

★ Topics

✓ ⊙ Static fields (Electrostatics & steady magnetics).

→ The fields are independent of time. is called static fields.

✓ ⊙ Time Varying fields

→ Maxwell Eqⁿ.

✓ ⊙ EM Waves.

- Defⁿ:

→ A wave is a physical phenomena which reproduces After certain instant of time get some other place, the time delay betⁿ the prior to the later locations is proportional to travelled distance. The whole phenomena constitutes a wave. Therefore, an EM waves is not only fⁿ of time but also a fⁿ of distance. Instead of distance we use space co-ordinates.

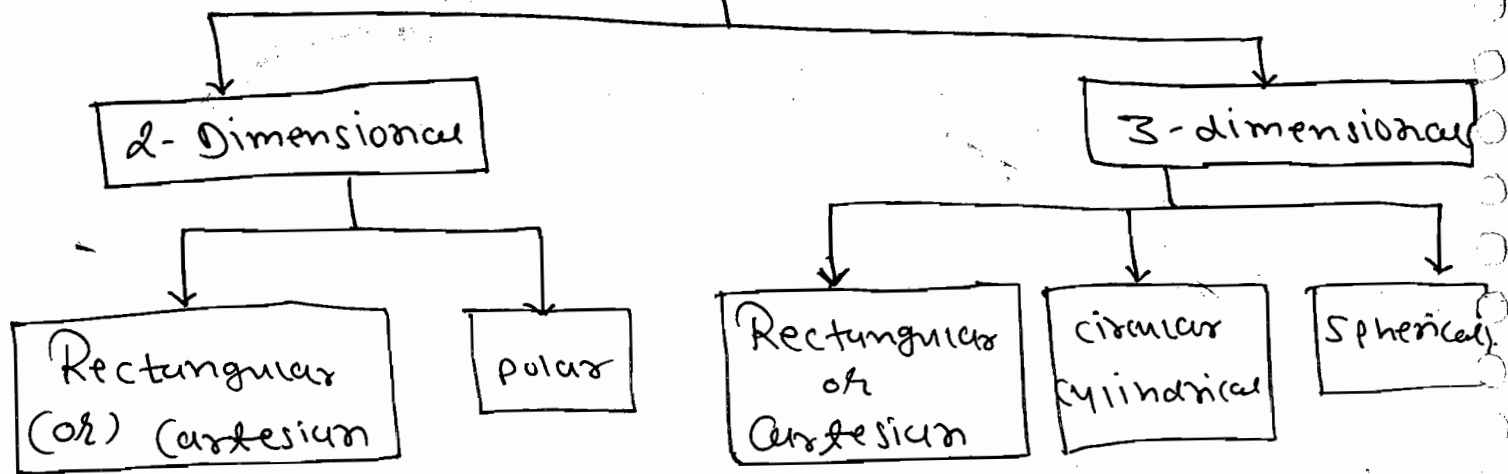
✓ ⊙ Waveguide (Rectangular)

⊙ Basics of Antennas

✓ ⊙ Two wire transmission lines.

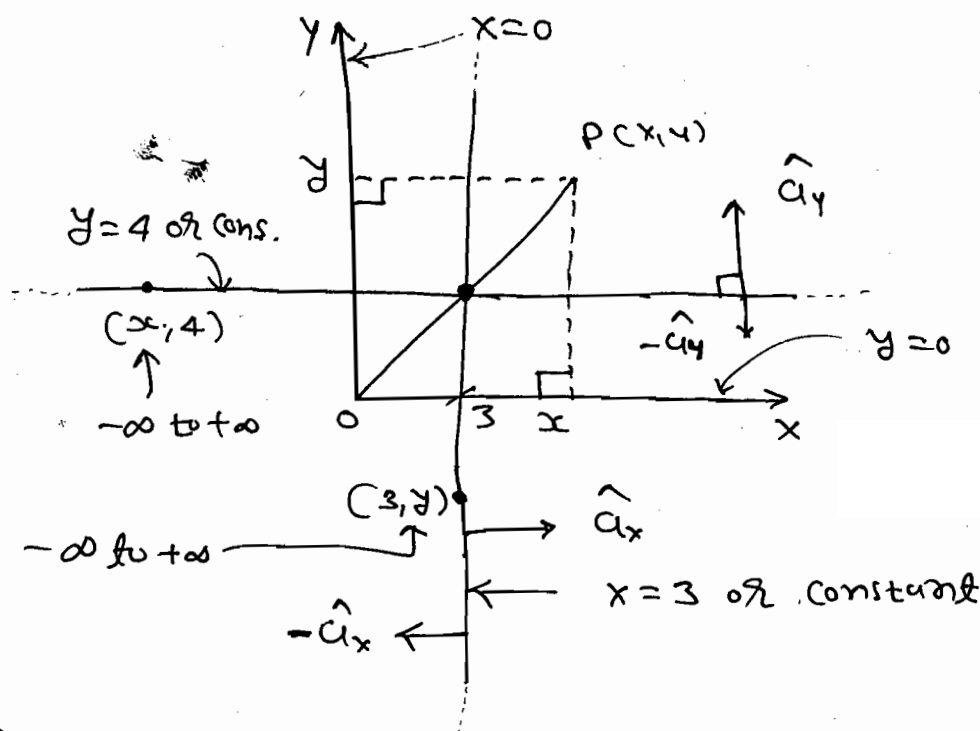
✓ ⊙ Scattering Parameters.

Coordinate Systems.



* 2-Dimensional:

(i) Rectangular (or) Cartesian



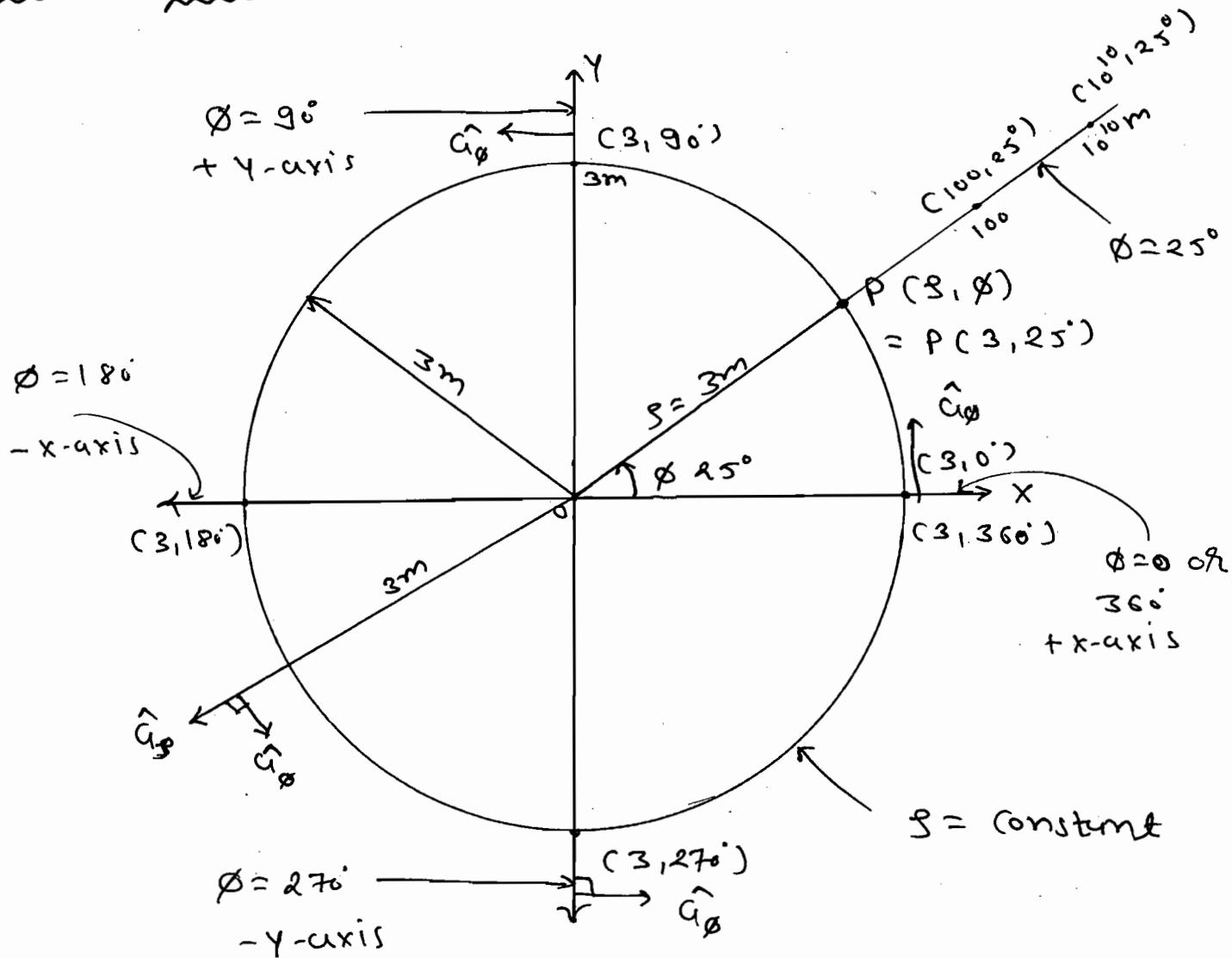
⇒ Integral	on $x = \text{const.}$	on $y = \text{const.}$
$x \rightarrow -\infty \text{ to } +\infty$	$y \rightarrow -\infty \text{ to } +\infty$	$x \rightarrow -\infty \text{ to } +\infty$
$y \rightarrow -\infty \text{ to } +\infty$		

→ $\hat{a}_x, \hat{a}_y$ are unit vectors and are orthogonal to each other. $\Rightarrow |\hat{a}_x| = |\hat{a}_y| = 1.$

→ They are represented + x-axis and along y-axis respectively.

→ They may be also represented as unit vectors normal to $x = \text{constant}$ and $y = \text{constant}$ respectively.

(ii) Polar



→ Locus of $s = \text{constant}$ represents a circle. Whose centre coincides should not origin. Therefore, s assumes all possible values ranging from 0 to ∞ . All

$s = \text{constant}$, ϕ assumes all possible values ranging from 0 to 2π .

Integrals	on $\rho = \text{const.}$	on $\phi = \text{const.}$
$\rho \rightarrow 0 \text{ to } \infty$	$\phi \rightarrow 0 \text{ to } 2\pi$	$\rho \rightarrow 0 \text{ to } \infty$
$\phi \rightarrow 0 \text{ to } 2\pi$		

→ Locus of $\phi = \text{constant}$ is a line emerging out from the origin. ϕ assumes all possible values ranging from 0 to 2π .

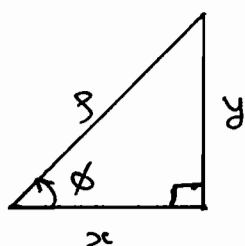
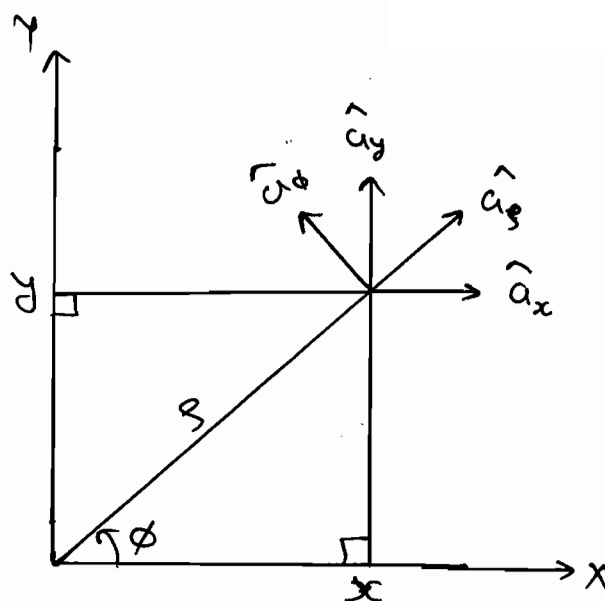
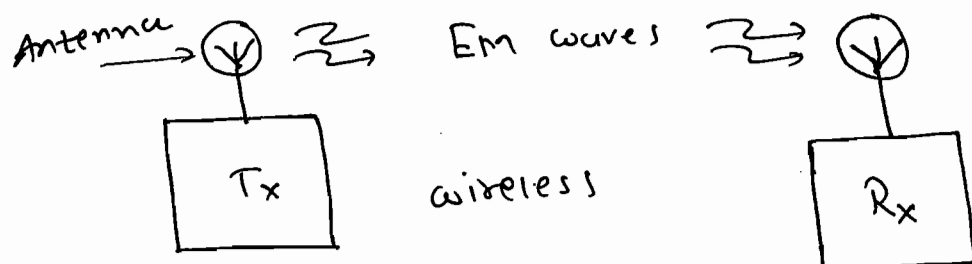
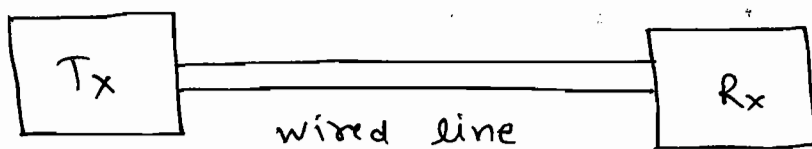
On $\phi = \text{constant}$ ρ assumes all possible values ranging from 0 to ∞ .

→ $\hat{a}_\rho$ & $\hat{a}_\phi$ are unit vectors orthogonal to each other.

→ $\hat{a}_\rho$ is represented normal to $\rho = \text{constant}$
(OR) normal to the circle.

→ Similarly, $\hat{a}_\phi$ is a unit vector
projecting normal to $\phi = \text{constant}$. and
it is projecting in the counter clock
wise direction as shown in figure.

→ $\hat{a}_\rho$ is normal to the circle and
 $\hat{a}_\phi$ is tangent to the circle.

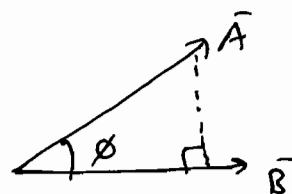


$$\rightarrow y = r \sin \phi$$

$$x = r \cos \phi$$

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$



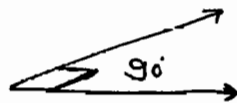
$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \phi$$

$$= AB$$

if $\phi = 0^\circ$.

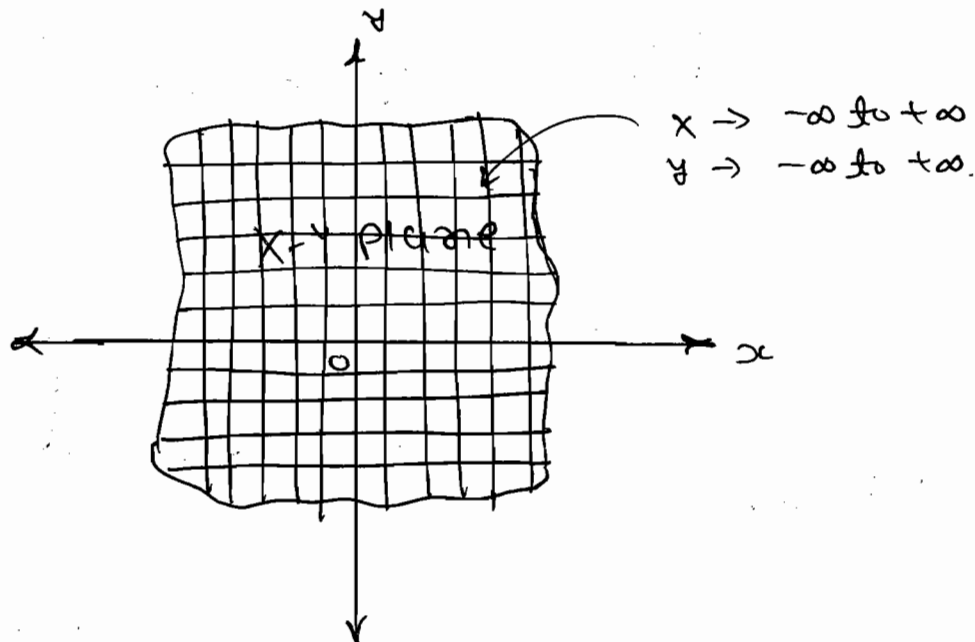


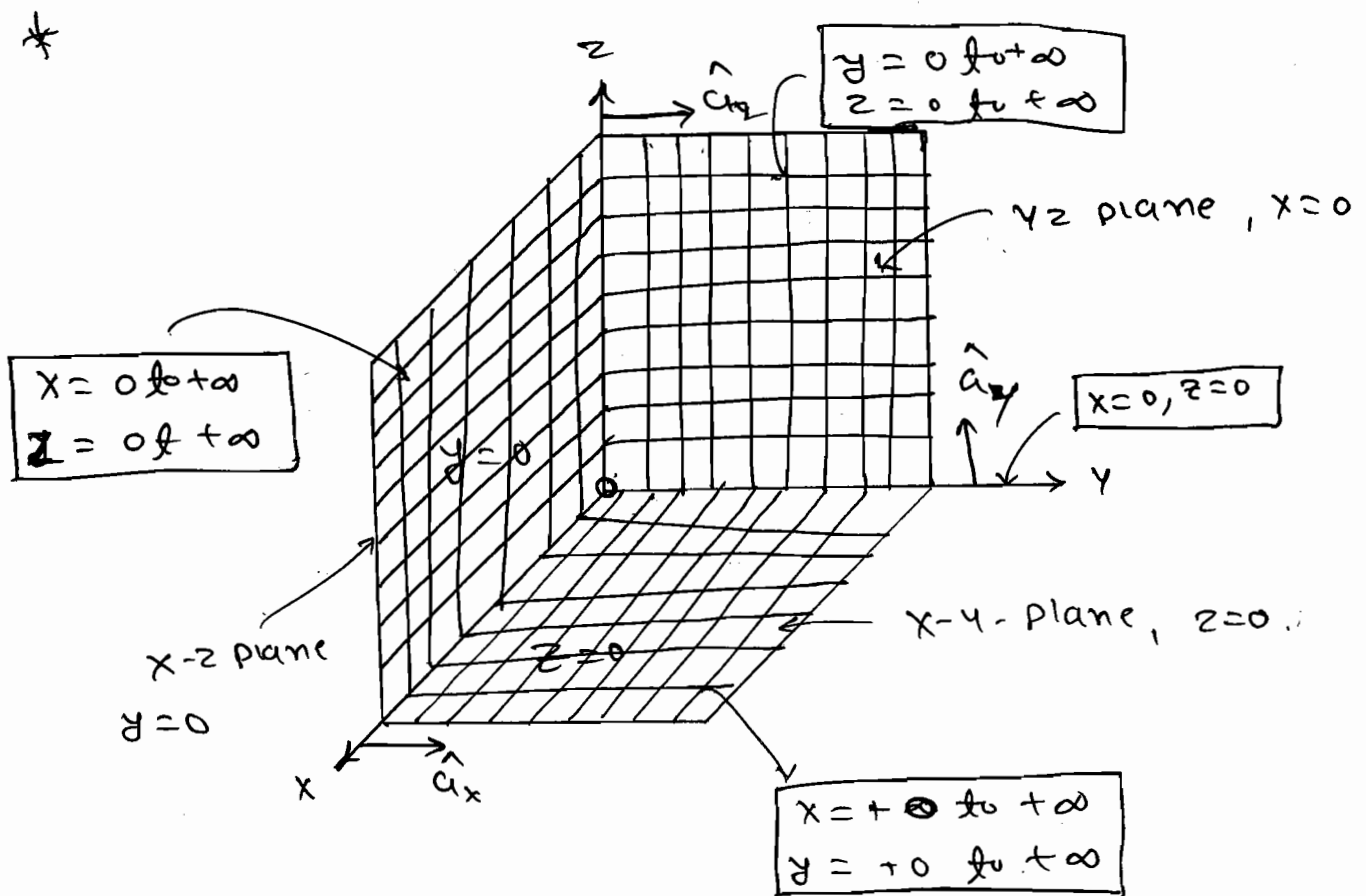
	$\hat{a}_1$	$\hat{a}_\phi$	$\hat{a}_2$
$\hat{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\hat{a}_y$	$\sin\phi$	$\cos\phi$	0
$\hat{a}_z$	0	0	1



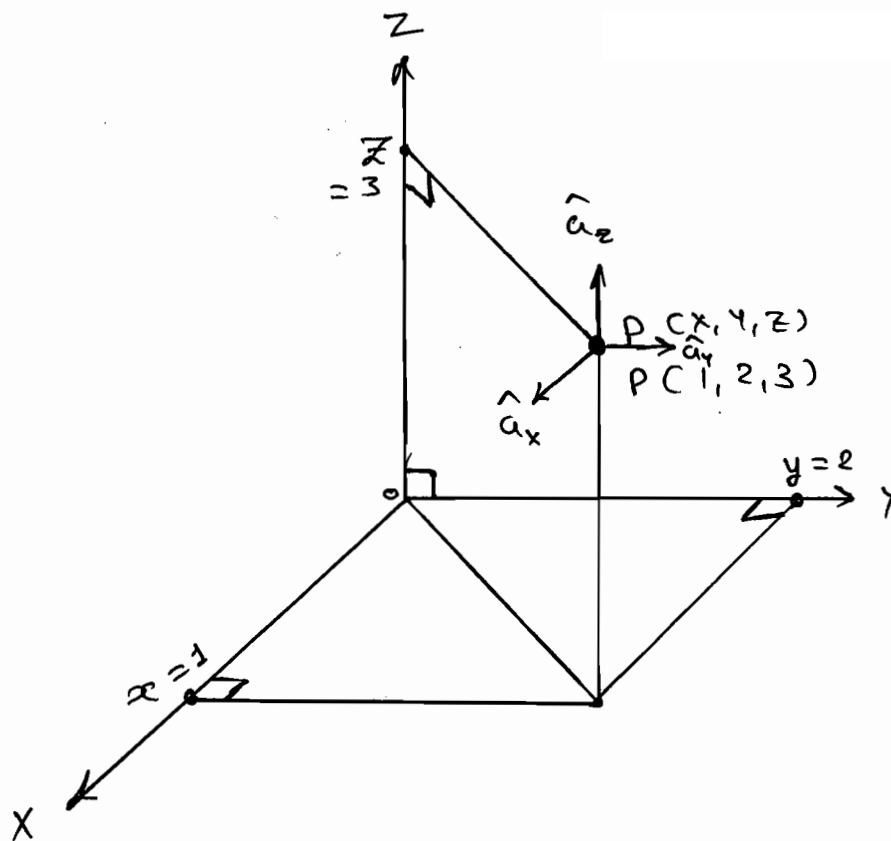
Plane:

→ Plane is a sheet like structure whose thickness is neglected.





* Cartesian Coordinates System



→ In general in a 3-D co-ordinates system fixing 3 coordinates that represents a point

- fixing 2 coordinates that represents a line
- fixing 1 co-ordinates that represents a plane.

⇒ In general

$$x \rightarrow -\infty \text{ to } +\infty$$

$$y \rightarrow -\infty \text{ to } +\infty$$

$$z \rightarrow -\infty \text{ to } +\infty$$

⇒ $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are unit vectors and orthogonal to each other.

$$\rightarrow |\hat{a}_x| = |\hat{a}_y| = |\hat{a}_z| = 1.$$

⇒ $\hat{a}_x, \hat{a}_y$, and $\hat{a}_z$ are unit vectors and orthogonal to each other.

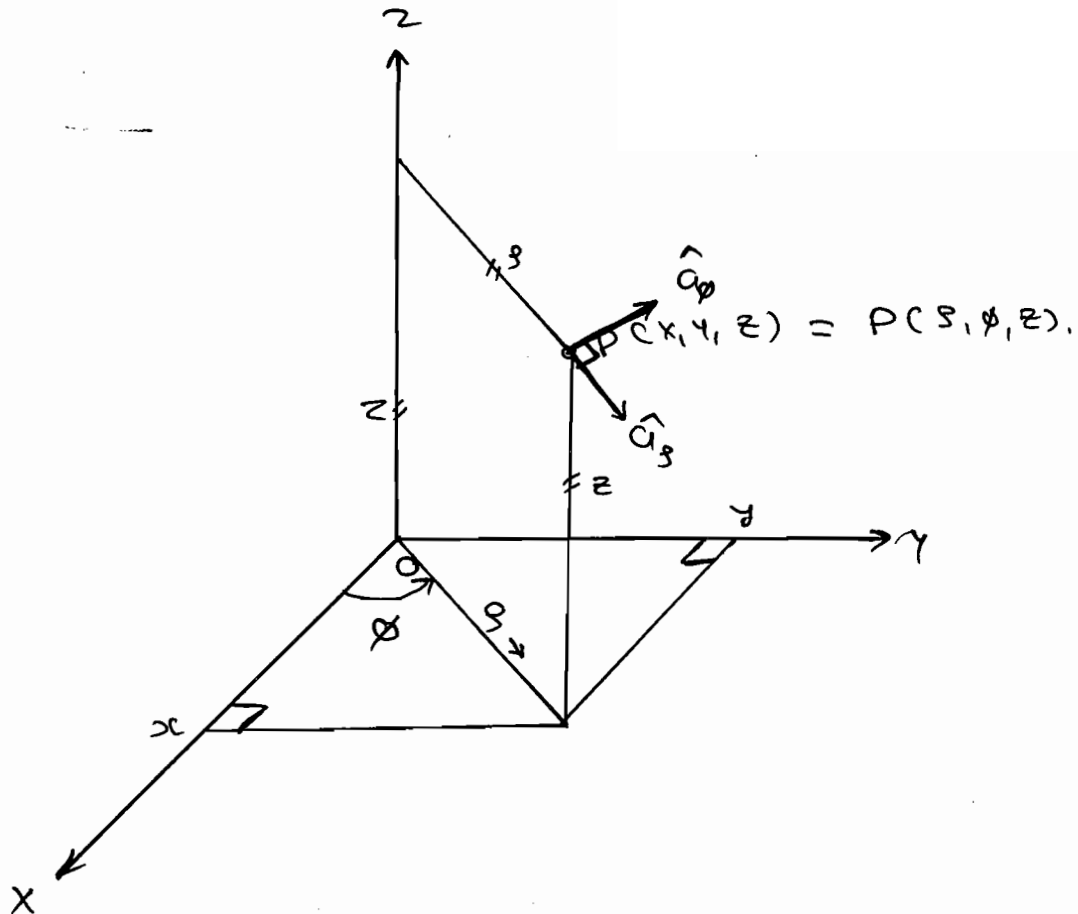
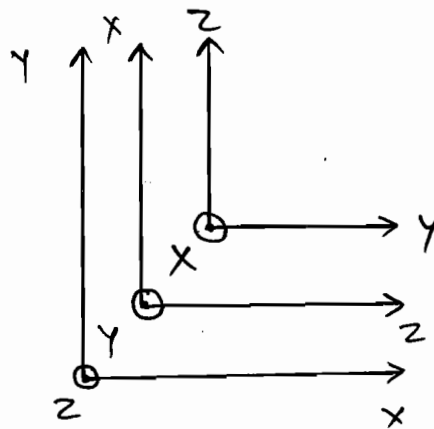
→ They are represented along x-axis, y-axis and z-axis.

→ They may be also represented as unit vectors normal to $x = \text{constant}$, $y = \text{constant}$, $z = \text{constant}$ planes respectively.

* $\odot$ vector/x-axis coming out of the paper

* $\odot$ vectors / is Coming out of the paper.

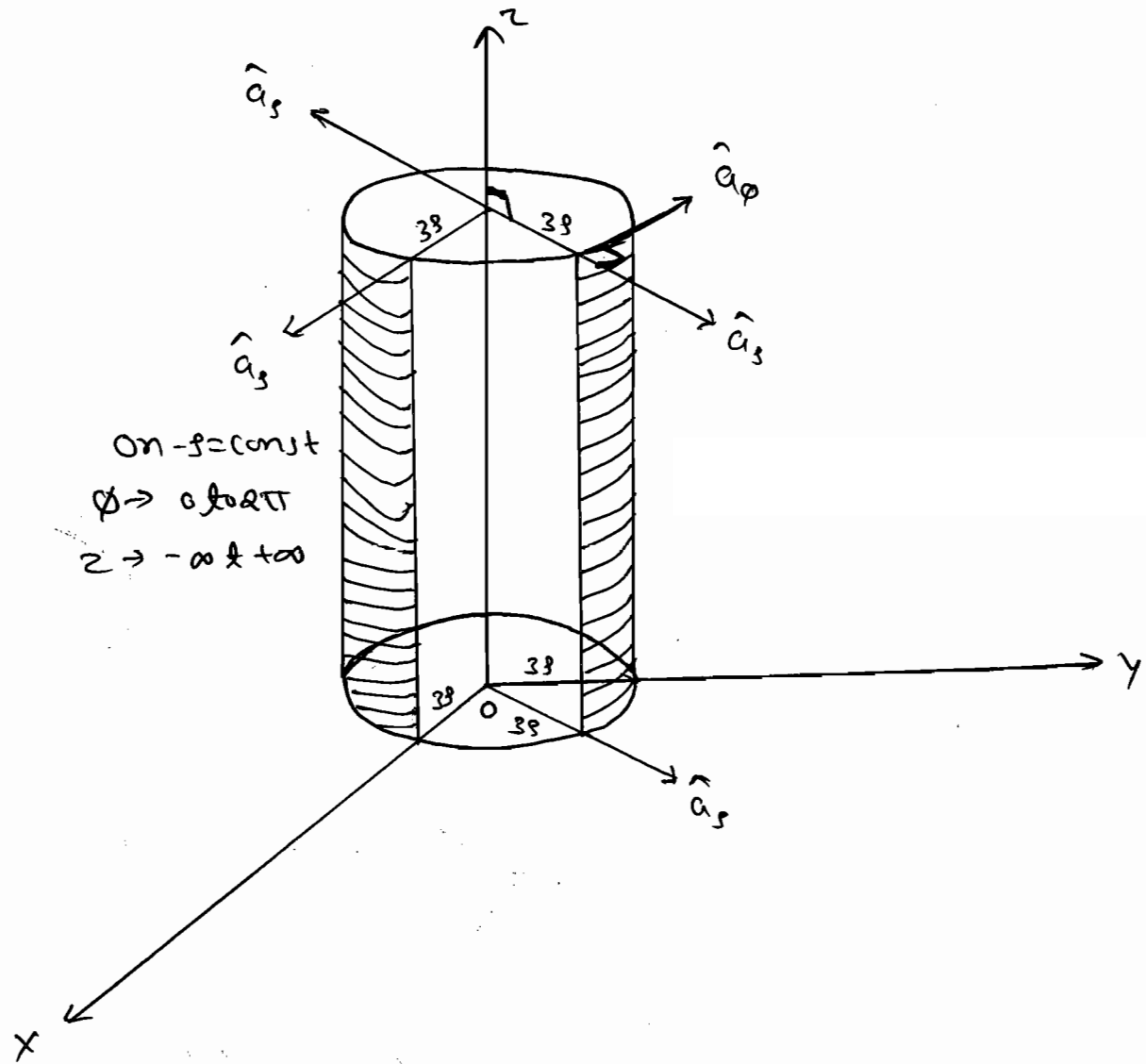
* circular or cylindrical
coordinate system.



$\rightarrow$ $\rho = \text{constant}$ represents a cylindrical plane whose cross-section is circular and whose axis coincides with z -axis. ρ is the distance measured normal to z -axis. ρ can assume all possible values ranging from 0 to ∞ .

→ $\hat{a}_s$ is a unit vector projecting normal to $s = \text{Constant}$ plane.

⇒

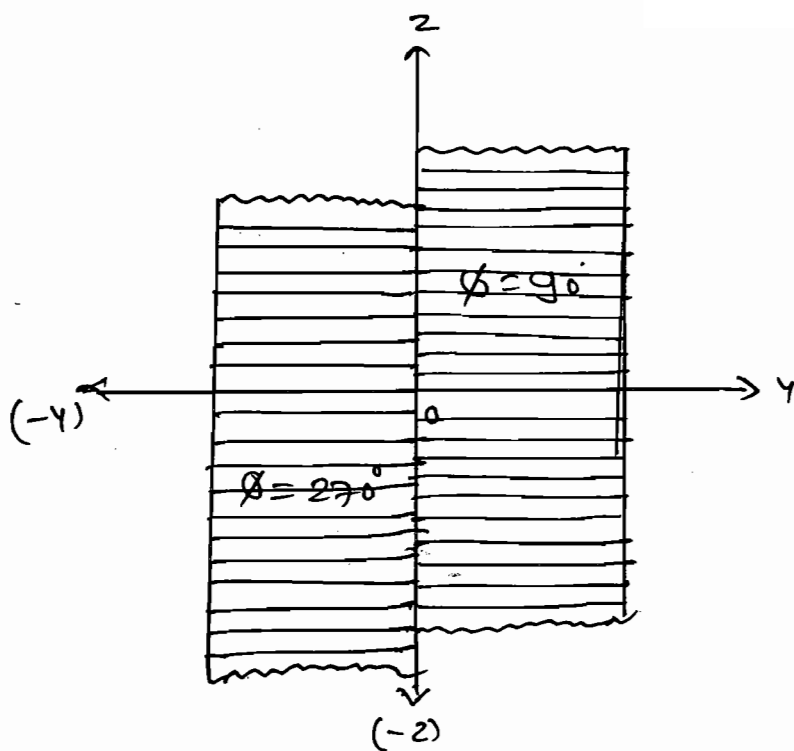
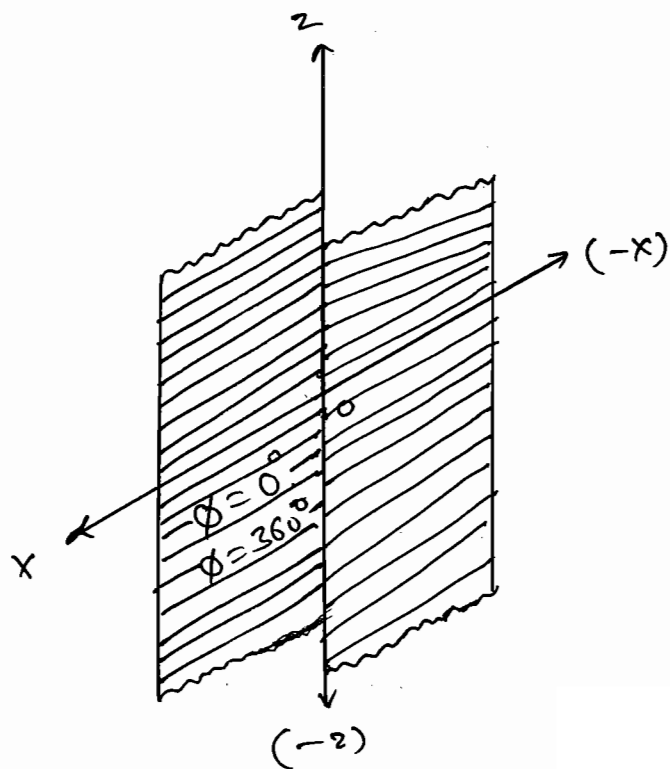


→ θ is called azimuth angle.

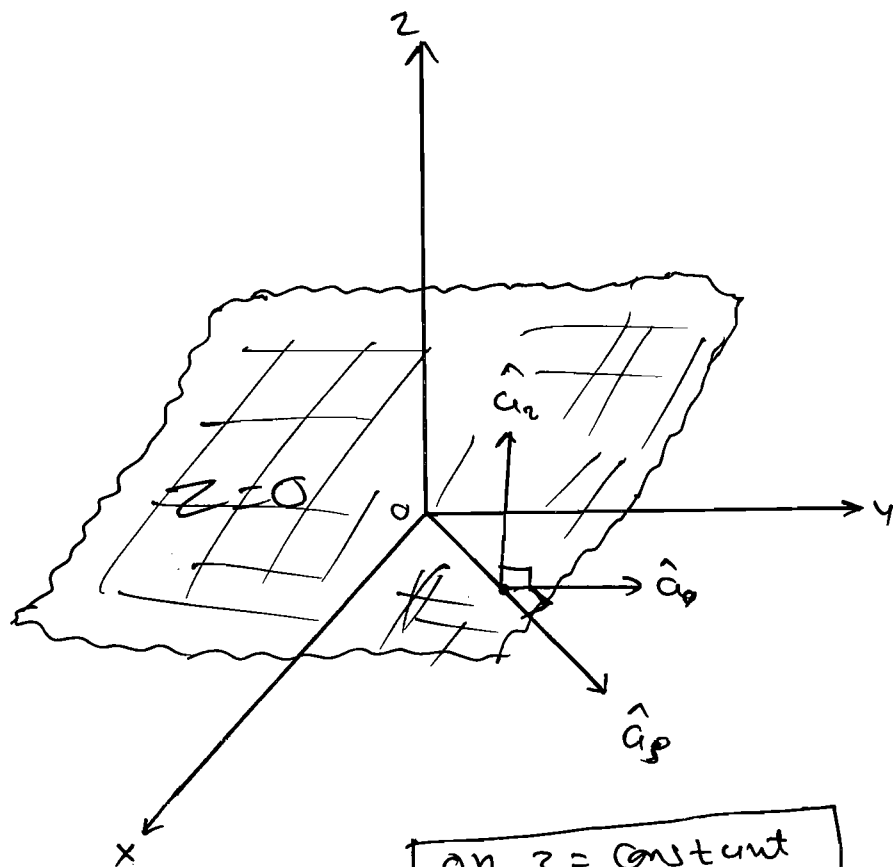
→ $\phi = \text{constant}$ plane is called elevation plane.

→ ϕ assumes all possible values ranging from 0 to 2π .

→ $\hat{a}_\phi$ is a unit vector projecting normal to $\phi = \text{constant}$ plane.



*



on $z = \text{constant}$
 $s \rightarrow 0 \text{ to } \infty$
 $\phi \rightarrow 0 \text{ to } 2\pi$

→ On a particular Constant plane that particular unit vector would project normal to the plane then remaining 2 unit vectors could be projecting tangential to the plane.

for e.g. On $s = \text{constant}$ plane $\hat{a}_s$ could be projecting normal to $s = \text{constant}$ and $\hat{a}_\phi$ & $\hat{a}_z$ could be projecting tangential to the plane.

In general

$$\rho \Rightarrow 0 \text{ to } \infty$$

$$\phi \Rightarrow 0 \text{ to } 2\pi$$

$$z \Rightarrow -\infty \text{ to } +\infty.$$

$\rightarrow \hat{a}_\rho, \hat{a}_\phi, \hat{a}_z$ are unit vectors and orthogonal to each other.

$\Rightarrow$ In general,

$$\begin{aligned} \vec{B} &= B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z \rightarrow \text{cartesian} \\ \vec{B} &= B_\rho \hat{a}_\rho + B_\phi \hat{a}_\phi + B_z \hat{a}_z \rightarrow \text{cylindrical} \end{aligned}$$

Ex-1 Let $\vec{B} = 2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z$ is define at a point $P(3, 4, 5)$ m. Convert this vector in cylindrical system.

Ans: $\vec{B} = B_\rho \hat{a}_\rho + B_\phi \hat{a}_\phi + B_z \hat{a}_z$

$$\therefore \vec{B} \cdot \hat{a}_\rho = B_\rho \underbrace{\hat{a}_\rho \cdot \hat{a}_\rho}_1 + B_\phi \underbrace{\hat{a}_\phi \cdot \hat{a}_\rho}_0 + B_z \underbrace{\hat{a}_z \cdot \hat{a}_\rho}_0$$

$$\therefore \vec{B} \cdot \hat{a}_\rho = B_\rho$$

$$\therefore B_\rho = \vec{B} \cdot \hat{a}_\rho$$

$$\therefore B_\rho = 2\hat{a}_x \cdot \hat{a}_\rho + 3\hat{a}_y \cdot \hat{a}_\rho + 4\hat{a}_z \cdot \hat{a}_\rho$$

$$B_\rho = 2 \cos \phi + 3 \sin \phi$$

Now, $P(3, 4, 5)$.

$$\therefore \rho = \sqrt{3^2 + 4^2} = 5$$

$$\phi = \tan^{-1} \left(\frac{4}{3} \right) = 53.13^\circ$$

$$\therefore B_3 = 26553.130^\circ + 25 \sin 53.130^\circ$$

$$\therefore \boxed{B_3 = 3.6}$$

Similarly,

$$\therefore B_\phi = \vec{B} \cdot \hat{a}_\phi, \quad B_z = \vec{B} \cdot \hat{a}_z$$

$$\therefore B_\phi = -2 \sin \phi + 3 \cos \phi.$$

$$\therefore \boxed{B_\phi = 0.2}$$

$$\therefore B_z = \vec{B} \cdot \hat{a}_z$$

$$\therefore B_z = B_z$$

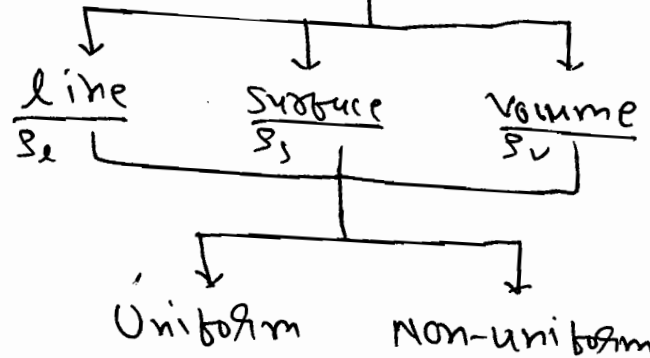
$$\therefore \boxed{B_z = 4}$$

$$\therefore \boxed{\vec{B} = 3.6 \hat{a}_3 + 0.2 \hat{a}_\phi + 4 \hat{a}_z}$$

Charge Configuration/Classification

Point Charge 'Q'.
(The Charge Combined to a point is called point charge).

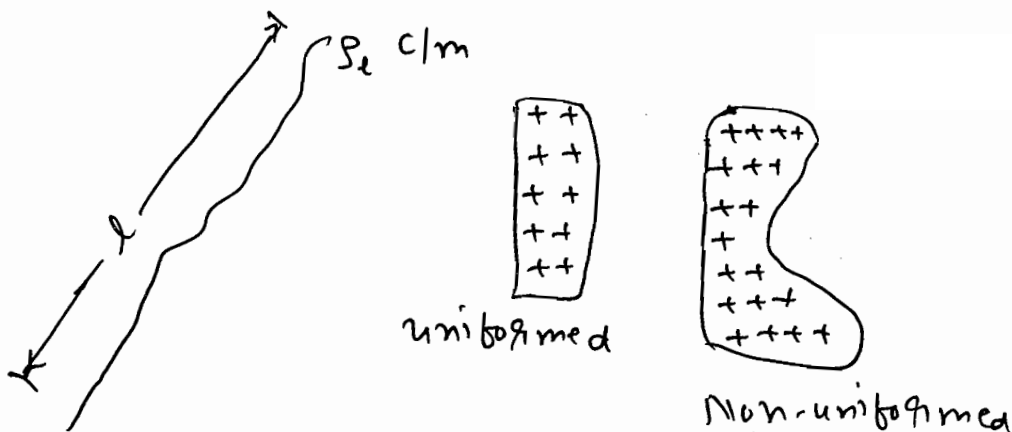
Charge Distribution



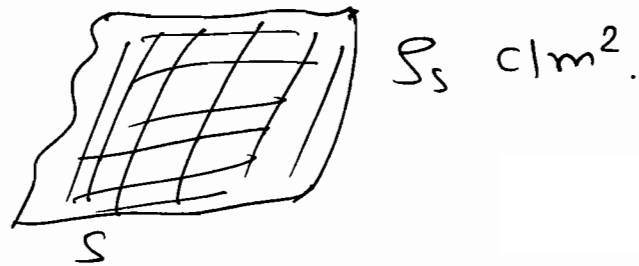
→ Distribution of Charge per unit line

is called line charge distribution it and is designated by S_L . If S_L is constant that may be called uniform otherwise nonuniform.

→ Having uniform charge distributions are impractical. The reason is due to mutual repulsion betⁿ the like charges.



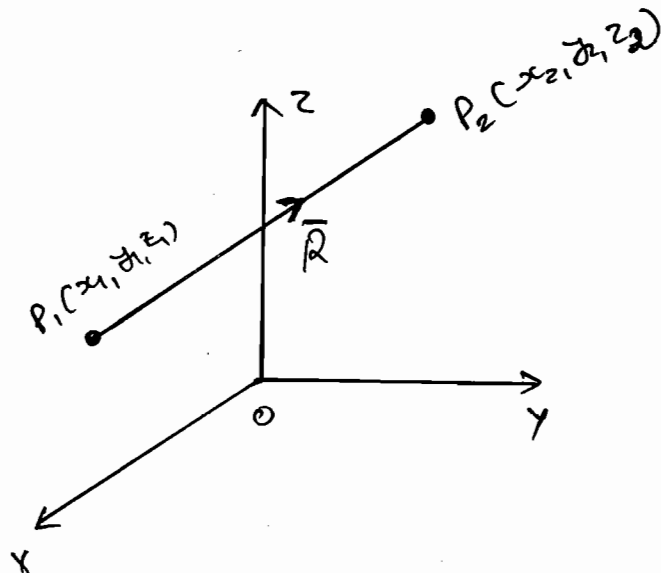
→ Distribution of charge per unit area is called surface charge density. and is designated by $\rho_s \text{ C/m}^2$



→ Distribution of charge per unit Volume is called Volume ^{charge} density and is designated by $\rho_v \text{ C/m}^3$.



*



→ R is vector drawn from P_1 to P_2 .

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$$\therefore \vec{R} = (x_2 - x_1) \hat{a}_x + (y_2 - y_1) \hat{a}_y + (z_2 - z_1) \hat{a}_z$$

$$\therefore |\vec{R}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

$\hat{a}_R$ = unit vector components of R.

$$|\hat{a}_R| = \frac{|\vec{R}|}{|\vec{R}|} = 1.$$

* ~~Newton's Force Law~~ Coulomb's Force Law:
→ it gives force of attraction (or) repulsion betⁿ charge conducting bodies.

→ if the charges are like the force is repulsive otherwise attractive.

→ Capacitivity (or) Permittivity specifies property
of a medium and that indicates ability
to store electrical energy.

$$F \propto Q_1 Q_2$$

$$\propto \frac{1}{|\vec{R}|^2}$$

$$\underline{E = \epsilon_0 \epsilon_r} \quad \text{F/m}$$

$$\therefore \boxed{|\vec{F}| = \frac{Q_1 Q_2}{4\pi\epsilon |\vec{R}|^2} \text{ N}}$$

$$\rightarrow E = \epsilon \epsilon_r \quad \text{F/m}$$

ϵ : Permittivity (or) capacitivity

ϵ_0 : Absolute permittivity $\rightarrow 8.854 \times 10^{-12} \text{ F/m}$

$$\checkmark = \frac{10^{-9}}{36\pi} \text{ F/m.}$$

ϵ_r = relative permittivity

(Q) Dielectric constant has no unit.

→ The vector force ^{acting} on Q_2 due to Q_1 ~~charge~~ is given by.

$$\therefore \vec{F} = \frac{Q_1 \cdot Q_2}{4\pi\epsilon |\vec{R}|^2} \cdot \hat{U}_R$$

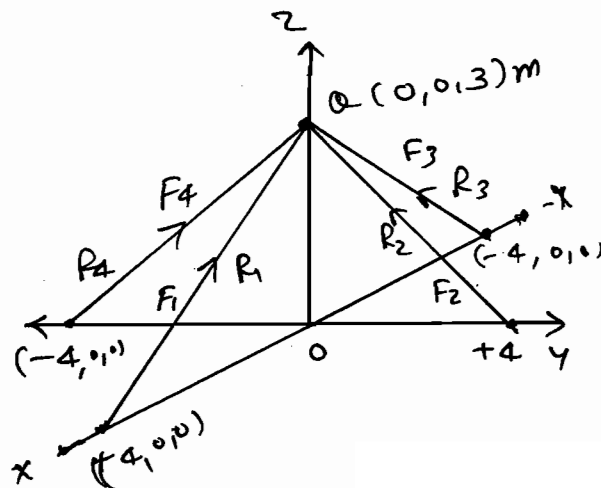
$$\boxed{\vec{F} = \frac{Q_1 \cdot Q_2}{4\pi\epsilon |\vec{R}|^2} \cdot \frac{\vec{R}}{|\vec{R}|} \cdot 1N}$$

→ The vector force acting on Q_1 due to Q_2 is $-\vec{F}$.

further we write $\boxed{|\vec{F}| = |-\vec{F}|}$

Ex-1 Four point charges of $1\mu C$ each one located on the x & y axis at $\pm 4m$. Find the vector force acting on $1\mu C$ charge which is located on z-axis at $z=3$ meters.

Ans:



$$\vec{F}_1 = \frac{10 \times 10^{-6} \times 10^{-3}}{4\pi \times \frac{10^{-9}}{36\pi} \times (5)^2} \times \frac{-4\hat{a}_x + 3\hat{a}_z}{(5)}$$

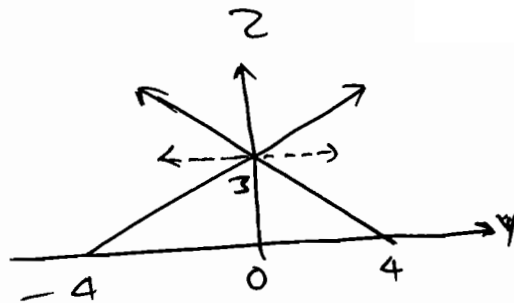
$$\vec{F}_2 = \frac{10 \times 10^{-6} \times 10^{-3}}{4\pi \times \frac{10^{-9}}{36\pi} \times (5)^2} \times \frac{-4\hat{a}_y + 3\hat{a}_z}{(5)}$$

$$\vec{F}_3 = \frac{10 \times 10^{-6} \times 10^{-3}}{4\pi \times \frac{10^{-9}}{36\pi} \times (5)^2} \times \frac{+4\hat{a}_x + 3\hat{a}_z}{(5)}$$

$$F_4 = \frac{10 \times 10^{-6} \times 10^{-3}}{4\pi \times \frac{10^{-9}}{36\pi} \times (5)^2} \times \frac{+4\hat{a}_y + 3\hat{a}_z}{(5)}$$

$$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$\boxed{\vec{F} = 8.64 \hat{a}_z \text{ N}}$$

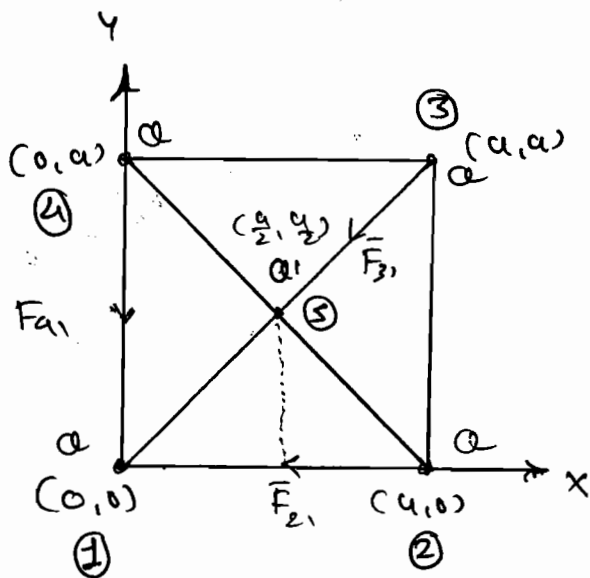


→ The 10 μC charges are located symmetrically on the x & y axis. about z-axis which results in cancellation of horizontal force components and the resultant force would be along $\hat{a}_z$ direction only.

Ex-2 4 point charges of $+q$ each are located at the corners of a square. What point charge to be kept at a centre of a square so that the resultant force acting on any charge which are located at a corner of a square is zero. (Or) it is required to hold 4 point charges of $+q$ each in equilibrium at corners of a square. What point charge to be kept at the centre of the square so that the charges would be in equilibrium.

Hint: equilibrium means the resultant force acting on any charge which are located at the corners of the square is zero.

Ans:



→ We have to find value of q in terms of q so that resultant force acting on any charge which are located at the corners of the square is zero.

for e.g.

Let's find force on ①

$$\therefore \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} + \vec{F}_{51} = 0.$$

$$\Rightarrow \vec{F}_{21} = \frac{Q^2}{4\pi\epsilon (a^2)} \times \frac{-a\hat{a}_x}{a}$$

$$\vec{F}_{41} = \frac{Q^2}{4\pi\epsilon (a^2)} \times \frac{-a\hat{a}_y}{a}$$

$$\vec{F}_{31} = \frac{Q^2}{4\pi\epsilon (\sqrt{2}a)^2} \times \frac{-a\hat{a}_x - a\hat{a}_y}{\sqrt{2}a}$$

$$\therefore \vec{F}_{51} = \frac{Q Q'}{4\pi\epsilon \left(\frac{a}{\sqrt{2}}\right)^2} \times \frac{-\hat{a}_x - \hat{a}_y}{\sqrt{2}}$$

→ Consider the sum and of $\hat{a}_x$ components of all forces and the same make equals to 0

$$\therefore \frac{-Q^2}{4\pi\epsilon a^2} - \frac{Q^2}{4\pi\epsilon \sqrt{2}a^2} - \frac{Q Q'}{4\pi\epsilon \frac{a^2}{\sqrt{2}}} = 0$$

$$\therefore Q' = -0.956 Q.$$

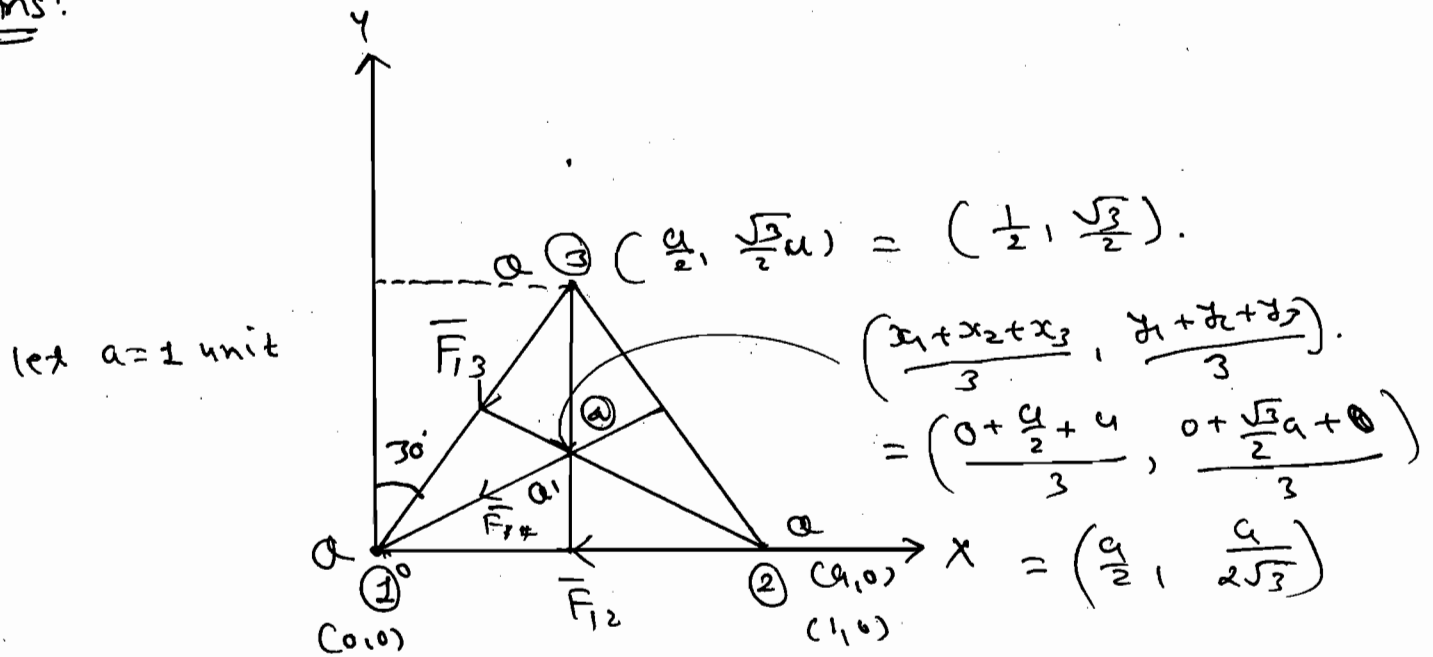
→ Even we consider the sum of $\hat{a}_y$ components the same ans is expected.

✓
→

Ex-3 3 point charges of +Q each are located at the corners of an equilateral triangle. What point charge to be kept at the centre of the triangle so that the resultant force acting on any charge which are located at the corners of the triangle is 0.

Ans: $Q' = -\frac{Q}{\sqrt{3}}$

Ans:



$$\Rightarrow \vec{F}_{13} = \frac{kQ^2}{1} \left(-\hat{a}_{x2} - \sqrt{\frac{3}{2}} \hat{a}_y \right) \quad \vec{R} = -\frac{Q}{2} \hat{a}_{x2} - \frac{\sqrt{3}}{2} Q \hat{a}_y$$

$$\vec{R} = -\frac{Q}{2} \hat{a}_{x2} - \frac{\sqrt{3}}{2} Q \hat{a}_y$$

$$\Rightarrow \vec{F}_{12} = \frac{-kQ^2}{1} (\hat{a}_{x2}) \quad \vec{R} = -\hat{a}_{x2}$$

$$|\vec{R}| = 1$$

$$\Rightarrow \vec{F}_{14} = \frac{-kQaQ'}{\frac{1}{\sqrt{3}}} \times \left(+\frac{\hat{a}_{x2}}{2} + \frac{1}{2\sqrt{3}} \hat{a}_y \right)$$

$$\vec{R} = -\frac{Q}{2} \hat{a}_{x2} - \frac{1}{2\sqrt{3}} Q \hat{a}_y$$

$$|\vec{R}| = \sqrt{\frac{1}{4} + \frac{1}{12}} = \sqrt{\frac{1}{3}}$$

$$\Rightarrow \overline{F_{14}} = -k\alpha\alpha' \times \frac{3}{2} \times \sqrt{3} \left(\hat{a}_x + \hat{a}_y \times \frac{1}{\sqrt{3}} \right)$$

$$\text{Now, } \overline{F_{12}} + \overline{F_6} + \overline{F_{14}} = 0.$$

$\therefore$ let, only $\hat{a}_x$ - a component

$$\therefore \frac{-k\alpha^2}{2} - k\alpha^2 - k\alpha\alpha' \times \frac{3\sqrt{3}}{2} = 0.$$

$$\therefore -\frac{3}{2}\alpha = \alpha' \times \frac{3\sqrt{3}}{2}$$

$$\alpha' = -\frac{\alpha}{\sqrt{3}}$$

$$\alpha' = \frac{-\alpha}{\sqrt{3}} c$$

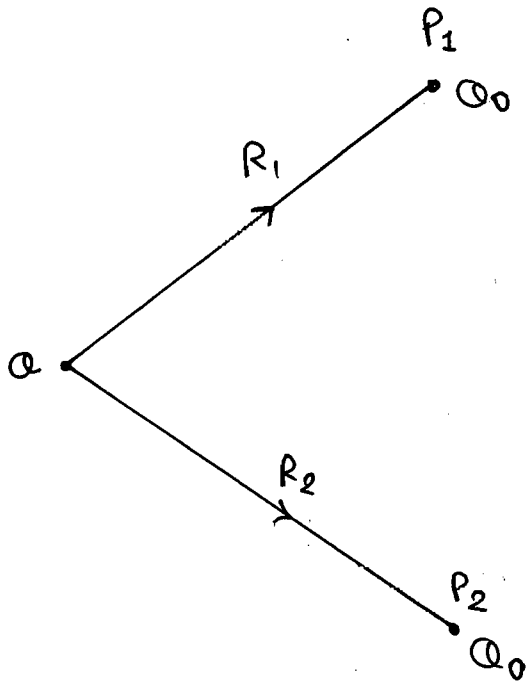
$\rightarrow$ If we considered only y-component then we also get same answers.

* Electric Field ($\vec{E}$):-

→ It is defined as force per unit charge.

→ Unit is N/C (or) V/m .

→



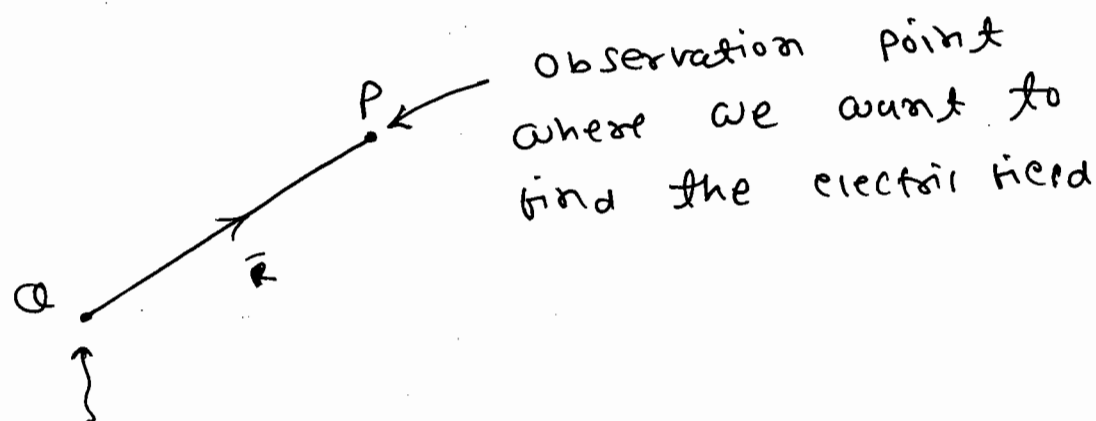
$$\rightarrow \vec{F}_1 = \frac{Q \cdot Q_0}{4\pi\epsilon |\vec{R}_1|^2} \cdot \hat{u}_{R_1}$$

$$\text{At } P_1 \Rightarrow \frac{\vec{F}_1}{Q_0} = \frac{Q}{4\pi\epsilon |\vec{R}_1|^2} \cdot \hat{u}_{R_1} = \text{Electric field}$$

$$\rightarrow \vec{F}_2 = \frac{Q \cdot Q_0}{4\pi\epsilon |\vec{R}_2|^2} \cdot \hat{u}_{R_2}$$

$$\text{At } P_2 \Rightarrow \frac{\vec{F}_2}{Q_0} = \frac{Q}{4\pi\epsilon |\vec{R}_2|^2} \cdot \hat{u}_{R_2} = \text{Electric field}$$

⇒ In general

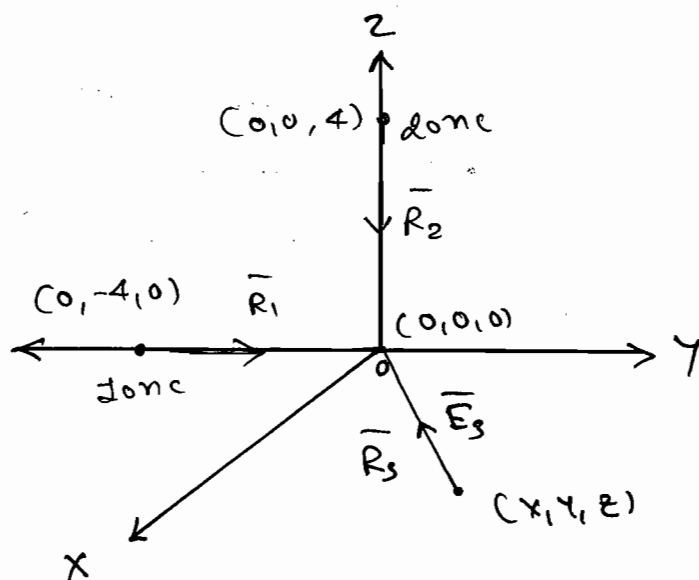


Source point,
Source of electric field
is electric charge.

$$\vec{E} = \frac{Q}{4\pi\epsilon |\vec{r}|^2} \hat{r}$$
$$\vec{E} = \frac{Q}{4\pi\epsilon |\vec{r}|^2} \times \frac{\vec{r}}{|\vec{r}|}$$

- ★
Ex-1 A point charge of 10 nC is located at $(0, -4, 0)\text{ m}$. Another charge of 20 nC is located at $(0, 0, 4)\text{ m}$.
- (i) find the electric field at the origin.
- (ii) Where should a 30 nC point charge be located so that electric field at origin is 0.

Ans:



$$\vec{E}_1 = \frac{10 \times 10^{-9}}{4\pi\epsilon_0 \times (4)^2} \times \frac{4\hat{a}_y}{(4)} = 5.625 \hat{a}_y \text{ V/m}$$

$$\vec{E}_2 = \frac{20 \times 10^{-9}}{4\pi \times \frac{20 \times 10^{-9}}{36\pi} (4)^2} \times \frac{-4\hat{a}_z}{4} = -11.25 \hat{a}_z \text{ V/m.}$$

① Electric field at the origin

$$= \vec{E}_1 + \vec{E}_2 = (5.625\hat{a}_y - 11.25\hat{a}_z) \text{ V/m.}$$

$$\textcircled{2} \quad \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \Rightarrow \vec{E}_3 = -(\vec{E}_1 + \vec{E}_2).$$

$$\therefore \vec{E}_3 = -[5.625\hat{a}_y - 11.25\hat{a}_z] \text{ V/m.}$$

$$\rightarrow \vec{E}_3 = \frac{30 \times 10^{-9}}{4\pi \times \frac{10^{-9}}{36\pi} \times (x^2 + y^2 + z^2)^{3/2}} \times (-x\hat{a}_x + (-y\hat{a}_y) - z\hat{a}_z)$$

$$\vec{E}_3 = \frac{270}{(x^2 + y^2 + z^2)^{3/2}} (-x\hat{a}_x - y\hat{a}_y - z\hat{a}_z)$$

$$\Rightarrow \text{Compare } \hat{a}_x \Rightarrow x=0$$

$$\text{Compare } \hat{a}_y \Rightarrow -\frac{270y}{(y^2+z^2)^{3/2}} = -5.625 \quad \text{--- (A)}$$

$$\text{Compare } \hat{a}_z \Rightarrow \frac{-270z}{(y^2+z^2)^{3/2}} = 11.25 \quad \text{--- (B)}$$

$$\frac{\text{(A)}}{\text{(B)}} \Rightarrow -\frac{y}{z} = \frac{1}{2} \Rightarrow \boxed{\frac{z}{y} = -2}$$

$$\text{from (A)} \quad \therefore \frac{270y}{y^3 [1 + (zy)^2]^{3/2}} = 5.625$$

$$\Rightarrow \frac{270y}{y^3 [1 + z^2]^{3/2}} = 5.625$$

$$\therefore y = \pm 2.07 \text{ m}$$

$$z = -2y = \mp 4.14 \text{ m}$$

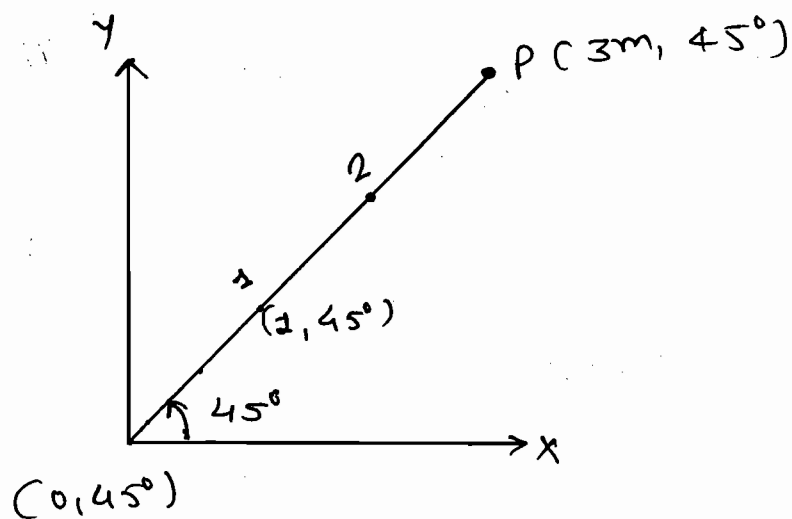
The possibilities.

$$(0, 2.07, -4.14) \checkmark$$

$$(0, -2.07, 4.14) \times$$

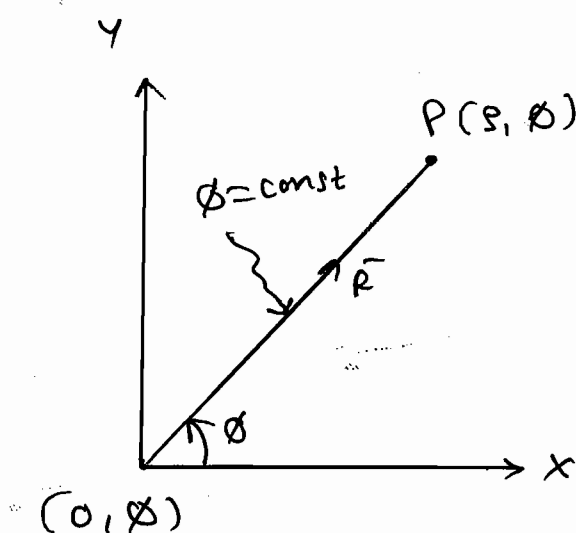
$$\therefore \text{Ans: } (0, 2.07, -4.14) \text{ m}$$

*



→ We assume that the origin lines are on $\phi = 45^\circ$ line.

→ With ref. to $P(3m, 45^\circ)$, the origin is coordinated as $(0m, 45^\circ)$



(origin lines on $\phi = \text{const. line}$)

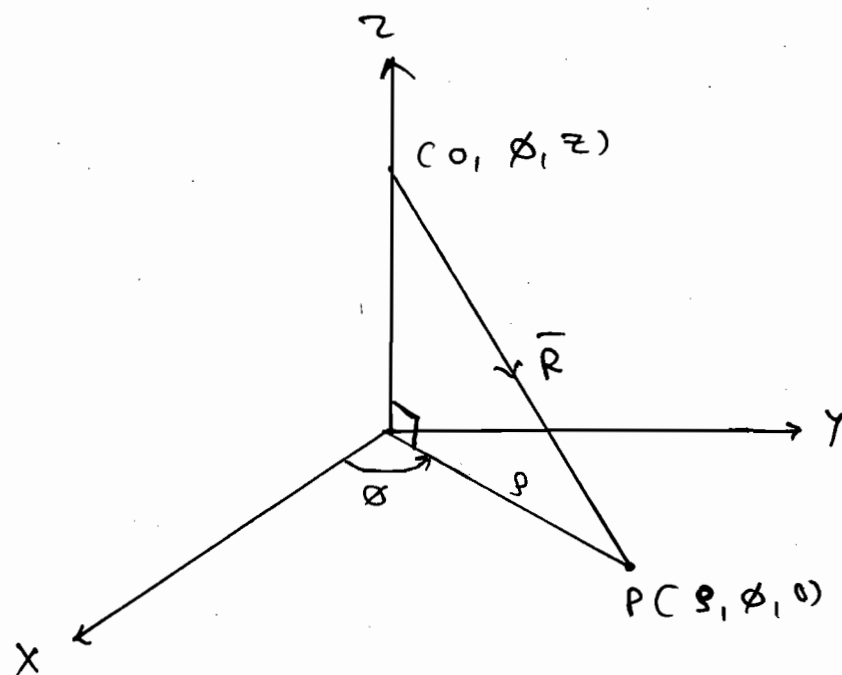
$$\vec{r} = (s-0) \hat{a}_s + (\phi-\phi) \hat{a}_\phi$$

$$\therefore \vec{r} = s \hat{a}_s$$

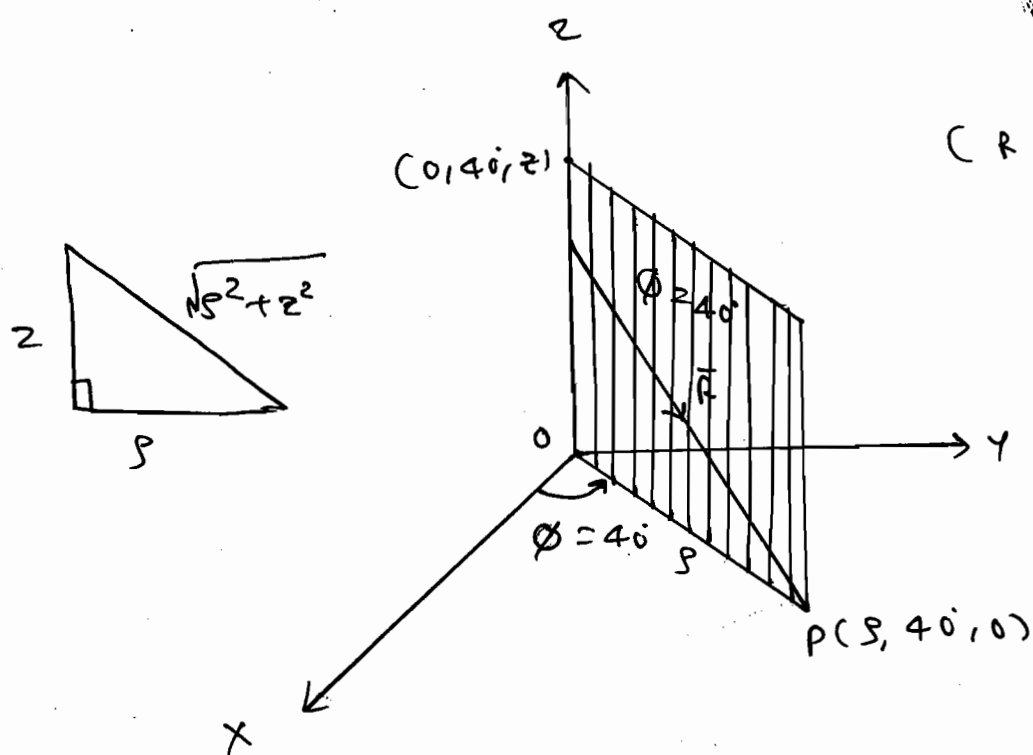
$$\therefore |\vec{r}| = s$$

$$\hat{a}_r = \frac{\vec{r}}{|\vec{r}|} = \hat{a}_s$$

*



→ With ref. to $P(\rho, \phi, 0)$ the point on the z -axis is co-ordinated as $(0, \phi, z)$.



(R lies on $\phi = 40^\circ$)

$$\vec{R} = \rho \hat{a}_\rho - z \hat{a}_z$$

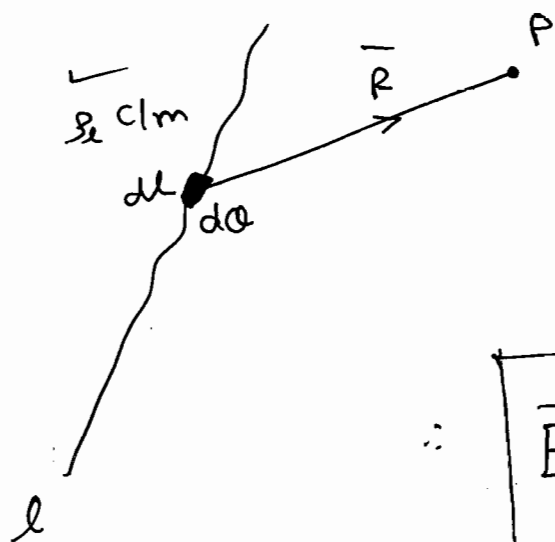
$$\therefore |\vec{R}| = \sqrt{\rho^2 + z^2}$$

→ The point on the z -axis also assumed to lying on $\phi = 40^\circ$ plane.

∴ with ref to $P(8, 40^\circ, 0)$, the point on the z -axis is co-ordinated as $(0, 40^\circ, z)$.

Ex-1 Expression for $\vec{E}$ due to a line with a charge density of ρ_e C/m.

⇒



$$d\vec{E} = \frac{dq}{4\pi\epsilon |\vec{R}|^2} \cdot \hat{G}_P$$

$$= \frac{\rho_e dl}{4\pi\epsilon |\vec{R}|^2} \times \hat{G}_P$$

$$\therefore \vec{E} = \int_L \frac{\rho_e dl}{4\pi\epsilon |\vec{R}|^2} \cdot \frac{\vec{R}}{|\vec{R}|} \text{ V/m.}$$

→ $dq = \rho_e dl$

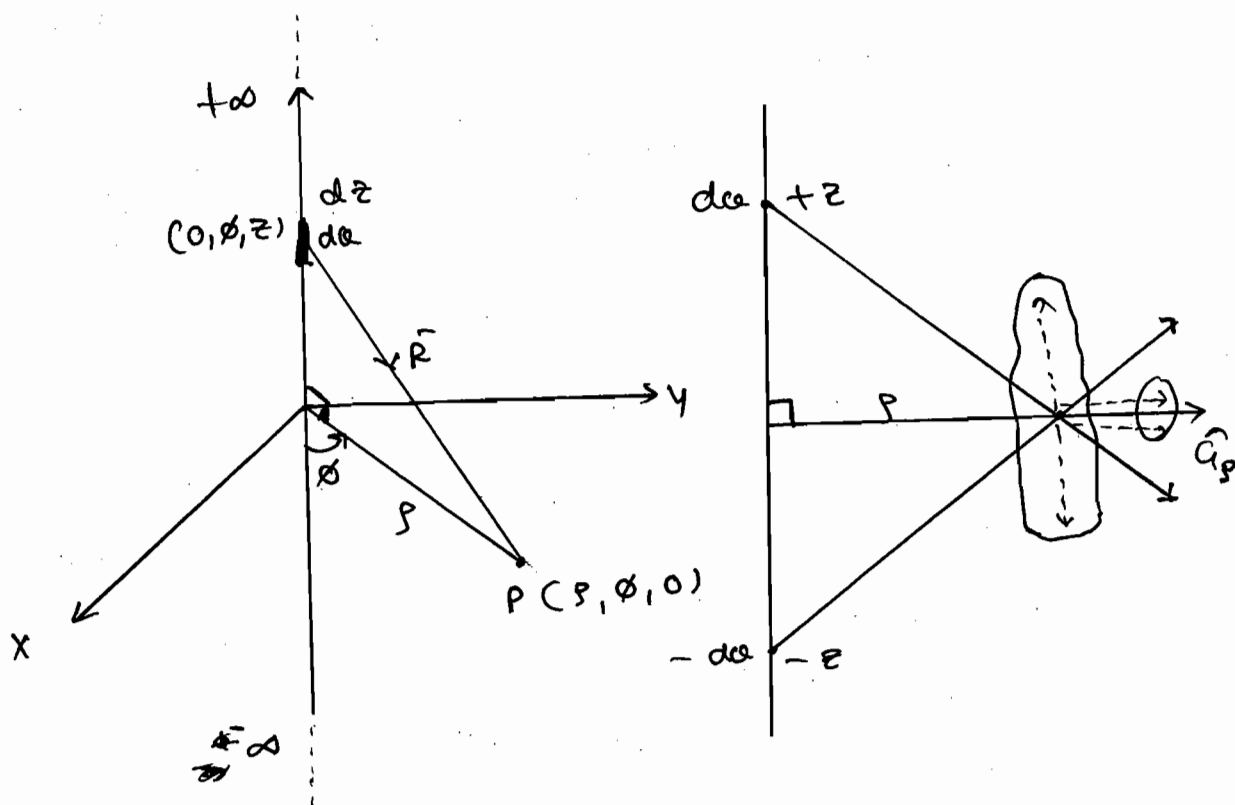
→ We assume that 'dl' is so small such that it shrinking to the point. when we say it is a point, we consider that the 'dq' is located then it can have coordinates.

→ At this point, we consider that the 'dq' is located.

Ex-1 Find an expression for the electric field due to an infinite line with a uniform charge density of λ C/m and show that

- (i) Magnitude of the $\vec{E}$ is inversely proportional to the distance betⁿ the infinite line and the observation ~~xxx~~ point.
- (ii) the direction of the $\vec{E}$ could be projecting in a direction normal to the infinite line.

Ans: We assume that the infinite line lies along z-axis, extending from $-\infty$ to $+\infty$. We find the electric field at some point on the x-y plane. for that convenience we further use cylindrical co-ordinates.



$$\rightarrow da = s_z dz$$

$dz \rightarrow$ Shown to point



At this point, we assume that 'da' is located.

$\therefore$ with ref. $(s, \phi, 0)$, the point on the z-axis is co-ordinated as $(0, 0, z)$.

At this point da is located

$$\vec{r} = s \hat{a}_s - z \hat{a}_z$$

$$\therefore d\vec{E} = \frac{s_z dz}{4\pi\epsilon (\sqrt{s^2 + z^2})^2} \cdot \frac{s \hat{a}_s - z \hat{a}_z}{\sqrt{s^2 + z^2}}$$

NOTE: for every da at +z on the +ve z-axis there exist an another da on the -ve z-axis at -z.

$\rightarrow$ The Charge Configuration is symmetry about X-Y plane. which result in cancellation of $\hat{a}_z$ components and the resultant field would be along $\hat{a}_s$ direction only. i.e.

$\rightarrow$ In general, no field component exist along the ~~line~~ length of the line resulting $\vec{E}$ exists normal to line ignoring $\hat{a}_z$ component

The ~~total~~

→ The total field given by.

$$\vec{E} = \frac{\rho_l \rho}{4\pi\epsilon} \hat{a}_\rho \int_{-\infty}^{\infty} \frac{dz}{(\rho^2 + z^2)^{3/2}}$$

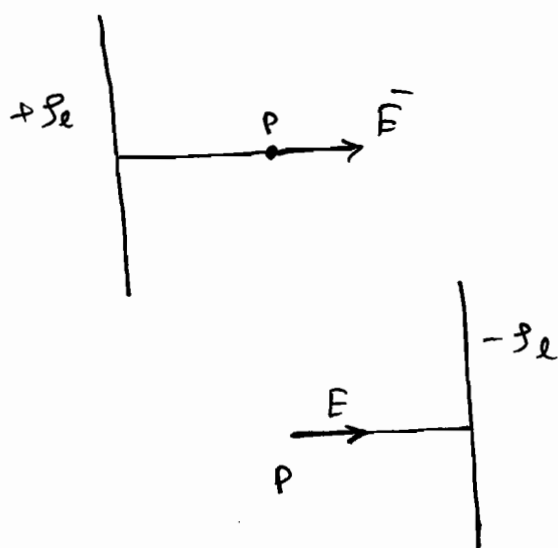
$$\begin{aligned} z &= \rho \tan \theta \\ dz &= \rho \sec^2 \theta d\theta \\ (\rho^2 + z^2)^{3/2} &= \rho^3 \sec^3 \theta. \end{aligned}$$

$$\therefore \vec{E} = \frac{\rho_l}{2\pi\epsilon \rho} \hat{a}_\rho$$

$$\therefore \boxed{\vec{E} = \frac{\rho_l}{2\pi\epsilon \rho} \hat{a}_\rho} \Rightarrow \boxed{|\vec{E}| \propto \frac{1}{\rho}}$$

→ where, ' ρ ' is the distance b/w the infinite line and the observation point.

⇒

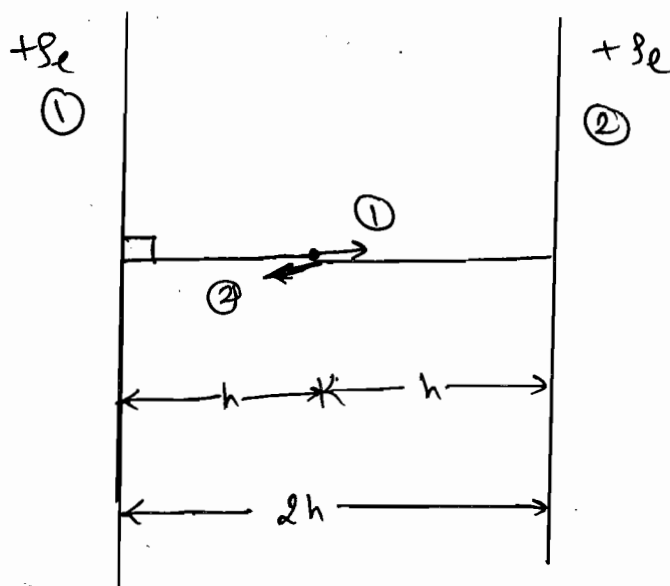


→ If ρ_l is +ve the direction of $\vec{E}$ would be away from the infinite line.

→ If ρ_l is -ve the direction of the $\vec{E}$ would be towards the infinite line.

Ex-1 Two infinite lines are parallel they are separated by $2h$ ($h > 0$). They are distributed with uniform line charge density of $+s_e$ clm each. Find mag. of the electric field betⁿ this infinite line and also find direction of the electric field.

Ans:

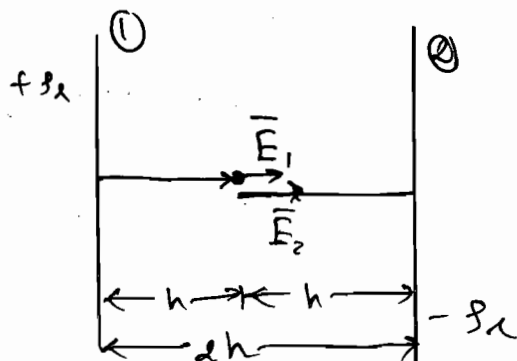


$$|\vec{E}| = 0$$

$\therefore$ No point in defining the direction of $\vec{E}$.

Ex-2 Repeat the above problem if they are distributed with $+s_e$ clm and $-s_e$ clm.

Ans:

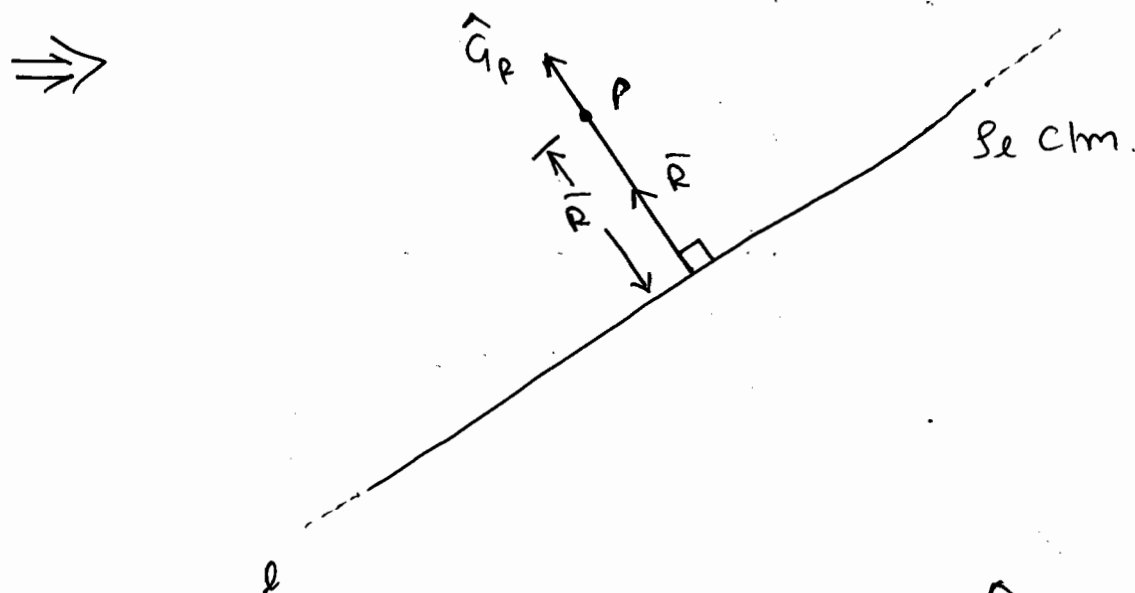


$$|\vec{E}| = \frac{\rho_l}{2\pi\epsilon h} + \frac{\rho_l}{2\pi\epsilon h}$$

$$|\vec{E}| = \frac{\rho_l}{\pi\epsilon h}$$

→ The direction of $\vec{E}$ would be towards the line which is having $-\rho_l$ clm.

(*) Expression for $\vec{E}$ due to an arbitrary oriented infinite line with a uniform charge density of ρ_l clm.

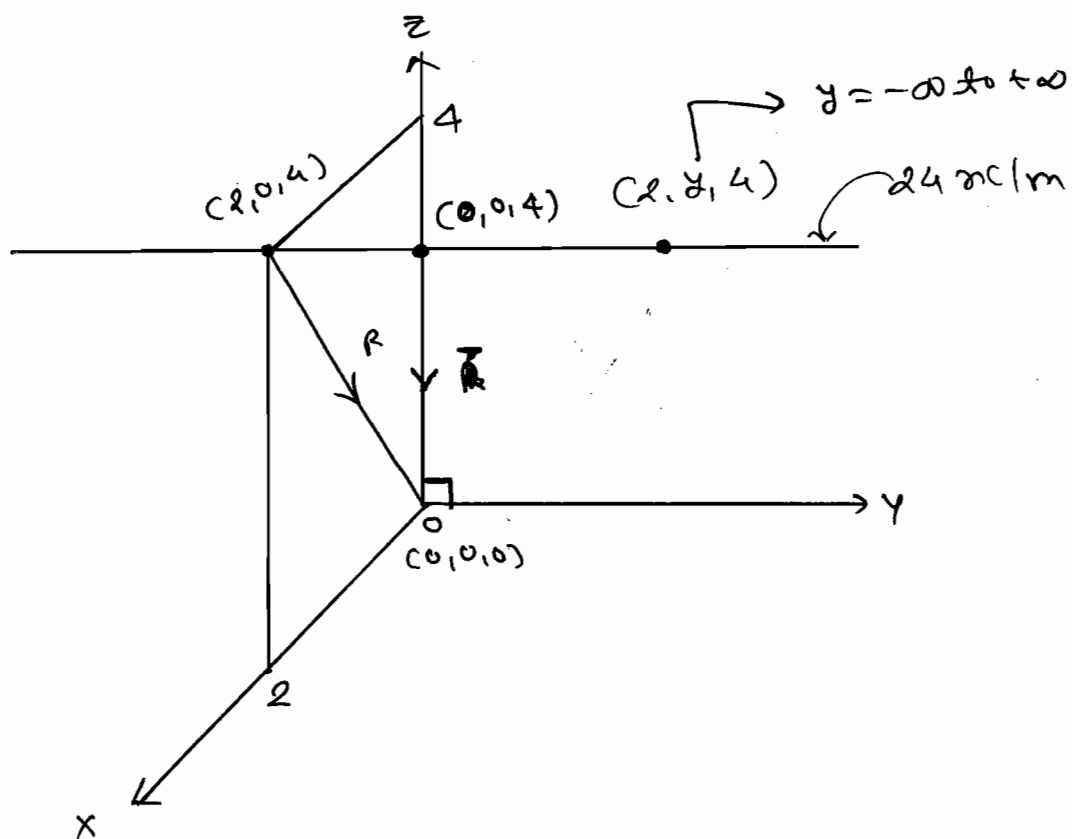


$$\vec{E} = \frac{\rho_l}{2\pi\epsilon |\vec{R}|} \cdot \hat{G}$$

$$\therefore \vec{E} = \frac{\rho_l}{2\pi\epsilon |\vec{R}|} \cdot \frac{\vec{R}}{|\vec{R}|} \text{ V/m.}$$

Ex-1 Find expression for the electric field at
 (a) origin (b) $(4, 5, 6)$ m (c) $(10, 10, 10)$ m.
 due to an infinite line with uniform
 charge density of 24 nC/m . which lies
 at $x=2, z=4 \text{ m}$.

Ans.



① at origin

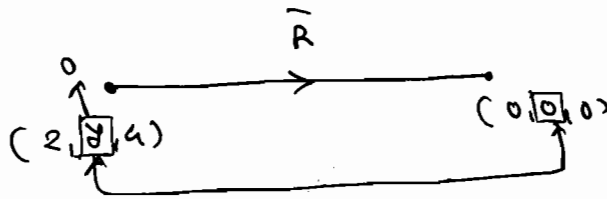
$$\therefore \vec{R} = -2\hat{a}_x - 4\hat{a}_z$$

$$\therefore \vec{E} = \frac{24 \times 10^{-9}}{\frac{2\pi \times 10^{-9}}{9 \times 10^9} \times \sqrt{20}} \times \frac{-2\hat{a}_x - 4\hat{a}_z}{\sqrt{20}}$$

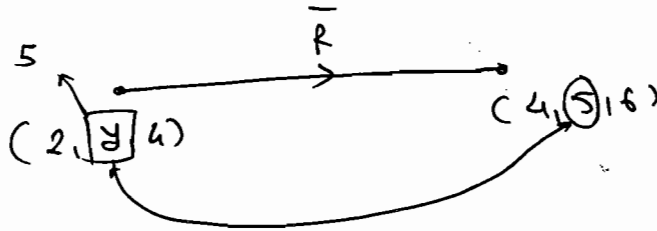
$$= \frac{216 \times 2}{20} \times (-2\hat{a}_x - 4\hat{a}_z)$$

$$\therefore \boxed{\vec{E} = -43.2 (\hat{a}_x + 2\hat{a}_z) \text{ V/m}}$$

Short cut:



(2)



$$\vec{R} = 2\hat{a}_x + 2\hat{a}_z$$

$$\therefore \vec{E} = \frac{q}{4\pi\epsilon |\vec{R}|} \times \frac{\vec{R}}{|\vec{R}|}$$

$$= \frac{24 \times 10^{-9}}{4\pi \times 10^{-9} \times \sqrt{8}} \times \frac{2\hat{a}_x + 2\hat{a}_z}{\sqrt{8}}$$

$$E = 54 (2\hat{a}_x + 2\hat{a}_z)$$

$$\therefore \boxed{\vec{E} = 108 (\hat{a}_x + \hat{a}_z)} \text{ v/m}$$

(3)

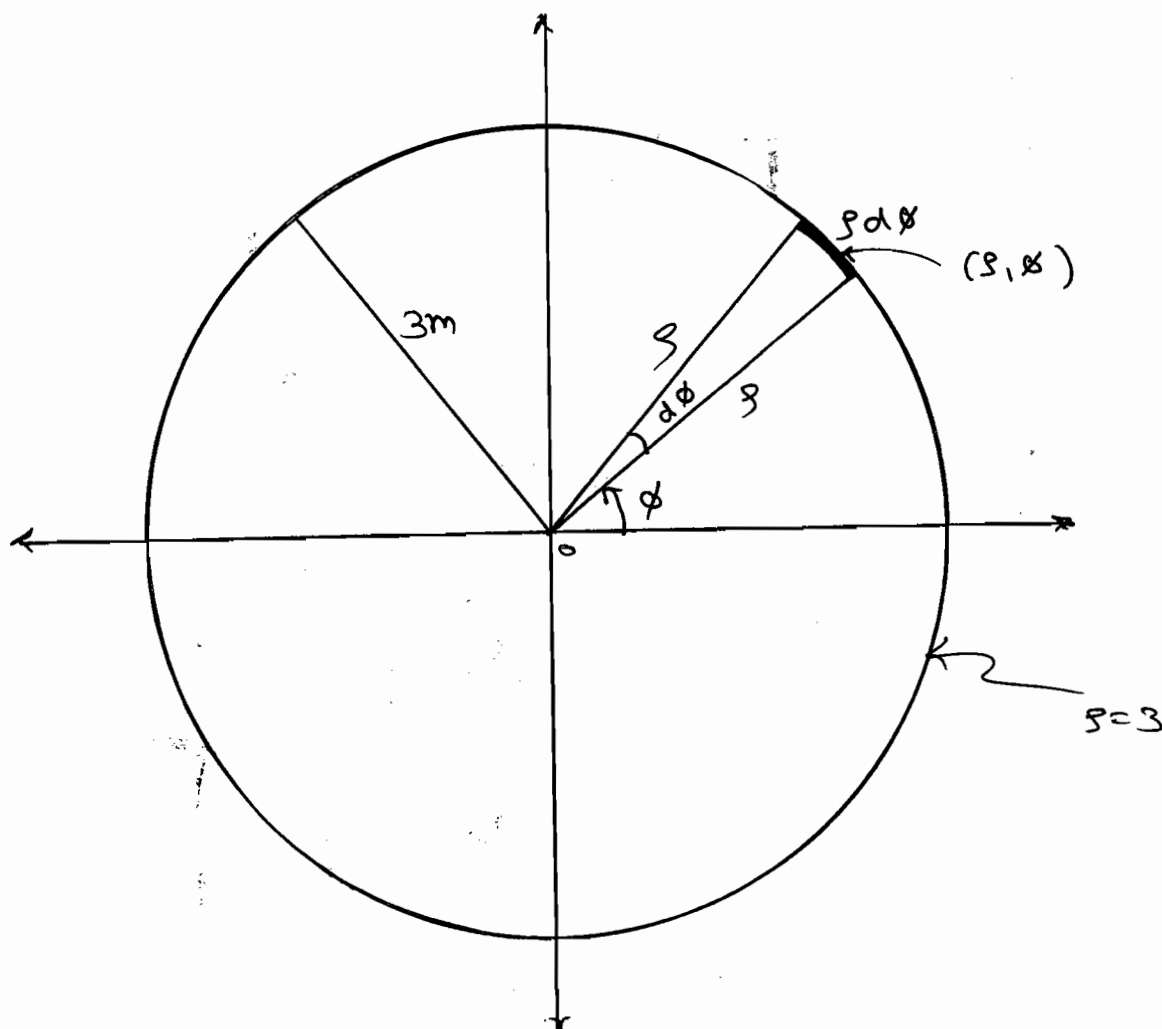


$$\vec{R} = 8\hat{a}_x + 6\hat{a}_z$$

$$\therefore \vec{E} = \frac{24 \times 10^{-9}}{4\pi \times 10^{-9} \times 10} \times \frac{8\hat{a}_x + 6\hat{a}_z}{10}$$

$$\therefore \boxed{\vec{E} = 4.32 (8\hat{a}_x + 6\hat{a}_z)} \text{ v/m}$$

*



$$\rightarrow r=3, 0 \leq \phi < 2\pi, z=0$$

$\Rightarrow$ This represents, there exists a circle of radius 3m centered at origin and is located in $z=0$ plane.

$$dl = r d\phi$$

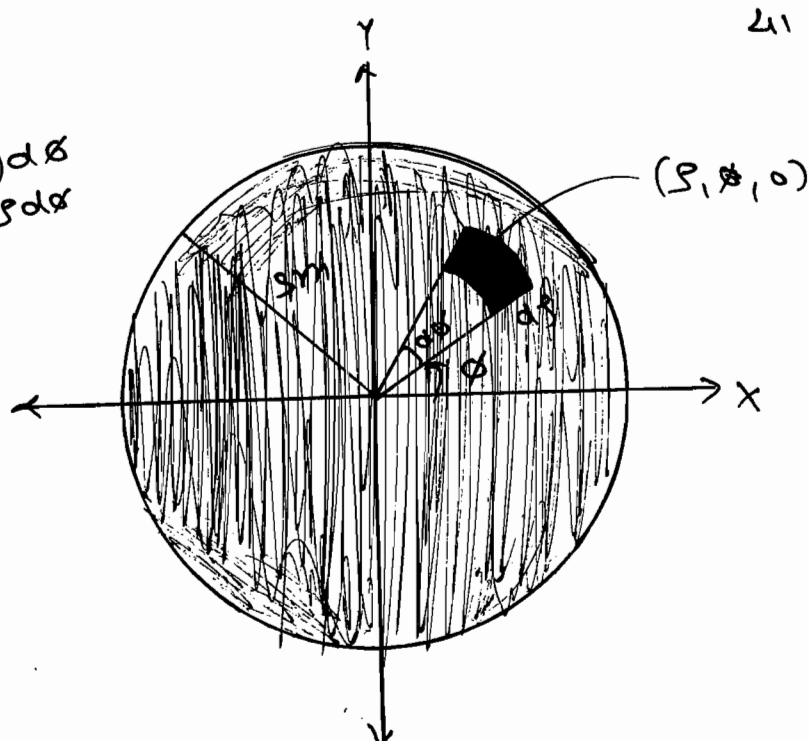
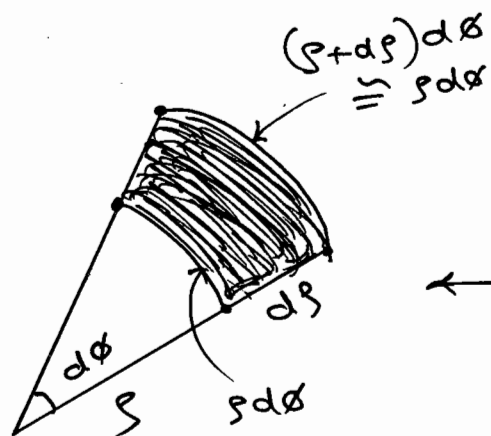
$$L = \int_0^{2\pi} r d\phi = 3 \int_0^{2\pi} d\phi = 6\pi m.$$

($z=0, r=3$)

$dl \rightarrow$ so small such that it is shrinking to a point (i.e) $d\phi \rightarrow 0$.
Then, that point is considered as (r, ϕ) .

→ *

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→ ① $0 \leq r \leq 3, 0 \leq \phi \leq 2\pi, z=0$
 ② $0 \leq r \leq 3, 0 \leq \phi \leq 2\pi, z=0$
 ③ $r \leq 3, \underline{z=0}$

This represents a circular disk of radius $3m$, centered at origin and is located in $z=0$ plane.

$$dS = r dr d\phi$$

$$S = \int_0^3 \int_0^{2\pi} r dr d\phi$$

$$= \int_0^3 [r^2]_0^{2\pi} dr$$

$$= \int_0^3 (2\pi) r dr$$

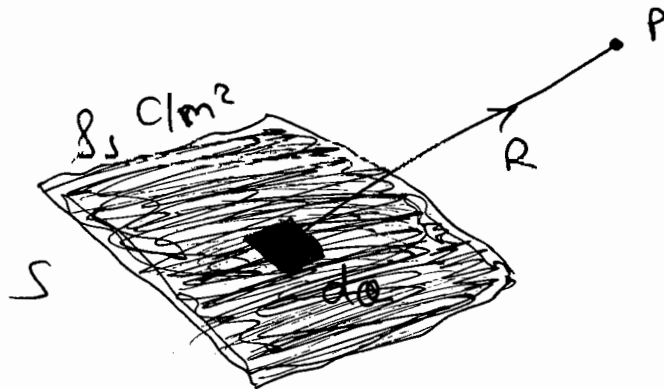
$$= \left[\frac{r^2}{2} \right]_0^3 \pi 2\pi$$

$$\therefore \boxed{S = 9\pi m^2}$$

$dS \rightarrow$ So small such that it shrinks to a point (i.e) $dr \rightarrow 0, d\phi \rightarrow 0$.
 $\therefore$ That point is coordinated as (r, ϕ) or $(r, \phi, 0)$.

* Electric field ($\vec{E}$) due to a sheet with a uniform charge density of $\sigma_s \text{ C/m}^2$.

$\Rightarrow$



$\rightarrow da = \sigma_s ds$

$ds \rightarrow$ shrunk to a point.

At this point, we consider that da is located. When we say, it is a point, then it can have co-ordinates.

$$d\vec{E} = \frac{da}{4\pi\epsilon |\vec{R}|^2} \hat{q}_R$$

$$\therefore \vec{E} = \int_S \frac{\sigma_s ds}{4\pi\epsilon |\vec{R}|^2} \times \frac{\vec{R}}{|\vec{R}|} \text{ V/m}$$

double integral.

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Ans!



$$\rightarrow ds = r dr d\theta.$$

$$d\alpha = r_s ds$$

$$d\alpha = r_s r dr d\theta$$

$ds \rightarrow$ Shrinks to a point



This point is coordinated as $(r, \theta, 0)$
[i.e. $dr \rightarrow 0, d\theta \rightarrow 0$].

At this point 'dα' is located,

$$d\vec{E} = \frac{r_s \cdot r dr d\theta}{4\pi\epsilon (\sqrt{r^2 + h^2})^2} \times \frac{-r\hat{q}_r + h\hat{q}_z}{\sqrt{r^2 + h^2}}.$$

→ As shown in figure for every dα on the sheet there exists an another dα diametrically opposite side. Therefore, the charge configuration is symmetry about z-axis. Which results in cancellation of horizontal field components. And the resultant $\vec{E}$ would be along $\hat{q}_z$ direction only. i.e. No field component exists $\parallel$ to the infinite sheet.

→ The resultant field exists in the direction to the normal to the sheet. Here, ignoring $\hat{q}_r$ components. The total field is given by,

→ Ignoring $\hat{a}_s$ Component,
the total field is given by,

$$\vec{E} = \frac{S_s h}{4\pi\epsilon} \hat{a}_z \int_0^\infty \int_0^{2\pi} \frac{s ds d\phi}{(s^2 + h^2)^{3/2}}$$

$$\int_0^{2\pi} d\phi = 2\pi$$

$$\int_0^\infty \frac{s ds}{(s^2 + h^2)^{3/2}} = \frac{1}{h}$$

$$\text{put } s^2 + h^2 = t$$

$$\therefore 2s ds = dt$$

$$\therefore s ds = \frac{1}{2} dt$$

$$\therefore \vec{E} = \frac{S_s}{2\epsilon} \hat{a}_z \text{ V/m}$$

→ In, general

$$\vec{E} = \frac{S_s}{2\epsilon} \hat{a}_n \text{ V/m}$$

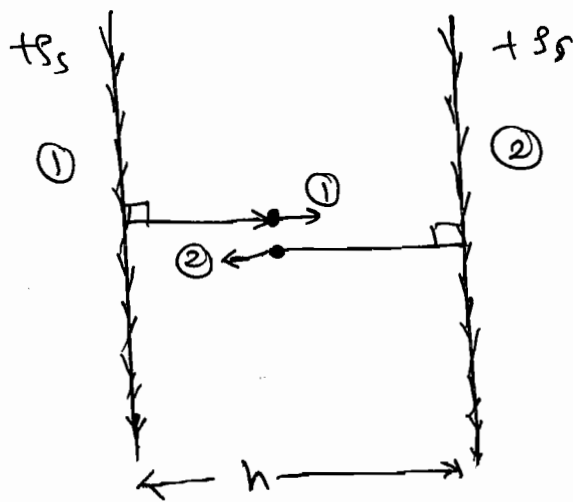
Where, $\hat{a}_n$ is the normal unit vector at the observation point with ref. to infinite sheet.

→ If S_s is +ve, the direction of $\vec{E}$ would be away from the infinite sheet.

→ If S_s is -ve, the direction of $\vec{E}$ would be towards to the infinite sheet.

Ex-1 Two infinite sheets are 11^{th} , they are separated by $2h$ m. they are distributed with uniform charge density of σ_s C/m^2 . Can find the electric field at any point betⁿ this two infinite sheet.

Ans:

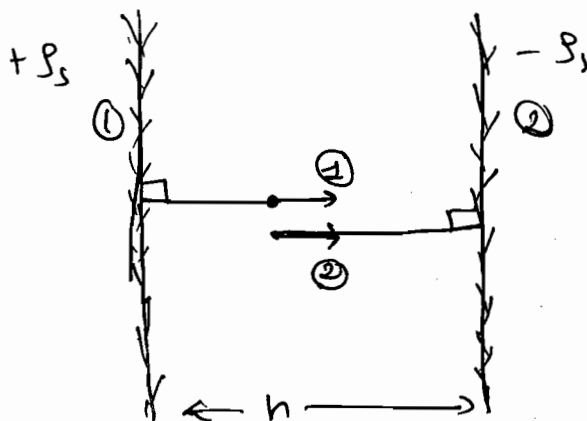


$$|\vec{E}| = 0.$$

The fields add in out of phase.

Ex-2 Repeat the above example If they are distributed with uniform charge density of $+\sigma_s$ C/m^2 and $-\sigma_s$ C/m^2 . The

Ans:

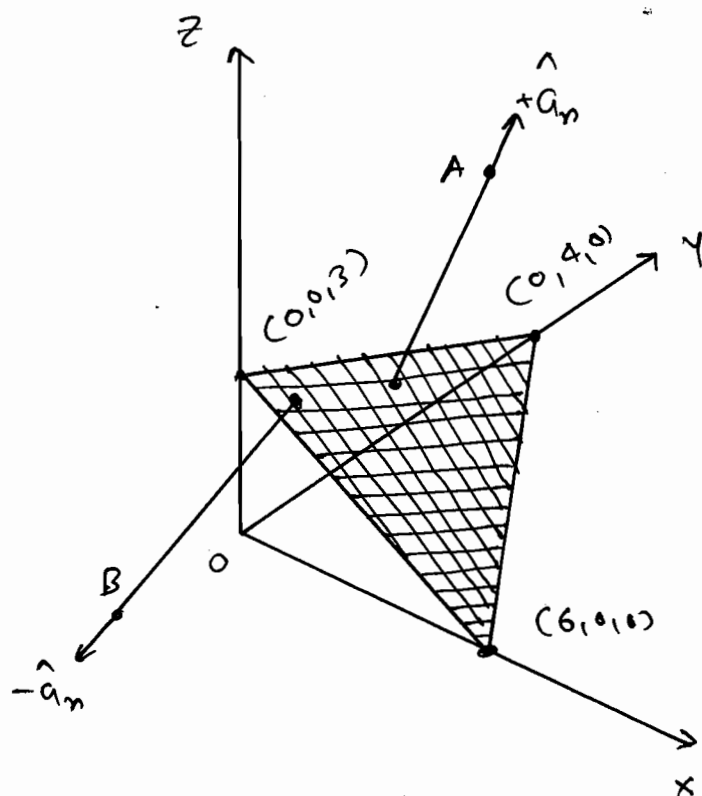


$$|\vec{E}| = \frac{S_s}{2\epsilon} + \frac{S_s}{2\epsilon} = \frac{S_s}{\epsilon}$$

→ The fields add in in-phase.
The direction of $\vec{E}$ would be towards the sheet which is having $-S_s \text{ C/m}^2$.

Ex-3 An Infinite Sheet with a uniform charge density of 24 nC/m^2 is lies in a plane define by $2x + 3y + 4z = 12$.
 find the $\vec{E}$ in all the regions.

Ans:



$$\rightarrow E \text{ (at A)} = \frac{S_s}{2\epsilon} \hat{a}_n$$

$$E \text{ (at B)} = \frac{S_s}{2\epsilon} (-\hat{a}_n)$$

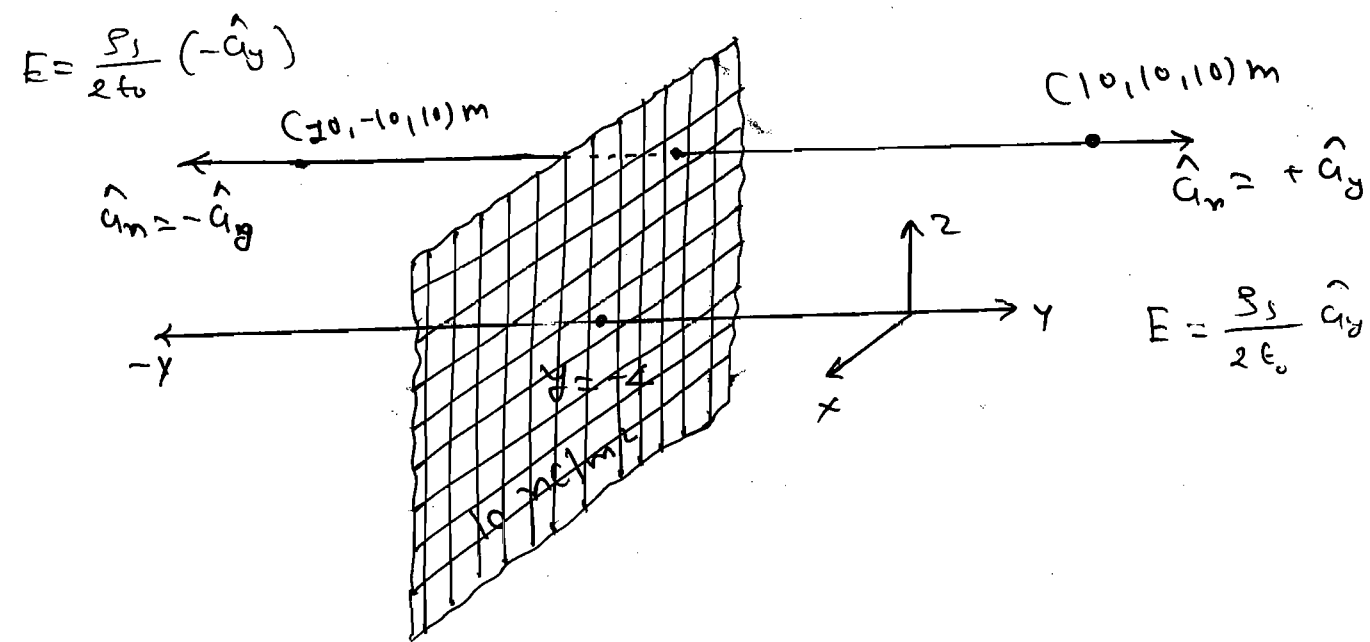
$$\hat{a}_n = \frac{2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z}{\sqrt{2^2 + 3^2 + 4^2}}$$

Ex-4 An Infinite Sheet with a uniform charge density of 10 nC/m^2 is lies at $y = -4 \text{ m}$. Find the $\vec{E}$ at

(i) $(10, 10, 10) \text{ m}$

(ii) $(10, -10, 10) \text{ m}$.

Ans: The infinite sheet is $\parallel$ to z - x plane.



(i) at $(10, 10, 10) \text{ m}$

$$\vec{E} = \frac{S_s}{2\epsilon_0} \hat{a}_y \text{ V/m}$$

(ii) at $(10, -10, 10)$

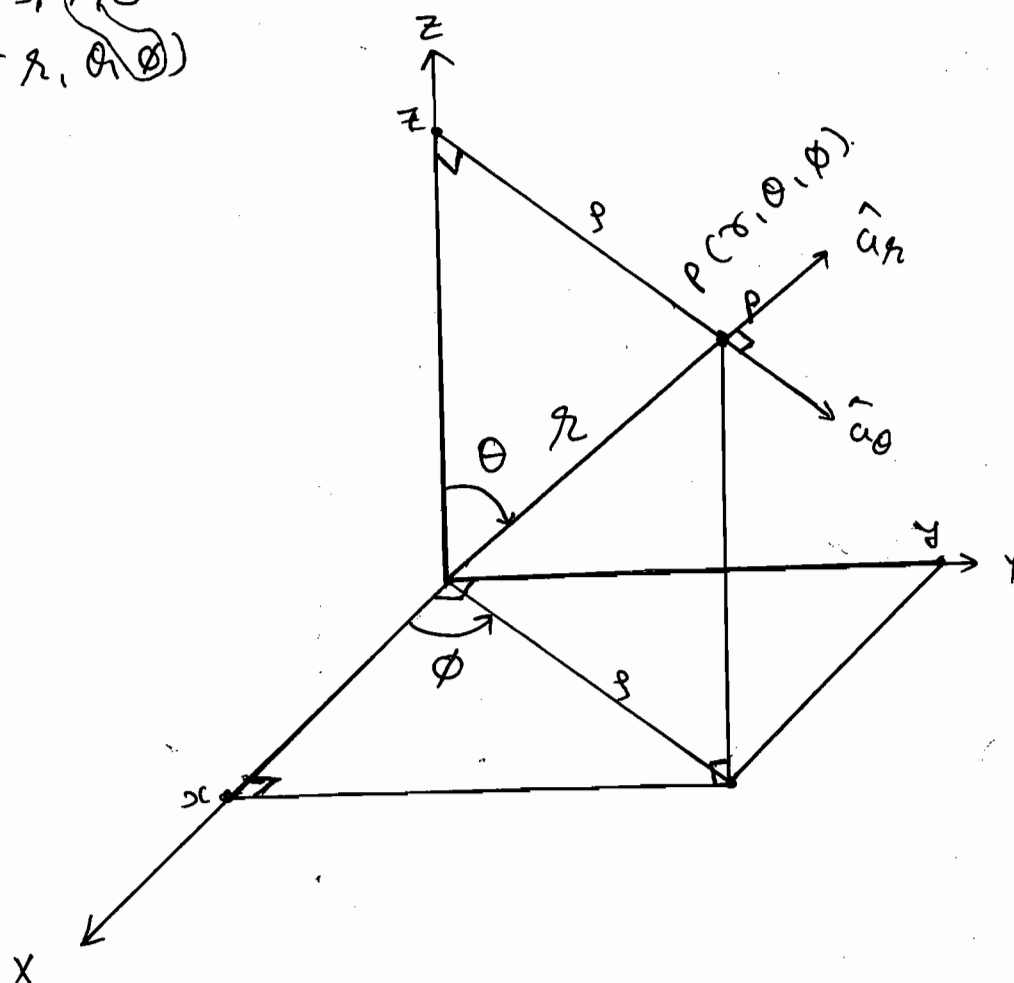
$$\vec{E} = \frac{S_s}{2\epsilon_0} (-\hat{a}_y) \text{ V/m.}$$

*

$$P(x, y, z)$$

$$P(\rho, \theta, \phi)$$

$$P(r, \theta, \phi)$$



$$\Rightarrow \rho = r \sin \theta$$

$$z = r \cos \theta$$

$$x = \rho \cos \phi$$

$$\therefore x = r \sin \theta \cdot \cos \phi$$

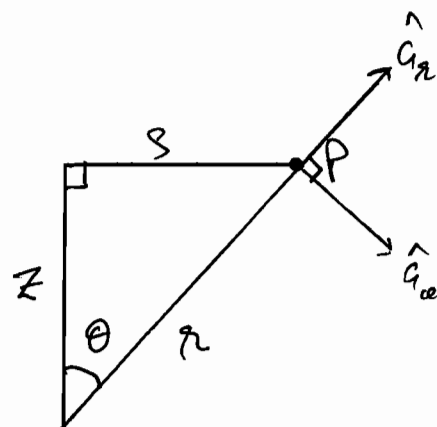
$$\therefore y = \rho \sin \phi$$

$$\therefore y = r \sin \theta \cdot \sin \phi$$

$$r = \sqrt{x^2 + y^2 + z^2} \quad 0 \leq r < \infty$$

$$\theta = \cos^{-1} \left(\frac{z}{r} \right) \quad 0 \leq \theta \leq \pi$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right) \quad 0 \leq \phi < 2\pi$$



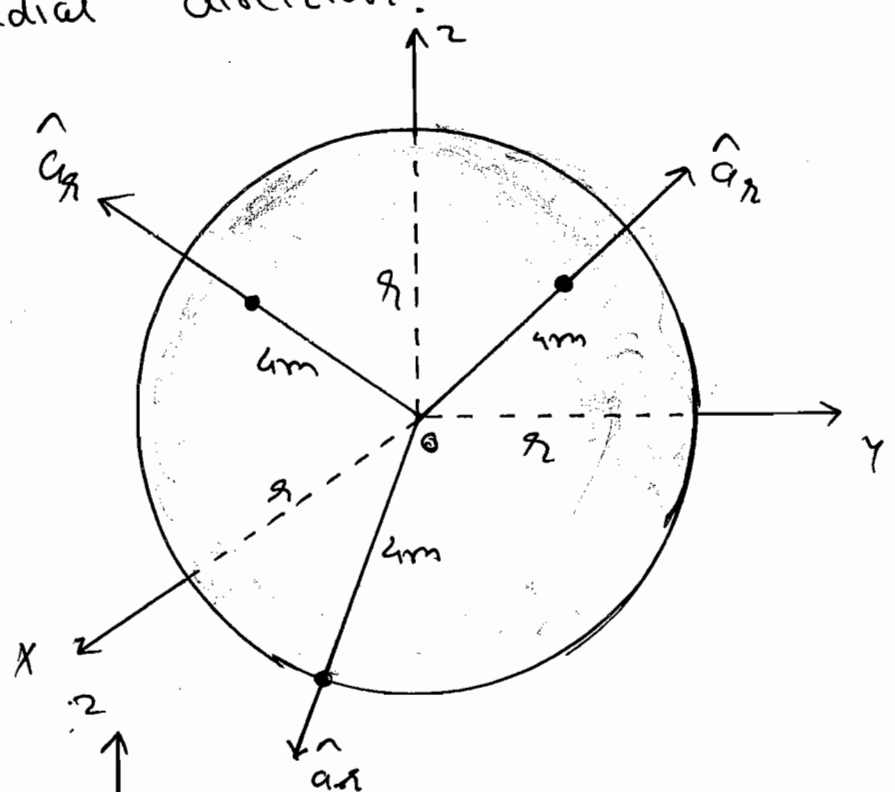
→ Locus of $r = \text{constant}$ represents a sphere (or) a sphericaal whose centre coincides with the origin. Therefore, r assume all possible values ranging from 0 to ∞ .

→ $\hat{a}_r$ is a unit vector projecting normal to $r = \text{constant}$ (or) normal to the sphere. also called radial direction.

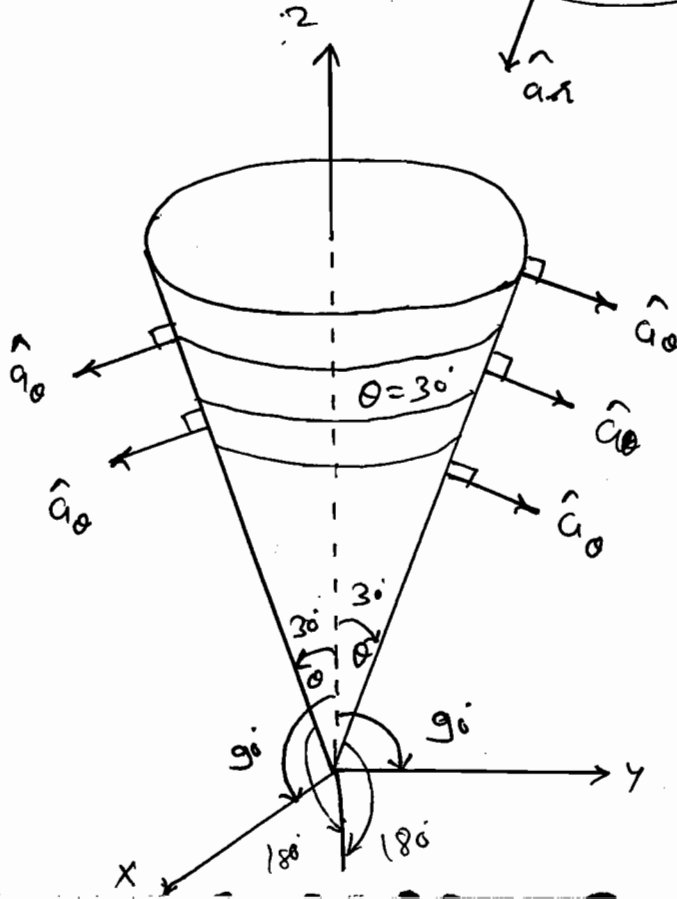
on $r = \text{const.}$

$\theta \rightarrow 0 \text{ to } \pi$

$\phi \rightarrow 0 \text{ to } 2\pi$



→ $\hat{a}_\theta$



on $\theta = \text{const.}$

$r \rightarrow 0 \text{ to } \infty$

$\phi \rightarrow 0 \text{ to } 2\pi$

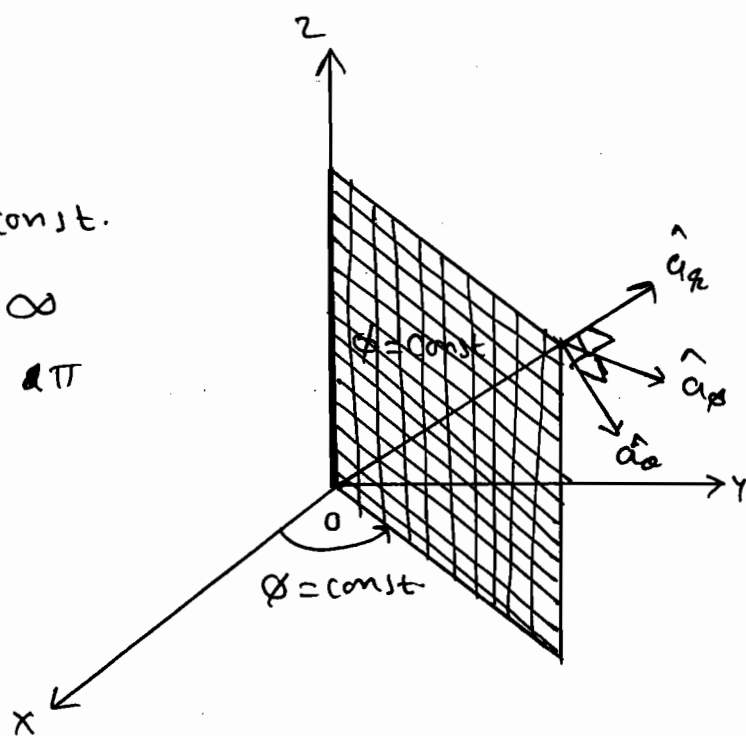
51
 $\rightarrow \theta = \text{Constant}$ for $0 < \theta < 90^\circ$ represents a
 Conical plane, $\theta = 90^\circ$ represents
 x-y plane, $\theta = 90^\circ$ is called azimuthal
 plane.

$\rightarrow$ As shown in the figure ϕ assumes all
 possible values ranging from 0 to π .

$\rightarrow \hat{a}_\phi$ is a unit vector projecting normal
 to $\theta = \text{Constant}$ plane. Further, $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi$
 orthogonal to each other.

$\rightarrow$

on $\phi = \text{const.}$
 $r \rightarrow 0 \text{ to } \infty$
 $\theta \rightarrow 0 \text{ to } \pi$

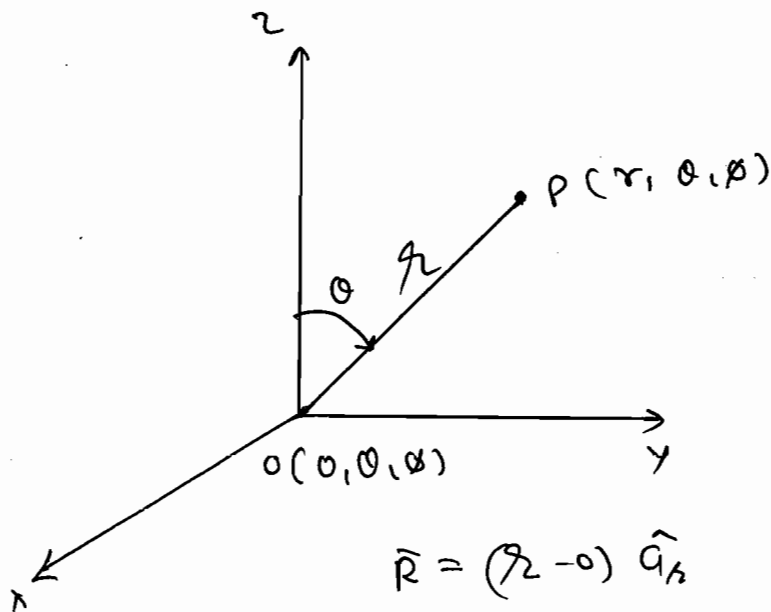


$\rightarrow \phi$ assumes all possible values ranging from
 0 to 2π . $\hat{a}_\phi$ is a unit vector projecting
 normal to $\phi = \text{const.}$ plane. further we write
 $\hat{a}_r, \hat{a}_\theta$ and $\hat{a}_\phi$ are orthogonal to each other.

→ $\phi = \text{const.}$ plane is also called Elevation plane.

①- With ref. to $P(r, \theta, \phi)$ what are the co-ordinates of the origin?

① = $O(0, 0, 0)$.



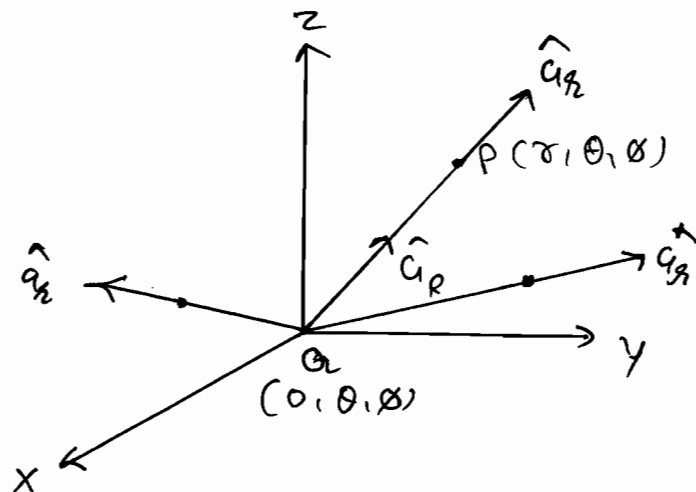
$$\vec{R} = (r - 0) \hat{a}_r$$

$$\therefore \vec{R} = r \hat{a}_r$$

$$\Rightarrow \hat{a}_R = \hat{a}_r$$

Q-1 A point charge of Q coulombs is located at the origin. find $\vec{E}$ at a distant point P in spherical co-ordinates.

Ans:



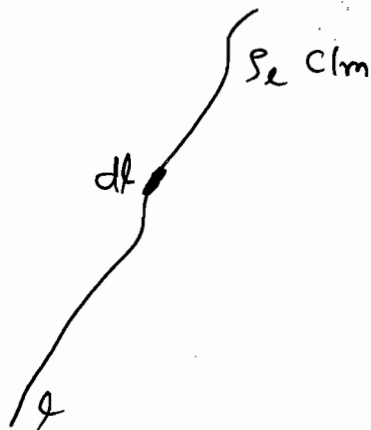
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \cdot \hat{a}_r$$

$$= \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

→ Thus, the direction of $\vec{E}$ would be along radial (or) $\hat{a}_r$ direction.

* Total Charge Calculation:

(1) Line Charge:



$$dQ = S_L dl$$

$$\therefore Q = \int S_L dl$$

single I

(2) Surface Charge:

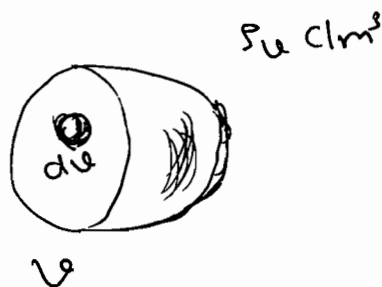


$$dQ = S_S ds$$

$$\therefore Q = \int S_S ds$$

double I

(3) Volume Charge:

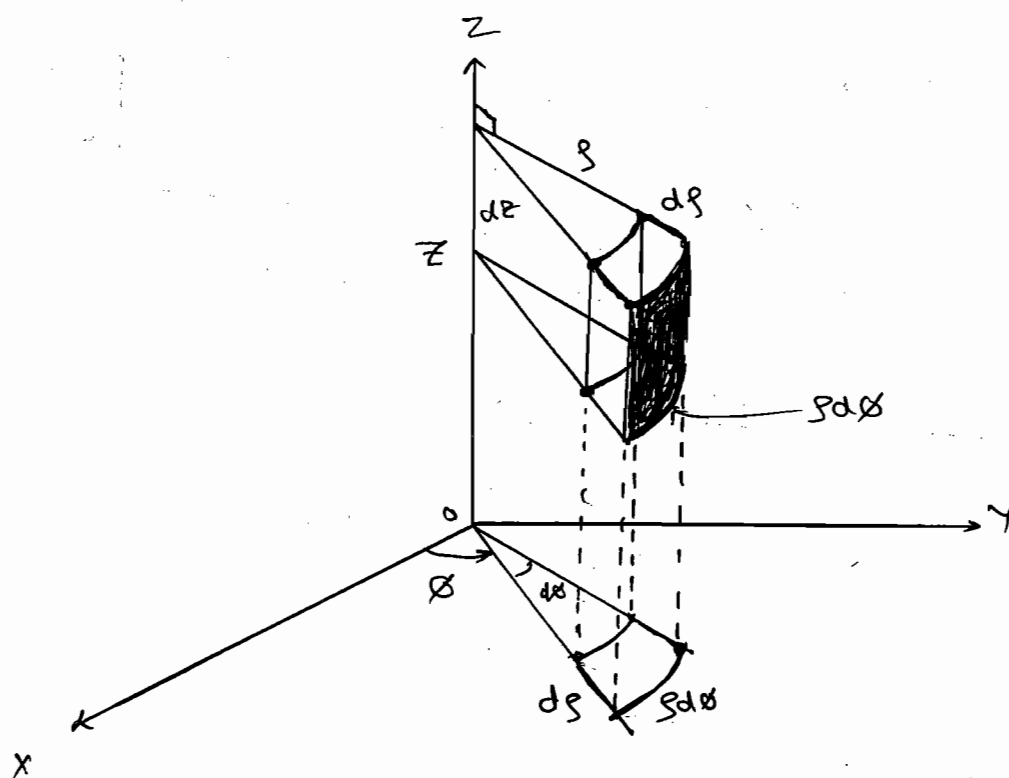
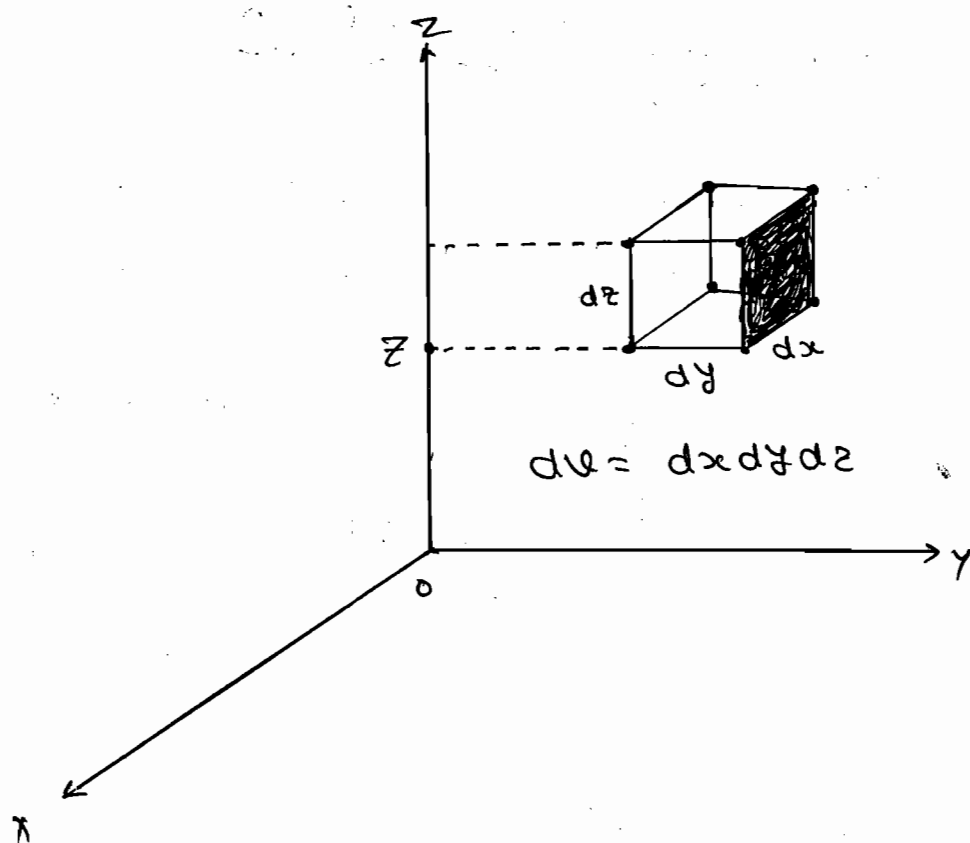


$$\therefore dQ = S_V dv$$

$$\therefore Q = \int S_V dv$$

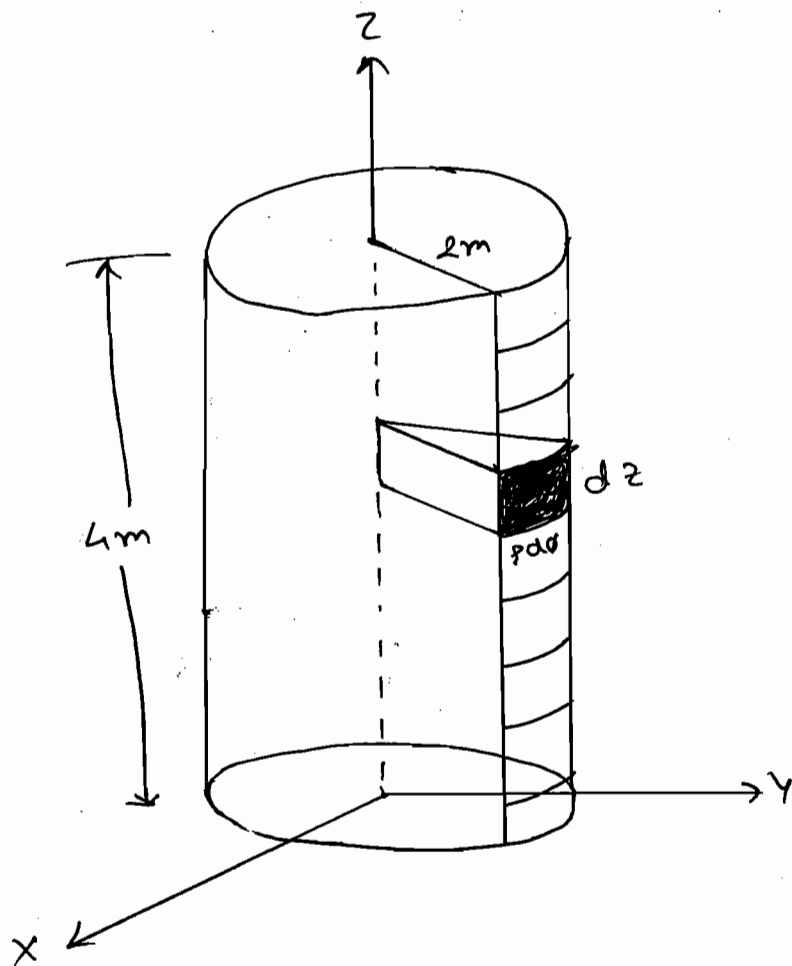
Triple I

* Construction of dV .



$$dV = ds \cdot s \cdot d\phi dz$$

$$dV = s ds d\phi dz$$



$$\rightarrow \boxed{S = 2m, \quad 0 \leq \phi \leq 2\pi, \quad 0 \leq z \leq 4}$$

→ This represents a cylindrical sheet with radius 2m and height 4m.

$$dS = S d\phi dz$$

$$\therefore S = \int_0^{2\pi} \int_0^4 \rho d\phi dz$$

$$\therefore S = 2 (\phi)_0^{2\pi} (z)_0^4$$

$$\therefore S = 2(2\pi)(4)m$$

$$\boxed{S = 16\pi \text{ m}^2}$$

Ex-1 Find the total charge with in each of the volume indicate below:

① $\rho_v = 10z^2 e^{-0.1x} \sin \pi y \text{ nC/m}^3$

$0 \leq x \leq 1, 1 \leq y \leq 2, 2.5 \leq z \leq 4.5.$

② $\rho_v = 2xyz^2 \text{ nC/m}^3.$

$0 \leq \rho \leq 2, 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq z \leq 1.$

③ $\rho_v = 10 e^{-10y} \cdot e^{-z} \text{ nC/m}^3$; 1st octant.

Ans: $dv = dx dy dz.$

$dQ = \rho_v dv$

$\therefore dQ = \rho_v dv$

① $Q = \int_{x=0}^1 \int_{y=1}^2 \int_{z=2.5}^{4.5} 10z^2 \cdot e^{-0.1x} \sin \pi y \cdot dx dy dz$

$\therefore Q = 10 \times \left[\frac{e^{-0.1x}}{-0.1} \right]_0^1 \times \left[-\frac{\cos \pi y}{\pi} \right]_1^2 \times \left[\frac{z^3}{3} \right]_{2.5}^{4.5}$

$\therefore Q = 10 \times \left[\frac{e^{-0.1} - 1}{-0.1} \right] \times \left[+\frac{1+1}{\pi} \right] \times \left[\frac{(4.5)^3 - (2.5)^3}{3} \right]$

$\therefore Q = \underline{\hspace{2cm}}$

② $x = \rho \cos \theta, y = \rho \sin \theta$

$\therefore \rho_v = \rho^2 \sin \theta \cdot z^2 \text{ nC/m}^3.$

$$\therefore dQ = \rho \, ds \, d\phi \, dz.$$

$$\therefore Q = \int_V \rho \, dV.$$

$$\therefore Q = \int_{s=0}^2 \int_{\phi=0}^{\pi/2} \int_{z=0}^1 \rho \cdot s^2 \cdot \sin \phi \cdot s \, ds \, d\phi \, dz$$

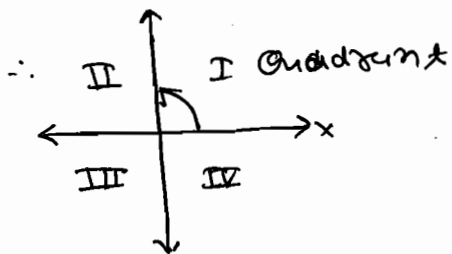
$$\therefore Q = \left[\frac{s^4}{4} \right]_0^2 \times \left[\frac{-\cos 2\phi}{2} \right]_0^{\pi/2} \times \left[\frac{z^3}{3} \right]_0^1$$

$$= \left[\frac{16}{4} \right] \times \left[\frac{1}{2} \right] \times \left[\frac{1}{3} \right].$$

$$\therefore Q = \frac{4}{3} \text{ nC}$$

$$\textcircled{3} \quad \rho = 10e^{-10s} \cdot e^{-z} \text{ nC/m}^3.$$

~~act~~ actual.



I - Octant: x, y are +ve

$$\text{i.e. } \left. \begin{array}{l} 0 \leq x \leq \infty \\ 0 \leq y \leq \infty \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} s \rightarrow 0 \text{ to } \infty \\ \phi \rightarrow 0 \text{ to } \frac{\pi}{2} \end{array} \right.$$

I - Octant $\rightarrow x, y, z$ are +ve

$\Downarrow$

$$\left. \begin{array}{l} x \rightarrow 0 \text{ to } \infty \\ y \rightarrow 0 \text{ to } \infty \\ z \rightarrow 0 \text{ to } \infty \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} s \rightarrow 0 \text{ to } \infty \\ \phi \rightarrow 0 \text{ to } \frac{\pi}{2} \\ z \rightarrow 0 \text{ to } \infty \end{array} \right.$$

$$dQ = \rho \, ds \, d\phi \, dz.$$

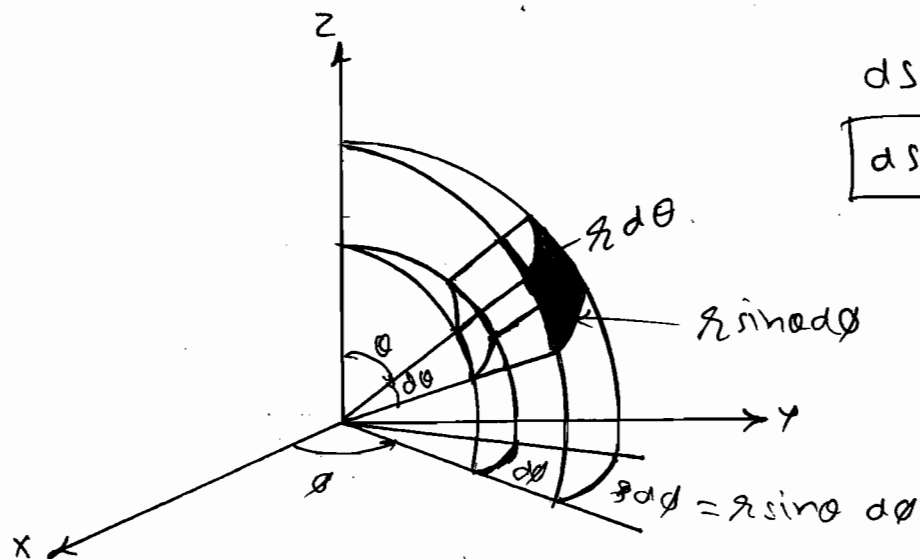
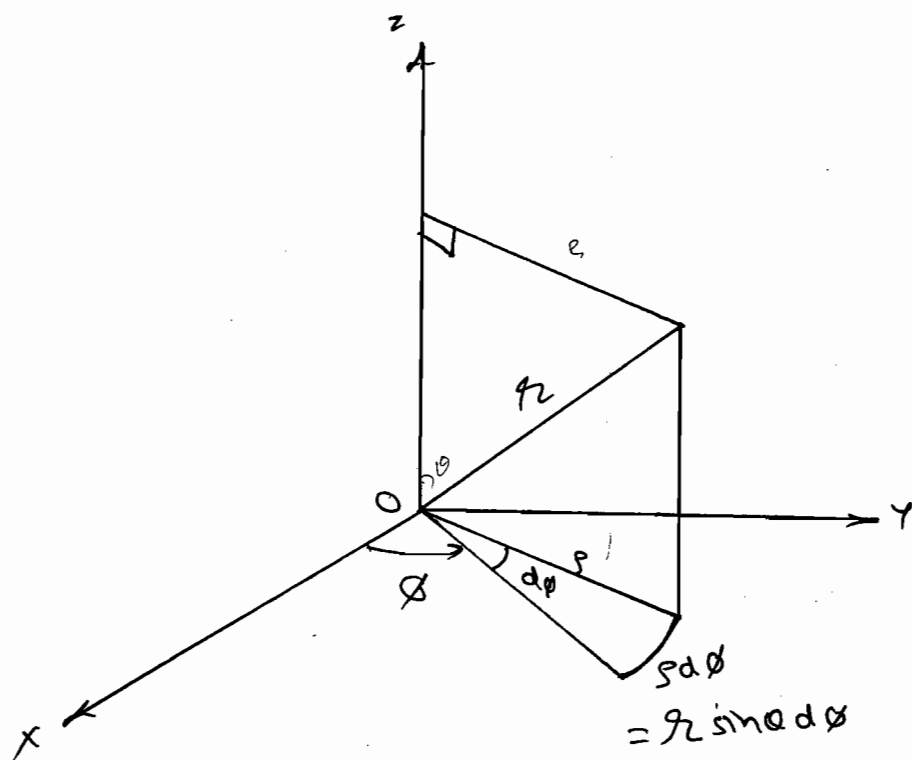
$$\therefore Q = 10 \int_0^{\infty} \int_0^{\pi/2} \int_0^{\infty} e^{-10s} \cdot e^{-z} \cdot s \, ds \, d\phi \, dz.$$

$$\therefore Q = 10 \left(\int_0^\infty r e^{-10r} dr \times \int_0^{\pi/2} d\phi \times \int_0^\infty e^{-z} dz \right) \text{ nC.}$$

$$\therefore Q = 10 \left[r \cdot \frac{e^{-10r}}{-10} + (1) \frac{e^{-10r}}{(10)^2} \right]_0^\infty \times \frac{\pi}{2} \times \left[\frac{e^{-z}}{-1} \right]_0^\infty$$

$$\therefore Q = 10 \left[0 + \frac{1}{100} \right] \times \frac{\pi}{2} \times 0 + 1.$$

$$\therefore Q = \frac{+\pi}{20} \text{ nC.}$$



$$dS = r d\theta \cdot r \sin \theta d\phi$$

$$dS = r^2 \sin \theta d\theta d\phi$$

$$\Rightarrow r = 2m, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi$$

$\Rightarrow$ This represents a sphere of $2m$ centered at origin.

$$ds = r^2 \sin\theta d\theta d\phi$$

$$\therefore S = \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\theta d\phi$$

$\begin{matrix} \uparrow \\ r=2m \end{matrix}$

$$\therefore S = (2)^2 [-\cos\theta]_0^\pi [\phi]_0^{2\pi}$$

$$\therefore S = 4\pi(2)^2 m^2.$$

$$\therefore \boxed{S = 16\pi m^2}$$

Ex-1 Let, $S_u = \frac{4}{3} \frac{\cos^2\theta \cdot \sin^2\phi}{r^2(r^2+1)}$ define for universe.

Find the total charge.

Ans: Universe $\rightarrow$ $r \rightarrow 0 \text{ to } \infty$
 $\theta \rightarrow 0 \text{ to } 2\pi$
 $\phi \rightarrow 0 \text{ to } \pi.$

$$du = r^2 \sin\theta d\theta d\phi.$$

$$\therefore Q = \int_u S_u \cdot du$$

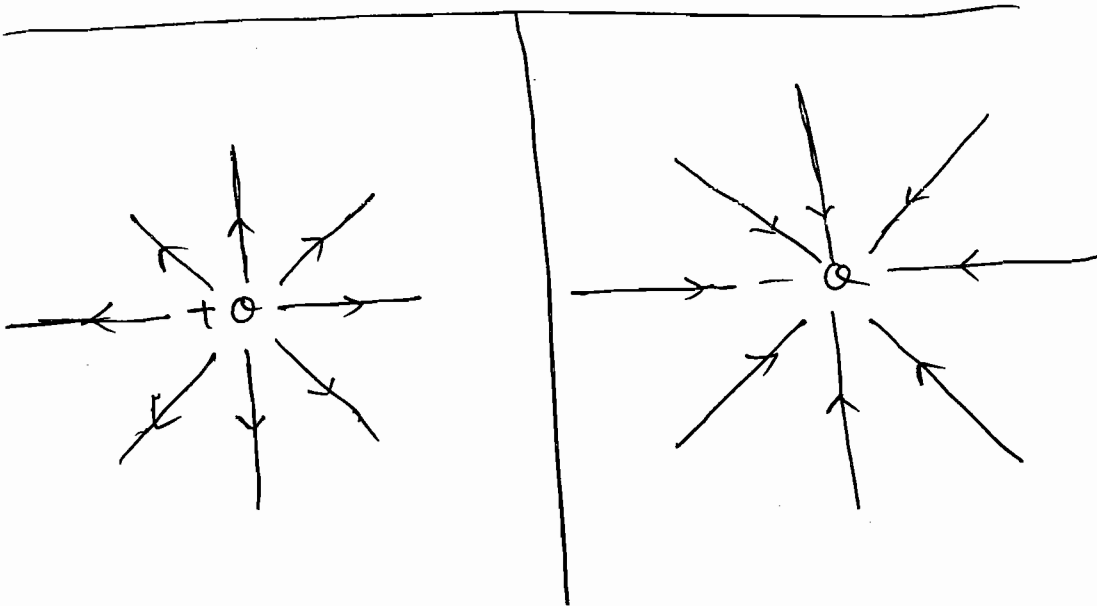
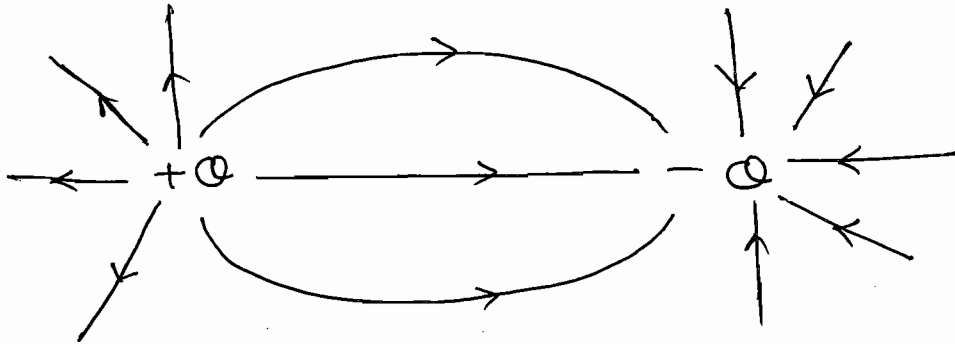
$$\therefore Q = \int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{4}{3} \cdot \frac{\cos^2\theta \cdot \sin^2\phi}{r^2(r^2+1)} \times r^2 \sin\theta d\theta d\phi dr.$$

$$= \frac{4}{3} [\tan^{-1} x]_0^{\infty} \times \left[-\frac{\cos^3 \theta}{3} \right]_0^{\pi} \times \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{2\pi}$$

$$= \frac{4}{3} \left[\frac{\pi}{2} \right] \times \left[\frac{1}{3} + \frac{1}{3} \right] \times [\pi]$$

$$\therefore \boxed{\phi = \frac{4}{9} \pi^2}$$

* Electric Flux (ψ).



→ An Electric flux originates from a +ve charge and ends with a negative charge. In the absence of -ve charge electric flux terminates at infinity.

→ 1C of electric charge would result
 1C of electric flux. rather ~~0C of~~
 → 0C of electric charge would result
 0C of electric flux.

$$\psi = Q \text{ C}$$

Ex-1 How much Electric flux would result
 from a non-uniform surface charge
 density $\frac{s}{s^2+1} \text{ nc/m}^2$ define for $s \leq 5\text{m}$,
 $z = 4\text{m}$.

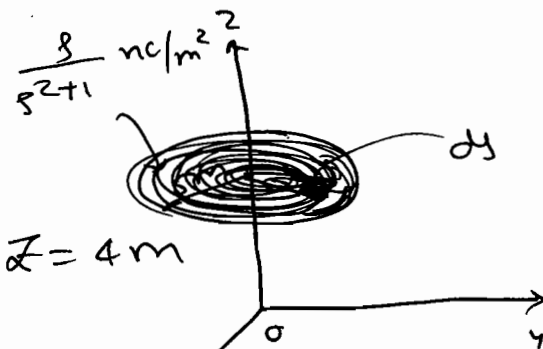
Ans:
 $\frac{s}{s^2+1}$

$$ds = s ds d\phi$$

$$\therefore Q = \int_S \vec{S} \cdot d\vec{s}$$

$$\therefore Q = \int_{s=0}^5 \int_{\phi=0}^{2\pi} \frac{s}{s^2+1} s ds d\phi$$

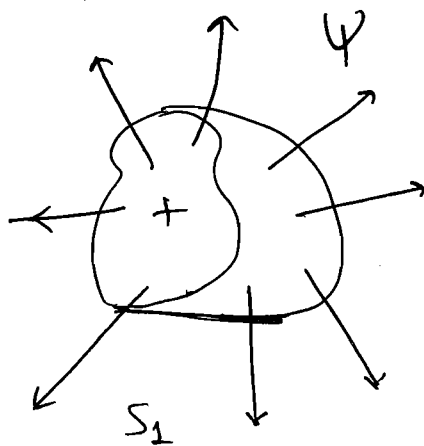
$z = 4\text{m}$



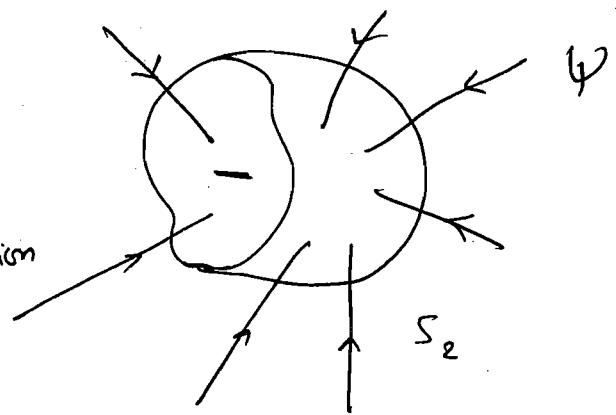
$$\therefore \psi = Q = \int_0^5 \frac{(s^2+1)-1}{(s^2+1)} ds \times [\phi]_0^{2\pi}$$

$$\therefore \psi = [s] - [\tan^{-1}s]_0^5 \times 2\pi$$

$$\therefore \psi = [5 - \tan^{-1}5] \times 2\pi \text{ nC.}$$



Q
 (or) S_1
 (or) S_2
 (or) any
 combination



S_1 & S_2 = arbitrary closed surfaces.

$$\phi_{net} = Q_{enc.}$$

→ S_1 & S_2 are two arbitrary closed surfaces. We assume that they enclosed some charge configuration i.e. either Q (or) S_1 (or) S_2 (or) any combination. Some how we have calculated the total charge within them.

→ Further, we assume that S_1 encloses +ve charge which would results flux leaving surface. The amount of electric flux leaving surface is equals to the charge enclosed within it.

→ Further we assume that S_2 encloses -ve charge. ~~which~~ Therefore, the flux enter the closed surface.

→ We then the electric flux leaving the surface or entering the surface, the electric flux passing through the closed surfaces.

→ Gauss's Law's state that the net electric flux passing through any closed surface is equal to the charge enclosed by that surface.

→ $\boxed{\Psi_{net} = Q_{enc.}}$

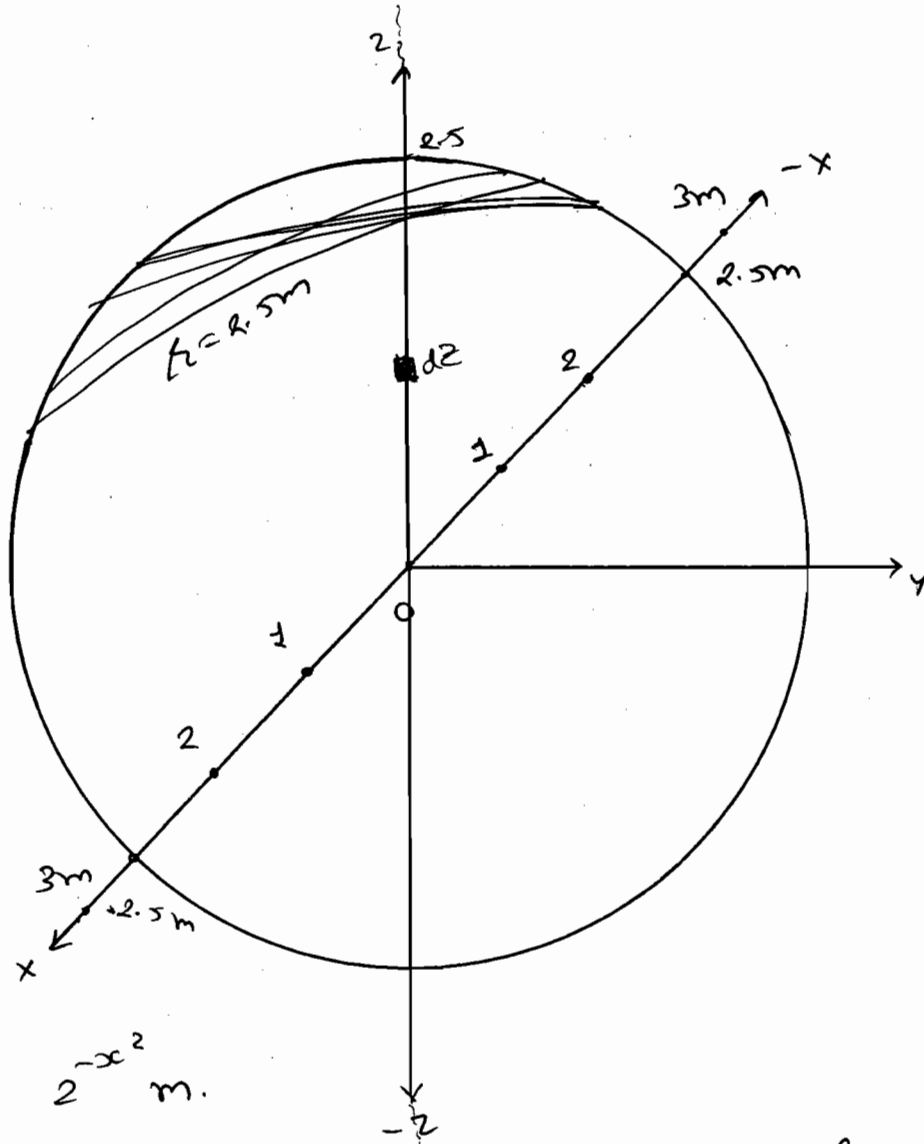
Ex-1/2 What net electric flux passes through a sphere of radius 2.5m centered at the origin. Given the charge configuration.

(1) point charges of $q = 2^{-x^2} \text{ nC}$, which are located ~~at~~ on the x-axis at $x=0, \pm 1, \pm 2, \pm 3 \text{ m}$
Ans: 2.125 nC

(2) An infinite line with a uniform charge density of $\frac{1}{2^2+1} = \frac{1}{z^2+1} \text{ nC/m}$ lies along z-axis.
Ans: 2.38 nC

(3) A 2m - uniform surface charge density of $\frac{1}{x^2+y^2+4} \text{ nC/m}^2$, lies in $z=0$ plane.
Ans: 2.95 nC.

(4) Uniform ^{line} charge density 20 nC/m, lies in $z=0$ plane and are located at $y=0, \pm 1, \pm 2, \pm 3 \text{ m}$.
Ans: 403 nC.



① $Q = 2^{-x^2} \text{ m.}$

→ The charges at $x = \pm 3\text{m}$ are not enclosed by the sphere

$$\psi_{\text{net}} = Q_{\text{enc.}}$$

$$\therefore \psi_{\text{net}} = \frac{-(-2)^2}{2} + \frac{-(-1)^2}{2} + \frac{-(-0)^2}{2} + \frac{-(-1)^2}{2} + \frac{-(-2)^2}{2} + 0$$

$$\therefore \psi_{\text{net}} = \frac{1}{16} + \frac{1}{2} + 1 + \frac{1}{16} + \frac{1}{2}$$

$$\boxed{\psi_{\text{net}} = 2.125 \text{ nC.}}$$

② Part of the infinite line is enclosed by sphere.

$$\text{For } |z| \leq 2.5 \text{ (or) } -2.5 \leq z \leq 2.5.$$

$$\therefore dQ = \lambda dz.$$

$$\therefore \psi_{\text{net}} = Q_{\text{enc.}}$$

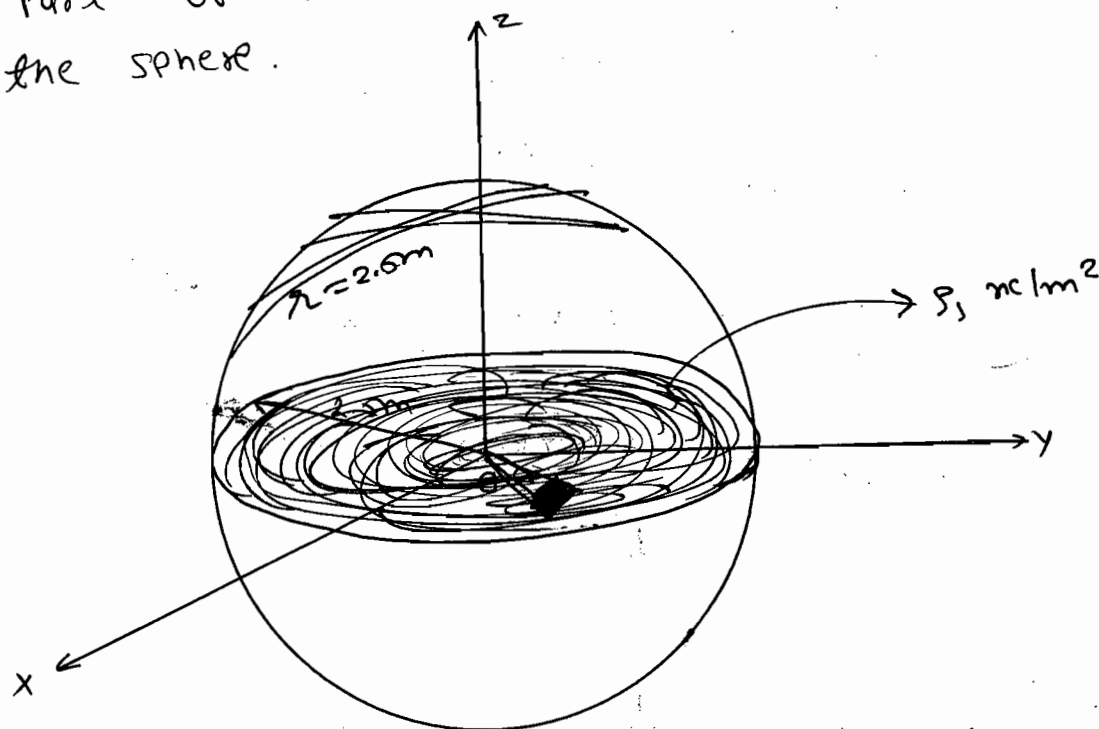
$$\therefore Q_{enc} = \int_{-2.5}^{2.5} \frac{1}{z^2+1} dz.$$

$$= \left[\tan^{-1} z \right]_{-2.5}^{2.5}$$

keep the calci
in Radian.

$$\therefore Q_{enc} = 2.38 \text{ nC}$$

(3) Part of the infinite sheet is enclosed by the sphere.



→ Sphere encloses a circular disk of radius 2.5 m centered at origin and it located in $z=0$ plane.

$$S_s = \frac{1}{x^2 + y^2 + z^2} \text{ nC/m}^2.$$

here, $\rho \leq 2.5 \text{ m}$; $z=0$

Put $x = \rho \cos \phi$, $y = \rho \sin \phi$

$$\therefore S_s = \frac{1}{\rho^2 + 4} \text{ nC/m}^2$$

$$\oint ds = \int ds d\theta$$

$$\therefore dQ = P_s ds$$

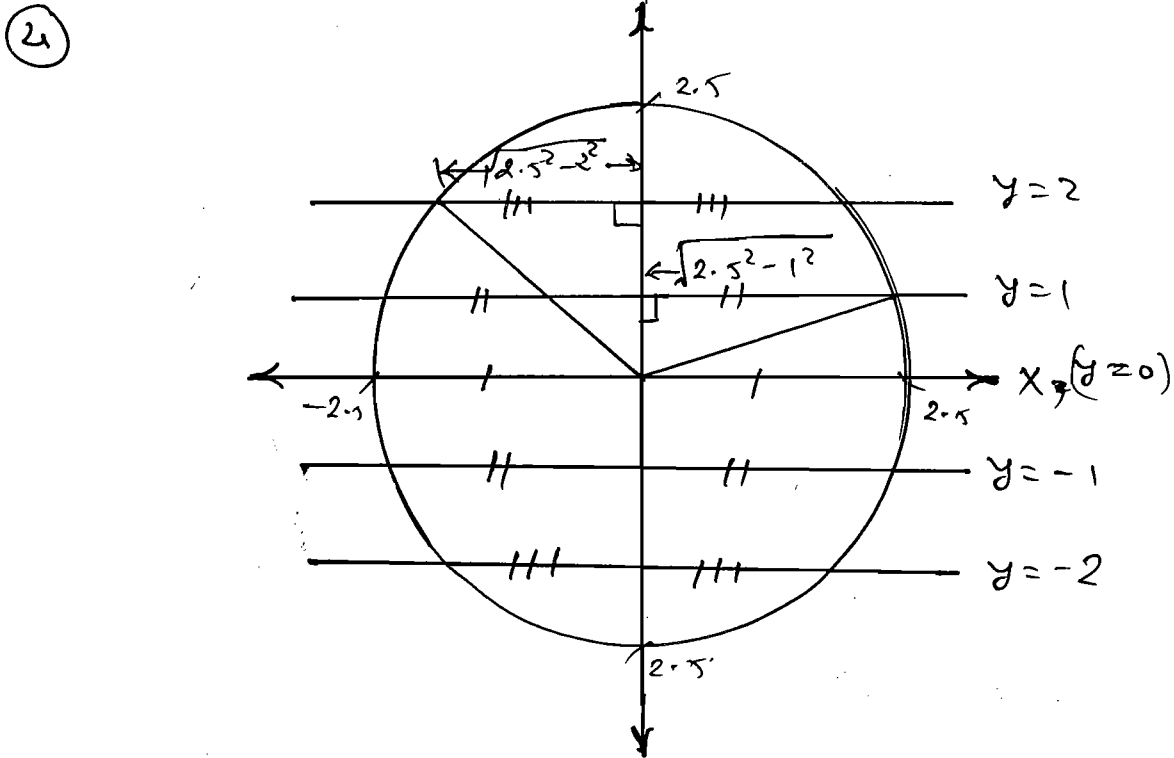
$$\therefore Q_{net} = Q_{enclosed}$$

$$= \int_S d\phi$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{0.5} \frac{r \, dr \, d\theta}{(r^2 + 4)}$$

$$= \frac{1}{2} \left[\ln(s^2+4) \right]_0^{2.5} \times [0]_0^{2\pi}$$

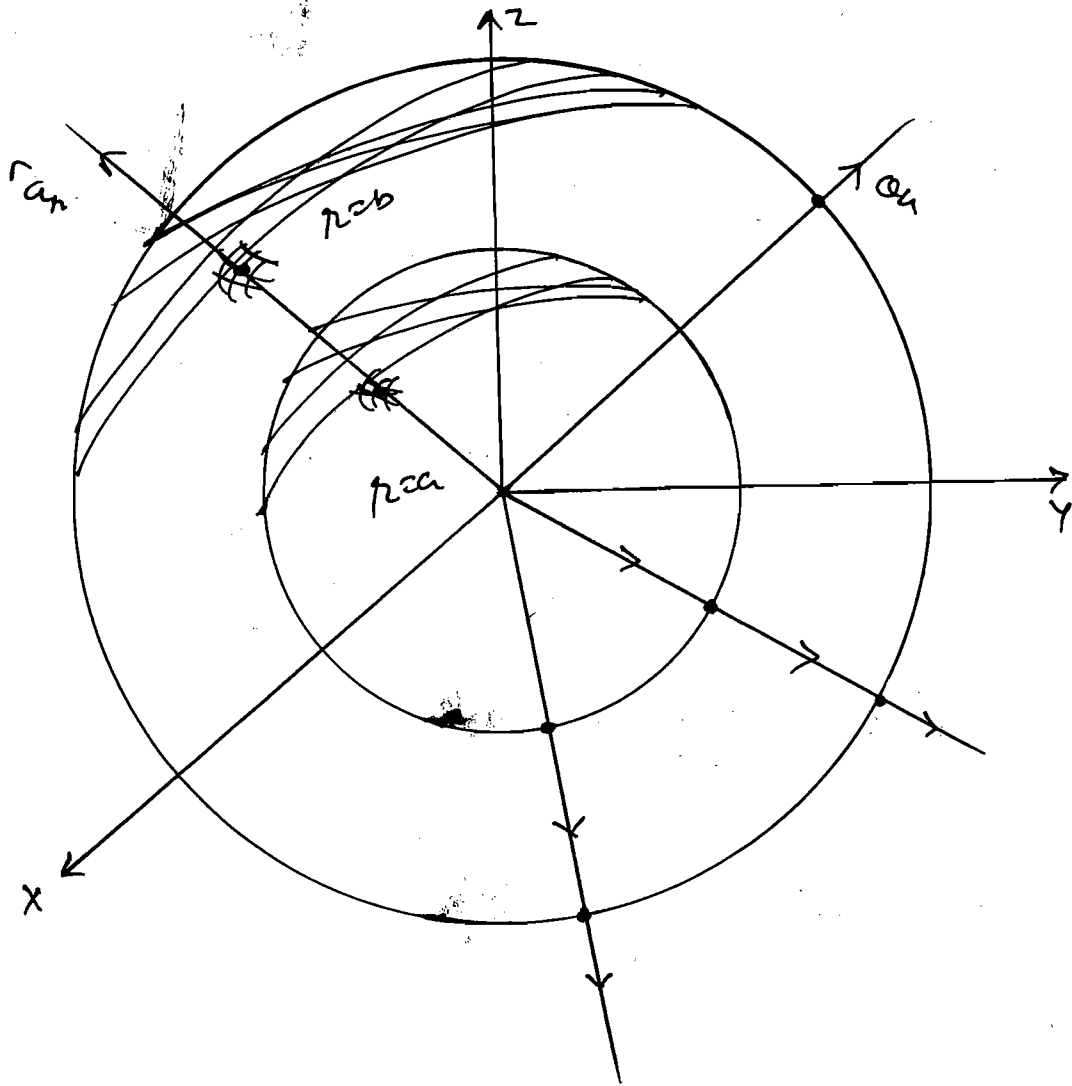
$$= 2.95 \text{ m.c.}$$



$$\rightarrow \phi_{\text{net}} = Q_{\text{enc.}} = 20 \times 10^{-9} \left[4 \sqrt{2.5^2 - 2^2} + 4 \sqrt{2.5^2 - 1^2} \right] + 5 \times 20 \times 10^{-9}$$

$$= 403 \text{ nC.}$$

* Electric Flux Density [$\vec{D}$ C/m²]



→ Flux per unit Area = Electric Flux Density

→ Through $r=a$

$$\psi_{\text{net}} = Q_{\text{enc}} = Q$$

$$\text{Surface area of Sphere} = 4\pi a^2$$

$$\therefore \text{Flux Density} = \frac{Q}{4\pi a^2} \text{ C/m}^2.$$

Through $r=b$.

$$\text{Flux density} = \frac{Q}{4\pi b^2} \text{ C/m}^2.$$

→ In general, the magnitude of the flux density through sphere of radius 'r' m ($r > 0$)

$$|\bar{D}| = \frac{Q}{4\pi r^2} \text{ C/m}^2$$

→ As shown, the flux density is changing its value along radial direction ($\hat{a}_r$)

∴ we write

$$\bar{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

$\bar{E}$ due to 'Q' located at the origin is given by

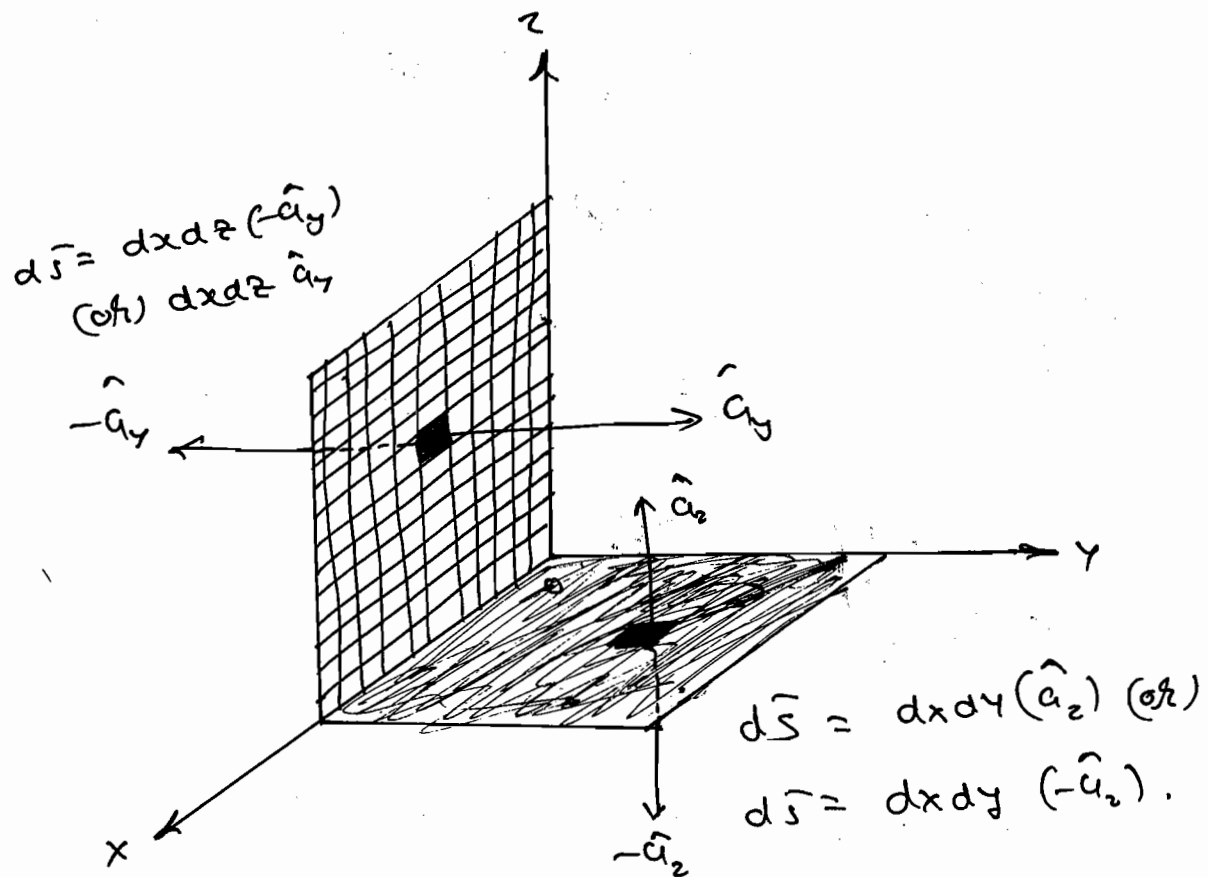
$$\bar{E} = \frac{Q}{4\pi \epsilon_0 r^2} \hat{a}_r$$

$$\therefore \bar{D} = \epsilon \bar{E}$$

Procedure for the calculation of $\bar{D}$ and $\bar{E}$ are identically same.

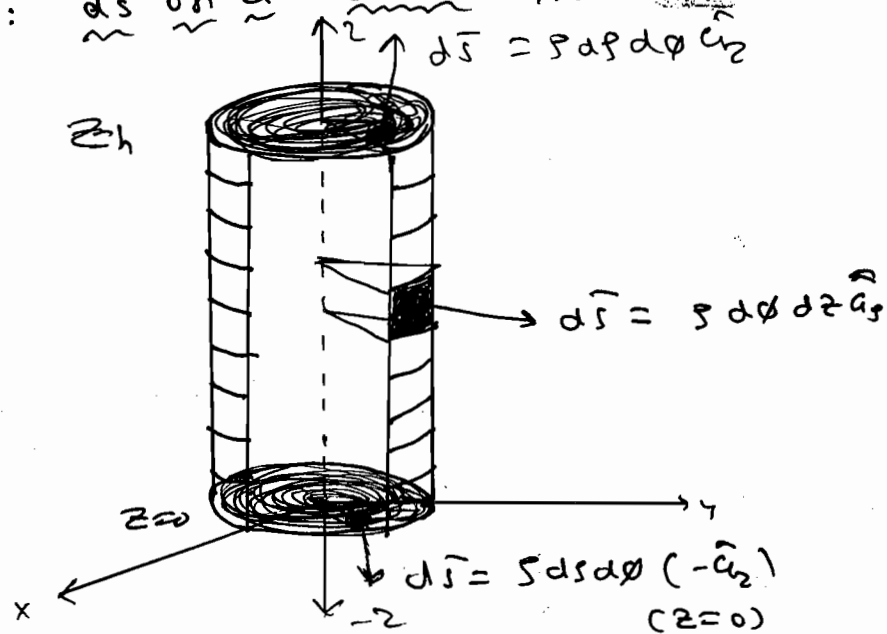
* Vector differential Surface element: $d\vec{s}$ 69

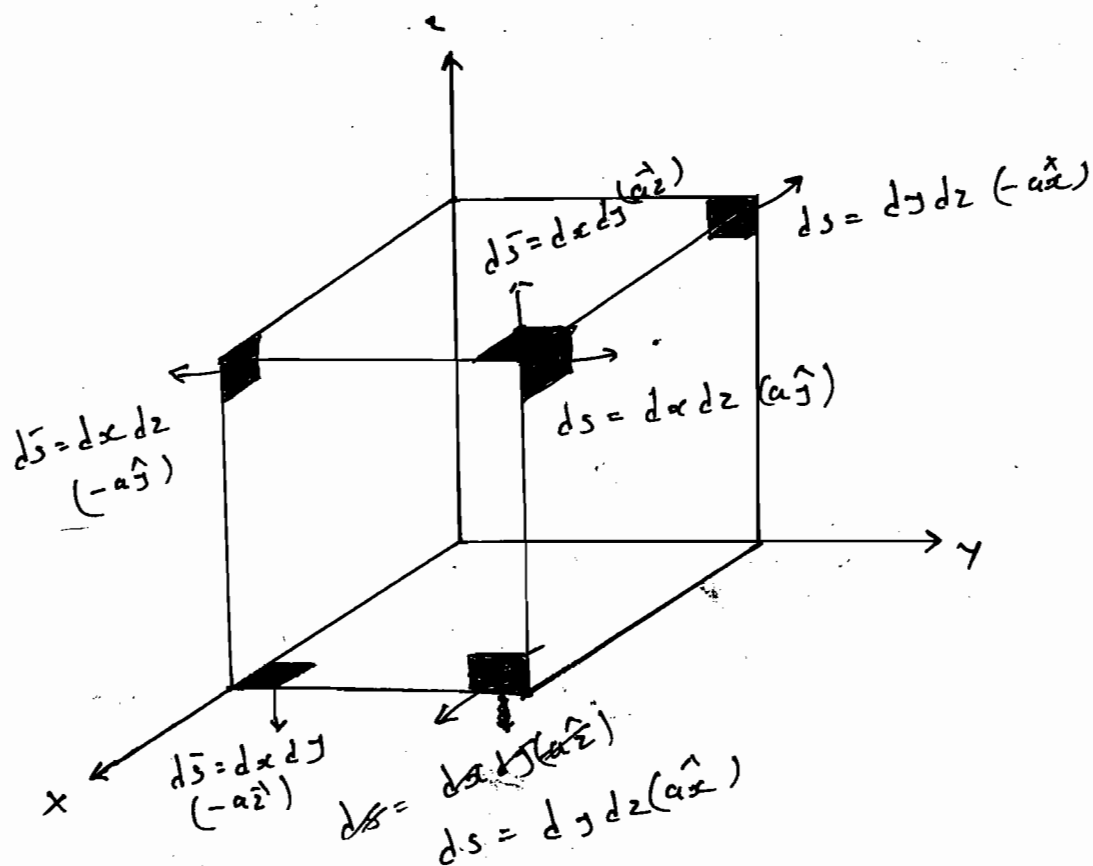
* Case-1: Open plane (or) open surface.



→ The Vector differential surface element $d\vec{s}$ at any point on the open plane would be projecting normal to the surface.

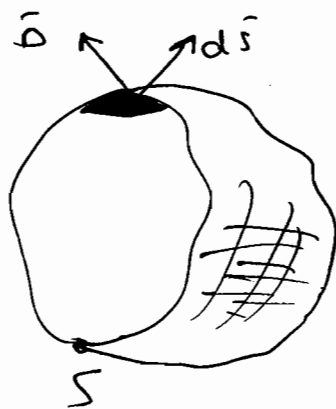
* Case-2: $d\vec{s}$ on a closed surface:





→ At any point on the closed surface $d\vec{s}$ could be projecting outward normal to the surface.

* Integral form of Gauss's Law:



→ Figure shows an arbitrary closed surface (S) at any point, on this surface $d\vec{s}$ would be projecting outward normal to the surface.

enclosing some charge configurations. Somehow we have calculated $\vec{D}$ at any point on the closed surface

for e.g. $\vec{D}$ makes an angle α with $d\vec{s}$

✓ (smiley face) C the direction of $\vec{D}$ at any point on the closed surface will solely depends upon the charge configuration within the closed surface. whereas direction of $d\vec{s}$ at any point on the closed surface would be projecting outward normal to the surface.]

→ α making any possible value betⁿ 0 to 180°.

→ The differential amount of flux passing through $d\vec{s}$ i.e. in a direction normal to the surface at that point is the projection of $\vec{D}$ on to the $d\vec{s}$.

→ Mathematical,

$$d\phi = |\vec{D}| \cdot |d\vec{s}| \cos \alpha.$$

$$= \vec{D} \cdot d\vec{s} \quad \text{if } \alpha$$

if $\alpha = 0$ or 180° max. amount of flux passes through $d\vec{s}$

→ If $\alpha = 90^\circ$, zero flux passes through $d\vec{s}$.

→ We write Gauss Law's Integral form.

$$\therefore \boxed{\Psi_{\text{net}} = \oint_S \vec{D} \cdot d\vec{s} = Q_{\text{enc.}}}$$

→ We assume that the closed surface is enclosing a volume charge density of $\rho_v \text{ C/m}^3$.

$$\therefore Q_{\text{enc}} = \int_V \rho_v dV$$

$$\therefore \oint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v dV. \quad \text{--- (1)}$$

Now, using Divergence theorem.

$$\therefore \oint_S \vec{D} \cdot d\vec{s} = \int_V \nabla \cdot \vec{D} dV. \quad \text{--- (2)}$$

$\therefore$ Comparing (1) & (2)

$$\boxed{\nabla \cdot \vec{D} = \rho_v}$$

→ This is a point form of Gauss Law.

(or) Gauss Divergence theorem.

$$\rightarrow \nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\rightarrow \nabla \cdot \vec{D} = \frac{1}{r} \frac{\partial}{\partial r} (r D_r) + \frac{1}{r} \frac{\partial D_\theta}{\partial \theta} + \frac{\partial D_z}{\partial z}$$

$$\rightarrow \nabla \cdot \vec{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta D_\theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$$

Ex-1 In a region the electric flux density is given by $\vec{D} = (2x \hat{a}_x + 3y \hat{a}_y - kz \hat{a}_z) \text{ C/m}^2$. Assume charge free region then find the value of k .

Ans: Charge free region $\rho_v = 0$

$$\therefore \nabla \cdot \vec{D} = \rho_v = 0$$

$$\therefore \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z = 0$$

$$\therefore 2 + 3 - k = 0$$

$$\boxed{k = 5}$$

$$-kz = \text{C/m}^2$$

$$\therefore +k(2) = \frac{5}{2} \text{ C/m}^2$$

$$\boxed{k = 5 \text{ C/m}^3}$$

(a) 5 C/m²

(b) 5 C/m

(c) 5 C-m.

✓ (d) 5 C/m³

Ex-2 The magnitude of the Electric flux density is proportional to r^k where k is constant. $r \rightarrow$ spherical coordinate. The $\vec{D}$ is projecting in the radial direction. Choose the value of k such that electric flux density has zero divergence.

Ans: (a) -2 (b) -4 (c) -8 (d) -16.

$\rightarrow |\vec{D}| \propto r^k$
 $\therefore \vec{D} = C_1 r^k \cdot \hat{a}_r = D_0 \hat{a}_r$

$\therefore \nabla \cdot \vec{D} = 0$

$\therefore \frac{1}{r^2} \left(r^2 \frac{\partial D_r}{\partial r} \right) = 0$

$\therefore \frac{1}{r^2} \times C_1 r^k$

$\therefore \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) = 0$

$\therefore \frac{\partial}{\partial r} (C_1 r^{k+2}) = 0$

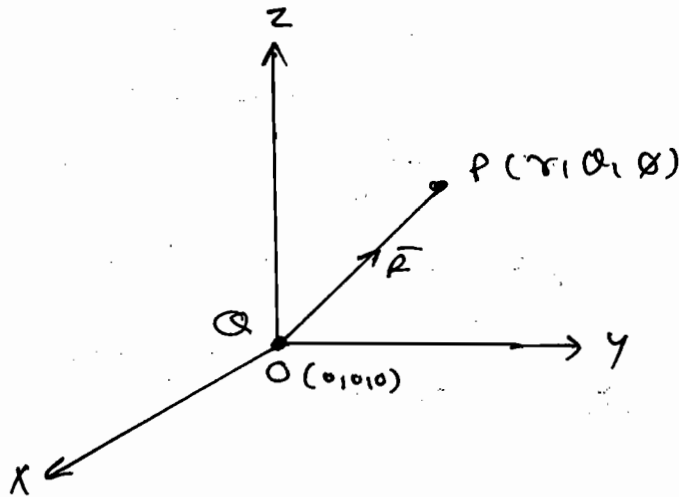
$\therefore (k+2) r^{k+1} = 0$

$\boxed{k = -2}$

Ex-3 A point charge of Q is located at origin. If no other charge is present what is the value $\nabla \cdot \vec{D}$ at any point other than the origin.

Ans:

$$\boxed{\nabla \cdot \vec{D} = 0}$$



$$\text{at } P \Rightarrow \vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

$$\vec{D} = D_r \hat{a}_r$$

$$\begin{aligned} \therefore \nabla \cdot \vec{D} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \dots \\ &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot \frac{Q}{4\pi r^2} \right) \end{aligned}$$

$$\therefore \boxed{\nabla \cdot \vec{D} = 0}$$

Ex-4 Let $\vec{D} = (4x^3 \hat{a}_x + x^2 z \hat{a}_y + 2xy \hat{a}_z) \text{ nC/m}^2$
 $0 \leq x, y, z \leq 1$. find the amount of $\vec{D}$ passing through closed surface defined by $0 \leq x, y, z \leq 1$. and also find the amount of charge enclosed within it. also indicate whether the flux entering a close surface or

Ans:

$$\phi_{\text{net}} = \oint_S \vec{D} \cdot d\vec{S} = \oint_S D_n dS = Q_{\text{enclosed}}$$

$$\rightarrow \phi_{net} = Q_{enc} = \oint_{dS} \vec{D} \cdot d\vec{S} = \int_{x=0} \vec{D} \cdot d\vec{S} + \int_{x=1} \vec{D} \cdot d\vec{S} + \int_{y=0} \vec{D} \cdot d\vec{S} + \int_{y=1} \vec{D} \cdot d\vec{S} + \int_{z=0} \vec{D} \cdot d\vec{S} + \int_{z=1} \vec{D} \cdot d\vec{S}$$

Surface is at $d\vec{S}$ surface $\vec{D} \cdot d\vec{S}$ $\vec{D} \cdot d\vec{S}$ on the surface

$x=0$	$dydz (\hat{a}_x)$	limit $0 \leq y, z \leq 1$	$-4x^3$ $dydz$	$-4(0)^3 dydz$	$-4 \int_0^1 \int_0^1 (0)^3 dydz = 0$
$x=1$	$dydz (\hat{a}_x)$	$0 \leq y, z \leq 1$	$+4x^3$ $dydz$	$+4 dydz$	$-4 \int_0^1 \int_0^1 dydz = 4$
$y=0$	$dx dz (-\hat{a}_y)$	$0 \leq x, z \leq 1$	$-x^2 z$ $-dx dz$	$-x^2 z dx dz$	$-\int_0^1 \int_0^1 -x^2 z dx dz = -1/8$
$y=1$	$dx dz (\hat{a}_y)$	$0 \leq x, z \leq 1$	$+x^2 z$ $+dx dz$	$+x^2 z dx dz$	$+\int_0^1 \int_0^1 x^2 z dx dz = 1/8$
$z=0$	$dx dy (-\hat{a}_z)$	$0 \leq x, y \leq 1$	$-2xy$ $-dx dy$	$-2xy dx dy$	$-2 \int_0^1 \int_0^1 xy dx dy = -1/2$
$z=1$	$dx dy (\hat{a}_z)$	$0 \leq x, y \leq 1$	$2xy$ $+dx dy$	$2xy dx dy$	$+2 \int_0^1 \int_0^1 xy dx dy = 1/2$

$$Q_{enc} = 4 \text{ nC}$$

$$\rightarrow Q_{enc} = +4 \text{ nC}$$

$\rightarrow$ The enclosed charge is +ve therefore, the electric flux ~~leaving~~ leaving the closed surface.

$$\rightarrow \nabla \cdot \vec{D} = 12x^2 + 0 + 0 = \rho_v$$

$$\rho_v = 12x^2$$

$$\therefore Q_{enclosed} = \int_V \rho_v dV = \int_0^1 \int_0^1 \int_0^1 12x^2 dy dz dx$$

$$\therefore Q_{enc} = 12 \times \left[\frac{y^3}{3} \right]_0^2 \left[z \right]_0^2 \left[x \right]_0^1$$

$$= \frac{12}{3}$$

$$\therefore \boxed{Q_{enc} = 4 \text{ nc.}}$$

Ex-5 $\rightarrow$ Let, $\vec{D} = 12x^2yz \hat{a}_x + 2xy \hat{a}_y + 3x^2z \hat{a}_z$ $\frac{\text{nc}}{\text{m}^2}$.
find the amount of Electric flux passing through a surface define by $x=1$, $0 \leq y, z \leq 2$.
 $\hat{a}_x$ direction.

$$\text{Ans: } \vec{D} = 12x^2yz \hat{a}_x + 2xy \hat{a}_y + 3x^2z \hat{a}_z$$

$$\therefore Q_{en1} = \int_0^2 \int_0^2 12yz \, dy \, dz$$

$$= \frac{12}{4} \left[y^2 \right]_0^2 \left[z^2 \right]_0^2$$

$$= \frac{12}{4} \times 4 \times 4$$

$$\therefore \boxed{Q_{enc} = 48 \text{ nc}}$$

$$\therefore \boxed{\Psi_{net} = Q_{enc} = 48 \text{ nc}}$$

$$d\vec{S} = dy \, dz \, \hat{a}_x$$

$$\vec{D} \cdot d\vec{S} = 12x^2yz$$

$$\text{at } x=1$$

$$\vec{D} \cdot d\vec{S} = 12yz$$

Ex-6 Let, $\vec{D} = \frac{r}{3} \hat{a}_r$ $\frac{\text{nc}}{\text{m}^2}$ where r is a Spherical Co-ordinate, $\hat{a}_r$ is a unit vector in the radial direction. find $\oint \vec{D} \cdot d\vec{S}$, amount of electric flux passing through a sphere of radius 1m. centered at the origin. also indicate the surface or entering

Ans:

$$\oint \vec{D} \cdot d\vec{S} = \nabla \cdot \vec{D}$$

$$\begin{aligned}
 \therefore \rho_v &= \nabla \cdot \vec{D} \\
 &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot D_r) \\
 &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot \frac{r}{3} \right) \\
 &= \frac{3r^2}{3r^2}
 \end{aligned}$$

$$\boxed{\rho_v = 1 \text{ nC/m}^3}$$

$$\begin{aligned}
 \therefore Q_{\text{enclosed}} &= \int_V \rho_v dV \\
 &= \int_0^1 \int_0^{2\pi} \int_0^\pi \rho_v \cdot r^2 \sin\theta \cdot dr d\alpha d\theta \\
 &= \int_0^1 \int_0^{2\pi} \int_0^\pi r^2 \cdot \sin\theta \cdot dr d\alpha d\theta \\
 &= \frac{1}{3} \times 2\pi \times [-\cos\theta]_0^\pi
 \end{aligned}$$

$$\boxed{Q_{\text{enclosed}} = \frac{4\pi}{3} = Q_{\text{net}} = n.c.}$$

$$\therefore Q_{\text{enclosed}} = \oint_S \vec{D} \cdot d\vec{S}$$

$$d\vec{S} = r^2 \sin\theta \cdot d\alpha d\theta \cdot \hat{e}_r$$

$$\therefore \vec{D} \cdot d\vec{S} = \frac{r^3}{3} \sin\theta \cdot d\alpha d\theta$$

$$\begin{aligned}
 \therefore Q_{\text{enclosed}} &= \int_0^\pi \int_0^{2\pi} \frac{r^3}{3} \sin\theta \cdot d\alpha d\theta \\
 &\quad r=1\text{m}
 \end{aligned}$$

$$= \frac{1}{3} [2\pi] [-60] 0] \pi$$

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$$= \frac{1}{3} \times 2\pi \times 2$$

$$\therefore \boxed{Q_{\text{enclosed}} = \frac{4\pi}{3} \rho c = \psi_{\text{net}}}$$

→ Leaving the sphere as Q is +ve.

Ex-6 Let, $\vec{D} = -\frac{20}{s^2} [\sin^2\theta \hat{a}_s + \sin 2\theta \hat{a}_\theta] \text{ nC/m}^2$

find the amount of electric flux passing through a closed region define by $1 \leq s \leq 2$,

$$0 \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq z \leq 1.$$

(a)

Ans:

$$\vec{D} = -\frac{20}{s^2} [\sin^2\theta \hat{a}_s + \sin 2\theta \hat{a}_\theta] \text{ nC/m}^2.$$

$$\therefore \nabla \cdot \vec{D} = \frac{1}{s} \frac{\partial}{\partial s} (s D_s) + \frac{1}{s} \frac{\partial}{\partial \theta} (D_\theta)$$

$$= \frac{1}{s} \cdot \frac{\partial}{\partial s} \left(-\frac{20}{s} \sin^2\theta \right) + \frac{1}{s} \frac{\partial}{\partial \theta} \left[-\frac{20}{s^2} \sin 2\theta \right]$$

$$\nabla \cdot \vec{D} = \frac{+20 \sin^2\theta}{s^3} + \left(-\frac{40}{s^3} \cos 2\theta \right)$$

$$\therefore \rho_v = \frac{20 \sin^2\theta}{s^3} - \frac{40}{s^3} \cos 2\theta = \frac{20}{s^3} \left[\frac{1}{2} - \frac{\cos 2\theta}{2} - 2 \cos 2\theta \right]$$

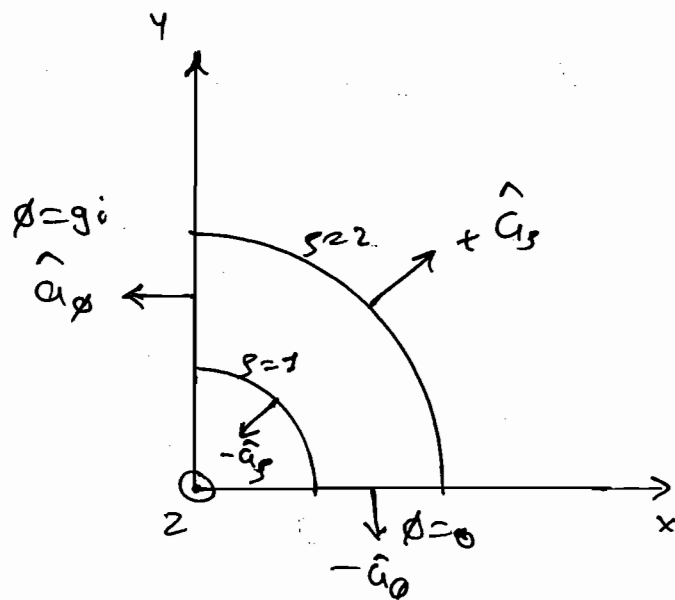
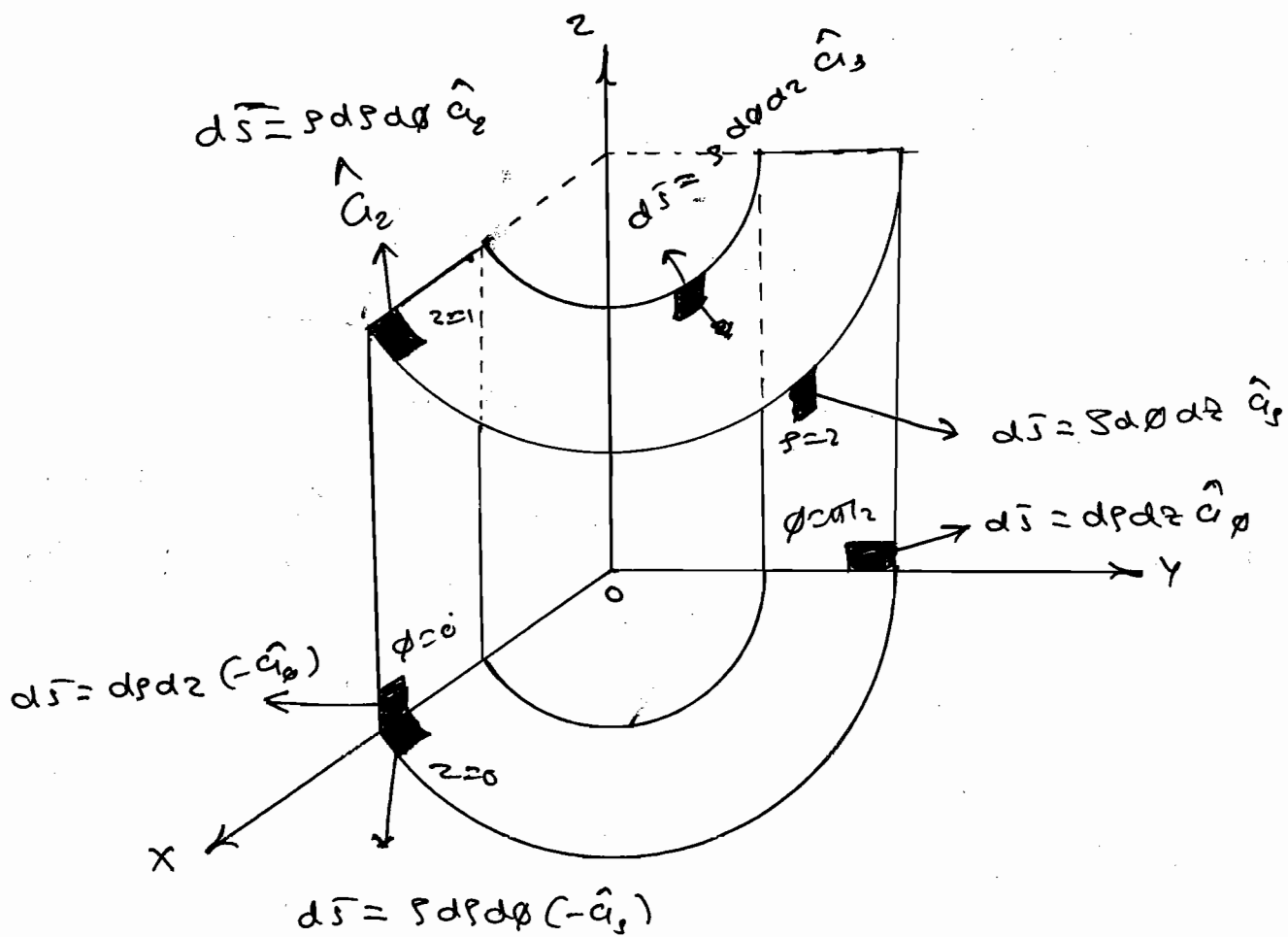
$$\boxed{\rho_v = \frac{20}{s^3} \left[\frac{1}{2} - \frac{5}{2} \cos 2\theta \right]}$$

$$\therefore Q_{\text{enclosed}} = \psi_{\text{net}} = \int_V \rho_v dV$$

$$= \int_1^2 \int_0^{\pi/2} \int_0^{2\pi} \frac{20 \sin^2\theta}{s^3} \cdot s ds d\theta dz$$

$$= 20 \left[-\frac{1}{s} \right]_1^2 \times \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi/2} \times \pi$$

$$= +10 \times \frac{\pi}{2} = +2.5\pi \text{ nC.}$$



* Work:

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→ Work is defined as a force acting over distance.

→ Work done in moving a charge of q Coulombs from an initial point to the final point in the vicinity of electric field is given by

$$W = -q \int_{\text{initial}}^{\text{final}} \vec{E} \cdot d\vec{l} \quad \text{Joules.}$$

Ex-1 find a w.o. in moving a 5 μC Charge from the origin to $(2, -1, 4)\text{m}$ through the field $(2xyz \hat{a}_x + x^2z \hat{a}_y + x^2y \hat{a}_z) \text{ V/m}$. via the path $(0, 0, 0)$ to $(2, 0, 0)$ to $(2, -1, 0)$ to $(2, -1, 4)$.

Ans: $d\vec{l} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$.

$$\vec{E} \cdot d\vec{l} = 2xyz dx + x^2z dy + x^2y dz.$$

$$\text{W.O.} = -q \int_{\text{initial}}^{\text{final}} 2xyz dx + x^2z dy + x^2y dz.$$

$$\rightarrow d\vec{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + dz \hat{a}_z$$

$$d\vec{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\alpha d\phi \hat{a}_\phi.$$

$$W.D. = -q \left[\int_1 \vec{E} \cdot d\vec{r} + \int_2 \vec{E} \cdot d\vec{r} + \int_3 \vec{E} \cdot d\vec{r} \right].$$

∴ ① Path-1 (0,0,0) to (2,0,0).

$$x \rightarrow 0 \text{ to } 2$$

$$\begin{aligned} y &= 0 \Rightarrow dy = 0 \\ z &= 0 \Rightarrow dz = 0 \end{aligned} \quad x=2.$$

$$\therefore I_1 = \int_0^2 \vec{E} \cdot d\vec{r} = \int_0^2 0 \cdot dz = 0.$$

② Path-2: (2,0,0) to (2,-1,0)

$$\therefore x=2 \Rightarrow dx=dz=0$$

$$y=0 \text{ to } -1.$$

$$z=0.$$

$$\therefore I_2 = \int_0^{-1} 4y \, dy = 0.$$

③ Path-3: (2,-1,0) to (2,-1,4).

$$x=2 \Rightarrow dx=0$$

$$y=-1 \Rightarrow dy=0.$$

$$z=0 \text{ to } 4.$$

$$I_3 = \int_0^4 -4 \cdot dz.$$

$$I_3 = -16.$$

$$\therefore W.D. = -q [I_1 + I_2 + I_3]$$

$$= -5 \times 10^{-6} [-16]$$

$$\therefore \boxed{W.D. = 80 \mu J}$$

Ex. 2 Repeat the above example ~~between~~ the

path $x = -2y, z = 2x$.

Ans: $d\vec{r} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$. (0, 0, 0) to (2, -1, 4).

$$\therefore \vec{E} \cdot d\vec{r} = 2xy z dx + x^2 z dy + x^2 y dz.$$

$$W = -q \int_{\text{initial}}^{\text{final}} \vec{E} \cdot d\vec{r} = -5 \times 10^{-6} \left[\int_0^2 xy z dx + \int_0^{-1} x^2 z dy + \int_0^4 x^2 y dz \right].$$

$$= -5 \times 10^{-6} \left[2 \int_0^2 -x^3 dx + \int_0^{-1} 2x^3 dx + \int_0^4 -x^3 dz \right].$$

$$= -5 \times 10^{-6} \left[-2 \left[\frac{1}{16} \right] - \left[\frac{1}{2} \right] - \left[\frac{1}{256} \right] \right].$$

$$= -5 \times 10^{-6} \left[\frac{1}{8} + 1 + \frac{1}{256} \right].$$

$$\neq 5 \times 10^{-6}$$

$$W = -5 \times 10^{-6} \left[2 \int_0^2 -x^3 dx + \int_0^{-1} 8y^3 dy + \int_0^4 -\frac{z^3}{8} dz \right].$$

$$\therefore W = -5 \times 10^{-6} \left[-\frac{1}{8} + \frac{8^2}{4} - \frac{648}{8} \right]$$

$$= -5 \times 10^{-6} \left[-6 - \frac{1}{8} \right].$$

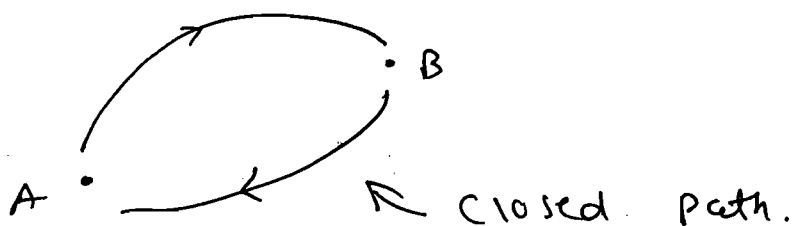
$$W = \frac{49 \times 5}{8} \times 10^{-6} \text{ J.}$$

$$\rightarrow (x_1, y_1, z_1) \text{ to } (x_2, y_2, z_2).$$

$$(x_1, y_1) \text{ to } (x_2, y_2) \quad (x_1, z_1), (x_2, z_2).$$

$$\therefore \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \quad | \quad \frac{z-z_1}{z_2-z_1} = \frac{x-x_1}{x_2-x_1}.$$

*
=



$$\rightarrow -q \oint \vec{E} \cdot d\vec{r} = 0 \Rightarrow '0' \text{ Can't be zero.}$$

$$\therefore \boxed{\oint \vec{E} \cdot d\vec{r} = 0} \Rightarrow \frac{\text{Work done over a closed path is ZERO.}}$$

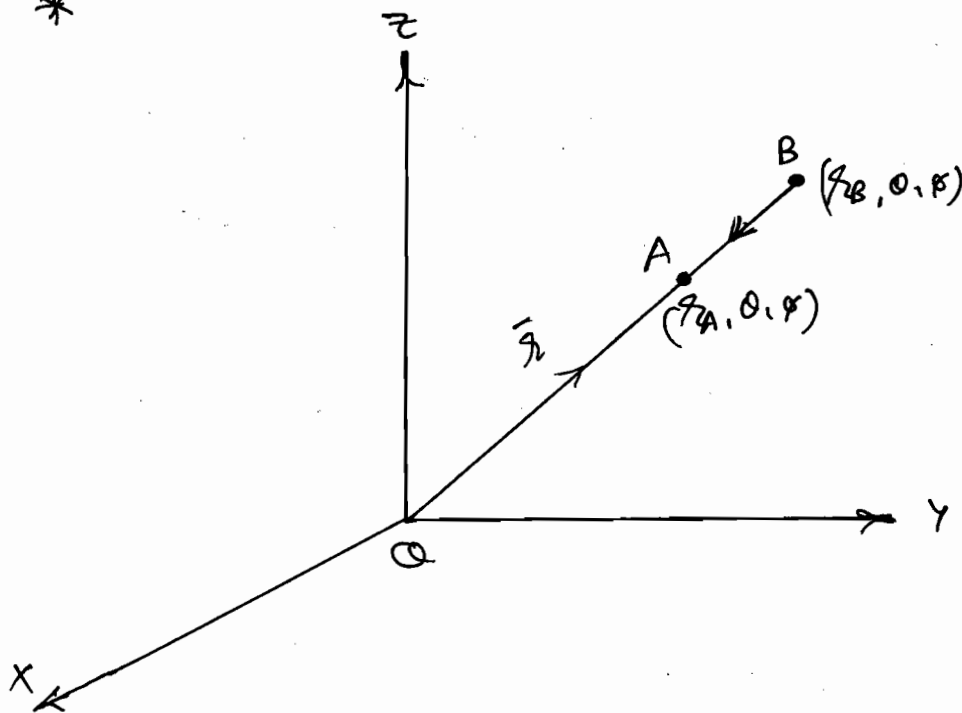
* Potential:

$$\rightarrow W = -q \int_B^A \vec{E} \cdot d\vec{r} \quad \text{J}$$

We define the potential at 'A' with ref. to 'B' is given by,

$$V_{AB} = \frac{W}{q} = - \int_B^A \vec{E} \cdot d\vec{r} \quad \text{J/C (or) Volts.}$$

*



$$\rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

From B to A $d\vec{r} = dr \hat{r}$.

$$\therefore \vec{E} \cdot d\vec{r} = \frac{q}{4\pi\epsilon_0 r^2} \cdot dr$$

$$V_{AB} = - \int_{r_B}^{r_A} \frac{q}{4\pi\epsilon_0 r^2} dr$$

$$\therefore V_{AB} = \frac{q}{4\pi\epsilon} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

if r_B is chosen at infinity,

$$r_B \rightarrow \infty$$

$$\frac{1}{r_B} = 0.$$

Then,

$$V_{AB} = \frac{q}{4\pi\epsilon r_A} = V_A.$$

In general

$$V_P = \frac{q}{4\pi\epsilon(r)} + C.$$

V_P is the potential at P due to 'Q'. $|r|$ is the distance b/w the charge 'Q' and the observation point 'P'.

$C=0$ if the ref. point is chosen at infinity.

* Potential Function:

→ Potentials is a functions of space

Co-ordinates $V(x, y, z)$ (or)

$V(r, \theta, \phi)$ (or)

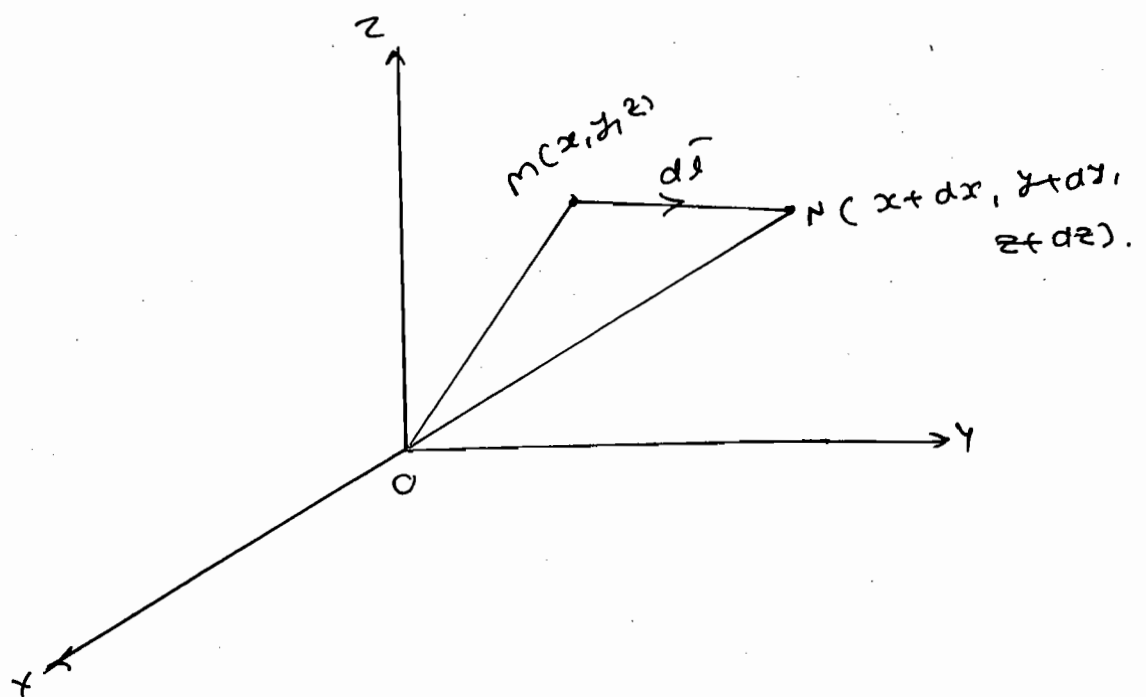
$V(r, \theta, \phi)$.

* Relation b/w Potential gradient and Electric field: 87

→ We assume that two neighbourhood points M, N because of some charge configuration we have known the potential function $V(x, y, z)$.

→ Further, we assume that potential at M is different from potential at N and there exist a potential difference of dV volts.

→



→ (1) $d\vec{r} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$.

$V(x, y, z)$ is known.

(2) $dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$.

• We introduce ∇ (or) Del (or) gradient operator

→ gradient of scalar function in vector.

(3) $\nabla V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z$.

② in terms of ① & ③.

$$dV = \nabla V \cdot d\vec{r} \quad - (4)$$

$$\therefore V = - \int \vec{E} \cdot d\vec{r}$$

$$\therefore dV = - \vec{E} \cdot d\vec{r} \quad - (5)$$

from (4) & (5)

$$\therefore - \vec{E} \cdot d\vec{r} = \nabla V \cdot d\vec{r}$$

suppressing $d\vec{r}$ on both sides.

$$\boxed{\vec{E} = - \nabla V.}$$

(I) Electric field is the gradient of the scalar electric potential functions.

(II) (a) From eqn (5) we can conclude that electric field projects normal to an equipotential surface.

(b) $\vec{E}$ would be project from a higher potential surface to towards lower potential surface.

* Equipotential surface:

→ It is that surface on which the potential difference betⁿ any two points is 0.

→ we assume that the points m and n lies on equipotential surface.

(c) From eqn (4) we can conclude that Potential can vary its value normal to an equipotential surface.

$$\rightarrow \nabla V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \rightarrow V(x, y, z) \quad 88$$

$$\rightarrow \nabla \cdot V = \frac{\partial V}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{\partial V}{\partial z} \hat{a}_z \rightarrow V(\rho, \phi, z)$$

$$\rightarrow \nabla \cdot V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi \rightarrow V(r, \theta, \phi)$$

Ex: ~~1~~ ~~2~~ ~~3~~ ✓

In a certain region the potential field distribution is given by $V(r) = 100\sqrt{r}$ volts where, r is spherical coordinates assume medium to be free space. Find $\vec{E}$, $\vec{D}$ & amount of flux passing through a sphere of radius 5m. centered at origin. also ^{find} indicate charge enclosed and also indicate the flux entering the surface or entering the surface.

Ans: $V(r) = 100(r)^{1/2}$

$$\therefore \vec{E} = -\nabla \cdot V$$

$$\therefore \vec{E} = -\frac{d}{dr} (100r^{1/2})$$

$$= -100 \times \frac{1}{2\sqrt{r}} \hat{a}_r$$

$$\therefore \vec{E} = \frac{-50}{\sqrt{r}} \hat{a}_r \text{ V/m.}$$

$$\therefore \vec{D} = \epsilon \vec{E}$$

$$\therefore \vec{D} = \frac{-50 \epsilon_0}{\sqrt{r}} \hat{a}_r \text{ V/m. C/m}^2$$

$$\therefore Q = \oint_S \vec{D} \cdot d\vec{S} = \oint_S \dots$$

$$\therefore Q = \int \dots d\theta$$

$$\therefore \nabla \cdot \vec{D} = + \frac{50}{2} \epsilon_0 r^{-3/2}$$

$$\therefore Q_{\text{enclosed}} = \int_0^5 \int_0^\pi \int_0^{2\pi} \epsilon_0 r^2 \sin \theta \cdot dr d\theta d\phi$$

$$= [2\pi] [-\cos \theta]_0^\pi \times 25 \times \int_0^5 (r)^{\frac{1}{2}} \cdot dr$$

$$= 100\pi \times \frac{2}{3} (5)^{3/2}$$

$$\vec{D} \cdot d\vec{S} = -50 \epsilon_0 r^{3/2} \sin \theta \cdot d\theta d\phi$$

at $r = 5m$

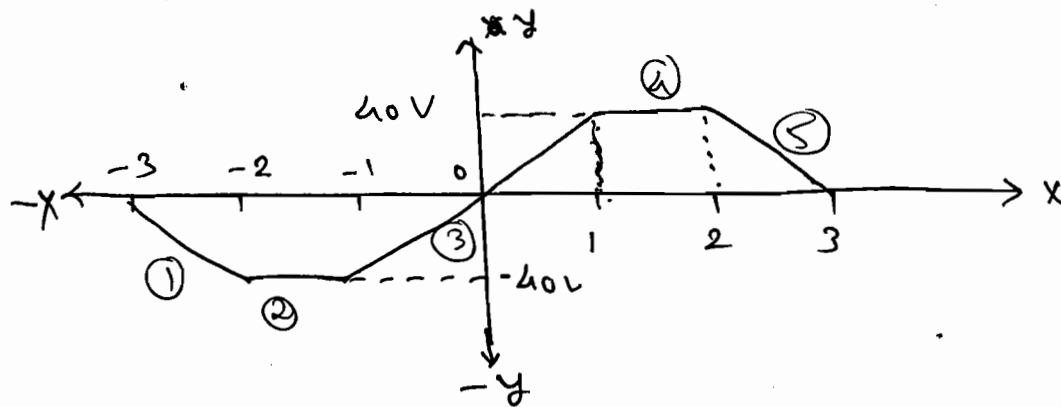
$$\vec{D} \cdot d\vec{S} = -50 \epsilon_0 (5)^{3/2} \sin \theta \cdot d\theta d\phi$$

$$\therefore \psi_{\text{net}} = Q_{\text{enc}} = \oint \vec{D} \cdot d\vec{S} \\ = -50 \epsilon_0 (5)^{3/2} \int_0^\pi \int_0^{2\pi} \sin \theta \cdot d\theta d\phi$$

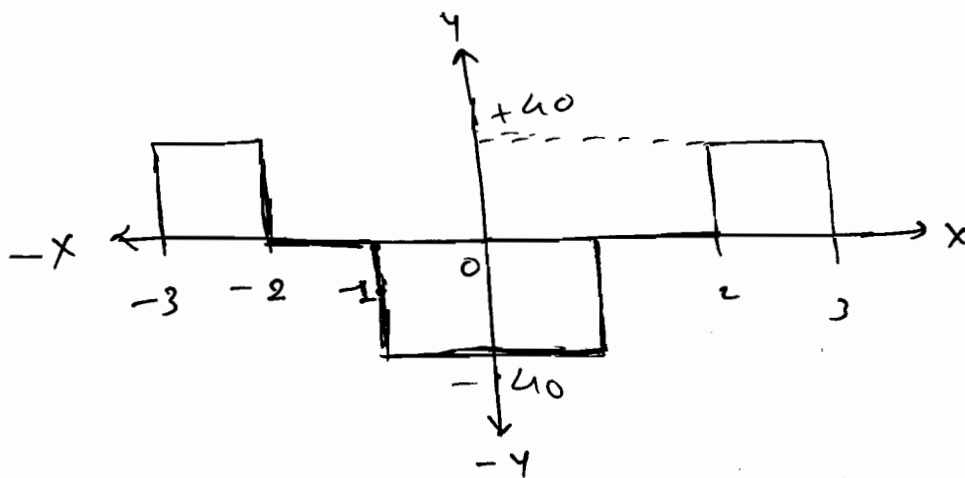
$$\psi_{\text{net}} = Q_{\text{enc}} = -50 \epsilon_0 (5)^{3/2} (4\pi) C$$

$\therefore -\psi'$ is entering to the closed surface.

Ex-2 In certain region the potential field ϕ is given distribution is given by the following sketch plot the corresponding electric field.



Ans:



→

seg-②

$$-2 < x < -1.$$

$$V(x) = -40 \text{ V}$$

$$E_x = -\frac{dV}{dx} = 0$$

seg-③ $-1 < x < 1.$

$$(-1, -40), (1, 40)$$

$$\Delta V = 40$$

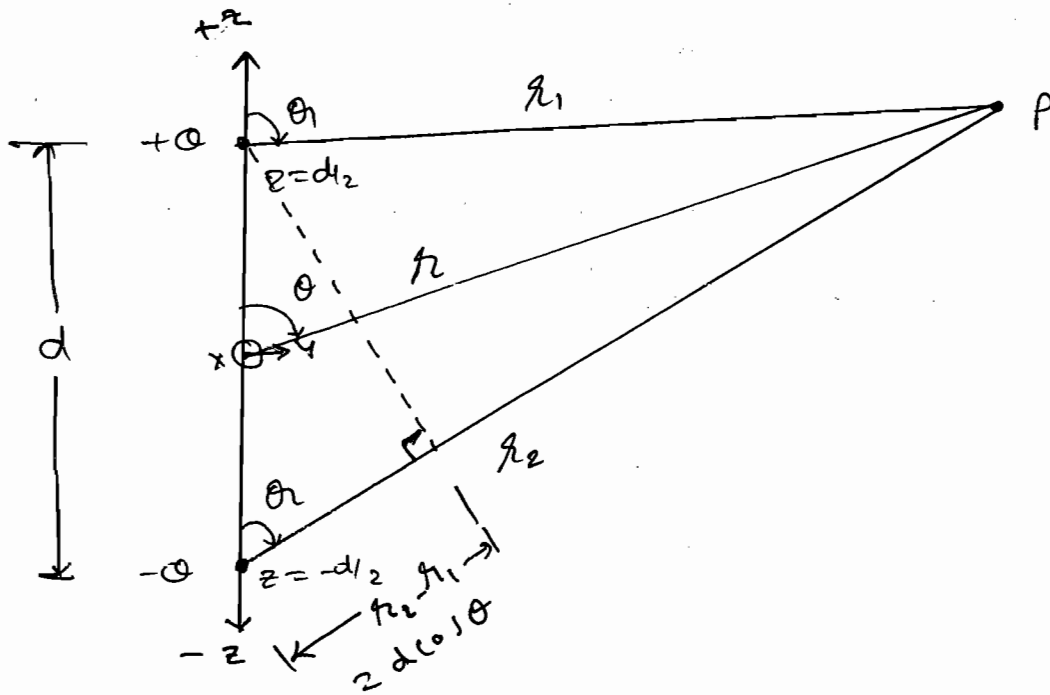
$$V(x) + 40 = 40(x + 1).$$

$$\therefore V(x) = 40x.$$

$$\therefore E_x = -\frac{\partial V}{\partial x}.$$

$$\therefore \boxed{E_x = -40 \text{ V/m}}$$

* Dipole:



$$\rightarrow V_P = \frac{q}{4\pi\epsilon} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$V_P = \frac{q}{4\pi\epsilon} \left[\frac{r_2 - r_1}{r_1 \cdot r_2} \right]$$

$$\therefore r \gg d$$

$$\Rightarrow \theta_1 \approx \theta_2 \approx \theta$$

$$\therefore r_2 - r_1 = d \cos \theta$$

$$\frac{1}{r_1 \cdot r_2} = \frac{1}{r^2}$$

$$\therefore V_P = \frac{q}{4\pi\epsilon} \times \frac{d \cos \theta}{r^2}$$

$$\therefore \boxed{V_P = \frac{q d \cos \theta}{4\pi\epsilon r^2}}$$

$$\therefore \boxed{V_P \propto \frac{1}{r^2}}$$

Ans:

$$\vec{E} = -\nabla V$$

$$\therefore \vec{E} = -\frac{\partial V}{\partial x} \hat{a}_x - \frac{\partial V}{\partial y} \hat{a}_y$$

$$\vec{E} = \left(\frac{qd \cos \theta}{2\pi \epsilon r^2} \hat{a}_r + \frac{qd \sin \theta}{4\pi \epsilon r^3} \hat{a}_\theta \right) \frac{V}{m}$$

$$\therefore \boxed{\vec{E} = \left(\frac{qd \cos \theta}{2\pi \epsilon r^3} \hat{a}_r + \frac{qd \sin \theta}{4\pi \epsilon r^3} \hat{a}_\theta \right) V/m}$$

✓

So, for the quantities $\vec{E}$, ψ , $\vec{D}$ & V have been analyzed. From the knowledge of given Charge Configuration, there are no procedure available for the measurement of this Charge Configuration but there are procedures available for the measurement of potentials at the given points.

→ From the known potentials if we are able to develop potential function i.e. $V(x, y, z)$ or $V(\rho, \phi, z)$ or $V(r, \theta, \phi)$ then

$$\begin{aligned} \vec{E} &= -\nabla V \\ \vec{D} &= \epsilon \vec{E} \\ \oint_S \vec{D} \cdot d\vec{s} &= Q_{enc} = Q_{net} \\ \nabla \cdot \vec{D} &= \rho_v \end{aligned}$$

→ For developing this potential ^m Poisson's eqⁿ and Laplace eqⁿ are used.

* Poisson's and Laplacian's Eqⁿ:

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \cdot (\epsilon \vec{E}) = \rho_v$$

(Homogeneous medium).

$$\epsilon \nabla \cdot \vec{E} = \rho_v$$

$$\therefore \nabla \cdot \vec{E} = \frac{\rho_v}{\epsilon}$$

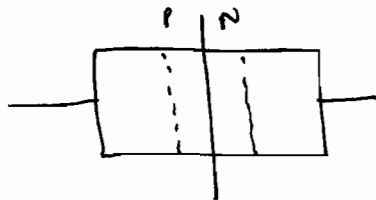
$$\therefore \nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}$$

$$\therefore \boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}} \rightarrow \text{Poisson Eqⁿ}$$

in a region of interest if $\rho_v = 0$

then $\boxed{\nabla^2 V = 0} \rightarrow \text{Laplacian Eqⁿ}$

→ For analysing junction characteristics of a PN diode one dimensional Poisson's Eqⁿ is used because junction has a charge i.e. it is an ionic region.

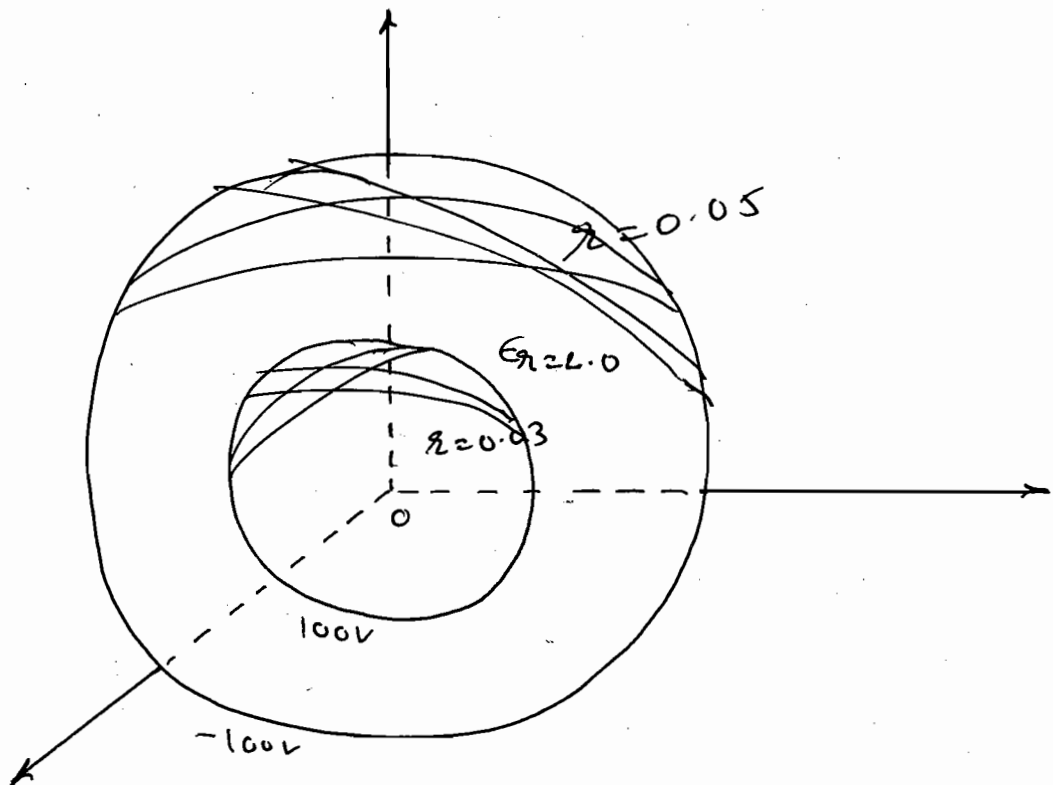


$$\frac{d^2 V}{dx^2} = -\frac{\rho}{\epsilon} \text{ (PE)}$$

Ex-1 Two concentric conducting spheres 95
 having radii 3 cm and 5 cm are
 centered at origin. The potential on the
 inner sphere is 100 V, while the
 outer sphere is at -100 V. The region
 betⁿ them is filled with a homogeneous
 dielectric having ϵ_r relative permittivity 2.0.
 Find

- (1) potential function.
- (2) potential mid way betⁿ the conducting
 sphere.
- (3) The value of r at which $V=0$.
- (4) Find the expression for electric field.

Ans:
 $\rightarrow$



$\rightarrow$ as shown in figure, there exists equipotential
 surfaces at $r = \text{constant}$ we know that
 potential varies normal to equipotential
 surface.

→ Therefore, Potential V must be a r alone.
 Since S_V is not mention betⁿ the sphere.
 We assume $S_V = 0$. Therefore, Laplacian eqⁿ
 reduces to

$$\nabla^2 V = 0$$

$$\therefore \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) = 0.$$

Cross multiply and integrate,

$$\therefore r^2 \frac{\partial V}{\partial r} = C_1$$

$$\therefore \frac{dV}{dr} = \frac{C_1}{r^2}.$$

$\therefore$ integrate again

$$\therefore V = -\frac{C_1}{r} + C_2.$$

$$V \text{ (at } r = 0.03) = 100 = -\frac{C_1}{0.03} + C_2.$$

$$V \text{ (at } r = 0.005) = -100 = -\frac{C_1}{0.05} + C_2.$$

Solve C_1 and C_2 .

$$\therefore C_1 = -15, C_2 = -400.$$

$$\boxed{V(r) = \left(\frac{15}{r} - 400 \right) \text{ Volts.}}$$

$$(1) V(r) = \left(\frac{15}{r} - 400 \right) \text{ V.}$$

$$(2) V \text{ (at } r = 0.04 \text{ m)}$$

$$V(0.04) = \left(\frac{15}{0.04} - 400 \right)$$

$$(3) \quad 0 = \frac{15}{r} - 400$$

$$\therefore V=0 \text{ at } r = \frac{15}{400} \text{ m.}$$

$$\therefore \boxed{r = 3.75 \text{ cm}}$$

$$(4) \quad \vec{E} = -\nabla \cdot V$$

$$= -\frac{\partial V}{\partial r} \hat{a}_r$$

$$\therefore \boxed{\vec{E} = +\frac{15}{r^2} \hat{a}_r \text{ V/m.}}$$

→ The obtained $\vec{E}$ is projecting along $\hat{a}_r$ direction and it is projecting from a higher potential surface to towards lower potential.

* Current: (I).

→ It is a rate of charge.

$$\rightarrow J = \rho_v \cdot \vec{v} = \sigma \vec{E}$$

$$\vec{J}_c = \rho_v \vec{v}_d \quad \text{A/m}^2$$

$$\vec{J}_c = \sigma \vec{E} \quad \text{A/m}^2$$

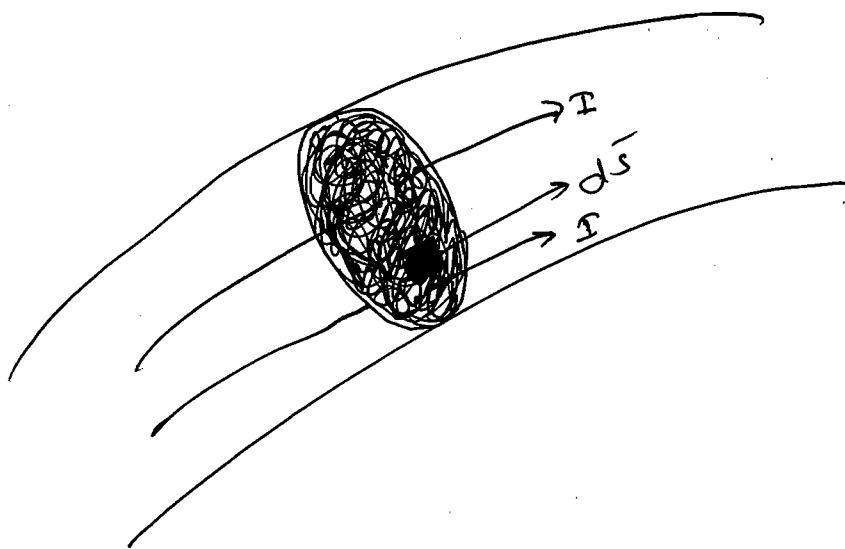
$\vec{J}_c$: Conduction current density (A/m²)

ρ_v : Volume charge density (C/m³).

σ : Conductivity (V/m).

$\vec{v}_d$: drift velocity (m/s).

$\vec{E}$ = Applied Electric field.



→ Across, S , we know the Conduction current density ($\vec{J}_c \text{ A/m}^2$).

→ The diff. amount of current dI passing length $d\vec{s}$ is given by

$$dI = \vec{J}_c \cdot d\vec{s}$$

$$\therefore I = \int_S \vec{J}_c \cdot d\vec{s}.$$

Ex-1 In certain region the conduction current density is given by $-10^5 \nabla V \text{ A/m}^2$ where $V = 10 e^{-x} \sin y$ volts. scalar electric potential function. find

- ① Conductivity of medium.
- ② Amount of current passing through $x=1, 0 \leq y \leq \pi$ in $\hat{u}_x$ direction.

Ans: ① $\vec{J}_c = \sigma \vec{E}$.

$$\vec{J}_c = \sigma (-\nabla V).$$

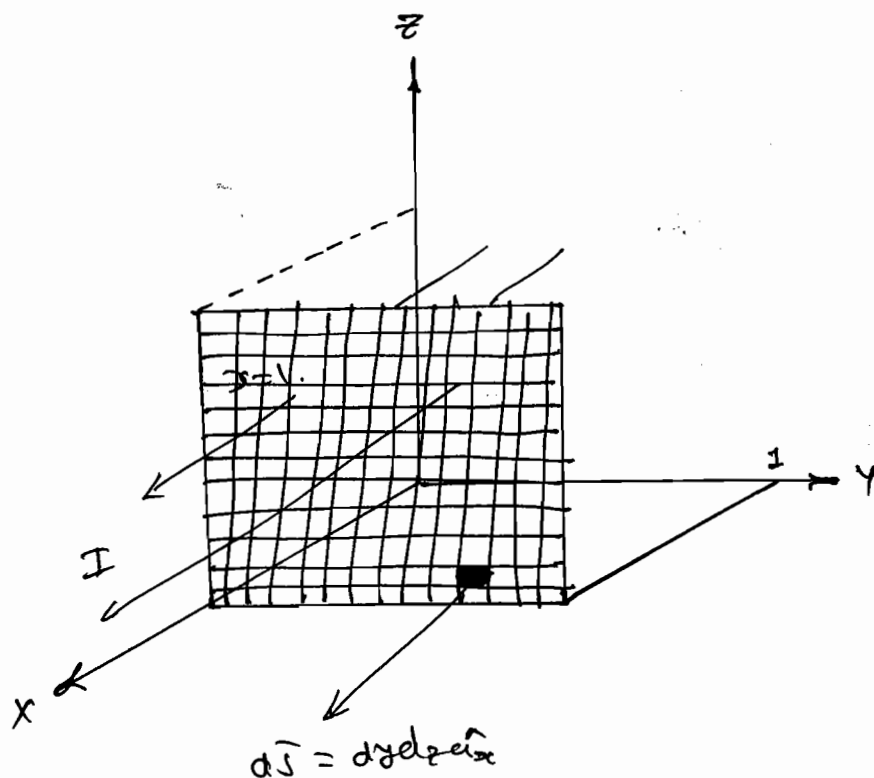
$$\therefore \vec{J}_c = 10^5 (-\nabla V).$$

$$\vec{J}_c \therefore \sigma = 10^5 \text{ u/m}.$$

② $\nabla V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y.$

$$\nabla V = 10^6 e^{-x} \sin y \hat{a}_x + 10^6 e^{-x} \cos y \hat{a}_y$$

$$\therefore \vec{J}_c = 10^6 e^{-x} \sin y \hat{a}_x - 10^6 e^{-x} \cos y \hat{a}_y.$$



$$\rightarrow \vec{J}_c \cdot d\vec{S} = 10^6 e^{-x} \sin y \, dy \, dz.$$

$$\vec{J}_c \cdot d\vec{S} = 10^6 e^{-1} \sin y \, dy \, dz.$$

At $x=1$

$$I = \int_S \vec{J}_c \cdot d\vec{S} = 10^6 e^{-1} \int_0^1 \int_0^1 \sin y \, dy \, dz$$

$$= 10^6 e^{-1} [1 - \cos 13] A.$$

↑ keep the calci in Rad.

$$\therefore \boxed{I = 169 \text{ KA}}$$

* Continuity of a Current Equation:

$$\rightarrow \oint_S \vec{J}_c \cdot d\vec{S} = - \frac{dq}{dt} = - \frac{d}{dt} \int_V \rho_v dV.$$

$$(or) \oint_S \vec{J}_c \cdot d\vec{S} = - \frac{\partial}{\partial t} \int_V \rho_v dV. \quad - (1)$$

Using divergence theorem.

$$\oint_S \vec{J}_c \cdot d\vec{S} = \int_V \nabla \cdot \vec{J}_c dV. \quad - (2)$$

$$\therefore \therefore \boxed{\nabla \cdot \vec{J} = - \frac{\partial \rho_v}{\partial t}}$$

Point form.

$$\begin{aligned} \rightarrow \vec{J} &= \sigma \cdot \vec{E} \\ \nabla \cdot \vec{J} &= \sigma \cdot \nabla \cdot \vec{E} \\ \nabla \cdot \vec{J} &= \sigma \cdot \nabla \cdot \left(\frac{\vec{D}}{\epsilon} \right) \\ \therefore \nabla \cdot \vec{J} &= \frac{\sigma}{\epsilon} \times \nabla \cdot \vec{D} \\ \therefore \nabla \cdot \vec{J} &= \frac{\sigma}{\epsilon} \rho_v \\ \therefore \frac{\sigma}{\epsilon} \rho_v &= - \frac{d\rho_v}{dt} \end{aligned}$$

$$\therefore \frac{d\rho_e}{dt} + \frac{\sigma}{\epsilon} \rho_e = 0$$

$$\therefore m + \frac{\sigma}{\epsilon} = 0$$

$$\therefore m = -\frac{\sigma}{\epsilon}$$

$$\therefore \rho_e = C_1 e^{-\frac{\sigma}{\epsilon} t}$$

$$\therefore \boxed{\rho_e = C_1 e^{-t/\tau}}$$

$$\text{Where, } \tau = \epsilon/\sigma$$

τ = Relaxation time.

→ We conclude that as the time progresses the charge density inside a conductor decays exponentially. The rate at which it decays exponentially depends upon conductivity of the conductor. If a conductor having infinite conductivity the density inside a conductor tends to zero within no time. In other words, for a transient time there may be some non zero charge inside a conductor.

→ Further we can conclude that if any charge is present in any conductor it resides on the surface of the conductor only.

Ex-1 Find the relaxation time for a copper conductor whose conductivity is 56 MS/m .
 assume $\epsilon = \epsilon_0$. also find % of charge density after 1 relaxation time and after 5 relaxation time.

Ans:

→ $\tau = \frac{\epsilon_0}{\sigma}$

$\therefore \tau = \frac{8.85 \times 10^{-12}}{56 \times 10^6}$

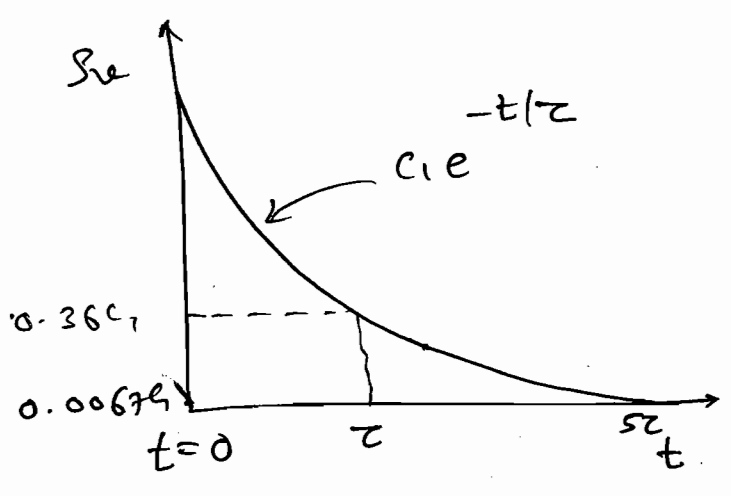
$\therefore \tau = 1.6 \times 10^{-19} \text{ s}$

→ $\rho_e \text{ (at } t = \tau) = C_1 e^{-\tau/\tau} = C_1/e = 0.36 C_1$

$\rho_e \text{ (at } t = \tau) = 36.1\% \text{ of } C_1$

$\rho_e \text{ (at } t = 5\tau) = C_1 e^{-5\tau/\tau} = C_1/e^5 = 0.0067 C_1$

$\therefore \rho_e \text{ (at } t = 5\tau) = 0.67\% \text{ of } C_1$

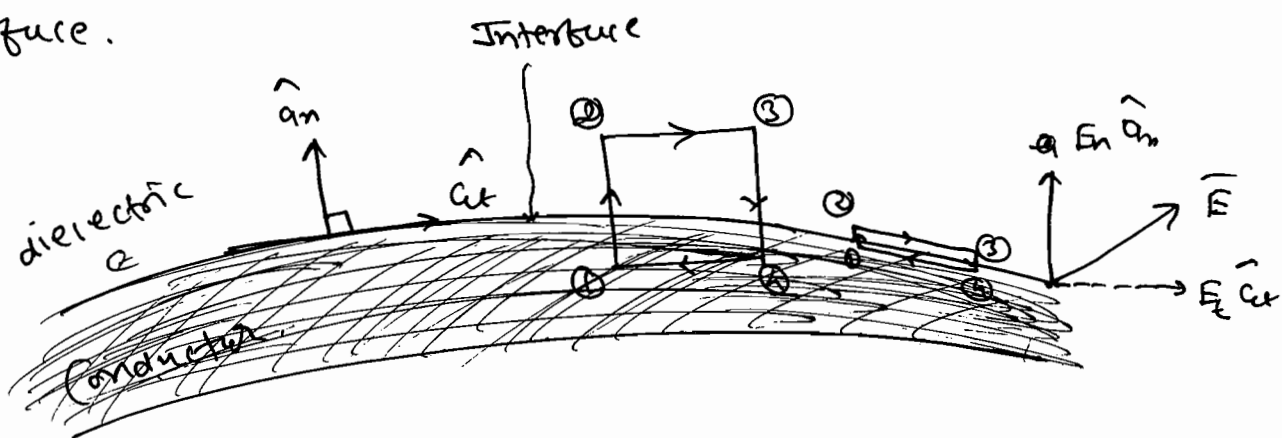


* Boundary Conditions:

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* Case-1: Conductor interface. (or) Conductor to dielectric interface.

→ An interface is a plane like structure where two mediums are interacting.
In this case we assume conductor interface (or) Conductor to dielectric interface. we have to investigate behaviour of electric field and electric flux density across the interface.



→ $\hat{n}$: Normal unit vector directed from conductor to dielectric.

→ $\hat{t}$: Unit vector tangential to the interface.

→ $\oint \vec{E} \cdot d\vec{l} = 0.$

$\Rightarrow \int_{1-2} \vec{E} \cdot d\vec{l} + \int_{2-3} \vec{E} \cdot d\vec{l} + \int_{3-4} \vec{E} \cdot d\vec{l} + \int_{4-1} \vec{E} \cdot d\vec{l} = 0.$

→ The $\int_{4-1}$ has to be computed inside a conductor.

→ The charge is zero inside a conductor.

Therefore, Electric field is zero inside a conductor and hence this integral vanishes.

→ We are interested to investigate behaviour of electric field across interface. To accomplish this we choose path line 1-2 and 3-4 so small such that the path 2-3 is grazing the interface, which would result the $\int_{1-2}$ and $\int_{3-4}$ vanishing.

$$\rightarrow \int_{2-3} \vec{E} \cdot d\vec{l} = 0.$$

→ For path (2-3) $\Rightarrow d\vec{l} = dl \hat{a}_t$

$$\text{Let } \vec{E} = E_n \hat{a}_n + E_t \hat{a}_t$$

This is assumed across the interface.

$$\therefore \vec{E} \cdot d\vec{l} = E_n \hat{a}_n \cdot dl \hat{a}_t + E_t \hat{a}_t \cdot dl \hat{a}_t$$

$$\therefore \vec{E} \cdot d\vec{l} = E_t dl$$

$$\therefore \Rightarrow \int E_t \cdot dl = 0.$$

$\therefore$ 'it' can't be zero.

$$\therefore \boxed{E_t = 0}$$

→ Tangential components of electric field across a conductor to dielectric interface vanishing.

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 $\Rightarrow$ Across a Conductor interface identify the correct one from the following: where $\vec{E}$ is the electric field across the interface.
 $\hat{a}_t$: unit vector tangential to interface.
 $\hat{a}_n$: the unit vector normal to interface.

(i) $\vec{E} \cdot \hat{a}_t = 0$

(v) $\vec{D} \cdot \hat{a}_n = \rho_s$

(ii) $\vec{E} \times \hat{a}_n = 0$

(vi) $\vec{D} \cdot \hat{a}_t = 0$

(iii) $\hat{a}_n \times \vec{E} = 0$

(vii) $\hat{a}_n \cdot \vec{D} = \rho_s$

(iv) All the above.

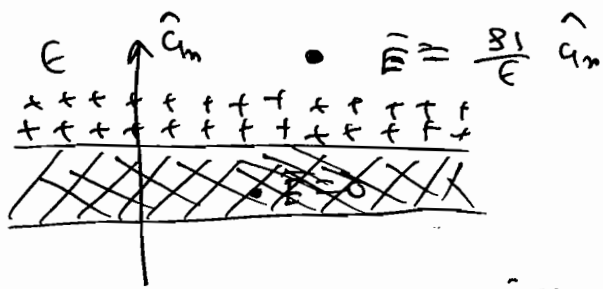
$\Rightarrow$ We assume that the Interface has a non-zero surface charge density of $\rho_s \text{ C/m}^2$. By using Gauss Law we can show that normal components of Electric flux densities are equal to surface charge density. By expression

$$D_n = \rho_s \quad (\text{or})$$

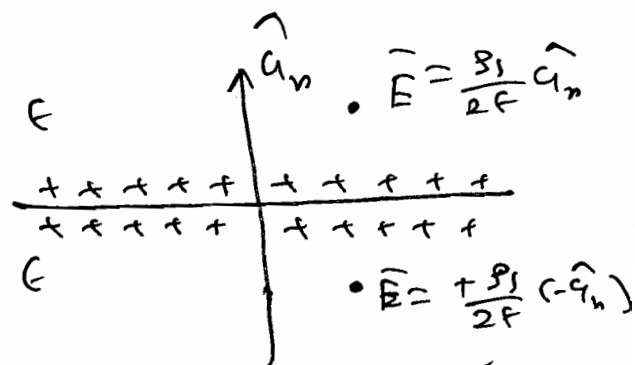
$$\vec{D} = \rho_s \hat{a}_n \quad \text{and}$$

$$\therefore \vec{E} = \frac{\rho_s}{\epsilon} \hat{a}_n$$

~~Imp~~
 *



$\rightarrow$ The entire charge lies on the top of the conductor surface.



$\rightarrow$ The charge is distributed on both sides of conductor sheet.

Ex-1 A charge density of 1 nC/m^2 is placed on a conductor surface. Assume interface is free space. Find the magnitude of the electric field.

Ans:
$$\vec{E} = \frac{S_s}{\epsilon} \hat{a}_n = \frac{1 \times 10^{-9}}{36\pi \times 10^9} \hat{a}_n$$

$$\therefore |\vec{E}| = 36\pi \text{ V/m.}$$

Ex-2 A positive charge is distributed on a conductor surface. Assume the interface is free space. Given that $\vec{D}$ at across interface is equal to $\vec{D} = 2(\hat{a}_x + \sqrt{3}\hat{a}_y)$ nC/m^2 . find the value of charge density across the interface.

Ans:
$$\vec{E} = \frac{S_s}{\epsilon} \hat{a}_n$$

$$\therefore \vec{D} = S_s \hat{a}_n$$

$$\therefore |\vec{D}| = S_s$$

$$\hat{a}_n = \frac{2\hat{a}_x + 2\sqrt{3}\hat{a}_y}{\sqrt{4+12}}$$

$$\hat{a}_n = \frac{\hat{a}_x}{2} + \frac{\sqrt{3}}{2}\hat{a}_y$$

$$\therefore |\vec{D}| = S_s$$

$$|\vec{D}| = \sqrt{2^2 + 2^2(3)} = 4$$

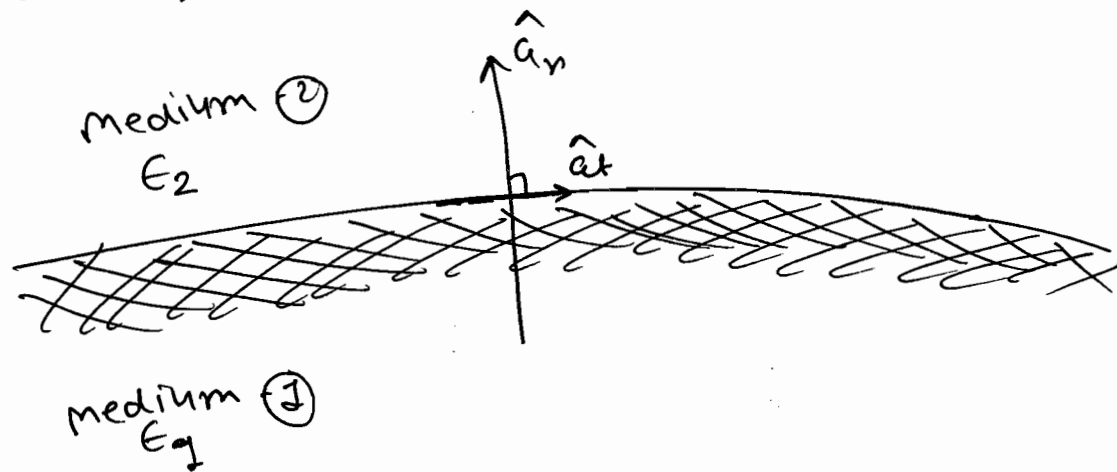
$$\therefore \vec{D} = 4 \left\{ \frac{2\hat{a}_x + 2\sqrt{3}\hat{a}_y}{4} \right\}$$

$$\vec{D} = S_s \hat{a}_n$$

$$\therefore S_s = 4 \text{ nC/m}^2$$

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Case-2: Dielectric to Dielectric interface:



- $\hat{a}_n$: Normal unit vector directed in from (1) to (2)
- $\hat{a}_t$: Tangential unit vector tangential to the interface.

We can show that

$$(1) \quad E_{t1} = E_{t2}$$

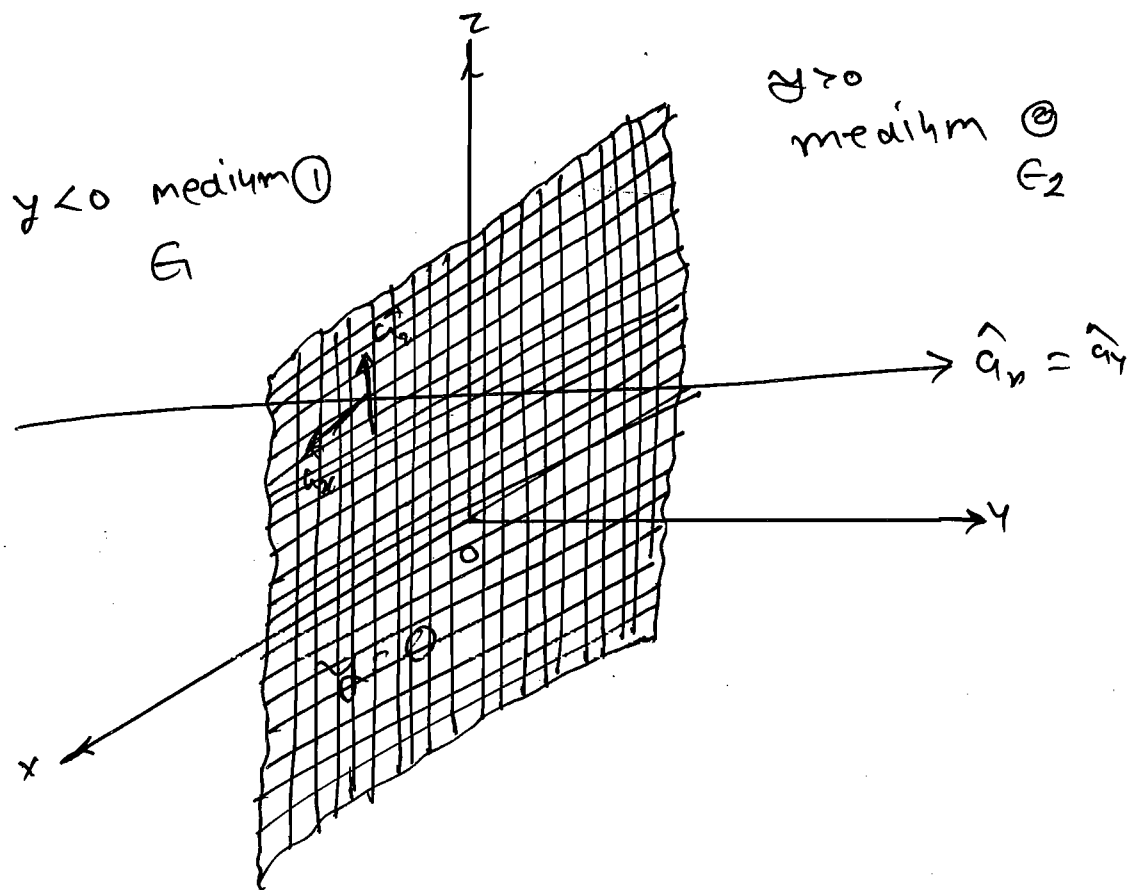
Tangential components of E-fields are continuous across a dielectric to dielectric interface.

$$(2) \quad (a) \quad D_{n2} - D_{n1} = \rho_s$$

→ The normal components of electric flux densities are discontinuous by an amount of surface charge density.

(b) if $\rho_s = 0$ (charge free interface).

→ The normal components of electric flux densities are continuous across a charge free interface.



→ figure shows that interface is defined by $y=0$. Medium -1 is defined for $y < 0$ and is characterised by ϵ_1 .

→ Medium -2 is defined for $y > 0$ and is characterised by ϵ_2 . To this interface the normal unit vector is $\hat{n} = \hat{y}$ and the unit vectors tangential to interface are $\hat{a}_x$ and $\hat{a}_z$.

Ex-1
→ With reference to the figure shown above let, $\epsilon_1 = 2\epsilon_0$, $\epsilon_2 = 3\epsilon_0$ and given that

$$\vec{E}_1 = (4\hat{a}_x + 5\hat{a}_y + 6\hat{a}_z) \text{ V/m.}$$

Find $\bar{D}_1$, $\bar{D}_2$ and $\bar{E}_2$. and assume that the interface is charge free: (i.e) $\rho_s = 0$ across, $y = 0$.

Ans: $\bar{E}_1 = \underbrace{4\hat{a}_x + 6\hat{a}_y}_{E_{t1}} + \underbrace{5\hat{a}_y}_{E_{n1}}$

$$\therefore \bar{E}_1 = E_{t1}\hat{a}_t + E_{n1}\hat{a}_n$$

→ (i.e) Any field vector across the interface can be represented as a vectorial sum of tangential and normal components.

$$\bar{E}_1 = 4\hat{a}_x + 6\hat{a}_y + 5\hat{a}_y$$

$$\bar{D}_1 = \bar{E}_1 \epsilon_1$$

$$\therefore \bar{D}_1 = 4\epsilon_1\hat{a}_x + 6\epsilon_1\hat{a}_y + 5\epsilon_1\hat{a}_y$$

$$E_{t1} = E_{t2}$$

$$\therefore \bar{E}_2 = 4\hat{a}_x + 6\hat{a}_y + E_{y2}\hat{a}_y$$

$$\therefore D_{n1} = D_{n2} \quad (\because \rho_s = 0)$$

$$\therefore \bar{D}_2 = D_{x2}\hat{a}_x + D_{z2}\hat{a}_z + 5\epsilon_2\hat{a}_y$$

$$\therefore \bar{D}_2 = \epsilon_2 \bar{E}_2$$

$$\therefore D_{x2}\hat{a}_x + D_{z2}\hat{a}_z + 5\epsilon_2\hat{a}_y = 4\epsilon_2\hat{a}_x + 6\epsilon_2\hat{a}_y + E_{y2}\epsilon_2\hat{a}_y$$

$$\therefore \left. \begin{array}{l} D_{x2} = 4\epsilon_2 \text{ C/m}^2 \\ D_{z2} = 6\epsilon_2 \text{ C/m}^2 \end{array} \right| E_{y2} = \frac{5\epsilon_2}{\epsilon_2} \text{ V/m}$$

Ex-2 Repeat the above problem if the interface has a non-zero surface charge density of $S_s \text{ C/m}^2$.

Ans:

$$\text{let, } \vec{D}_2 = D_{x2} \hat{a}_x + D_{z2} \hat{a}_z + D_{y2} \hat{a}_y$$

$$\therefore D_{n2} - D_{n1} = S_s$$

$$D_{y2} - D_{y1} = S_s$$

$$\therefore D_{y2} = S_s + 5\epsilon_1$$

$$\therefore \vec{D}_2 = D_{x2} \hat{a}_x + D_{z2} \hat{a}_z + (S_s + 5\epsilon_1) \hat{a}_y$$

$$\therefore \vec{D}_2 = \epsilon_2 \vec{E}_2$$

$$\therefore D_{x2} = 4\epsilon_2 \text{ C/m}^2$$

$$D_{z2} = 6\epsilon_2 \text{ C/m}^2$$

$$\therefore E_{y2} = \frac{S_s + 5\epsilon_1}{\epsilon_2} \text{ V/m.}$$

Ex-3 Repeat above 2 example by assuming the interface as $z=0$. $z < 0$ is medium 1 and is characterized by ϵ_1 whereas $z > 0$ is medium 2 and is characterized by ϵ_2 .

Ans:

$$\vec{E} = \underbrace{4\hat{a}_x + 5\hat{a}_y}_{\vec{E}_{t1}} + \underbrace{6\hat{a}_z}_{\vec{E}_{n1}}$$

$$\therefore \vec{E} = \vec{E}_{n1} + \vec{E}_{t1}$$

$$\therefore \vec{E}_{t1} = 4\hat{a}_x + 5\hat{a}_y$$

$$\vec{E}_{n1} = 6\hat{a}_z$$

is directed in $\hat{a}_z$ direction. unit vector $\hat{a}_z$ is
 $\hat{a}_z$. & unit vector tangential to interface
 are $\hat{a}_x$ and $\hat{a}_y$.

$$\vec{E}_{t2} = \vec{E}_{t1}$$

$$\therefore \vec{E}_{t2} = 4\hat{a}_x + 5\hat{a}_y.$$

$$\therefore \vec{E}_2 = \vec{E}_{t2} + \vec{E}_{n2}$$

$$\therefore \vec{D}_{n2} = \vec{D}_{n1}$$

$$\therefore \epsilon_2 \vec{E}_{n2} = \epsilon_1 \vec{E}_{n1}$$

$$\therefore \vec{E}_{n2} = \frac{\epsilon_1}{\epsilon_2} \vec{E}_{n1}$$

$$\therefore \vec{E}_{n2} = \frac{2}{3} \times 6\hat{a}_z = 4\hat{a}_z$$

$$\therefore \vec{E}_2 = 4\hat{a}_x + 5\hat{a}_y + 4\hat{a}_z$$

$$\therefore \vec{D}_1 = \epsilon_1 \vec{E}_1$$

$$\vec{D}_1 = \epsilon_0 (8\hat{a}_x + 10\hat{a}_y + 12\hat{a}_z)$$

$$\therefore \vec{D}_2 = \epsilon_2 \vec{E}_2$$

$$\vec{D}_2 = \epsilon_0 (12\hat{a}_x + 15\hat{a}_y + 12\hat{a}_z)$$

Now, there are normal components of $\vec{D}$ that are discontinuous by surface charge density ρ_s C/m².

$$\therefore D_{n2} - D_{n1} = \rho_s \text{ C/m}^2$$

$$\therefore \epsilon_2 E_{n2} - \epsilon_1 E_{n1} = \rho_s$$

$$\epsilon_2 E_{n2} = \epsilon_1 E_{n1} + \beta_1$$

$$\therefore E_{n2} = \frac{\epsilon_1 E_{n1} + \beta_1}{\epsilon_2}$$

$$\therefore E_{n2} = \frac{6\epsilon_1 + \beta_1}{\epsilon_2}$$

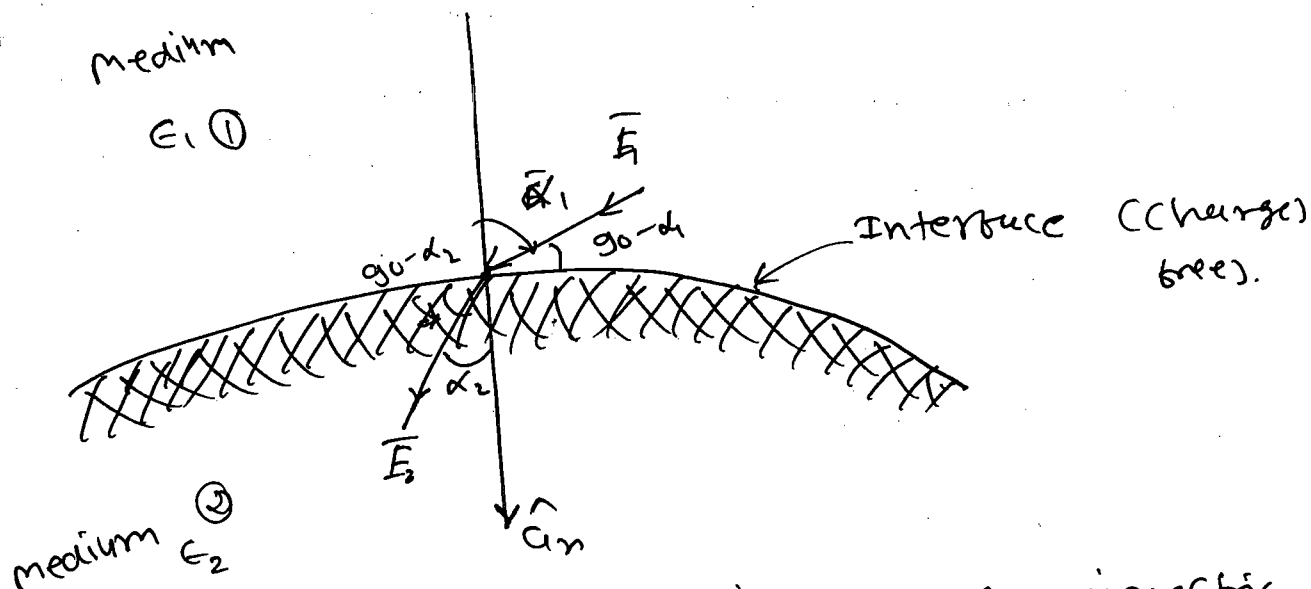
$$\therefore \bar{E}_2 = 4\hat{a}_x + 5\hat{a}_y + \left(\frac{6\epsilon_1 + \beta_1}{\epsilon_2}\right)\hat{a}_z$$

$$\therefore \bar{D}_2 = 4\epsilon_2\hat{a}_x + 5\epsilon_2\hat{a}_y + (6\epsilon_1 + \beta_1)\hat{a}_z$$

$$\bar{D}_1 = \epsilon_1 \bar{E}_1$$

$$\therefore \bar{D}_1 = \epsilon_0 (8\hat{a}_x + 10\hat{a}_y + 12\hat{a}_z)$$

Ex-3



→ Fig. Shows charge free dielectric to dielectric interface. Further it is shown normal unit vector directed in from medium ① to medium ② of P. Relate an expression $\alpha_1, \alpha_2, \epsilon_1, \epsilon_2$.

Ans::

$$|\vec{E}_{t1}| = |\vec{E}_{t2}|$$

$$\therefore E_2 \cos(90 - \alpha_2) = E_1 \cos(90 - \alpha_1)$$

$$\therefore E_2 \sin \alpha_2 = E_1 \sin \alpha_1$$

$$\therefore \cancel{E_1 \sin \alpha_1} = \cancel{E_2 \sin \alpha_2} \cdot \epsilon_2$$

$$\therefore \cancel{E_2 \sin \alpha_2} = \cancel{E_1 \sin \alpha_1} \cdot \epsilon_1$$

$$\therefore \boxed{\epsilon_1 \tan \alpha_1 = \epsilon_2 \tan \alpha_2}$$

$$D_{n1} - D_{n2} = \rho_s$$

$$\text{But } \rho_s = 0$$

$$D_{n1} = D_{n2}$$

$$\therefore \epsilon_1 |E_1| \cos \alpha_1 = \epsilon_2 |E_2| \cos \alpha_2$$

$$\therefore \frac{\tan \alpha_1}{\epsilon_1} = \frac{\tan \alpha_2}{\epsilon_2}$$

$$\therefore \boxed{\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\epsilon_1}{\epsilon_2}}$$

Ex-1 An interface is defined by $2x + 3y + 4z = 12$.
 origin side of the interface is medium-①
 and is characterised by $\epsilon_1 = 2\epsilon_0$. Other side
 of the interface is medium-② and is
 characterised by free space. Let, the electric
 field in the medium-② is given by
 $\vec{E}_1 = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z$ V/m. Assume charge
 free interface. Find $\vec{D}_1$, $\vec{E}_2$, $\vec{D}_2$.

Ans:

$$2x + 3y + 4z = 12$$

$$\therefore \frac{x}{6} + \frac{y}{4} + \frac{z}{3} = 1$$

$$\vec{r}_{AB} \times \vec{r}_{BC}$$

$$\vec{r}_{AB} = -6\hat{a}_x + 3\hat{a}_z$$

$$\vec{r}_{BC} = 4\hat{a}_y - 3\hat{a}_z$$

$$\vec{r}_{AB} \times \vec{r}_{BC}$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -6 & 0 & 3 \\ 0 & 4 & -3 \end{vmatrix}$$

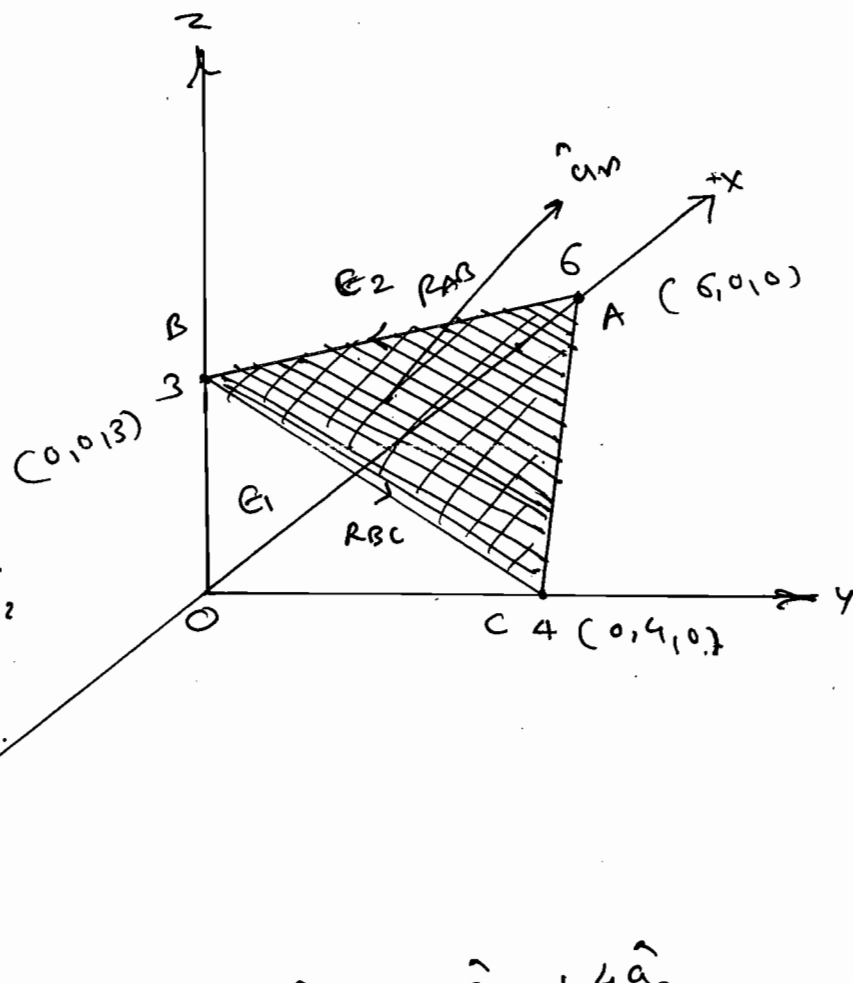
$$= -12\hat{a}_x - 18\hat{a}_y - 24\hat{a}_z$$

$$\hat{a}_n = -3[4\hat{a}_x + 6\hat{a}_y + 8\hat{a}_z]$$

$$\vec{r}_n = -6(2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z)$$

$$\hat{a}_n = \frac{-6(2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z)}{36.31} \quad \hat{a}_n = \frac{2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z}{\sqrt{2^2 + 3^2 + 4^2}}$$

$$\hat{a}_n = -\frac{(2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z)}{5.385}$$



$$\rightarrow \bar{E}_1 = E_{n1} \hat{a}_n + E_{t2} \hat{a}_t.$$

$$\bar{E}_2 = E_{n2} \hat{a}_n + E_{t2} \hat{a}_t.$$

$$\rightarrow \bar{E}_1 \cdot \hat{a}_n = E_{n1} \underbrace{\hat{a}_n \cdot \hat{a}_n}_1 + E_{t2} \underbrace{\hat{a}_t \cdot \hat{a}_n}_0.$$

$$\therefore \boxed{E_{n1} = \bar{E}_1 \cdot \hat{a}_n} \rightarrow (1)$$

$$\therefore \boxed{\bar{E}_{n1} = E_{n1} \hat{a}_n} \rightarrow (2)$$

$$\therefore \boxed{\bar{E}_{t2} = \bar{E} - \bar{E}_{n1}} \rightarrow (3)$$

$\therefore$ Now, tangential components ^{of $\bar{E}$} are continuous.

$$\text{So, } \bar{E}_{t1} = \bar{E}_{t2}$$

$$\therefore \boxed{\bar{E}_{t2} = \bar{E}_{t1}} \rightarrow (4)$$

$$\therefore \text{Now, } D_{n1} = \epsilon_1 E_{n1}.$$

$$\therefore \bar{E}_{n2} = \frac{\bar{D}_{n2}}{\epsilon_2} =$$

normal components at $\bar{O}$ are continuous
($\because$ charge free
so $\rho_s = 0$).

$$\therefore \boxed{\bar{D}_{n1} = \bar{D}_{n2}} \rightarrow (5)$$

$$\therefore \bar{D}_{n1} = \epsilon_1 \bar{E}_{n1}$$

$$\therefore \boxed{\bar{D}_{n2} = \epsilon_1 \bar{E}_{n1}} \rightarrow (6)$$

$$\bar{E}_{n2} = \frac{\bar{D}_{n2}}{\epsilon_2} = \frac{\epsilon_1}{\epsilon_2} \bar{E}_{n1}$$

$$\therefore \boxed{\bar{E}_{n2} = \frac{\epsilon_1}{\epsilon_2} \bar{E}_{n1}} \rightarrow (7)$$

$$\text{So, } \boxed{\bar{E}_2 = \bar{E}_{t2} + \bar{E}_{n2}} \rightarrow (8)$$

$$\therefore \bar{D}_1 = \bar{D}_{n1} + \bar{D}_{t1}$$

$$\boxed{\bar{D}_1 = \epsilon_1 \bar{E}_1} \rightarrow (10)$$

$$\boxed{\bar{D}_2 = \epsilon_2 \bar{E}_2} \rightarrow (9)$$

$$\rightarrow \bar{E}_1 = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z$$

$$\therefore \hat{a}_n = \frac{5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z}{10.49}$$

$$\hat{a}_n \cdot \bar{E}_1 = E_{n1} \cdot \hat{a}_n$$

$$= E_{n1} = \bar{E}_1 \cdot \hat{a}_n$$

$$\therefore E_{n1} = \frac{25 + 36 + 49}{10.49}$$

$$\boxed{E_{n1} = 10.49}$$

$$\therefore \bar{E}_{n1} = E_{n1} \hat{a}_n$$

$$\therefore \bar{E}_{n1} = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z$$

$$\therefore \bar{E}_{t1} = 0$$

$$\therefore \bar{E}_{t2} = 0$$

$$\therefore \bar{D}_{n1} = \bar{D}_{t2}$$

$$\therefore \epsilon_2 \bar{E}_{n2} = \epsilon_1 \bar{E}_{n1}$$

$$\therefore \bar{E}_{n2} = \frac{\epsilon_1}{\epsilon_2} (5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z)$$

$$\therefore \bar{E}_{n2} = \frac{10}{3} \hat{a}_x + 4\hat{a}_y + \frac{14}{3} \hat{a}_z$$

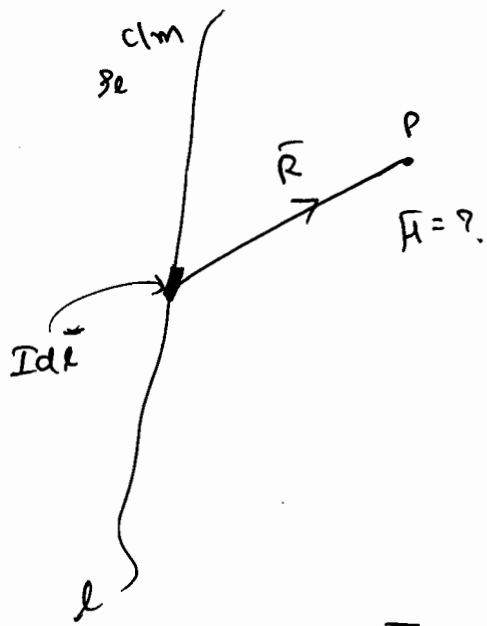
$$\therefore \bar{E}_2 = \bar{E}_{n2} + \bar{E}_{t2}$$

$$\therefore \bar{E}_2 = \frac{10}{3}$$

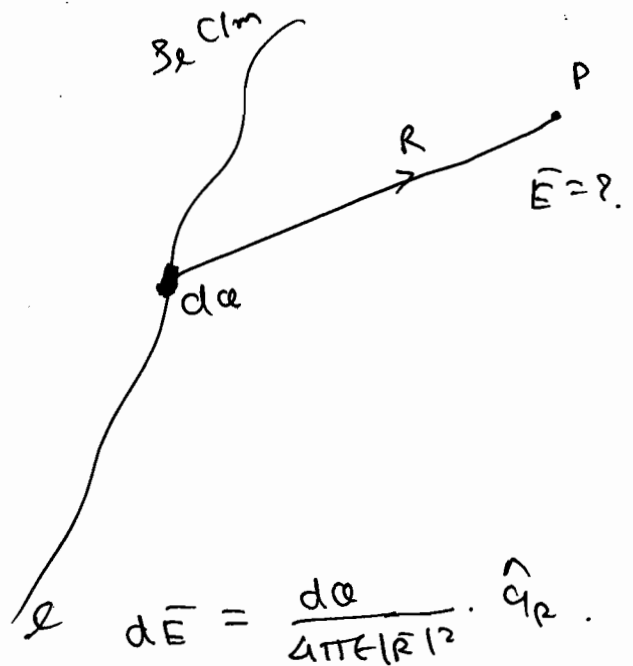
★ Magnetic Fields (Steady)

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Fields R Independent of time.



$$d\vec{H} = \frac{I d\vec{l} \times \vec{r}}{4\pi |\vec{r}|^2}$$



$$d\vec{E} = \frac{da}{4\pi\epsilon |\vec{r}|^2} \cdot \hat{r}$$

Current Element = $I d\vec{l}$

- Current multiplied vector dist. length
- This is vector quantity.
- Source of magnetic field.

→ There exists a similarity betⁿ electric and magnetic fields.

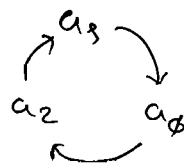
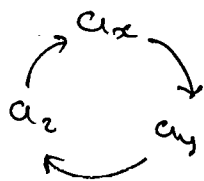
→ Both fields are proportional to the corresponding sources.

→ Both fields are inversely proportional to square of distance from their corresponding sources.

→ Both fields are vector fields.

⇒ Bio Savart's Law:

$$\vec{H} = \int \frac{I d\vec{l} \times \hat{r}}{4\pi |\vec{r}|^2} \quad \text{Alm.}$$



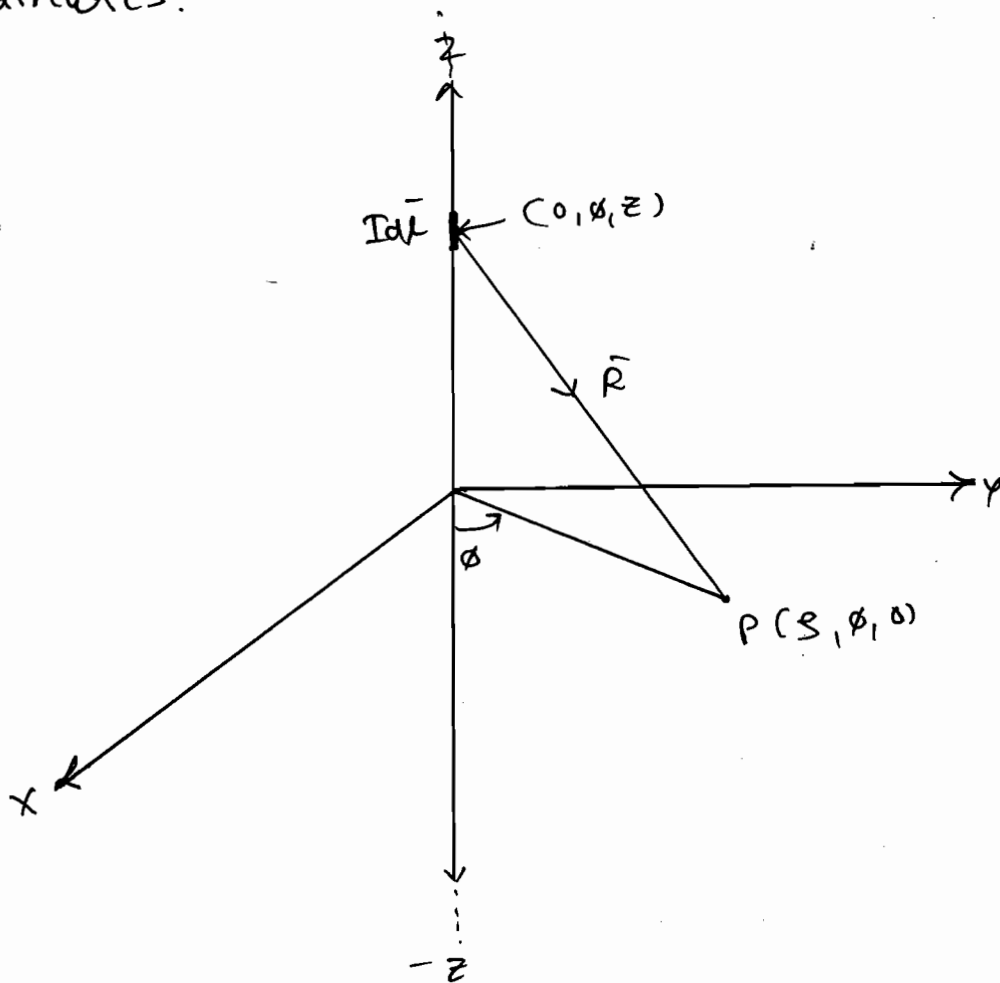
Ex-1 Find an expression for the magnetic field intensity due to a long straight infinite filamentary conductor which carries a direct current of I A. Show that the magnitude of the $\vec{H}$ magnetic field intensity is inversely proportional to the distance between infinite current filament and the observation point.

Ans: We assume that the infinite current filament lies along z axis and is extending from $-\infty$ to $+\infty$. We find the magnetic field intensity at some

point on the $x-y$ plane.

→ Say at s point $P(s, \phi, 0)$.

for the convenience we circular cylindrical
Co-ordinates.



$$\rightarrow d\vec{l} = dz \hat{a}_z$$

$$\therefore Id\vec{l} = Idz \hat{a}_z$$

$$\rightarrow \vec{R} = s \hat{a}_s - z \hat{a}_z \quad \hat{a}_R = \frac{s \hat{a}_s - z \hat{a}_z}{\sqrt{s^2 + z^2}}$$

$$\therefore Id\vec{l} \times \hat{a}_R = \frac{Isdz}{\sqrt{s^2 + z^2}} \cdot \hat{a}_\phi$$

$$\text{So, } d\vec{H} = \frac{Id\vec{l} \times \hat{a}_R}{4\pi |\vec{R}|^2}$$

$$d\vec{H} = \frac{I dz \hat{a}_\phi}{4\pi (s^2 + z^2)^{3/2}} \cdot \hat{a}_\phi$$

$$\therefore \vec{H} = \frac{I}{4\pi} \int_{-\infty}^{\infty} \frac{s}{(s^2 + z^2)^{3/2}} dz \cdot \hat{a}_\phi$$

Now, let $z = s \tan \theta$

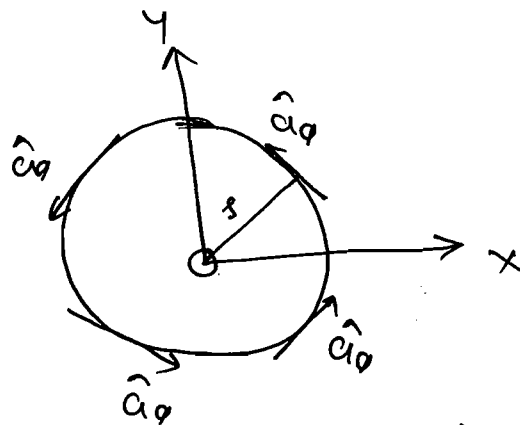
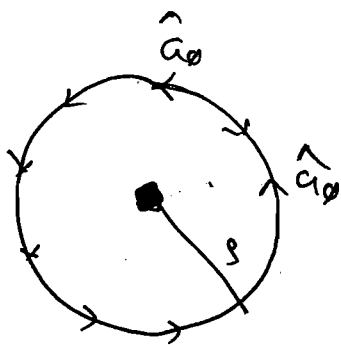
$$\therefore dz = s \sec^2 \theta$$

$$z \in (-\infty, \infty) \Rightarrow (-\pi/2, \pi/2)$$

$$\vec{H} = \frac{I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{s^2 \sec^2 \theta}{s^3 \sec^3 \theta} \cdot d\theta \cdot \hat{a}_\phi$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi s} \cdot \hat{a}_\phi}$$

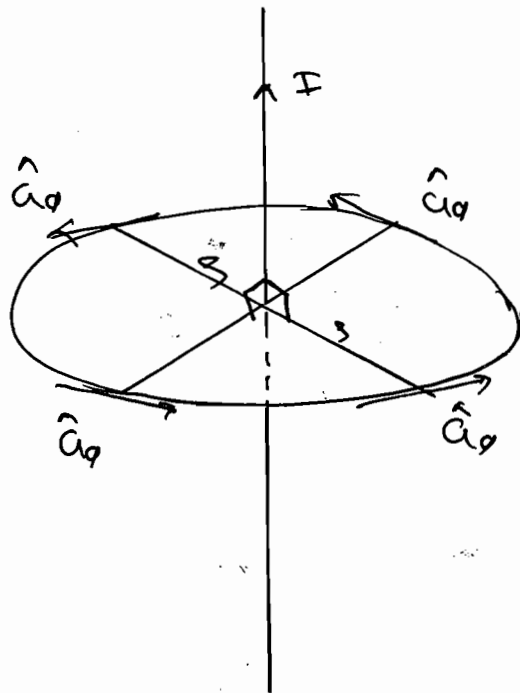
$$\therefore \boxed{|\vec{H}| \propto \frac{1}{s}}$$



→ Magnitude of magnetic field intensity is inversely proportional to the distance betⁿ the infinite current filament and the observation point. The direction of the Magnetic field is around the conductor.

it encircles the conductor.

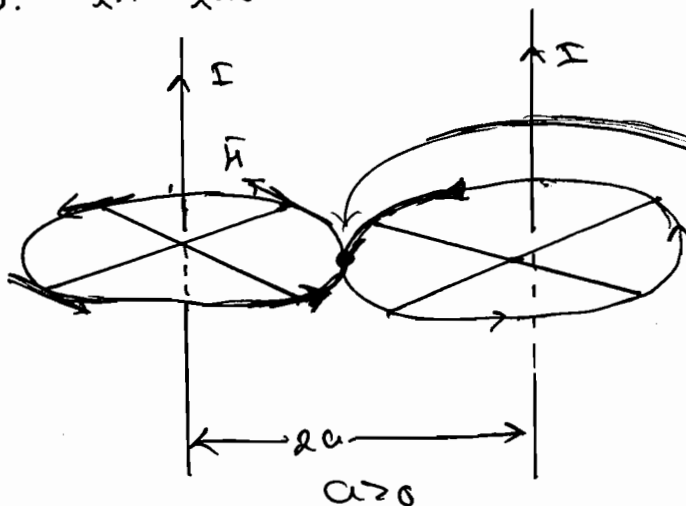
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⇒ Two infinite current filaments are

parallel: Case-1: currents are in same direction:

→ They are separated by $2a$ m. ($a > 0$). They carry equal current of I amp. In same direction. find the magnitude of the ~~magnetic field intensity~~ magnetic field intensity at the middle point betⁿ this two infinite current filaments. Assume that this conductor's carry equal currents of I amps. in the same direction.

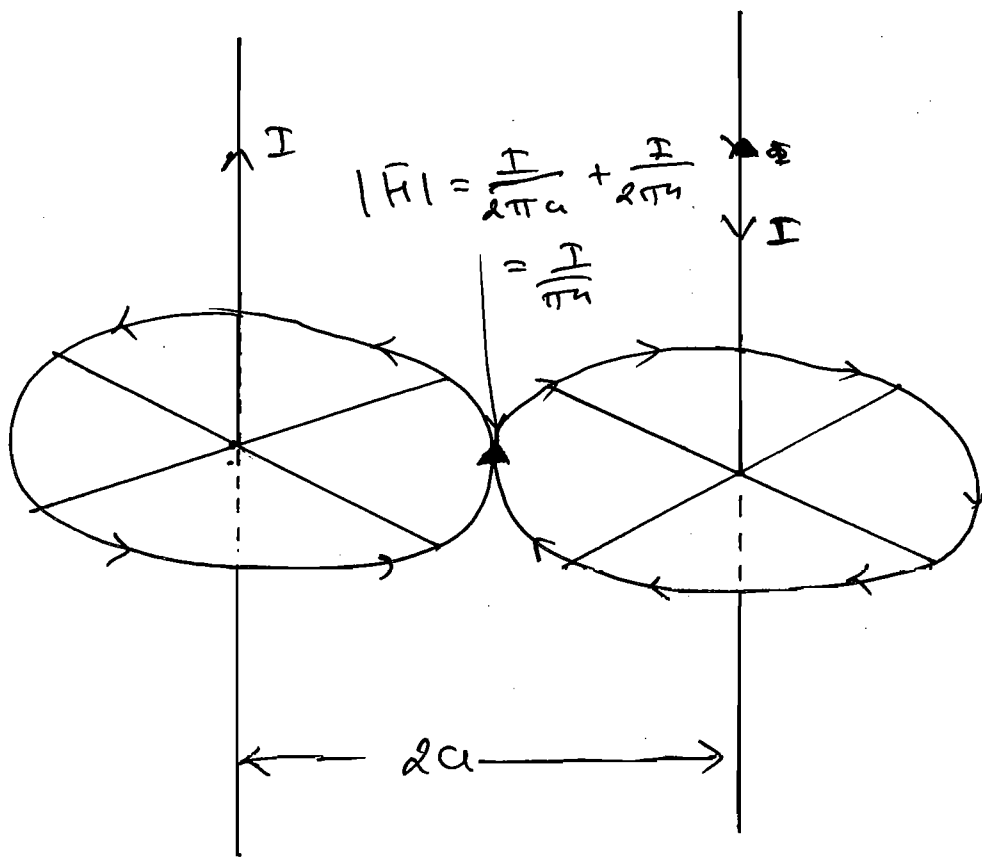


$$|H| = 0$$

The fields add in out of phase

$$\therefore |H| = 0.$$

Case-2: Currents are in opposite direction.

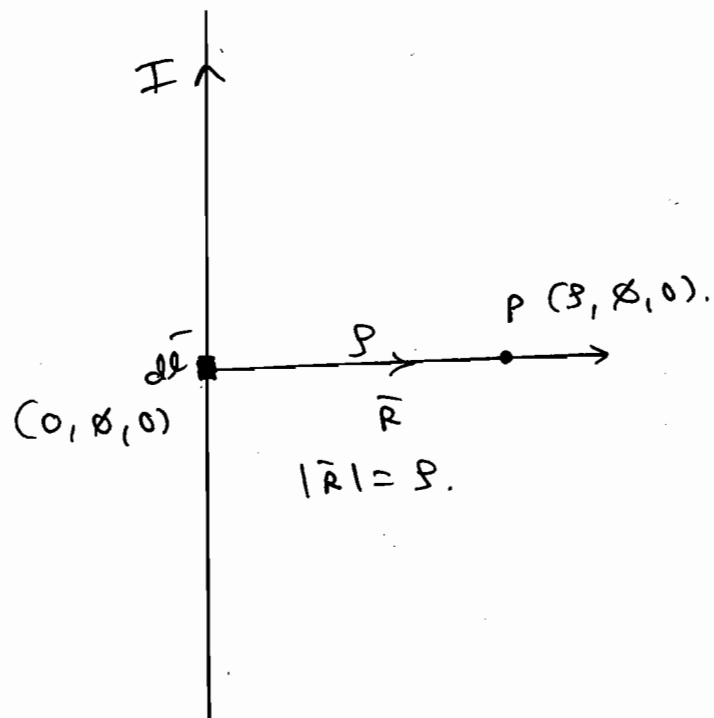


$$\rightarrow |\vec{H}| = \frac{I}{2\pi a} + \frac{I}{2\pi a} = \frac{I}{\pi a}$$

The fields are added in phase.

* General Expression for the Magnetic Field Intensity due to an infinite current filament.

⇒



$$\rightarrow dl = dz \hat{a}_z$$

$$\therefore \vec{R} = s \hat{a}_s$$

$$\hat{a}_R = \hat{a}_s$$

$$\therefore dl \times \hat{a}_R = \hat{a}_\phi dz$$

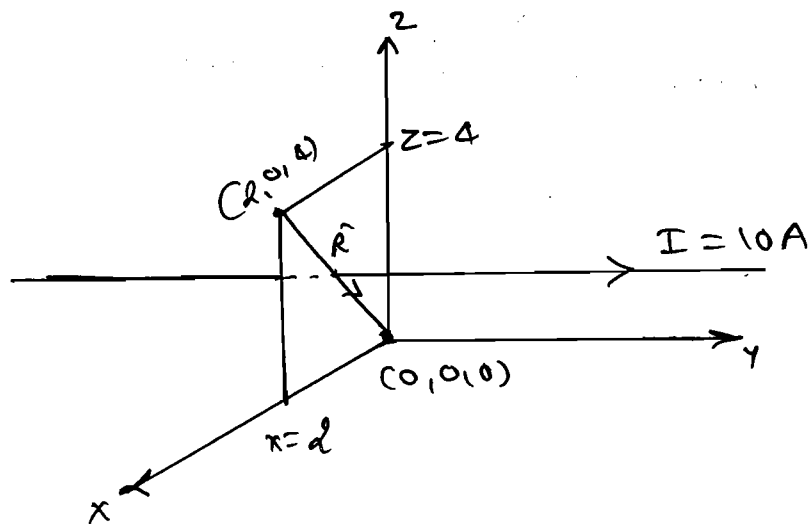
The unit vector of $dl \times \hat{a}_R = \hat{a}_\phi$

$$\rightarrow \boxed{\vec{H} = \frac{I}{2\pi |\vec{R}|} \cdot \text{Unit vector of } (dl \times \hat{a}_R)}$$

Short-cut formula.

Ex 1 An infinite current filament is lies at $x=2$, $y=2$ $z=4$ m it carries a current of 10 A along +ve z direction. Find H at the origin.

Ans:



$$\Rightarrow \quad \vec{R} = -2\hat{a}_x - 4\hat{a}_z, \quad d\vec{l} = dy\hat{a}_y$$
$$|\vec{R}| = \frac{-2\hat{a}_x - 4\hat{a}_z}{\sqrt{20}} \cdot \sqrt{20}$$

$$\therefore \quad \hat{a}_R = \frac{-2\hat{a}_x - 4\hat{a}_z}{\sqrt{20}}, \quad d\vec{l} = dy\hat{a}_y$$

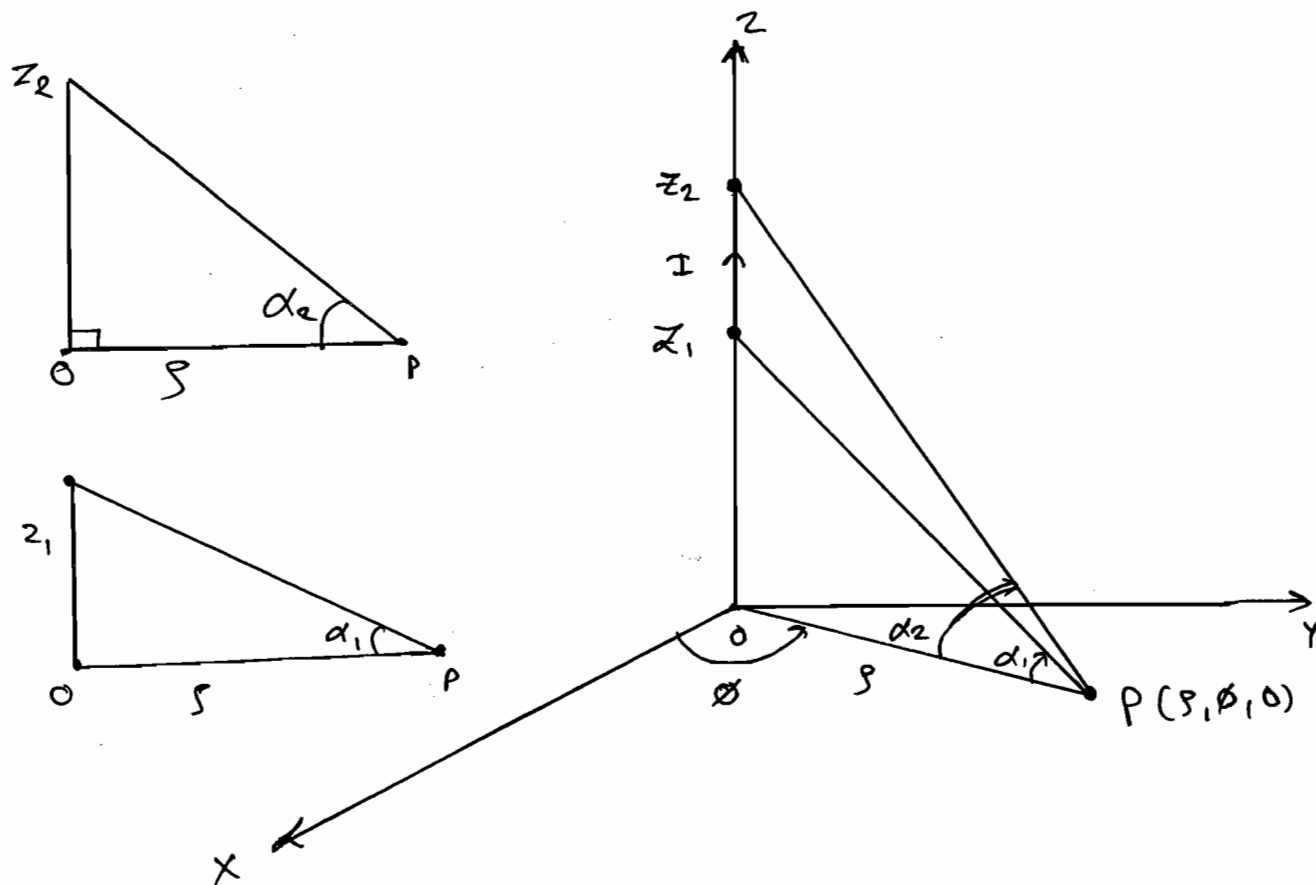
$$\therefore \quad d\vec{l} \times \hat{a}_R = \frac{2\hat{a}_z - 4\hat{a}_x}{\sqrt{20}} dy$$

$$\text{Unit vector of } (d\vec{l} \times \hat{a}_R) = \frac{2\hat{a}_z - 4\hat{a}_x}{\sqrt{20}}$$

$$\therefore \quad H = \frac{I}{2\pi\sqrt{20}} \cdot \frac{2\hat{a}_z - 4\hat{a}_x}{\sqrt{20}}$$

$$\therefore \quad \boxed{H = \frac{I(\hat{a}_z - 2\hat{a}_x)}{20\pi} \text{ A/m.}}$$

*



- Figure shows a finite length current filament lies along z -axis. It carries a current of I A.
- find the magnetic field intensity at $P(s, \phi, 0)$ in terms of α_1 & α_2 .
- We know that for infinitesimal

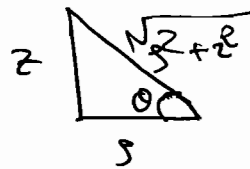
$$d\vec{H} = \frac{I ds dz}{4\pi (s^2 + z^2)^{3/2}} \cdot \hat{a}_\phi$$

∴ Now, for finite length (z_1 to z_2).

$$\therefore \vec{H} = \int_{z_1}^{z_2} \frac{I ds dz}{4\pi (s^2 + z^2)^{3/2}} \hat{a}_\phi$$

$$\therefore \vec{H} = \frac{I}{4\pi} \hat{a}_\phi \int_{z_1}^{z_2} \frac{s dz}{(s^2 + z^2)^{3/2}}$$

Put $z = \rho \tan \theta$
 $\therefore dz = \rho \sec^2 \theta$



$$\therefore \vec{H} = \frac{I}{4\pi} \int_{z_1}^{z_2} \frac{\rho^2 \sec^2 \theta \cdot d\theta}{\rho^3 \sec^3 \theta} \hat{a}_\phi$$

$$\sin \theta = \frac{z}{\sqrt{z^2 + \rho^2}}$$

$$\cos \theta = \frac{\rho}{\sqrt{z^2 + \rho^2}}$$

$$\vec{H} = \frac{I}{4\pi\rho} \cdot [\sin \theta]_{z_1}^{z_2}$$

$$\vec{H} = \frac{I}{4\pi\rho} \left[\frac{z}{(z^2 + \rho^2)^{1/2}} \right]_{z_1}^{z_2} \hat{a}_\phi$$

$$\vec{H} = \frac{I}{4\pi\rho} \left[\frac{z_2}{(z_2^2 + \rho^2)^{1/2}} - \frac{z_1}{\sqrt{z_1^2 + \rho^2}} \right] \hat{a}_\phi$$

$$\therefore \boxed{\vec{H} = \frac{I}{4\pi\rho} [\sin \alpha_2 - \sin \alpha_1] \hat{a}_\phi} \quad \text{A/m.}$$

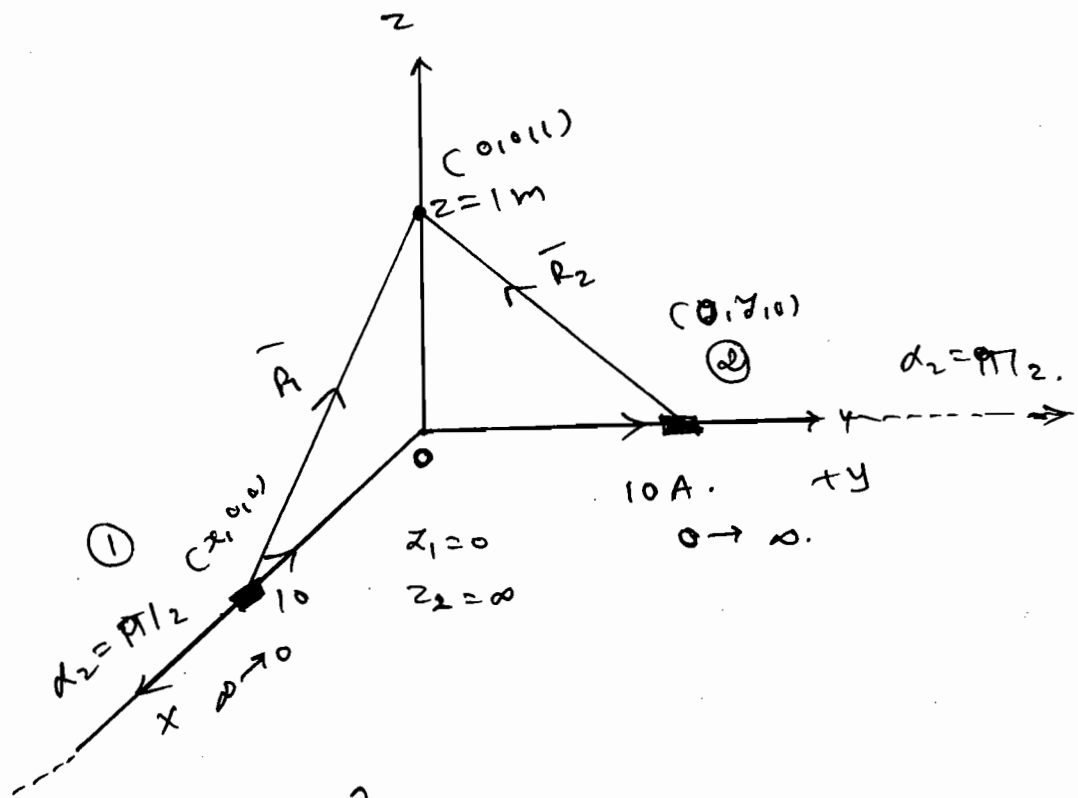
$$\rightarrow \text{If } z_2 \rightarrow \infty \Rightarrow \alpha_2 = \frac{\pi}{2} \Rightarrow \sin \frac{\pi}{2} = 1.$$

$$\rightarrow \text{If } z_1 \rightarrow -\infty \Rightarrow \alpha_1 = -\frac{\pi}{2} \Rightarrow \sin \frac{\pi}{2} = -1.$$

$$\therefore \vec{H} = \frac{I}{4\pi\rho} [1 - (-1)] \hat{a}_\phi$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

* $\rightarrow$ A current of $10A$ is directed in from $-\infty$ towards origin on the positive x-axis and then and then map to ∞ on the +ve y-axis find magnitude of magnetic field intensity on the z-axis. at $z=1m$.



$$\vec{H} = \frac{I d\vec{l} \times \hat{a}_r}{4\pi |\vec{r}|^2}$$

$$\vec{H} = \frac{I}{4\pi |\vec{r}|} \cdot \text{Unit vector of } (\hat{a}_r \times d\vec{l})$$

①

$$d\vec{l}_1 = dx \hat{a}_x$$

$$\vec{r}_1 = -x \hat{a}_x + \hat{a}_z$$

$$|\vec{r}_1| = \sqrt{x^2 + 1}$$

$$\hat{a}_{r_1} = \frac{-x \hat{a}_x + \hat{a}_z}{\sqrt{x^2 + 1}}$$

$$\hat{a}_{r_1} \times d\vec{l}_1 = \frac{-dx}{\sqrt{x^2 + 1}} \hat{a}_y$$

②

$$d\vec{l}_2 = dy \hat{a}_y$$

$$\vec{r}_2 = \frac{-y \hat{a}_y + \hat{a}_z}{\sqrt{1 + y^2}}$$

$$|\vec{r}_2| = \sqrt{1 + y^2}$$

$$\hat{a}_{r_2} = \frac{-y \hat{a}_y + \hat{a}_z}{\sqrt{1 + y^2}}$$

$$\hat{a}_{r_2} \times d\vec{l}_2 = \frac{dy}{\sqrt{1 + y^2}} \hat{a}_x$$

$$\therefore d\vec{H} = \frac{I dy}{4\pi(y^2+1)^{3/2}} \hat{a}_x - \frac{I dx}{4\pi(x^2+1)^{3/2}} \hat{a}_y.$$

$$\therefore \vec{H} = \frac{I}{4\pi} \left[\int_0^\infty \hat{a}_x \frac{dy}{(y^2+1)^{3/2}} - \hat{a}_y \int_\infty^0 \frac{dx}{(x^2+1)^{3/2}} \cdot \hat{a}_y \right]$$

$$\therefore \vec{H} = \frac{I}{4\pi} [\hat{a}_x + \hat{a}_y].$$

$$\because \int_{-\infty}^{\infty} \frac{1}{\sqrt{x^2+1}} dx = 1$$

$$\int_0^\infty \frac{dy}{\sqrt{y^2+1}} = 1.$$

$$\boxed{\vec{H} = \frac{I}{4\pi} [\hat{a}_x + \hat{a}_y].}$$

Ex-2 Find the magnetic field intensity on the axis of a circular current loop of radius a which carries a direct current of I . Also find the magnetic field intensity at the centre of the circular current loop.

→ We assume that the circular current loop is located in $z=0$ plane and centred at origin. Therefore z axis would become axis of the circular current loop on the z .

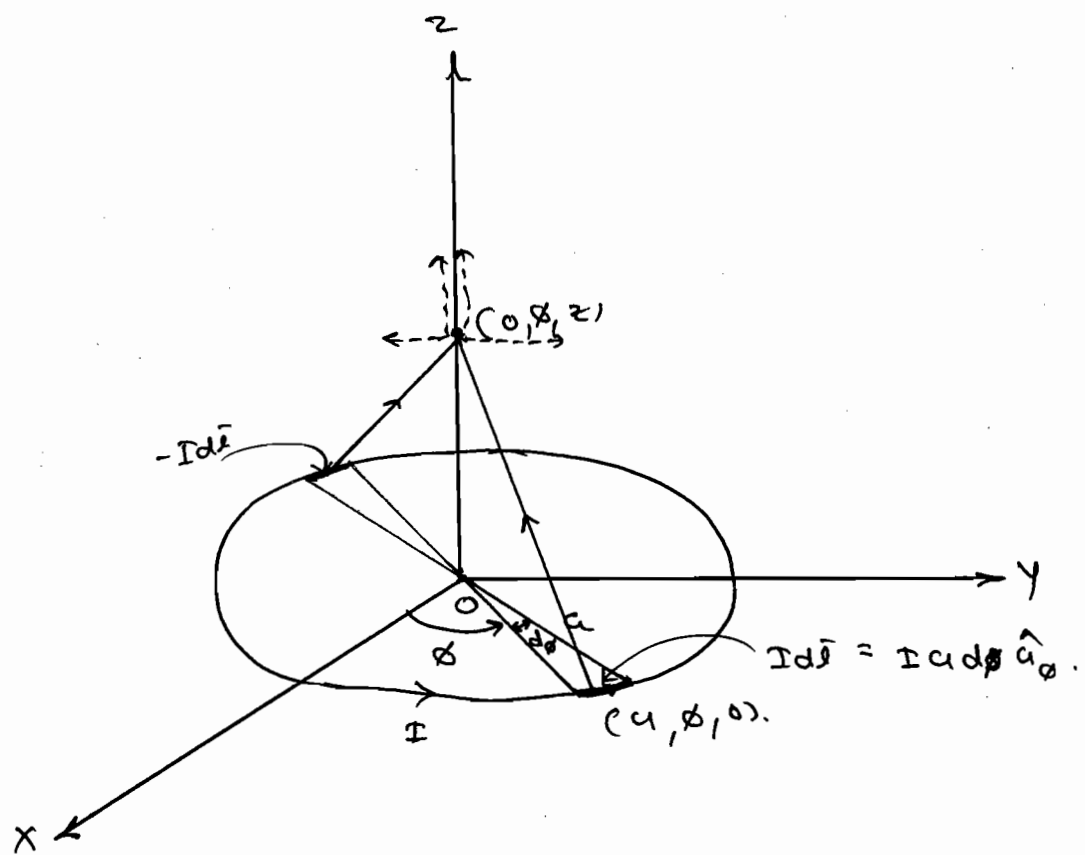
$$\therefore I d\vec{l} = I a d\phi \hat{a}_\phi$$

$$\therefore \vec{R} = -a \hat{a}_1 + b \hat{a}_2$$

$$\hat{a}_R = \frac{-a \hat{a}_1 + b \hat{a}_2}{\sqrt{a^2 + b^2}}$$

$$I d\vec{l} \times \hat{a}_R$$

$$= \frac{I a^2 d\phi \hat{a}_2 + I a b \hat{a}_1}{\sqrt{a^2 + b^2}}$$



→ As shown in the figure for every differential current filament on the circular current loop, there exists an another differential current filament diametrically opposite side which results in cancellation of a horizontal field components and the resultant field would be along $\hat{a}_z$ direction only.

→ Ignoring $\hat{a}_z$ components the total field is given by

$$d\vec{H} = \frac{I d\vec{l} \times \hat{a}_r}{4\pi |\vec{r}|^2}$$

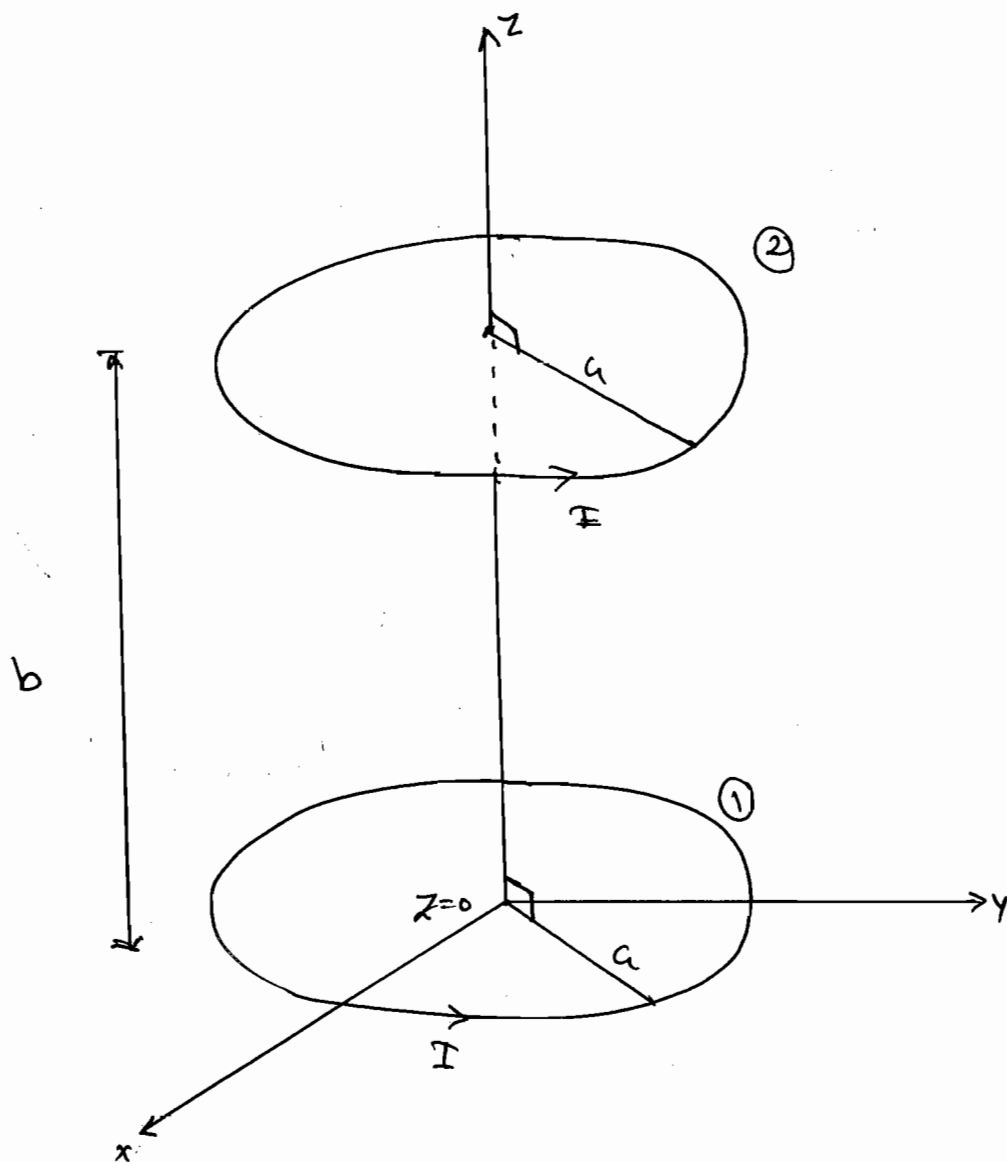
$$\therefore d\vec{H} = \frac{I a^2 \hat{a}_z + I a b \hat{a}_\phi}{4\pi (a^2 + b^2)^{3/2}}$$

$$\vec{H} = \frac{Ia^2}{4\pi (a^2 + b^2)^{3/2}} \int_0^{2\pi} d\phi.$$

$$\therefore \vec{H} = \frac{Ia^2}{2(a^2 + b^2)^{3/2}} \hat{C}_2 \text{ Alm.}$$

→ At the centre of the loop (put $b=0$), the expression for magnetic field intensity is given by

$$\vec{H} = \frac{I}{2a} \hat{C}_2 \text{ Alm.}$$



→ Figure Shows parallel circular current loops ① & ② find $\vec{H}$ at $z=b$ at the centre of loop ②.

$$\vec{H}_1 = \frac{Ia^2}{2(a^2+b^2)^{3/2}} \hat{a}_2$$

$$\vec{H}_2 = \frac{I}{2a} \hat{a}_2$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2$$

$$\therefore \vec{H} = \left(\frac{Ia^2}{2(a^2+b^2)^{3/2}} + \frac{I}{2a} \right) \hat{a}_2$$

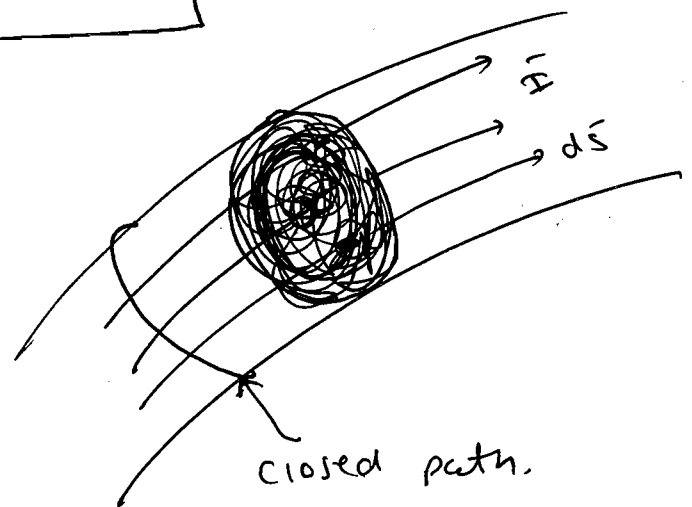
* Ampere's Law:

→ The line Integral of Magnetic field Intensity around a closed path is equal to current enclosed by the path.

$$\oint_L \vec{H} \cdot d\vec{l} = I_{enc.}$$

$$\therefore I = \int_S \vec{J}_c \cdot d\vec{S}$$

$$\therefore \oint_L \vec{H} \cdot d\vec{l} = \int_S \vec{J}_c \cdot d\vec{S}$$



Now, By Stoke's theorem.

$$\therefore \oint_L \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot d\vec{S} = \int_S \vec{J}_c \cdot d\vec{S}$$

$$\nabla \times \vec{H} = \vec{J}_c$$

point form of Ampere's Law.

→ The closed path is touching the conductor

∴ the total current is enclosed by the path $I_{enc} = I$.

$$\rightarrow \nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$\rightarrow \nabla \times \vec{H} = \frac{1}{r} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & \hat{a}_z \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial z \\ H_r & rH_\theta & H_z \end{vmatrix}$$

$$\rightarrow \nabla \times \vec{H} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ H_r & rH_\theta & r\sin\theta H_\phi \end{vmatrix}$$

Ex-1 Let, $\vec{H} = -y(x^2+y^2)\hat{a}_x + x(x^2+y^2)\hat{a}_y$ Alm. 133

Find the amount of current passing through $z=0$, $-1 \leq x \leq 1$, $-2 \leq y \leq 2$ in $\hat{a}_z$ direction.

Ans:

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y(x^2+y^2) & x(x^2+y^2) & 0 \end{vmatrix}$$

$$\therefore \nabla \times \vec{H} = 0 - 0 + \hat{a}_z (2x^2+y^2) + (x^2+2y^2).$$

$$\therefore \nabla \times \vec{H} = 4(x^2+y^2)\hat{a}_z \text{ Alm.}$$

$$\therefore \vec{J}_c \cdot d\vec{s} = (4(x^2+y^2)\hat{a}_z) (dx dy \hat{a}_z).$$

$$\vec{J}_c \cdot d\vec{s} = 4(x^2+y^2) dx dy.$$

$$\therefore I = \int \vec{J}_c \cdot d\vec{s} = 4 \int_{-1}^1 \int_{-2}^2 (x^2+y^2) dx dy.$$

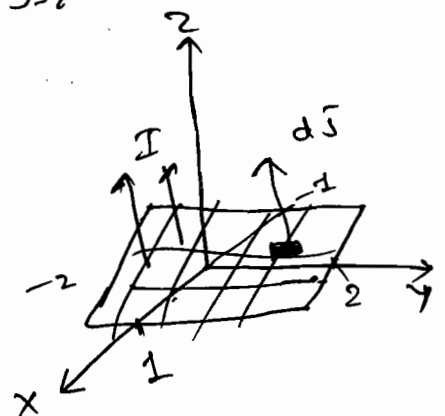
$$= 4 \left[\int_{-2}^2 \int_{-1}^1 x^2 dx dy + \int_{-1}^1 \int_{-2}^2 y^2 dx dy \right]$$

$$= 4 \left[\left[\frac{x^3}{3} \right]_{-1}^1 [y]_{-2}^2 + \left[\frac{y^3}{3} \right]_{-2}^2 [x]_{-1}^1 \right].$$

$$I = 4 \left[\left(\frac{2}{3} \times 4 \right) + \frac{16}{3} \times 2 \right].$$

$$= 4 \left[\frac{8}{3} + \frac{32}{3} \right].$$

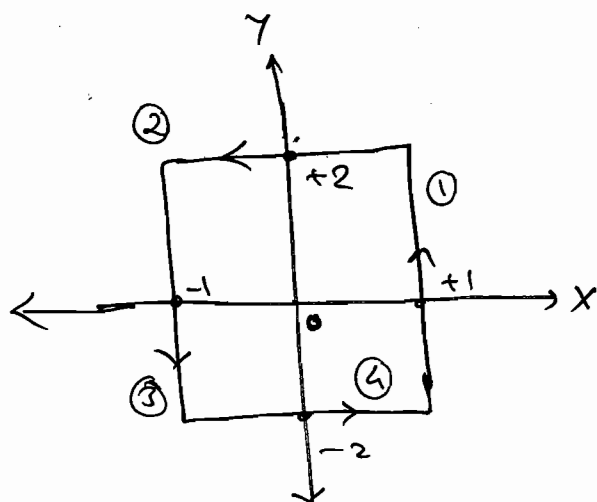
$$\therefore I = 160 \text{ A}$$



→ Method-2: by Integral form

$$I_{enc} = \oint_C \vec{H} \cdot d\vec{l}$$

$$\therefore I_{enc} = \int_{(1)} \vec{H} \cdot d\vec{l} + \int_{(2)} \vec{H} \cdot d\vec{l} + \int_{(3)} \vec{H} \cdot d\vec{l} + \int_{(4)} \vec{H} \cdot d\vec{l}$$



Path No	$d\vec{l}$	Path is at	Path limit	$\vec{H} \cdot d\vec{l}$	$\vec{H} \cdot d\vec{l}$ at $x=1$
①	$dy \hat{a}_y$	$x=1$	$-2 \leq y \leq 2$	$x(x^2+y^2)dy$	$(1+y^2)dy$
②	$dx \hat{a}_x$	$y=2$	$+1 \leq x \leq -1$	$-y(x^2+y^2)dx$	$-2(x^2+4)dx$
③	$dy \hat{a}_y$	$x=-1$	$2 \leq y \leq -2$	$x(x^2+y^2)dy$	$-(1+y^2)dy$
④	$dx \hat{a}_x$	$y=-2$	$-1 \leq x \leq 1$	$-y(x^2+y^2)dx$	$2(x^2+4)dx$

$$\oint_C \vec{H} \cdot d\vec{l} = \int_{-2}^2 (y^2+1) dy = \left[\frac{y^3}{3} + y \right]_{-2}^2 = \frac{8}{3} + 2 + \frac{8}{3} - 2 = \frac{16}{3} + 4$$

$$\int_{(1)} \vec{H} \cdot d\vec{l} = \int_{-2}^2 -2(x^2+4) dx = 2 \left[\frac{x^3}{3} + 4x \right]_{-1}^1 = \frac{4}{3} + 16$$

$$\int_{(2)} \vec{H} \cdot d\vec{l} = \int_2^{-2} -(1+y^2) dy = \left[\frac{y^3}{3} + y \right]_{-2}^2 = \frac{16}{3} + 4$$

$$\int_{(4)} \vec{H} \cdot d\vec{l} = \int_{-1}^1 2(x^2+4) dx = 2 \left[\frac{x^3}{3} + 4x \right]_{-1}^1 = \frac{4}{3} + 16$$

$$\therefore I = \oint \vec{H} \cdot d\vec{\ell}$$

$$= \frac{16}{3} + \frac{16}{3} + \frac{4}{3} + \frac{4}{3} + 20$$

$$\therefore I = \frac{40}{3} + 20$$

$$\boxed{I = \frac{160}{3} \text{ A}}$$

* Application of Ampere's Law:

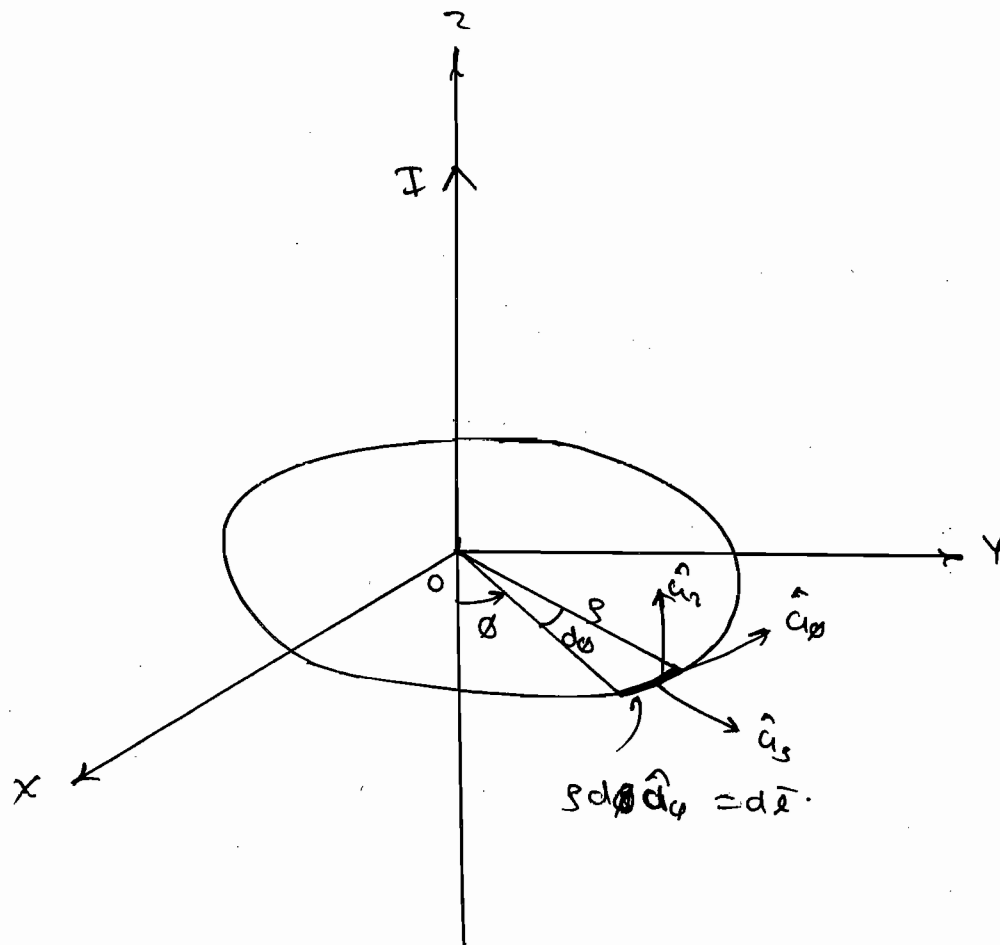
⇒ For Symmetrical current distribution where we have an idea about the direction of magnetic field Intensity then one can find out magnitude of the magnetic field intensity by using the following procedure.

- ① one has to choose a suitable appropriate closed path which is enclosing partially (or) fully the given current distribution
- ② $d\vec{\ell}$ always lies along the path ✓
- ③ The closed path is so chosen in such a way that $\vec{H}$ may lie along the path or normal to the path
- ④ $\vec{H} \cdot d\vec{\ell} = |\vec{H}| |d\vec{\ell}|$ if $\vec{H}$ lies along the path,
 $\vec{H} \cdot d\vec{\ell} = 0$ if $\vec{H}$ normal to the path.
- ⑤ over that part of the path where $\vec{H}$ lies along the path, on that part of the path

$\vec{H}$ is constant.

Ex-1 Find $\vec{H}$ due to a long straight infinite filamentary conductor which carries a direct current of I A.

Ans: We assume that the infinite current filament lies along z axis



① A ^{closed} circular path is chosen at $s = \text{const.}$ ($s > 0$) is chosen.

② $d\vec{l} = s d\phi \hat{e}_\phi$

③ $\vec{H} = H_\phi \hat{e}_\phi$ only $\rightarrow \vec{H}$ would be around the conductor.

$$\vec{H} \cdot d\vec{l} = \int H_\phi d\phi$$

④ $|\vec{H}| = H_\phi$ must be const. on $s = \text{const.}$
 $\therefore \vec{H}$ lies along the path.

$I_{enc} = I$ (The total current is enclosed). 137

$$\therefore \oint \vec{H} \cdot d\vec{r} = I_{enc}.$$

$$\therefore I_{enc} = H \int_0^{2\pi} d\phi.$$

$$\therefore I = H \cdot 2\pi.$$

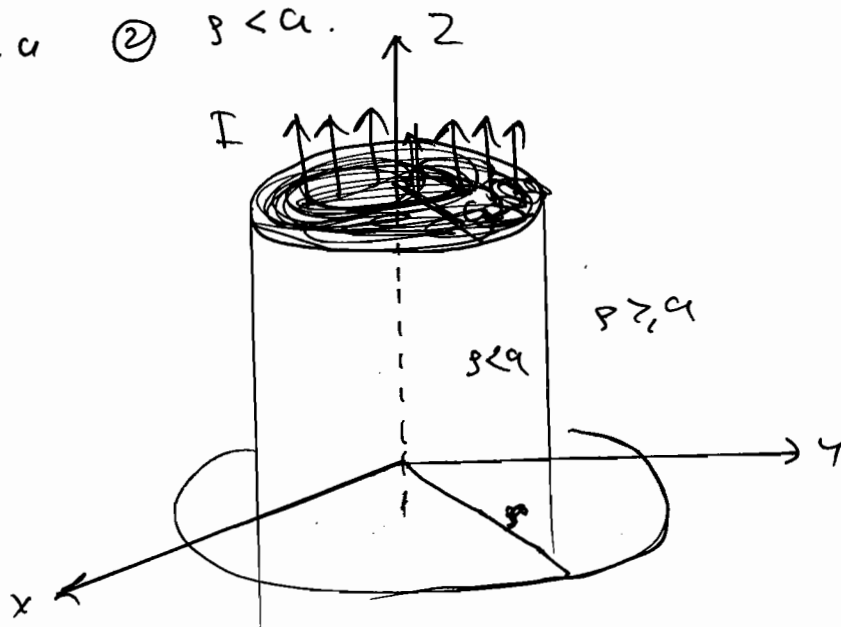
$$\therefore H = \frac{I}{2\pi r}.$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi r} \hat{\phi}}$$

Ex? Find $\vec{H}$ due to a ~~along~~ infinite solid cylindrical conductor of radius a where current I uniformly distributed throughout the cross section.

Ans? We assume that the solid cylindrical conductor position along z -axis as shown in the figure. where the current I is uniformly distributed throughout the cross section one can find $\vec{H}$ for

- ① $r \geq a$ ② $r < a$.



① The circular (closed) path of $s = \text{constant}$ ($s > 0$) is chosen.

② $d\vec{r} = a d\phi \hat{a}_\phi$.

③ $\vec{H} = H_\phi \hat{a}_\phi$.

$$\vec{H} \cdot d\vec{r} = a H_\phi d\phi.$$

④ $|\vec{H}| = H_\phi$ must be const. on $s = \text{const.}$

$\therefore \vec{H}$ lies along the path.

$I_{\text{enc}} = I$ (The total current is enclosed).

$$\therefore \oint \vec{H} \cdot d\vec{r} = I_{\text{enc}}.$$

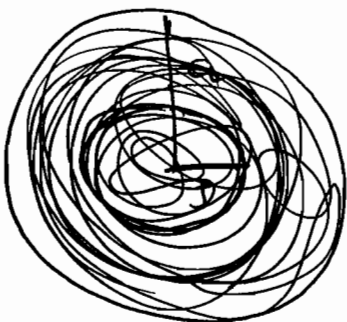
$$\therefore a H_\phi \int_0^{2\pi} d\phi = I.$$

$$\therefore 2\pi a H_\phi = I.$$

$$\therefore H_\phi = \frac{I}{2\pi a}.$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi a} \hat{a}_\phi}$$

② $s < a$.



$$\pi a^2 \rightarrow I$$

$$\pi e^2 \rightarrow I'$$

$$\therefore I_{\text{enc}} = \frac{\pi e^2}{\pi a^2} \times I$$

$$\therefore I_{\text{enc}} = \frac{I e^2}{a^2}.$$

$$\oint \vec{H} \cdot d\vec{r} = I_{\text{enc}} \Rightarrow H_\phi (2\pi s) = \frac{I e^2}{a^2}.$$

$$\therefore H_s = \frac{Is}{2\pi a^2}$$

$$\therefore \boxed{\bar{H} = \frac{Is}{2\pi a^2} \hat{a}_\phi} \text{ for } s < a.$$

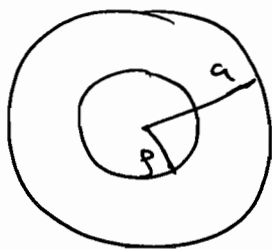
$$(i) \quad s \geq a \quad \bar{H} = \frac{I}{2\pi s} \hat{a}_\phi \Rightarrow |H| \propto 1/s.$$

$$(ii) \quad s < a \quad \bar{H} = \frac{Is}{2\pi a^2} \hat{a}_\phi \Rightarrow |H| \propto s.$$

Ex-2 Repeat the above problem if it is a hollow cylindrical conductor of radius a .

$$\underline{\text{Ans:}} \quad (i) \quad s \geq a \quad \bar{H} = \frac{I}{2\pi s} \hat{a}_\phi =$$

$$(ii) \quad s < a \quad \bar{H} = 0$$



$s < a$

$I_{enc} = 0.$

★ Magnetic Flux Density ($\bar{B}$)

→ Unit $\Rightarrow$ T (or) wb/m^2 .

$$\bar{B} = \mu \bar{H}.$$

$$\mu = \mu_0 \mu_r \text{ H/m.}$$

$\therefore \mu$: permeability / Inductivity (H/m).

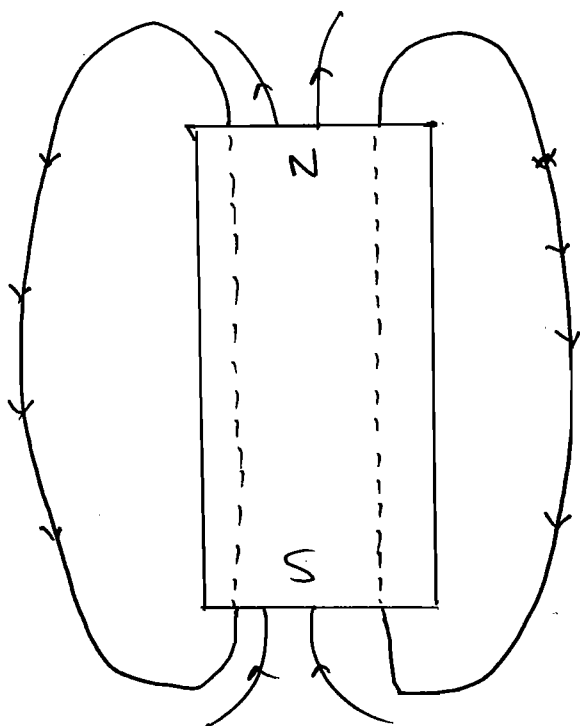
μ_0 : Absolute permeability / Inductivity (H/m).

μ_r : Relative " "

→ Ⓚ Permeability or Inductivity specifies
property of a medium and that indicates
the ability to store the magnetic energy.

~~Here~~ ~~the~~ ~~area~~ ~~is~~ ~~not~~

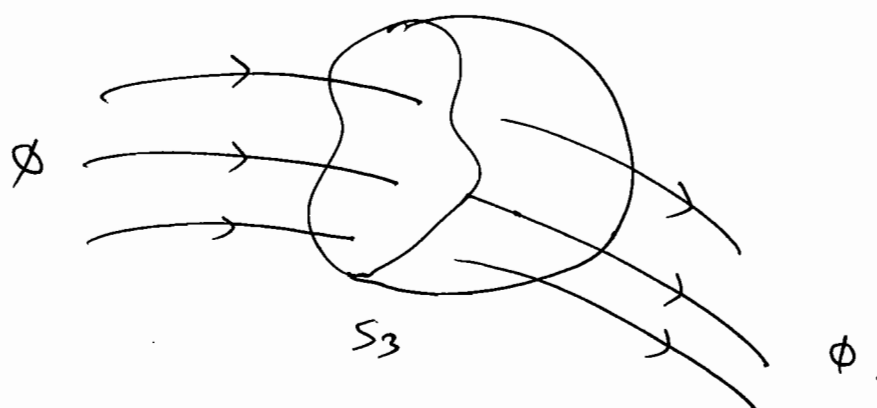
* Magnetic flux: Φ wb.



→ The amount of magnetic flux passing through a cross sectional surface 's' is given by.

$$\phi = \int_s \vec{B} \cdot d\vec{s}$$

← cross sectional



S_3 = arbitrary closed surface.

→ $\oint_s \vec{B} \cdot d\vec{s} = 0.$

$\therefore \oint_v \nabla \cdot \vec{B} \cdot d\vec{v} = 0.$

$\therefore \boxed{\nabla \cdot \vec{B} = 0.}$

Gauss law for H-fields.

→ for electric field

$$\psi_{net} = q_{enc} = \oint_s \vec{D} \cdot d\vec{s} = \int_v \nabla \cdot \vec{D} \cdot d\vec{v}.$$

$\therefore \boxed{\nabla \cdot \vec{D} = \rho_v}$

Gauss field for E-fields.

→ Unlike a electric flux the magnetic flux would not have starting point and an ending point it enters the closed surface and leaves the same closed surface as shown above. one can find out the amount of magnetic flux passing through a cross sectional surface.

Ex-1 Find the amount of magnetic flux.

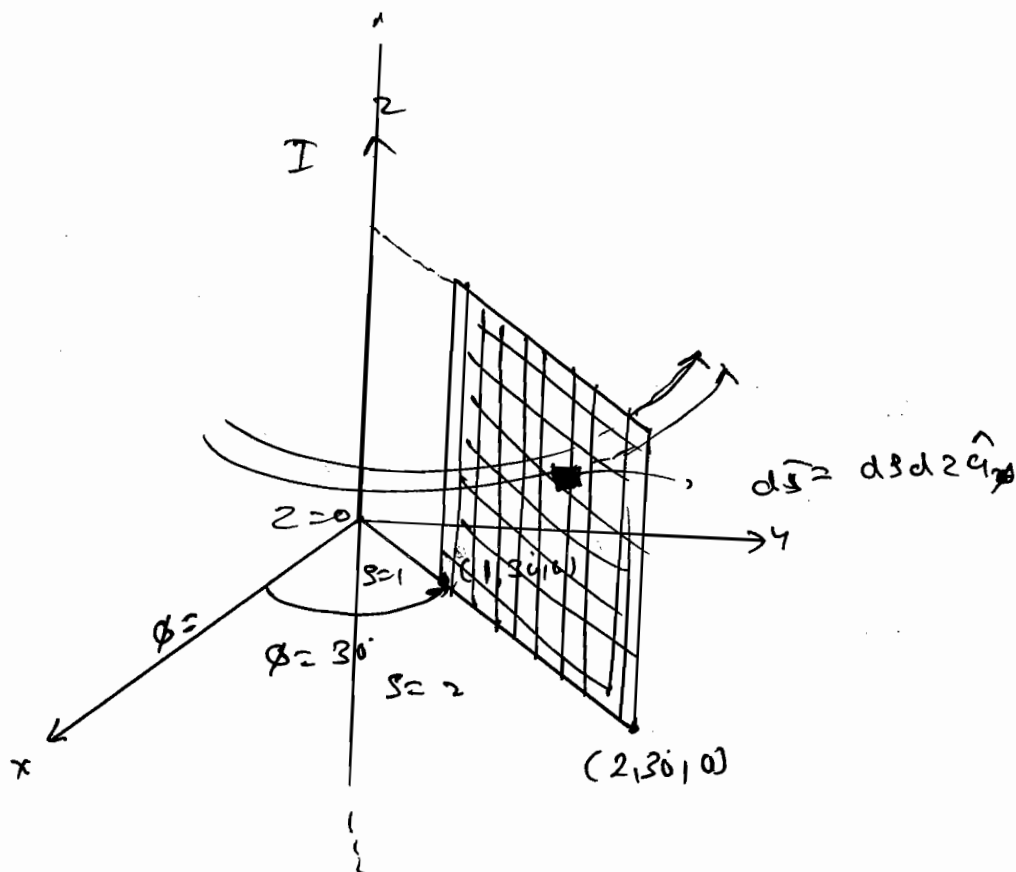
passing through a cross-sectional surface

define by $\phi = 30^\circ$, $1 \leq \rho \leq 2$, $0 \leq z \leq 3$.

due to an infinite current filament lies along z -axis. which carries a direct current of 2.5 A along $+ve$ z -direction.

Assume $\mu = \mu_0$.

Ans:



$$\rightarrow d\vec{s} = ds dz \hat{a}_\phi.$$

$$\vec{B} = \mu \vec{H}$$

$$\therefore \vec{B} = \mu_0 \vec{H}$$

$$\therefore \vec{B} = \frac{\mu_0 I}{2\pi s} \hat{a}_\phi.$$

$$\therefore \vec{B} \cdot d\vec{s} = \frac{\mu_0 I}{2\pi s} ds dz.$$

$$\therefore \phi = \int_S \vec{B} \cdot d\vec{s}$$

$$= \int_1^2 \int_0^3 \frac{\mu_0 I}{2\pi s} ds dz.$$

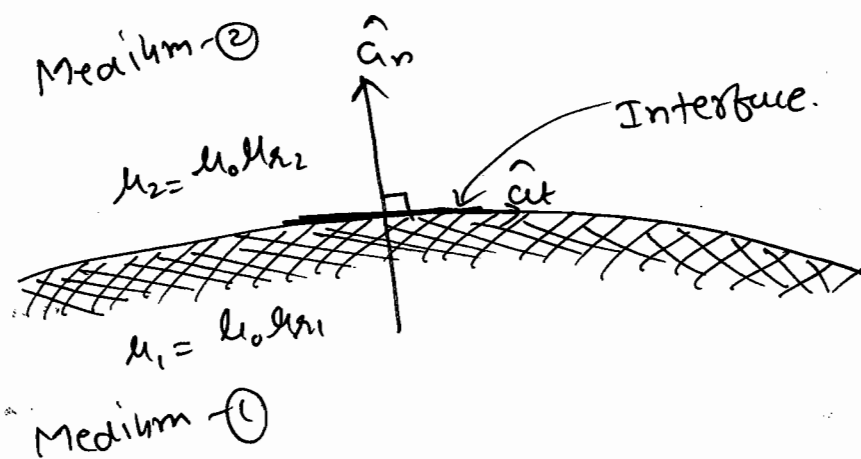
$$= \frac{\mu_0 I}{2\pi s} \times [2-1] \times [3].$$

$$\therefore \boxed{\phi = \frac{3\mu_0 I \ln 2}{2\pi s} \omega_b}$$

$$\therefore \boxed{\phi = \frac{3\mu_0 I \ln 2}{2\pi} \omega_b}$$

$$\phi = \frac{7.5 \mu_0 \ln 2}{2\pi} \omega_b.$$

* Boundary Condition:



(I) Using Ampere's Law, one can show that

(a) $H_{t1} = H_{t2}$

Tangential components of H-fields are continuous across a current free interface.

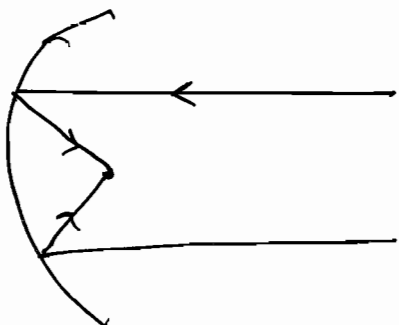
(b) $(\vec{H}_1 - \vec{H}_2) \times \hat{n} = \vec{J}_s$ (A/m)

→ Tangential components of H-fields are discontinuous by an amount of surface current densities.

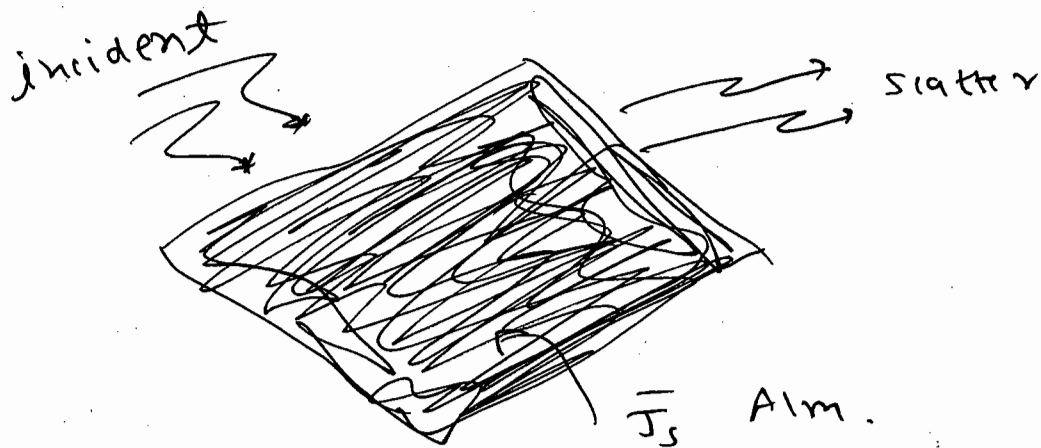
(II) Using Gauss Law for H-field,

$B_{n1} = B_{n2}$

i.e. Normal component of magnetic flux densities are continuous across the interface.



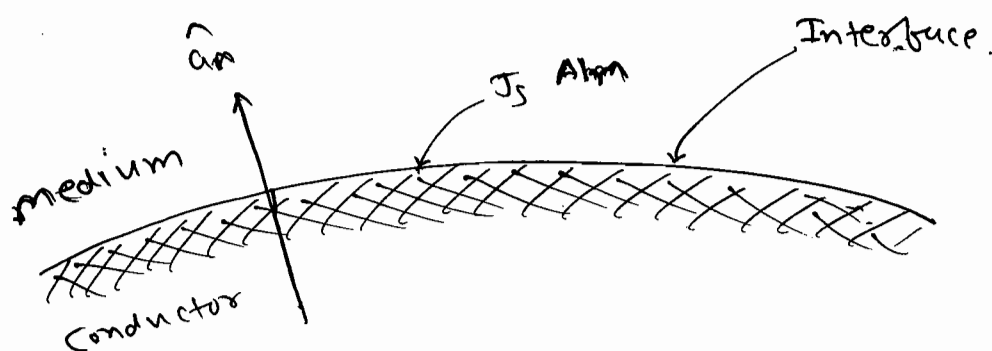
Perpendicular entering.



PEC: Perfect electric Conductor

$\bar{J}_s$: current per unit width (Alm)

* $\rightarrow$ Behaviour of Magnetic field intensities
across a conductor interface.



① $\hat{n} \times \bar{H} = \bar{J}_s$

$\rightarrow$ Tangential components of magnetic fields are equal to surface current density.

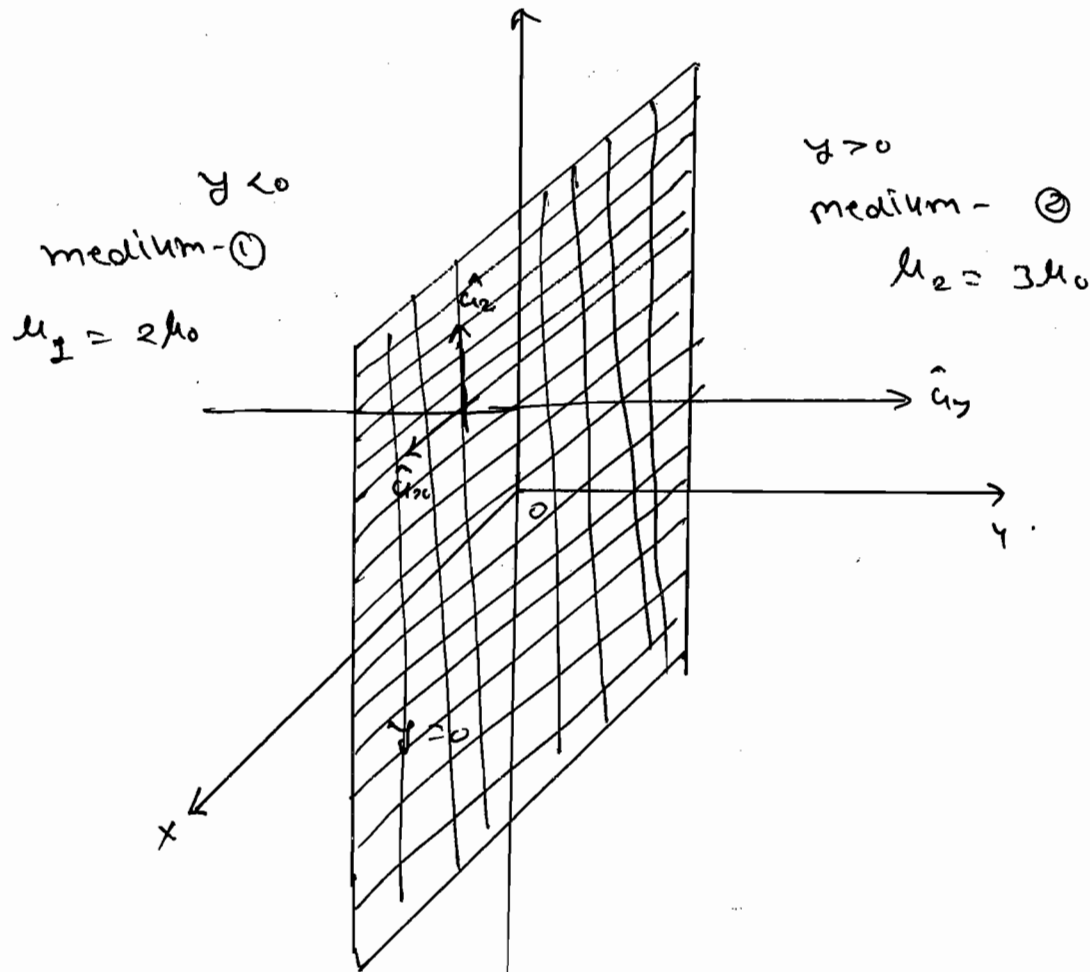
② $B_n = 0$ (or) $H_n = 0$

$\rightarrow$ Normal components of magnetic field intensities are vanished across the conductor interface.

$\bar{J}_s$: Surface current density across the conductor interface.

Ex-1 The plane $y=0$ separates two mediums $y < 0$ is medium-① and is characterised by $2\mu_0$ and $y > 0$ is medium-② and is characterised by $3\mu_0$ given that $\vec{H}_1 = 10\hat{a}_2$ and $\vec{H}_2 = 10\hat{a}_x$. Find the surface current density across the interface.

Ans:



→ $\vec{H}_1 = 10\hat{a}_2$, $\vec{H}_2 = 10\hat{a}_x$.

$$(\vec{H}_1 - \vec{H}_2) \times \hat{a}_n = \vec{J}_s$$

$$\therefore \vec{J}_s = (10\hat{a}_2 - 10\hat{a}_x) \hat{a}_y$$

$$\boxed{\vec{J}_s = -10\hat{a}_x - 10\hat{a}_2 \text{ A/m.}}$$

Ex-2 Retesting to the above figure 147

Let, $\vec{H}_1 = (3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z) \text{ A/m}$

$\vec{H}_2 =$

assume current free interface find $\vec{B}_1, \vec{H}_2$ and $\vec{B}_2$

Ans:

$\vec{B}_1 = \mu_1 [\vec{H}_1]$

$\therefore \vec{B}_1 = \mu_0 [6\hat{a}_x + 8\hat{a}_y + 10\hat{a}_z]$

$\therefore \vec{H}_1 = 3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z$

$\therefore \vec{H}_1 = H_{t1}\hat{a}_t + H_{n1}\hat{a}_n$

$\therefore H_{t1} = 3\hat{a}_x + 5\hat{a}_z, \quad H_{n1} = 4\hat{a}_y$

$\therefore \text{Now, } H_{t1} = H_{t2} \quad (\because J_s = 0)$

($\because$ tangential components are equal).

$\vec{H}_{t2} = 3\hat{a}_x + 5\hat{a}_z$

Now, $\vec{H}_2 = H_{t2}\hat{a}_t + H_{n2}\hat{a}_n$

$\therefore B_{n1} = B_{n2} \quad (\because J_s = 0)$

$\therefore B_{n2} = B_{n1}$

$\therefore \mu_2 H_{n2} = \mu_1 H_{n1}$

$\therefore H_{n2} = \frac{\mu_1}{\mu_2} \times 4\hat{a}_y$

$\therefore H_{n2} = \frac{2}{3} \times 4\hat{a}_y$

$\therefore H_{n2} = \frac{8}{3}\hat{a}_y$

$\therefore \vec{H}_2 = 3\hat{a}_x + \frac{8}{3}\hat{a}_y + 5\hat{a}_z$

$$\therefore \vec{B}_2 = \mu_2 \vec{H}_2$$

$$\therefore \vec{B}_2 = \mu_0 [9\hat{a}_x + 8\hat{a}_y + 15\hat{a}_z]$$

Ex-3 In the above problem assume that the interface has non zero surface current density of $\vec{J}_s = (5\hat{a}_x + 10\hat{a}_z)$ A/m. Find $\vec{B}_1$, $\vec{H}_2$ and $\vec{B}_2$.

Ans:

$$\vec{H}_1 = 3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z$$

$$\vec{B}_1 = \mu_1 \vec{H}_1$$

$$\vec{B}_1 = \mu_0 [6\hat{a}_x + 8\hat{a}_y + 10\hat{a}_z]$$

$$\text{Now, } (\vec{H}_1 - \vec{H}_2) \times \hat{a}_n = \vec{J}_s$$

$$\therefore \mu_1 \vec{H}_1 - \mu_2 \vec{H}_2 = \vec{J}_s$$

$$[(\mu_1 - \mu_2) \hat{a}_x + (\mu_1 - \mu_2) \hat{a}_z] \times \hat{a}_n = \vec{J}_s$$

$$\therefore (\mu_1 - \mu_2) (\hat{a}_x \times \hat{a}_n) = \vec{J}_s$$

$$\therefore \hat{a}_t = \frac{3}{\sqrt{34}} \hat{a}_x + \frac{5}{\sqrt{34}} \hat{a}_z$$

$$\therefore \hat{a}_n = \hat{a}_y$$

$$\therefore \hat{a}_t \times \hat{a}_n = \frac{12}{\sqrt{34}} \hat{a}_z - \frac{20}{\sqrt{34}} \hat{a}_x$$

$$\therefore (\mu_1 - \mu_2) \left(\frac{12}{\sqrt{34}} \hat{a}_z - \frac{20}{\sqrt{34}} \hat{a}_x \right) = 5\hat{a}_x + 10\hat{a}_z$$

$$\therefore \frac{12}{\sqrt{34}} (\mu_1 - \mu_2) = 10, \quad -\frac{20}{\sqrt{34}} (\mu_1 - \mu_2) = 5$$

$$(H_{t1} - H_{t2}) = \frac{5}{\sqrt{34}} \sqrt{34}$$

$$H_{t2} - H_{t1} = \frac{\sqrt{34}}{4}$$

$$H_{t1} - H_{t2} = \frac{\sqrt{34}}{4}$$

$$\bar{H}_{t1} = H_{t1} \cdot \hat{a}_t$$

$$H_{t1} = \bar{H}_{t1} \cdot \hat{a}_{t1}$$

$$\bar{H} = 3\hat{a}_x + 5\hat{a}_y$$

$$(\bar{H}_1 - \bar{H}_2) \times \hat{a}_n = \vec{r}_s$$

$$\therefore [(3 - H_{x2})\hat{a}_x + (4 - H_{y2})\hat{a}_y + (5 - H_{z2})\hat{a}_z] \times \hat{a}_y = 5\hat{a}_x + 10\hat{a}_z$$

$$\therefore 7 - 3 - H_{x2} = 10 \Rightarrow H_{x2} = -7$$

$$\therefore -(5 - H_{z2}) = 5$$

$$\therefore H_{z2} = 10$$

$$\therefore \bar{H}_2 = -7\hat{a}_x + H_{y2}\hat{a}_y + 10\hat{a}_z$$

$$\therefore B_2 = \mu_2 \bar{H}_2$$

$$\therefore B_2 = \mu_2 [-7\hat{a}_x + H_{y2}\hat{a}_y + 10\hat{a}_z]$$

$$-7\mu_2\hat{a}_x + \mu_2 H_{y2}\hat{a}_y + 10\mu_2\hat{a}_z = B_{x2}\hat{a}_x + 4\mu_1\hat{a}_y + B_{z2}\hat{a}_z$$

$$\therefore B_{x2} = -7\mu_2$$

$$B_{z2} = 10\mu_2$$

$$H_{y2} = \frac{4\mu_1}{\mu_2} \text{ A/m}$$

$$\therefore H_{y2} = \frac{8}{3} \hat{a}_y$$

$$\therefore \bar{H}_2 = -7\hat{a}_x + \frac{8}{3}\hat{a}_y + 10\hat{a}_z$$

$$\bar{B}_2 = (-21\hat{a}_x + 8\hat{a}_y + 30\hat{a}_z) \mu_2$$

$$\bar{B}_1 = \mu_1 \bar{H}_1$$

$$\bar{B}_1 = (6\hat{a}_x + 8\hat{a}_y + 10\hat{a}_z) \mu_1$$

☆ Time Varying Fields:

→ The existing Ampere's Law, when it is applied to the time-varying field in a non-conducting medium the Law is having some inconsistency or unsatisfaction. This inconsistency has been eliminated by adding a new term $\vec{J}_D$ as follows:

$$\boxed{\nabla \times \vec{H} = \vec{J}_c + \vec{J}_D} \rightarrow \text{Modified Ampere's Law}$$

∴ take ∇ on both the side.

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J}_c + \nabla \cdot \vec{J}_D$$

∴ Divergence of curl is zero.

$$\nabla \cdot \vec{J}_c = - \nabla \cdot \vec{J}_D$$

$$\nabla \cdot \vec{J}_c = - \frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \vec{J}_D = + \frac{\partial \rho_v}{\partial t}$$

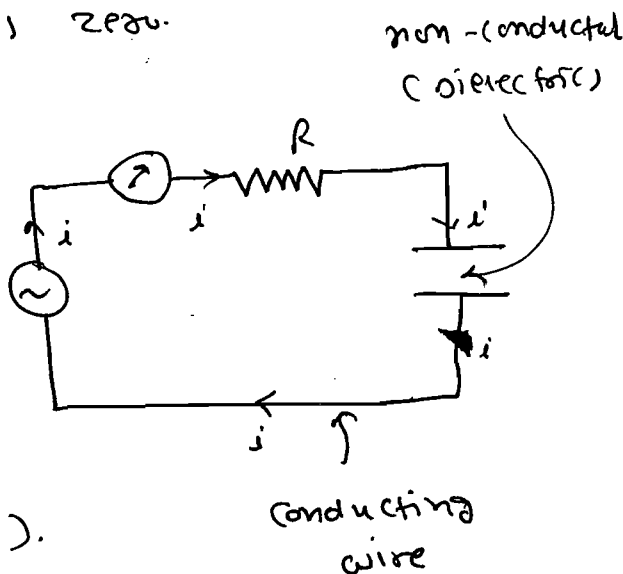
$$\nabla \cdot \vec{J}_D = \frac{\partial}{\partial t} (\nabla \cdot \vec{D})$$

$$\nabla \cdot \vec{J}_D = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

∴ Suppressing ∇ both the side.

$$\boxed{\vec{J}_D = \frac{\partial \vec{D}}{\partial t}}$$

$\vec{J}_D$ = Displacement current density.



→ $\bar{J}_D$ is defined as time rate of change of electric flux density.

→ $\bar{J}_C$ dominates in a conducting medium and is zero in perfect dielectric.

→ $\bar{J}_D$ dominates in a dielectric medium and is zero in perfect conductor.

→ The modified Ampere's Law is written

as $\boxed{\bar{J}_C + \bar{J}_D = \nabla \times \bar{H}}$

$$\therefore \nabla \times \bar{H} = \bar{J}_C + \bar{J}_D.$$

$$\therefore \boxed{\oint_L \bar{H} \cdot d\bar{l} = \int_S (\bar{J}_C + \bar{J}_D) \cdot d\bar{S}}$$

$$\therefore \boxed{\begin{aligned} \bar{J}_C &= \sigma \bar{E} \quad \text{A/m}^2. \\ \bar{J}_D &= \frac{\partial D}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t} \quad \text{A/m}^2. \end{aligned}}$$

* Faraday's Law of Electromagnetic Conduction:

→ When a Stationary Conductor cuts by a moving magnetic flux then or vice versa then emf will be induced. This induced emf will in turn produces a magnetic flux which opposes original flux [Lenz's law]. Mathematically we write

$$\text{emf} = - \frac{\partial \Phi}{\partial t}$$

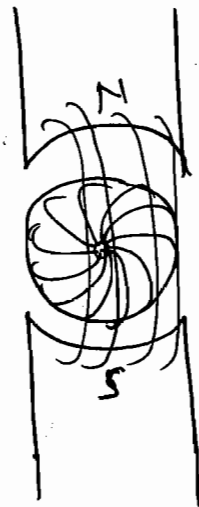
$$\therefore \oint_L \vec{E} \cdot d\vec{l} = - \frac{\partial \Phi}{\partial t}$$

⇓

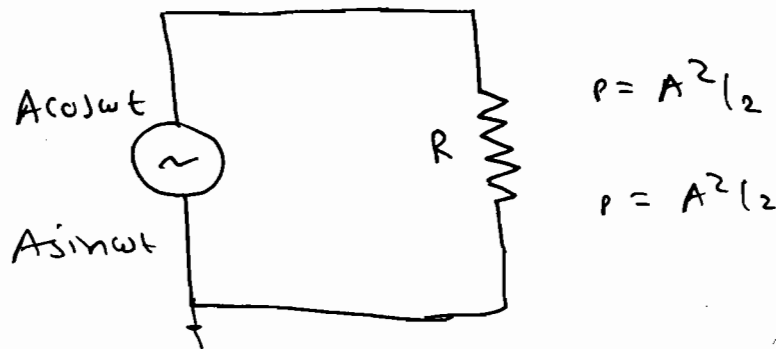
$$\therefore \oint_L \vec{E} \cdot d\vec{l} = - \frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{s}$$

$$\therefore \int_S (\nabla \times \vec{E}) \cdot d\vec{s} = - \frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{s}$$

$$\therefore \boxed{\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}}$$



→ Whether source is $A \cos \omega t$ or $A \sin \omega t$ the average power deliver to 1Ω Resistor is identically same which is equal to $A^2/2$.



$0.1 \angle 45^\circ$ volts $\rightarrow$ phasor form.

$\rightarrow 0.1 \cos(\omega t + 45^\circ)$ (or)

$\rightarrow 0.1 \sin(\omega t + 45^\circ)$.

$\rightarrow \text{or } \text{Im} [0.1 e^{j(\omega t + 45^\circ)}]$ (or)

$\rightarrow \text{Re} [0.1 e^{j(\omega t + 45^\circ)}]$.

→ When the quantities are represented in the phasor form we suppress the time variation for the mathematical convenience this time variations are approximated as cosine or sine or $e^{j\omega t}$.

$\rightarrow \vec{E} \rightarrow \vec{E}(x, y, z)$ (or) $\vec{E}(r, \theta, \phi, t)$ or $\vec{E}(r, \theta, \phi, t)$.

$\hookrightarrow$ It is a function of time and space coordinates.

$\vec{E} = \text{Re} [\vec{E}_a \cdot e^{j\omega t}]$.

$\bar{E}_x$ is called phasor form of $\bar{E}$.

$$\bar{E}_x \rightarrow \bar{E}_x(x, y, z) \text{ (or) } \bar{E}_x(r, \theta, z) \text{ (or) } \bar{E}_x(r, \phi, \theta).$$

↳ It is a fn of space coordinates only.

$$\rightarrow \nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t}.$$

$$\therefore \nabla \times [\text{Re}\{\bar{E}_x \cdot e^{j\omega t}\}] = - \frac{\partial}{\partial t} [\text{Re}\{\bar{B}_x \cdot e^{j\omega t}\}].$$

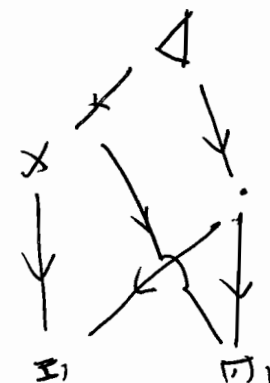
$$\therefore \nabla \times [\text{Re}\{\bar{E}_x \cdot e^{j\omega t}\}] = -j\omega [\text{Re}\{\bar{B}_x \cdot e^{j\omega t}\}].$$

$\therefore$ Suppressing time variation both the side.

$$\therefore \boxed{\nabla \times \bar{E}_x = -j\omega \bar{B}_x}$$

$$\text{So, } \frac{\partial}{\partial t} = j\omega = s$$
$$\underline{\int dt = \frac{1}{j\omega} = 1/s.}$$

→ This is Faraday's law in phasor form.

Name of the Law	Integral form	Point form	Phasor form
1. Faraday's Law	$\oint \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{s}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $= -\mu \frac{\partial \vec{H}}{\partial t}$	$\nabla \times \vec{E}_s = -j\omega \vec{B}_s$
2. Modified Ampere's Law	$\oint \vec{H} \cdot d\vec{r} = \int (\vec{J}_c + \vec{J}_d) \cdot d\vec{s}$	$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_d$ $= \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{H}_s = \vec{J}_{cs} + \vec{J}_{ds}$ $= \sigma \vec{E}_s + \epsilon \frac{\partial \vec{E}_s}{\partial t}$
3. Gauss Law for E-field	$\oint \vec{D} \cdot d\vec{s} = Q_{encl}$ $= \int \rho_v dV$	$\nabla \cdot \vec{D} = \rho_v$	$\nabla \cdot \vec{D}_s = \rho_s$
Gauss Law for H-field	$\oint \vec{B} \cdot d\vec{s} = 0$	$\nabla \cdot \vec{B} = 0$	$\nabla \cdot \vec{B}_s = 0$
Maxwell's Equation	$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t}$ $\vec{J}_c = \sigma \vec{E}$		

→ Maxwell had proved that any Electromagnetic problem can be solved by using above four eqns.

Ex-1 Let $\vec{D} = (3x\hat{a}_x + ky\hat{a}_y + 7z\hat{a}_z) \text{ nC/m}^2$

Assume Charge free region.
 $\rho_v = 0$

Ans:

$$\therefore \nabla \cdot \vec{D} = \rho_v$$

But, $\rho_v = 0$.

$$\therefore 3 + k + 7 = 0$$

$$\therefore k = -11 \text{ nC/m}^3$$

$k \hat{y} = 11 \text{ nC/m}^2$
 $\downarrow$
 $m \quad k = 11/y \text{ nC/m}^2$
 $\therefore \boxed{k = 11 \text{ nC/m}^3}$

Ex-2 Let, $\vec{E} = (kx - 100t)\hat{a}_y \text{ V/m}$, $\vec{H} = (x + 20t)\hat{a}_z$
 A/m. Assume $\mu = 0.25 \text{ H/m}$.

$$\rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\therefore \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & kx - 100t & 0 \end{vmatrix} = -\mu (0 + 20)\hat{a}_z$$

$$\therefore 0 - 100(k(1) - 0)\hat{a}_z = -20\mu \hat{a}_z$$

$$k = -20\mu$$

$$\therefore k = -20 \times \frac{1}{4}$$

$$\therefore \boxed{k = -5 \text{ V/m}^2}$$

unit is meter
 $\nearrow m$
 $k(x) = -50 \text{ V/m}$
 $\therefore \boxed{k = -50 \text{ V/m}^2}$

* EM Waves:

⇒ Linear Medium:-

→ A medium is said to be linear in that medium $\underline{\bar{D}}, \underline{\bar{E}}$ must have same direction (or) $\underline{\bar{B}}, \underline{\bar{H}}$ must have same direction.
It does not mean that all are having same direction.

⇒ Homogeneous Medium:-

→ Usually at high frequency medium^{properties} is characterised by μ and ϵ . If these are constant throughout the medium then the medium is said to be a homogeneous medium.

⇒ Isotropic Medium:-

→ In this medium μ and ϵ scalar constant.

In general,

$$\begin{aligned} \mu &= \mu' - j\mu'' \\ \epsilon &= \epsilon' - j\epsilon'' \\ \mu &= \mu_0 \mu_r \\ \epsilon &= \epsilon_0 \epsilon_r \end{aligned}$$

Real part
↓
Imag. part

①	②	③
$\mu_1 = \mu_0 \mu_{r1}$	$\mu_2 = \mu_2' - j\mu_2''$	$\mu_3 = \mu_0 \mu_{r3}$
$\epsilon_1 = \epsilon_0 \epsilon_{r1}$	$\epsilon_2 = \epsilon_2' - j\epsilon_2''$	$\epsilon_3 = \epsilon_0 \epsilon_{r3}$
homo 1	homo 2	homo 3

↑ Isotropic ↑

- Real part of μ & ϵ indicate storage property of medium.
- Imaginary part of μ & ϵ indicate, dissipate property of medium.
- An isotropic medium is homogenous
 ✓ whereas homogenous medium need not be isotropic.
- Charge free medium: $\rho_e = 0$.
- Non-Conducting medium: $\sigma = 0$.

* Unbounded medium:

- There are no boundaries to meet in any direction.
- We assume that the wave is propagating through a linear homogenous isotropic charge free non-conducting and unbounded medium.
- ⇒ Writing the Maxwell's eqⁿ for the above assumed medium.

$$(1) \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$(2) \nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \quad (\text{Non conducting medium is assumed } \sigma = 0).$$

(3) $\nabla \cdot \bar{D} = 0$ ($\because$ Charge free medium is assumed $\epsilon_r = 0$).

$$\Rightarrow \nabla \cdot \epsilon \bar{E} = 0$$

$\Rightarrow \nabla \cdot \bar{E} = 0$ ($\because$ homogeneous medium is assumed).

$$(4) \quad \nabla \cdot \bar{H} = 0.$$

$$\nabla \cdot \bar{B} = 0.$$

$\rightarrow$ Taking curl on (1) both the sides.

$$\nabla \times \nabla \times \bar{E} = -\mu \nabla \times \frac{\partial \bar{H}}{\partial t}.$$

$$\therefore \underbrace{\nabla(\nabla \cdot \bar{E})}_0 - \nabla^2 \bar{E} = -\mu \frac{\partial}{\partial t} \underbrace{(\nabla \times \bar{H})}_{+\frac{\partial \bar{E}}{\partial t} \times \epsilon}.$$

$$\therefore \boxed{\nabla^2 \bar{E} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}} \rightarrow \text{Vector wave eqn.}$$

Similarly taking curl on (2) both sides.

$$\therefore \boxed{\nabla^2 \bar{H} = \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2}}$$

$\rightarrow$ For simplicity Let us solved the problem in Cartesian coordinate.

$\rightarrow$ Expanding wave eqn in Cartesian coordinate.

$$\therefore \left. \begin{aligned} \frac{\partial^2 \bar{E}_x}{\partial x^2} + \frac{\partial^2 \bar{E}_x}{\partial y^2} + \frac{\partial^2 \bar{E}_x}{\partial z^2} &= \mu \epsilon \frac{\partial^2 \bar{E}_x}{\partial t^2} \\ \frac{\partial^2 \bar{H}_x}{\partial x^2} + \frac{\partial^2 \bar{H}_x}{\partial y^2} + \frac{\partial^2 \bar{H}_x}{\partial z^2} &= \mu \epsilon \frac{\partial^2 \bar{H}_x}{\partial t^2} \end{aligned} \right\} \begin{array}{l} \text{These are} \\ \text{2nd order} \\ \text{4 dim.} \\ \text{PDE'S.} \end{array}$$

→ $\bar{E}$ and $\bar{H}$ are fns of time and Space Coordinates.

→ For simplicity Let us assumed that wave is propagating along z -direction in unbounded medium.

→ Since there are no boundaries to meet along x & y directions. Then we conclude the partial variation of any field Component with respect to x and y i.e.

$$\frac{\partial}{\partial y} () = 0, \quad \frac{\partial}{\partial x} () = 0.$$

$\xrightarrow{\hspace{10em}} (z).$

→ The eqⁿ reduced to

$$\frac{\partial^2 \bar{E}}{\partial z^2} = \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t^2}.$$

Simillary $\frac{\partial^2 \bar{H}}{\partial z^2} = \mu \epsilon \frac{\partial^2 \bar{H}}{\partial t^2}.$

These are 2nd order
2-Dim
PDES.

→ We assumed Charge free region:

$$\nabla \cdot \bar{E} = 0.$$

$$\therefore \frac{\partial (E_x)}{\partial x} + \frac{\partial (E_y)}{\partial y} + \frac{\partial (E_z)}{\partial z} = 0.$$

$$0 + 0 + \frac{dE_z}{dz} = 0.$$

$\therefore E_z$ may be zero (or) constant.

→ Max. Value of D.D. = $|\nabla \phi|$.

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(OR)
Greatest rate of increase.

* Angle betⁿ the two surfaces:

→ Let, $\phi_1(x, y, z) = c_1$ &

$\phi_2(x, y, z) = c_2$

$$\therefore \cos \theta = \frac{\nabla \phi_1 \cdot \nabla \phi_2}{|\nabla \phi_1| \cdot |\nabla \phi_2|}.$$

* For Solenoidal vector $\nabla \cdot \vec{F} = 0$.

* For Irrotational vector $\nabla \times \vec{F} = 0$.

* Green Theorem in a plane.

$$\rightarrow \oint M(x, y) dx + N(x, y) dy = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] dx dy.$$

* Del operator (∇).

$$\rightarrow \nabla = \frac{\partial}{\partial x} \bar{a}_x + \frac{\partial}{\partial y} \bar{a}_y + \frac{\partial}{\partial z} \bar{a}_z.$$

* gradient of a scalar field:

$$\rightarrow \text{grad } V = \nabla V = \frac{dV}{dV} \bigg|_{\max} \bar{a}_n.$$

* Divergence of a vector:

$$\therefore \text{grad } V = \nabla V = \frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z.$$

* Divergence of a vector:

$$\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

*

	Coordinate	Multiplier	Unit vector
General	$u \ v \ w$	$h_1 \ h_2 \ h_3$	$\bar{a}_u \ \bar{a}_v \ \bar{a}_w$
Cartesian	$x \ y \ z$	$1 \ 1 \ 1$	$\bar{a}_x \ \bar{a}_y \ \bar{a}_z$
Cylindrical	$\rho \ \phi \ z$	$1 \ \rho \ 1$	$\bar{a}_\rho \ \bar{a}_\phi \ \bar{a}_z$
Spherical	$r \ \theta \ \phi$	$1 \ r \ r \sin \theta$	$\bar{a}_r \ \bar{a}_\theta \ \bar{a}_\phi$

$$\rightarrow \nabla \phi = \frac{1}{h_1} \frac{\partial \phi}{\partial u} \hat{a}_u + \frac{1}{h_2} \frac{\partial \phi}{\partial v} \bar{a}_v + \frac{1}{h_3} \frac{\partial \phi}{\partial w} \bar{a}_w.$$

$$\rightarrow \nabla \cdot \bar{A} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} (h_2 h_3 A_u) + \frac{\partial}{\partial v} (h_3 h_1 A_v) + \frac{\partial}{\partial w} (h_1 h_2 A_w) \right].$$

$$\rightarrow \nabla \times \bar{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \bar{a}_u & h_2 \bar{a}_v & h_3 \bar{a}_w \\ \frac{\partial}{\partial v} & \frac{\partial}{\partial w} & \frac{\partial}{\partial u} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$$

$$\rightarrow \nabla^2 \phi = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} \left(\frac{h_2 h_3}{h_1} \frac{\partial \phi}{\partial u} \right) + \frac{\partial}{\partial v} \left(\frac{h_1 h_3}{h_2} \frac{\partial \phi}{\partial v} \right) + \frac{\partial}{\partial w} \left(\frac{h_1 h_2}{h_3} \frac{\partial \phi}{\partial w} \right) \right]$$

$$\rightarrow \nabla^2 \bar{A} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial}{\partial u} \left(\frac{h_2 h_3}{h_1} \frac{\partial \bar{A}_1}{\partial u} \right) + \frac{\partial}{\partial v} \left(\frac{h_1 h_3}{h_2} \frac{\partial \bar{A}_1}{\partial v} \right) + \frac{\partial}{\partial w} \left(\frac{h_1 h_2}{h_3} \frac{\partial \bar{A}_1}{\partial w} \right) \right]$$

* Divergence Theorem:

$$\int_V (\nabla \cdot \bar{A}) dv = \oint_S \bar{A} \cdot d\bar{s}$$

surface to
volume

$$\therefore \oint_S \bar{A} \cdot d\bar{s} = \int_V \nabla \cdot \bar{A} dv$$

* Stoke's Theorem:

$$\int_S (\nabla \times \bar{A}) \cdot d\bar{s} = \oint_C \bar{A} \cdot d\bar{l}$$

line to
surface

$$\therefore \oint_C \bar{A} \cdot d\bar{l} = \int_S \nabla \times \bar{A} \cdot d\bar{s}$$

* Directional derivative (D.O):

→ The Directional derivative of a diffⁿ scalar function in the direction of vector $\bar{a}$ is

$$D.O = \nabla \phi \cdot \frac{\bar{a}}{|\bar{a}|}$$

* Unit Vector, Vector and its magnitude. 165

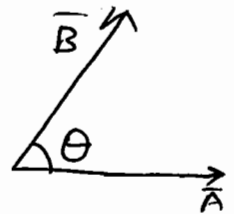
$$\rightarrow \hat{a}_{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

$$\rightarrow \vec{AB} = (x_2 - x_1) \hat{a}_x + (y_2 - y_1) \hat{a}_y + (z_2 - z_1) \hat{a}_z$$

$$\rightarrow |\vec{AB}| \text{ or } AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

* Scalar or Dot Product:

$$\rightarrow \boxed{\vec{A} \cdot \vec{B} = AB \cos \theta}$$

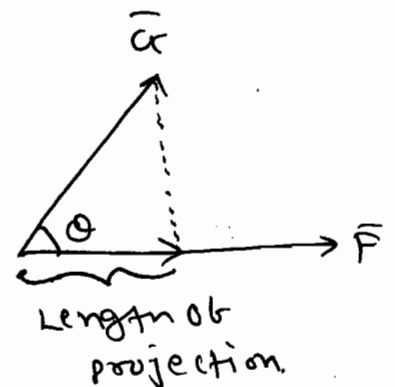


where θ = angle betⁿ $\vec{A}$ & $\vec{B}$

* Length of Projection:

per

$$\rightarrow \text{Length of projection} = \vec{C} \cdot \vec{a}_F$$



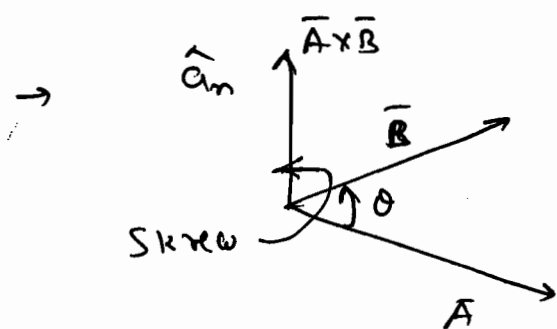
$$\rightarrow \text{Vector projection} = (\vec{C} \cdot \vec{a}_F) \vec{a}_F$$

$$= \frac{\vec{C} \cdot \vec{F}}{F^2} \times \vec{F}$$

* Cross Product:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\rightarrow \bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$$



$$\therefore \bar{A} \times \bar{B} = AB \sin \theta \hat{u}_m$$

* Application of cross product:

$$\rightarrow \text{Area of parallelogram} = |\bar{AB} \times \bar{AC}| \checkmark$$

$$\rightarrow \text{Area of the triangle ABC} = \frac{1}{2} |\bar{AB} \times \bar{AC}| \checkmark$$

* Scalar Triple Product:

$$\rightarrow \bar{A} \times (\bar{B} \times \bar{C}) = \bar{A} \cdot \bar{C} (\bar{B}) - \bar{A} \cdot \bar{B} (\bar{C}).$$

$$\rightarrow \bar{A} \cdot \bar{B} \times \bar{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

* Coordinates Systems:

1. Cartesian (or) rectangular coordinate system
2. Cylindrical coordinate system
3. Spherical coordinate system.

① Cartesian (or) Rectangular Coordinate System. 167

→ Any point in Cartesian system is the intersection of $x = \text{constant}$, $y = \text{constant}$ and the $z = \text{constant}$ planes.

→ point in Cartesian system = $P(x, y, z)$

Unit vectors are $\bar{a}_x, \bar{a}_y, \bar{a}_z$.

$$d\bar{l} = dx, dy, dz$$

$$\rightarrow d\bar{l} = dx \bar{a}_x + dy \bar{a}_y + dz \bar{a}_z.$$

$$\rightarrow d\bar{s} = dy dz \hat{a}_x \quad (x = \text{constant})$$

$$d\bar{s} = dz dx \hat{a}_y \quad (y = \text{constant}).$$

$$d\bar{s} = dx dy \hat{a}_z \quad (z = \text{constant}).$$

$$\rightarrow dV = dx dy dz.$$

② Cylindrical System:

→ Point is (ρ, ϕ, z) .

→ Unit vectors $\bar{a}_\rho, \bar{a}_\phi, \bar{a}_z$.

→ Differential lengths are $d\rho, \rho d\phi, dz$.

$$\rightarrow d\bar{l} = d\rho \bar{a}_\rho + \rho d\phi \bar{a}_\phi + dz \bar{a}_z.$$

$$d\bar{s} = \rho d\phi dz \hat{a}_\rho \quad (\rho = \text{constant}).$$

$$d\bar{s} = d\rho dz \hat{a}_\phi \quad (\phi = \text{constant}).$$

$$d\bar{s} = \rho d\rho d\phi \hat{a}_z \quad (z = \text{constant}).$$

③ Spherical coordinate system:

$$\rightarrow \rho(r, \theta, \phi).$$

$$\rightarrow \hat{a}_r, \hat{a}_\theta, \hat{a}_\phi.$$

$$\rightarrow dr, r d\theta, r \sin\theta d\phi.$$

$$\rightarrow d\vec{s} = r^2 \sin\theta d\theta d\phi \hat{a}_r \quad (r = \text{constant})$$

$$\rightarrow d\vec{s} = r \sin\theta dr d\phi \hat{a}_\theta \quad (\theta = \text{constant}).$$

$$\rightarrow d\vec{s} = r d\theta d\phi \hat{a}_\phi \quad (\phi = \text{constant}).$$

$$\rightarrow d\vec{r} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin\theta d\phi \hat{a}_\phi.$$

* Transformation from Cartesian to cylindrical vectors and vice versa:

	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
$\hat{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\hat{a}_y$	$\sin\phi$	$\cos\phi$	0
$\hat{a}_z$	0	0	1

$$x = \rho \cos\phi$$

$$y = \rho \sin\phi$$

$$z = z$$

$$[\vec{B}]_y = [A^T][\vec{B}]_x$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$

* Transformation of vectors from Cartesian to spherical or vice versa:

	$\hat{a}_r$	$\hat{a}_\theta$	$\hat{a}_\phi$
$\hat{a}_x$	$\sin\theta \cos\phi$	$\cos\theta \cos\phi$	$-\sin\phi$
$\hat{a}_y$	$\sin\theta \sin\phi$	$\cos\theta \sin\phi$	$\cos\phi$
$\hat{a}_z$	$\cos\theta$	$-\sin\theta$	0

$$x = \rho \sin\theta \cos\phi$$

$$y = \rho \sin\theta \sin\phi$$

$$z = \rho \cos\theta$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \cos^{-1}(z/\rho)$$

$$\phi = \tan^{-1}(y/x).$$