

## CHAPTER 9

### DIFFERENTIAL EQUATIONS

#### VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the order and degree of the following differential equations.

(i)  $\frac{dy}{dx} + \cos y = 0.$

(ii)  $\left(\frac{dy}{dx}\right)^2 + 3\frac{d^2y}{dx^2} = 4.$

(iii)  $\frac{d^4y}{dx^4} + \sin x = \left(\frac{d^2y}{dx^2}\right)^5.$

(iv)  $\frac{d^5y}{dx^5} + \log\left(\frac{dy}{dx}\right) = 0.$

(v)  $\sqrt{1 + \frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}.$

(vi)  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2}.$

(vii)  $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 = \sin x.$

(viii)  $\frac{dy}{dx} + \tan\left(\frac{dy}{dx}\right) = 0$

2. Write the general solution of following differential equations.

$$(i) \quad \frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}.$$

$$(ii) \quad (e^x + e^{-x}) dy = (e^x - e^{-x}) dx$$

$$(iii) \quad \frac{dy}{dx} = x^3 + e^x + x^e.$$

$$(iv) \quad \frac{dy}{dx} = 5^{x+y}.$$

$$(v) \quad \frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}.$$

$$(vi) \quad \frac{dy}{dx} = \frac{1 - 2y}{3x + 1}.$$

3. Write integrating factor of the following differential equations

$$(i) \quad \frac{dy}{dx} + y \cos x = \sin x$$

$$(ii) \quad \frac{dy}{dx} + y \sec^2 x = \sec x + \tan x$$

$$(iii) \quad x^2 \frac{dy}{dx} + y = x^4.$$

$$(iv) \quad x \frac{dy}{dx} + y \log x = x + y$$

$$(v) \quad x \frac{dy}{dx} - 3y = x^3$$

$$(vi) \quad \frac{dy}{dx} + y \tan x = \sec x$$

$$(vii) \quad \frac{dy}{dx} + \frac{1}{1+x^2} y = \sin x$$

4. Write order of the differential equation of the family of following curves

$$(i) \quad y = Ae^x + Be^{x+c}$$

$$(ii) \quad Ay = Bx^2$$

$$(iii) \quad (x-a)^2 + (y-b)^2 = 9$$

$$(iv) \quad Ax + By^2 = Bx^2 - Ay$$

$$(v) \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0.$$

$$(vi) \quad y = a \cos (x + b)$$

$$(vii) \quad y = a + be^{x+c}$$

### SHORT ANSWER TYPE QUESTIONS (4 MARKS)

5. (i) Show that  $y = e^{m \sin^{-1} x}$  is a solution of

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0.$$

- (ii) Show that  $y = \sin(\sin x)$  is a solution of differential equation

$$\frac{d^2 y}{dx^2} + (\tan x) \frac{dy}{dx} + y \cos^2 x = 0.$$

- (iii) Show that  $y = Ax + \frac{B}{x}$  is a solution of  $\frac{x^2 d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$ .

- (iv) Show that  $y = a \cos(\log x) + b \sin(\log x)$  is a solution of

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0.$$

- (v) Verify that  $y = \log(x + \sqrt{x^2 + a^2})$  satisfies the differential equation :

$$(a^2 + x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0.$$

- (vi) Find the differential equation of the family of curves

$$y = e^x (A \cos x + B \sin x), \text{ where } A \text{ and } B \text{ are arbitrary constants.}$$

- (vii) Find the differential equation of an ellipse with major and minor axes  $2a$  and  $2b$  respectively.

- (viii) Form the differential equation representing the family of curves  $(y - b)^2 = 4(x - a)$ .

6. Solve the following differential equations.

$$(i) \quad \frac{dy}{dx} + y \cot x = \sin 2x. \quad (ii) \quad x \frac{dy}{dx} + 2y = x^2 \log x.$$

$$(iii) \quad \frac{dx}{dy} + \frac{1}{x} \cdot y = \cos x + \frac{\sin x}{x}, \quad x > 0.$$

$$(iv) \quad \cos^3 x \frac{dy}{dx} + \cos x = \sin x.$$

$$(v) \quad ydx + (x - y^3) dy = 0$$

$$(vi) \quad ye^y dx = (y^3 + 2xe^y) dy$$

7. Solve each of the following differential equations :

$$(i) \quad y - x \frac{dy}{dx} = 2 \left( y^2 + \frac{dy}{dx} \right).$$

$$(ii) \quad \cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0.$$

$$(iii) \quad x\sqrt{1-y^2} \, dy + y\sqrt{1-x^2} \, dx = 0.$$

$$(iv) \quad \sqrt{(1-x^2)(1-y^2)} \, dy + xy \, dx = 0.$$

$$(v) \quad (xy^2 + x) \, dx + (yx^2 + y) \, dy = 0; \, y(0) = 1.$$

$$(vi) \quad \frac{dy}{dx} = y \sin^3 x \cos^3 x + xy e^x.$$

$$(vii) \quad \tan x \tan y \, dx + \sec^2 x \sec^2 y \, dy = 0$$

8. Solve the following differential equations :

$$(i) \quad x^2 y \, dx - (x^3 + y^3) \, dy = 0.$$

$$(ii) \quad x^2 \frac{dy}{dx} = x^2 + xy + y^2.$$

$$(iii) \quad (x^2 - y^2) \, dx + 2xy \, dy = 0, \quad y(1) = 1.$$

$$(iv) \left( y \sin \frac{x}{y} \right) dx = \left( x \sin \frac{x}{y} - y \right) dy. \quad (v) \frac{dy}{dx} = \frac{y}{x} + \tan \left( \frac{y}{x} \right).$$

$$(vi) \frac{dy}{dx} = \frac{2xy}{x^2 + y^2} \quad (vii) \frac{dy}{dx} = e^{x+y} + x^2 e^y.$$

$$(viii) \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}.$$

$$(ix) (3xy + y^2) dx + (x^2 + xy) dy = 0$$

9. (i) Form the differential equation of the family of circles touching y-axis at (0, 0).  
 (ii) Form the differential equation of family of parabolas having vertex at (0, 0) and axis along the (i) positive y-axis (ii) positive x-axis.  
 (iii) Form differential equation of family of circles passing through origin and whose centre lie on x-axis.  
 (iv) Form the differential equation of the family of circles in the first quadrant and touching the coordinate axes.

10. Show that the differential equation  $\frac{dy}{dx} = \frac{x+2y}{x-2y}$  is homogeneous and solve it.

11. Show that the differential equation :

$$(x^2 + 2xy - y^2) dx + (y^2 + 2xy - x^2) dy = 0 \text{ is homogeneous and solve it.}$$

12. Solve the following differential equations :

$$(i) \frac{dy}{dx} - 2y = \cos 3x.$$

$$(ii) \sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x \text{ if } y\left(\frac{\pi}{2}\right) = 1$$

$$(iii) 3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$$

13. Solve the following differential equations :

$$(i) \quad (x^3 + y^3) dx = (x^2y + xy^2)dy.$$

$$(ii) \quad x dy - y dx = \sqrt{x^2 + y^2} dx.$$

$$(iii) \quad y \left\{ x \cos \left( \frac{y}{x} \right) + y \sin \left( \frac{y}{x} \right) \right\} dx \\ - x \left\{ y \sin \left( \frac{y}{x} \right) - x \cos \left( \frac{y}{x} \right) \right\} dy = 0.$$

$$(iv) \quad x^2 dy + y(x + y) dx = 0 \text{ given that } y = 1 \text{ when } x = 1.$$

$$(v) \quad x e^{\frac{y}{x}} - y + x \frac{dy}{dx} = 0 \text{ if } y(e) = 0$$

$$(vi) \quad (x^3 - 3xy^2) dx = (y^3 - 3x^2y)dy.$$

$$(vii) \quad \frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec} \left( \frac{y}{x} \right) = 0 \text{ given that } y = 0 \text{ when } x = 1$$

16. Solve the following differential equations :

$$(i) \quad \cos^2 x \frac{dy}{dx} = \tan x - y.$$

$$(ii) \quad x \cos x \frac{dy}{dx} + y (x \sin x + \cos x) = 1.$$

$$(iii) \quad \left( 1 + e^{\frac{x}{y}} \right) dx + e^{\frac{x}{y}} \left( 1 - \frac{x}{y} \right) dy = 0.$$

$$(iv) \quad (y - \sin x) dx + \tan x dy = 0, y(0) = 0.$$

### LONG ANSWER TYPE QUESTIONS (6 MARKS EACH)

17. Solve the following differential equations :

$$(i) \quad (x \, dy - y \, dx) y \sin\left(\frac{y}{x}\right) = (y \, dx + x \, dy) x \cos\left(\frac{y}{x}\right)$$

$$(ii) \quad 3e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0 \text{ given that } y = \frac{\pi}{4}, \text{ when } x = 1.$$

$$(iii) \quad \frac{dy}{dx} + y \cot x = 2x + x^2 \cot x \text{ given that } y(0) = 0.$$

### ANSWERS

- 1.(i) order = 1, degree = 1      (ii) order = 2, degree = 1  
(iii) order = 4, degree = 1      (iv) order = 5, degree is not defined.  
(v) order = 2, degree = 2      (vi) order = 2, degree = 2  
(vii) order = 3, degree = 2      (viii) order = 1, degree is not defined

$$2.(i) \quad y = \frac{x^6}{6} + \frac{x^3}{6} - 2 \log |x| + c \quad (ii) \quad y = \log_e |e^x + e^{-x}| + c$$

$$(iii) \quad y = \frac{x^4}{4} + e^x + \frac{x^{e+1}}{e+1} + c. \quad (iv) \quad 5^x + 5^{-y} = c$$

$$(v) \quad 2(y - x) + \sin 2y + \sin 2x = c. \quad (vi) \quad 2 \log |3x + 1| + 3 \log |1 - 2y| = c.$$

$$3.(i) \quad e^{\sin x} \quad (ii) \quad e^{\tan x}$$

$$(iii) \quad e^{-1/x} \quad (iv) \quad e^{\frac{(\log x)^2}{2}}$$

$$(v) \quad \frac{1}{x^3} \quad (vi) \quad \sec x$$

(vii)  $e^{\tan^{-1} x}$

4.(i) 1

(ii) 1

(iii) 2

(iv) 1

(v) 1

(vi) 1

(vii) 2

5.(vi)  $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$

(vii)  $x \left( \frac{dy}{dx} \right)^2 + xy \frac{d^2 y}{dx^2} = y \frac{dy}{dx}$

(viii)  $2 \frac{d^2 y}{dx^2} + \left( \frac{dy}{dx} \right)^3 = 0$

6.(i)  $y \sin x = \frac{2 \sin^3 x}{3} + c$

(ii)  $y = \frac{x^2 (4 \log_e x - 1)}{16} + \frac{c}{x^2}$

(iii)  $y = \sin x + \frac{c}{x}, x > 0$

(iv)  $y = \tan x - 1 + ce^{-\tan x}$

(v)  $xy = \frac{y^4}{4} + c$

(vi)  $x = -y^2 e^{-y} + cy^2$

7.(i)  $cy = (x + 2)(1 - 2y)$

(ii)  $(e^x + 2) \sec y = c$

(iii)  $\sqrt{1 - x^2} + \sqrt{1 - y^2} = c$

(iv)  $\frac{1}{2} \log \left| \frac{\sqrt{1 - y^2} - 1}{\sqrt{1 - y^2} + 1} \right| = \sqrt{1 - x^2} - \sqrt{1 - y^2} + c$

(v)  $(x^2 + 1)(y^2 + 1) = 2$



$$\begin{aligned} \text{(vi)} \quad \log y &= -\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x + xe^x - e^x + c \\ &= \frac{1}{16} \left[ \frac{\cos^3 2x}{3} - \cos 2x \right] + (x-1)e^x + c \end{aligned}$$

$$\text{(vii)} \quad \log |\tan y| - \frac{\cos 2x}{y} = c$$

$$\text{8.(i)} \quad \frac{-x^3}{3y^3} + \log |y| = c$$

$$\text{(ii)} \quad \tan^{-1} \left( \frac{y}{x} \right) = \log |x| + c$$

$$\text{(iii)} \quad x^2 + y^2 = 2x$$

$$\text{(iv)} \quad y = ce^{\cos(x/y)} \quad [\text{Hint : Put } \frac{1}{x} = v]$$

$$\text{(v)} \quad \sin \left( \frac{y}{x} \right) = cx$$

$$\text{(vi)} \quad c(x^2 - y^2) = y$$

$$\text{(vii)} \quad -e^{-y} = e^x + \frac{x^3}{3} + c$$

$$\text{(viii)} \quad \sin^{-1} y = \sin^{-1} x + c$$

$$\text{(ix)} \quad x \log(x^3 y) + y = cx$$

$$\text{9.(i)} \quad x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

$$\text{(ii)} \quad 2y = x \frac{dy}{dx}, \quad y = 2x \frac{dy}{dx}$$

$$\text{(iii)} \quad x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

$$\text{(iv)} \quad (x-y)^2 (1+y')^2 = (x+yy')^2$$

$$\text{10.} \quad \log |x^2 + xy + y^2| = 2\sqrt{3} \tan^{-1} \left( \frac{x+2y}{\sqrt{3x}} \right) + c$$

$$\text{11.} \quad \frac{x^3}{x^2 + y^2} = \frac{c}{x} (x+y)$$

$$12.(i) \quad y = \frac{3 \sin 3x}{13} - \frac{2 \cos 3x}{13} + ce^{2x} \quad (ii) \quad y = \frac{2}{3} \sin^2 x + \frac{1}{3} \operatorname{cosec} x$$

$$(iii) \quad \tan y = k(1 - e^x)^3$$

$$13.(i) \quad -y = x \log \{c(x - y)\}$$

$$(ii) \quad cx^2 = y + \sqrt{x^2 + y^2}$$

$$(iii) \quad xy \cos\left(\frac{y}{x}\right) = c$$

$$(iv) \quad 3x^2y = y + 2x$$

$$(v) \quad y = -x \log(\log|x|), \quad x \neq 0$$

$$(vi) \quad c(x^2 + y^2) = \sqrt{x^2 - y^2}.$$

$$(vii) \quad \cos \frac{y}{x} = \log|x| + 1$$

$$16. \quad (i) \quad y = \tan x - 1 + ce^{\tan^{-1} x}$$

$$(ii) \quad y = \frac{\sin x}{x} + c \frac{\cos x}{x}$$

$$(iii) \quad x + ye^{\frac{x}{y}} = c$$

$$(iv) \quad 2y = \sin x$$

$$17. \quad (i) \quad cxy = \sec\left(\frac{y}{x}\right)$$

$$(ii) \quad (1 - e)^3 \tan y = (1 - e^x)^3$$

$$(iii) \quad y = x^2.$$