PLAYING WITH NUMBERS

NUMBER IN GENERALIZED FORM

- **2-DIGIT NUMBER IN GENERALIZED FORM**

2 digit number whose tens digit is x and unit digit is y can be written as $10x + y$ where $x\ (\neq 0)$ and y are whole numbers. Where x is any digit from 1 to 9 in the tens place and y is any of the digit 0 to 9 at the ones place.

(2 Digit) $xy = 10x + y$

Ĝ $10 \times$ tens digit + units digit

(Interchanging) $yx = 10y + x$

Ĝ $10 \times$ tens digit + units digit.

Ex. $= 10 \times \underset{\substack{\text{Tens digit}\\(x)}}{\downarrow 6} + \underset{\substack{\text{Unit digit}\\(y)}}{\downarrow 9}$

☞ The sum of a 2-digit number and its reversing number is always a multiple of 11.

$(10x + y) + (10y + x) = 11x + 11y = 11(x + y)$

☞ When we divide the resulting number by 11, we get always quotient as $(x + y)$.

Ex. $52 + 25 = 77$

$77 \div 11 = 7$

y $x + y = 5 + 2 = 7$

☞ The difference of a two digit number and its reversing number is always a multiple of 9.

(a) If $x > y$: Then $(10x + y) - (10y + x)$

$= 10x + y - 10y - x = 9x - 9y = 9(x - y)$, which is divisible by 9.

(b) If $y > x$: Then $(10y + x) - (10x + y)$

$= 10y + x - 10x - y = 9y - 9x = 9(y - x)$, which is divisible by 9.

(c) If $y = x$: Then $(10x + y) - (10x + y)$

Thus, the quotient is divisible by 9 and remainder is zero.

- **3 DIGIT NUMBERS IN GENERALIZED FORM**

A 3 digit number whose hundreds digit is x tens digit is y and units digit is z can be written as $100x + 10y + z$ where $x\ (\neq 0)$ y and z are whole numbers.

(3 digit number) $xyz = 100x + 10y + z$

(Interchanging) $zyx = 100z + 10y + x$

$100 \times$ hundreds digits + $10 \times$ ten digit + units digit

Ex. $= 100 \times \underset{\substack{\text{hundred digit}\\(x)}}{\downarrow 4} + 10 \times \underset{\substack{\text{Ten digit}\\(y)}}{\downarrow 5} + \underset{\substack{\text{Unit digit}\\(z)}}{\downarrow 8}$

☞ Subtraction of a 3-digit number from its reversed number is a multiple of 99.

(a) If $x > z$: Then $(100x + 10y + z) - (100z + 10y + x) = 99\ (x - z)$ which is multiple of 99.

(b) If $z > x$: Then $(100z + 10y + x) - (100x + 10y + z) = 99\ (z - x)$ which is multiple of 99.

(c) If $x = z$: Then difference is 0.

☞ In general form, 3-digit number is :

$xyz = 100 \times x + 10 \times y + 1 \times z = 100x + 10y + z$

$zyx = 100 \times z + 10 \times y + 1 \times x = 100z + 10y + x$ and

$yzx = 100 \times y + 10 \times z + 1 \times x = 100y + 10z + x$

Ex.2 Solve the cryptarithm: $\overline{AB} + \overline{BA} = \overline{DAD}$

Sol. Clearly, $\overline{AB}$ and $\overline{BA}$ are two digit numbers. So, maximum value of their sum is 99 + 99 + 198. This means that the number $\overline{DAD}$ is at most equal to 198. So, D must be equal to 1. Note that D can not be zero as $\overline{DAD}$ is a three digit number.

Now, $\overline{AB} + \overline{BA} = \overline{DAD}$

$\Rightarrow$ $(10A + B) + (10B + A) = \overline{1A1}$

$\Rightarrow$ $11A + 11B = \overline{1A1}$

$\Rightarrow$ $11(A + B) = \overline{1A1}$ ...(i)

Clearly, LHS of this equation is a multiple of 11. So, RHS must be a multiple of 11 having digits at units and hundreds place as unity. RHS can take ten values viz. 101, 111, 121, 131, ... , 191. Out of these values only 121 is a multiple of 11. Therefore, A = 2.

Substituting A = 2 in (i), we get

$11(2 + B) = 121 \Rightarrow 2 + B = 11 \Rightarrow B = 9$

Hence, A = 2, B = 9 and D = 1.

Ex.3 Solve the following Cryptarithms:

(i) $\begin{array}{r} 1A \\ \times A \\ \hline 9A \\ \hline \end{array}$ (ii) $\begin{array}{r} AB \\ \times 6 \\ \hline BBB \\ \hline \end{array}$ (iii) $\begin{array}{r} AB \\ \times 5 \\ \hline CAB \\ \hline \end{array}$

Sol. (i) We have, $\begin{array}{r} 1A \\ \times A \\ \hline 9A \\ \hline \end{array}$

This means that the product of A with itself is either A or it has units digit as A. Since A = 1 satisfies A × A = 1 but it is not possible as the product is 9A. The other value of A is 6 whose product with itself is a number having 6 at units place.

Taking A = 6 , we have

$$\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \\ \hline \end{array}$$

Clearly, it satisfies the given product. Hence, A = 6.

(ii) We have, $\begin{array}{r} AB \\ \times 6 \\ \hline BBB \\ \hline \end{array}$

This means that 6 × B is a number having its ones digit as B. Such values of B are 2, 4, 6 and 8, because 6 × 2 = 12, 6 × 4 = 24, 6 × 6 = 36 and 6 × 8 = 48. So, we have following cases:

Case-I When B = 2

In this case, we have

$\overline{AB} \times 6 = \overline{BBB} \Rightarrow \overline{A2} \times 6 = 222$

$\Rightarrow$ $(10A + 2) \times 6 = 222$ $\Rightarrow$ $60A + 12 = 222$ $\Rightarrow$ $60A = 210$

$\Rightarrow$ $2A = 7$ $\Rightarrow$ $A = \frac{7}{2}$ which is not possible.

Case-II When B = 4

In this case, we have

$\overline{AB} \times 6 = \overline{BBB}$

$\Rightarrow$ $\overline{A4} \times 6 = 444 \Rightarrow (10A + 4) \times 6 = 444$

$\Rightarrow$ $10A + 4 = \frac{444}{6}$

(iii) To find C, we observe that 2 × 7 = 14 so C = 4, 2 × A + 1 (carried from units place) = 5.

$\Rightarrow$ 2A + 1 = 5 $\Rightarrow$ 2A = 5 – 1 = 4 $\Rightarrow$ A = $\frac{4}{2}$ = 2,

$$\begin{array}{r} 127 \\ \times 312 \\ \hline 254 \\ 1270 \\ 38100 \\ \hline 39624 \end{array}$$

Similarly, you can find the replacements for other letters as B = 2, D = 1, E = 7, F = 8, G = 1, H = 9, K = 6.

(iv) 34 goes into 99, 2 times and 34 × 2 = 68, so A = 2 and D = 6, 9 – 0 = E $\Rightarrow$ E = 9.

Now, 34 goes into 319. 9 times (34 × 9 = 319), so B = 9, H = 6 – 0 = 6, E (= 9) – G = 3, so G = 9 – 3 = 6, 3 – F = 0, so F = 3. H (= 6) – K = 0, so K = 6, 3 – J = 0, so J = 3, 1 – L = 0, so L = 1.

$$\begin{array}{r} 297 \\ 34\overline{)9996} \\ 6800 \\ \hline 3196 \\ 3060 \\ \hline 136 \\ 136 \\ \hline 0 \end{array}$$

TEST OF DIVISIBILITY

- TEST OF DIVISIBILITY BY 2.

A number is divisible by 2 if its units digit is even, i.e., if its units digit is any of the digits 0, 2, 4, 6 or 8. For a number in the generalized form :

(i) A two-digit number 10a + b : is divisible by 2 if 'b' is any of the digits 0, 2,4,6 or 8.

(ii) A three-digit number 100a + 10b + c : is divisible by 2 if 'c' is any of the digits 0, 2, 4, 6 or 8.

Ex. The numbers 12, 68, 120, 854 are all divisible by 2.

- TEST OF DIVISIBILITY BY 3.

A number is divisible by 3, if the sum of its digits is divisible by 3. For a number in the generalized form :

(i) A two-digit number 10a + b : is divisible by 3 if (a + b) is divisible by 3.

(ii) A three-digit number 100a + 10b + c : is divisible if (a + b + c) is divisible by 3.

Ex. 21, 54, 123, 351 are all divisible by 3 but none of the numbers 22, 56, 76, 359, 835 divisible by 3.

- TEST OF DIVISIBILITY BY 5.

A number is divisible by 5, if its units digit is either 0 or 5. For a number in the generalized form :

(i) A two-digit number l0a + b : is divisible by 5 if 'b' is either 0 or 5.

(ii) A three-digit number 100a + 10b + c : is divisible by 5, if 'c' is divisible by 5.

Ex. The numbers 15, 80, 110 are all divisible by 5.

- TEST OF DIVISIBILITY BY 9.

A number is divisible by 9 if the sum of its digits is divisible by 9. For a number in the generalized form :

(i) A two-digit number 10a + b : is divisible by 9 if 'a + b' is divisible by 9.

(ii) A three-digits number 100a + 10b + c : is divisible by 9 if 'a + b + c' is divisible by 9.

Ex. The numbers 18, 27, 225, 801 are all divisible by 9.

- TEST OF DIVISIBILITY BY 10.

A number is divisible by 10, if its units digit is 0. For a number in the generalized form :

(i) A two-digit number 10a + b : is divisible by 10, if 'b' is equal to 0.

(ii) A three-digit number 100a + 10b + c : is divisible by 10, if 'c' is equal to 0.

$(7^{153} \times 1^{72}) = [(7^4)^{38} \times 7 \times 1]$

Required unit digit = $(1 \times 7 \times 1) = 7$.

Ex.9 What is the unit digit in $(264)^{102} + (264)^{103}$?

Sol. $(264)^{102} + (264)^{103} = (264)^{102} \times (1 + 264) = (264)^{102} \times 265$

Required unit digit = unit digit in $[4^{102} \times 5] = [(4^4)^{25} \times 4^2] \times 5 = 6 \times 6 \times 5 = 0$

Ex.10 Find the total number of prime factors in the product $(4^{11} \times 7^5 \times 11^2]$?

$(2 \times 2)^{11} \times 7^5 \times (11)^2 \Rightarrow (22)^{11} \times 7^5 \times (11)^2 \Rightarrow 2^{22} \times 7^5 \times 11^2$

Required number of factors = $(22 + 5 + 2) = 29$

Ex.11 Find the remainder when 2^{31} is divided by 5 ?

Sol. $2^{31} = (2^{10} \times 2^{10} \times 2^{10}) \times 2 \Rightarrow (2^{10})^3 \times 2 = (1024)^3 \times 2$

Unit digit in 2^{31} = unit digit in $[(1024)^3 \times 2] = 4 \times 2 = 8$

Now, 8 when divided by 5 gives 3 as remainder

$\therefore$ 2^{31} when divided by 5 given remainder = 3.

NUMBER PUZZLES AND GAMES

Ex.12 Use the numeral 5 only three times and the signs +, –, ÷, × and $\sqrt{\ }$ make the numeral 1.

Sol. $5 \div (\sqrt{5} \times \sqrt{5}) = 1$ or $(\sqrt{5} \times \sqrt{5}) \div 5$

Ex.13 Insert the symbols +, –, × , ÷, and parenthesis in the following sequence of numbers so that the expression equals 100.

1, 2, 3, 4, 5, 6, 7, 8, 9

Sol. The desired expression is $1 + (2 \times 3) - 4 + (56 \div 7) + 89$

Clearly, the value of the above expression is 100.