Vector Algebra

Que 1: ***Marks :(1)***

If $\vec{a}$ is a unit vector perpendicular to $\vec{b}$ and $\vec{c}$

Then find a second unit vector perpendicular to $\vec{b}$ and $\vec{c}$

Ans:

If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, then $-\vec{a}$ is

also perpendicular to $\vec{b}$ and $\vec{c}$

Que 2: ***Marks :(1)***

What is the angle between $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$

Ans:

$\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$ have opposite directions. So angle between them is 180^0

Que 3: ***Marks :(3)***

If $\vec{a}$ and$\vec{b}$ are unit vectors such that $\left|\vec{a}+\vec{b}\right| = \sqrt{3}$. Find the value of$(\vec{a}+\vec{b}).(\vec{a}-2\vec{b})$

Ans:

$$\left|\vec{a}+\vec{b}\right| = \sqrt{3}. \left|\vec{a}+\vec{b}\right|^2 = |\vec{a}|^2 + \left|\vec{b}\right|^2 + 2\vec{a}.\vec{b} = 3$$

$$1 + 1 + 2\vec{a}.\vec{b} = 3$$

$$\vec{a}.\vec{b} = \frac{1}{2}$$

$$(\vec{a}+\vec{b}).(\vec{a}-2\vec{b}) = \vec{a}.\vec{a} - 2\vec{a}.\vec{b} + \vec{a}.\vec{b} - 2\vec{b}.\vec{b}$$

$$= |\vec{a}|^2 - \vec{a}.\vec{b} - 2\left|\vec{b}\right|^2 = 1 - \frac{1\cdot}{2} - 2 = \frac{-3}{2}$$

Que 4: ***Marks :(3)***

Given vectors $\vec{a} = 3\hat{i} - 6\hat{j} - \hat{k}, \vec{b} = \hat{i} + 4\hat{j} - 3\hat{k}, \vec{c} = 3\hat{i} - 4\hat{j} - 12\hat{k}$. Find projection of $\vec{a} \times \vec{b}$ on $\vec{c}$

Ans:

$$Projection\ of\ \vec{a}\times\vec{b}\ on\ \vec{c}=\frac{(\vec{a}\times\vec{b}).\vec{c}}{|\vec{c}|}=\frac{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}{|\vec{c}|}$$

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}3 & -6 & -1\\ 1 & 4 & -3\\ 3 & -4 & -12\end{vmatrix}=-182$$

$$|\vec{c}|=\sqrt{9+16+144}=\sqrt{169}=13$$

$$Hence\ projection=\frac{-182}{13}=-14$$

Que 5: ***Marks :(3)***

Find a vector of magnitude 12 *units perpendicular to the plane containing the vectors* $4\hat{i}+6\hat{j}-\hat{k}$ *and* $3\hat{i}+8\hat{j}+\hat{k}$

Ans:

A vector perpendicular to given vectors is $\vec{a}\times\vec{b}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}i & j & k\\ 4 & 6 & -1\\ 3 & 8 & 1\end{vmatrix}=14\hat{i}-7\hat{j}+14\hat{k}$$

$$unit\ vector\ along\ \vec{a}\times\vec{b}=\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}=\frac{14\hat{i}-7\hat{j}+14\hat{k}}{21}=\frac{2\hat{i}-\hat{j}+2\hat{k}}{3}$$

$$Vector\ of\ magnitude\ 12\ along\ \vec{a}\times\vec{b}=12(\frac{2\hat{i}-j+2\hat{k}}{3})$$

$$=8\hat{i}-4\hat{j}+8\hat{k}$$