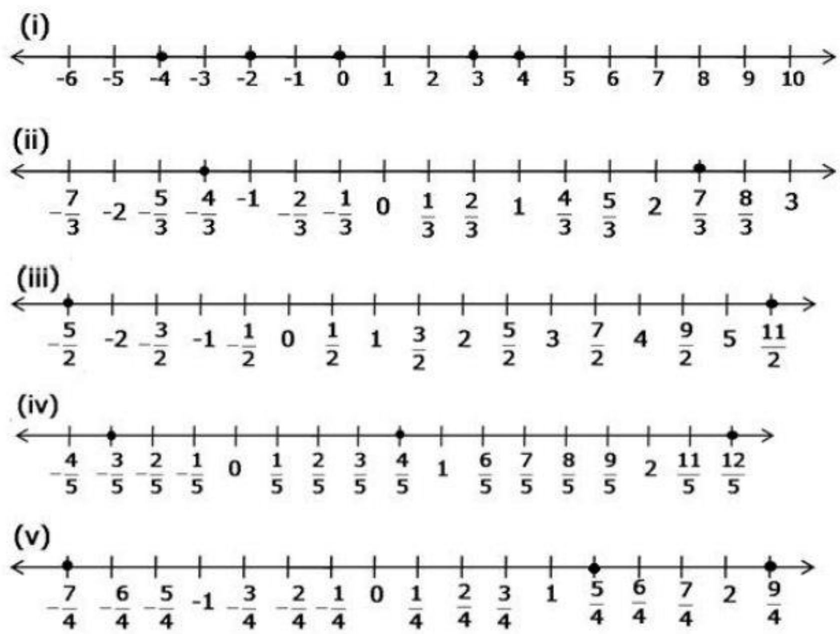
Real Numbers

Exercise – 2.1

Solution 1:



Solution 2(i):

$$\frac{227}{400} = 227 \div 400$$

$$
\begin{array}{r}
0.5675 \\
400\overline{)\ 227.0000} \\
\underline{-2000} \\
2700 \\
\underline{-2400} \\
3000 \\
\underline{-2800} \\
2000 \\
\underline{-2000} \\
0
\end{array}
$$

Hence, $\frac{227}{400} = 0.5675$

Solution 2(ii):

$\frac{27}{99} = 27 \div 99$

$$
\begin{array}{r}
0.2727 \\
99\overline{)\,27.0000} \\
-198 \\
\hline
720 \\
-693 \\
\hline
270 \\
-198 \\
\hline
720 \\
-693 \\
\hline
27
\end{array}
$$

$\frac{27}{99} = 0.2727...$

Hence, $\frac{27}{99}$ is a recurring decimal

Hence, $\frac{27}{99} = 0.2727.... = 0.\overline{27}$

Solution 2(iii):

$\frac{15}{7} = 15 \div 7$

$$
\begin{array}{r}
2.142857 \\
7\overline{)\,15.000000} \\
-14 \\
\hline
10 \\
-7 \\
\hline
30 \\
-28 \\
\hline
20 \\
-14 \\
\hline
60 \\
-56 \\
\hline
40 \\
-35 \\
\hline
50 \\
-49 \\
\hline
1
\end{array}
$$

$\frac{15}{7} = 2.142857.....$ is a non terminating decimal

$\therefore \frac{15}{7} = 2.142857...$

Solution 2(iv):

$$\frac{3}{5} = 3 \div 5$$

$$\begin{array}{r} 0.6 \\ 5\overline{)\ 3.0} \\ -30 \\ \hline 0 \end{array}$$

Hence, $\frac{3}{5} = 0.6$

Solution 2(v):

$$\frac{17}{125} = 17 \div 125$$

$$\begin{array}{r} 0.136 \\ 125\overline{)\ 17.00} \\ -125 \\ \hline 450 \\ -375 \\ \hline 750 \\ -750 \\ \hline 0 \end{array}$$

Hence, $\frac{17}{125} = 0.136$

Solution 2(vi):

$$\frac{2}{11} = 2 \div 11$$

$$\begin{array}{r} 0.1818 \\ 11 \overline{)\ 2.00} \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \\ \hline 2 \end{array}$$

$$\frac{2}{11} = 0.1818\ldots$$

Hence, $\frac{2}{11} = 0.1818\ldots$ is a recurring decimal

$$\therefore \frac{2}{11} = 0.\overline{18}$$

Solution 2(vii):

$$\frac{17}{8} = 17 \div 8$$

$$\begin{array}{r} 2.125 \\ 8 \overline{)\ 17.000} \\ -16 \\ \hline 10 \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Hence, $\frac{17}{18} = 2.125$

Solution 3(i):

$0.\bar{6} = 0.6666....$

Let $x = 0.6666....$

$\therefore 10x = 6.6666... = 6.\bar{6}$

$10x - x = 6.\bar{6} - 0.\bar{6}$

$\therefore 9x = 6$

$\therefore x = \frac{6}{9} = \frac{2}{3}$

Solution 3(ii):

$0.\overline{37} = 0.3737....$

Let $x = 0.3737....$

$\therefore 100x = 37.3737... = 37.\overline{37}$

$100x - x = 37.\overline{37} - 0.\overline{37}$

$\therefore 99x = 37$

$\therefore x = \frac{37}{99}$

Solution 3(iii):

$2.\overline{17} = 2.1717....$

Let $x = 2.1717.... = 2.\overline{17}$

$\therefore 100x = 217.1717... = 217.\overline{17}$

$100x - x = 217.\overline{17} - 2.\overline{17}$

$\therefore 99x = 215$

$\therefore x = \frac{215}{99}$

Solution 3(iv):

$17.\overline{89} = 17.8989\ldots$

Let $x = 17.8989\ldots = 17.\overline{89}$

$\therefore 100x = 1789.89\ldots = 1789.\overline{89}$

$100x - x = 1789.\overline{89} - 17.\overline{89}$

$\therefore 99x = 1772$

$\therefore x = \frac{1772}{99}$

Solution 3(v):

$13.\overline{514} = 13.\overline{514}\ldots$

Let $x = 13.\overline{514}\ldots = 13.\overline{514}$

$\therefore 1000x = 13514.514\ldots = 13514.\overline{514}$

$1000x - x = 13514.\overline{514} - 13.\overline{514}$

$\therefore 999x = 13501$

$\therefore x = \frac{13501}{999}$

Exercise – 2.2

Solution 1:

i. $|11-25|$

$= |-14|$

$= 14$

ii. $|9|+|-9|$

$= 9+9 = 18$

iii. $|6\times3+(-6)\times3|$

$= |18+(-18)|$

$= |0| = 0$

iv. $|x+5-(7+x)|$

$= |x+5-7-x|$

$= |5-7|$

$= |-2|$

$= 2$

v. $|4|\times|-8|$

$= 4\times8 = 32$

vi. $|3(3-8)+(-5)|$

$= |3(-5)+(-5)|$

$= |-15-5|$

$= |-20| = 20$

Solution 2:

i. $|x-2| = 6$

$\therefore x-2 = +6$ or $x-2 = -6$

$\therefore x = 6+2$ or $x = -6+2$

$\therefore x = 8$ or $x = -4$

ii. $|3x-6| = 21$

$\therefore 3x-6 = +21$ or $3x-6 = -21$

$\therefore 3x = 21+6$ or $3x = -21+6$

$\therefore 3x = 27$ or $3x = -15$

$\therefore x = 9$ or $x = -5$

iii. $|4x-2| = 10$

$\therefore 4x-2 = +10$ or $4x-2 = -10$

$\therefore 4x = 10+2$ or $4x = -10+2$

$\therefore 4x = 12$ or $4x = -8$

$\therefore x = 3$ or $x = -2$

iv. $|-(2x-3)| = 7$

$\therefore |-2x+3| = 7$

$\therefore -2x+3 = +7$ or $-2x+3 = -7$

$\therefore -2x = 7-3$ or $-2x = -7-3$

$\therefore -2x = 4$ or $-2x = -10$

$\therefore x = -2$ or $x = 5$

v. $\left|x-\frac{1}{2}\right| = \frac{3}{2}$

$\therefore x-\frac{1}{2} = +\frac{3}{2}$ or $x-\frac{1}{2} = -\frac{3}{2}$

$\therefore x = \frac{3}{2}+\frac{1}{2}$ or $x = -\frac{3}{2}+\frac{1}{2}$

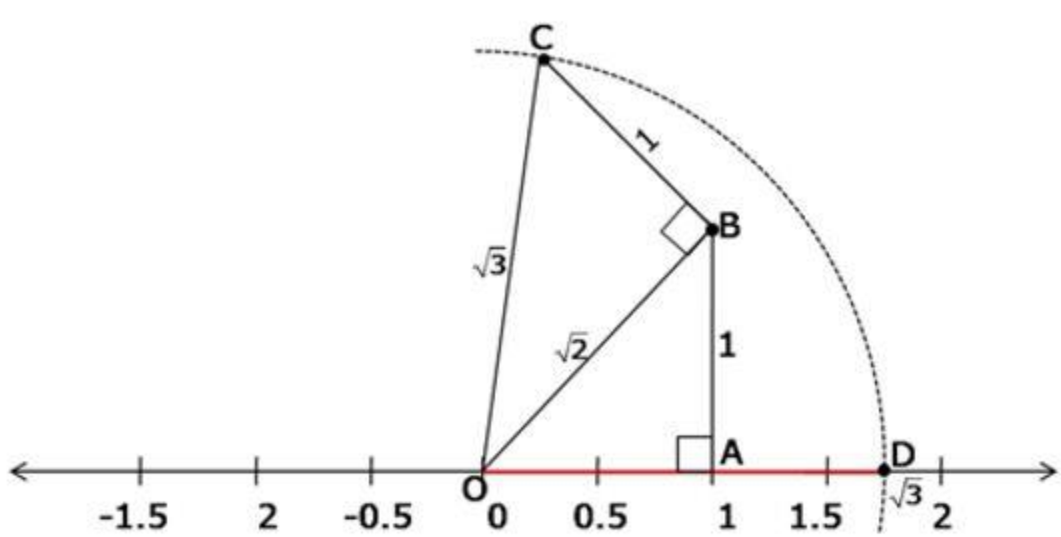
$\therefore x = 2$ or $x = -1$

Solution 3:

i. x > 2 ; y < 2
x > 2
∴2 < x
But y < 2 and 2 < x
∴y < 2 < x
∴x > y
ii. x = 4 ; 4 < y
x = 4
But 4 < y
∴x < y
iii. x > -3 ; -6 > y
x > -3 and y < -6
y < x
∴x > y
iv. -x = 5 ; -5 < y
x = -5 and y > -5
Substituting -5 for x
∴y > x
∴x < y
x > 5 ; y < -5
v. x > 5 and y < -5
∴x > y

Solution 4:

$\sqrt{3}$ can be shown on the number line as follows:



Draw a number line.

Mark a point O representing zero.
Take a point OA such that OA = 1.
Now draw a perpendicular line segment AB to OA, such that AB = 1.
Join the points O and B.

Let us find $\sqrt{2}$

Consider ΔOAB,

Since AB is perpendicular to OA, ΔOAB is a right triangle.

In ΔOAB, $OB^2 = OA^2 + AB^2$ (Pythagoras' Theorem)

$\therefore OB^2 = 1^2 + 1^2$

$\therefore OB^2 = 2, \therefore OB = \sqrt{2}$

We know that 3 can be written as,

$3 = 2 + 1$

$\therefore 3 = \left(\sqrt{2}\right)^2 + 1^2$

$\therefore \left(\sqrt{3}\right)^2 = (OB)^2 + 1^2$

$\therefore$ Taking OB as the base draw a line segment BC = 1 unit, such that $BC \perp OB$.

$\therefore \Delta OBC$ is a right triangle.

In ΔOBC,

$OC^2 = OB^2 + OC^2$ (Pythagoras' Theorem)

$\therefore OC^2 = \left(\sqrt{2}\right)^2 + 1^2$

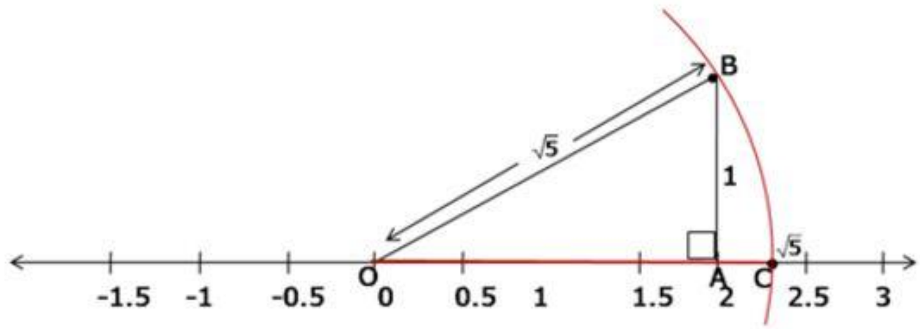
$\therefore OC^2 = 2 + 1 = 3$

$\therefore OC = \sqrt{3}$

Draw an arc of radius OC and centre O

The arc intersects the number line at point D

$\therefore \sqrt{3}$ is thus marked at point D on the number line.



Take point A on numberline such that OA = 2

Construct AB, of length of 1 unit, ⊥ to OA.

$\therefore$ ΔOAB is a right triangle.

In ΔOAB $(OB)^2 = (OA)^2 + (AB)^2$ (Pythagoras' Theorem)

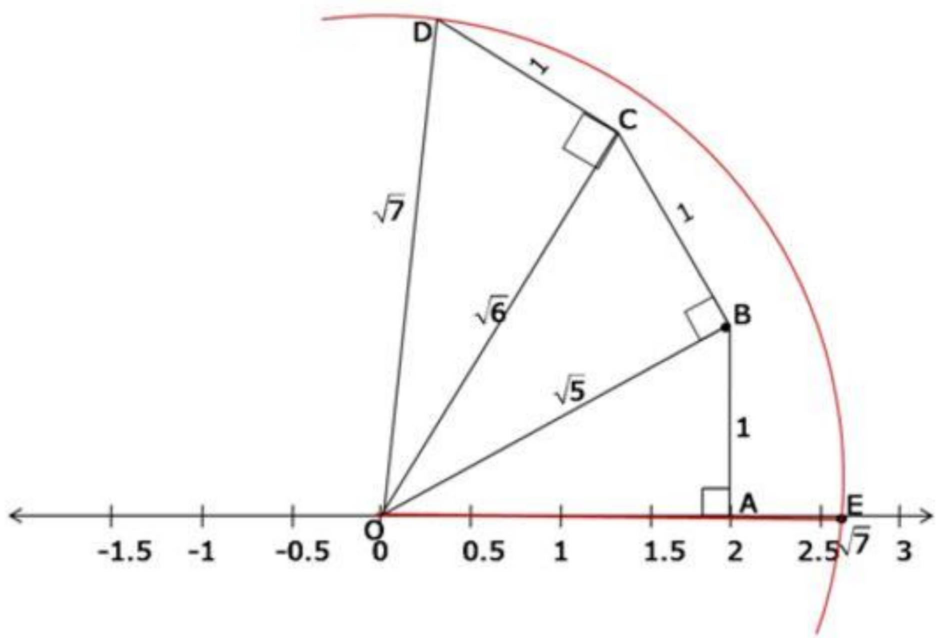
$\therefore (OB)^2 = 2^2 + 1^2$

$\therefore (OB)^2 = 5, \therefore OB = \sqrt{5}$

Draw an arc of radius OB taking Oas centre.

The arc intersects the number line at point C

$\therefore \sqrt{5}$ is thus marked at point C on the number line.



Let us find $\sqrt{5}$

Draw a number line.

Mark a point O representing zero.

Take point A on numberline such that OA = 2

Construct AB ⊥ OA such that AB = 1 unit.

$\therefore$ ΔOAB is a right triangle.

In ΔOAB, $(OB)^2 = (OA)^2 + (AB)^2$ (Pythagoras' Theorem)

$\therefore (OB)^2 = 2^2 + 1^2$

$\therefore (OB)^2 = 5, \therefore OB = \sqrt{5}$

Now, let us find $\sqrt{6}$

Construct $BC \perp OB$, such that $BC = 1$ unit.

$\therefore \Delta OBC$ is a right triangle.

In ΔOBC, $OC^2 = OB^2 + BC^2$ (Pythagoras' Theorem)

$\therefore OC^2 = \left(\sqrt{5}\right)^2 + 1^2$

$\therefore OC^2 = 5 + 1 = 6$

$\therefore OC = \sqrt{6}$

Now, let us find $\sqrt{7}$

Construct $CD \perp OC$, such that $CD = 1$ unit.

In ΔOCD, $OD^2 = OC^2 + CD^2$ (Pythagoras' Theorem)

$\therefore OD^2 = \left(\sqrt{6}\right)^2 + 1^2$

$\therefore OD^2 = 6 + 1 = 7$

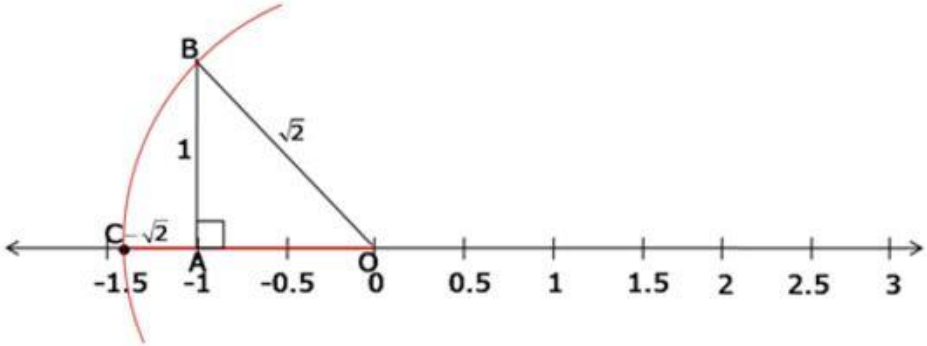
$\therefore OD = \sqrt{7}$

Draw an arc of radius OD and centre O

The arc intersects the number line at E.

$\therefore \sqrt{7}$ is thus marked at point E on the number line.

Solution 5:



Take point A on a numberline such that $A = -1$

$\therefore OA = 1$

Construct $AB \perp OA$ such that $AB = 1$ unit.

Since $AB \perp OA$, ΔOAB is a right triangle

In ΔOAB $(OB)^2 = (OA)^2 + (AB)^2$ (Pythagoras' Theorem)

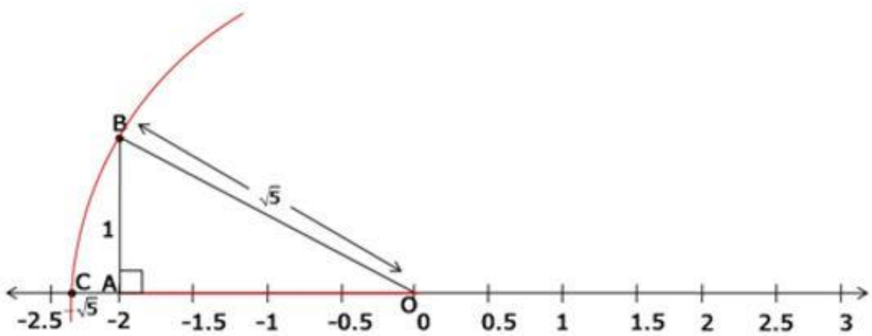
$\therefore (OB)^2 = 1^2 + 1^2$

$\therefore (OB)^2 = 2, \therefore OB = \sqrt{2}$

Draw an arc of radius OB taking O as centre.

The arc intersects the number line, in the left direction at C.

$\therefore -\sqrt{2}$ is thus marked at point C on the number line.



Take point A on a numberline such that $A = -2$

$\therefore OA = 2$

Construct $AB \perp OA$ such that $AB = 1$.

In ΔOAB $(OB)^2 = (OA)^2 + (AB)^2$ (Pythogoras' Theorem)

$\therefore (OB)^2 = 2^2 + 1^2$

$\therefore (OB)^2 = 5, \therefore OB = \sqrt{5}$

Draw an arc of radius OB taking O as centre.

The arc intersects the number line in the left direction at C.

$\therefore -\sqrt{5}$ is thus marked at point C on the number line.

Solution 6:

Let us prove this by the method of contradiction.

Suppose $5+\sqrt{5}$ is a rational number.

$\therefore 5+\sqrt{5}=\frac{a}{b}$ $\left(\begin{array}{l}b \neq 0, \text{and a and b has no} \\ \text{common factor except 1}\end{array}\right)$

$\therefore \sqrt{5}=\frac{a}{b}-5$

$=\frac{a-5b}{b}$

$\because$ a and b are integers, $\frac{a-5b}{b}$ is a rational number.

But $\frac{a-5b}{b}=\sqrt{5}$, which is not a rational number,

which is a contradiction.

Hence our assumption that $5+\sqrt{5}$ is a rational number is wrong.

$\therefore 5+\sqrt{5}$ is an irrational number.

Solution 7:

Numbers

i. -0.2 and -0.22

$-0.2=-\frac{2}{10}=-\frac{20}{100}=-\frac{200}{1000}$ and $-0.22=-\frac{22}{100}=-\frac{220}{1000}$

Hence the numbers between $-\frac{200}{1000}$ and $-\frac{220}{1000}$:

$\therefore -\frac{200}{1000}>-\frac{201}{1000}>-\frac{202}{1000}-\frac{203}{1000}>\ldots.>\frac{219}{1000}>-\frac{220}{1000}$

$\therefore -0.2>-0.201>-0.202>-0.203\ldots.>0.219>-0.22$

ii. -5 and -6

$-5=\frac{-5}{1}=\frac{-50}{10}=$ and $-6=\frac{-6}{1}=\frac{-60}{10}$

Hence the numbers between $\frac{-50}{10}$ and $\frac{-60}{10}$:

$\therefore -\frac{-50}{10}>\frac{-51}{10}>\frac{-52}{10}>\frac{-53}{10}\ldots..>\frac{-59}{10}>\frac{-60}{10}$

$\therefore -5>-5.1>-5.2>-5.3\ldots..>-5.9>-6$

iii. 0 and 1

$1=\frac{1}{1}=\frac{10}{10}$

Hence the numbers between $0=$ and $\frac{10}{10}$:

$\therefore 0<\frac{1}{10}<\frac{2}{10}<\frac{3}{10}<\ldots..\frac{9}{10}<\frac{10}{10}$

$\therefore 0<0.1<0.2<0.3<\ldots..0.9<1$

iv. 2 and 3

$2 = \frac{2}{1} = \frac{20}{10}$ and $3 = \frac{3}{1} = \frac{30}{10}$

Hence the numbers between $\frac{20}{10}$ and $\frac{30}{10}$:

$\therefore \frac{20}{10} < \frac{21}{10} < \frac{22}{10} < \frac{23}{10} < \ldots\ldots < \frac{29}{10} < \frac{30}{10}$

$\therefore 2.0 < 2.1 < 2.2 < 2.3 < \ldots\ldots < 2.9 < 3.0$

v. $\frac{1}{2}$ and $\frac{3}{2}$

$\frac{1}{2} = \frac{10}{20} =$ and $\frac{3}{2} = \frac{30}{20}$

Hence the numbers between $\frac{10}{20}$ and $\frac{30}{20}$:

$\therefore \frac{10}{20} < \frac{11}{20} < \frac{12}{20} < \frac{13}{20} < \ldots\ldots < \frac{29}{20} < \frac{30}{20}$

$\therefore \frac{1}{2} < \frac{11}{20} < \frac{12}{20} < \frac{13}{20} < \ldots\ldots < \frac{29}{20} < \frac{3}{2}$

Solution 8:

Solve $\frac{x-3}{x^2+4} \geq \frac{5}{x^2+4}$

Let $x^2 + 4 = a$

$\therefore \frac{x-3}{a} \geq \frac{5}{a}$

$\therefore x - 3 \geq 5$

$\therefore x \geq 5 + 3$

$\therefore x \geq 8$

Exercise – 2.3

Solution 1:

A surd in the form of $\sqrt[n]{a}$ is an irrational root of a positive rational number 'a', where n ≠1, is a positive natural number.

i. $\sqrt[3]{7}$

7 is a positive rational number.

$\sqrt[3]{7}$ is an irrational number.

$\therefore \sqrt[3]{7}$ is a surd.

ii. $\sqrt[4]{0.16}$

$= \sqrt[4]{0.4 \times 0.4}$

$= \sqrt[2]{0.4}$

0.4 is a positive rational number.

$\sqrt{0.4}$ is an irrational number.

$\therefore \sqrt[4]{0.16}$ is a surd

iii. $\sqrt[3]{\sqrt{8}}$

$= \sqrt[2]{\sqrt[3]{8}}$

$= \sqrt{2}$

2 is a positive rational number.

$\sqrt{2}$ is an irrational number.

$\therefore \sqrt[3]{\sqrt{8}}$ is a surd

iv. $\sqrt{-5}$

-5 is not a positive rational number.

$\therefore \sqrt{-5}$ is not a surd.

v. $\frac{1}{\sqrt{15}}$

$= \frac{1}{\sqrt{3 \times 5}}$

$= \frac{1 \times \sqrt{3 \times 5}}{\sqrt{3 \times 5} \times \sqrt{3 \times 5}}$

$= \frac{\sqrt{15}}{15} = \frac{1}{15}\sqrt{15}$

$\therefore \frac{1}{\sqrt{15}}$ is a surd.

vi. $\sqrt{0.333}$

$= 0.3\sqrt{3.7}$

3.7 is a positive rational number.

$\sqrt{3.7}$ isan irrational number.

$\therefore \sqrt{0.333}$ is a surd.

vii. $\sqrt{576}$

$= 24$

24 is a positive rational number.

$\therefore \sqrt{576}$ is not a surd.

viii. $\sqrt{\frac{5}{4}}$

$= \frac{\sqrt{5}}{2}$

$= \frac{1}{2}\sqrt{5}$

5 is a positive rational number.

$\sqrt{5}$ is an irrational number.

$\therefore \sqrt{\frac{5}{4}}$ is a surd

ix. $\sqrt{\frac{22}{7}}$

$\frac{22}{7}$ is a positive rational number.

$\sqrt{\frac{22}{7}}$ is an irrational number.

$\therefore \sqrt{\frac{22}{7}}$ is a surd

x. $\sqrt[3]{2+\sqrt{9}}$

$= \sqrt[3]{2+3}$

$= \sqrt[3]{5}$

5 is a positive rational number.

$\sqrt[3]{5}$ is an irrational number.

$\therefore \sqrt[3]{2+\sqrt{9}}$ is a surd

Solution 2:

i. $\sqrt{101}$

$\sqrt{101}$ is a surd of order 2

ii. $\sqrt[3]{5}$

$\sqrt[3]{5}$ is a surd of order 3

iii. $\sqrt[4]{29}$

$\sqrt[4]{29}$ is a surd of order 4

iv. $\sqrt[4]{7}$

$\sqrt[4]{7}$ is a surd of order 4

v. $\sqrt[7]{4^3}$

$\sqrt[7]{4^3}$ is a surd of order 7

Solution 3:

i. $4\sqrt{5}$

$= \sqrt{16 \times 5}$

$= \sqrt{80}$

ii. $3\sqrt{6}$

$= \sqrt{9 \times 6}$

$= \sqrt{54}$

iii. $\frac{4}{3}\sqrt{6}$

$= \sqrt{6 \times \frac{16}{9}}$

$= \sqrt{\frac{32}{3}}$

iv. $5\sqrt[3]{5}$

$= \sqrt[3]{5 \times 125}$

$= \sqrt[3]{625}$

v. $2\sqrt[4]{3}$

$= \sqrt[4]{3 \times 16}$

$= \sqrt[4]{48}$

Solution 4:

i. $\sqrt{27}$

$= \sqrt{9 \times 3}$

$= 3\sqrt{3}$

ii. $\sqrt{50}$

$= \sqrt{25 \times 2}$

$= 5\sqrt{2}$

iii. $\sqrt[3]{40}$

$= \sqrt[3]{8 \times 5}$

$= 2\sqrt[3]{5}$

iv. $\sqrt[4]{32}$

$= \sqrt[4]{16 \times 2}$

$= 2\sqrt[4]{2}$

v. $\sqrt[5]{15625}$

$= \sqrt[5]{5^6} = \sqrt[5]{5 \times 5^5}$

$= 5\sqrt[5]{5}$

Solution 5:

i. $\sqrt{11}, \sqrt{13}$

Both surds have the same order, i.e. 2

$\therefore \sqrt{11} < \sqrt{13}$

ii. $\sqrt{3}, \sqrt[3]{2}$

Let us convert the surds in the same order

The order of $\sqrt{3}$ is 2 and the order of $\sqrt[3]{2}$ is 3

L.C.M of 2 and 3 is 6 $(2 \times 3 = 6)$

$\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$

$27 > 4, \therefore \sqrt[6]{27} > \sqrt[6]{4}$

$\therefore \sqrt{3} > \sqrt[3]{2}$

iii. $\sqrt[3]{4}, \sqrt[4]{6}$

Let us convert the surds in the same order

The order of $\sqrt[3]{4}$ is 3 and the order of $\sqrt[4]{6}$ is 4

L.C.M of 3 and 4 is 12 $(3 \times 4 = 12)$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{6} = \sqrt[12]{6^3} = \sqrt[12]{216}$

$256 > 216, \therefore \sqrt[12]{256} > \sqrt[12]{216}$

$\therefore \sqrt[3]{4} > \sqrt[4]{6}$

iv. $\sqrt[8]{80}, \sqrt[4]{40}$

Let us convert the surds in the same order

The order of $\sqrt[8]{80}$ is 8 and the order of $\sqrt[4]{40}$ is 4

L.C.M of 8 and 4 is 8

$\sqrt[4]{40} = \sqrt[8]{40^2} = \sqrt[8]{1600}$

As, 1600>80, $\therefore \sqrt[8]{1600} > \sqrt[8]{80}$

$\therefore \sqrt[4]{40} > \sqrt[8]{80}$

v. $\sqrt[4]{64}, \sqrt[6]{128}$

Let us convert the surds in the same order

The order of $\sqrt[4]{64}$ is 4 and the order of $\sqrt[6]{128}$ is 6

L.C.M of 4 and 6 is 12

$\sqrt[4]{64} = \sqrt[12]{64^3} = \sqrt[12]{262144}$

$\sqrt[6]{128} = \sqrt[12]{128^2} = \sqrt[12]{16384}$,

$\because 262144 > 16384$,

$\therefore \sqrt[12]{262144} > \sqrt[12]{16384}$

$\therefore \sqrt[4]{64} > \sqrt[6]{128}$

Solution 6:

Consider the surds, $\sqrt[6]{4}, \sqrt[3]{3}$ and $\sqrt[4]{5}$

The above surds are not in the same order.

To arrange them in the descending order,

Convert the surds into surds of the same order.

The order of the surds are 4, 3 and 6 respectively.

L.C.M of 4, 3, 6 is 12.

$\therefore \sqrt[4]{5} = \sqrt[12]{5^3}$

$= \sqrt[12]{125}$

And $\sqrt[3]{3} = \sqrt[12]{3^4}$

$= \sqrt[12]{81}$

Also, $\sqrt[6]{4} = \sqrt[12]{4^2}$

$= \sqrt[12]{16}$

Compare the surds by comparing their radicands.

125> 81>16

$\therefore \sqrt[12]{125} > \sqrt[12]{81} > \sqrt[12]{16}$

$\therefore$ The surds in the descending order are $\sqrt[4]{5} > \sqrt[3]{3} > \sqrt[6]{4}$

Exercise – 2.4

Solution 1:

i. $3\sqrt{3} + 10\sqrt{3}$

$= \sqrt{3}(3 + 10)$

$= 13\sqrt{3}$

ii. $7\sqrt{5} - 4\sqrt{5} + \sqrt{125}$

$= \sqrt{5}(7 - 4) + \sqrt{25 \times 5}$

$= 3\sqrt{5} + 5\sqrt{5}$

$= \sqrt{5}(5 + 3)$

$= 8\sqrt{5}$

iii. $4\sqrt{8} + \sqrt{32} - 3\sqrt{2}$

$= 4\sqrt{4 \times 2} + \sqrt{16 \times 2} - 3\sqrt{2}$

$= 4 \times 2\sqrt{2} + 4\sqrt{2} - 3\sqrt{2}$

$= 8\sqrt{2} + 4\sqrt{2} - 3\sqrt{2}$

$= \sqrt{2}(8 + 4 - 3)$

$= 9\sqrt{2}$

iv. $\sqrt{50} - \sqrt{98} + \sqrt{162}$

$= \sqrt{25 \times 2} - \sqrt{49 \times 2} + \sqrt{81 \times 2}$

$= 5\sqrt{2} - 7\sqrt{2} + 9\sqrt{2}$

$= \sqrt{2}(5 - 7 + 9)$

$= 7\sqrt{2}$

v. $7\sqrt{48} - \sqrt{72} - \sqrt{27} + 3\sqrt{2}$

$= 7\sqrt{16 \times 3} - \sqrt{36 \times 2} - \sqrt{9 \times 3} + 3\sqrt{2}$

$= 7 \times 4\sqrt{3} - 6\sqrt{2} - 3\sqrt{3} + 3\sqrt{2}$

$= 28\sqrt{3} - 3\sqrt{3} - 6\sqrt{2} + 3\sqrt{2}$

$= \sqrt{3}(28 - 3) + \sqrt{2}(-6 + 3)$

$= 25\sqrt{3} - 3\sqrt{2}$

vi. $5\sqrt{3} + 2\sqrt{27} + 4\sqrt{\frac{1}{3}}$

$= 5\sqrt{3} + 2\sqrt{9 \times 3} + 4\sqrt{\frac{1}{3}} \times \sqrt{\frac{3}{3}}$

$= 5\sqrt{3} + 2 \times 3\sqrt{3} + 4\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 5\sqrt{3} + 6\sqrt{3} + \frac{4\sqrt{3}}{3}$

$= \sqrt{3}\left[5 + 6 + \frac{4}{3}\right]$

$= \sqrt{3}\left[11 + \frac{4}{3}\right]$

$= \sqrt{3}\left[\frac{33 + 4}{3}\right]$

$= \sqrt{3}\left[\frac{37}{3}\right]$

vii. $\frac{1}{2}\sqrt{243}-\sqrt{\frac{27}{4}}$

$=\frac{1}{2}\sqrt{81\times 3}-\sqrt{\frac{27}{4}}$

$=\frac{1}{2}\times 9\sqrt{3}-\frac{3}{2}\sqrt{3}$

$=\frac{9}{2}\sqrt{3}-\frac{3}{2}\sqrt{3}$

$=\sqrt{3}\left(\frac{9}{2}-\frac{3}{2}\right)$

$=\sqrt{3}\,(3)=3\sqrt{3}$

viii. $\sqrt{\frac{8}{9}}-3\sqrt{\frac{1}{2}}+5\sqrt{\frac{9}{8}}$

$=\sqrt{\frac{4\times 2}{9}}-3\sqrt{\frac{1\times 2}{2\times 2}}+5\sqrt{\frac{9}{4\times 2}}$

$=\frac{2}{3}\sqrt{2}-\frac{3}{2}\sqrt{2}+5\times\frac{3}{2}\sqrt{\frac{1}{2}}$

$=\frac{2}{3}\sqrt{2}-\frac{3}{2}\sqrt{2}+\frac{15}{2}\sqrt{\frac{1\times 2}{2\times 2}}$

$=\frac{2}{3}\sqrt{2}-\frac{3}{2}\sqrt{2}+\frac{15}{2}\times\frac{1}{2}\sqrt{2}$

$=\sqrt{2}\left(\frac{2}{3}-\frac{3}{2}+\frac{15}{4}\right)$

$=\frac{35}{12}\sqrt{2}=\frac{35\sqrt{2}}{12}$

$$= \frac{35\sqrt{2} \times \sqrt{2}}{12 \times \sqrt{2}}$$

$$= \frac{35 \times 2}{12 \times \sqrt{2}} = \frac{35}{6\sqrt{2}}$$

ix. $4\sqrt{2} - 2\sqrt{8} + \frac{3}{\sqrt{2}}$

$$= 4\sqrt{2} - 2\sqrt{4 \times 2} + \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= 4\sqrt{2} - 2 \times 2\sqrt{2} + \frac{3\sqrt{2}}{2}$$

$$= 4\sqrt{2} - 4\sqrt{2} + \frac{3}{2}\sqrt{2}$$

$$= \sqrt{2}\left(4 - 4 + \frac{3}{2}\right)$$

$$= \sqrt{2}\left(\frac{3}{2}\right) = \sqrt{2}\left(\frac{3}{\sqrt{2} \times \sqrt{2}}\right)$$

$$= \frac{3}{\sqrt{2}}$$

x. $8\sqrt[3]{16} + \sqrt[3]{54} + \sqrt[3]{250}$

$$= 8\sqrt[3]{8 \times 2} + \sqrt[3]{27 \times 2} + \sqrt[3]{125 \times 2}$$

$$= 8 \times 2\sqrt[3]{2} + 3\sqrt[3]{2} + 5\sqrt[3]{2}$$

$$= 16\sqrt[3]{2} + 3\sqrt[3]{2} + 5\sqrt[3]{2}$$

$$= \sqrt[3]{2}\,(16 + 3 + 5)$$

$$= 24\sqrt[3]{2}$$

Exercise – 2.5

Solution 1:

(i) $\sqrt{3} \times \sqrt{7}$

$= \sqrt{3 \times 7}$

$= \sqrt{21}$

(ii) $3\sqrt{11} \times \sqrt{10}$

$= 3\sqrt{11 \times 10}$

$= 3\sqrt{110}$

(iii) $4\sqrt{12} \times 7\sqrt{16}$

$= 4\sqrt{3 \times 4} \times 7 \times 4$

$= 4 \times 2\sqrt{3} \times 28$

$= 8\sqrt{3} \times 28$

$= 224\sqrt{3}$

(iv) $\sqrt[3]{2} \times \sqrt[4]{3}$

Convert the surds in the same order

The order of $\sqrt[3]{2}$ is 3 and the order of $\sqrt[4]{3}$ is 4

L.C.M of 3 and 4 is 12 $(3 \times 4 = 12)$

$\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

$\sqrt[4]{3} = \sqrt[12]{3^3} = \sqrt[12]{27}$

$\therefore \sqrt[3]{2} \times \sqrt[4]{3} = \sqrt[12]{16 \times 27}$

$= \sqrt[12]{432}$

(v) $\sqrt[3]{3} \times \sqrt[6]{2}$

Convert the surds in the same order

The order of $\sqrt[3]{3}$ is 3 and the order of $\sqrt[6]{2}$ is 6

L.C.M of 3 and 6 is 6

$\therefore \sqrt[3]{3} = \sqrt[6]{3^2} = \sqrt[6]{9}$

$\therefore \sqrt[3]{3} \times \sqrt[6]{2} = \sqrt[6]{9 \times 2}$

$= \sqrt[6]{18}$

Solution 2:

(i) $\sqrt[3]{2} \times \sqrt[3]{3} \times \sqrt[3]{4}$

$= \sqrt[3]{2 \times 3 \times 4}$

$= \sqrt[3]{24}$

(ii) $\sqrt[3]{3} \times \sqrt{3} \times \sqrt[3]{2}$

Convert the surds in the same order

L.C.M of 3 and 2 is 6 $(3 \times 2 = 6)$

$\therefore \sqrt[3]{3} = \sqrt[6]{3^2} = \sqrt[6]{9};$

$\sqrt{3} = \sqrt[2]{3} = \sqrt[6]{3^3} = \sqrt[6]{27};$

$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$

$\therefore \sqrt[3]{3} \times \sqrt{3} \times \sqrt[3]{2} = \sqrt[6]{9} \times \sqrt[6]{27} \times \sqrt[6]{4}$

$= \sqrt[6]{9 \times 27 \times 4} = \sqrt[6]{9 \times 27 \times 4}$

$= \sqrt[6]{972}$

(iii) $\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt{2}$

Convert the surds in the same order

L.C.M of 3, 4 and 2 is 12

$\therefore \sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

$\sqrt[4]{2} = \sqrt[12]{2^3} = \sqrt[12]{8}$

$\sqrt{2} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\therefore \sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt{2} = \sqrt[12]{16} \times \sqrt[12]{8} \times \sqrt[12]{64}$

$= \sqrt[12]{16 \times 8 \times 64} = \sqrt[12]{16 \times 8 \times 64}$

$= \sqrt[12]{8192}$

(iv) $2 \times \sqrt[3]{4} \times 3 \times \sqrt[3]{16}$

$= 6 \times \sqrt[3]{4} \times \sqrt[3]{16}$

The order of $\sqrt[3]{4}$ and $\sqrt[3]{16}$ is 3

$\therefore 6 \times \sqrt[3]{4} \times \sqrt[3]{16}$

$= 6 \times \sqrt[3]{4 \times 16} = 6 \times \sqrt[3]{64}$

$= 6 \times 4 = 24$

(v) $3 \times \sqrt[3]{32} \times 3 \times \sqrt{4}$

$= 9 \times \sqrt[3]{32} \times 2 = 18 \times \sqrt[3]{32}$

Convert the surds in the same order

$\therefore \sqrt[3]{32} = \sqrt[3]{2^5} = 2 \times \sqrt[3]{2^2}$

$\therefore 18 \times \sqrt[3]{32} = 18 \times 2 \times \sqrt[3]{4} = 36 \times \sqrt[3]{4}$

Solution 3:

(i) $\sqrt{98} \div \sqrt{2} = \sqrt{98 \div 2} = \sqrt{49} = 7$

(ii) $8\sqrt{28} \div 2\sqrt{7} = 8\sqrt{7 \times 4} \div 2\sqrt{7} = \dfrac{8\sqrt{7 \times 4}}{2\sqrt{7}} = \dfrac{8 \times 2\sqrt{7}}{2\sqrt{7}} = 8$

(iii) $\sqrt[4]{27} \div \sqrt[4]{3} = \sqrt[4]{3^3} \div \sqrt[4]{3} = \sqrt[4]{\dfrac{3 \times 3 \times 3}{3}} = \sqrt[4]{9}$

(iv) $\sqrt[3]{5} \div \sqrt[4]{3}$

Convert the surds into surds of same order,

L.C.M of 3 and 4 is 12

$\sqrt[3]{5} = \sqrt[12]{5^4}$; $\sqrt[4]{3} = \sqrt[12]{3^3}$

$\therefore \sqrt[3]{5} \div \sqrt[4]{3} = \sqrt[12]{5^4} \div \sqrt[12]{3^3} = \dfrac{\sqrt[12]{5^4}}{\sqrt[12]{3^3}} = \sqrt[12]{\dfrac{5 \times 5 \times 5 \times 5}{3 \times 3 \times 3}} = \sqrt[12]{\dfrac{625}{27}}$

(v) $5\sqrt[3]{4} \div \sqrt[3]{2}$

$= 5 \times \sqrt[3]{\dfrac{4}{2}} = 5\sqrt[3]{2}$

Solution 4:

(i) $\sqrt[6]{24} \div \left(\sqrt[3]{3}.\sqrt{2}\right)$ (L.C.M of 6, 3 and 2 = 6)

$= \dfrac{\sqrt[6]{24}}{\sqrt[6]{3^2} \times \sqrt[6]{2^3}}$

$= \dfrac{\sqrt[6]{2 \times 2 \times 2 \times 3}}{\sqrt[6]{3 \times 3 \times 2 \times 2 \times 2}} = \dfrac{1}{\sqrt[6]{3}}$

(ii) $\sqrt[3]{24} \div \left(\sqrt[4]{2}.\sqrt[3]{3}\right)$ (L.C.M of 3, 4 = 12)

$= \dfrac{\sqrt[12]{(24)^4}}{\sqrt[12]{2^3} \times \sqrt[12]{3^4}}$

$= \sqrt[12]{\dfrac{24 \times 24 \times 24 \times 24}{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}} = \sqrt[12]{512}$

(iii) $27\ \sqrt[3]{18} \div 3\ \sqrt[3]{9}$

$= \dfrac{27\ \sqrt[3]{18}}{3\ \sqrt[3]{9}}$

$= \dfrac{9\ \sqrt[3]{9 \times 2}}{\sqrt[3]{9}}$

$= 9\ \sqrt[3]{\dfrac{9 \times 2}{9}} = 9\ \sqrt[3]{2}$

(iv) $\sqrt{x^3y^3} \div \sqrt[3]{x^4y^3}$

$$= \frac{\sqrt{x^3y^3}}{\sqrt[6]{x^4y^3}}$$

$$= \frac{\sqrt[6]{\left(x^3y^3\right)^3}}{\sqrt[6]{\left(x^4y^3\right)^2}} = \frac{\sqrt[6]{\left(x^9y^9\right)}}{\sqrt[6]{\left(x^8y^6\right)}} \quad \text{(L.C.M of 2 and 3 is 6)}$$

$$= \sqrt[6]{\frac{\left(x^9y^9\right)}{x^8y^6}}$$

$$= \sqrt[6]{\frac{\left(x^{9-8}y^{9-6}\right)}{1}} = \sqrt[6]{xy^3}$$

(v) $\sqrt{m^2n^2} \times \sqrt[6]{m^2n^2} \times \sqrt[3]{m^2n^2}$ (L.C.M of 2, 6 and 3 is 6)

$$= \sqrt[6]{\left(m^2n^2\right)^3} \times \sqrt[6]{m^2n^2} \times \sqrt[6]{\left(m^2n^2\right)^2}$$

$$= \sqrt[6]{m^6n^6} \times \sqrt[6]{m^2n^2} \times \sqrt[6]{m^4n^4}$$

$$= \sqrt[6]{m^6n^6 \times m^2n^2 \times m^4n^4}$$

$$= \sqrt[6]{m^{6+2+4}n^{6+2+4}} = \sqrt[6]{m^{12}n^{12}}$$

$$= m^2n^2$$

Exercise – 2.6

Solution 1(i):

Square root of $7-2\sqrt{10}$

$= \sqrt{7-2\sqrt{10}}$

$= \sqrt{(5+2)-2\sqrt{5\times 2}}$

$= \sqrt{(\sqrt{5})^2+(\sqrt{2})^2-2\sqrt{5}\times\sqrt{2}}$

$= \sqrt{(\sqrt{5}-\sqrt{2})^2}$.......$\left[\because a^2+b^2-2ab=(a-b)^2\right]$

$= \sqrt{5}-\sqrt{2}$

$\therefore$ Square root of $7-2\sqrt{10}$ is $\sqrt{5}-\sqrt{2}$

Solution 1(ii):

Square root of $8+2\sqrt{15}$

$= \sqrt{8+2\sqrt{15}}$

$= \sqrt{(5+3)+2\sqrt{5\times 3}}$

$= \sqrt{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\times\sqrt{3}}$

$= \sqrt{(\sqrt{5}+\sqrt{3})^2}$ ....$\left[\because a^2+b^2+2ab=(a+b)^2\right]$

$= \sqrt{5}+\sqrt{3}$

$\therefore$ Square root of $8+2\sqrt{15}$ is $\sqrt{5}+\sqrt{3}$

Solution 1(iii):

Square root of $5-2\sqrt{6}$

$= \sqrt{5-2\sqrt{6}}$

$= \sqrt{(3+2)-2\sqrt{3\times 2}}$

$= \sqrt{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{3}\times\sqrt{2}}$

$= \sqrt{(\sqrt{3}-\sqrt{2})^2}$ ....$\left[\because a^2+b^2-2ab=(a-b)^2\right]$

$= \sqrt{3}-\sqrt{2}$

$\therefore$ Square root of $5-2\sqrt{6}$ is $\sqrt{3}-\sqrt{2}$

Solution 1(iv):

Square root of $47 + 4\sqrt{33}$

$= \sqrt{47 + 4\sqrt{33}}$

$= \sqrt{(44+3) + 2 \times \sqrt{132}}$

$= \sqrt{(\sqrt{44})^2 + (\sqrt{3})^2 + 2 \times \sqrt{44} \times \sqrt{3}}$

$= \sqrt{(\sqrt{44} + \sqrt{3})^2}$

$= \sqrt{44} + \sqrt{3}$

$\therefore$ Square root of $47 + 4\sqrt{33}$ is $\sqrt{44} + \sqrt{3}$.

Solution 1(v):

Square root of $31 + 4\sqrt{21}$

$= \sqrt{31 + 4\sqrt{21}}$

$= \sqrt{(28+3) + 2 \times \sqrt{21 \times 4}}$

$= \sqrt{(\sqrt{28})^2 + (\sqrt{3})^2 + 2 \times \sqrt{28} \times \sqrt{3}}$

$= \sqrt{(\sqrt{28} + \sqrt{3})^2}$

$= \sqrt{28} + \sqrt{3}$

$\therefore$ Square root of $31 + 4\sqrt{21}$ is $\sqrt{28} + \sqrt{3}$.

Solution 1(vi):

Square root of $19 + 8\sqrt{3}$

$= \sqrt{19 + 8\sqrt{3}}$

$= \sqrt{(16+3) + 2 \times 4\sqrt{3}}$

$== \sqrt{(16+3) + 2 \times \sqrt{3 \times 16}}$

$= \sqrt{(\sqrt{16})^2 + (\sqrt{3})^2 + 2 \times \sqrt{16} \times \sqrt{3}}$

$= \sqrt{(\sqrt{16} + \sqrt{3})^2}$

$= \sqrt{16} + \sqrt{3}$

$= 4 + \sqrt{3}$

$\therefore$ Square root of $19 + 8\sqrt{3}$ is $4 + \sqrt{3}$.

Solution 1(vii):

Square root of $17+12\sqrt{2}$

$= \sqrt{17+12\sqrt{2}}$

$= \sqrt{(17)+2\times 6\sqrt{2}}$

$= \sqrt{(17)+2\times \sqrt{2\times 36}}$

$= \sqrt{(9+8)+2\times \sqrt{9\times 8}}$

$= \sqrt{(\sqrt{9})^2+(\sqrt{8})^2+2\times \sqrt{9}\times \sqrt{8}}$

$= \sqrt{(\sqrt{9}+\sqrt{8})^2}$

$= \sqrt{9}+\sqrt{8}$

$= 3+\sqrt{8}$

$\therefore$ Square root of $17+12\sqrt{2}$ is $3+\sqrt{8}$.

Solution 1(viii):

Square root of $73-12\sqrt{35}$

$= \sqrt{73-12\sqrt{35}}$

$= \sqrt{(45+28)-2\times 6\sqrt{35}}$

$= \sqrt{(\sqrt{45})^2+(\sqrt{28})^2-2\times \sqrt{35\times 36}}$

$= \sqrt{(\sqrt{45})^2+(\sqrt{28})^2-2\times \sqrt{45\times 28}}$

$= \sqrt{(\sqrt{45})^2+(\sqrt{28})^2-2\times \sqrt{45}\times \sqrt{28}}$

$= \sqrt{(\sqrt{45}-\sqrt{28})^2}$

$= \sqrt{45}-\sqrt{28}$

$\therefore$ Square root of $73-12\sqrt{35}$ is $\sqrt{45}-\sqrt{28}$.

Solution 1(ix):

Square root of $21-\sqrt{440}$

$= \sqrt{21-\sqrt{440}}$

$= \sqrt{(11+10)-2\sqrt{\frac{440}{4}}}$

$= \sqrt{\left(\sqrt{11}\right)^2+\left(\sqrt{10}\right)^2-2\times\sqrt{110}}$

$= \sqrt{\left(\sqrt{11}\right)^2+\left(\sqrt{10}\right)^2-2\times\sqrt{11}\times\sqrt{10}}$

$= \sqrt{\left(\sqrt{11}-\sqrt{10}\right)^2}$

$= \sqrt{11}-\sqrt{10}$

$\therefore$ Square root of $21-\sqrt{440}$ is $\sqrt{11}-\sqrt{10}$.

Solution 1(x):

Square root of $3+\sqrt{5}$

$= \sqrt{3+\sqrt{5}}$

$= \sqrt{3+\left(2\times\sqrt{\frac{5}{4}}\right)}$

$= \sqrt{\left(\frac{5}{2}+\frac{1}{2}\right)+\left(2\times\sqrt{\frac{5}{2}\times\frac{1}{2}}\right)}$

$= \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2+\left(\sqrt{\frac{1}{2}}\right)^2+2\times\sqrt{\frac{5}{2}}\times\sqrt{\frac{1}{2}}}$

$= \sqrt{\left(\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}\right)^2}$

$= \sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}$

$\therefore$ Square root of $3+\sqrt{5}$ is $\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}$.

Solution 2(i):

$$\frac{4}{\sqrt{5}} = \frac{4}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{4\sqrt{5}}{\left(\sqrt{5}\right)^2}$$
$$= \frac{4\sqrt{5}}{5}$$

Solution 2(ii):

$$\frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{1\sqrt{3}}{2\left(\sqrt{3}\right)^2}$$
$$= \frac{\sqrt{3}}{6}$$

Solution 2(iii):

$$\frac{2}{3\sqrt{3}} = \frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{2\sqrt{3}}{3\left(\sqrt{3}\right)^2}$$
$$= \frac{2\sqrt{3}}{9}$$

Solution 2(iv):

$$\frac{3\sqrt{3}}{2\sqrt{8}} = \frac{3\sqrt{3}}{2\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}}$$
$$= \frac{3\sqrt{3} \times \sqrt{8}}{2\left(\sqrt{8}\right)^2}$$
$$= \frac{3\sqrt{24}}{16}$$
$$= \frac{3\sqrt{6 \times 4}}{16}$$
$$= \frac{6\sqrt{6}}{16}$$
$$= \frac{3\sqrt{6}}{8}$$

Solution 2(v):

$$\frac{3\sqrt[3]{5}}{\sqrt[3]{9}} = \frac{3\sqrt[3]{5}}{\sqrt[3]{9}} \times \frac{\sqrt[3]{9}}{\sqrt[3]{9}} \times \frac{\sqrt[3]{9}}{\sqrt[3]{9}}$$
$$= \frac{3\sqrt[3]{5} \times \sqrt[3]{9} \times \sqrt[3]{9}}{\left(\sqrt[3]{9}\right)^3}$$
$$= \frac{3\sqrt[3]{5 \times 9 \times 9}}{9}$$
$$= \frac{3 \times 3\sqrt[3]{15}}{9}$$
$$= \sqrt[3]{15}$$

Solution 2(vi):

$$\frac{5+\sqrt{6}}{5-\sqrt{6}} = \frac{5+\sqrt{6}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}}$$
$$= \frac{\left(5+\sqrt{6}\right)^2}{(5)^2-\left(\sqrt{6}\right)^2}$$
$$= \frac{25+10\sqrt{6}+6}{25-6}$$
$$= \frac{31+10\sqrt{6}}{19}$$

Solution 2(vii):

$$\frac{3}{\sqrt{6}-\sqrt{7}} = \frac{3}{\sqrt{6}-\sqrt{7}} \times \frac{\sqrt{6}+\sqrt{7}}{\sqrt{6}+\sqrt{7}}$$
$$= \frac{3\left(\sqrt{6}+\sqrt{7}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{7}\right)^2}$$
$$= \frac{3\left(\sqrt{6}+\sqrt{7}\right)}{6-7}$$
$$= -3\left(\sqrt{6}+\sqrt{7}\right)$$

Solution 2(viii):

$$\frac{2\sqrt{8}}{2\sqrt{5}-\sqrt{3}} = \frac{4\sqrt{2}}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$$
$$= \frac{4\sqrt{2}\left(2\sqrt{5}+\sqrt{3}\right)}{\left(2\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$
$$= \frac{4\sqrt{2}\left(2\sqrt{5}+\sqrt{3}\right)}{20-3}$$
$$= \frac{4\sqrt{2}\left(2\sqrt{5}+\sqrt{3}\right)}{17}$$

Solution 2(ix):

$$\frac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}} = \frac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}} \times \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{\sqrt{6}+\sqrt{5}+\sqrt{11}}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{\left(\sqrt{6}+\sqrt{5}\right)^2-\left(\sqrt{11}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{\left(\sqrt{6}+\sqrt{5}\right)^2-\left(\sqrt{11}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{6+5+2\sqrt{6}\sqrt{5}-11}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}} \times \frac{\sqrt{30}}{\sqrt{30}}$$

$$= \frac{\sqrt{30}\left(\sqrt{6}+\sqrt{5}+\sqrt{11}\right)}{2\left(\sqrt{30}\right)^2}$$

$$= \frac{\sqrt{30}\left(\sqrt{6}+\sqrt{5}+\sqrt{11}\right)}{60}$$

Solution 2(x):

$$\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} = \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} \times \frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$$

$$= \frac{\left(\sqrt{a+x}+\sqrt{a-x}\right)^2}{\left(\sqrt{a+x}\right)^2-\left(\sqrt{a-x}\right)^2}$$

$$= \frac{a+x+a-x+2\sqrt{a+x}\sqrt{a-x}}{a+x-a+x}$$

$$= \frac{2a+2\sqrt{a^2-x^2}}{2x}$$

$$= \frac{a+\sqrt{a^2-x^2}}{x}$$

Solution 3:

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = a+b\sqrt{3}$$

$$\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2} = a+b\sqrt{3}$$

$$\frac{3+1-2\sqrt{3}}{3-1} = a+b\sqrt{3}$$

$$\frac{4-2\sqrt{3}}{2} = a+b\sqrt{3}$$

$$2-\sqrt{3} = a+b\sqrt{3}$$

Equating the values of both the sides we get,

$a = 2$ and $b = -1$

$\therefore$ The values of a and b are 2 and -1 respectively.

Solution 4:

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$$

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = a-b\sqrt{6}$$

$$\frac{\left(\sqrt{2}+\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2} = a-b\sqrt{6}$$

$$\frac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12} = a-b\sqrt{6}$$

$$\frac{12+5\sqrt{6}}{6} = a-b\sqrt{6}$$

$$2+\frac{5}{6}\sqrt{6} = a-b\sqrt{6}$$

Equating the values of both the sides we get,

$a = 2$ and $b = -\frac{5}{6}$

$\therefore$ The values of a and b are 2 and $-\frac{5}{6}$ respectively.