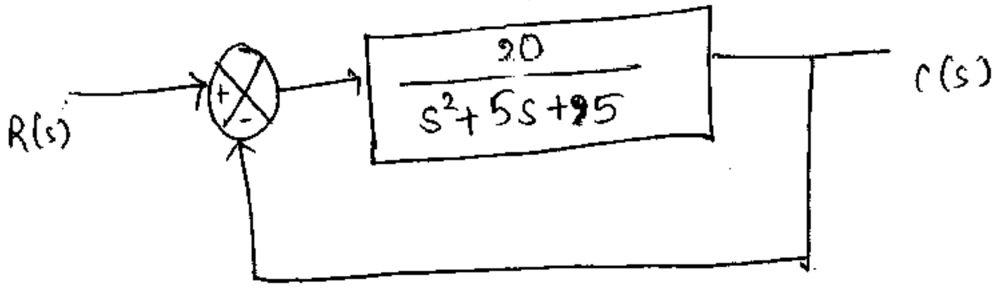
Q $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$



$\frac{C(s)}{R(s)} = k \cdot \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$



Q:- Find time-domain specifications:-



$\text{CLTF} = \frac{C(s)}{R(s)} = \frac{20}{s^2+5s+5+20} = \frac{20}{s^2+5s+25}$

$c(\infty) = \frac{20}{25} = 0.8$

$\boxed{\omega_n = 5 \text{ rad/sec.}}$

$2\xi\omega_n = 5$

$\boxed{\xi = 0.5}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$= 5\,(0.866)$

$\boxed{\omega_d = }$ 4.33 rad/sec

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

$$= e^{\frac{-1.571}{0.866}} \times 100$$

$$= e^{-1.8141} \times 100$$

$$=0.16\overset{3}{0}0 \times 100$$

$$\boxed{M_p = 16.30\%}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7\cdot0.5}{5} = 0.27 \text{ sec.}$$

$t_s = 1.6$ sec.

$$t_p = \frac{\pi}{\omega_d} = \quad = 0.725 \text{ sec.}$$

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d} = \frac{3.141 - 1.047}{4.33} = 0.484 \text{ sec.}$$

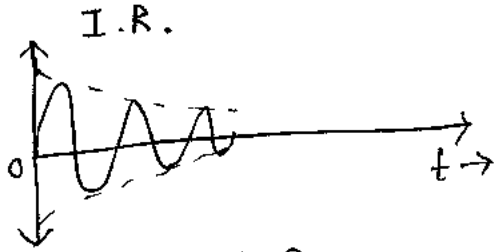
Steady state response

① Impulse response

$R(s) = 1$

$$c(\infty) = \lim_{s\to0} s\cdot1\,\frac{20}{s^2+5s+25}$$

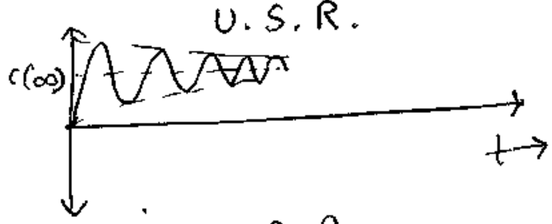
$c(\infty) = 0$



② Unit step response

$$c(\infty) = \lim_{s\to0} s\cdot\frac{1}{s}\,\frac{20}{s^2+5s+25}$$

$c(\infty) = 0.8$



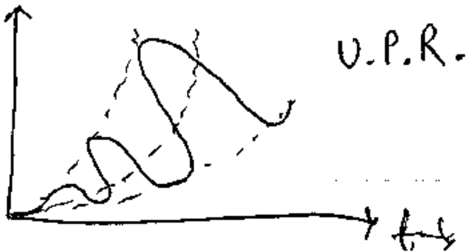
③ Unit ramp response

$$c(\infty) = \lim_{s\to0} s\cdot\frac{1}{s^2}\cdot\frac{20}{s^2+5s+25}$$

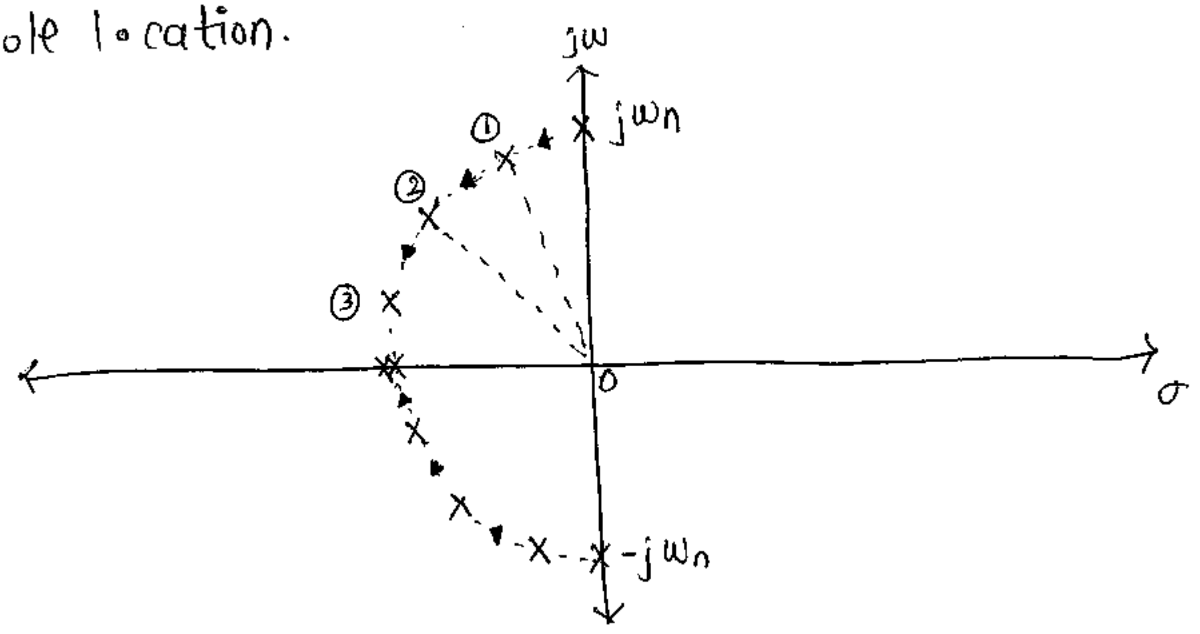
$c(\infty) = \infty$

④ Unit parabola response

$c(\infty) = \infty$



*Variation in time-domain specifications w.r.t to variation in pole location.



$\tau\downarrow \longrightarrow t_s\downarrow$

$\uparrow\cos\theta\downarrow = \xi\uparrow \longrightarrow M_p\downarrow$

$\uparrow t_d = \frac{1+0.7\xi\uparrow}{\omega_n\ (\text{constant})}$

$\uparrow t_r = \frac{\pi-\theta\downarrow}{\omega_d\downarrow}$

$\uparrow t_p = \frac{\pi}{\omega_d\downarrow}$

$\uparrow T_{osc} = \frac{2\pi}{\omega_d\downarrow}$

$\downarrow N_{osc} = \frac{t_s\downarrow}{T_{osc}\uparrow}$

① $t_d = \frac{1+0.7\xi}{\omega_n}$

② $t_r = \frac{\pi-\cos^{-1}\xi}{\omega_d}$

③ $t_p = \frac{\pi}{\omega_d}$

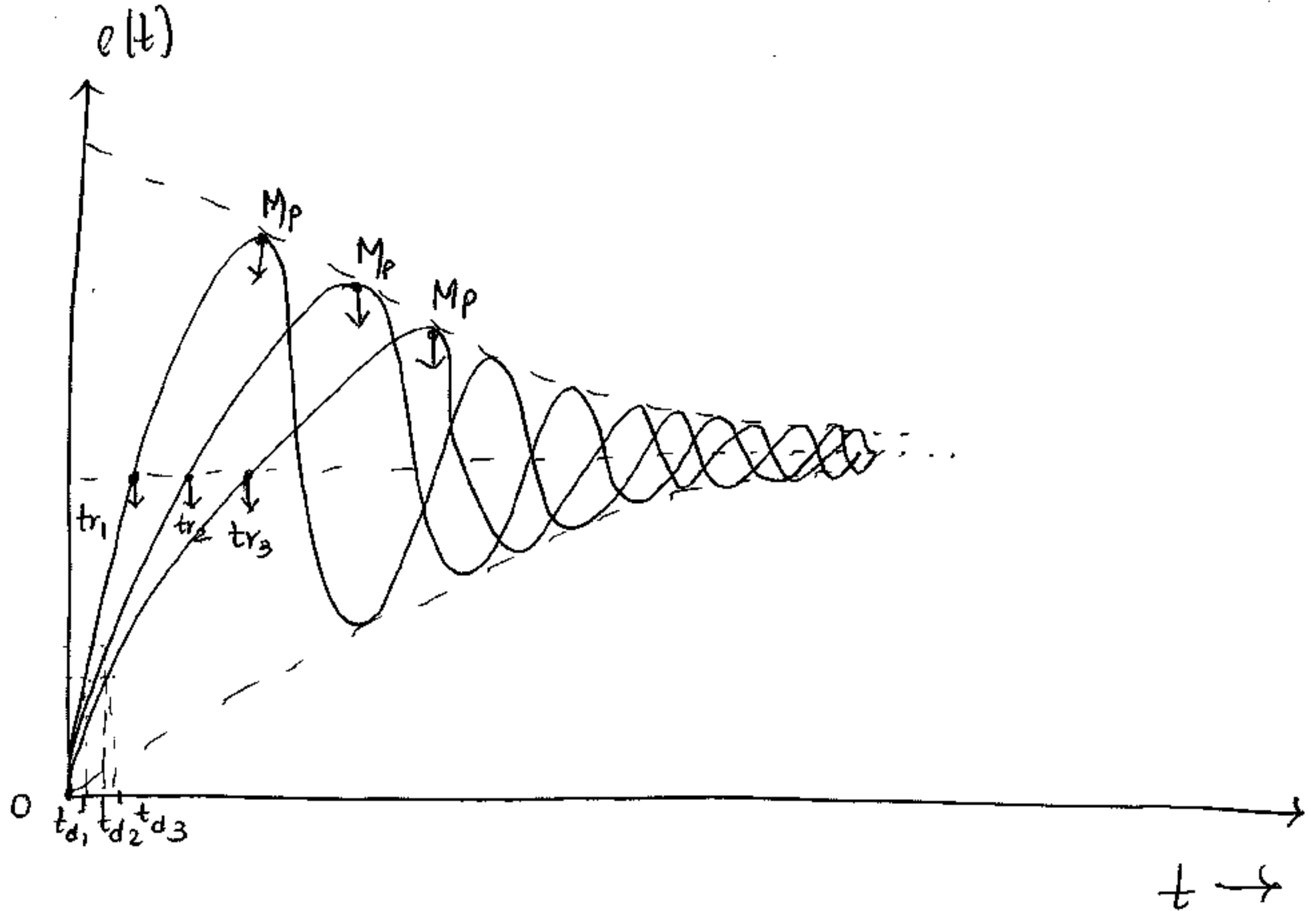
④ $\%M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\times 100\%$

⑤ $t_s = 4\tau = \frac{4}{\xi\omega_n}$

⑥ $T_{osc} = \frac{2\pi}{\omega_d}$

⑦ $N_{osc} = \frac{t_s}{T_{osc}}$

⑧ $\cos\theta = \xi$

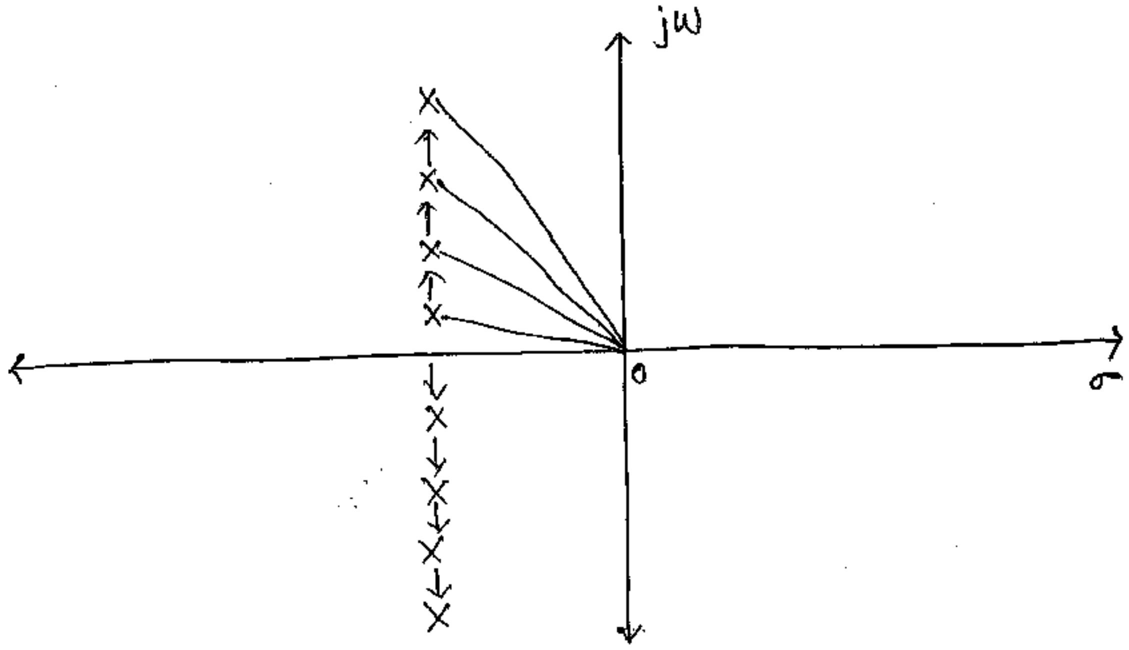


- As θ increases from 0 to 1, the poles move ~~from~~ towards left and near to real axis. In this case time constant is decreasing hence t_s also decreases.

= In this case ω_n is constant and ω_d decreases ξ increases hence t_p, t_d, t_r increases.

- As ξ increases from 0 to 1 % M_p decreases and system is less oscillatory and become more relative stable.

Q:-



τ = constant , $\theta\uparrow$ $\downarrow\cos(\theta)\uparrow = \xi\downarrow$, $M_p\uparrow$

-As real part is constant, τ is also constant. Hence T_s is also constant.

-As imaginary part is increasing damped oscillation ω_d increases. as ω_d increases ω_n also increases hence time domain specifications t_r, t_d & t_p decreases

-As inclination of pole θ increases, ξ (damping ratio) decreases. Hence %M_p ~~d~~ increases

-So the large M_p makes system less relative stable and more oscillatory.

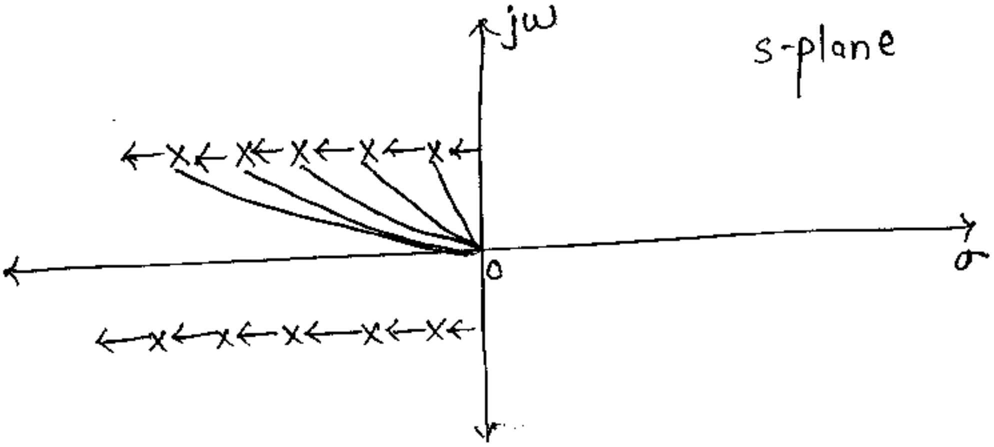
-The optimum ~~valt~~range of %M_p is 5% to 25%

$$\downarrow t_d = \frac{1+0.7\xi\downarrow}{\omega_n\uparrow} \qquad \uparrow t_r = \frac{\pi-\theta\uparrow}{\omega_d\uparrow} \qquad \downarrow t_p = \frac{\pi}{\omega_d\uparrow}$$

$$\downarrow T_{osc} = \frac{2\pi}{\omega_d\uparrow} \qquad \uparrow N_{osc} = \frac{t_s}{T_{osc}\downarrow}$$

If the peak overshoot is more than 25% system is less relative stable if the peak overshoot is less than 5% then system has slow response.

Q:-



$\omega_d \rightarrow$ constant

$\tau \rightarrow \downarrow$

$T_s \rightarrow \downarrow$

$\uparrow \cos(\theta\downarrow) = \xi\uparrow = M_p\downarrow$

$\omega_n\uparrow$

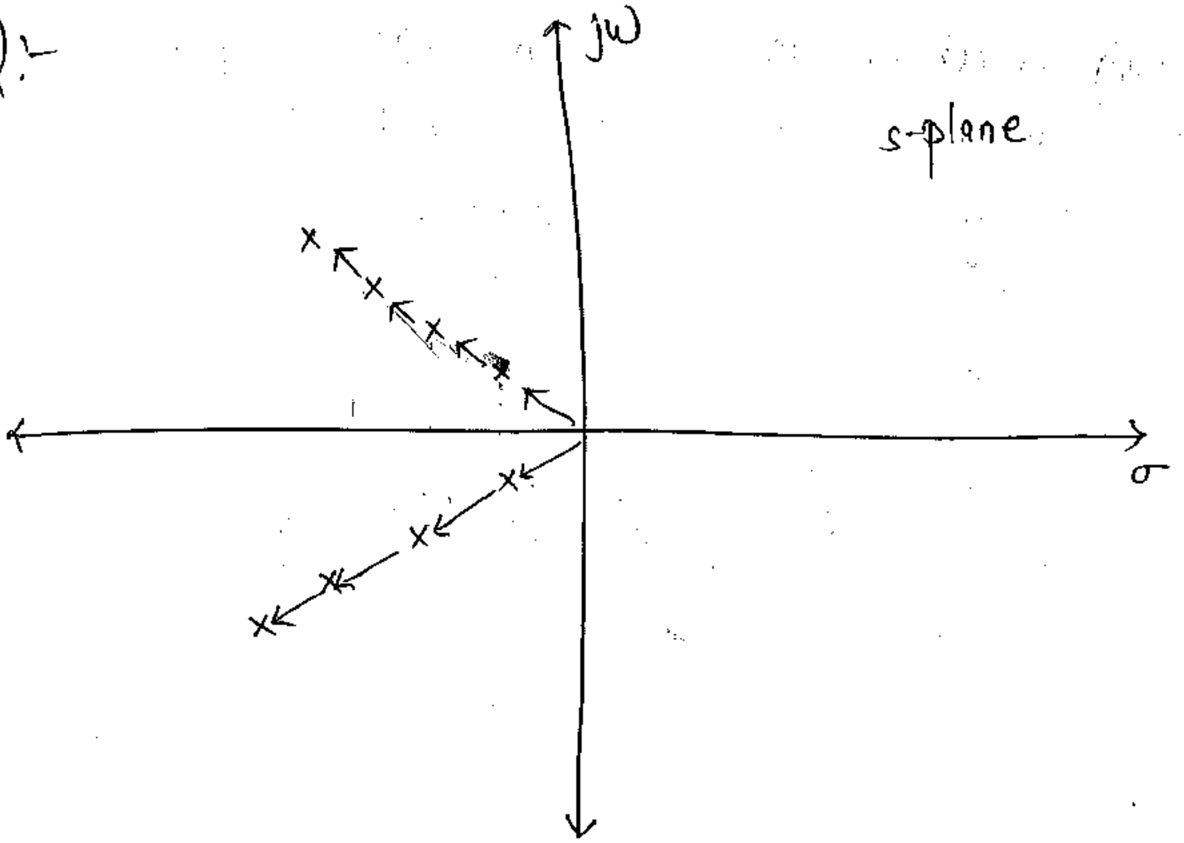
$\downarrow t_d = \frac{1+0.7\xi\uparrow}{\omega_n\uparrow}$, $\uparrow t_r = \frac{\pi-\theta\downarrow}{\omega_d}$, $t_p = \frac{\pi}{\omega_d}$ = constant

$T_{osc} = \frac{2\pi}{\omega_d}$ = constant , $\downarrow N_{osc} = \frac{t_s\downarrow}{T_{osc}}$

- Poles moving towards left side hence τ decreases and t_s also decreases. Imaginary part is constant so ω_d also constant hence t_p is also constant

- As inclination of pole θ decreases hence ξ increases hence %M_p decreases. The system become more relative stable.

Q:-



θ = constant

∴ ξ = constant ∴ $M_p\%$ = constant

$\omega_n \uparrow$, $\omega_d \uparrow$, $t_s \downarrow = 4T = \frac{4}{\xi\omega_n \uparrow}$

$\downarrow t_d = \frac{1+0.7\xi}{\omega_n \uparrow}$, $\downarrow t_r = \frac{\pi - \cos^{-1}\xi}{\omega_d \uparrow}$; $\downarrow t_p = \frac{\pi}{\omega_d \uparrow}$

$\downarrow T_{osc} = \frac{2\pi}{\omega_d \uparrow}$, $N_{osc} = \frac{t_s \downarrow}{T_{osc} \downarrow}$ = constant

$$= \frac{\frac{4}{\xi\omega_n}}{\frac{2\pi}{\omega_d}}$$

$$= \frac{4 \cancel{\omega_n}\sqrt{1-\xi^2}}{\xi \cancel{\omega_n} 2\pi}$$

$$N_{osc} = \frac{4\sqrt{1-\xi^2}}{2\pi\xi} = \text{constant}$$

$$\boxed{N_{osc} = \text{constant}}$$

No change in relative stability

* Steady state error (e_{ss})

$e(t) = r(t) - c(t)$

$E(s) = R(s) - C(s)$

$$e_{ss} = \lim_{t\to\infty} e(t)$$

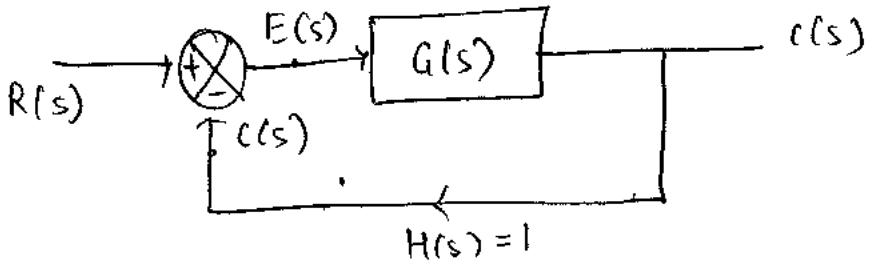
$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$e_{ss} = \lim_{s\to 0} s\left[R(s) - C(s)\right]$$

→ e_{ss} is calculated for closed UFS only. → if it is non-unity feedback then convert it into unity and then solve it

→ only applicable for stable system

UFS : (unity feedback system)



OLTF = G[s]

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$\rightarrow \frac{E(s)}{R(s)} = \frac{1\cdot[1]}{1+G(s)}$$

$$\frac{E[s]}{e} = \frac{1}{1+G[s]}\cdot R[s]$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s\,R[s]}{1+G[s]}}$$

depends on type of input → R(s)

type of system → $\underset{\underset{G(s)}{\|}}{\text{OLTF}}$

Type of input

① Step input

$$r(t) = \underset{\text{Amplitude}}{A} \cdot t^{\textcircled{0}} u(t) \xrightarrow{} \text{Type of input}$$

Type-0 input

$$R[s] = \frac{A}{s}$$

② Ramp input

$$r(t) = \underset{\text{amplitude}}{A} \cdot t^{\textcircled{1}} u(t) \xrightarrow{} \text{Type-1 input}$$

$$R[s] = \frac{A}{s^2}$$

③ Parabolic input

$$r(t) = \underset{\text{Amplitude}}{A} \cdot \frac{t^{\textcircled{2}}}{2} u(t) \xrightarrow{} \text{Type-2 input}$$

$$R[s] = \frac{A}{s^3}$$

For eg:- $r(t) = \frac{t^2}{10} u(t)$

$A = 1/5$

Type of system

$$\text{OLTF} = \frac{K(1+sT_1)(1+sT_2)(1+sT_3)\cdots}{s^{\textcircled{n}}(1+sT_a)(1+sT_b)(1+sT_c)\cdots}$$

$\downarrow$

n = Type of system

No. of poles @ origin = Type of system

n = 0 ⟶ Type 0

n = 1 ⟶ Type 1

n = 2 ⟶ Type 2

Type	i/p
↓ Type of system	↓ Type of i/p

*Static error co-efficient

$$e_{ss} = \lim_{s\to0} \frac{sR(s)}{1+G(s)}$$

① Step input

$$r(t) = A\,t^{0}u(t)$$

$$R(s) = \frac{A}{s}$$

$$e_{ss} = \lim_{s\to0} \frac{\cancel{s}\cdot A/\cancel{s}}{1+G(s)}$$

$$e_{ss} = \lim_{s\to0} \frac{A}{1+G(s)}$$

$$e_{ss} = \frac{A}{1+\underbrace{\lim_{s\to0} G(s)}_{k_p}}$$

$$\boxed{k_p = \lim_{s\to0} G(s)}$$

positional error co-efficient

$$\boxed{e_{ss} = \frac{A}{1+k_p}}$$

② Ramp input

$$r(t) = A\cdot t^{1}\,u(t)$$

$$R'(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s\to0} \frac{\cancel{s}\cdot A/s^{\cancel{2}}}{1+G(s)}$$

$$= \lim_{s\to0} \frac{A}{s+sG(s)}$$

$$= \frac{A}{0+\underbrace{\lim_{s\to0} sG(s)}_{k_v}}$$

$$\boxed{k_v = \lim_{s\to0} sG(s)}$$

velocity error co-efficient

$$\boxed{e_{ss} = \frac{A}{k_v}}$$

③ Parabolic input

$$r(t) = A\cdot\frac{t^2}{2}u(t)$$

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s\to0} \frac{\cancel{s}\cdot A/s^{\cancel{3}2}}{1+G(s)}$$

$$e_{ss} = \lim_{s\to0} \frac{A}{s^2+s^2G(s)}$$

$$e_{ss} = \frac{A}{0+\underbrace{\lim_{s\to0} s^2G(s)}_{k_a}}$$

$$\boxed{k_a = \lim_{s\to0} s^2G(s)}$$

acceleration error coefficient

$$\boxed{e_{ss} = \frac{A}{k_a}}$$

NOTE:

Type	k_p	k_v	k_a
$n=0$	k	0	0
$n=1$	∞	k	0
$n=2$	∞	∞	k
$n=3$	∞	∞	∞

① Step input

$r(t) = A \cdot t^0 u(t)$

Type of ~~system~~ input $= 0$

$$e_{ss} = \frac{A}{1+k_p}$$

$$k_p = \lim_{s \to 0} G(s)$$

Type of system	k_p	e_{ss}	
$n=0$	k	$\frac{A}{1+k}$	Type = i/p
$n=1$	∞	$\frac{A}{1+\infty} = 0$	Type > i/p
$n=2$	∞	0	Type > i/p

NOTE:

-As type of system increases steady state error decreases

② Ramp input

$r(t) = A \cdot t^{1} u(t)$

Type of i/p = 1

$e_{ss} = \frac{A}{K_v}$

$K_v = \lim_{s \to 0} s\,G(s)$

Type of system	K_v	e_{ss}	
$n=0$	0	$\frac{A}{K_v} = \infty$	Type < i/p
$n=1$	K	$\frac{A}{K}$	Type = i/p
$n=2$	∞	0	Type > i/p

③ Parabolic i/p

$r(t) = A \cdot \frac{t^2}{2} u(t)$; Type of i/p = 2

$e_{ss} = \frac{A}{K_a}$

$K_a = \lim_{s \to 0} s^2 G(s)$

Type of system	K_a	e_{ss}	
$n=0$	0	∞	Type < i/p
$n=1$	0	∞	Type < i/p
$n=2$	K	A/K	Type = i/p
$n=3$	∞	0	Type > i/p

IMP

Type = i/p → e_{ss} = constant

Type > i/p → $e_{ss} = 0$

Type < i/p → $e_{ss} = \infty$

*Summary:-

	Step	Ramp	Parabola
$r(t)$	$A \cdot t^0 u(t)$	$A \cdot t^1 u(t)$	$A \frac{t^2}{2} u(t)$
e_{ss}	$\frac{A}{1+K_P}$	$\frac{A}{K_V}$	$\frac{A}{K_a}$
Static error co-efficient	$K_P = \lim_{s\to 0} G(s)$	$K_V = \lim_{s\to 0} sG(s)$	$K_a = \lim_{s\to 0} s^2 G(s)$

Q:- Find e_{ss} for given unity feedback system.

$G[s] = \frac{10(s+1)}{s^2(s+2)(s+10)}$ and $H(s) = 1$ to the

following input $r(t) = \left(10 + 2t + \frac{t^2}{2}\right)u(t)$

Sol:-

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

$$R(s) = \frac{10}{s} + \frac{2}{s^2} + \frac{1\cancel{2}}{s^3 \cdot \cancel{2}}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{10}{s} + \frac{2}{s^2} + \frac{1}{s^3}\right)}{1 + \frac{10(s+1)}{s^2(s+2)(s+10)}}$$

$$e_{ss} = \lim_{s\to 0} \frac{10s^2+2s+1}{s^2 + \frac{10(s+1)}{(s+2)(s+10)}} = \frac{1}{\frac{\cancel{10}\cdot 1}{2\cdot\cancel{10}}} = 2$$

$$\boxed{e_{ss} = 2}$$

Short-cut

	Type of system		i/p	ess
Error due to 10	2	>	0	0
Error due to 2t	2	>	1	+ 0
Error due to $\frac{t^2}{2}$	2	=	2	+ $\frac{A}{K} = \frac{1}{1/2} = 2$

Q:- $G[s] = \frac{10}{s(s+5)}$; $H[s] = 1$

(1) $10u(t)$

(2) $10tu(t)$

(3) $10t^2u(t)$

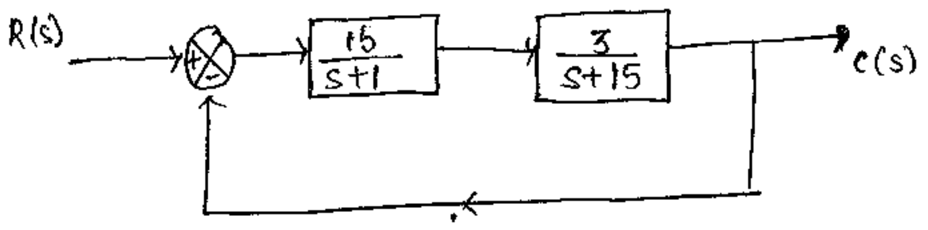
(4) $(1+t)u(t)$

(5) $(1+t+t^2)u(t)$

	Type of system		Type of i/p	ess	
(1)	1	>	0	0	
(2)	1	=	1	$e_{ss} = \frac{A}{k} = \frac{10}{2} = 5$	
(3)	1	<	2	$e_{ss} = \infty$	
(4)	1	>	0	0	
	1	=	1	+ $\frac{A}{k} = \frac{1}{2} = 0.5$	$e_{ss} = 0.5$
(5)	1	>	0	0	
	1	=	1	$\frac{1}{2} = 0.5$	$e_{ss} = \infty$
	1	<	1	∞	

Q:- $G[s] = \frac{s+1}{\quad}$

Q:- Calculate steady state error for the system to the unity feedback system for unit step input.



$$\frac{C(s)}{R(s)} = \frac{\frac{15\cdot 3}{(s+1)(s+15)}}{1+\frac{45}{(s+1)(s+15)}} = \frac{45}{s^2+16s+15+45} = \frac{45}{s^2+16s+60}$$

$$G(s) = \frac{45}{(s+1)(s+15)}$$

Now,

$$e_{ss} = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)} = \lim_{s\to 0} \frac{s\cdot 1/s}{1+\frac{45}{s^2+16s+15}} = \frac{1}{1+\frac{45}{15}}$$

$$e_{ss} = \frac{1}{1+3} = \frac{1}{4} = 0.25$$

2nd Method:-

$$r(t) = t^n \cdot u(t)$$

Type:- i/p

0 = 0 $\rightarrow e_{ss} = \text{constant}$

$$e_{ss} = \frac{A}{1+\frac{\cancel{45}3}{\cancel{15}}} = \frac{1}{4} = 0.25$$

3rd Method:-

$$e_{ss} = \lim_{s\to 0} s\left[R(s) - C(s)\right]$$

$$= \lim_{s\to 0} s\,R(s)\left[1 - \frac{C(s)}{R(s)}\right]$$

$$\boxed{e_{ss} = \lim_{s\to 0} s\,R(s)\left[1 - CLTF\right]}$$

$$CLTF = \frac{45}{s^2+16s+60}$$

$$e_{ss} = \lim_{s\to 0} s\cdot\frac{1}{s}\left[1 - \frac{45}{s^2+16s+60}\right]$$

$$e_{ss} = 1 - \frac{\cancel{45}3}{\cancel{60}4} = 1/4$$

$$e_{ss} = 1/4 = 0.25$$

- Steady state errors are calculated for OLTF (H(s)=1)
- If any block diagram (unity or non-unity feedback system), SFG directly CLTF or differential eq^n are given then follow 3rd method.

Q:- The OLTF of unity feedback system $G(s) = \frac{K}{S(s+1)(s+2)}$ The value of K to get steady state error 0.1 to the unit ramp input is ____.

Sol:- $r(t) = t^1 \cdot u(t)$

$R(s) = 1/s^2$ $\quad \text{Type}_1 = \text{i/p}_1 \therefore e_{ss} = \text{constant}$

$$e_{ss} = \lim_{s \to 0} \frac{s\,R(s)}{1+G(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot 1/s^2}{1+\frac{K}{s(s+1)(s+2)}}$$

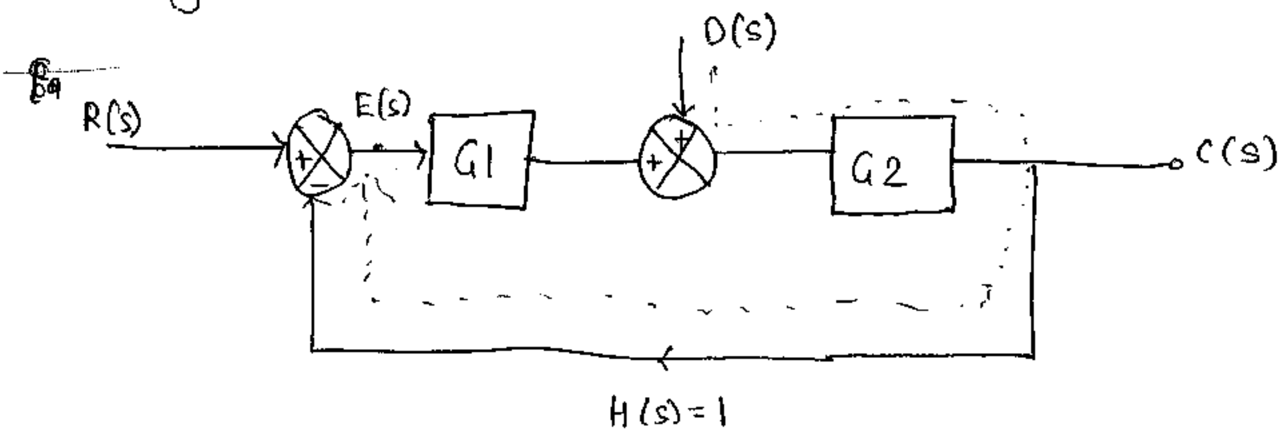
$$= \lim_{s \to 0} \frac{1/\cancel{s}\,(s+1)(s+2)}{s(s+1)(s+2)+K}$$

$$0.1 = \frac{2}{K}$$

$$K = \frac{2}{0.1} = 20$$

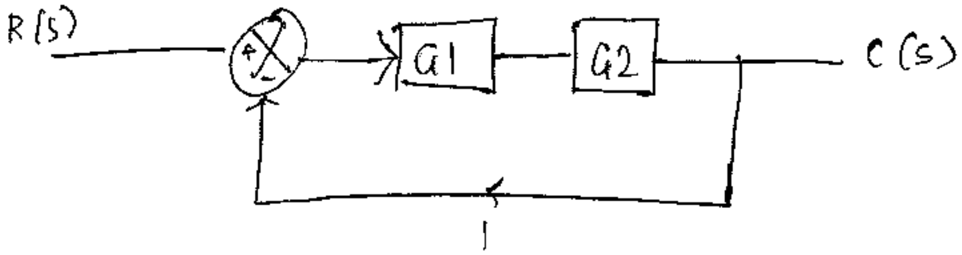
$$\boxed{K = 20}$$

* Steady state error to disturbance input



$$\frac{C(s)}{D(s)} = ? \quad , \quad \frac{C(s)}{R(s)} = ?$$

$\frac{C(s)}{R(s)}$ Put $D(s)=0$



$$\frac{C(s)}{R(s)} = \frac{G1G2}{1+G1G2}$$

Now,

$$C(s) = \frac{G1G2}{1+G1G2} R(s) + \frac{G2}{1+G1G2} \cdot D(s)$$

① e_{ss} due to R(s)

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$E(s) = \frac{R(s)}{1+G1G2}$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G1G2}}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G1G2}$$

② e_{ss} due to D(s)

$$e_{ss} = \lim_{s\to 0} \_\_\_\_\_\_\_\_$$

$$\frac{E(s)}{D(s)} = \frac{-G2}{1+G2G1}$$

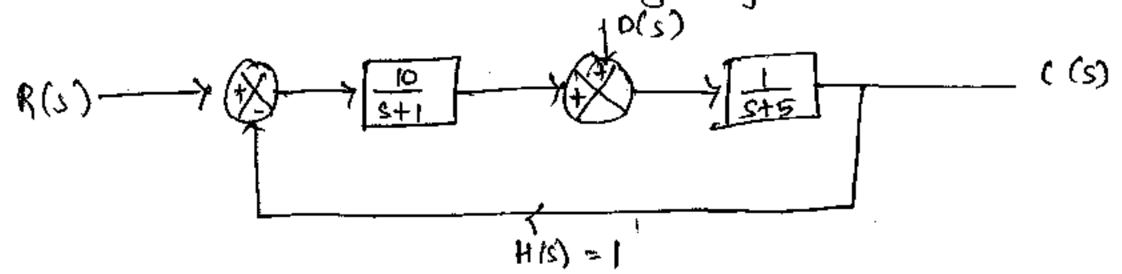
$$\Rightarrow E(s) = \frac{-G2\,D(s)}{1+G1G2}$$

$$e_{ss} = \lim_{s\to 0} \frac{-s\,G_2\,D(s)}{1+G1G2}$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s[R(s) - G_2 D(s)]}{1+G1G2}}$$

Overall error

Q:- Find e_{ss} due to unit step and unit step disturbance to following system.



$$e_{ss} = \lim_{s\to 0} \frac{s[R(s) - G_2 D(s)]}{1+G1G2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\left[1/s - \frac{1}{s+5}\cdot\frac{1}{s}\right]}{1+\frac{10}{s+1}\cdot\frac{1}{s+5}}$$

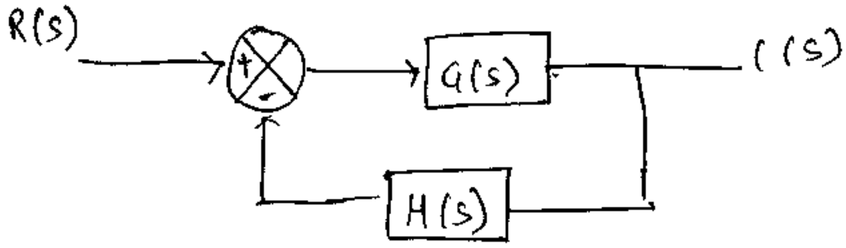
$$e_{ss} = \lim_{s\to 0} \frac{1-\frac{1}{s+5}}{\frac{(s+1)(s+5)+10}{(s+1)(s+5)}} = \lim_{s\to 0} \frac{\frac{s+5-1}{\cancel{s+5}}}{\frac{(s^2+6s+5+10)}{(s+1)\cancel{(s+5)}}}$$

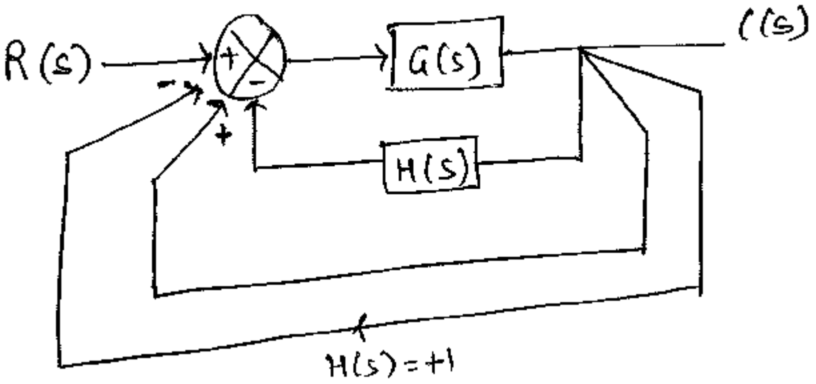
$$e_{ss} = \lim_{s\to 0} \frac{(s+4)(s+1)}{s^2+6s+15}$$

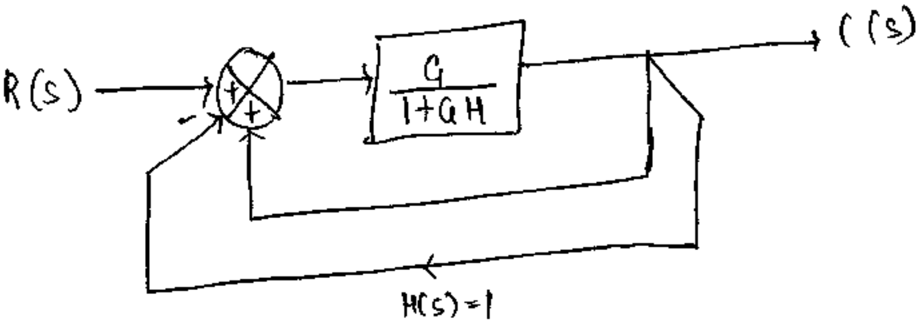
$$e_{ss} = \frac{4}{15} = 0.266$$

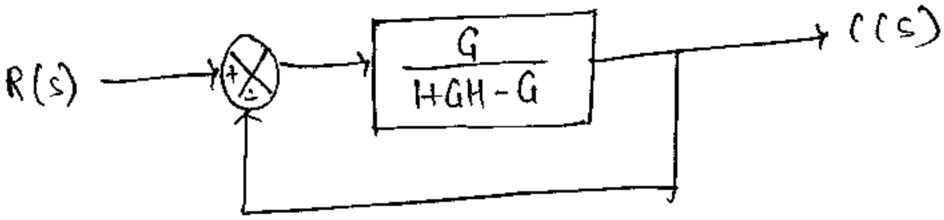
*Steady state error to the non-unity feedback system

- Steady state error are calculated to closed loop stable unity feedback only if non-unity feedback system is given then it should be converted into unity feedback system as follows:-









$$G_{NUF} = \frac{G}{1+GH-G}$$

non-unity feedback

Compare type of G_{NUF} & order & then find steady state erro

$$\boxed{e_{ss} = \lim_{s\to0} \frac{sR(s)}{1+G_{NUF}(s)}}$$

$$= \lim_{s\to0} \frac{sR(s)}{1+\frac{G}{1+GH-G}}$$

$$e_{ss} = \lim_{s\to0} \frac{sR(s)\,(1+GH-G)}{1+GH}$$

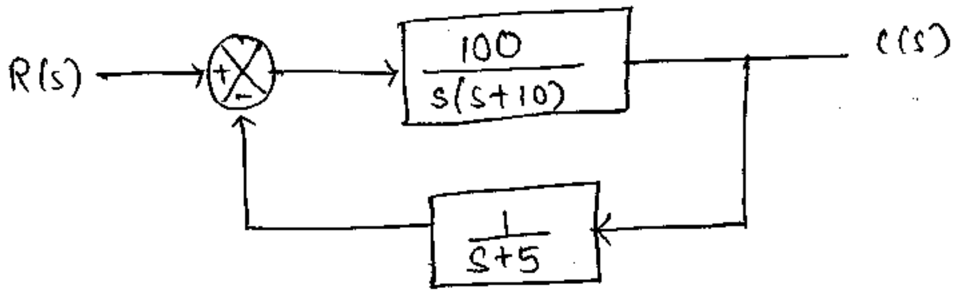
$$\text{CLTF} = \frac{G}{1+GH}$$

$$\boxed{e_{ss} = \lim_{s\to0} sR[s]\left[1-\text{CLTF}\right]}$$

$$= \lim_{s\to0} sR[s]\left[1-\frac{G}{1+GH}\right]$$

$$e_{ss} = \lim_{s\to0} \frac{sR[s]\left[1+GH-G\right]}{1+GH}$$

Q:- Find e_{ss} of non-unity feedback system. Take i/p as unit step input



$$C_{NUF} = \frac{G}{1+GH-G}$$

$$= \frac{\frac{100}{s(s+10)}}{1+\frac{100}{(s+10)s}\cdot\frac{1}{s+5}-\frac{100}{s(s+10)}}$$

$$= \frac{100(s+5)}{(s+5)s(s+10)+100-100(s+5)}$$

$$= \frac{100(s+5)}{(s+5)\left[s^2+10s-100\right]+100}$$

$$= \frac{100(s+5)}{s^3+15s^2-50s-500+100}$$

$$C_{NUF} = \frac{100(s+5)}{s^3+15s^2-50s-400}$$

Type : 0 i/p : 0

$$e_{ss} = \frac{A}{1+k} = \frac{1}{1-\frac{500}{400}} = -4$$

Method: 2
CLTF

$$\text{CLTF} = \frac{\frac{100}{S(s+10)}}{1+\frac{100}{s(s+10)}\cdot\frac{1}{(s+5)}}$$

$$= \frac{100(s+5)}{s(s+10)(s+5)+100}$$

$(s^2+15s+50)$

$$\text{CLTF} = \frac{100(s+5)}{s^3+15s^2+50s+100}$$

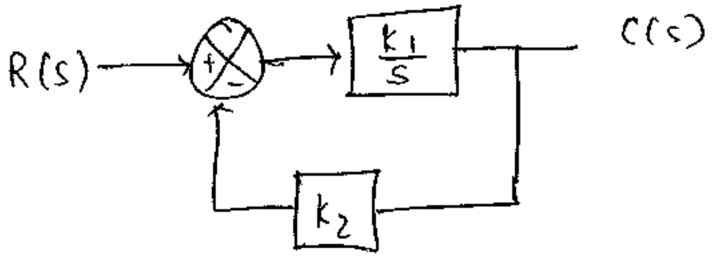
$$e_{ss} = \lim_{s\to 0} s\left\{R(s)\left[1 - \text{CLTF}\right]\right.$$

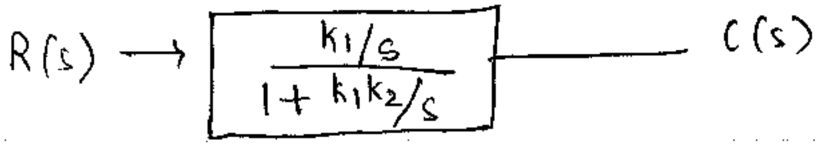
$$= \lim_{s\to 0} s\cdot 1/s\left[1 - \frac{100(s+5)}{s^3+15s^2+50s+100}\right]$$

$$= 1 - \frac{500}{100}$$

$$e_{ss} = -4$$

Q:- For the system shown in figure the steady state o/p is 2 for unit step input and system time constant is 0.4 sec. The values of k_1 & k_2 are:-





$$\frac{C(s)}{R(s)} = \frac{k_1}{s+k_1k_2}$$

⇒ For time constant,

$$\tau = \frac{+1}{k_1k_2} = 0.4$$

$$k_1k_2 = \frac{5}{2}$$

$$\boxed{k_2 = \frac{1}{2}}$$

⇒ Steady state output value

$$\lim_{t\to\infty} c(t)$$

$$\lim_{s\to 0} s\cdot C(s)$$

$$2 = \lim_{s\to 0} \cancel{s}\cdot\frac{k_1}{s+k_1k_2}\cdot\frac{1}{\cancel{s}}$$

$$2 = \frac{k_1}{k_1k_2}$$

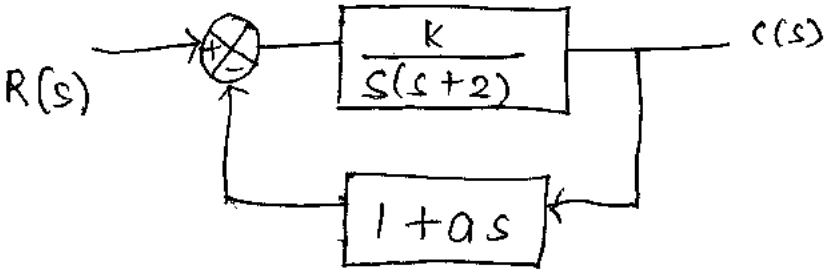
$$2 = \frac{k_1}{5}\cdot 2$$

$$\boxed{k_1 = 5}$$

NOTE:- For steady state o/p value

$$\lim_{t\to\infty} c(t) = \lim_{s\to 0} s\cdot C(s)$$

Q:- For the system shown in figure natural frequency of oscillation is 4rad/sec & daming ratio is 0.7. The values of k_1 and a are ______.



$$\frac{C(s)}{R(s)} = \frac{\dfrac{k}{s(s+2)}}{1 + \dfrac{k}{s(s+2)}(1+as)}$$

$$= \frac{k}{s(s+2) + k(1+as)}$$

$$= \frac{k}{s^2+2s+k+(ka)s}$$

$$= \frac{k}{s^2+s(2+ka)+k}$$

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = K$

$\boxed{k = 16}$

$2\xi\omega_n = 2+ka$

$2 \cdot 0.7 \cdot 4 = 2+16a$

$5.6 = 2+16a$

$3.6 = 16a$

$\boxed{a = 0.225}$

~~$2\xi\omega_n = (2+k+a)$~~

~~$2 \cdot 0.7 \cdot 4 = 2+16+a$~~

~~$\frac{2 \cdot 7}{10} = 18+a$~~

~~$2.8 = 18+a$~~

~~$\boxed{a = -1.2}$~~

Q:- Control system is described by following eq^n $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+5y=10\left(1-e^{-2t}\right)$. The respose at $t\to\infty$ is.

$$s^2Y(s)+5sY(s)+5Y(s)=10\left[\frac{1}{s}-\frac{1}{s+2}\right]$$

$$Y(s)\left[s^2+5s+5\right]=\frac{10\cdot 2}{s(s+2)}$$

$$Y(s)=\frac{20}{s(s+2)(s^2+5s+5)}$$

$$\lim_{s\to 0}\cancel{s}\cdot\frac{20}{\cancel{s}(s+2)(s^2+5s+5)}$$

$$\text{The response}=\frac{\cancel{20}\,\cancel{4}\,2}{\cancel{2}\cdot\cancel{8}}$$

$$\boxed{\begin{array}{l}\text{Response}=2\\ \text{at } t\to\infty\end{array}}$$

Q:- Match List-I with List-II and select correct answer using codes given below:-

List-I (system function)	List-II (name of damping)
A. $T(s)=\frac{64}{3s^2+4s+5}$	1. Underdamped
B. $T(s)=\frac{s^2-2s}{s^2+6s+9}$	2. Overdamped

C. $\frac{9s^2+3s+10}{s^2+5s+2}$ 3. Critically damped

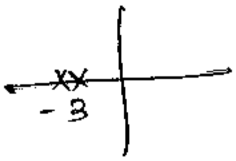
D. $\frac{19s-20}{s^2+s+100}$ 4. Oscillatory

Sol^n:- Check pole location

s^2+6s+9

$(s+3)^2$

$-3 \quad -3$



Critically damped

B —— (3)

$3s^2+4s+5$

$$s = \frac{-4 \pm \sqrt{16-60}}{2(3)} = \frac{-4 \pm \sqrt{44}}{6}$$

Complex conjugate

Underdamped

A —— (1)

s^2+5s+2

$$s = \frac{-5 \pm \sqrt{25-8}}{2}$$

$$s = \frac{-5 \pm \sqrt{17}}{2} \qquad \frac{-5+\sqrt{17}}{2}, \frac{-5-\sqrt{17}}{2}$$

$$\frac{-5+4.12}{2}, \frac{-5-4.12}{2}$$

-0.44, -4.56

Overdamped

C — (2)

D — (1)

Q:- The system with T.F. $T(s) = \dfrac{20s}{3s^2+2s+k+4}$ is to have $\xi = 0.1$. The constant k is ____.

$$T(s) = \frac{20s}{3s^2+2s+k+4}$$

$$\therefore \quad s^2 + \frac{2}{3}s + \frac{k+4}{3} = 0$$

$$s^2 + 2s\xi\omega_n + \omega_n^2 = 0$$

$$\xi\omega_n = \frac{1}{3}$$

$$\omega_n = \frac{1}{3 \cdot 0.1} = \frac{10}{3}$$

$$\omega_n^2 = \frac{k+4}{3}$$

$$\frac{100}{\cancel{9}3} = \frac{k+4}{\cancel{3}}$$

$$100 = 3k+12$$

$$88 = 3k$$

$$k = \frac{88}{3}$$

$$\boxed{k = 29.33}$$

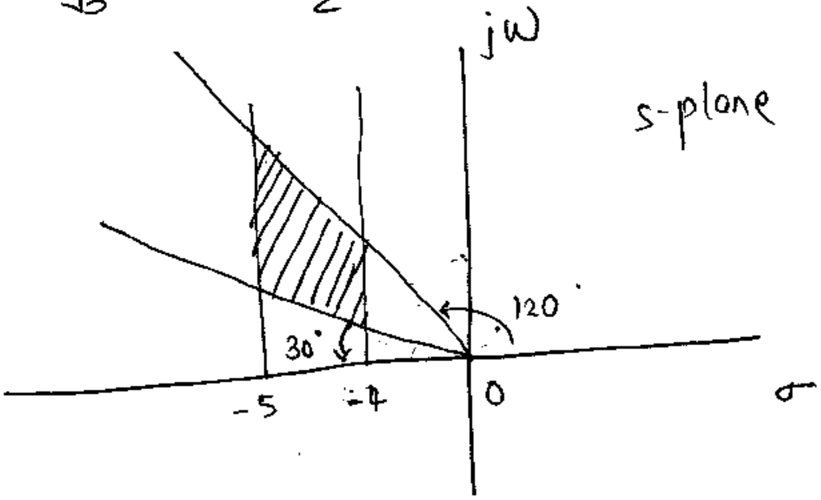
Q:- The roots of 2^{nd} order system lies in the shaded region in s-plane as shown below. If ξ represen damping ratio and t_s represents settling time in seconds for 2% tolerance band then

(a) $\frac{1}{2} < \xi < \frac{\sqrt{3}}{2}$ and $0.8 < t_s < 1$

(b) $\frac{1}{\sqrt{3}} < \xi < \frac{\sqrt{3}}{2}$ and $1 < t_s < 1.25$

(c) $\frac{1}{2} < \xi < \frac{\sqrt{3}}{2}$ and $1 < t_s < 1.25$

(d) $\frac{1}{\sqrt{3}} < \xi < \frac{\sqrt{3}}{2}$ and $0.8 < t_s < 1$



$$\xi = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

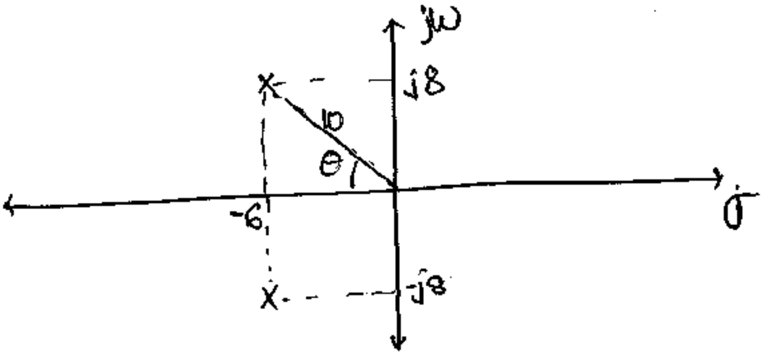
$$\xi = \cos 60^\circ = 1/2$$

$$t_s = 4T_1 = \frac{4}{4} = 1$$

$$t_s = 4T_2 = \frac{4}{5} = 0.8$$

(A)

Q:-



$j8 = j\omega_d$

$\omega_d = 8$ rad/sec

$\xi\omega_n = 6$

$\cos\theta = \xi$

$\cos\frac{6}{10} = \xi$

$\xi = 0.6$

$\theta = \cos^{-1}\left(\frac{6}{10}\right) = 0.927^{\circ}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$8 = \omega_n\sqrt{1-0.36}$

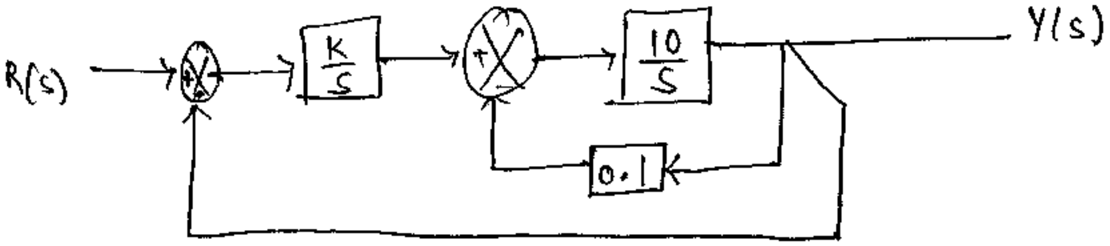
$8 = \omega_n\, 0.8$

$\omega_n = 10$ rad/s.

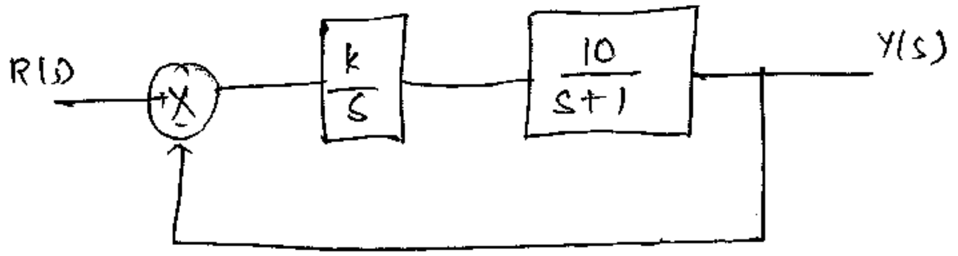
$$t_r = \frac{1+0.7\xi}{\omega_n}$$

$$t_r = \frac{1+0.7(0.6)}{10} =$$

Q: In the control system shown below, the steady state error $e(\infty) = \lim_{t\to\infty} e(t)$ is required to be less than or equal to 0.01 for unit ramp input r(t)=tu(t). What is the restriction on K?



$$\frac{\frac{10}{s}}{1+\frac{\cancel{10}}{s}\cdot\frac{0.1}{\cancel{10}}}$$



$$\frac{Y(s)}{R(s)} = \frac{\frac{10k}{s(s+1)}}{1+\frac{10k}{s(s+1)}} = \frac{10k}{s(s+1)+10k}$$

$$G(s) = OLTF = \frac{10k}{s(s+1)}$$

Now, Type i/p

1 = 1

$$e_{ss} = \frac{A}{k} = \frac{1}{10k} \leq 0.01$$

$$\frac{100}{10} \leq k$$

$$\underline{k \geq 10}$$

Q:- A 2nd order system with no zeros ~~are located as~~ at unity feedback system with $\xi = 1.25$ and undamped natural frequency $\frac{10}{2\pi}$ Hz. The unit step response of system is ____.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$f = \frac{10}{2\pi}$$

$$\frac{C(s)}{R(s)} = \frac{100}{s^2+25s+100}$$

$$\frac{\omega}{2\pi} = \frac{10}{2\pi}$$

$\omega = 10$ rad/s

$$\frac{C(s)}{R(s)} = \frac{100}{s^2+25s+100}$$

$$C(s) = \frac{100\cdot 1/s}{(s+5)(s+20)}$$

$$= \frac{A}{s} + \frac{B}{s+5} + \frac{C}{s+20}$$

$$\frac{-25 \pm \sqrt{625-400}}{2}$$

$$\frac{-25\pm 15}{2} = -5, -20$$

$$\frac{\cancel{100}\,20}{-5\,15} \qquad \frac{\cancel{100}\,4}{-20(-15)} \qquad \frac{\cancel{100}\,5}{-20\,(-15)}$$

$$= \frac{1}{s} + \frac{-4}{3(s+5)} + \frac{1}{s+20(3)}$$

$$c(t) = \left(1 - \frac{4}{3}e^{-5t} + \frac{1}{3}e^{-20t}\right)u(t)$$

Q: A system placed in unity feedback when excited by $r(t)=a\,u(t)$ exhibits steady state error of 0.4 while position error constant $k_p=4$ the same system when excited by $r(t)=1.5t\,u(t)$ again exhibits steady state error e then value of e is ______.

Sol:- $r(t) = a\,u(t)$

$e = 0.4$

$k_p = 4$

$$\Rightarrow e_{ss} = \frac{A}{1+k_p} = 0.4$$

$$= \frac{a}{1+4} = 0.4$$

$$= 0.4 + 1.6 = a$$

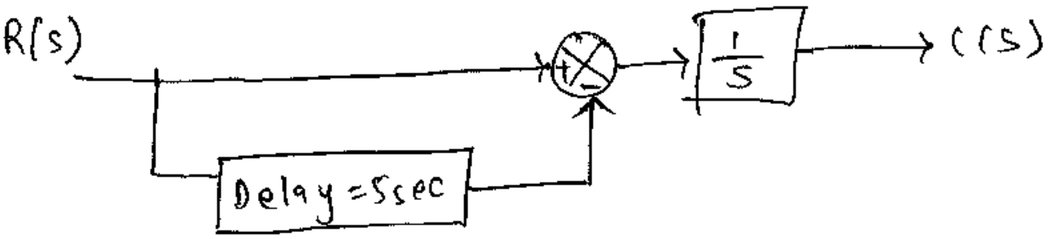
$$\boxed{a=2}$$

$r(t)=1.5t\,u(t)$

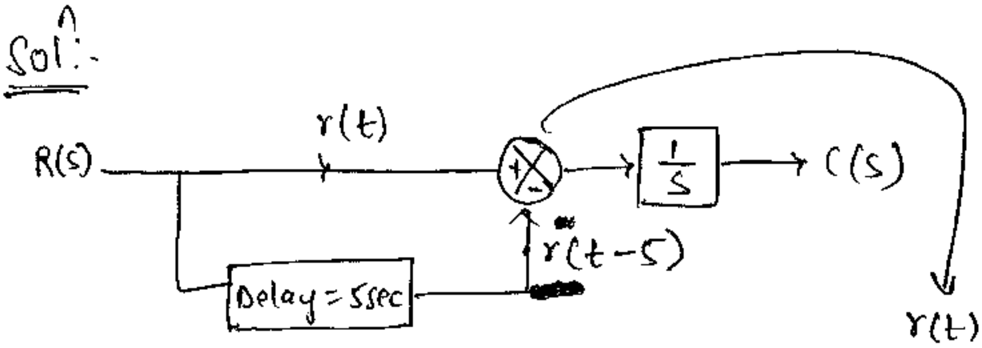
→ Type $\quad$ i/p

$0 < 1 \qquad e_{ss} = \infty$

Q:- Consider the system below :- where $r(t) = \delta(t)$, $\delta(t)$ is unit impulse function. The response $c(t)$ of system is ____ .



Sol:-



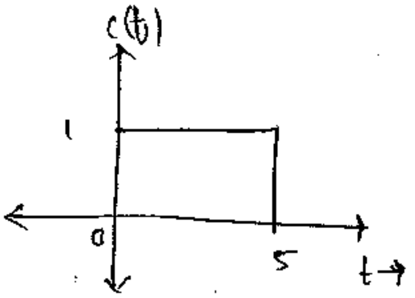
$r(t) - r(t-5)$

$R(s) - e^{-5s}R(s)$

$$C(s) = \frac{R(s)}{s} - \frac{e^{-5s}}{s}R(s)$$

$$c(s) = \frac{1}{s} - \frac{e^{-5s}}{s}$$

$$\boxed{c(t) = u(t) - u(t-5)}$$



Q:- The closed loop T.F. T(s) of unity feedback sys. is given by $T(s) = \frac{as+b}{s^2+as+b}$ then the steady state error due to $r(t) = t^2u(t)$ is ____ .

(a) 2/b ✓ (b) b/2 (c) 2/a (d) a/2

$$T(s)_{OLTF} = \frac{as+b}{s^2}$$

Type = i/p
2 = 2

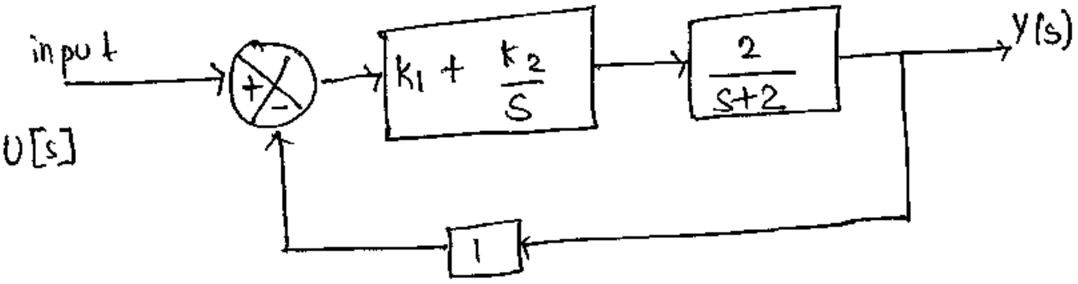
$$e_{ss} = \frac{A}{K} = \frac{2}{b}$$

$r(t) = t^2u(t)$

$R(s) = \frac{2}{s^3}$

For k remove pole at origin and put s=0

Q:- The design of unity feedback control system is shown below is so attempted that sys. response to unit step input is. oscillatory with damping ratio 0.7, and undamped natural frequency ω_n of 10 rad/sec. The value of K_1 and k_2 are ____.

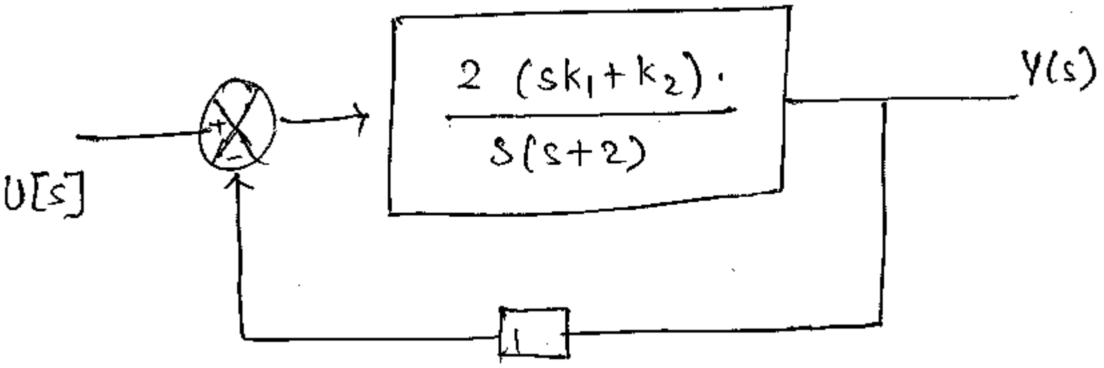


(a) $K_1 = 7$ & $k_2 = 100$

✓(b) $K_1 = 6$ & $K_2 = 50$

(c) $K_1 = 100$ & $K_2 = 7$

(d) $K_1 = 14$ & k_2 100



$$\frac{\cancel{\omega_n^2}}{s^2 \cancel{+2s\xi\omega_n} + \cancel{\omega_n^2}} \qquad \text{CLTF} = \frac{\frac{2(sk_1+k_2)}{s(s+2)}}{1 + \frac{2(sk_1+k_2)}{s(s+2)}}$$

$$\frac{Y(s)}{U(s)} = \frac{2\ (sK_1+k_2)}{s(s+2)+2(sk_1+k_2)} = \frac{2sK_1+2k_2}{s^2+s(2+2k_1)+2k_2}$$

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\xi\omega_n = 1+k_1$$

$$\omega_n^2 = 2k_2$$

$$100 = 2k_2$$

$$\boxed{k_2=50}$$

$$\xi\omega_n = 1+k_1$$

$$(0.7)(10) = 1+k_1$$

$$7 = 1+k_1$$

$$\boxed{k_1=6}$$

★ ★ IMP

①:- System has input $r(t)$ & output $y(t)$ such that

★ ★ $T[s] = \frac{Y[s]}{R[s]} = \frac{4}{s+3}$ and $r(t)=u(t)$, $u(t)$ is unit step function and $y[0^-]=-2$. Find complete response, zero state response, zero input response, forced response and natural response.

$$\Rightarrow \quad \frac{Y[s]}{R[s]} = \frac{4}{s+3}$$

$$Y[s][s+3] = 4R[s]$$

$$sY[s] - y(0^-) + 3Y[s] = 4R[s]$$

$$(s+3)Y[s] = y(0^-) + 4R[s]$$

$$Y[s] = \underbrace{\frac{y(0^-)}{s+3}}_{ZIR} + \underbrace{\frac{4}{s+3}R[s]}_{ZSR}$$

$$Y[s] = \underbrace{\frac{-2}{s+3}}_{ZIR} + \underbrace{\frac{4}{s(s+3)}}_{ZSR}$$

$$= \frac{-2}{s+3} + \frac{4}{3}\left[\frac{1}{s} - \frac{1}{s+3}\right]$$

$$y(t) = \left(-2e^{-3t} + \frac{4}{3} - \frac{4}{3}e^{-3t}\right)u(t)$$

$\downarrow$

complete response

$$y_{ZIR}(t) = -2e^{-3t}u(t)$$

$$y_{ZSR}(t) = \frac{4}{3}\left[1 - e^{-3t}\right]u(t)$$

✯ constant term = forced response $(y(t))$ ✯

$y_F(t) = \frac{4}{3}$ → Forced response

$y_N(t) = \frac{-10}{3} e^{-3t} u(t)$ → Natural response

Q:-

$T(s) = \frac{Y(s)}{R(s)} = \frac{10}{s+4}$. If $r(t) = \delta(t)$ where $\delta(t)$ is unit impulse function and $y(0^-) = 0$ then forced and natural response components are respectively

$Y[s](s+4) = 10R(s)$

$r(t) = \delta(t)$

$R(s) = 1$

$sY[s] - y(0^-) + 4Y(s) = 10R(s)$

$Y[s][s+4] = 10$

$Y[s] = \frac{10}{s+4} \xrightarrow{} ZSR$

$\underline{y(t) = 10e^{-4t}u(t)}$

forced response = 0

Natural response = $10e^{-4t}u(t)$