Chapter 28. Distance Formula

Exercise 1(A)

Solution 1:

(i) (-3, 6) and (2, -6)

Distance between the given points

$= \sqrt{(2+3)^2 + (-6-6)^2}$

$= \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

(ii) (-a, -b) and (a, b)

Distance between the given points

$= \sqrt{(a+a)^2 + (b+b)^2}$

$= \sqrt{(2a)^2 + (2b)^2}$

$= \sqrt{4a^2 + 4b^2}$

$= 2\sqrt{a^2 + b^2}$

(iii) $\left(\frac{3}{5}, 2\right)$ and $\left(-\frac{1}{5}, 1\frac{2}{5}\right)$

Distance between the given points

$= \sqrt{\left(-\frac{1}{5} - \frac{3}{5}\right)^2 + \left(1\frac{2}{5} - 2\right)^2}$

$= \sqrt{\left(-\frac{4}{5}\right)^2 + \left(\frac{7-10}{5}\right)^2}$

$= \sqrt{\frac{16}{25} + \frac{9}{25}}$

$= \sqrt{\frac{25}{25}}$

$= 1$

(iv) $\left(\sqrt{3}+1, 1\right)$ and $\left(0, \sqrt{3}\right)$

Distance between the given points

$= \sqrt{(0-\sqrt{3}-1)^2 + (\sqrt{3}-1)^2}$

$= \sqrt{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}$

$= \sqrt{8}$

$= 2\sqrt{2}$

Solution 2:

Coordinates of origin are O (0, 0).

(i) A (-8, 6)

AO = $\sqrt{(0+8)^2+(0-6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

(ii) B (-5, -12)

BO = $\sqrt{(0+5)^2+(0+12)^2} = \sqrt{25+144} = \sqrt{169} = 13$

(iii) C (8, -15)

CO = $\sqrt{(0-8)^2+(0+15)^2} = \sqrt{64+225} = \sqrt{289} = 17$

Solution 3:

It is given that the distance between the points A (3, 1) and B (0, x) is 5.

$\therefore$ AB = 5

$AB^2 = 25$

$(0-3)^2+(x-1)^2 = 25$

$9+x^2+1-2x = 25$

$x^2-2x-15 = 0$

$x^2-5x+3x-15 = 0$

$x(x-5)+3(x-5) = 0$

$(x-5)(x+3) = 0$

$x = 5, -3$

Solution 4:

Let the coordinates of the point on x-axis be (x, 0).

From the given information, we have:

$\sqrt{(x-11)^2+(0+8)^2} = 17$

$(x-11)^2+(0+8)^2 = 289$

$x^2+121-22x+64 = 289$

$x^2-22x-104 = 0$

$x^2-26x+4x-104 = 0$

$x(x-26)+4(x-26) = 0$

$(x-26)(x+4) = 0$

$x = 26, -4$

Thus, the required co-ordinates of the points on x-axis are (26, 0) and (-4, 0).

Solution 5:

Let the coordinates of the point on y-axis be (0, y).
From the given information, we have:

$$\sqrt{(0+8)^2+(y-4)^2} = 10$$
$$(0+8)^2+(y-4)^2 = 100$$
$$64+y^2+16-8y = 100$$
$$y^2-8y-20=0$$
$$y^2-10y+2y-20=0$$
$$y(y-10)+2(y-10)=0$$
$$(y-10)(y+2)=0$$
$$y = 10, -2$$

Thus, the required co-ordinates of the points on y-axis are (0, 10) and (0, -2).

Solution 6:

It is given that the co-ordinates of point A are such that its ordinate is twice its abscissa.
So, let the co-ordinates of point A be (x, 2x).
We have:

$$\sqrt{(x-4)^2+(2x-3)^2} = \sqrt{10}$$
$$(x-4)^2+(2x-3)^2 = 10$$
$$x^2+16-8x+4x^2+9-12x = 10$$
$$5x^2-20x+15=0$$
$$x^2-4x+3=0$$
$$x^2-x-3x+3=0$$
$$x(x-1)-3(x-1)=0$$
$$(x-1)(x-3)=0$$
$$x = 1, 3$$

Thus, the co-ordinates of the point A are (1, 2) and (3, 6).

Solution 7:

Given that the point P (2, -1) is equidistant from the points A (a, 7) and B (-3, a).

$$\therefore PA = PB$$
$$PA^2 = PB^2$$
$$(a-2)^2+(7+1)^2 = (-3-2)^2+(a+1)^2$$
$$a^2+4-4a+64 = 25+a^2+1+2a$$
$$42 = 6a$$
$$a = 7$$

Solution 8:

Let the co-ordinates of the required point on x-axis be P (x, 0).
The given points are A (7, 6) and B (-3, 4).
Given, PA = PB

$$PA^2 = PB^2$$
$$(x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2$$
$$x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$$
$$60 = 20x$$
$$x = 3$$

Thus, the required point is (3, 0).

Solution 9:

Let the co-ordinates of the required point on y-axis be P (0, y).
The given points are A (5, 2) and B (-4, 3).
Given, PA = PB

$$PA^2 = PB^2$$
$$(0-5)^2 + (y-2)^2 = (0+4)^2 + (y-3)^2$$
$$25 + y^2 + 4 - 4y = 16 + y^2 + 9 - 6y$$
$$2y = -4$$
$$y = -2$$

Thus, the required point is (0, -2).

Solution 10:

(i) Since, the point P lies on the x-axis, its ordinate is 0.
(ii) Since, the point Q lies on the y-axis, its abscissa is 0.
(iii) The co-ordinates of P and Q are (-12, 0) and (0, -16) respectively.

$$PQ = \sqrt{(-12-0)^2 + (0+16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$$

Solution 11:

$$PQ = \sqrt{(5-0)^2 + (10-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$
$$QR = \sqrt{(6-5)^2 + (3-10)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$
$$RP = \sqrt{(0-6)^2 + (5-3)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

Since, PQ = QR, Δ PQR is an isosceles triangle.

Solution 12:

$$PQ = \sqrt{(6-0)^2 + (2+4)^2} = 6\sqrt{2}\text{units}$$

$$QR = \sqrt{(6-3)^2 + (2-5)^2} = 3\sqrt{2}\text{units}$$

$$RS = \sqrt{(3+3)^2 + (5+1)^2} = 6\sqrt{2}\text{units}$$

$$PS = \sqrt{(-3-0)^2 + (-1+4)^2} = 3\sqrt{2}\text{units}$$

$$PR = \sqrt{(3-0)^2 + (5+4)^2} = 3\sqrt{10}\text{units}$$

$$QS = \sqrt{(6+3)^2 + (2+1)^2} = 3\sqrt{10}\text{units}$$

$\because PQ = RS$ and $QR = PS$,

Also $PR = QS$

$\therefore$ PQRS is a rectangle.

Solution 13:

$$AB = \sqrt{(-3-1)^2 + (0+3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$BC = \sqrt{(4+3)^2 + (1-0)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$CA = \sqrt{(1-4)^2 + (-3-1)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$\because AB = CA$

A, B, C are the vertices of an isosceles triangle.

$AB^2 + CA^2 = 25 + 25 = 50$

$BC^2 = (5\sqrt{2})^2 = 50$

$\therefore AB^2 + CA^2 = BC^2$

Hence, A, B, C are the vertices of a right – angled triangle.

Hence, ΔABC is an isosceles right-angled triangle.

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times CA$$

$$= \frac{1}{2} \times 5 \times 5$$

$$= 12.5 \text{ sq.units}$$

Solution 14:

$$AB = \sqrt{(1-5)^2 + (5-6)^2} = \sqrt{16+1} = \sqrt{17}$$
$$BC = \sqrt{(2-1)^2 + (1-5)^2} = \sqrt{1+16} = \sqrt{17}$$
$$CD = \sqrt{(6-2)^2 + (2-1)^2} = \sqrt{16+1} = \sqrt{17}$$
$$DA = \sqrt{(5-6)^2 + (6-2)^2} = \sqrt{1+16} = \sqrt{17}$$

$$AC = \sqrt{(2-5)^2 + (1-6)^2} = \sqrt{9+25} = \sqrt{34}$$
$$BD = \sqrt{(6-1)^2 + (2-5)^2} = \sqrt{25+9} = \sqrt{34}$$

Since, AB = BC = CD = DA and AC = BD,
A, B, C and D are the vertices of a square.

Solution 15:

Let the given points be A (-3, 2), B (-5, -5), C (2, -3) and D (4, 4).

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{4+49} = \sqrt{53}$$
$$BC = \sqrt{(2+5)^2 + (-3+5)^2} = \sqrt{49+4} = \sqrt{53}$$
$$CD = \sqrt{(4-2)^2 + (4+3)^2} = \sqrt{4+49} = \sqrt{53}$$
$$DA = \sqrt{(-3-4)^2 + (2-4)^2} = \sqrt{49+4} = \sqrt{53}$$

$$AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{25+25} = 5\sqrt{2}$$
$$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{81+81} = 9\sqrt{2}$$

Since, AB = BC = CD = DA and AC $\neq$ BD
The given vertices are the vertices of a rhombus.

Solution 16:

AB = CD
$AB^2 = CD^2$
$(-6+3)^2 + (a+2)^2 = (0+3)^2 + (-1+4)^2$
$9 + a^2 + 4 + 4a = 9 + 9$
$a^2 + 4a - 5 = 0$
$a^2 - a + 5a - 5 = 0$
$a(a-1) + 5(a-1) = 0$
$(a-1)(a+5) = 0$
$a = 1$ or -5

It is given that a is negative, thus the value of a is -5.

Solution 17:

Let the circumcentre be P (x, y).

Then, PA = PB

$PA^2 = PB^2$

$(x - 5)^2 + (y - 1)^2 = (x - 11)^2 + (y - 1)^2$

$x^2 + 25 - 10x = x^2 + 121 - 22x$

$12x = 96$

$x = 8$

Also, PA = PC

$PA^2 = PC^2$

$(x - 5)^2 + (y - 1)^2 = (x - 11)^2 + (y - 9)^2$

$x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 121 - 22x + y^2 + 81 - 18y$

$12x + 16y = 176$

$3x + 4y = 44$

$24 + 4y = 44$

$4y = 20$

$y = 5$

Thus, the co-ordinates of the circumcentre of the triangle are (8, 5).

Solution 18:

AB = 5

$AB^2 = 25$

$(0 - 3)^2 + (y - 1 - 1)^2 = 25$

$9 + y^2 + 4 - 4y = 25$

$y^2 - 4y - 12 = 0$

$y^2 - 6y + 2y - 12 = 0$

$y(y - 6) + 2(y - 6) = 0$

$(y - 6)(y + 2) = 0$

$y = 6, -2$

Solution 19:

AB = 17

$AB^2 = 289$

$(11 - x - 2)^2 + (6 + 2)^2 = 289$

$x^2 + 81 - 18x + 64 = 289$

$x^2 - 18x - 144 = 0$

$x^2 - 24x + 6x - 144 = 0$

$x(x - 24) + 6(x - 24) = 0$

$(x - 24)(x + 6) = 0$

$x = 24, -6$

Solution 20:

Distance between the points A (2x - 1, 3x + 1) and B (-3, -1) = Radius of circle

$\therefore$ AB = 10 (Since, diameter = 20 units, given)

$AB^2 = 100$

$(-3 - 2x + 1)^2 + (-1 - 3x - 1)^2 = 100$

$(-2 - 2x)^2 + (-2 - 3x)^2 = 100$

$4 + 4x^2 + 8x + 4 + 9x^2 + 12x = 100$

$13x^2 + 20x - 92 = 0$

$$x = \frac{-20 \pm \sqrt{400 + 4784}}{26}$$

$$x = \frac{-20 \pm 72}{26}$$

$$x = 2, -\frac{46}{13}$$

Solution 21:

Let the co-ordinates of point Q be (10, y).

PQ = 10

$PQ^2 = 100$

$(10 - 2)^2 + (y + 3)^2 = 100$

$64 + y^2 + 9 + 6y = 100$

$y^2 + 6y - 27 = 0$

$y^2 + 9y - 3y - 27 = 0$

$y(y + 9) - 3(y + 9) = 0$

$(y + 9)(y - 3) = 0$

$y = -9, 3$

Thus, the required co-ordinates of point Q are (10, -9) and (10, 3).

Solution 22:

(i) Given, radius = 13 units

$\therefore$ PA = PB = 13 units

Using distance formula,

$$\begin{aligned} PT &= \sqrt{(-2-2)^2 + (-4+7)^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Using Pythagoras theorem in Δ PAT,

$AT^2 = PA^2 - PT^2 = 169 - 25 = 144$

AT = 12 units

(ii) We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore$ AB = 2AT = 2 $\times$ 12 units = 24 units

Solution 23:

$$PQ = \sqrt{(5-2)^2 + (4-2)^2}$$
$$= \sqrt{9 + 4}$$
$$= \sqrt{13}$$
$$= 3.6055$$
$$= 3.61 \text{ units}$$

Solution 24:

We know that any point on x-axis has coordinates of the form (x, 0).
Abscissa of point B = 11
Since, B lies of x-axis, so its co-ordinates are (11, 0).

$$AB = \sqrt{(11-7)^2 + (0-3)^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

Solution 25:

We know that any point on y-axis has coordinates of the form (0, y).
Ordinate of point B = 9
Since, B lies of y-axis, so its co-ordinates are (0, 9).

$$AB = \sqrt{(0-5)^2 + (9+3)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

Solution 26:

Let the required point on y-axis be P (0, y).

$$PA = \sqrt{(0-6)^2 + (y-7)^2}$$
$$= \sqrt{36 + y^2 + 49 - 14y}$$
$$= \sqrt{y^2 - 14y + 85}$$
$$PB = \sqrt{(0-4)^2 + (y+3)^2}$$
$$= \sqrt{16 + y^2 + 9 + 6y}$$
$$= \sqrt{y^2 + 6y + 25}$$

From the given information, we have:

$$\frac{PA}{PB} = \frac{1}{2}$$
$$\frac{PA^2}{PB^2} = \frac{1}{4}$$
$$\frac{y^2 - 14y + 85}{y^2 + 6y + 25} = \frac{1}{4}$$
$$4y^2 - 56y + 340 = y^2 + 6y + 25$$
$$3y^2 - 62y + 315 = 0$$
$$y = \frac{62 \pm \sqrt{3844 - 3780}}{6}$$
$$y = \frac{62 \pm 8}{6}$$
$$y = 9, \frac{35}{3}$$

Thus, the required points on y-axis are (0, 9) and $\left(0, \frac{35}{3}\right)$.

Solution 27:

It is given that PA: PB = 2: 3

$$\frac{PA}{PB} = \frac{2}{3}$$

$$\frac{PA^2}{PB^2} = \frac{4}{9}$$

$$\frac{(x-1)^2 + (y+3)^2}{(x+2)^2 + (y-2)^2} = \frac{4}{9}$$

$$\frac{x^2 + 1 - 2x + y^2 + 9 + 6y}{x^2 + 4 + 4x + y^2 + 4 - 4y} = \frac{4}{9}$$

$$9(x^2 - 2x + y^2 + 10 + 6y) = 4(x^2 + 4x + y^2 + 8 - 4y)$$

$$9x^2 - 18x + 9y^2 + 90 + 54y = 4x^2 + 16x + 4y^2 + 32 - 16y$$

$$5x^2 + 5y^2 - 34x + 70y + 58 = 0$$

Hence, proved.

Solution 28:

$$AB = \sqrt{(a-3)^2 + (-2-0)^2} = \sqrt{a^2 + 9 - 6a + 4} = \sqrt{a^2 - 6a + 13}$$

$$BC = \sqrt{(4-a)^2 + (-1+2)^2} = \sqrt{a^2 + 16 - 8a + 1} = \sqrt{a^2 - 8a + 17}$$

$$CA = \sqrt{(3-4)^2 + (0+1)^2} = \sqrt{1+1} = \sqrt{2}$$

Since, triangle ABC is a right-angled at A, we have:

$AB^2 + AC^2 = BC^2$

$\Rightarrow a^2 - 6a + 13 + 2 = a^2 - 8a + 17$

$\Rightarrow 2a = 2$

$\Rightarrow a = 1$