05

Arithmetic Progression

5.01 Introduction

We must have observed that in nature, many things follows a certain pattern, such as the holes of a beehive represents a definite pattern of such type.

A pipe rod is fixed at definte gap of a steel ladder used in shop or home. In mathematical language we can say that patterns increase or decrease in a fixed number and there is same relation in this first, second or third sequence. This definite series of numbers is called sequence.

For Example : Consider the sequence of following numbers :

(i) $2, 4, 6, 8, 10, ...$

(ii) $8, 5, 2, -1, -4, ...$

(iii) $3^o, 3^1, 3^2, 3^3, 3^4 ...$

In sequence (i) each term is 2 less than the term succeeding it.

In sequence (ii) each term is 3 less than the term preceeding it.

Similarly in sequence (iii) each term is in increasing power of 3.

From above examples, it is clear that all the patterns follows a definite pattern. In this chapter we will discuss such type of pattern in which succeding terms are obtained by adding a fixed number to the preceding terms. Here we will study the general term (n^{th} term) of this pattern and methods to find sum of consecutive terms.

5.02. Arithmetic Progression

Consider the following sequence of numbers:

(i) $1, 4, 7, 10, 13, ...$

(ii) $100, 70, 40, 10, ...$

(iii) $-5, -3, -1, 1, ...$

In the above sequences, each term is obtained by adding a definite number (positive or negative) except the first term. Such sequence of numbers is said to form an arithmetic progression.

Difference between each number with its preceeding number is same. This fixed number is called the common difference of the A.P. Let $a_1, a_2, ..., a_n$ be terms of any sequence if they are in A.P. then each term is obtained by adding a fixed number to the preceding term except the first term.

Let common difference be d, then

$$\begin{aligned} a_2 &= a_1 + d \\ a_3 &= a_2 + d \\ \vdots & \quad \vdots \\ a_n &= a_{n-1} + d \end{aligned}$$

So, $a_2 - a_1 = a_3 - a_2 = ... = a_n - a_{n-1} = d$

In general $a_n - a_{n-1} = d$, where $n = 1, 2, 3, ...$

We can say that if first term of sequence is a and common difference is d then general form of A.P. can be written as

$$a, a+d, a+2d, a+3d, ..., a+(n-1)d, ...$$

Above concepts can be understand by the followign examples

Example 1. Write the first term and common difference of the following A.P.

$$-5, -1, 3, 7, ...,$$

Solution : On comparing given A.P. by its general form, we get first term $a = -5$

Common difference d = difference between two consecutive terms

i.e., $-5-(-1) = 4$, $3-(-1) = 4$

Example 2. Check, which of the following sequence are in the form an A.P.,?

(i) $4, 10, 16, 22, ...$

(ii) $-2, 2, -2, 2, -2, ...$

Solution : (i) Let us find the common difference to check an A.P. of first sequence i.e., $4, 10, 16, 22, ...$

$$a_2 - a_1 = 10 - 4 = 6$$

$$a_3 - a_2 = 16 - 10 = 6$$

$$a_4 - a_3 = 22 - 16 = 6$$

Same difference is obtained each time so this sequence is in on A.P. with common difference = 6

(ii) Let us find out the common difference to check an A.P. of second sequence i.e., $-2, 2, -2, 2, -2, ...$

$$a_2 - a_1 = 2 - (-2) = 4$$

$$a_3 - a_2 = -2 - (2) = -4$$

$$a_4 - a_3 = 2 - (-2) = 4$$

Same difference is not obtained each time, so given sequence does not form an A.P.

Example 3. Find the common difference of the followig A.P. and write next four terms.

(i) $0, -3, -6, -9, ...$ (ii) $-1, \frac{-5}{6}, \frac{-2}{3}, ...$

Solution : (i) Let A.P. is $a_1, a_2, a_3, ...$ So, here

$$a_2 - a_1 = -3 - 0 = -3$$

$$a_3 - a_2 = -6 - (-3) = -3$$

$$a_4 - a_3 = -9 - (-6) = -3$$

It is clear that, the difference between two consecutive terms is equal i.e. –3. So, common difference $d=-3$ and next four terms will be as follows:

$$a_5=a_4+d=-9+(-3)=-12$$

$$a_6=a_5+d=-12+(-3)=-15$$

$$a_7=a_6+d=-15+(-3)=-18$$

$$a_8=a_7+d=-18+(-3)=-21$$

(ii) Let A.P. is expressed as $a_1, a_2, a_3, \ldots$ then

$$a_2-a_1=\frac{-5}{6}-(-1)=\frac{1}{6}$$

$$a_3-a_2=\frac{-2}{3}-\frac{(-5)}{6}=\frac{-4+5}{6}=\frac{1}{6}$$

It is clear that the difference of two consecutive terms '$\frac{1}{6}$' is same so common difference $d=\frac{1}{6}$

So next four terms will be as follows:

$$a_4=a_3+d=\frac{-2}{3}+\frac{1}{6}=\frac{-4+1}{6}=\frac{-3}{6}=\frac{-1}{2}$$

$$a_5=a_4+d=\frac{-1}{2}+\frac{1}{6}=\frac{-3+1}{6}=\frac{-2}{6}=\frac{-1}{3}$$

$$a_6=a_5+d=\frac{-1}{3}+\frac{1}{6}=\frac{-2+1}{6}=\frac{-1}{6}$$

$$a_7=a_6+d=\frac{-1}{6}+\frac{1}{6}=0$$

Exercise 5.1

1. Find the first term *a* and common difference *d* for the following A.P.

(i) $6, 9, 12, 15, \ldots$ (ii) $-7, -9, -11, -13, \ldots$ (iii) $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}, \ldots$

(iv) $1, -2, -5, -8, \ldots$ (v) $-1, \frac{1}{4}, \frac{2}{3}, \ldots$ (vi) $3, 1, -1, -3, \ldots$

(vii) $3, -2, -7, -12, \ldots$

2. If first term *a* and common difference *d* of A.P. is given then find the first four terms of that progression.

(i) $a=-1,\ d=\frac{1}{2}$ (ii) $a=\frac{1}{3},\ d=\frac{4}{3}$ (iii) $a=0.6,\ d=1.1$

(iv) $a=4,\ d=-3$ (v) $a=11,\ d=-4$ (vi) $a=-1.25,\ d=-0.25$

(vii) $a=20,\ d=\frac{-3}{4}$

3. Check the following list of numbers for an A.P. If any of then form an A.P., then find its common difference and write the next four terms:

(i) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

(ii) $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \ldots$

(iii) $a, a^2, a^3, a^4, \ldots$

(iv) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$

(v) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

(vi) $a, 2a, 3a, 4a, \ldots$

(vii) 0.2, 0.22, 0.222, ...

(viii) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

5.03 n^{th} term 'general term' of an Arithmetic Progression

In previous section we have studied about finding of consecutive terms of A.P. by first term a and common difference d. Here we consider the following examples.

Let basic salary of an employee is ₹ 1000 and he gets ₹ 300 as annual increment then after 20 yrs what will be his sarly. To find this we calculate salary of first five years.

	Monthly salary in I year	= ₹ 10,000
	Monthly salary in II year	= ₹ (10000 + 300)
i.e.	Salary in II year	= ₹ 10,300
∴	Monthly salary in III year	= ₹ (10300 + 300)
		= ₹ (10,000 + 300 + 300)
		= ₹ [10,000 + 2 × 300]
i.e.	Salary in III year	= ₹ (10,000 + (3–1) × 300)
		= ₹ 10600
∴	Monthly salary in IV year	= ₹ (10600 ? 300)
		= ₹ (10,000 + 300 + 300 + 300)
		= ₹ [10,000 + 3 × 300]
i.e.	Salary in IV year	= ₹ (10,000 + (4 – 1) × 300)
		= ₹ 10,900
	Similarly, salary in V year	= ₹ (10,900 + 300)
		= ₹ (10,000 + 300 + 300 + 300 + 300)
		= ₹ (10,000 + 4 × 300)
		= ₹ (10,000 + (5–1) × 300)
		= ₹ 11,200

Here we write data of 5 years of annual salary in the following order.

10000, 10300, 10600, 10900, 11200 ...

This sequence is an A.P. because common difference between two consecutive terms is 300. From above it is clear that by adding ₹ 300 in previous year salary, we can find salary of required year. From above it is clear that in 20^{th} year salary of employee will be

Salary of 19^{th} year + ₹ 300

= ₹ [10,000 + (300 + 300 + . . . + 300) + 300]

= ₹ [10,000 + (20 – 1) × 300]

i.e. Salary in 20^{th} year = ₹ 15,700

So it is clear that as we have find salary in II, III, IV, V and at last in 20th year so in general this relation can be written in the following form.

Salary for 20th year = I (Basic) salary + (20 –1) annual increment

We can generalize this example as

If first term is a, common difference is d, n^{th} term (general term) is a_n then it can be written as $a_n = a + (n-1)d$

Let A.P. is in the form $a_1, a_2, a_3, ..., a_n, ...$ and $a_1 = a$ = first term and common difference is d then

Second term $\quad a_2 = a + d = a + (2-1)d$

and $\quad a_3 = a_2 + d = (a+d) + d = a + 2d$

or $\quad a_3 = a + (3-1)d$

Similary n^{th} term $\quad a_n = a_{n-1} + d = a + (n-1)d$

So, **General term = first term + (no. of terms –1) × common difference**

Here it is necessary to mention, if there are m terms in A.P., i.e., last term is a_m (or ℓ) then n^{th} term from last will as follows:

n^{th} term from last $\quad = a_{m?n?1}$

$= a + (m - n + 1 - 1)d$

$= a + (m - n)d$

If we take last term 'ℓ' as first term and reducing common difference as $-d$ then n^{th} term from last can be written as

n^{th} term from last = last term $+(n-1)(-d)$

$= \ell - (n-1)d$

By the following examples we can easily understand about the general term of Arithemetic Progression.

Example 4. Find the 30th and n^{th} (general terms) of an A.P. 10, 7, 4, ...

Solution : Given A.P. is

$$10, 7, 4, \ldots$$

Its first term $\quad a = 10$

Common difference $\quad d = 7 - 10 = -3$

So, n^{th} term of given A.P. is a_n

i.e. $\quad a_n = a + (n-1)d$

Similarly, 30th term $\quad a_{30} = 10 + (30-1) \times (-3)$

$= 10 - 29 \times 3 = -77$

and general term (n^{th} term) $\quad a_n = 10 + (n-1) \times (-3)$

$= 10 - 3(n-1) = 13 - 3n$

Thus, required 30th term $= -77$ and n^{th} term $= 13 - 3n$.

Example 5. Which term of an A.P. 3, 15, 27, 39, ... is 639?

Solution : Give A.P. is 3, 15, 27, 39, ...

$\therefore$ First term $a = 3$ and common difference $d = 12$. Let n^{th} term $= 639$, then general term

$$a_n = a + (n-1)d$$

Here, $639 = 3 + (n-1) \times 12$

or $639 = 3 + 12n - 12$

or $648 = 12n$

or $n = \frac{648}{12} = 54$

Hence, 54^{th} term of A.P. is 639.

Example 6. Find the number of terms in A.P. 7, 13, 19, ..., 205.

Solution : Given A.P. is 7, 13, 19, ..., 205. Here first term $a = 7$, and common difference $d = 6$. Let n^{th} term is last term, then

$$a_n = 205.$$

$\therefore$ n^{th} term $a_n = a + (n-1) \times d$

$205 = 7 + (n-1) \times 6$

$205 = 7 + 6n - 6$

or $204 = 6n$

or $n = \frac{204}{6} = 34$

Hence there are 34 terms in given A.P.

Example 7. If third term of an A.P. is 12 and 50^{th} term in 106. Find its 29^{th} term.

Solution : General term of A.P. $= n^{th}$ term

$\therefore$ $a_n = a + (n-1)d$

Where a is first term of A.P. and d is common difference

Here $a_3 = 12$ and $a_{50} = 106$

So, $a_3 = a + (3-1)d$

or $12 = a + 2d$ . . . (i)

and $a_{50} = a + (50-1)d$

or $106 = a + 49d$ . . . (ii)

By subtracting equation (i) from (ii), we get

$$106 - 12 = 49d - 2d$$

or $94 = 47d$

or $d = \frac{94}{47} = 2$

Putting the value of d in equation (i)

$$12 = a + 2 \times 2$$

or $$a = 8$$

$\therefore$ 29^{th} term $$a_{29} = a + (29-1)d$$

$$= 8 + 28 \times 2 = 64$$

Thus, 29^{th} term of A.P. will be 64.

Example 8. Is 184 any term of A.P. 3, 7, 11, ... ?

Solution : Given A.P. is 3, 7, 11, ... Here first term $a = 3,$ and common differece $d = 4$

Let n^{th} term of A.P is 184

So, $$a_n = a + (n-1)d$$

$\therefore$ $$184 = 3 + (n-1) \times 4$$

or $$184 = 3 + 4n - 4$$

or $$185 = 4n$$

or $$n = \frac{185}{4} = 46\frac{1}{4}$$

Since value of n is not a natural number. So, 184 is not any term of given A.P.

Example 9. How many two digit numbers are divisible by 7?

Solution : We know that smallest number of two digit (positive) which is divisible by 7 is 14. So following will be the sequence of two digit number divisible by 7.

$$14, 21, 28, \ldots, 98$$

This is an A.P. whose first term is $a = 14$ and common difference $d = 7$.

Let A.P. has n terms then n^{th} term $a_n = 98$ can be expressed as:

$$a_n = a + (n-1)d$$

i.e. $$98 = 14 + (n-1) \times 7$$

or $$98 = 14 + 7n - 7$$

or $$91 = 7n$$

or $$n = \frac{91}{7} = 13$$

Thus, there are 13 two digit number which are divisible by 7.

Example 10. Find the 20^{th} term of A.P. 3, 8, 13, ..., 253 from the last.

Solution : Here last term of A.P. is $\ell = 253$. First term $a = 3$ and common difference $d = 5$. So 20^{th} term from last term

$$= \ell - (20-1)d$$

$$= 253 - 19 \times 5 = 253 - 95 = 158$$

Thus 20^{th} term from the last is 158.

Example 11. How many multiples of 4 between 10 and 250?

Solution : Clearly, first number divisible by 4, between 10 and 250 is 12. When we divide 250 by 4 then remainder 2 obtained. So last term which is divisible by 4 is 250 ? 2 ? 248 i.e., numbers which are divisible by 4 between 10 and 250 makes following A.P.

$$12, 16, \ldots, 248$$

Now we have to find number of multiples of 4. Let this is n, then a_n ? 248 i.e.,

$$a_n = a + (n-1)d$$

or $$248 = 12 + (n-1) \times 4$$

or $$248 = 12 + 4n - 4$$

or $$240 = 4n$$

or $$n = 60$$

Thus, there will be 60 multiples of 4 between 10 and 250.

5.04 Selection of terms of Arithmetic Progression

To find numbers (odd or even term) in A.P. terms of progression can be conveniently find by the following way

Numbers	**Terms**
3	$a-d, a, a+d$
4	$a-3d, a-d, a+d, a+3d$
5	$a-2d, a-d, a, a+d, a+2d$
6	$a-5d, a-3d, a-d, a+d, a+3d, a+5d$

Here, it is clear that if number of terms is odd then mid term is a and common difference is d and if number of terms is even then $a - d$ and $a + d$ are two mid terms and common difference is $2d$. Problems related to numbers can be clear by following examples.

Examples 12. Three numbers are in A.P. If their sum is –3 and product is 8 then find the numbers.

Solution : Let three numbers in A.P. are

$$a-d, a, a+d$$

Given that sum of numbers is – 3

i.e. $$(a-d) + a + (a+d) = -3$$

or $$3a = -3$$

or $$a = -1$$

It is also given that product of numbers is 8

∴ $$(a-d) \times a \times (a+d) = 8$$

or $$(a^2 - d^2) \times a = 8$$

Putting $$a = -1$$

$$[(-1)^2 - \mathrm{d}^2] \times (-1) = 8$$

or $$d^2 - 1 = 8$$

or $d^2 = 9$

or $d = \pm 3$

By putting vlaue of a and d, required numbers are obtained. If d ? 3 then $-1-3, -1, -1+3$ i.e., $-4, -1, +2$ and If $d = -3$ then $-1+3, -1, -1-3$ i.e., $2, -1, -4$ are required numbers

Exercise 5.2

1. Find:
   (i) 10^{th} term of A.P., 2, 7, 12, ...
   (ii) 18^{th} term of A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$
   (iii) 24^{th} term of A.P. 9, 13, 17, 21 ...
2. Solve :
   (i) Which term of A.P., 21, 18, 15, ... is –81?
   (ii) Which term of A.P., 84, 80, 76, ... is zero?
   (iii) Is 301 any term of sequence 5, 11, 17, 23, ...?
   (iv) Is –150 any term of A.P. 11, 8, 5, 2, ...?
3. If 6^{th} and 17^{th} term of an A.P. are 19 and 41 respectively then find 40^{th} term.
4. If 3^{rd} and 9^{th} term of an A.P. are 4 and –8 respectively then which term of its will be zero?
5. If third term of an A.P. is 16 and 7^{th} term is 12 more than 5^{th} term, then find A.P.
6. How many three digits numbers are divisible by 7?
7. Find the 11^{th} term from the last of A.P. 10, 7, 4, ... –62.
8. Find the 12^{th} term from the last of an A.P. 1, 4, 7, 10, ... 88
9. There are 60 terms in an A.P. If its first and last terms are 7 and 125 respectively then find its 32^{th} term.
10. Four numbers are in A.P. If sum of numbers is 50 and largest number is four times the smaller one,then find the numbers.

5.05 Sum of First *n* terms of Arithmetic Progression

In this section, we will obtain the formula of sum of arithemtic progression. To understand this consider an example. Lata's mother gives Rs. 500 on her birthday, Rs 600 on IInd birthday, Rs700 onIIIrd, Rs. 800 on IVth birthday and will continue in the same way upto 18 years of Lata's age. How much money will be collected at the age of 18 years?

Here we see that 500, 600, 700, 800, ... these numbers are in arithemetic progression so for adding these 18 terms i.e., for adding the terms of A.P. we get formula by following method. In this formula, by substituting values of variables we will find sum of A.P. easily.

Let a be first term of A.P. and d is common difference and S_n is sum of its n terms. So first n terms of A.P. are written as

$$a, a+d, a+2d, ..., a+(n-1)d$$

then $$S_n = a + (a+d) + (a+2d) + ... + \left[a+(n-2)d\right] + \left[a+(n-1)d\right] \quad ... \text{(i)}$$

By reversing the order of terms, sum does not effected so we can write as

$$S_n = [a+(n-1)d]+[a+(n-2)d]+...+(a+2d)+(a+d)+a \qquad ...(ii)$$

On adding corresponding terms of (i) and (ii)

$$2S_n = [2a+(n-1)d]+[2a+(n-1)d]+...+[2a+(n-1)d]+[2a+(n-1)d]$$

($\because$ It contains n terms)

$\therefore \qquad 2S_n = n[2a+(n-1)d]$

or $\qquad S_n = \frac{n}{2}[2a+(n-1)d]$

This formula shows the sum of n terms,when first term and common difference of an A.P. is given.

If last term of A.P. is ℓ then formula $S_n = \frac{n}{2}[2a+(n-1)d]$ can be written in the following form and sum can be obtained.

i.e. $\qquad S_n = \frac{n}{2}[a+a+(n-1)d]$

or $\qquad S_n = \frac{n}{2}[a+\ell] \qquad [\because \ \ell =$ last term = n^{th} term $= a+(n-1)d$]

Thus,there are n terms in A.P. then $a_n = \ell$ will be last term, so

Sum of n terms $= S_n = \frac{n}{2}$ [First term + last term]

Here, it is necessary to understand that n^{th} term ofA.P. is equal to the difference of sum of first n terms and first $(n-1)$ terms.

i.e., $\qquad a_n = S_n - S_{n-1}$

By above formula, problems based on sum of terms of A.P., can be easily understand by following examples.

Example 13. Find the sum of

(i) A.P. 1, 4, 7, 10, ... upto 20 terms

(ii) A.P. 2, 7, 12, ... upto 10 terms

Solution : (i) Given A.P. is 1, 4, 7, 10, ...

Here first terms $a = 1$ and common difference $d = 3$

$\therefore \qquad$ Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$

Sum of 20 terms $S_{20} = \frac{20}{2}[2\times1+(20-1)\times3]$

$$=10[2+57]=590$$

Thus, required sum = 590.

(ii) Given A.P. is 2, 7, 12, ...

Here first term a = 2 common difference $d = 5$. Since sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$

So, Sum of 10 terms $S_{10} = \frac{10}{2}[2\times 2+(10-1)\times 5]$

or $S_{10} = 5[4+45] = 5\times 49 = 245$

Thus, required sum $= 245$

Example 14. Find the sum of the following :

(i) $34+32+30+....+10$

(ii) $(-5)+(-8)+(-11)+....+(-230)$

Solution : (i) Given series $34+32+30+....+10$ is an A.P. whose first term $a = 34$ last term $\ell = a_n = 10$ and common difference $d = -2$

Thus, $a_n = a+(\mathrm{n}-1)\,\mathrm{d}$

or $10 = 34+(n-1)(-2)$

or $10 = 34-2n+2$

or $2n = 26$

or $n = 13$

Sum of series $S_n = \frac{n}{2}[a+\ell]$

Thus, $S_{13} = \frac{13}{2}[34+10] = 13\times 22 = 286$

(ii) Given series $(-5)+(-8)+(-11)+...+(-230)$ is an A.P. whose first term $a = -5$ and common difference $d = -3$ last term $a_n = \ell = -230$

$\therefore$ $a_n = a+(n-1)d$

Here $-230 = -5+(n-1)\times(-3)$

or $-230 = -5-3n+3$

or $3n = 228$

or $n = \frac{228}{3} = 76$

$\because$ $S_n = \frac{n}{2}[a+\ell]$

$\therefore$ $S_{76} = \frac{76}{2}[-5+(-230)] = 38\times(-235) = -8930$

Example 15. Sum of how many terms of an A.P. 54, 51, 48, ... will be 513 ?

Solution : First term of an A.P. 54, 51, 48, ... is a = 54 and common difference is d = -3

Let sum of n terms is $S_n = 513$ then sum of n terms of A.P

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Here $$513 = \frac{n}{2}[2\times 54+(n-1)\times(-3)]$$

or $$513 = \frac{n}{2}[108-3n+3]$$

or $$513\times 2 = n(111-3n)$$

or $$3n^2 - 111n + 1026 = 0$$

or $$n^2 - 37n + 342 = 0$$

On factorize, we get

or $$n^2 - 18n - 19n + 342 = 0$$

or $$n(n-18) - 19(n-18) = 0$$

or $$(n-19)(n-18) = 0$$

or $$n = 19 \quad \text{and} \quad n = 18$$

Here common difference $d = -3$ (is negative)

and 19^{th} term $= a_{19} = a+(n-1)d = 54+(19-1)(-3) = 0$

Here 19^{th} term is zero. So, sum of 18 terms and sum of 19 terms will be same 513.

Example 16. Find the sum of first 15 terms of A.P. whose n^{th} term is $a_n = 9-5n$.

Solution : $\because$ n^{th} term of series $a_n = 9-5n$

$\therefore$ $$a_1 = 9-5\times 1 = 4$$

$$a_2 = 9-5\times 2 = -1$$

$$a_3 = 9-5\times 3 = -6$$

So sequence of number obtained are $4, -1, -6, \ldots$

which is an A.P. whose first term $a = 4$ and common difference is $d = -5$.

Thus, sum of n terms of this series

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Here $$S_{15} = \frac{15}{2}[2\times 4+(15-1)\times(-5)] = \frac{15}{2}[8-70]$$

$$= -(15\times 31) = -465$$

Thus, sum of first 15 terms of A.P. will be –465.

Example 17. If sum of first 7 terms of A.P. is 49 and sum of first 17 terms is 289, then find the sum of first n terms of A.P.

Solution : Given that $S_7 = 49$ and $S_{17} = 289$

Sum of n terms of series

$\because$ $$S_n = \frac{n}{2}[2a + (n-1)d]$$

Here, $$S_7 = \frac{7}{2}[2a + (7-1)d] = 49$$

and $$S_{17} = \frac{17}{2}[2a + (17-1)d] = 289$$

Thus, on writing above two equations in simplest form, we get first equation

$$2a + 6d = \frac{49 \times 2}{7}$$

or $$a + 3d = 7 \qquad \text{. . . (i)}$$

and second equation $$2a + 16d = \frac{289 \times 2}{17}$$

or $$a + 8d = 17 \qquad \text{. . . (ii)}$$

On subtracting (ii) from (i), we get

$$5d = 10$$

or $$d = 2$$

Putting value of d in equation (i), we get

$$a + 3 \times 2 = 7$$

or $$a = 7 - 6$$

or $$a = 1$$

Thus, putting value of a and d in the formula of A.P sum of n terms

$$S_n = \frac{n}{2}[2 \times 1 + (n-1) \times 2] = \frac{n}{2}[2 + 2n - 2] = n^2$$

Thus, sum of n terms of A.P is n^2.

Example 18. If sum of n terms of A.P is $4n - n^2$ then what is the first term ? What is the sum of first two terms? What is second term? Similarly find 3rd, 10th and nth term.

Solution : Given that sum of n terms of A.P., $S_n = 4n - n^2$

Putting $n = 1$

$$S_1 = 4 \times 1 - (1)^2 = 4 - 1 = 3$$

So, first term is 3

For sum of two terms

$$S_2 = 4 \times 2 - (2)^2 = 8 - 4 = 4$$

So, sum of first two terms is 4

Second term $a_2 = S_2 - S_1 = 4 - 3 = 1$

i.e., Second term of A.P. is 1.

Here, sum of first three terms $S_3 = 4 \times 3 - (3)^2 = 12 - 9 = 3$

$\therefore$ Third term of A.P. $a_3 = S_3 - S_2 = 3 - 4 = -1$

Thus obtained A.P. is $3, 1, -1, ...$ whose common difference $d = a_3 - a_2 = -1 - 1 = -2$

$$n^{\text{th}} \text{ term } = a_n = a + (n-1)d$$

Here, first term $a = 3$, common difference $d = -2$, So

$$a_n = 3 + (n-1) \times (-2) = 3 - 2n + 2 = 5 - 2n$$

i.e., n^{th} term $a_n = 5 - 2n$ So for 10^{th} term, putting $n = 10$

$$a_{10} = 5 - 2 \times 10 = 5 - 20 = -15$$

Thus, 10th term will be -15.

Example 19. Find the sum of natural number divisible by 3 between 250 and 1000.

Solution : It it clear that between 250 and 1000, numbers divisible by 3 are 252, 255, 258, ... 999 which is in an A.P. Its first term a = 252,last term $a_n = l$ = 999 and common difference d = 3

Here $a_n = a + (n-1)d$

$\therefore$ $999 = 252 + (n-1) \times 3$

or $999 = 252 + 3n - 3$

or $999 = 249 + 3n$

or $3n = 750$

or $n = 250$

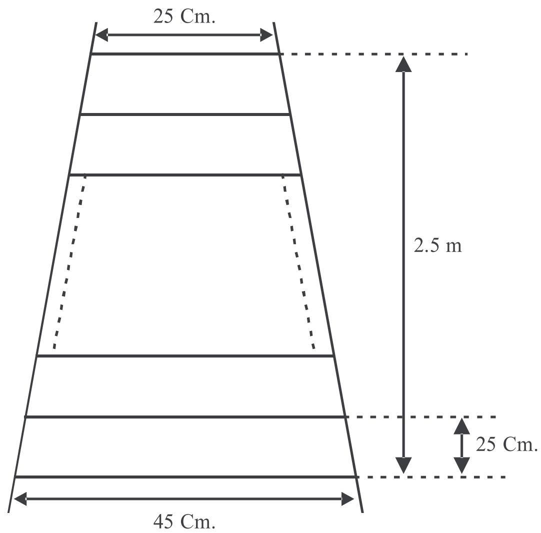
$\therefore$ Required sum $S_n = \frac{n}{2}(a + \ell)$

$\therefore$ $S_{250} = \frac{250}{2}(252 + 999)$

$$= 125 \times 1251 = 156375$$

Thus required sum will be 156375.

Example 20. A ladder has rungs 25 cm apart (See fig.). Length of lowest rung is 45 cm and it goes to decrease as goes to high and the rungs 25 cm at the top. If the top and the bottom rungs are 2.5m apart, what is the length of the wood required for the rungs ?



Solution : Given that distance between two consecutive rungs is 25 cm and distance between first and last rungs is 2.5 m *i.e.,* 250 cm.

So number of rungs in ladder

$$= \frac{250}{25} + 1 = 10 + 1 = 11$$

It is given that rungs decrease uniformly from bottom to top and length of rungs at bottom is 45 cm and at top 25 cm. So it is clear that length of rungs are in an A.P. whose first term a = 45 cm and 11^{th} term (last term) l = 25 cm.

So, total length of wood required = Sum of 11 terms of A.P.

$$\because \qquad S_n = \frac{n}{2}(a + \ell)$$

$$\therefore \qquad S_n = \frac{11}{2}(45 + 25)\text{cm} = 11 \times 35 = 385\,\text{cm}$$

Thus, total length of wood will be 3.85 m.

Exercise 5.3

1. Find the sum of the follwing arithmetic progression :
   (i) 1, 3,5, 7, ... upto 12 terms   (ii) 8, 3, -2, ... upto 22 terms
   (iii) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots,$ upto 11 terms
2. Find the sum of the following :
   (i) $3 + 11 + 19 + \ldots + 803$ (ii) $7 + 10\frac{1}{2} + 14 + \ldots + 84$
3. Find the number of terms :
   (i) How many terms of an A.P. $9, 17, 25, \ldots$ taken so that their sum is 636?
   (ii) How many terms of an A.P. $63, 60, 57, \ldots$ taken so that their sum is 693?
4. Find the sum of first 25 terms of following progression whose n^{th} term is given :
   (i) $a_n = 3 + 4n$ (ii) $a_n = 7 - 3n$

5. Find the sum of 51 terms of an A.P. in which 2^{nd} and 3^{rd} terms are 14 and 18 respectively.
6. First and last term of A.P. are 17 and 350 respectively. If common difference is 9 then how many terms are in A.P. what is their sum?
7. Find the sum of all odd numbers divisible by 3 between 1and 1000.
8. First term of an arithmetic progression is 8, n^{th} term is 33 and sum of first n terms is 123, then find n and common difference d.
9. A sum of ₹280 is to be used to give four prizes. If each prize is ₹20 less than its precceding prize, find the value of each of the prizes.
10. A manufacturer of T.V. sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find
    (i) the production in the 1^{st} year (ii) the production in the 10^{th} year.
    (iii) the total production in first 7^{th} year.

Miscellaneous Exercise-5

1. The common difference of two Arithmetic progression is same out of two, one A.P. has first term is 8 and other is 3. The difference between their 30^{th} terms is :
   (a) 11 (b) 3 (c) 8 (d) 5
2. If $18, a, b, -3$ are in an A.P., then $a+b=$
   (a) 19 (b) 7 (c) 11 (d) 15
3. If 7^{th} and 13^{th} terms of an A.P. are 34 and 64 respectively then its 18^{th} term is :
   (a) 89 (b) 88 (c) 87 (d) 90
4. In an A.P. first term is 2 and common differnce is 8 , if sum of n terms is 90 then value of n will be:
   (a) 3 (b) 4 (c) 5 (d) 6
5. If sum of n terms of an A.P. is $3n^2+5n$, then which term of series is 164?
   (a) 12^{th} (b) 15^{th} (c) 27^{th} (d) 20^{th}
6. If S_n is sum of first n terms of A.P. and $S_{2n}=3S_n$ then $S_{3n}:S_n$ will be :
   (a) 10 (b) 11 (c) 6 (d) 4
7. The first and last term of A.P. are1 and 11 respectively. If sum of its terms is 36 then number of terms will be
   (a) 5 (b) 6 (c) 99 (d) 11
8. Write 5^{th} term form last of A.P. $3, 5, 7, 9, ..., 201$.
9. If three consecutive terms of A.P. are $\frac{4}{5}, a, 2$, then find the value of a.
10. Find the sum of first 1000 positive integers.
11. Is 299 be any terms of sequence of numbers $5, 11, 17, 23, ...$?
12. Which term of an A.P. $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is first negative term?
13. Four numbers are in arithmetic progression. If their sum is 20 and sum of their squares is 120, then find the numbers.
14. If sum of n terms of an A.P. is $\frac{3n^2}{2}+\frac{5n}{2}$, then find its 25^{th} term.

15. Houses in a row are numbered serially from 1 to 49. Show that one value of x in such that sum of numbers of houses preceeding the marked house is equal to the sum of numbers of houses succeeding the marked house. Find the valur of x.

Important Points

1. $a, a+d, a+2d, \ldots$ is general form of an A.P., where a is first term and d is common difference.
2. Sequence of numbers is in arithmetic progression. If difference $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ are same. This is called common difference of A.P.
3. General term of an A.P. (n^{th} term) $a_n = a + (n-1)d$, where a is first term and d is common difference.
4. Sum of n terms of an A.P. $a, a+d, a+2d, \ldots, a+(n-1)d$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or $$S_n = \frac{n}{2}[a + \ell], \text{ where } \ell = \text{last term} = n^{th} \text{ term } = a + (n-1)d.$$

5. Choose the terms of an A.P in the following from

No. of terms	Term
3	$a-d, a, a+d$
4	$a-3d, a-d, a+d, a+3d$
5	$a-2d, a-d, a, a+d, a+2d$

6. If sum of series of an A.P is given then n^{th} term of series can be find by following formula.

$$a_n = S_n - S_{n-1}$$

Answersheet

Exercise 5.1

1. (i) $a = 6, d = 3$ (ii) $a = -7, d = -2$ (iii) $a = \frac{3}{2}, d = -1$ (iv) $a = 1, d = -3$

(v) $a = -1, d = \frac{5}{4}$ (vi) $a = 3, d = -2$ (vii) $a = 3, d = -5$

2. (i) $-1, \frac{-1}{2}, 0, \frac{1}{2}$ (ii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$ (iii) $0.6, 1.7, 2.8, 3.9$ (iv) $4, 1, -2, -5$

(v) $11, 7, 3, -1$ (vi) $-1.25, -1.50, -1.75, -2.00$ (vii) $20, \frac{77}{4}, \frac{74}{4}, \frac{71}{4}$

3. (i) Yes, $d = \frac{1}{2}; 4, \frac{9}{2}, 5, \frac{11}{2}$ (ii) Yes, $d = 0; \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}$ (iii) No (iv) No

(v) Yes, $d = \sqrt{2}; \sqrt{50}, \sqrt{72}, \sqrt{98}, \sqrt{128}$ (vi) Yes, $d = a; 5a, 6a, 7a, 8a$

(vii) No (viii) Yes, $d=\sqrt{2}; 3+4\sqrt{2}, 3+5\sqrt{2}, 3+6\sqrt{2}, 3+7\sqrt{2}$

Exercise 5.2

1. (i) 47 (ii) $35\sqrt{2}$ (iii) 101
2. (i) 35^{th} (ii) 22^{th} (iii) No (iv) No
3. 87
4. 5^{th}
5. 4, 10, 16, 22
6. 128
7. -32
8. 55
9. 69
10. 5, 10, 15, 20

Exercise 5.3

1. (i) 144 (ii) -979 (iii) $\frac{33}{20}$
2. (i) 40703 (ii) $1046\frac{1}{2}$
3. (i) 12 (ii) 21, 22
4. (i) 1375 (ii) -800
5. 5610
6. 38, 6973
7. 83667
8. $n=6, d=5$
9. ₹ 100, ₹ 80, ₹ 60, ₹ 40
10. (i) 550 (ii) 775 (iii) 4375

Miscellaneous Exercise 5

1. (d) 2- (d) 3. (a) 4. (c) 5. (c) 6. (c) 7. (b)

8. 193 9. $\frac{7}{5}$ 10. 500500 11. Yes 12. 28^{th}

13. 2, 4, 6, 8 or 8, 6, 4, 2 14. 76 15. $x=35$