GRAPHS →

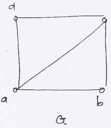
A graph G is defined as a pair of sets

$G = (V, E)$

where V → set of all vertices/nodes/points.

& E → set of all edges in the graph.

|V| → no. of vertices in G.
OR
<u>Oredeř of graph G.</u>



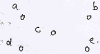
$|V| = 4$

$|E| = 5.$

|E| → no. of edges in the graph.
OR
<u>Size of the graph G.</u>

* <u>NULL Graph</u> →

A graph no edges is called a "Null graph".

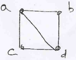


* <u>Trivial graph</u> →

A null graph with only one vertex is called "trivial graph."

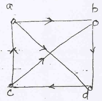
* Non directed Graph → (Undirected Graph)

In a non-directed graph, each edge is represented by a set of two vertices $\{V_i, V_j\}$



$\{V_i, V_j\}$ = an edge betn V_i and V_j.

* Directed Graph (Di-Graph) →

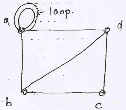


In a digraph, each edge is represented by an ordered pair of two vertices V_i and V_j.

(V_i, V_j) = An edge from V_i to V_j

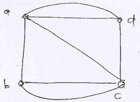
* Loop →

An edge drawn from a vertex to itself is called a "loop".



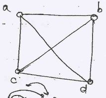
* Parallel edges →

In a graph, if a pair of vertices are allowed to join by more than one edge, then those edges are called "Parallel edges" and the resulting or corresponding graph is called a "multigraph".



Simple Graph →

A graph with no loops and no parallel edges is called a "Simple Graph".



max. no of edges here = 6.

$\{a, b, c, d\}$. : 6

Max. no. of edges possible in a simple graph with 'n' vertices

$$= {}^{n}C_2 = \boxed{\frac{n(n-1)}{2}}$$

No. of simple graphs possible with 'n' vertices

no. of edges.

$= 2^{nC_2} = 2^{\left(\frac{n \cdot (n-1)}{2}\right)}$

Q.1. No. of simple graphs poss with 5 vertices & 4 edges?

→ Max no. of edges possible with 5 vertices

$= 5C_2 = 10.$

No. of ways we can choose we can choose any 4 edges.

$= 10C_4 = 10 \times 9 \times 8 \times 7 = 210.$

∴ Required no. of graphs $= 210$

Q.2. No. of simple graphs poss. with n vertices & m edges?

→ With n vertices, max. no. of edges poss.

$= nC_2 = \frac{n \cdot (n-1)}{2}.$

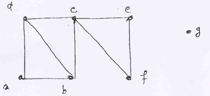
∴ we can select m edges from $\frac{n \cdot (n-1)}{2}$

edges in $\boxed{\frac{n \cdot (n-1)}{2} C_m}$ ways.

$\boxed{C\left(\frac{n \cdot (n-1)}{2}, m\right)}$

Connected Graph →

A graph 'G' is said to be "connected" if there exists a path betⁿ every pair of vertices.



A graph which is not connected will have two or more connected components

(OR)

One or more connected components and an isolated vertex.

Degree of a vertex (V) →

denoted by deg (v)

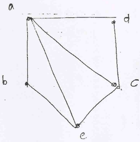
= number of edges incident with the vertex V.

A vertex with degree '0' is called an 'isolated vertex'.

A vertex with degree '1' is called 'pendant vertex'.

In a simple graph with 'n' vertices, degree of any vertex V is less than or equal to (n-1) i.e. $\deg(V) \leq (n-1)$ (for all vertices).

$\forall V \in G$



In an undirected graph, a loop at a vertex is counted as 2 edges.

In a digraph,

Indegree of a vertex $V = \deg^{+}(V)$

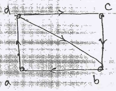
= no. of edges incident to the vertex

(or) no. of incoming edges.

Outdegree of a vertex $V = \deg^{-}(V)$

= no. of edges incident from the vertex

(or) no. of outgoing edges.



$\deg^{+}(a) = 1$

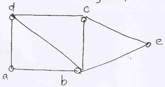
$\deg^{-}(a) = 1$

* Loops are not allowed in a "digraph"

⊛ Adjacency →

neighbours ↓

*In a graph, two vertices are said to be adjacent if there exists an edge betⁿ the two vertices.



a and c are not adjacent.

*In a graph, two edges are said to adjacent, if there exists a common vertex for the two edges.

⊛ Degree Sequence →

If the degrees of all the vertices in the graph G are arranged in ascending or descending order, then the sequence so obtained is called "degree sequence" of the graph.

$\{4, 3, 3, 2, 2\}$.

⊛ $\delta(G)$ → min of the degrees of all vertices in G.

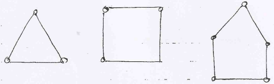
⊛ $\Delta(G)$ → max of the degrees of all vertices in G.

Regular Graph →

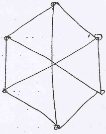
A graph is said to "Regular" if all the vertices have same degree.

In a graph,
If degree of each vertex is 'k', then the graph is called "k-regular graph".

ex → a polygon is a 2-regular graph.



3-regular graph →

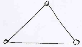


Complete Graph →

A simple graph with 'n' mutually adjacent vertices is called a complete graph and it is denoted by 'K_n'.

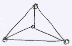
ex. → K_2 → complete graph with two mutually adjacent vertices.

K_2



K_3

K_4.



K_4.

K_5

In a complete graph K_n, degree of each vertex is $(n-1)$.

(***) :- every complete graph is a "Regular Graph".

No. of edges in K_n = $^nC_2$ = $\frac{n(n-1)}{2}$.

Every complete graph is a simple graph with max no. of edges.

(*) Cycle Graph →

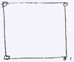
$(n \geq 3)$

A simple graph with 'n' vertices and n edges is called a "cycle graph" if all the edges form a cycle of length 'n'. A cycle graph with n vertices is denoted by 'C_n'

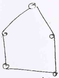
(**)

C_3

→ the only cycle graph which is also complete is C_3.



C_4



C_5

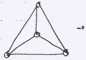
C_6.

(*) Wheel Graph →

A wheel graph with n vertices $(n \geq 4)$ can be obtained from a cycle graph C_{n-1} by adding a new vertex (called hub) which is adjacent to all vertices in C_{n-1}.

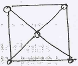
It is denoted by 'W_n'.

ex-

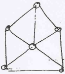


→ The only wheel graph which is also complete graph.

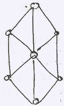
$W_4 = K_4$.



W_5



W_6.



W_7.

In a wheel graph, degree of hub is $(n-1)$

No. of edges in $W_n = 2(n-1)$

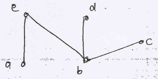
Cyclic graph →

A graph with at least one cycle is called as 'cyclic graph'.

Every cycle graph and wheel graph and complete graph are cyclic graphs too.

Acyclic graph →

A graph with no cycles is called acyclic graph.



Tree →

A connected acyclic graph is called a 'tree'

(OR)

A connected graph with no cycles is called a 'tree'.

A tree with 'n' vertices has 'n-1' edges

Every tree has at least two vertices with degree '1'

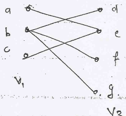
* Forest →

A disconnected acyclic graph is called a 'forest'

* Bipartite Graph →

A simple graph $G = (V, E)$ with vertex partition $V = \{V_1, V_2\}$ is called a bipartite graph if every vertex

A simple graph $G = (V, E)$ with vertex partition $V = \{V_1, V_2\}$ is called 'bipartition graph' if every edge of E, joins a vertex in V_1 to a vertex in V_2.

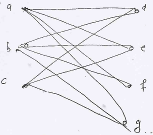


In a biparted graph, no 2 vertices in V_1 and V_2 are adjacent.

9/10/13

* Complete Bipartite graph →

A bipartite graph G with vertex partition, $V = \{V_1, V_2\}$ $(G = (V, E))$, is said to be a "complete bipartite graph" if every vertex in V_1 is adjacent to every vertex in V_2.



$K_{3,4}$.

In general, if $|V_1| = m$ and $|V_2| = n$, then a complete bipartite graph is denoted by "$K_{m,n}$".

$K_{m,n}$ has '$m+n$' vertices and '$m \times n$' edges.

$K_{m,n}$ is a regular graph iff $m = n$

In general, a complete bipartite graph is not a complete graph.

Exception → $K_{1,1}$ is the only complete graph which is also complete bipartite graph.

Max. no. of edges possible in a complete bipartite graph with 'n' vertices is $\left\lfloor \frac{n^2}{4} \right\rfloor$.

ex:-

if $n=10$, then max. no. of edges

$$= \frac{100}{4} = \underline{25}$$

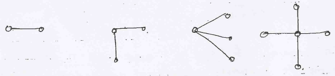
using $K_{5,5}$.

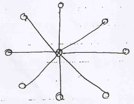
(**) G is a bipartite graph <u>iff</u> G has no cycles of odd length.

A special case of bipartite graph → "star graph".

* Star Graph →

A bipartite graph of the form $K_{1,n-1}$ is a star graph with n vertices. $(n \geq 2)$.



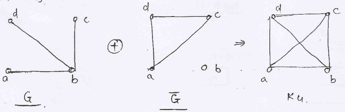


Complement of a graph →

Let 'G' be a simple graph with 'n' vertices.

Complement of G, denoted by $\bar{G}$ is also a simple graph with same vertices as that of G, but an edge $\{u,v\}$ is present in $\bar{G}$ iff the edge is absent in G. i.e. $\{u,v\} \notin G$.

i.e. two vertices are adjacent in $\bar{G}$ iff they are not adjacent in G.



* If G is any simple graph, then no. of edges in G + no. of edges in $\bar{G}$ is always equal to no. of edges in complete graph K_n. when $n = |V(G)|$.

Q.1. Let G be a simple graph with 9 vertices and 12 edges. find no. of edges in $\bar{G}$.

→ we have,

$$n(E(G)) + n(E(\bar{G})) = n(E(K_n))$$

∴ no. of edges in $K_n = \frac{n(n-1)}{2} = \frac{9 \times 8}{2} = 36.$

$n(n-1) = 156$ $n^2 - n + 156 = 0$ 184

$n^2 - n = 156$

$\therefore$ no. of edges in $\bar{G} = 36 - 12 = \boxed{24}$

Q.2. G is a simple graph with 40 edges and $\bar{G}$ has 38 edges. Find no. of vertices in graph.

→ let no. of vertices in graph = n.

we have, $|E(G)| + |E(\bar{G})| = |E(K_n)|$

$= 40 + 38 = 78.$

$\therefore \frac{n(n-1)}{2} = 78. \quad \therefore n(n-1) = 156$

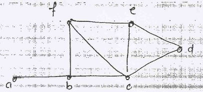
$\therefore n(n-1) = 13 \times 12$

$\therefore \boxed{n = 13}.$

(*) <u>Distance betn two vertices u and v</u> →

$= d(u,v)$

= no. of edges in the shortest path betn u and v.



* Eccentricity of a vertex V → $e(v)$ →

$$e(v) = \max.\ d(u,v) \quad \forall u \in G$$

from prev. graph,

$e(a) = 3$ ⇒ 3. 2 2 3 3 2

$e(b) = 2$

$e(c) = 2$

$e(d) = 3$

$e(e) = 3$

$e(f) = 2$.

* Radius of a connected graph → $r(G)$.

$r(G)$ = min. of the eccentricities of all vertices

∴ For the graph previous,

$r(G) = 2$.

* Diameter of a connected graph → $d(G)$ →

$d(G)$ = max. of eccentricities of all vertices

∴ $d(G) = 3$.

* Central point →

If $e(v) = r(G)$ then v is called as a "central point of G"

For the previous graph, b, c and f are central points of G.

* Centre of a graph →

Set of all central points of a graph is called "Center of the graph".

* Circumference / perimeter of G →

The no. of edges in a longest cycle of G is called circumference of G.

For the prev. graph, circumference is 5.

* Girth of a graph G →

No. of edges in a smallest cycle of G is called Girth of the graph G and is denoted by $g(G)$.

For the prev. graph, girth of the graph is 3.

Sum of degrees of vertices theorem :-

If $G = (V, E)$ be a nondirected graph with vertices $V = \{v_1, v_2, \dots, v_n\}$, then

$$\sum_{i=1}^{n} \deg(v_i) = 2 \times \text{no. of edges.}$$

$$= 2 \times |E|.$$

* Corollary 1) →
Let G be a directed graph with $V = \{V_1, V_2, \dots, V_n\}$ then

$$\sum_{i=1}^{n} \deg^{+}(V_i) = |E|$$

and

$$\sum_{i=1}^{n} \deg^{-}(V_i) = |E|.$$

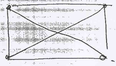
* Corollary 2) →
In any non-directed graph, the no. of vertices with odd degree is always even.

$$\sum_{i=1}^{n} \deg(V_i) = 2 \times E_i$$

* because of this, the sum of degrees of all vertices is always even.
So, no. of vertices with odd degree can not be odd in no.

* Corollary 3) →
In a nondirected graph, if degree of each vertex is 'k', then sum of degrees of all vertices is $k \cdot |V|$.

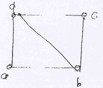
$\therefore k \cdot |V| = 2 \cdot |E|$.



Corollary 4) →

In a nondirected graph, if degree of each vertex is atleast k ($\geq k$), then

$$k \cdot |V| \leq 2 \cdot |E|.$$



$\{3, 3, 2, 2\}$.
d b a c

$\therefore$ $k = 2$ here. $\therefore$ $k \cdot |V| \leq 2 \cdot |E|$

$\therefore$ $2(4) < 10$.

* k is denoted by $\delta(G)$.

Corollary 5) →

In a nondirected graph, if degree of each vertex is atmost k ($\leq k$) then

$$k \cdot |V| \geq 2 \cdot |E|.$$

$\therefore$ In above graph, $k = \Delta(G) = 3$.

$\therefore$ $3(4) \geq 5 \times 2$. ⇒ $12 > 10$.

* Here, k is denoted by $\Delta(G)$.

Note# → For any graph G,

$\delta(G) \cdot |V| \leq 2 \cdot |E|$ and — (cor. 4)

$\Delta(G) \cdot |V| \geq 2 \cdot |E|$ — (cor. 5)

$\delta(G) \cdot |V|$ $|V| \delta(G) \cdot$ $\delta(G)|V| \leq 2E$ $= \frac{14}{5}$

Q.1. Let G be a simple nondirected graph with 5 vertices and 7 edges. Which of the foll. statements are true?

✓a) $\delta(G) \leq 2$

b) $\delta(G) \geq 2$

c) $\Delta(G) \leq 3$

✓d) $\Delta(G) \geq 3$.

a) and d) both are correct.

$\rightarrow \delta(G) \cdot |V| \leq 2 \times 7$.

$\therefore \delta(G) \leq 14/5$

$\therefore \delta(G) \leq 2.8$

$\therefore \boxed{\delta(G) \leq 2}$

Also,

$\Delta(G) \cdot |V| \geq 7 \times 2$.

$\therefore \Delta(G) \geq 14/5 \Rightarrow \Delta(G) \geq 2.8$

$\boxed{\therefore \Delta(G) \geq 3.}$

Q.2. Let G be a simple graph with 21 edges, 3 vertices with degree 4 and rem. vertices with degree 2. find no of vertices in the graph

$21 \times 2 = \frac{42}{12}$ $12+2x = 42$ $2x = 30$ $x = 15$ $K|V| = 2|E|$ $3 \times 4 = 12$

$\rightarrow \sum_{i=1}^{n} \deg(V_i) = 2|E|$

$\therefore 3 \times (4) + x(2) = 21 \times 2$

$\therefore 12 + 2x = 42 \Rightarrow \therefore 2x = 42 - 12$

$\Rightarrow \therefore 2x = 30 \quad \therefore x = 15$

$\therefore$ total no. of vertices $= 15 + 3 = \boxed{18}$.

Q.3. Let G be a simple graph with 38 edges and degree of each vertex is 4. Then $|V(G)| = ?$

$\rightarrow$ By cor. 3, $K \cdot |V| = 2 \cdot |E|$.

$\therefore 4 \cdot |V| = 2 \times 38$

$\therefore |V| = \frac{2 \times 38}{4} = \boxed{19}$.

Q.4. Let G be a simple graph with 24 edges and degree of each vertex is k. Which of the foll. is possible no. of vertices.

a) 6

b) 10

✓ c) 12

~~d) 15~~

$\because$ by cor. 3), degree of each vertex is 'K',

$K \cdot |V| = 2 \cdot |E|$

$\Rightarrow K \cdot |V| = 48.$

$\Rightarrow |V| = \frac{48}{K}$ $(K = 1, 2, 3, 4, 6)$

$K = 5$ not possible

$K = 7, 8$ not possible.

$\therefore$ possible no. of vertices are

48, 24, 16, 12, 8

Q.5. Max. no. of ~~edges~~ vertices possible in a simple graph with 35 edges and degree of each vertex is 3 is ?

$\rightarrow$ by corolary 4), if degree of each vertex is at least k,

then $K \cdot |V| \leq 2 \cdot |E|$

$\therefore 3 \cdot |V| \leq 2 \times 35$

$\therefore |V| \leq 70/3.$

$\therefore |V| \leq 23.$

$\therefore$ max. no. of vertices possible $= \boxed{23}$.

Q.6. Min. no of edges necessary in a simple graph with 13 vertices and degree of each vertex is at least 4.

By cor. 4) $\rightarrow$ $k \cdot |V| \leq 2 \cdot |E|$.

$13 \times 4 \leq 2 \cdot |E|$

$\therefore |E| \geq \frac{13 \times 4}{2}$

$\therefore |E| \geq 26.$

$\therefore$ Min. no. of edges $= \boxed{26.}$

Q.7. Min. no. of vertices necessary in a simple graph with 17 edges and degree of each vertex at most 5.

by cor. 5) $\rightarrow$ $5|V| \geq 34.$

$= |V| \geq 34/5$

$|V| \geq 6.8.$

$\therefore |V| \geq 7$

$\therefore$ Min. no. of vertices $= \boxed{7}$.

Q.8. which of the foll. degree sequences represent a simple nondirected graph. ?

a) $\{2, 3, 3, 4, 4, 5\}$
   a b c d e f

→ sum of degrees of all vertices is odd.

$\boxed{\text{not possible}}$

* (or) no. of odd degree vertices is not even)

b) $\{2, 3, 4, 4, 5\}$.

* The graph has 5 vertices. So, degree of each vertex ≤ 4 always.

∴ $\boxed{\text{not possible.}}$

c) $\{1, 3, 3, 4, 5, 6, 6\}$.
   a b c d e f g.

→ Cannot represent a simple nondirected graph because in a simple graph with 7 vertices if we have 2 vertices with deg. 6 then deg. of every other vertex should be ≥ 2.

∴ a vertex with deg. 1 not possible.

∴ $\boxed{\text{not possible}}$

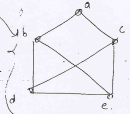
d) $\{0, 1, 2, \ldots, n-1\}$.
$\quad v_1 \quad v_2 \quad v_3 \quad v_n$

→ Cannot represent a simple nondirected graph, because in a simple graph with n vertices, if ther is a vertex with deg 'n-1', then that vertex is adjacent to all other vertices.

∴ a vertex with deg. 0 is not possible.

Note ⇒ In a simple graph with n vertices, $(n \geq 2)$, at least two vertices should have same degree.

e) $\{2, 3, 3, 3, 3\}$.
$\quad a \quad b \quad c \quad d \quad e$



∴ the given degree seq. is possible.

f) $\{3, 3, 3, 1\}$.

→ deg. of 4th vertex cannot be 1.

∴ not possible.

Havel-Hakimi's Result →

Consider the foll. degree sequences I and II and assume that sequence I is in descending order

I : $\{s, t_1, t_2, \ldots, t_s, d_1, d_2, \ldots, d_n\}$.

II : $\{t_1-1, t_2-1, \ldots, t_s-1, d_1, d_2, \ldots, d_n\}$.

⊛ I is graphic (simple nondirected graph) iff II is graphic.

Q.g. which of the foll. deg. sequences represent a simple nondirected graph.?

$S_1)$ $\{6, 6, 6, 6, 4, 3, 3, 0\}$ ← isolated
a b c d e f g h

∴ deg. (f) and deg (g) cannot be 3.

→ $S_1)$ $\{\cancel{6}, \underbrace{6, 6, 6, 4, 3, 3}_{6}, 0\}$

⇓

$S_2)$ $\{\cancel{5}, \underbrace{5, 5, 3, 2, 2}_{5}, 0\}$.

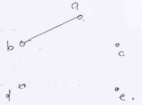
⇓

$S_3)$ $\{\cancel{4}, \underbrace{4, 2, 1, 1}_{4}, 0\}$.

⇓

$S_4)$ $\{\cancel{3}, 1, 0, 0, 0\}$.
a b c d e

can't reduce any further.



The reduced sequence cannot be represented by a simple non-directed graph.

∴ the given sequence also cannot be represented by a simple nondirected graph.

8e) $\{6, 5, 5, 4, 3, 3, 2, 2, 2\}$.

$\rightarrow$ 1) $\{\cancel{6}, 5, 5, 4, 3, 3, 2, 2, 2\}$

$\Downarrow$

2) $\{4, 4, 3, 2, 2, 1, 2, 2\}$

$\Rightarrow \{4, \underbrace{4, 3, 2, 2}, 2, 2, 1\}$

$\Downarrow$

3) $\{3, \underbrace{2, 1, 1}, 2, 2, 1\}$

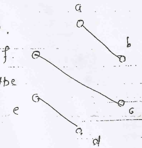
$\Downarrow$

$\Rightarrow \{3, \underbrace{2, 2, 2}, 1, 1, 1\}$

$\Downarrow$

4) $\{1, 1, 1, 1, 1, 1\}$.

So, we can get a simple nondirected graph from the last seq.



∴ we can also draw a simple nondirected graph from the given sequence.

Isomorphic graphs →

Two graphs G_1 and G_2 are said to be isomorphic if there exists a funcⁿ $f : V(G_1) \to V(G_2)$ such that

i) f is a bijection (one-one onto funcⁿ)

and

ii) funcⁿ f preserves adjacency of vertices.
i.e. if any two vertices are adjacent in graph G_1 then the images of these vertices should be adjacent in G_2.

i.e. if the edge $\{u, v\} \in G_1$ then

edge $\{f(u), f(v)\} \in G_2$.

then $G_1 \cong G_2$

isomorphic to

* If G_1 and G_2 are isomorphic, then the foll. condⁿs must hold good →

1) No. of vertices in graph G_1 must be equal to no. of vertices in graph G_2.

i.e. $|V(G_1)| = |V(G_2)|$.

2) $|E(G_1)| = |E(G_2)|$

$cond^{ns}$ 3) The degree sequences of G_1 and G_2 are same.

4) If the vertices $\{v_1, v_2, \cdots, v_k\}$ form a cycle of length 'k' in G_1, then the images of these vertices i.e. $\{f(v_1), f(v_2), \ldots, f(v_k)\}$ should form a cycle of length 'k' in G_2.

Note ⇒ All the above conditions are necessary for graphs G_1 and G_2 to be isomorphic. But these $cond^{ns}$ are not sufficient to prove that graphs are isomorphic.

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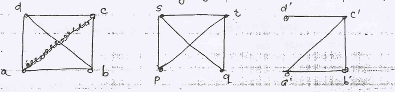
⊛ $(G_1 \cong G_2)$ iff $(\overline{G_1} \cong \overline{G_2})$

⊛ $(G_1 \cong G_2)$ iff the adjacency matrices of G_1 and G_2 are same.

⊛ $(G_1 \cong G_2)$ iff the corresponding subgraphs of G_1 and G_2 ~~are~~ obtained ~~by~~ deleting some vertices in G_1 and there images in G_2) are isomorphic.

Q.1. Which of the following graphs are isomorphic?

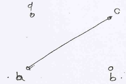


G_1 G_2 G_3

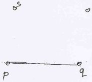
→ In graph G_3, we have only 4 edges.
∴ G_3 is not isomorphic to G_1 or G_2.

Taking the complements of G_1 and G_2, we have,

$\overline{G_1}$ =



$\overline{G_2}$ =



$\overline{G_1} \cong \overline{G_2}$.

$\therefore \boxed{G_1 \cong G_2}$

(OR) by using adjacency matrices,

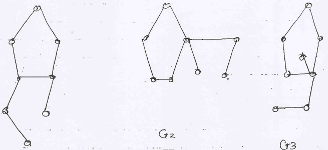
G_1	a	b	c	d
a	0	1	0	1
b	1	0	1	1
c	0	1	0	1
d	1	1	1	0

G_2:

	p	q	r	s
p	0	0	1	1
q	0	1	0	1
r	1	1	1	0
s	1	0	1	1

$\therefore G_1$ & G_2 are isomorphic.

Q.2. Which of the foll. graphs are isomorphic.?



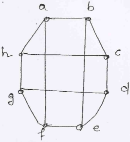
G_1 G_2 G_3

→ G_1 is not isomorphic to G_2, because degree sequences of G_1 and G_2 are not same.

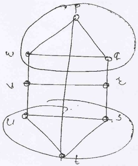
But G_2 and G_3 are isomorphic

$\boxed{G_2 \cong G_3}$

Q.3. which of the foll. graphs are isomorphic?



G_1



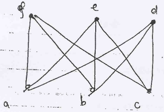
G_2

In G_1, no cycle of length 3 present.

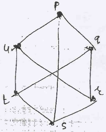
In G_2, cycle of length 3 is there

$\therefore$ G_1 and G_2 are not isomorphic.

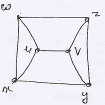
Q.4. Which of the foll. graphs are isomorphic?



G_1

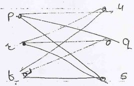


G_2



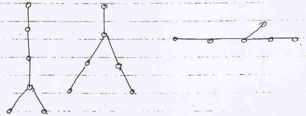
G_3

→ G_1 and G_2 are isomorphic because G_2 can be drawn as an bipartite graph.



G_3 is not isomorphic to G_1 or G_2, in G_3, we have cycles of odd length. So, it cannot be a bipartite graph.

Q.5 Which of the foll. graphs are isomorphic?

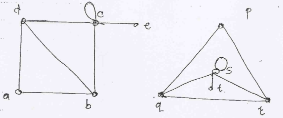


G_1 G_2 G_3

→ G_2 and G_3 are isomorphic.

G_1 and G_2 are not isomorphic bcoz, in the graph G_1 the vertex with degree 3 has two neighbours with degree 1. Whereas, in graph G_2, the vertex with deg. 3 has only one neighbour with deg. 1.

Q 6. Find whether the foll. graphs are isomorphic ?

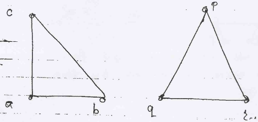


G_1 G_2

→ Comparing vertices of degree 5, we have,
image of c = s.

Further c and s have similar neighbours.

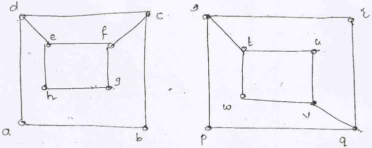
∴ Deleting c and s from G_1 and G_2, we get



The subgraphs are isomorphic.

∴ $\boxed{G_1 \cong G_2}$

Q.7. Find whether the foll. graphs are isomorphic.



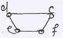
G_1 G_2

→ In G_1, all the vertices form a cycle of length 8. whereas, in G_2, there is no cycle of length 8.

∴ G_1 and G_2 are not isomorphic.

$\{c, d, e, f\}$ $\{q, s, t, v\}$.

G_1



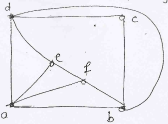
G_2

Here, the 4 vertices of deg 3 form a cycle in G_1 whereas, no cycle is formed by the vertices of deg. 3 in G_2

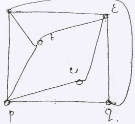
∴ $G_1 \not\cong G_2$.

In graph G_1, a pair of vertices of deg. 2 are adjacent, whereas, in G_2, no two vertices of deg. 2 are adjacent.

Q.8 Find whether the foll. graphs are isomorphic



G1

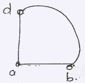


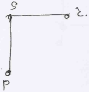
G2.

→ Comparing vertices of deg. 4.

{a, b, d} {p, r, s}.

G1. G2.

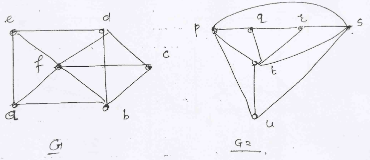




In G_1, the three vertices of deg. 4 form a cycle, whereas, in the second graph, no cycle is formed by the vertices of deg. 3.

$\boxed{G_1 \ncong G_2}$

Q.9. Find whether the foll. graphs are isomorphic?



→ Deleting f and t from G1 and G2.

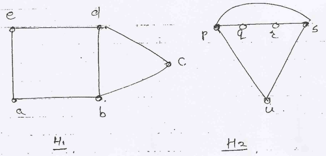
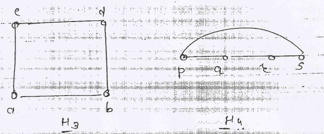


Image of c = u.

∴ Deleting c and u from H1 and H2.

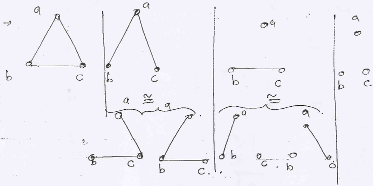


$\therefore H_3 \cong H_4$.

$\therefore \boxed{G_1 \cong G_2}$

Q.10 How many simple nonisomorphic graphs are possible with 3 vertices?

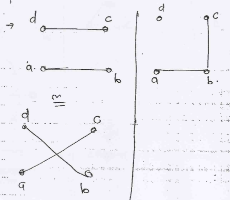
→



$\therefore \boxed{4}$

Q.11 How many simple nonisomorphic graphs are possible with 4 vertices and 2 edges.

→

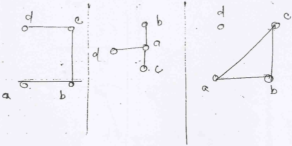


In a simple graph with 4 vertices and 2 edges, the two edges may be adjacent or nonadjacent.

∴ only 2 are possible.

Q. 12. How many simple nonisomorphic graphs are possible with 4 vertices and 3 edges?

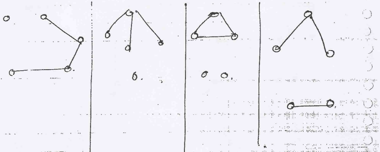
→



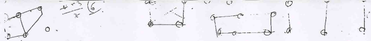
∴ only $\boxed{3}$ possible.

Q.13. How many " — " are poss. with 5 vertices & 3 edges?

→



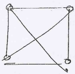
∴ $\boxed{4}$ nonisomorphic graphs possible.



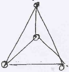
Q.14. " — " with 4 vertices ?

①

4V, 6E.

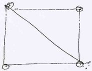


$\cong$ K_4.

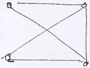


①

4V 5E.

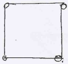


$\cong$



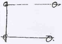
⑤

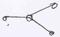
4V 4E.

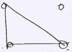


③

4V 3E.

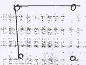


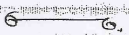




②

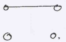
4V 2E.



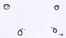


①

4V 1E



4V 0E.



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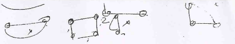
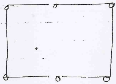


∴ total = 1 + 1 + 2 + 3 + 2 + 1 + 1

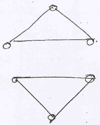
= $\boxed{11}$.

Q.15. How many simple nonisomorphic graphs pos. with 6 vertices, 6 edges and deg. of each vertex is 2?

→ C_6



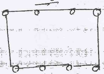
C_6



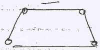
∴ Ans. $\boxed{2}$.

Q.16. " — " with 8 vertices, 8 edges and deg. of each vertex 2.?

→ C_8

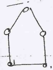


C_4



C_4

C_5

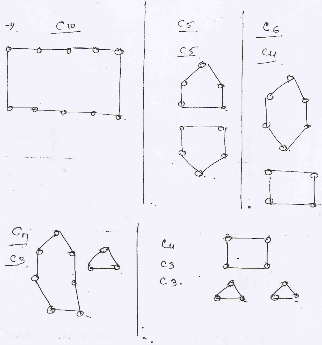


C_3

Ans $\boxed{3}$

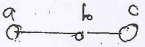
16. " — " 10 vectices & 10 edges and deg of each vectex 2 ?

→



17. How many simple nonisomorphic trees are possible with 3 vectices ?

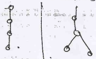
→



ans $\boxed{1}$

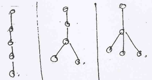
18. " — 4 vectices ?

→

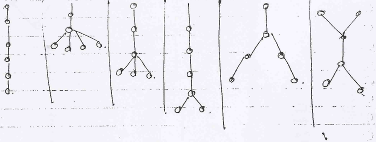


∴ ans. $\boxed{2}$

Q.19. " – " 5 vertices ?

→  ∴ ans $\boxed{3}$.

Q.20 " – " 6 vertices ?

→ 

∴ ans $\boxed{6}$.

Note :

If a simple graph G is isomorphic to its complement $\bar{G}$, then G is said to be "Self-Complementary" Graph.

Results :

Let G be a simple graph with 'n' vertices.

If G is isomorphic to $\bar{G}$ i.e. $G \cong \bar{G}$, then

i) No. of edges in the graph $= \frac{n(n-1)}{4}$

i.e. $\boxed{|E(G)| = \frac{n(n-1)}{4}}$

(ii) No. of vertices in the graph

$$\boxed{|V(G)| = 4k \text{ (or) } (4k+1)} \quad (k = 1, 2, 3, \ldots)$$

Q.21. If a cycle graph C_n is isomorphic to $\overline{C_n}$, $C_n \cong \overline{C_n}$, then $n = ?$

→ By the above result, $|E(C_n)| = \frac{n \cdot (n-1)}{4}$

⇒ ∴ $n = \frac{n(n-1)}{4}$ (By defn of C_n). ($\because C_n \cong \overline{C_n}$)

∴ $n - 1 = 4$.

$\therefore \boxed{n = 5}$

Note: → ∴ C_5 is the only cyclic graph isomorphic to itself.

Q.22. If a tree T with n vertices is isomorphic to $\overline{T}$, then $n = ?$

→ By the above result, no. of edges in tree $= \frac{n \cdot (n-1)}{4}$ ($\because T$ is $\cong \overline{T}$). i.e. $|E(T)| = \frac{n(n-1)}{4}$.

⇒ $(n-1) = \frac{n \cdot (n-1)}{4}$

$\boxed{n = 4}$

* Note → A tree with 4 vertices is isomorphic to its complement.

Q.23. If G is a simple graph with n vertices and $G \cong \bar{G}$, then which of the foll. is not true?

✓ a) $n=6$ b) $n=8$ c) $n=9$ d) $n=13$.

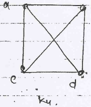
→ Given that $G \cong \bar{G}$. So, no of vertices $= 4K$ (or) $4K+1$

∴ 8 possible 9 ✓ 13 ✓

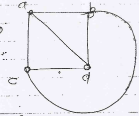
but 6 not possible.

Planar Graph →

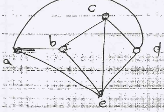
A graph 'G' is said to "planar graph" if it can b drawn on a plane (or) (sphere) so that no two edge cross each other at a nonvertex point.



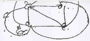
→



So, planar graph.

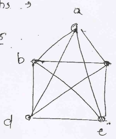


planar graph.

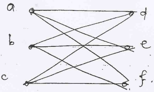


The following are some examples of non planar graphs →

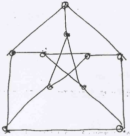
1) K_5



2) $K_{3,3}$.

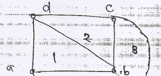


3).

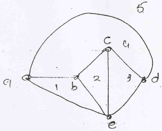


Peterson's Graph.

Regions → Every planar graph divides the plane into connected areas called "Regions".



G_1 ← unbounded region.



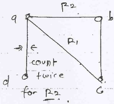
$deg \cdot (1) = 3$

$deg \cdot (2) = 3$

$deg \cdot (3) = 3$

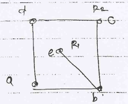
$deg \cdot (4) = 4$

$deg \cdot (5) = 3.$



$deg \cdot (R_1) = 3$

$deg \cdot (R_2) = 5.$ (count ad twice).



$deg \cdot (R_1) = 6.$ ('be' twice)

$deg \cdot (R_2) = 4.$

Degree of a bounded region → deg (R) →

No. of edges enclosing that region R.

Degree of an unbounded (or) exterior region R → deg (R) →

No. of edges exposed to the region R.

$\{a, b, c, d, e, f\}$

$\underline{b}$ should speak after $\underline{a}$

$\overset{\uparrow}{4}$ a $\overset{\uparrow}{4}$ $\overset{\uparrow}{3}$ $\overset{\uparrow}{2}$ 1.

$(4 \times (4!))$

$\overset{\uparrow}{4}$ $\overset{\uparrow}{3}$ a $\overset{\uparrow}{3}$ $\overset{\uparrow}{2}$ $\overset{\uparrow}{1}$.

$(4 \times 3)\ 3!$

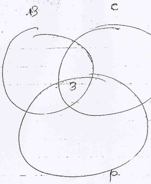
$(4 \times 3 \times 2) \times 2!$

$(4 \times 3 \times 2 \times 1) \times 1$

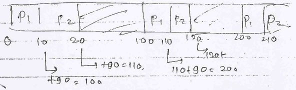
3! +

6 - people. Speeches of a & b → $^6C_2$ ways

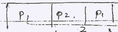
30-


B
C
3
P

FCFS


P1
P2
P1
P2
P1
P2
0
10
20
100
110
120
200
210
+90 = 100
+90 = 110
110+90 = 200
120+

SRTF


P1
P2.
P1
0
1
2
3