Using Variables in Generalising Patterns

We know that letter of the English alphabet like *a*, *b*, *c*, *x*, *y*, *z*, *l*, *m*, *n*, etc. can be used as variables to form expressions.

In our daily life, we see various patterns of objects or numbers which help us to predict or determine some information.



Now, observe the following design made by a number of As.



We can find the total number of match sticks involved in this design by counting them, which is a time consuming process.

There is one more way to solve such problems.

Now, we can easily find the number of match sticks involved in the above design as follows:

Number of match sticks in one A = 5

Number of A's in the design = 8

Number of match sticks involved in design = $5 \times$ Number of A's in design = $5 \times 8 = 40$

Now, let us learn how to use variables to generalize patterns found in real life.

Suppose there are 15 flowering plants in one row in a garden. How many plants are there in the garden, if there are 30 such rows?

The total number of flowering plants in the garden depends on the number of rows in the garden.

If the number of rows = 3 then total number of plants = $15 \times 3 = 45$

If number of rows = 4 then total number of plants = $15 \times 4 = 60$

: Total number of plants = $15 \times$ number of rows in the garden

If we substitute the variable *n* in place of the number of rows, then the above rule becomes:

Total number of plants = $15 \times n = 15n$, where $n = 1, 2, 3, 4 \dots$

According to the considered situation, n = 30

Thus, total number of plants = $15 \times 30 = 450$

In the same way, we can apply the concept of variables in many real life situations.

Let us now understand this concept better by solving some more examples.

Example 1:

In a class, students have been seated in few rows such that 5 students are there in each row. Generalize this pattern for n rows. Also, find the total number of students in the class if there are 8 rows in total.

Solution:

We can find the total number of students as follows:

Total number of students in class = Number of students in one row \times Number of rows

 \Rightarrow Total number of students in class = 5 × Number of rows

When the number of rows is n.

Total number of students in class = $5 \times n = 5n$

When the number of rows is 8.

Total number of students in class = $5n = 5 \times 8 = 40$

Example 2:

Observe the following pattern made by match sticks.





Solution:

From the figure, we have

Number of match sticks in 1^{st} column = 4

Number of match sticks in 2^{nd} column = 4 + 3

Number of match sticks in 3^{rd} column = 4 + 3 + 3 = 4 + 3 × 2

Number of match sticks in 4^{th} column = $4 + 3 + 3 + 3 = 4 + 3 \times 3$

Thus, it can be generalized as follows:

Number of match sticks in a column = $4 + 3 \times ($ Number of that column - 1)

: Number of match sticks in n^{th} column = 4 + 3 × (n – 1)

Thus, number of match sticks in 20^{th} column = $4 + 3 \times (20 - 1) = 4 + 57 = 61$

Using Variables in Geometric Formulae

Algebraic formulae have applications in many areas. They are also used extensively in geometry, where they are used to shorten the geometric formulae.

For example, we know that the perimeter of an equilateral triangle is three times the length of its side. If we denote the perimeter of the triangle by variable P and the length of the side of the triangle by l, then we can restate the formulae as:

P = 3l

Here, the value of P will change with a change in the value of l. In this formula, P and l are variables and 3 is a constant.

Similarly, the perimeter of a square is written as:

P = 4l

Here, the variables P and l represent the perimeter and length of the side of the square respectively, whereas 4 is a constant.

The perimeter of a regular pentagon is written as:

the variables P and l represent the perimeter and length of the side of

P = 5l

Again, the variables P and l represent the perimeter and length of the side of the pentagon respectively and 5 is a constant.

Let us now write the formula for the perimeter of a rectangle using variables. We know that the perimeter of a rectangle is twice the sum of its length and breadth.

If we denote variables P, l, and b for the respective perimeter, length, and breadth of the rectangle, then the formula of the perimeter of the rectangle in terms of algebraic expression will be written as:

$$P=2\ (l+b)$$

Here, *P*, *l*, and *b* are variables and 2 is a constant.

Now, let us try to denote the formulae of areas of some geometrical figures like triangle, rectangle, square, etc in terms of algebraic expression and variables. Let us start with the area of a rectangle.

We know that the area of a rectangle is the product of its length and breadth.

If we denote the variables A, l, and b for the area, length, and breadth of the rectangle, then the formula of area of rectangle in terms of variables is

$$A = l \times b = lb$$

Here, A, l, and b are variables. This expression does not contain any constant.

We know that, area of a square = side \times side

If we denote the variables *A* and *l* for the area and length of the side of the square, then the formula of area of the square in terms of variables is

$$A = l \times l = l^2$$

Here, A and l are variables. This expression also does not contain any constant.

We know that the area of a triangle equals one-half the product of the length of its base and its corresponding height.

If we denote the variables *A*, *b*, and *h* for the area, base, and height of the triangle, then the formula of area of triangle is

$$A = \frac{1}{2} \times b \times h = \frac{bh}{2}$$

Here, A, b, and h are variables and $\frac{1}{2}$ is a constant.

Now, we can easily find the perimeter or area of a polygon by putting in the values of the variables in the given formula. For example, for a rectangle of length 7 cm and breadth 4 cm, the perimeter is obtained by putting the value l = 7 cm and b = 4 cm in the formula, P = 2 (l + b).

Now, the perimeter of the given rectangle, P = 2 (7 + 4) cm = 22 cm

We can also find the area of this rectangle by putting the value l = 7 cm and b = 4 cm in the formula, A = lb.

Now, the area of the given rectangle, $A = lb = (7 \times 4) \text{ cm}^2 = 28 \text{ cm}^2$

Similarly, we can express the formulae used in different areas in terms of variables.

For example, the formula of loss in any transaction is as follows:

Loss = Cost price – Selling price

If we denote loss by *l*, cost price by *c* and selling price by *s* then the above formula can be expressed as follows:

l = c - s

Now, let us discuss some examples to have better understanding of use of variables in geometric formulae.

Example 1:

The perimeter of a regular octagon (an eight-sided polygon) is given by the formula

L = 8s, where L and s are the perimeter and length of the side of the octagon.

Identify the variables and constant in this formula.

Solution:

The formula for the perimeter of a regular octagon is L = 8s.

Here, the value of *L* changes with a change in the value of *s*. However, the value 8 does not change. Therefore, *L* and *s* are variables and 8 is a constant.

Example 2:

Derive the rule to find the perimeter of a regular pentagon by representing the length of one side by a variable *l*.

Solution:

The length of one side of a regular pentagon is *l*, where *l* is a variable.



Perimeter of a regular pentagon = $5 \times \text{length of one side}$

: Perimeter of a regular pentagon = $5 \times l = 5l$

Let p be the perimeter of the pentagon. Then, the following rule is obtained.

P = 5l

Example 3:

Derive the rule to find the diameter of a ball by taking the radius of the ball as the variable *r*.



Solution:

We know that the diameter of a ball is twice the radius of the ball.

: Diameter of the ball = $2 \times \text{radius}$ of the ball = $2 \times r = 2r$

Using Variables In Generalising Properties

We know that whole numbers are commutative under addition. According to this property, if the order of numbers is interchanged in addition, it does not affect the result.

For example,

2 + 3 = 5 and 3 + 2 = 5 $\therefore 2 + 3 = 3 + 2$ 11 + 14 = 25 and 14 + 11 = 25 $\therefore 11 + 14 = 14 + 11$

Is there any other way to write commutative property of whole numbers?

We can easily write this property in a general way by the use of variables. Therefore, let us take two variables p and q, where p and q are any two whole numbers.

Therefore, we can write the commutative property of whole numbers as follows: p + q = q + p, where *p* and *q* are whole numbers.

Similarly, the commutative property also holds true under multiplication. We can write this property in a general way by making use of variables, say p and q, where p and q are whole numbers.

We can write the rule for the **commutativity of multiplication** of whole numbers as

follows: $p \times q = q \times p$, where *p* and *q* can be any two whole numbers.

In a similar way, we can make use of variables in generalising the distributive and the associative property also. Let us see how.

Distributive property of multiplication over addition

Consider three numbers 6, 25, and 5. We have, $6 \times (25 + 5) = 6 \times 30 = 180$ Also, $6 \times 25 + 6 \times 5 = 150 + 30 = 180$ $\therefore 6 \times (25 + 5) = 6 \times 25 + 6 \times 5$

This property is known as distributive property and is applicable for all whole numbers. Let us now write the generalised form of this property by using variables. Let us take three variables *a*, *b*, and *c* where *a*, *b*, and *c* are whole numbers. Using these variables, we can write the distributive property as follows: $a \times (b + c) = a \times b + a \times c$, where *a*, *b*, and *c* are any three whole numbers.

Associative property under addition and multiplication

Consider the whole numbers 9, 15, and 46. Then, according to the associative property, we obtain

9 + (15 + 46) = 9 + 61 = 70Also, (9 + 15) + 46 = 24 + 46 = 70 $\therefore 9 + (15 + 46) = (9 + 15) + 46$

By using the variables p, q, and r, the associative property can be written as follows: p + (q + r) = (p + q) + r, where p, q, and r are whole numbers.

Similarly, the associative property under multiplication can be written as follows: $p \times (q \times r) = (p \times q) \times r$, where *p*, *q*, and *r* are whole numbers.

Now, let us observe the results, when a whole number is multiplied with 0. $1 \times 0 = 0$; $5 \times 0 = 0$; $99 \times 0 = 0$; $625 \times 0 = 0$, etc.

It can be seen that the product of any number with 0, gives 0 as result. Thus, the rule can be written as follows: number $\times 0 = 0$ or, $x \times 0 = 0$, where *x* is a whole number.

Example 1.

Identify the property used in the a(b + c) = ab + ac. Answer:

Distributive property of multiplication over addition is used in the given expression.

Example 2.

Fill in the blanks in the following expression and mention the property used.

- 1. $3a \times (2b + c) = 6ab + _$
- 2. $a \times (-c) = (-c) \times _$
- $3. \quad (x \times 2y) \times 9z = x \times (_)$
- 4. $2p + _ = 4r + _$

Answer:

- 1. $3a \times (2b + c) = 6ab + 3ac$ (Distributive property of multiplication over addition)
- 2. $a \times (-c) = (-c) \times a$ (Commutativity of multiplication)
- 3. $(x \times 2y) \times 9z = x \times (2y \times 9z)$ (Associative property under multiplication)
- 4. 2p + 4r = 4r + 2p (Commutativity of addition)

Introduction of Variables and Expressions

We know that there are infinitely many numbers. However, there are only 26 alphabets in the English language.

Can we represent numbers with the help of letters?

Yes, we can represent the numbers with the help of alphabets(letters). Alphabets like a, b, c, x, y, z, l, m, n, etc. are used in Mathematics to denote variables.

Let us take an example.

In a class, the number of boys are 20 more than the number of girls. How many boys are there in the class?

The number of boys in the class varies with the number of girls. If there are 10 girls in the class, then the number of boys = 10 + 20 = 30If there are 55 girls in the class, then the number of boys = 55 + 20 = 75Therefore, we can write a rule to find the number of boys as follows: Number of boys = Number of girls + 20

Here, number of girls in the class can vary, and can take different values. On the other hand, number of boys in the class varies according to the number of girls in the class. If we replace the number of boys and girls in the class with

letters *m* and *n* respectively, then we get following expression:

$$m = n + 20$$

Here, n can take any value such as 0, 1, 2, 3... etc., and the value of m will change accordingly. Since the values of *m* and *n* can vary, these are known as variables.

A variable is something that does not have a fixed value. The value of a variable varies.

Also, a symbol with a fixed numeric value is known as a constant.

For example, 2, -4, $\sqrt{8}$, -3.4, $\frac{1}{2}$ etc., are constants as each of them have a fixed numeric value.

A combination of variables, numbers, and operators $(+, -, \times, \text{ and } \div)$ is known as an algebraic expression.

For example,

1.
$$x + 7$$

2.
$$2 - y$$

3.
$$(5 \times y) +$$

2. $2^{-5}y$ 3. $(5 \times y) + 9$ 4. $11xyz + ab^2 - 2p^4q^3 + \frac{3}{4}$ 5. $\frac{a^2b}{cd} + 2p^5 - 1.5z$

Let us try to form few simple mathematical expressions by applying the four operations on numbers.

> 58 + 4225

- 1. 42 is added to 58 and then the result is divided by 25 =
- 2. The product of 24 and 15 is subtracted from the product of 30 and $43 = (30 \times 43) -$ (24×15)

In the same way, we can form algebraic expressions by applying the four operations on variables.

$$\frac{z}{5}$$
 + 5

- 1. z divided by 5 and 5 added to the result = 5
- 2. x multiplied by 6 and 4 added to the product = 6x + 4
- 3. 39 subtracted from 5m = 5m 39
- 4. 5 added to 6*m* and the result is subtracted from 8n = 8n (6m + 5)
- 5. 16 subtracted from 7x and the result is subtracted from -2y = -2y (7x 16)

In this way, we can represent a given real-life situation by using variables. Let us now solve some more examples to understand the concept better.

Example 1: Write down the following expressions in words.

1.
$$5x + 19$$

2. $6y - 2$
3. $\frac{z}{2} + 2y$
4. $(2y + 5) - 8$
5. $6 - (m + 7)$
 $\frac{2x - 5}{5}$

Solution:

(i) x is multiplied with 5 and then 19 is added to the product.

(ii) y is multiplied by 6 and then 2 is subtracted from the product.

(iii) z is divided by 2 and then 2y is added to the result.

(iv) 5 is added to 2y and 8 is subtracted from the result.

Or, 5 is added to the product of 2 and y and then 8 is subtracted from the result.

(v) 7 is added to m and the result is subtracted from 6.

(vi) 5 is subtracted from the product of 2 and x and then the result is divided by 5.

Example 2:

Form six expressions using two numbers 8 and 11 and variable *a*.

Solution:

- 1. 8a + 11
- 2. 11a + 8
- 3. 8a 11

4.
$$\frac{a}{8} + 11$$

5.
$$\frac{a}{11} + 8$$

6. $\frac{a}{11} - 8$

Example 3: Write down the expressions for the following situations.

- 1. Diganta's age is 2 years more than 4 times Arjun's age.
- 2. What is the length of a rectangular field, if its length is 3 m less than twice its breadth?
- 3. The number of boys in a class is 8 less than 3 times the number of girls. Find the total number of students in the class.
- 4. Sonu is two times taller than Monu. Find the height of Sonu.
- 5. What will be the age of Aman after 14 years from now?

Solution:

- 1. Let Arjun's age be z years. Therefore, four times Arjun's age is 4z.
 - \therefore Diganta's age = 4z + 2
- 2. Let the breadth of the rectangular field be *a* m.

: Length of the rectangular field = (2a - 3) m

- 3. Let the number of girls be *p*.
 - \therefore Number of boys = 3p 8

Thus, total number of students in the class = p + 3p - 8 = 4p - 8

- 4. Let the height of Monu be h cm.
 - : The height of Sonu will be 2h cm.
- 4. Let the present age of Aman be *y* years.
 - : The age of Aman after 14 years will be y + 14 years.

Example 4:

Sonu is twice as old as Monu. Find the rule to find Sonu's age if Monu's age is taken as x.

Solution:

It is given that Sonu is twice as old as Monu.

The rule can be written as: Sonu's age = $2 \times$ Monu's age = 2x, where $x = 1, 2, 3, 4, 5 \dots$

Example 5:

The price of Mohit's book is Rs 3 less than 3 times the price of Rohit's book. Find the rule to find the price of Mohit's book.

Solution:

Here, the price of Mohit's book is given in terms of the price of Rohit's book. Let the price of Rohit's book be Rs x.

: Price of Mohit's book = Rs $(3 \times \text{price of Rohit's book} - 3)$ = Rs (3x - 3)

Example 6:

Sachin has 5 apples more than Suhaan. Find the rule to find the number of apples with Suhaan.

Solution:

Sachin has 5 apples more than Suhaan. This means that Suhaan has 5 apples less than Sachin.

Let the number of apples with Sachin be n.

∴ Number of apples with Suhaan = Number of apples with Sachin -5 = n-5

Example 7:

The speed of a train is 70 km/h. Find a rule for the total distance covered by the train in *x* hours.

Solution:

Speed of the train = 70 km/h

Total time taken by the train to cover the distance = x h

- \therefore Total distance covered by the train = speed × total time taken
- $= 70 \times \text{total time taken}$
- $= 70 \times x$
- = 70x km

Introduction to Equations

Consider the expressions $27 \div 3$ and 5 + 4. Both give the same value i.e., 9.

Thus, $27 \div 3 = 5 + 4$

In the above statement, value of the expression on either side of = sign is equal. Such a statement is known as **equality**.

Few more examples of equality are as follows:

 $5 \times 3 = 8 + 7$ $46 - 20 = 2 \times 13$ $6 \times 4 = 72 \div 3$

Properties of equality:

If we perform the same operation on both sides of the equality, then the values so obtained are equal.

Let us consider the equality $5 \times 3 = 8 + 7$ to verify these properties.

Addition property of equality:

If the same number is added to both sides of the equality then the values so obtained are equal.

For example, $(5 \times 3) + 4 = (8 + 7) + 4$ $\Rightarrow 15 + 4 = 15 + 4$ $\Rightarrow 19 = 19$

Subtraction property of equality:

If the same number is subtracted from both sides of the equality then the values so obtained are equal.

For example, $(5 \times 3) - 10 = (8 + 7) - 10$ $\Rightarrow 15 - 10 = 15 - 10$ $\Rightarrow 5 = 5$

Multiplication property of equality:

If the same number is multiplied to both sides of the equality then the values so obtained are equal.

For example, $(5 \times 3) \times 6 = (8 + 7) \times 6$ $\Rightarrow 15 \times 6 = 15 \times 6$ $\Rightarrow 90 = 90$

Division property of equality:

If both sides of the equality are divided by the same number then the values so obtained are equal.

For example, $(5 \times 3) \div 3 = (8 + 7) \div 3$ $\Rightarrow 15 \div 3 = 15 \div 3$ $\Rightarrow 5 = 5$

These properties can be generalised as follows:

If we have p = q, then

(1) Addition property: p + r = q + r

- (2) Subtraction property: p r = q r
- (3) Multiplication property: $p \times r = q \times r$
- (4) Division property: $p \div r = q \div r$

These properties are very basic, but very useful.

Now, let us consider a situation. Suppose Ritu has 12 marbles more than Raj. If Ritu has 40 marbles, then how many marbles does Raj have?

Let the number of marbles with Raj be *x*. The mathematical statement for the given situation is: 40 = x + 12This is an equation.

Thus, we can say

"An equation is a condition to find the value of a variable".

It is to be noted that an equation must have an 'equal sign' (=). The value on the right hand side (R.H.S.) and the left hand side (L.H.S.) of the 'equal sign' (=) must be equal, i.e., L.H.S. = R.H.S.

x + 15 = 25, 2y = 32, 3p + 1 = 4 etc. are the examples of equations.

Note: If L.H.S. is greater than R.H.S. or vice-versa, i.e., if L.H.S. > R.H.S. or L.H.S. < R.H.S., then it is not an equation. For example, x + 3 > 9, 2y < 14 etc. are not equations.

Let us now solve some examples to understand the concept of equations better.

Example 1:

Shalini is 7 years younger than Sandhya. If Shalini is 18 years old, then what will be the equation for this situation?

Solution:

Let Sandhya be y years old. According to the question, Sandhya's age -7 = Shalini's age i.e., y - 7 = 18

Example 2:

The length of a rectangular park is twice its breadth. If the breadth is 26 m, then what will be the equation for this situation?

Solution:

Let the length of the rectangular park be *l*. According to the question, Length of the park = $2 \times$ Breadth of the park i.e., $l = 2 \times 26$

Example 3:

Sanjay got 5 marks less than twice the marks that Vinay got in Mathematics. Sanjay got 81 marks. Which of the following equations satisfy this situation?

- 1. 2m 5 = 81
- 2. 5*m* − 2 = 81
- 3. 2*m* 81 = 5
- 4. 5m 81 = 5

Here, *m* is the marks obtained by Vinay.

Solution:

Given, marks obtained by Vinay = mNow, the equation for the above situation is as follows. $2 \times \text{Vinay's marks} - 5 = \text{Sanjay's marks}$ i.e., 2m - 5 = 81Thus, the first equation is the correct answer.

Example 4:

Which of the following expressions are equations containing some variable? Also, write the variable present in those equations.

(i) a + 5 = 11(ii) 2a + 5 < 2(iii) 2p + 18 = 14(iv) $-\frac{x}{5} - 22 = 39$ (v) $9 - 5 = 2 \times 2$ (vi) $25 - 5 > \frac{q}{2}$ (vii) 3 = 4x

Solution:

Equation (v), $9-5=2 \times 2$, contains only numbers. Therefore, this is a numerical equation without any variable.

The expressions (ii), 2a + 5 < 2, and (vi), $25 - 5 > \frac{q}{2}$, do not contain an 'equal sign' (=). They contain '>' and '<' signs. Therefore, (ii) and (vi) are not equations.

The equations (i), (iii), (iv) and (vii) are equations with a variable. The variables present in these equations are given in the following table.

Equation	Variable
(i) $a + 5 = 11$ (iii) $2p + 18 = 14$ (iv) $-\frac{x}{5} - 22 = 39$	a p x
(iv) $3 = 4x$	x

Solution of Equations by Trial and Error Method

Consider the equation 2x - 5 = 7.

Can we solve this equation?

Solving an equation means finding the value of the variable contained in that equation for which the L.H.S. and R.H.S. of the equation are equal. The value of the variable that satisfies the equation is called the **solution** of the equation.

Let us solve the above equation, 2x - 5 = 7.

For this, we have to find the value of the variable for which the L.H.S. and R.H.S. are equal.

Now, for x = 6, the L.H.S. of the equation becomes $2 \times 6 - 5 = 12 - 5 = 7$

Thus, for x = 6, the L.H.S. and R.H.S. of the above equation are equal,

i.e., for x = 6, the equation 2x - 5 = 7 is satisfied.

 $\therefore x = 6$ is the solution of the equation 2x - 5 = 7.

Observe that if we put any other number instead of 6 as the value of the variable x, then it will not satisfy the equation.

For example,

If x = 5, then $2x - 5 = 2 \times 5 - 5 = 10 - 5 = 5 \neq 7$

If x = 7, then $2x - 5 = 2 \times 7 - 5 = 14 - 5 = 9 \neq 7$

Thus, the equation will be satisfied only by x = 6. Therefore, 6 is the only solution of the given equation.

But a question that arises is how do we know the solution of a given equation?

A method known as **trial and error method** is used to find the solution of a given equation. In this method, we find the solution of the given equation by putting different values for the variable. The value of the variable that satisfies the equation is the solution of the equation.

Let us solve some more examples to understand this concept better.

Example 1:

Solve the following equations using trial and error method.

1.
$$x - 5 = 37$$

2. $\frac{k}{2} = 6$

Solution:

1. x - 5 = 37

If x = 30, then $x - 5 = 30 - 5 = 25 \neq 37$

If x = 40, then $x - 5 = 40 - 5 = 35 \neq 37$

If x = 42, then x - 5 = 42 - 5 = 37

Now, x = 42 satisfies the given equation.

Therefore, the solution of the given equation is x = 42.

2. $\frac{k}{2} = 6$

If
$$k = 8$$
, then $\frac{k}{2} = \frac{8}{2} = 4 \neq 6$
If $k = 10$, then $\frac{k}{2} = \frac{10}{2} = 5 \neq 6$
If $k = 12$, then $\frac{k}{2} = \frac{12}{2} = 6$

Now, k = 12 satisfies the given equation.

Therefore, the solution of the given equation is k = 12.

Example 2:

State whether the following statements are correct or incorrect.

1. x = 6 is the solution to the equation 3x = 16.

- 2. t = 3 is the solution to the equation $\frac{t}{3} + 5 = 6$ 3. u = 9 is the solution to the equation $\frac{t}{3} + 5 = 6$ 3. u = 9 is the solution to the equation u + 21 = 36.
- 4. l = 0 is the solution to the equation 7l + 8 = 8.

Solution:

- 1. 3x = 16
 - If x = 6, then $3x = 3 \times 6 = 18 \neq 16$

Therefore, the statement is incorrect.

2.
$$\frac{t}{3} + 5 = 6$$

If $t = 3$, then $\frac{t}{3} + 5 = \frac{3}{3} + 5 = 1 + 5 = 6$

Therefore, the statement is correct.

3. u + 21 = 36

If u = 9, then $u + 21 = 9 + 21 = 30 \neq 36$

Therefore, the statement is incorrect.

4. 7l + 8 = 8

If l = 0, then $7l + 8 = 7 \times 0 + 8 = 0 + 8 = 8$

Therefore, the statement is correct.